Proton GPDs from lattice QCD at the physical point

Martha Constantinou

Temple University

In collaboration with:

C. Alexandrou, S. Bacchio, M.-H. Chu, K. Cichy,
G. Koutsou, J. Miller, G. Pierini

Extended Twisted Mass Collaboration

The 43rd International Symposium on Lattice Field Theory (Lattice 2026)

Outline

1 Motivation

2 Updates in analysis

3 Calculation Setup

4 Preliminary Results

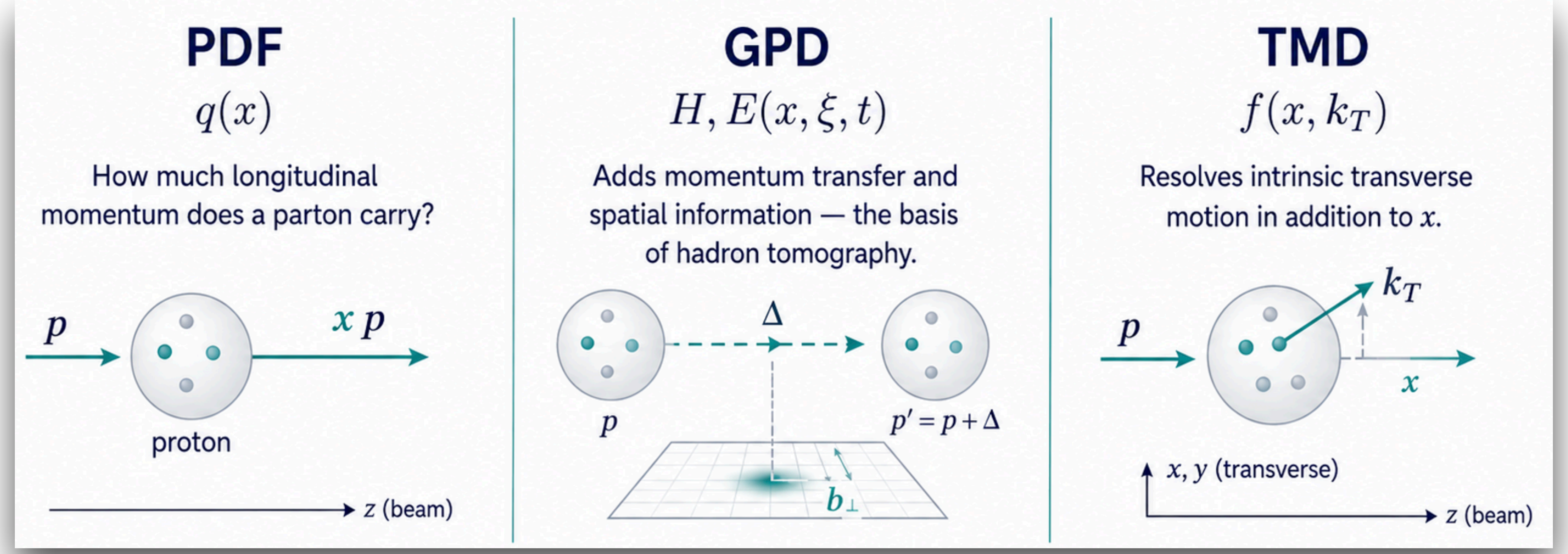
5 Synergies & Complementarity

6 Conclusions & Future work

1 Generalized Parton Distributions

★ GPDs additional information on partonic structure compared to PDFs

★ Along with TMDs give access to tomography of hadrons



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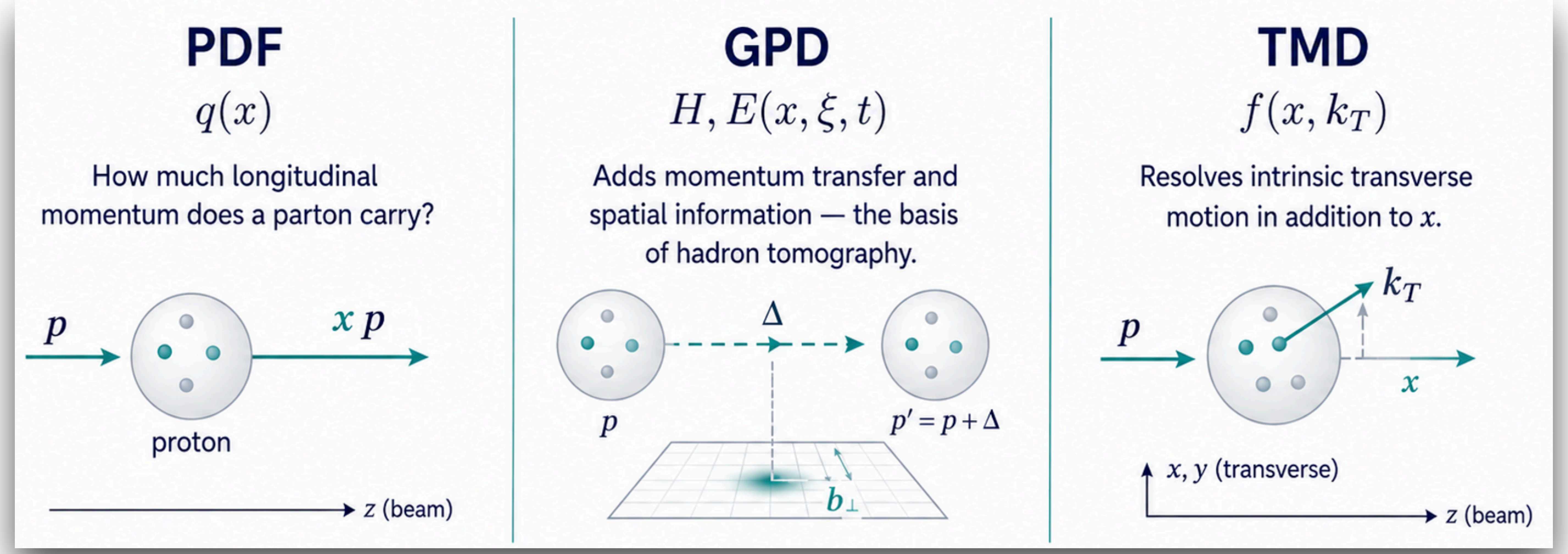

★ GPDs are not well-constrained experimentally:

- x-dependence extraction is not direct. DVCS amplitude:

$$\mathcal{H} = \int_{-1}^{+1} \frac{H(x, \xi, t)}{x - \xi + i\epsilon} dx$$

(SDHEP gives access to x) J.-W. Qiu et al, ***JHEP*** **08 (2022) 103; PRD 113 (2026) 5, 054037**

- independent measurements to disentangle GPDs
- GPDs phenomenology more complicated than PDFs (multi-dimensionality)
- and more challenges …

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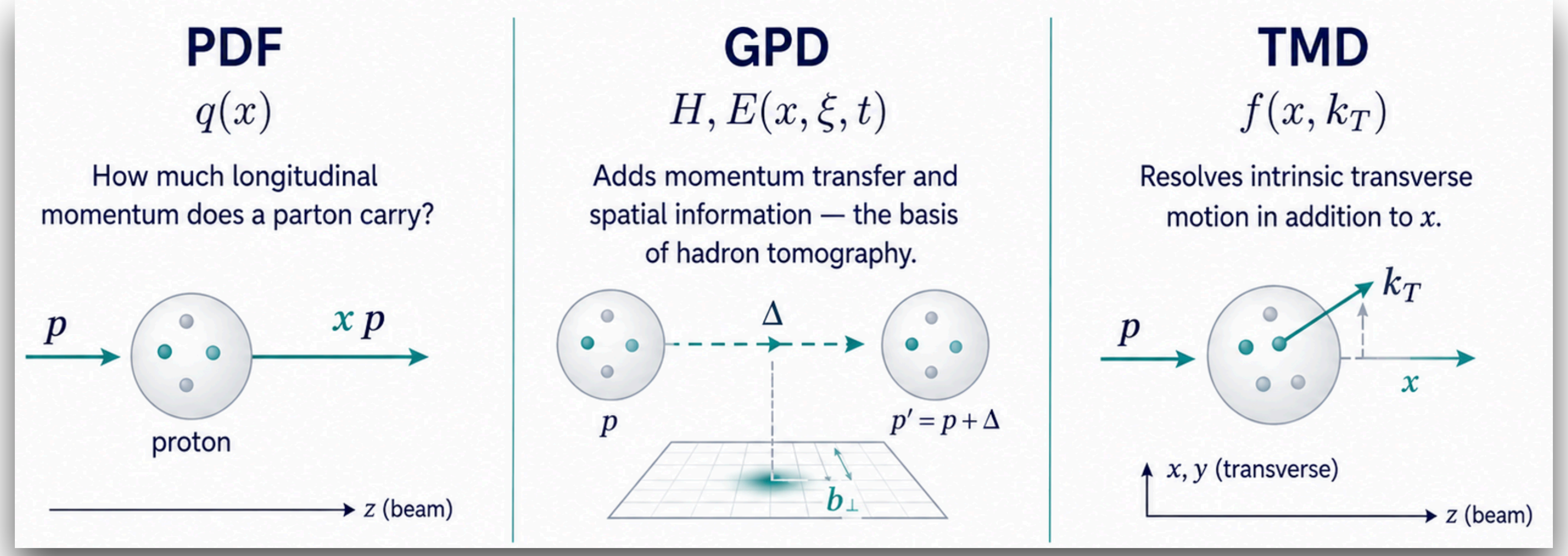

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Essential to complement the knowledge on GPD from lattice QCD

Generalized Parton Distributions within LaMET

Matrix elements of nonlocal operators

X. Ji, **PRL 110 (2013) 262002; Sci. China M.A. 57 (2014) 1407**

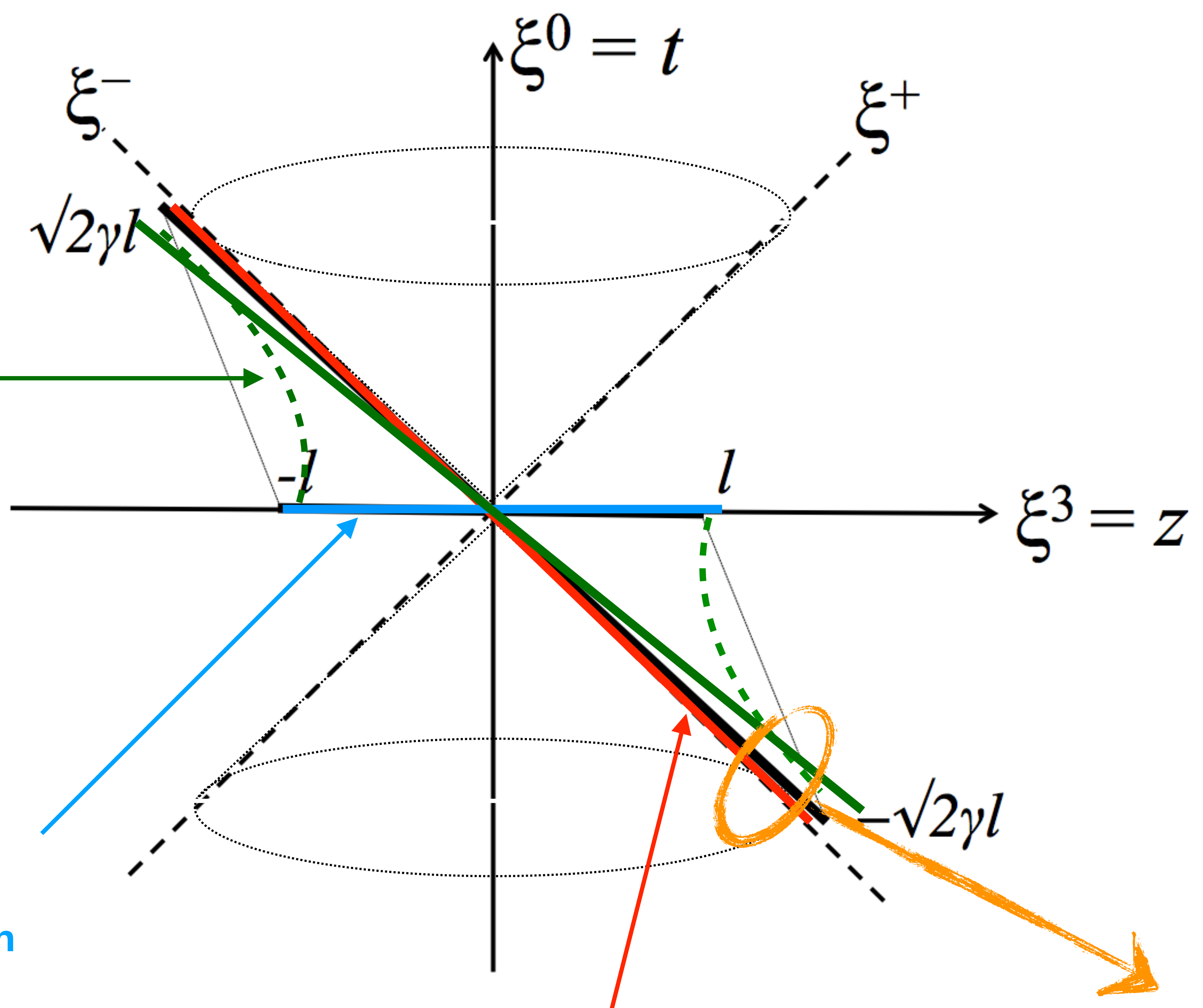
$$\mathcal{M}(P_f, P_i, z) = \langle N(P_f)\,|\,\underline{\bar{\Psi}(z)\,\Gamma\,\mathcal{W}(z,0)\Psi(0)}\,|\,N(P_i)\rangle_\mu$$

Nonlocal operator with Wilson line

$$\langle N(P')|O_V^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu H(x,\xi,t) + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m_N}E(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

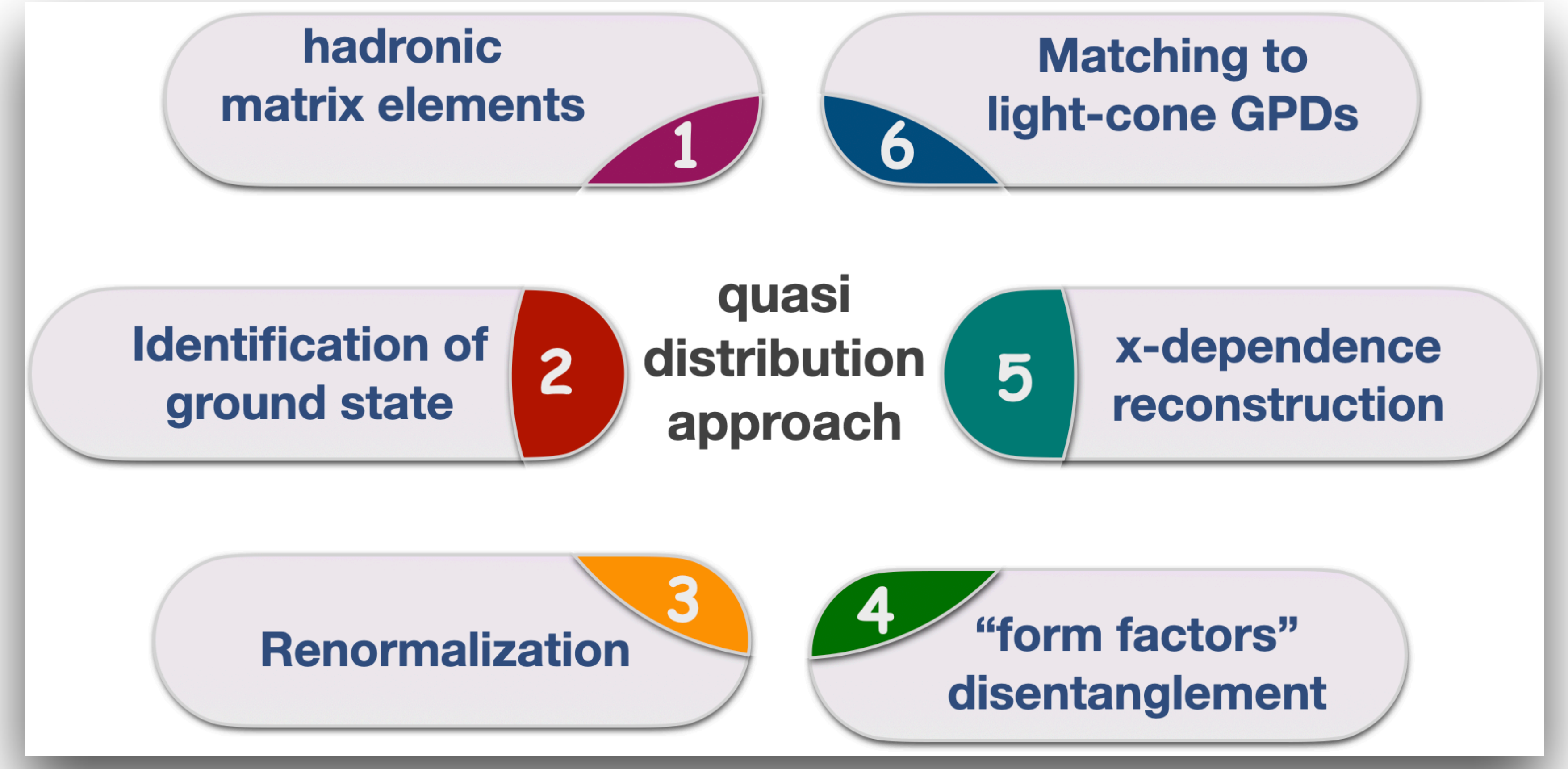
$$\langle N(P')|O_A^\mu(x)|N(P)\rangle = \overline{U}(P')\left\{\gamma^\mu\gamma_5\widetilde{H}(x,\xi,t) + \frac{\gamma_5\Delta^\mu}{2m_N}\widetilde{E}(x,\xi,t)\right\}U(P) + \text{ht}\,,$$

$$\langle N(P')|O_T^{\mu\nu}(x)|N(P)\rangle = \overline{U}(P')\left\{i\sigma^{\mu\nu}H_T(x,\xi,t) + \frac{\gamma^{[\mu}\Delta^{\nu]}}{2m_N}E_T(x,\xi,t) + \frac{\overline{P}^{[\mu}\Delta^{\nu]}}{m_N^2}\widetilde{H}_T(x,\xi,t) + \frac{\gamma^{[\mu}\overline{P}^{\nu]}}{m_N}\widetilde{E}_T(x,\xi,t)\right\}U(P) + \text{ht}$$



Difference reduced as P_3 increases $\mathcal{O}\left(\Lambda^2_{\rm QCD}/P_3^2, m_N^2/P_3^2, t/P_3^2, \ldots\right)$

A number of non-trivial steps



2 Lorentz-invariant parametrization of nonlocal matrix elements

★ Historically, calculations of GPDs in symmetric frame $\vec{p}_f = \overrightarrow{P}_{Boost} + \overrightarrow{\Delta}/2, \;\; \vec{p}_i = \overrightarrow{P}_{Boost} - \overrightarrow{\Delta}/2$

$$F^{[\gamma^+]}(x, \Delta; \lambda, \lambda') = \frac{1}{2P^+}\bar{u}(p', \lambda')\left[\gamma^+ H(x, \xi, t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}E(x, \xi, t)\right]u(p, \lambda)$$

→ highly impractical - computationally costly

ETM Collaboration **PRD 105 (2022) 3, 034501**

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→ Frame-independent - computational efficiency: $\vec{p}_f = \overrightarrow{P}_{Boost}, \;\; \vec{p}_i = \overrightarrow{P}_{Boost} - \overrightarrow{\Delta}$

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^\mu}{M}A_1 + z^\mu MA_2 + \frac{\Delta^\mu}{M}A_3 + i\sigma^{\mu z}MA_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{M}A_6 + \frac{z^\mu i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

S. Bhattacharya et al., **PRD 106 (2022) 11, 114512**

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Key idea: extract A_i from asymmetric frame → reconstruct H and E GPDs

→ A_i relate to H and E quasi-GPDs, e.g.,

(Definition not unique; here LI)

$$\mathscr{H}^{LI} = A_1 + \frac{\Delta^{s/a}\cdot z}{P^{s/a}\cdot z}A_3$$

$$\mathscr{E}^{LI} = -A_1 - \frac{\Delta^{s/a}\cdot z}{P^{s/a}\cdot z}A_3 + 2A_5 + 2P^{s/a}\cdot zA_6 + 2\Delta^{s/a}\cdot zA_8$$

3 Calculation Setup

★ Nf=2+1+1 twisted-mass fermions with a clover term

Ensemble	a [fm]	$L^3 \times T$	m_π [MeV]
cB211.072.64	0.0801(4)	$64^3 \times 128$	139.3(7)

★ Zero skewness calculation, $\Delta_3 = 0$

P_z [GeV]	$N_{\rm conf}$	$N_{\rm src}$	N_Δ	$-t$ range [GeV^2]
0.00	652	16	12	0.058–1.114
0.48	762	2	12	0.058–1.163
0.72	762	4	12	0.058–1.207
0.96	762	6	12	0.058–1.251
1.21	762	12	12	0.058–1.289
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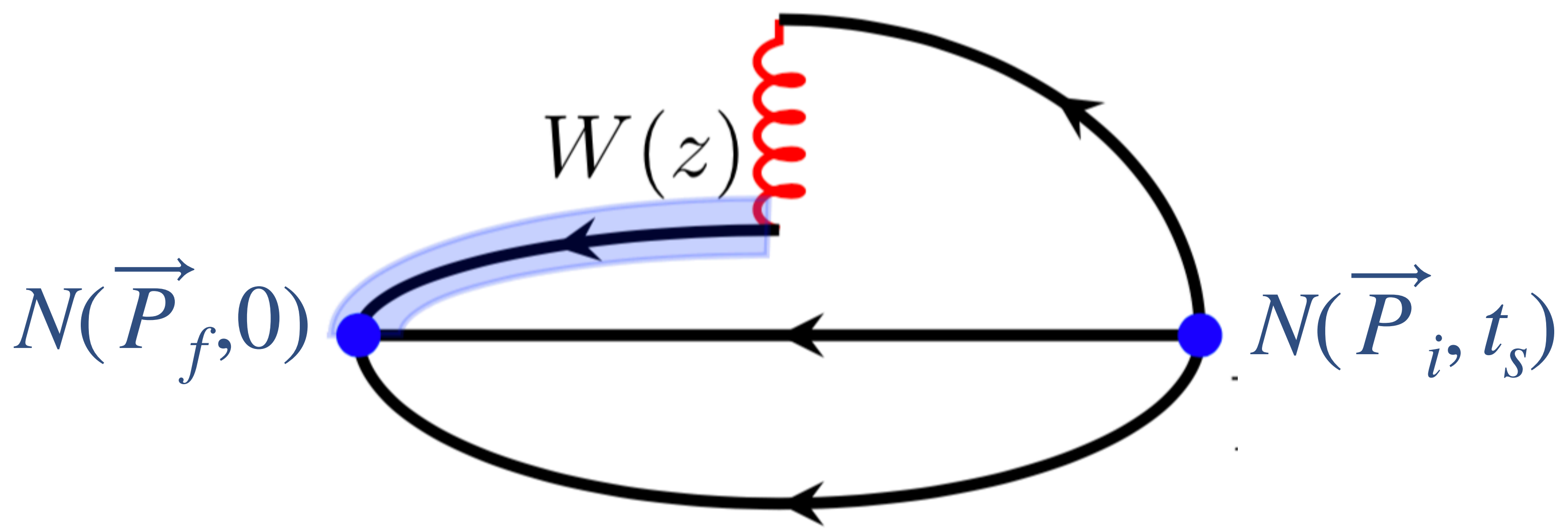
★ Connected diagram



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★ Asymmetric frame calculation:

$$\vec{p}_f = \vec{P}_{Boost}, \quad \vec{p}_i = \vec{P}_{Boost} - \vec{\Delta}$$

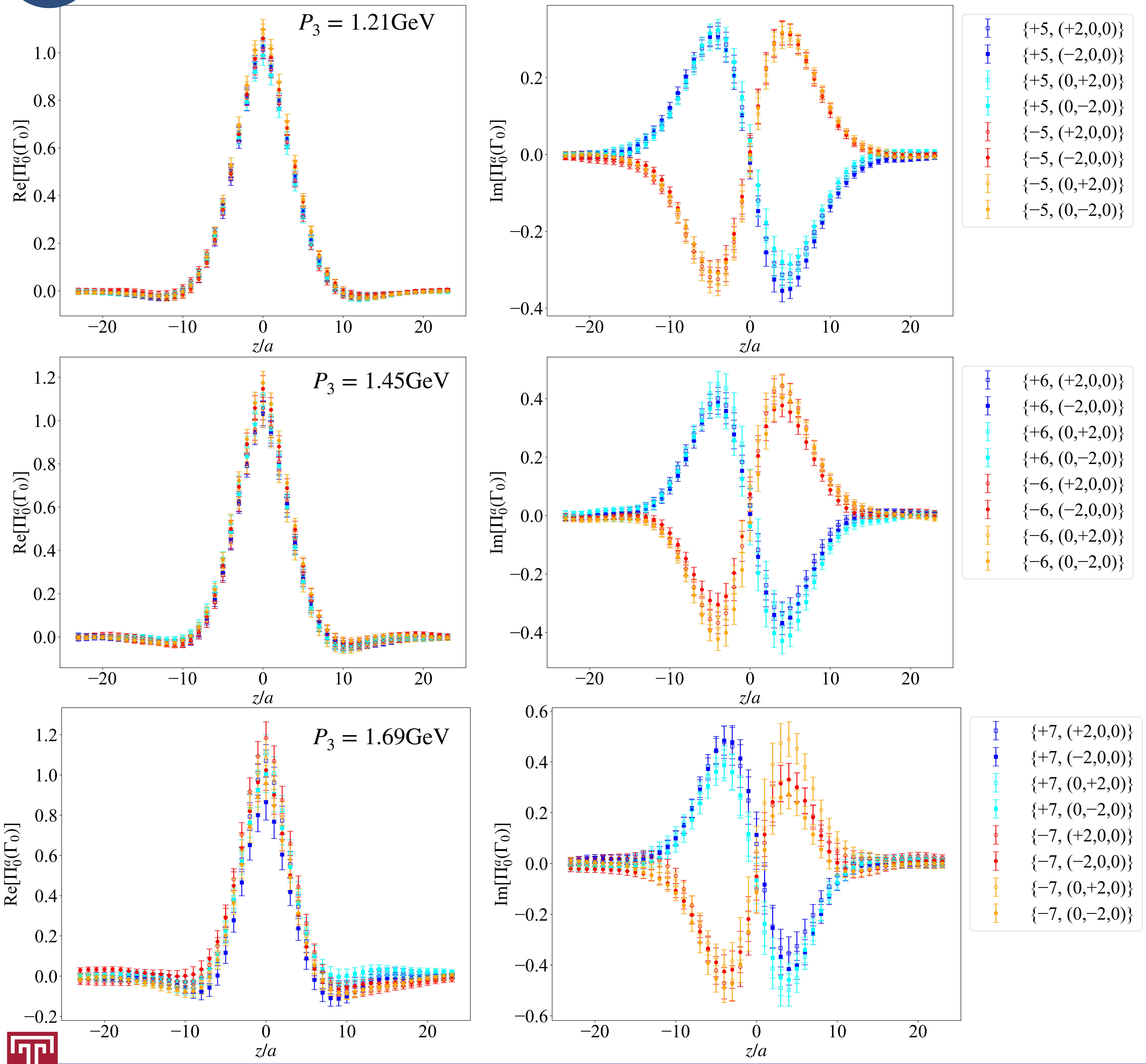
$$\vec{P}_{Boost} = (0,0,P_3)$$

★ Several values of momentum transfer

★ Tsink = 10a

$$-t = \Delta^2 - (E_i - Ef)^2 \qquad E_{i/f} = \sqrt{m^2 + \vec{p}_{i/f}^{\,2}}$$

4 Matrix elements - kinematic dependence



★ Π^{asymm} do not have definite symmetries

→ First decompose $A_1 - A_8$, e.g.:

$$A_1 = \frac{2m^2}{E_f(E_i+m)}\frac{\Pi_0^a(\Gamma_0)}{K} + i\,\frac{2(E_f-E_i)P_3m^2}{E_f(E_f+m)(E_i+m)\Delta}\frac{\Pi_0^a(\Gamma_2)}{K} + \frac{2(E_i-E_f)P_3m^2}{E_f(E_f+E_i)(E_f+m)(E_i+m)}\frac{\Pi_1^a(\Gamma_2)}{K}$$
$$+i\,\frac{2(E_i-E_f)m^2}{E_f(E_i+m)\Delta}\frac{\Pi_1^a(\Gamma_0)}{K} + \frac{2(E_i-E_f)P_3m^2}{E_f(E_f+E_i)(E_f+m)(E_i+m)}\frac{\Pi_2^a(\Gamma_1)}{K} + \frac{2(E_f-E_i)m^2}{E_f(E_i+m)\Delta}\frac{\Pi_2^a(\Gamma_3)}{K},$$

$$A_5 = \frac{m^2P_3}{E_f(E_f+m)(E_i+m)}\frac{\Pi_2^a(\Gamma_1)}{K} - \frac{(E_f+E_i)m^2}{E_f(E_i+m)\Delta}\frac{\Pi_2^a(\Gamma_3)}{K}$$

$$A_6 = \frac{P_3m^2}{E_f^2(E_f+m)(E_i+m)z}\frac{\Pi_0^a(\Gamma_0)}{K} + i\,\frac{(E_f-E_i-2m)m^2}{E_f^2(E_i+m)\Delta z}\frac{\Pi_0^a(\Gamma_2)}{K} + i\,\frac{(E_i-E_f)P_3m^2}{E_f^2(E_f+m)(E_i+m)\Delta z}\frac{\Pi_1^a(\Gamma_0)}{K}$$
$$+\frac{(-E_f+E_i+2m)m^2}{E_f^2(E_f+E_i)(E_i+m)z}\frac{\Pi_1^a(\Gamma_2)}{K} + \frac{2(m-E_f)m^2}{E_f^2(E_f+E_i)(E_i+m)z}\frac{\Pi_2^a(\Gamma_1)}{K} + \frac{2P_3m^2}{E_f^2(E_i+m)\Delta z}\frac{\Pi_2^a(\Gamma_3)}{K},$$

★ Symmetries of A_i

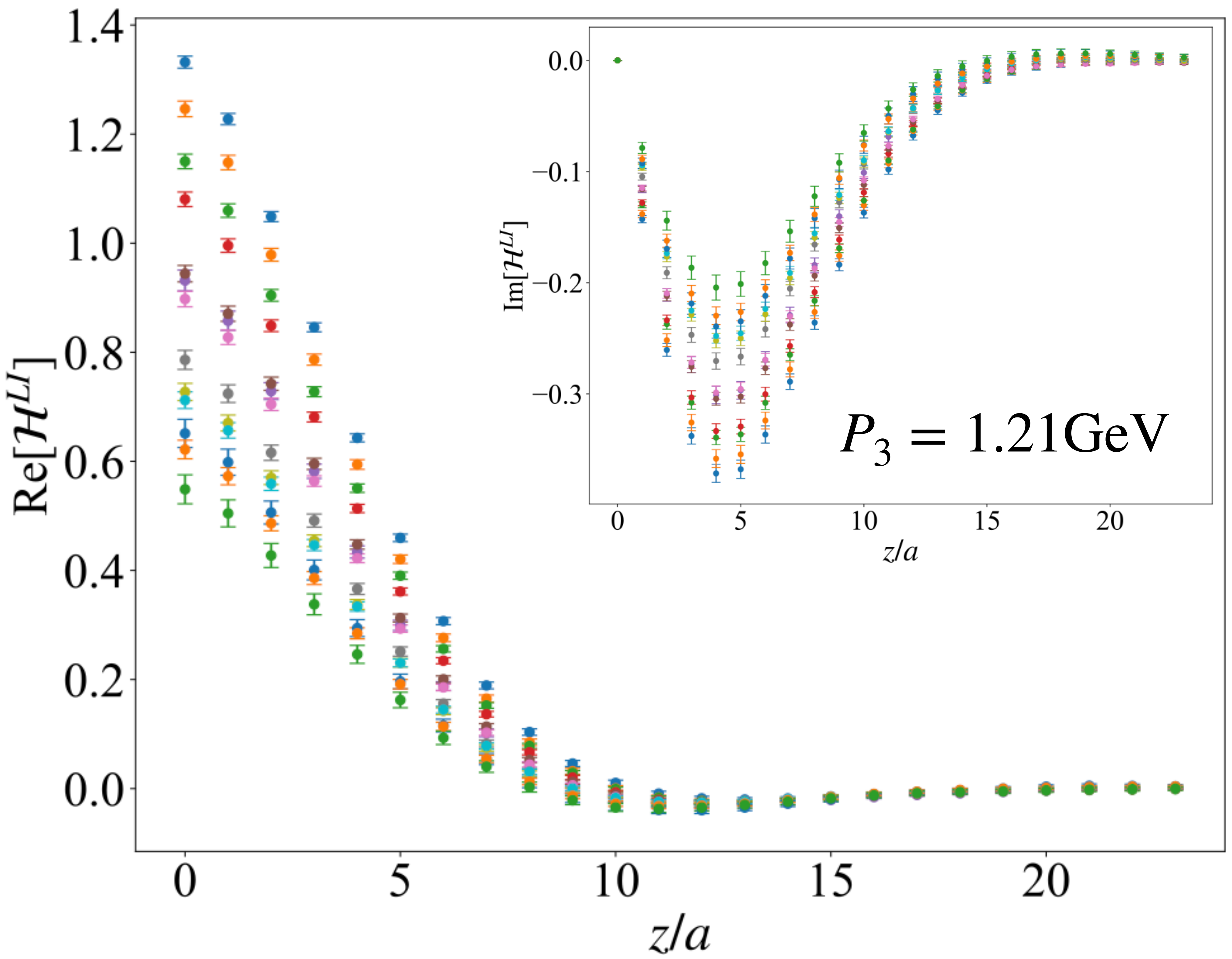
$$A_1^*(-z\cdot P, z\cdot\Delta, \Delta^2, z^2) = A_1(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$$
$$A_5^*(-z\cdot P, z\cdot\Delta, \Delta^2, z^2) = A_5(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$$
$$-A_6^*(-z\cdot P, z\cdot\Delta, \Delta^2, z^2) = A_6(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$$

★ Increase of statistics per kinematics:

$\times 8,\ \vec{Q} = (\pm Q, \pm Q, 0)$

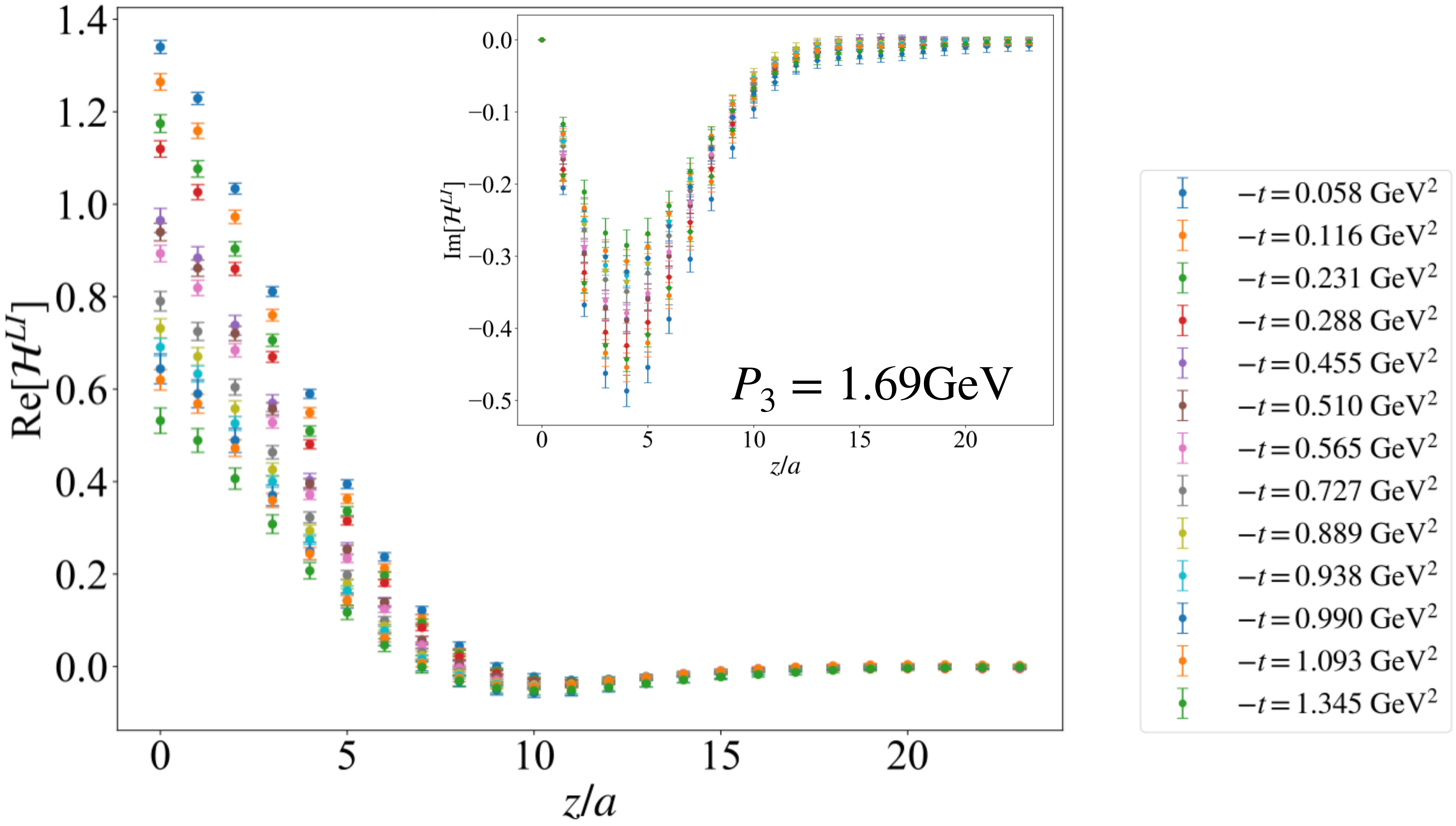
$\times 16,\ \vec{Q} = (\pm Q_x, \pm Q_y, 0), (\pm Q_y, \pm Q_x, 0)$

Coordinate-space quasi-GPDs



★ Statistical errors controlled for all kinematics

★ Standard and LI definition (for H quasi-GPD): similar



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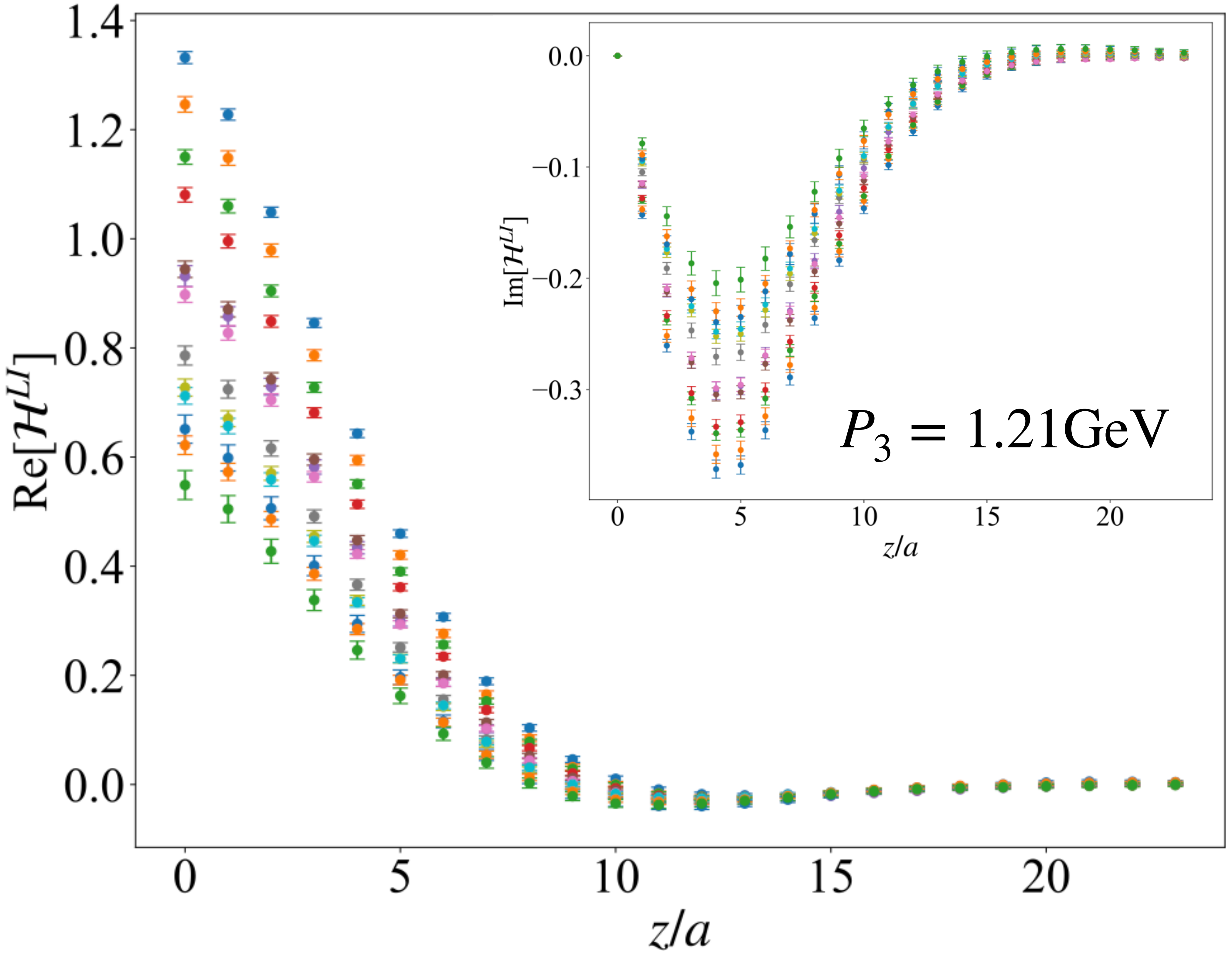

★ Possible definitions:

Standard: $$\mathcal{H}_0^a(A_i^a;z) = i\frac{2m^2(E_f-E_i)}{KP_3\Delta(E_i+m)}\Pi_0^a(\Gamma_2) + \frac{2m^2(E_f+E_i+2m)}{K(E_f+E_i)(E_f+m)(E_i+m)}\Pi_0^a(\Gamma_0)$$

LI: $$\mathcal{H}(A_i^a;z) = \frac{2m^2}{E_f(E_i+m)}\frac{\Pi_0^a(\Gamma_0)}{K} + i\frac{2(E_f-E_i)P_3m^2}{E_f(E_f+m)(E_i+m)\Delta}\frac{\Pi_0^a(\Gamma_2)}{K} + \frac{2(E_i-E_f)P_3m^2}{E_f(E_f+E_i)(E_f+m)(E_i+m)}\frac{\Pi_1^a(\Gamma_2)}{K}$$
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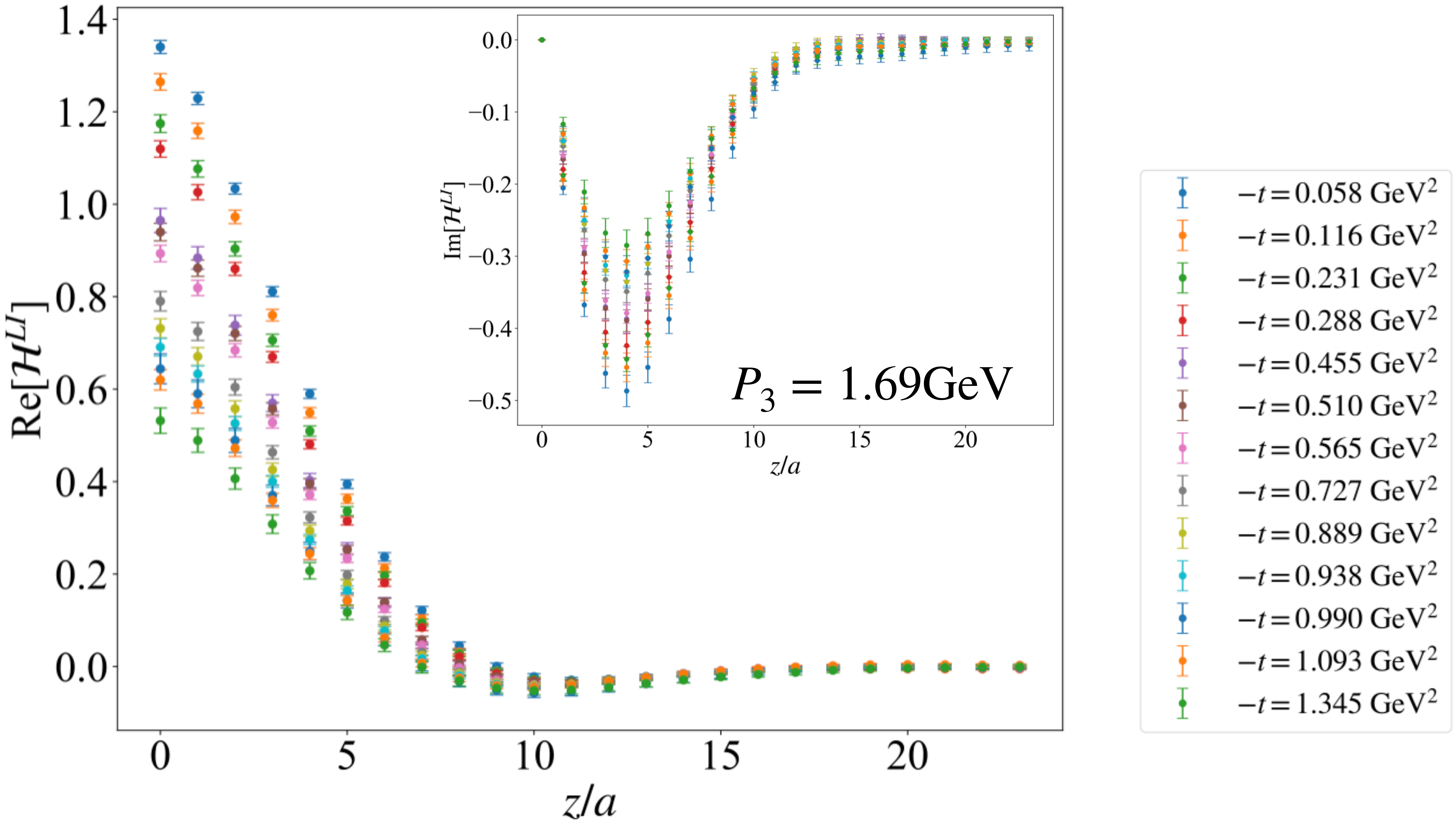

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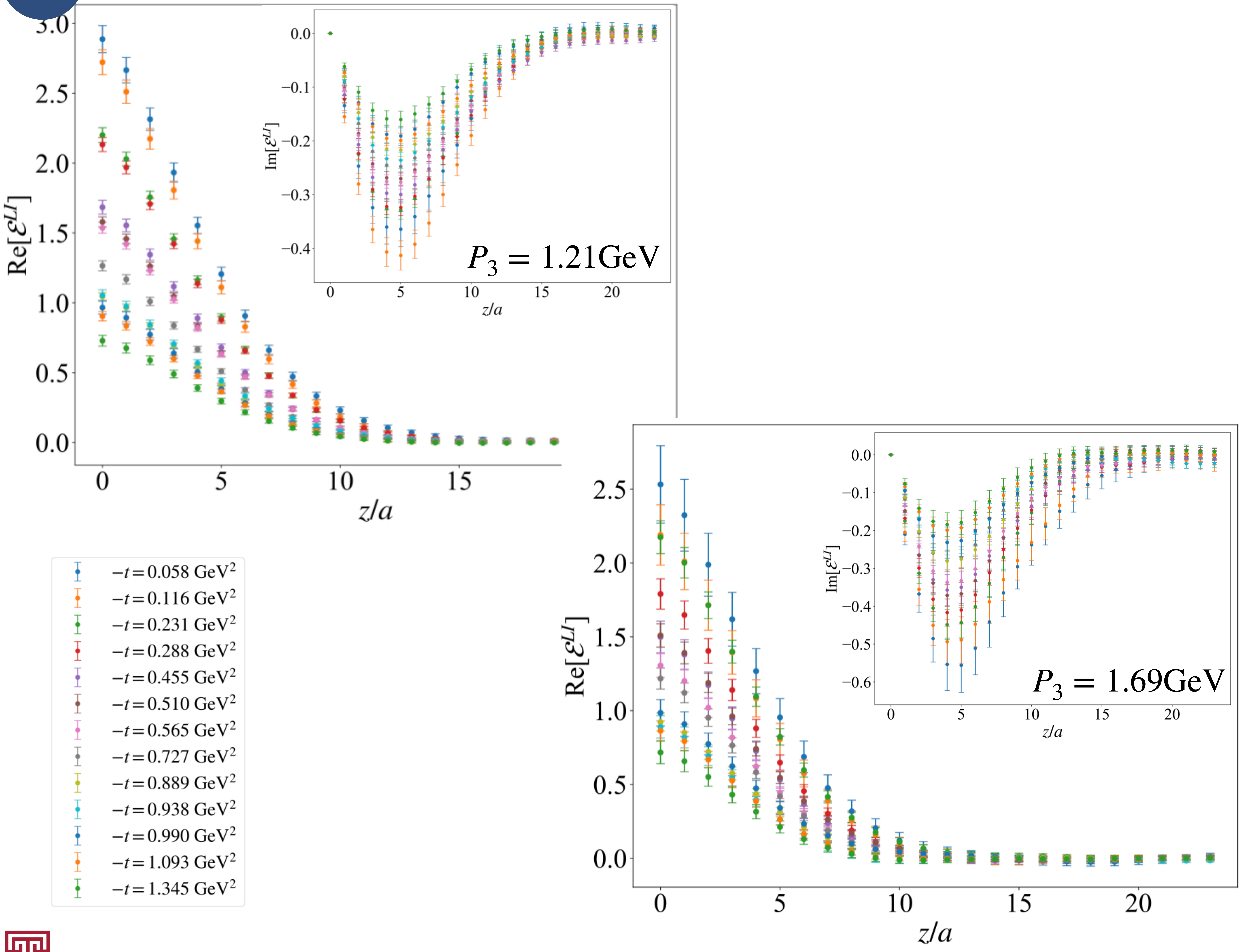

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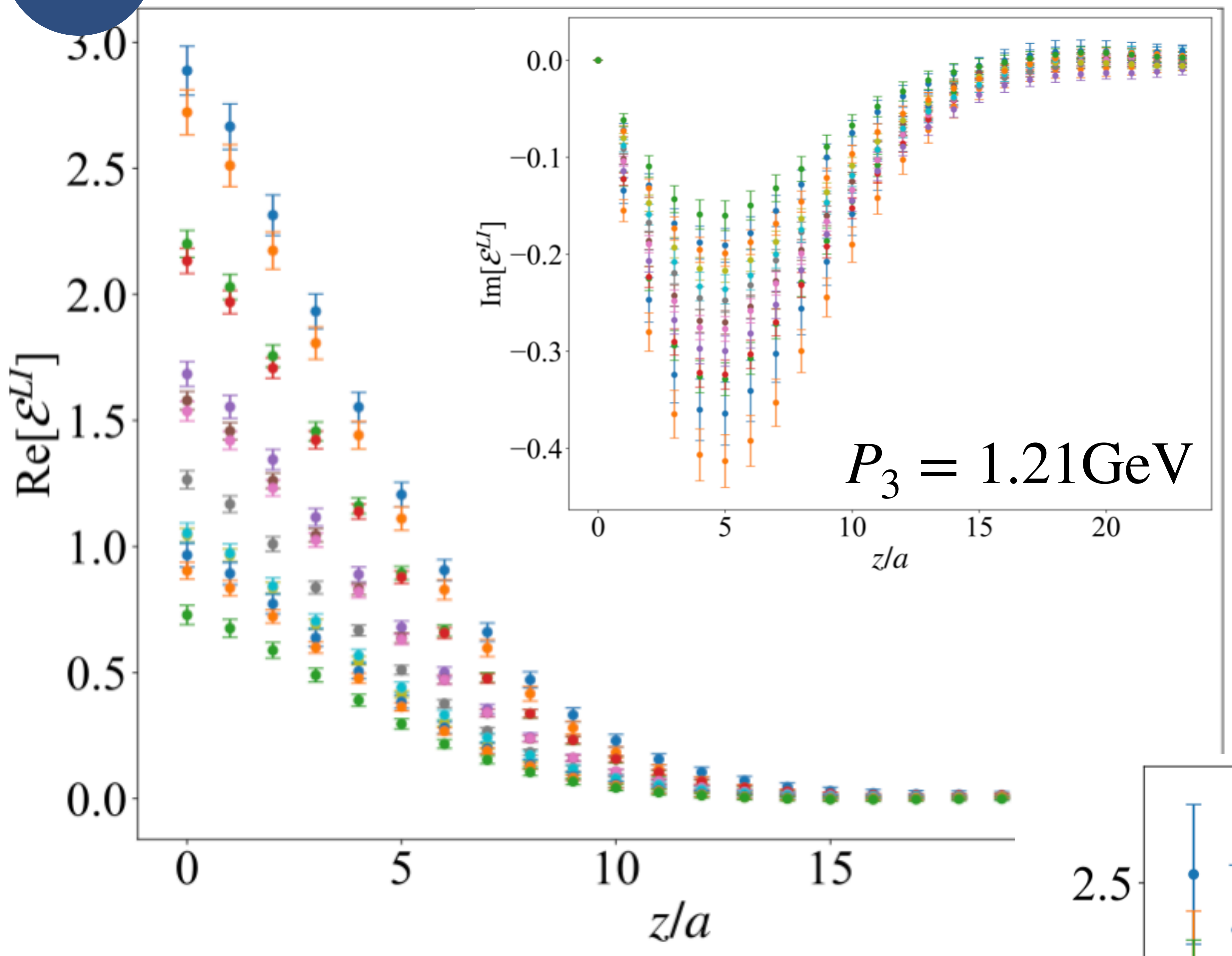

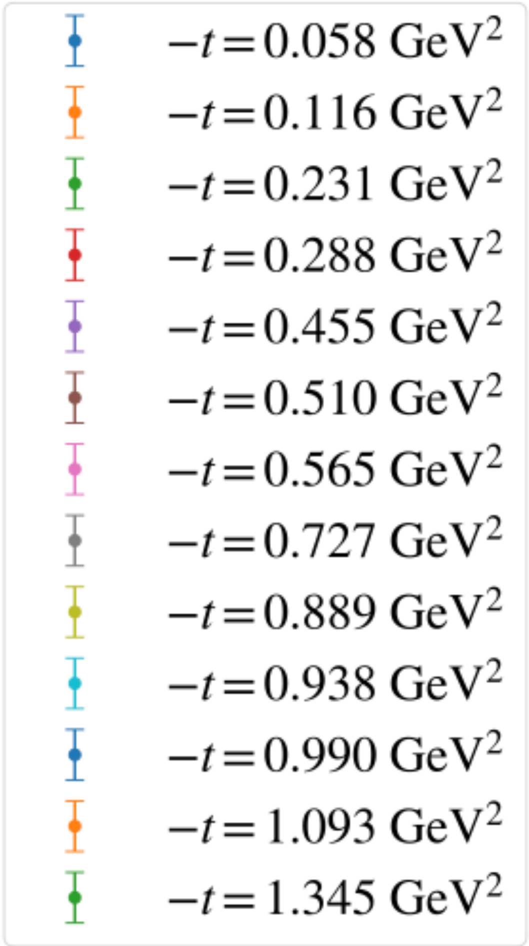

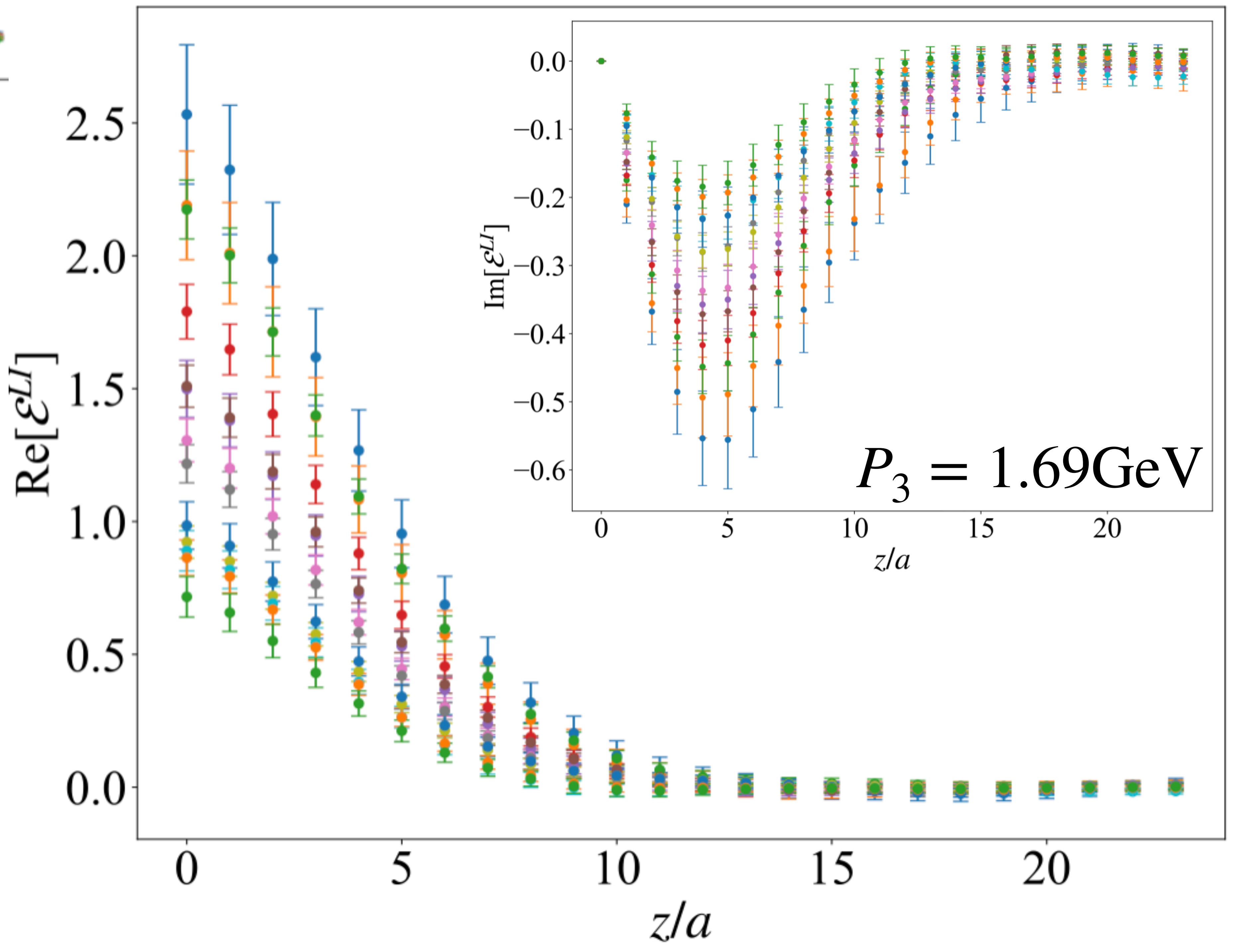

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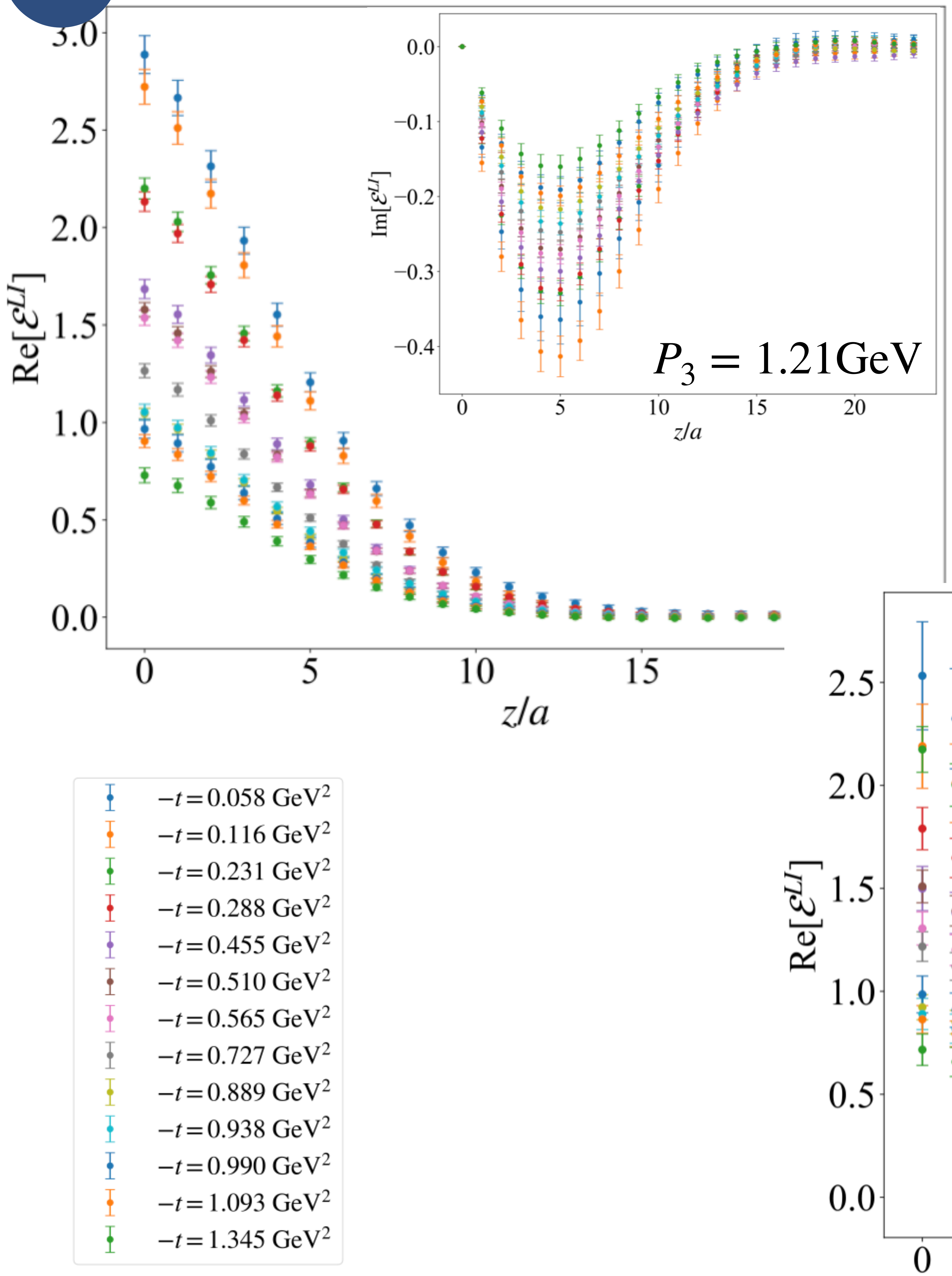

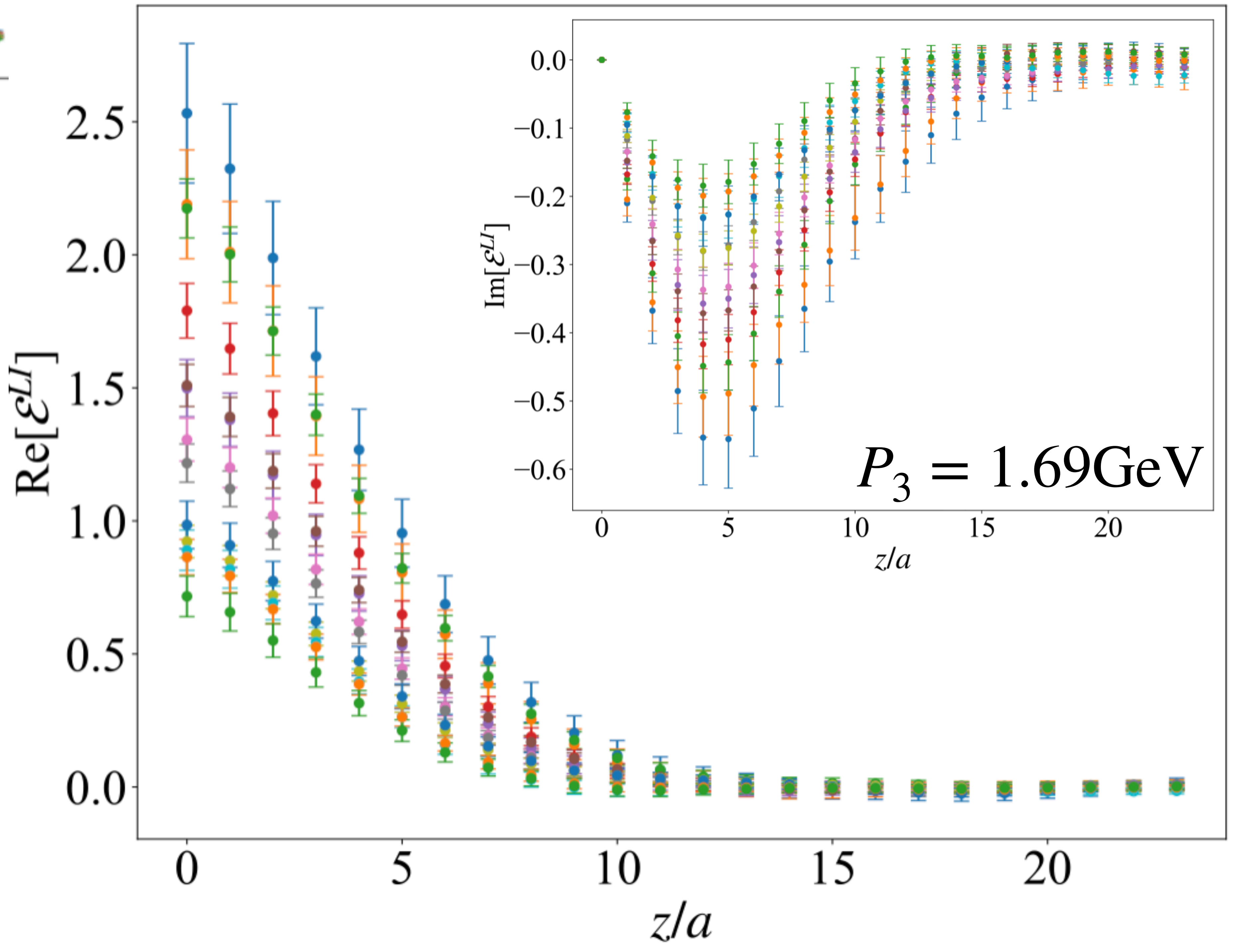

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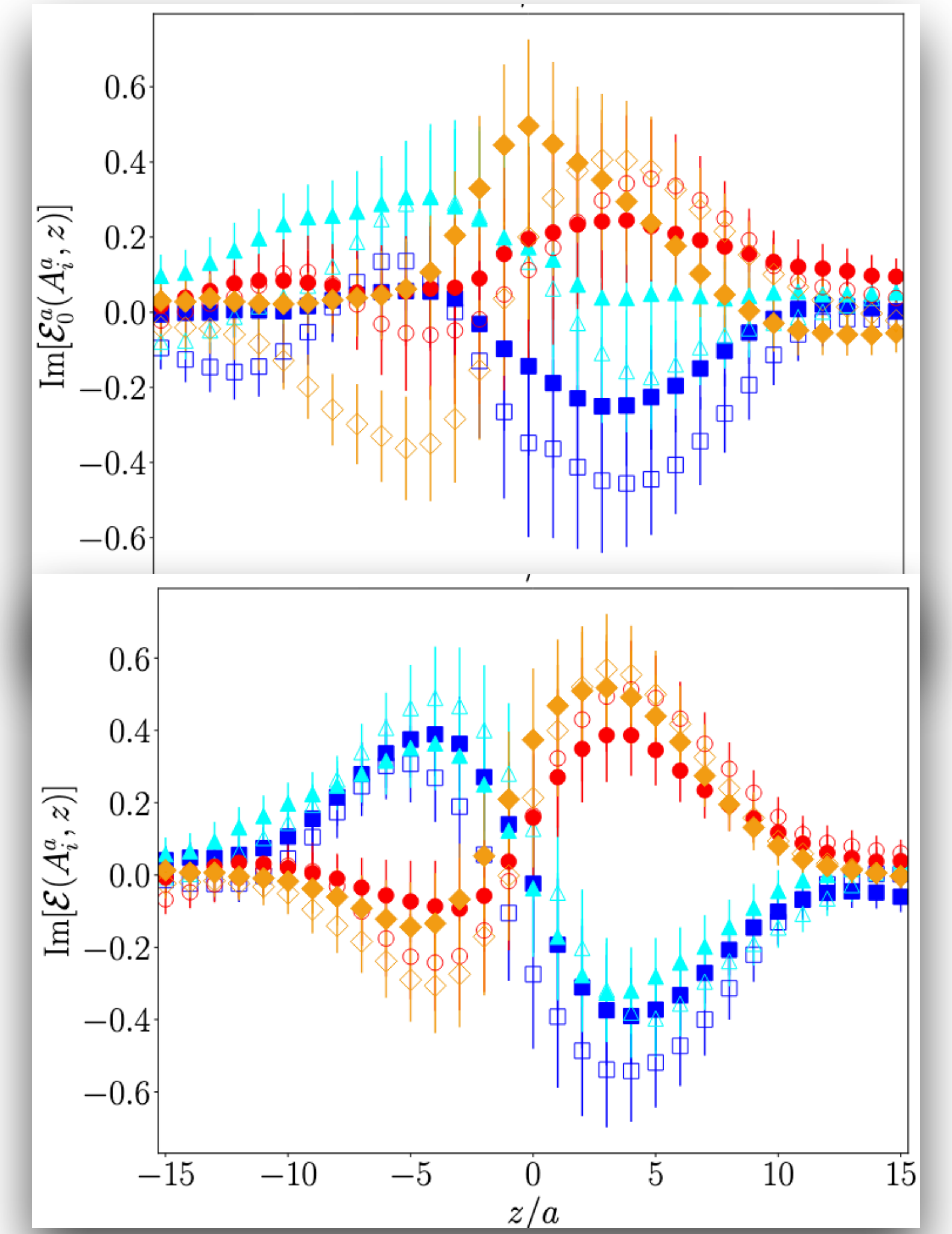
★ LI definition for E improves signal substantially



S. Bhattacharya et al., **PRD 106 (2022) 11, 114512**

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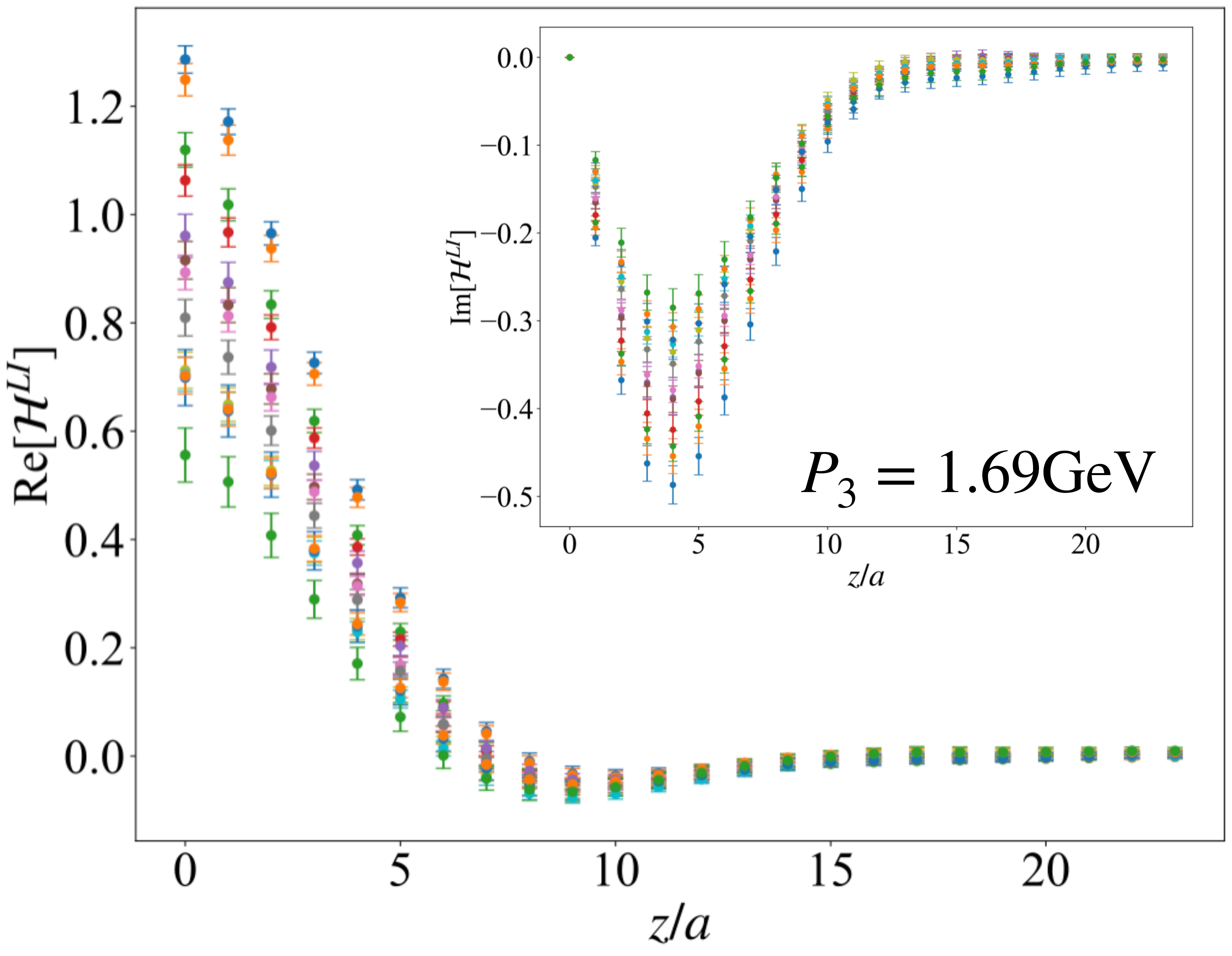

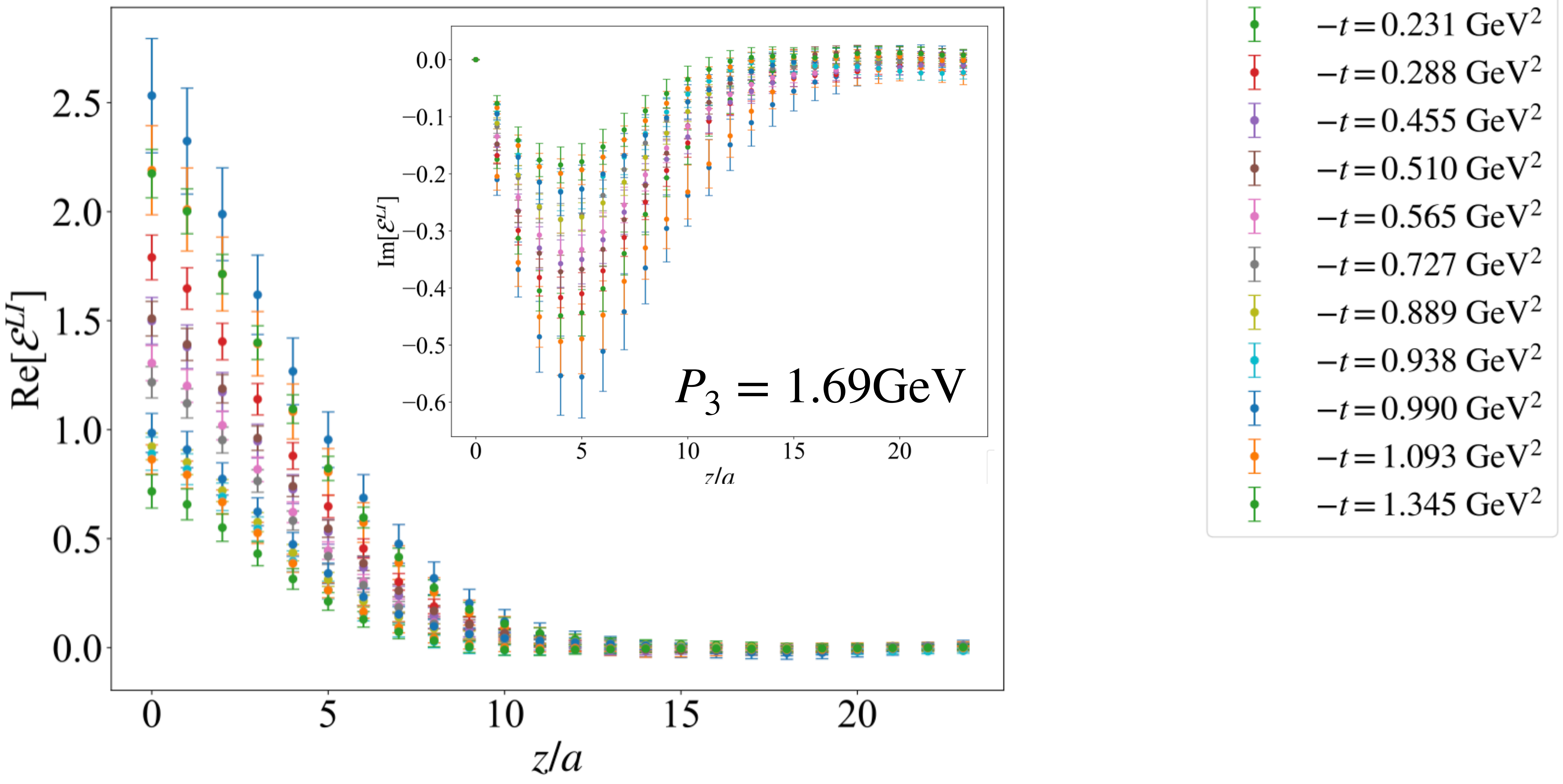

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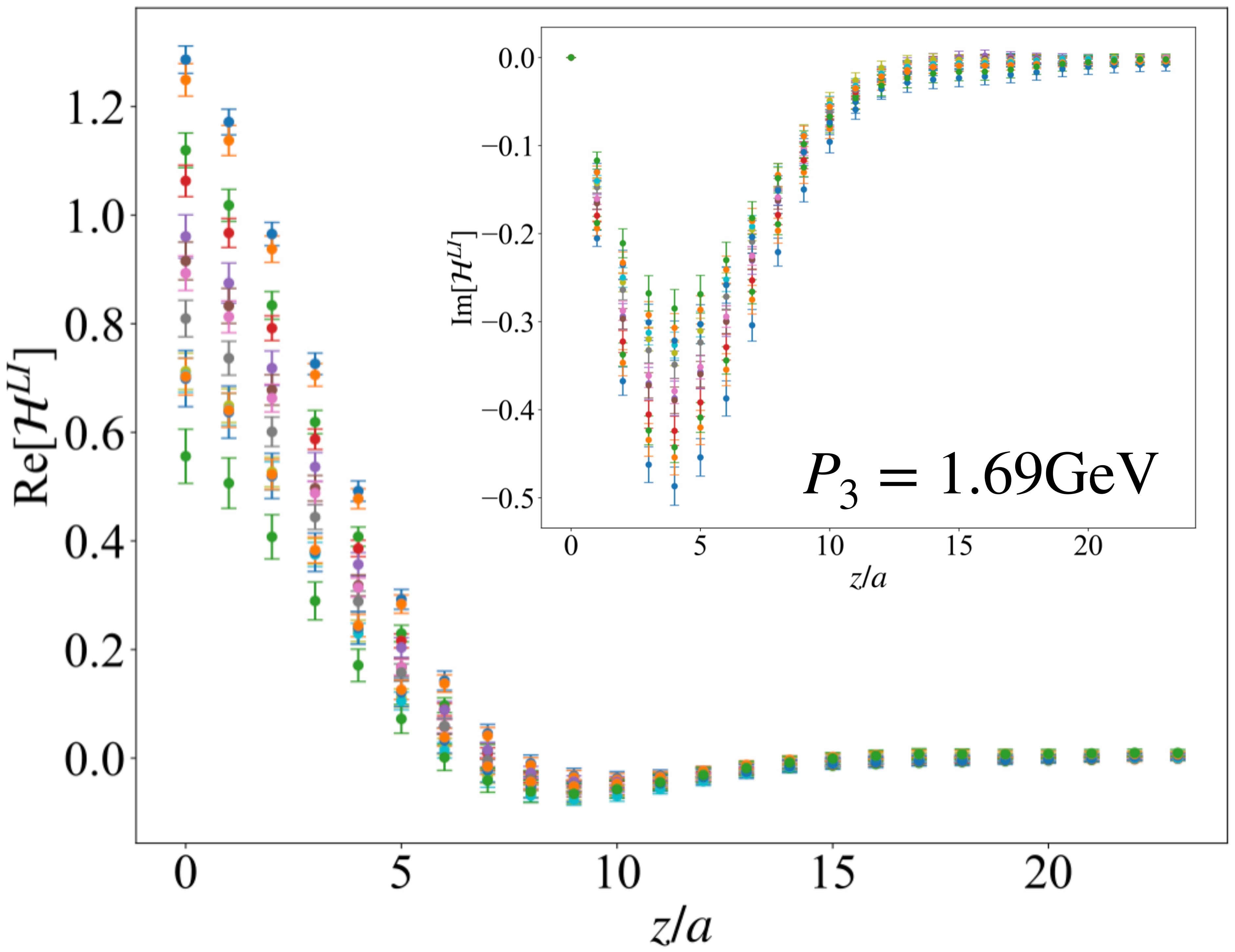
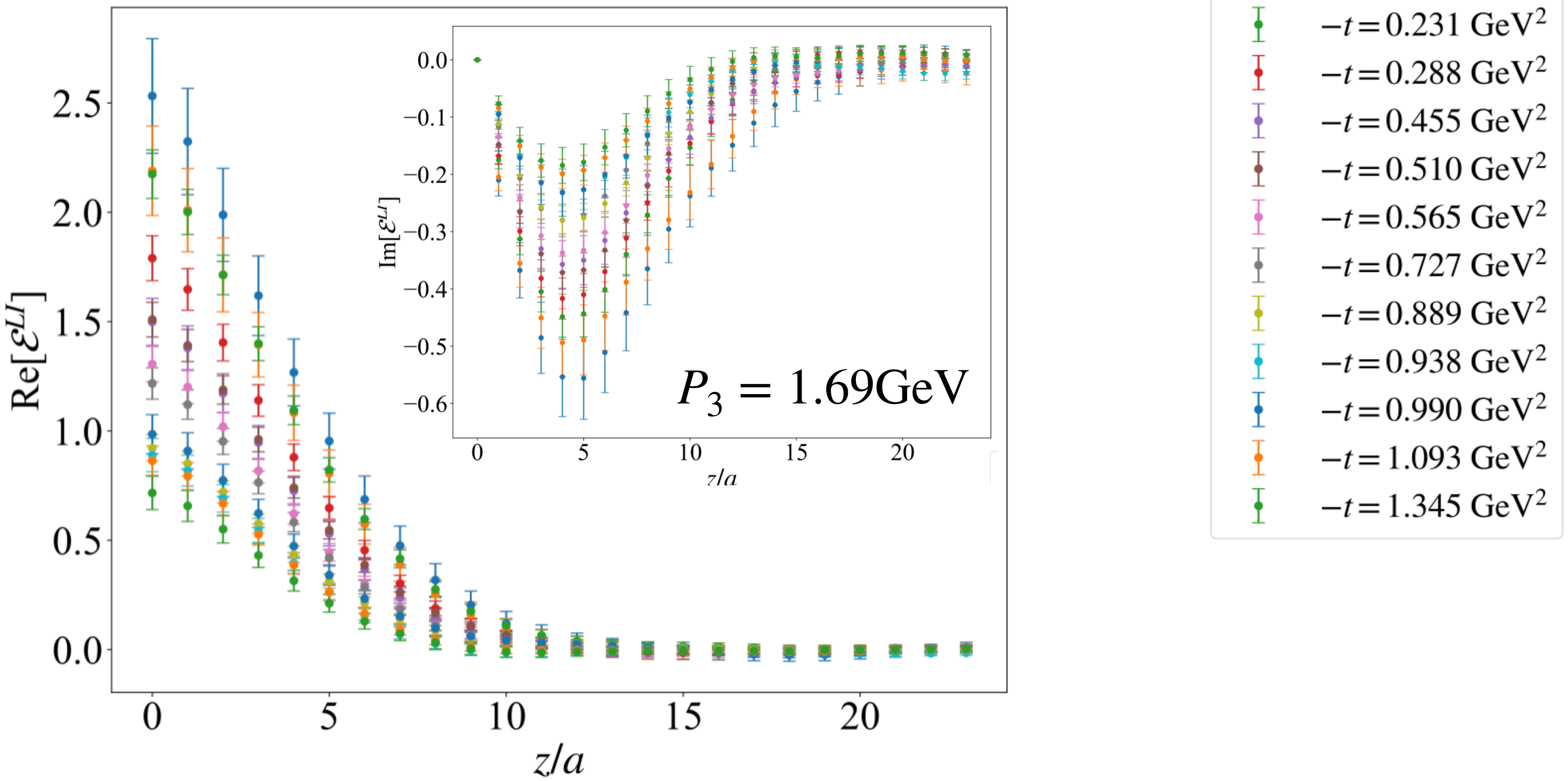
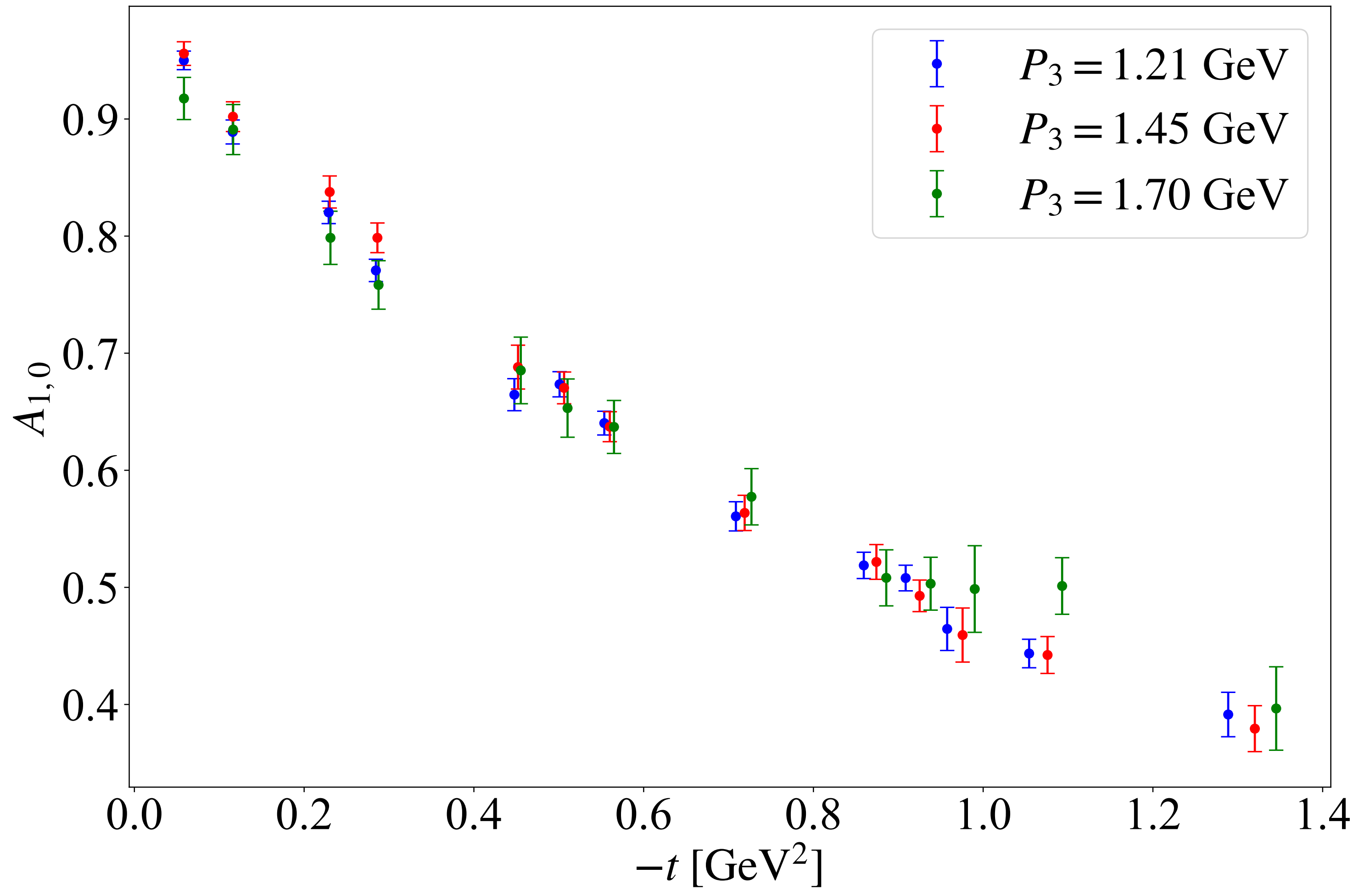
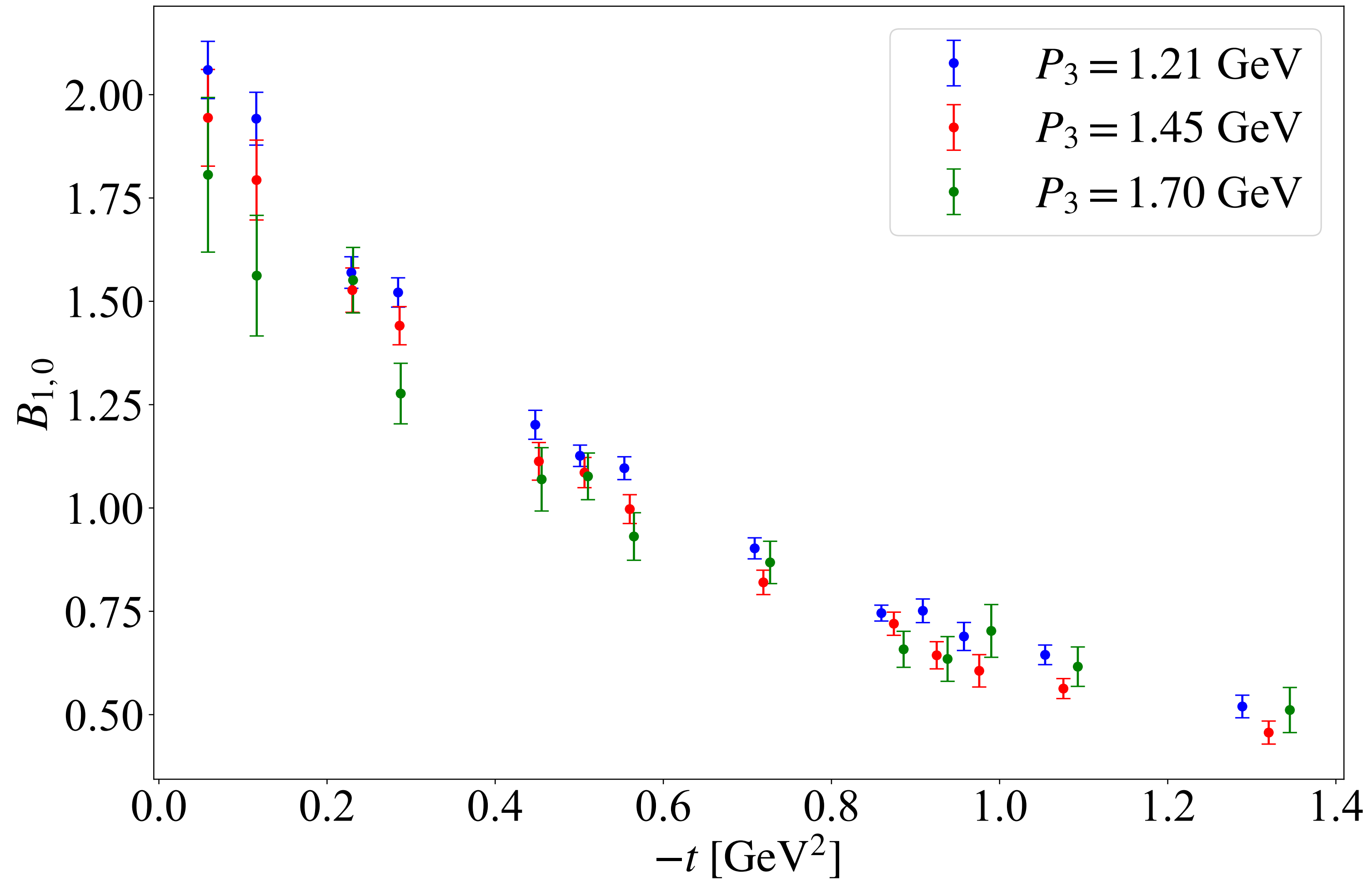




★ Sanity check: Form factors from local case (z=0)





★ Can be compared with rest-frame calculation, as well as OPE on nonlocal

2 Hybrid renormalization (HR)

X. Ji et al., **NPB 964 (2021) 115311**

★ RI-type/ratio scheme is natural at short distances; non-perturbative effects in long distances → treat Wilson-line divergence with self-energy subtraction

M. Constantinou et al., **PRD 96 (2017) 5, 054506**

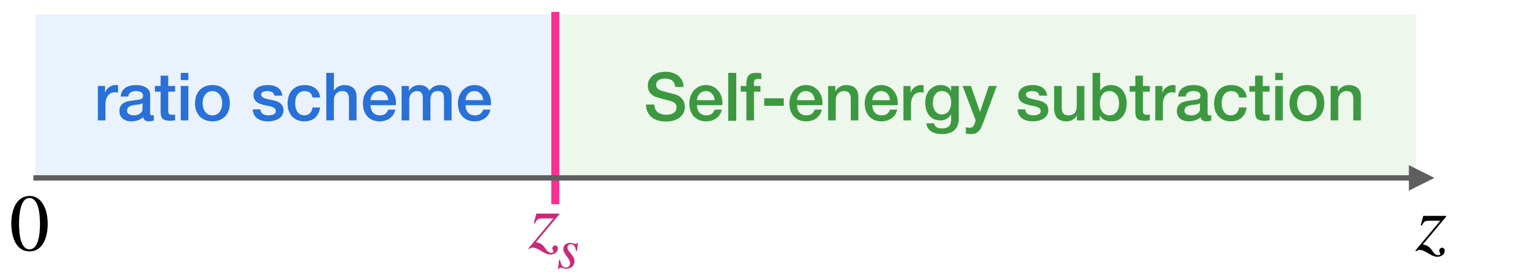
★ Calculating δm, m_0 <u>not unique</u>. Here:

➡ Large z (non-perturbative):
no renormalon ambiguity, thus extraction of δm,

$$f(z,0) = A' e^{-\delta m z}$$

➡ Short z: m_0 relevant along δm

$$e^{(\delta m + m_0) z} f(z,0) = C_0(z,\mu)$$ 1-loop at 2 GeV



$$F^R(z,P_z) = \begin{cases} \frac{F(z,P_z)}{f(z,0)} & z \le z_s \\ e^{(\delta m + m_0)(z - z_S)} \frac{F(z,P_z)}{f(z_S,0)} & z \ge z_s \end{cases}$$

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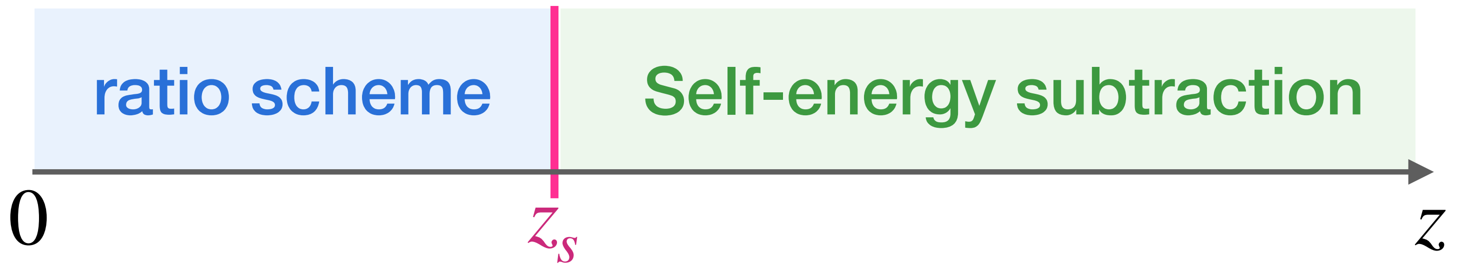

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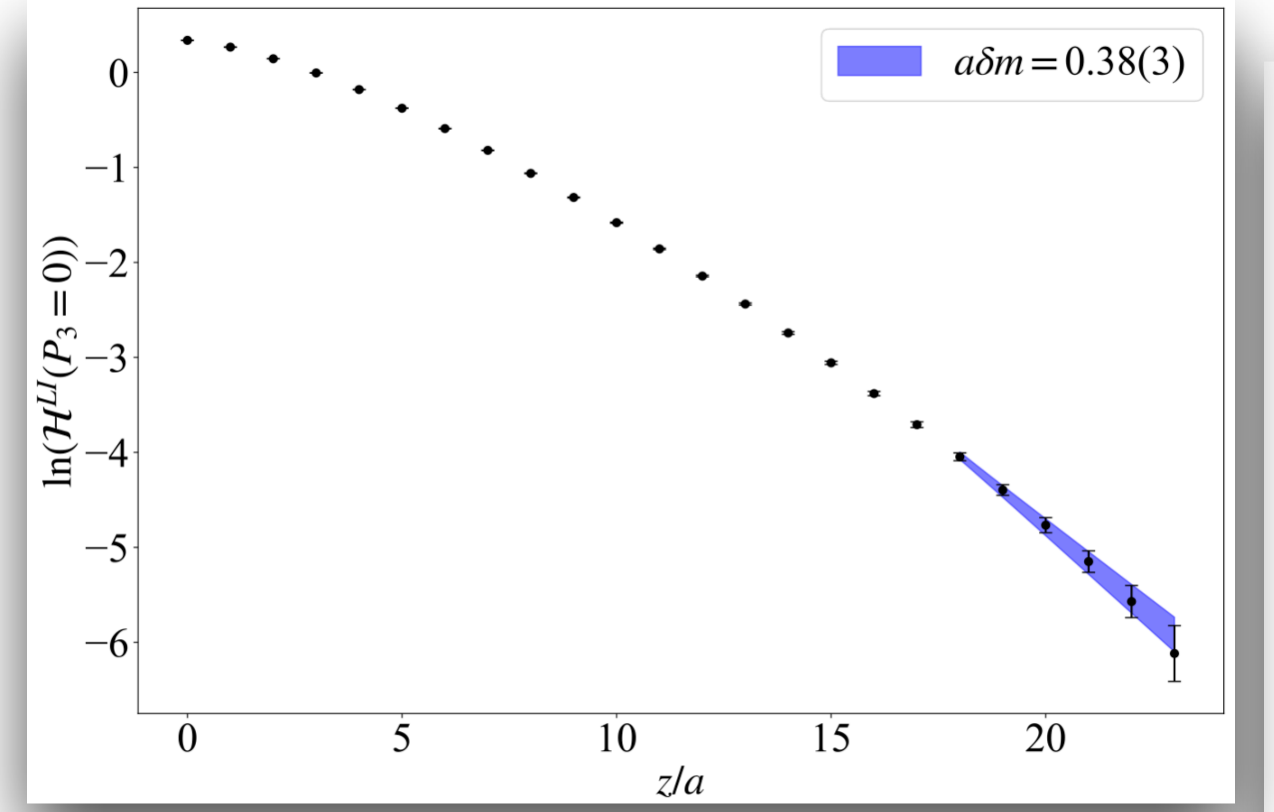
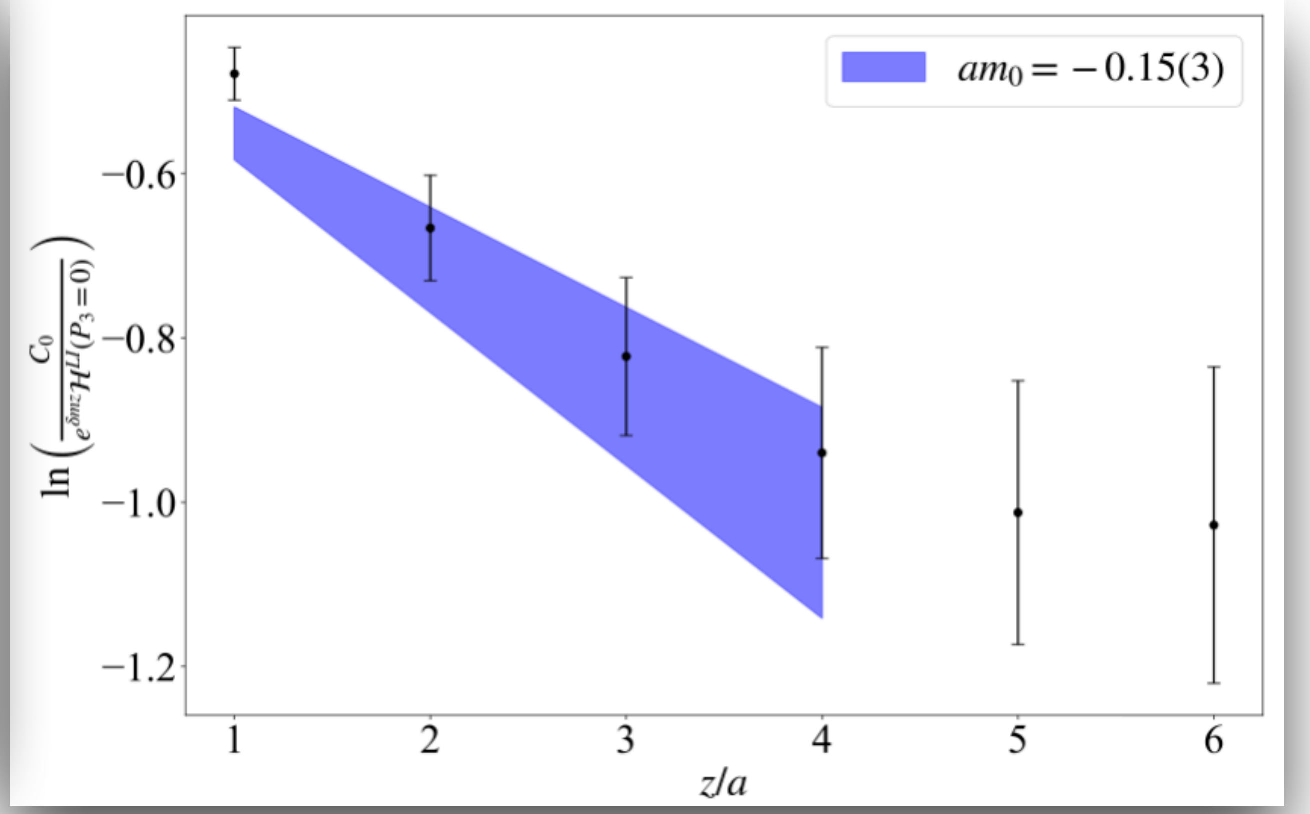
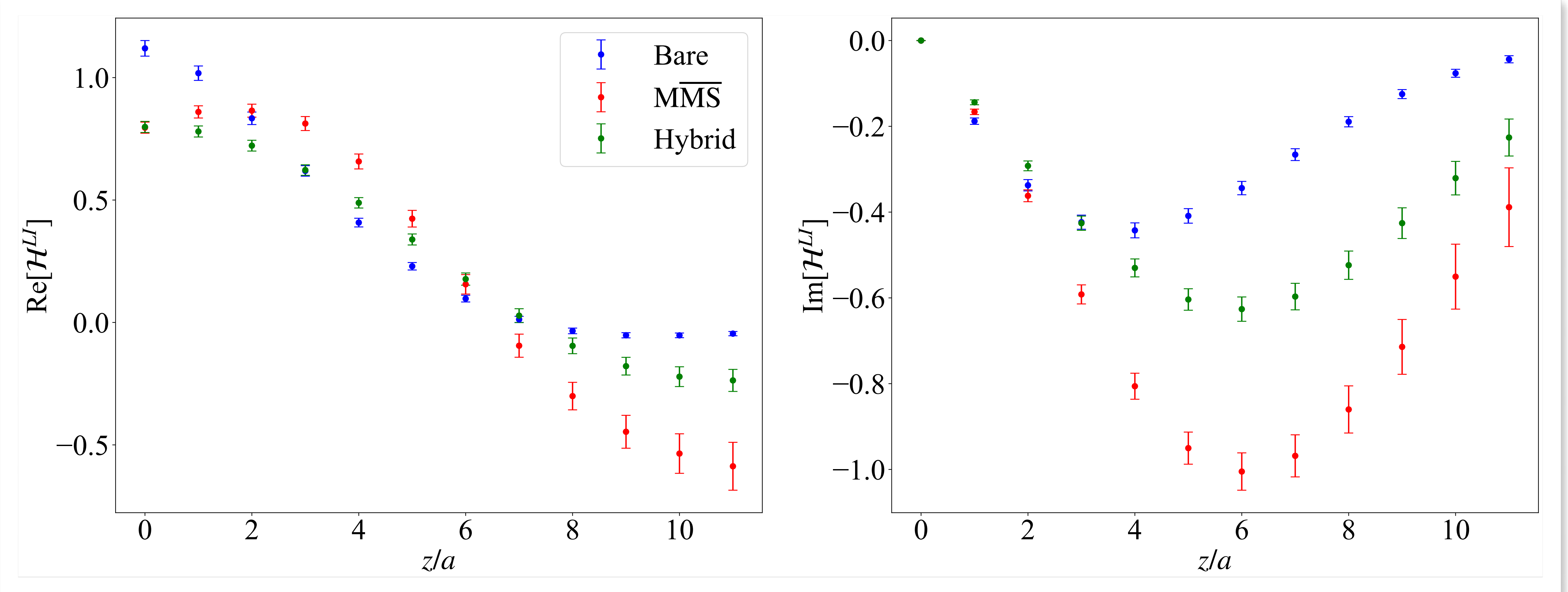






- Effect of HR essential in large-z region
- Matching specific to HR scheme

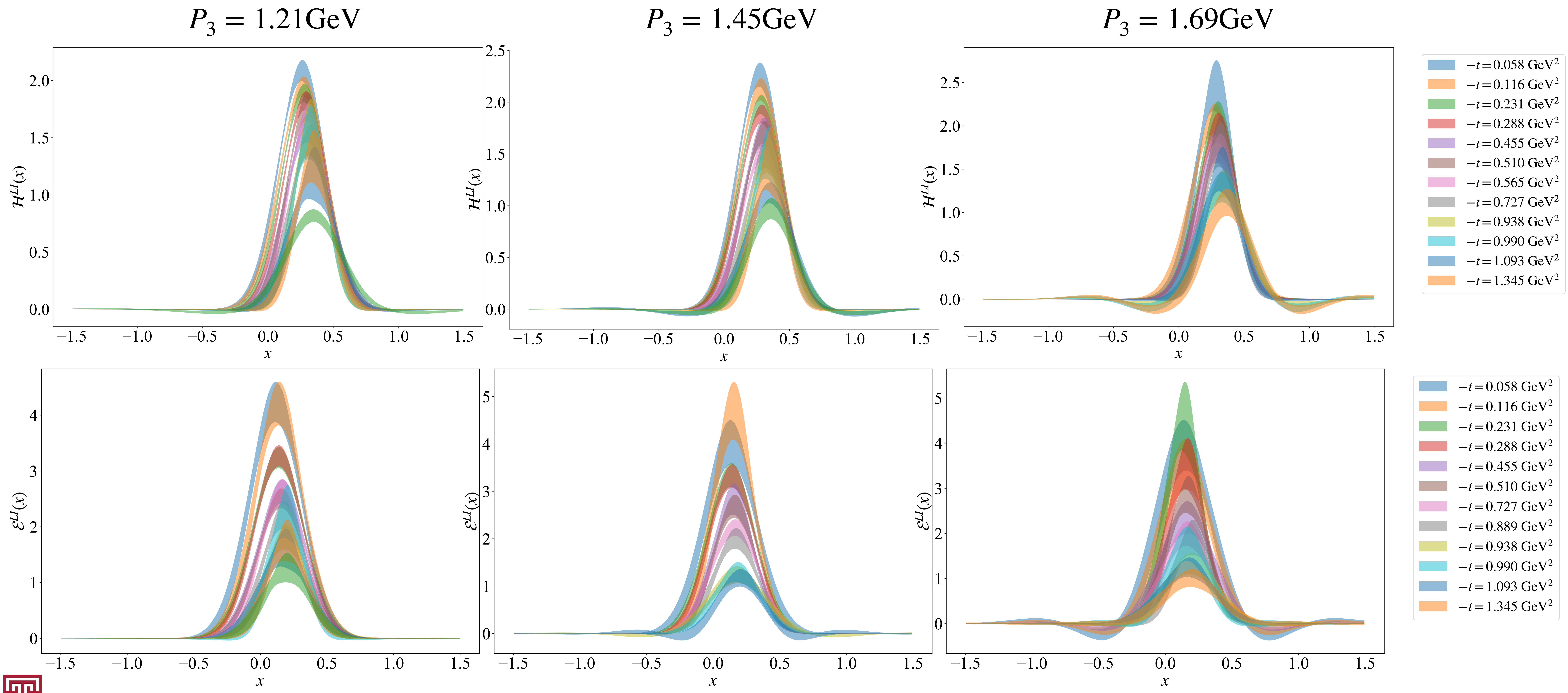
4

x-dependence reconstruction

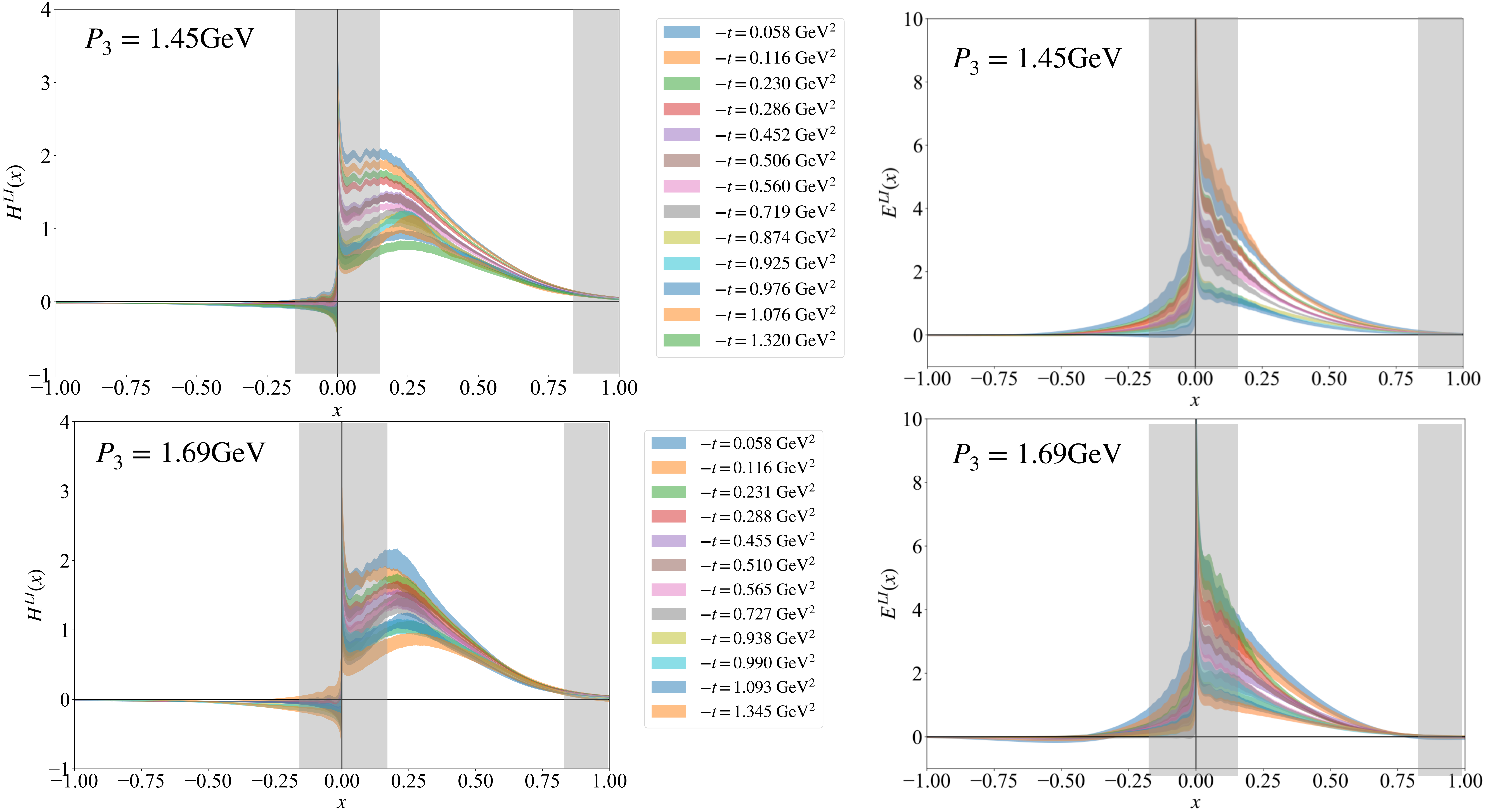
C. Alexandrou et al., **PRD 102, 094508 (2020)**

★ Gaussian-process regression and Fourier transform

Talk by G. Pierini



4 Light-cone GPDs



★ Next step: -t parametrization for impact-parameter space distributions

5

Complementarity and synergies drive progress

https://qgtcollab.github.io/

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- Theoretical frameworks
- Advances
- Lattice within global analysis

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- **Yuxun Guo, Fatma P. Aslan, Xiangdong Ji, M. Gabriel Santiago,**
  First Global Extraction of Generalized Parton Distributions from Experiment and Lattice Data with Next-to-Leading-Order Accuracy,
  Phys. Rev. Lett. 135 (Sep 2025) 261903
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  Generalized Parton Distributions from Lattice QCD with Asymmetric Momentum Transfer: Tensor Case,
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  Skewness-dependent Moments of Pion GPD from Nonlocal Quark-Bilinear Correlators,
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See also talk by J. Miller, J. Torsiello, posters by I. Anderson, N. Khosla

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Conclusions

★ The Lorentz-invariant amplitude parametrization enables the use of the computationally efficient asymmetric frame and allows symmetry-related kinematics to be combined consistently

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Future work

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★ Helicity and Transversity GPDs. LI parametrization completed:

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★ Mellin moments through short-distance OPE on nonlocal operators — J. Miller et al., **arXiv: 2606.28102**

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