2- and 3-Pion Scattering in Multiple 3-Body Isospin Channels

Jake Sitison, Nicholas Chambers, Rana Urek

Raúl Briceño, Robert Edwards, Max Hansen, Andrew Jackura

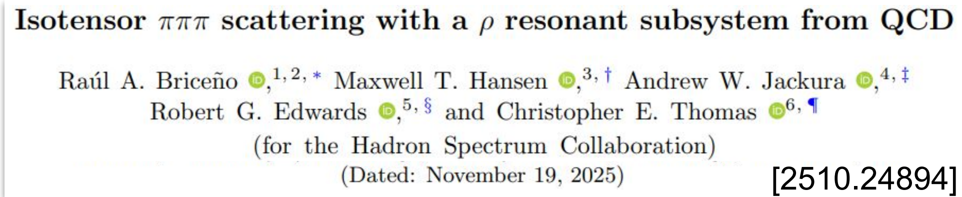
Lattice 2026, UMD, College Park

July 30, 2026

University of Colorado
Boulder

Background and Motivation

Review of Isotensor 3π Analysis



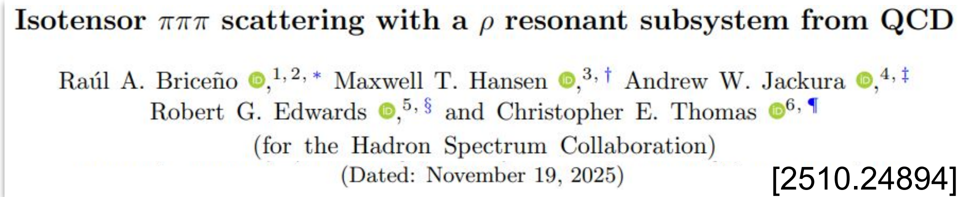
Isotensor $\pi\pi\pi$ scattering with a ρ resonant subsystem from QCD

Raúl A. Briceño[1,2,*] Maxwell T. Hansen[3,†] Andrew W. Jackura[4,‡]
Robert G. Edwards[5,§] and Christopher E. Thomas[6,¶]
(for the Hadron Spectrum Collaboration)
(Dated: November 19, 2025)

[2510.24894]

Talk by A. Jackura:
July 30, 2:20 pm

Review of Isotensor 3π Analysis

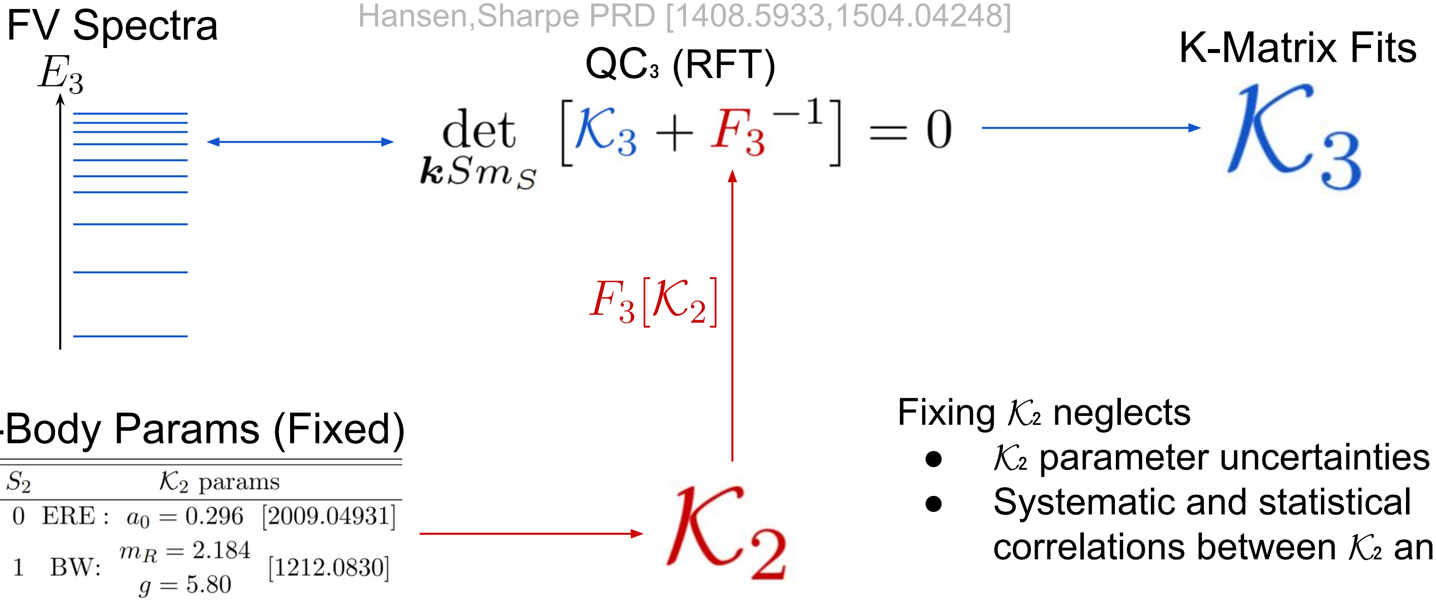


Isotensor $\pi\pi\pi$ scattering with a ρ resonant subsystem from QCD

Raúl A. Briceño[1,2,*] Maxwell T. Hansen[3,†] Andrew W. Jackura[4,‡]
Robert G. Edwards[5,§] and Christopher E. Thomas[6,¶]
(for the Hadron Spectrum Collaboration)
(Dated: November 19, 2025)

[2510.24894]

Talk by A. Jackura:
July 30, 2:20 pm



Hansen,Sharpe PRD [1408.5933,1504.04248]

FV Spectra

QC₃ (RFT)

$$\det_{\boldsymbol{k}Sm_S}\left[\mathcal{K}_3 + F_3{}^{-1}\right] = 0$$

K-Matrix Fits

$$\mathcal{K}_3$$

$$F_3[\mathcal{K}_2]$$

$$\mathcal{K}_2$$

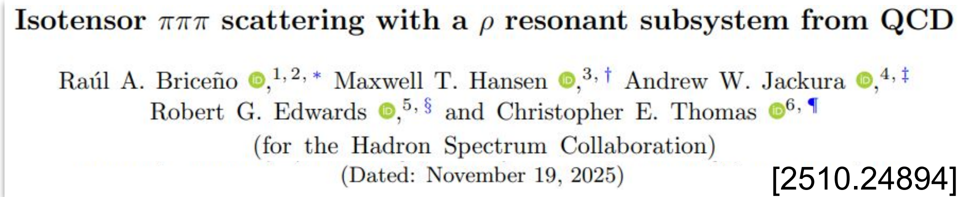
2-Body Params (Fixed)

I_2	S_2	$\mathcal{K}_2$ params
2	0	ERE : $a_0 = 0.296$ [2009.04931]
1	1	BW: $m_R = 2.184$ $g = 5.80$ [1212.0830]

Fixing $\mathcal{K}_2$ neglects

- $\mathcal{K}_2$ parameter uncertainties
- Systematic and statistical correlations between $\mathcal{K}_2$ and $\mathcal{K}_3$

Review of Isotensor 3π Analysis

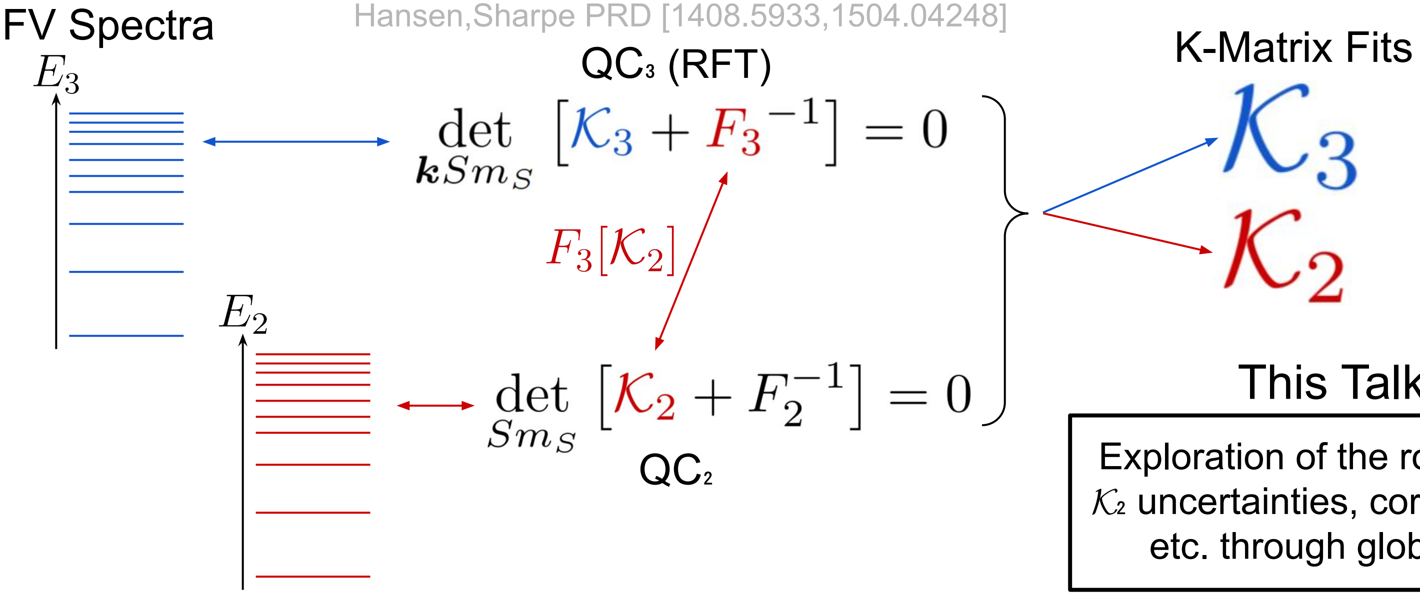


Isotensor $\pi\pi\pi$ scattering with a ρ resonant subsystem from QCD

Raúl A. Briceño,[1,2,*] Maxwell T. Hansen,[3,†] Andrew W. Jackura,[4,‡]
Robert G. Edwards,[5,§] and Christopher E. Thomas[6,¶]
(for the Hadron Spectrum Collaboration)
(Dated: November 19, 2025)

[2510.24894]

Talk by A. Jackura:
July 30, 2:20 pm



FV Spectra

Hansen,Sharpe PRD [1408.5933,1504.04248]

QC$_3$ (RFT)

$$\det_{\boldsymbol{k}Sm_S}\left[\mathcal{K}_3 + F_3^{-1}\right] = 0$$

$F_3[\mathcal{K}_2]$

$$\det_{Sm_S}\left[\mathcal{K}_2 + F_2^{-1}\right] = 0$$

QC$_2$

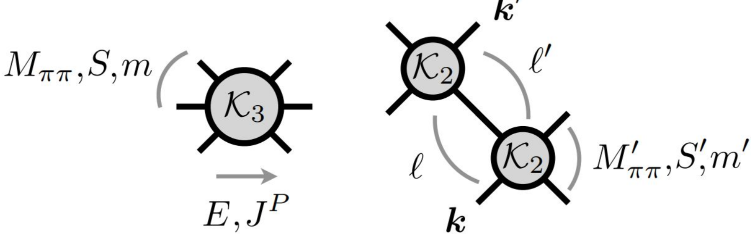
K-Matrix Fits

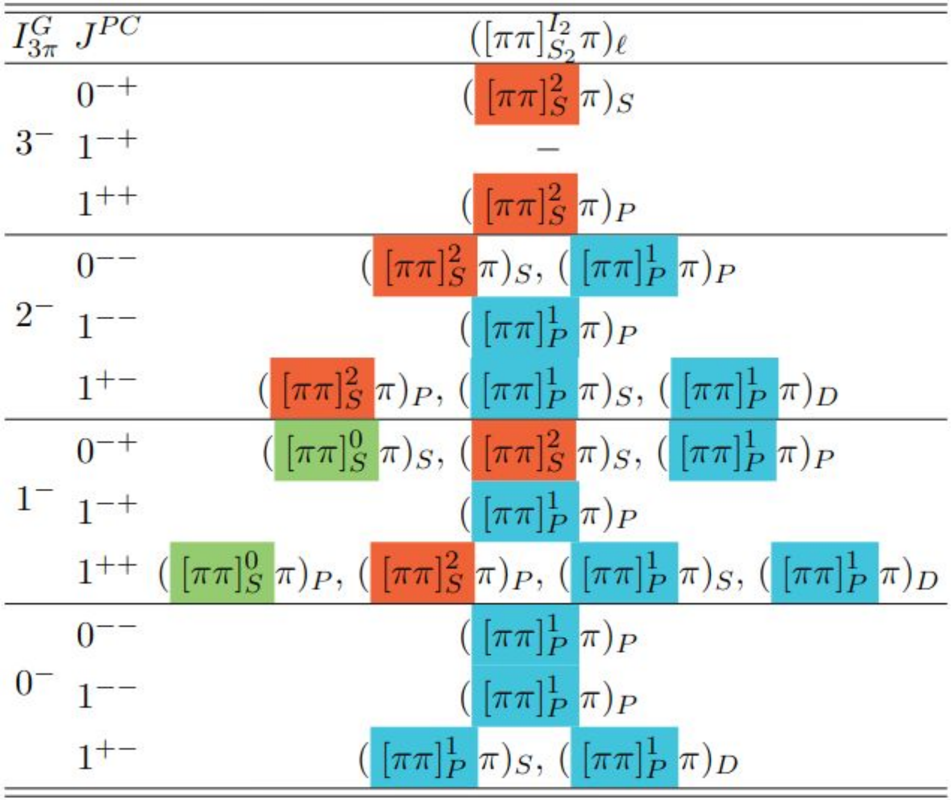
This Talk

Exploration of the role of the $\mathcal{K}_2$ uncertainties, correlations, etc. through global fits

Correlations Between 3π Channels

- $\mathcal{K}_3$ matrix factorizes across channels (isospin, total angular momentum, parity, …)
- Different 3π channels correlated through 2π sub-channels



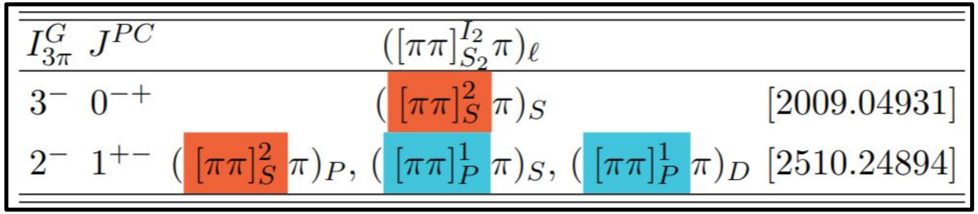


$I^G_{3\pi}$	J^{PC}	$([\pi\pi]^{I_2}_{S_2}\pi)_\ell$
3^-	0^{-+}	$([\pi\pi]^2_S\,\pi)_S$
	1^{-+}	—
	1^{++}	$([\pi\pi]^2_S\,\pi)_P$
2^-	0^{--}	$([\pi\pi]^2_S\,\pi)_S$, $([\pi\pi]^1_P\,\pi)_P$
	1^{--}	$([\pi\pi]^1_P\,\pi)_P$
	1^{+-}	$([\pi\pi]^2_S\,\pi)_P$, $([\pi\pi]^1_P\,\pi)_S$, $([\pi\pi]^1_P\,\pi)_D$
1^-	0^{-+}	$([\pi\pi]^0_S\,\pi)_S$, $([\pi\pi]^2_S\,\pi)_S$, $([\pi\pi]^1_P\,\pi)_P$
	1^{-+}	$([\pi\pi]^1_P\,\pi)_P$
	1^{++}	$([\pi\pi]^0_S\,\pi)_P$, $([\pi\pi]^2_S\,\pi)_P$, $([\pi\pi]^1_P\,\pi)_S$, $([\pi\pi]^1_P\,\pi)_D$
0^-	0^{--}	$([\pi\pi]^1_P\,\pi)_P$
	1^{--}	$([\pi\pi]^1_P\,\pi)_P$
	1^{+-}	$([\pi\pi]^1_P\,\pi)_S$, $([\pi\pi]^1_P\,\pi)_D$

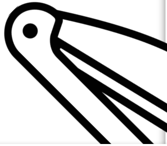
Correlations Between 3π Channels

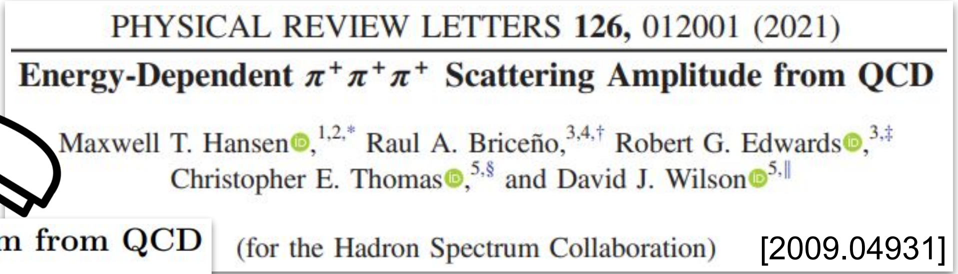
- $\mathcal{K}_3$ matrix factorizes across channels (isospin, total angular momentum, parity, …)
- Different 3π channels correlated through 2π sub-channels

This Talk



$I^G_{3\pi}$	J^{PC}	$([\pi\pi]^{I_2}_{S_2}\pi)_\ell$	
3^-	0^{-+}	$([\pi\pi]^2_S\pi)_S$	[2009.04931]
2^-	1^{+-}	$([\pi\pi]^2_S\pi)_P$, $([\pi\pi]^1_P\pi)_S$, $([\pi\pi]^1_P\pi)_D$	[2510.24894]





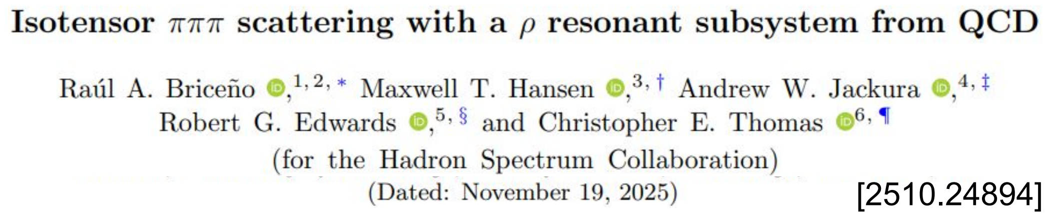
PHYSICAL REVIEW LETTERS **126,** 012001 (2021)

Energy-Dependent $\pi^+\pi^+\pi^+$ Scattering Amplitude from QCD

Maxwell T. Hansen,[1,2,*] Raul A. Briceño,[3,4,†] Robert G. Edwards,[3,‡]
Christopher E. Thomas,[5,§] and David J. Wilson[5,‖]

(for the Hadron Spectrum Collaboration)

[2009.04931]



Isotensor $\pi\pi\pi$ scattering with a ρ resonant subsystem from QCD

Raúl A. Briceño,[1,2,*] Maxwell T. Hansen,[3,†] Andrew W. Jackura,[4,‡]
Robert G. Edwards,[5,§] and Christopher E. Thomas[6,¶]

(for the Hadron Spectrum Collaboration)

(Dated: November 19, 2025)

[2510.24894]

* and references therein

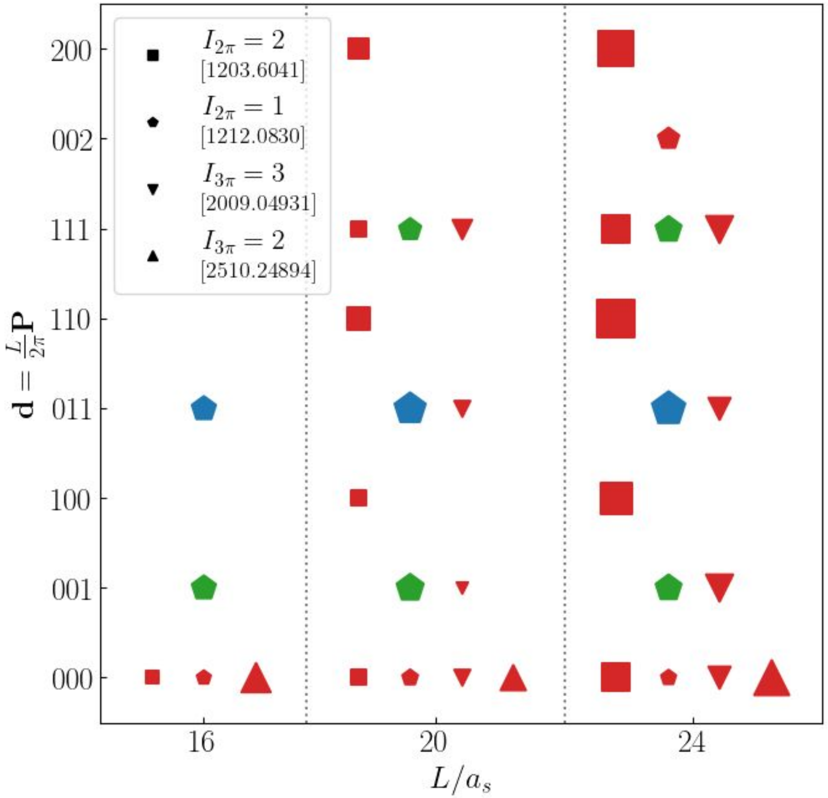
FV Lattice Spectra

Data Summary

$$m_\pi \approx 400\ \text{MeV}$$
$$a_t m_\pi = 0.06906(13)$$
$$\xi = a_s/a_t = 3.444(6)$$

- 96 levels (66 2π+30 3π) included in fits

- Shape: Isospin Channel
  - ■: $I_2 = 2$, S-wave 2π
  - ⬟: $I_2 = 1$ P-wave 2π
  - ▼: $I_3 = 3$, S-wave 3π
  - ▲: $I_3 = 2$, 3π and ρπ
- Size ∝ $n_{\text{states}} \times n_{\text{cfgs}}$ ~ stat power
- Color: R/G/B for 1/2/3 irreps
- Original papers in legend



Spectrum Systematics

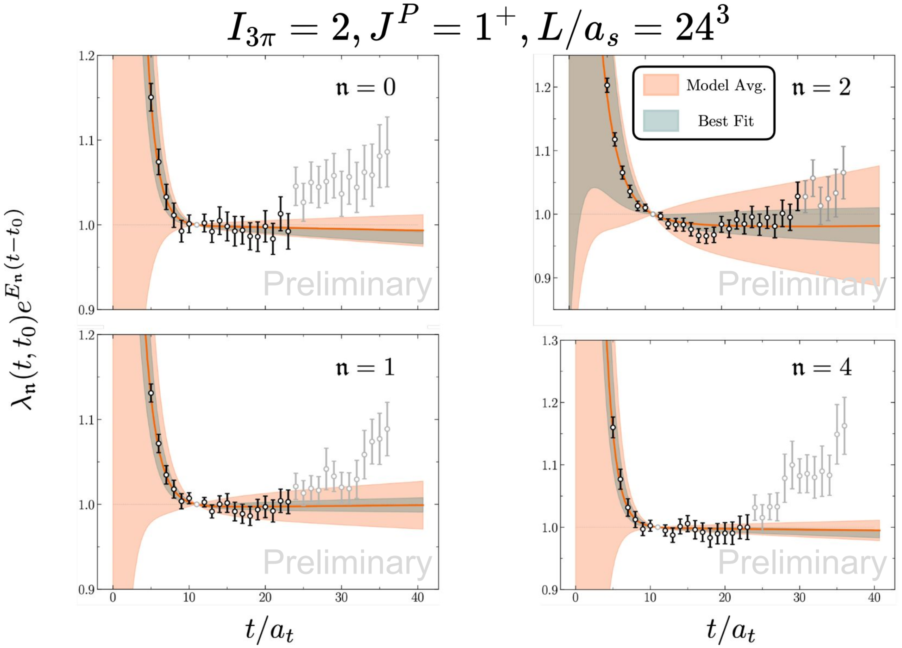
FV Pion Masses

- Added (in quadrature) to all E levels

$$\sigma_L = E_L \frac{|m_\pi(L) - m_\pi(\infty)|}{m_\pi(L)}$$

Fit Variations

- Previous analyses use conservative systematic for GEVP variations
- Reanalysis with model averaging (AIC) to marginalize over hyperparameter choices underway
- Significant changes unlikely



Plot: N. Chambers

Error Analysis and Fitting Strategy

Analysis Workflow and Error Propagation

$$\mathrm{QC}_2 = \det_{Sm}\left[\mathbf{F}_2^{-1}(E, L, P) + \mathbf{K}_2(E|\eta_2)\right] \qquad \mathrm{QC}_3 = \det_{I_2\boldsymbol{k}Sm}\left[\mathbf{F}_3^{-1}(E, L, P|\eta_2) + \mathbf{K}_3(E|\eta_3)\right]$$

Analysis Workflow and Error Propagation

$$\mathrm{QC}_2 = \det_{Sm}\left[\mathbf{F}_2^{-1}(E,L,P) + \mathbf{K}_2(E|\eta_2)\right] \qquad \mathrm{QC}_3 = \det_{I_2\boldsymbol{k}Sm}\left[\mathbf{F}_3^{-1}(E,L,P|\eta_2) + \mathbf{K}_3(E|\eta_3)\right]$$

Analysis Workflow

1. Compute QC's as a function of E each volumes, irreps, etc. for given parameters η_2,η_3

2. Find zeros of the QC's (FV E levels)

3. Minimize the χ^2 function between the QC and lattice spectra w.r.t. η_2,η_3

$$\chi^2 = r_i \Sigma_{ij}^{-1} r_j$$

$$r_i(\boldsymbol{\eta}) = E_i^{\mathrm{data}} - E_i^{\mathrm{QC}}(\boldsymbol{\eta})$$

Analysis Workflow and Error Propagation

$$\mathrm{QC}_2 = \det_{Sm}\left[\mathbf{F}_2^{-1}(E,L,P) + \mathbf{K}_2(E|\eta_2)\right]$$

$$\mathrm{QC}_3 = \det_{I_2\boldsymbol{k}Sm}\left[\mathbf{F}_3^{-1}(E,L,P|\eta_2) + \mathbf{K}_3(E|\eta_3)\right]$$

Analysis Workflow

1. Compute QC's as a function of E each volumes, irreps, etc. for given parameters η_2,η_3
2. Find the zeros of the QC's
3. Minimize the χ^2 function between the QC and lattice spectra w.r.t. η_2,η_3

$$\chi^2 = r_i \Sigma_{ij}^{-1} r_j$$

$$r_i(\boldsymbol{\eta}) = E_i^{\mathrm{data}} - E_i^{\mathrm{QC}}(\boldsymbol{\eta})$$

Error Propagation

- Linear approx of derived quantities $\boldsymbol{f}$ w.r.t. primary sources of uncertainty $\boldsymbol{p}$

$$f_i(\boldsymbol{p}) \approx \bar{f}_i + \overbrace{\left(\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{p}}\right)_{ij}}^{J_{ij}}(p_j - \bar{p}_j)$$

$$\Longrightarrow \Sigma_{\boldsymbol{f}} \approx J \Sigma_{\mathbf{p}} J^T$$

- Good approx in the limit of small errors or Gaussian distributions
- Cross-checks suggest good approx (see backup)

QC Building Blocks

- Global fits are more expensive
- To minimize computation in each fit, pre-compute “QC Building Blocks”

QC Building Blocks

- Global fits are more expensive
- To minimize computation in each fit, pre-compute “QC Building Blocks”

Building Blocks

$$\mathbf{K}_2^{-1} = \sum_n \eta_2^{(n)} \boldsymbol{\kappa}_2^{(n)} \qquad \mathbf{K}_3 = \sum_m \eta_3^{(m)} \boldsymbol{\kappa}_3^{(m)}$$

Fit Params

- Fixed η_2:

$$\mathrm{QC}_3 = \det_{I_2 \boldsymbol{k} S m} \left[\mathbf{F}_3^{-1}(\eta_2) + \sum_m \eta_3^{(m)} \boldsymbol{\kappa}_3^{(m)} \right]$$

QC Building Blocks

- Global fits are more expensive
- To minimize computation in each fit, pre-compute “QC Building Blocks”

Building Blocks

$$\mathbf{K}_2^{-1} = \sum_n \eta_2^{(n)} \boldsymbol{\kappa}_2^{(n)} \qquad \mathbf{K}_3 = \sum_m \eta_3^{(m)} \boldsymbol{\kappa}_3^{(m)}$$

Fit Params

- Fixed η_2:

$$\mathrm{QC}_3 = \det_{I_2 \boldsymbol{k} S m} \left[\mathbf{F}_3^{-1}(\eta_2) + \sum_m \eta_3^{(m)} \boldsymbol{\kappa}_3^{(m)} \right]$$

- Variable η_2:

$$\mathrm{QC}_3 = \det_{I_2 \boldsymbol{k} S m} \left[L^3 \left((\mathbf{F} + \mathbf{G})^{-1} + \underbrace{\left(\sum_n \eta_2^{(n)} \boldsymbol{\kappa}_2^{(n)} \right)}_{\mathbf{K}_2} \right)^{-1} + \sum_m \eta_3^{(m)} \boldsymbol{\kappa}_3^{(m)} \right]$$

- Similar idea for QC_2

$\mathbf{K}_2$ Matrix Parameterizations

Building Blocks

$$\mathbf{K}_2^{-1} = \sum_n \eta_2^{(n)} \boldsymbol{\kappa}_2^{(n)}$$

Fit Params

$$[\mathbf{K}_2^{-1}]_{I_2'\mathbf{k}'S'm',I_2\mathbf{k}Sm} \supset q_k^{\star\,2S+1}\cot\delta_S(q_k^\star)$$

- Kinematic factors, cutoff functions, phase space shifts, etc. not shown for simplicity

$$\left(\frac{q^\star}{m_\pi}\right)\cot\delta_0^{\mathrm{ERE}}(q_k^\star) = -\frac{1}{a_0} + \frac{r_0}{2}\left(\frac{q^{\star 2}}{m_\pi^2}\right) + \cdots$$

- S-wave: Effective range expansion
- P-wave: Breit Wigner

$$\left(\frac{q^\star}{m_\pi}\right)^3 \cot\delta_1^{\mathrm{BW}}(q_k^\star) = \frac{6\pi}{m_\pi^3}\left(\frac{m_{R,1}^2}{g_1^2} E_2^\star - \frac{1}{g_1^2} E_2^{\star\,3}\right)$$

- Others parameterizations ongoing

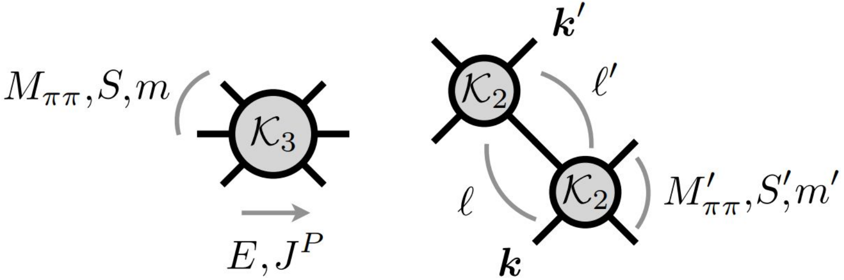
$\mathbf{K}_3$ Matrix Parameterizations: $I_3 = 2$

Building Blocks

$$\mathbf{K}_3 = \sum_m \eta_3^{(m)} \boldsymbol{\kappa}_3^{(m)}$$

Fit Params

- 3π analog of threshold expansion
- Sub-channel momenta dependence included in $I_3 = 2$



$\boldsymbol{k}$: Spectator momentum
$q_k^\star$: Momentum of π in pair rest frame

$$[\mathbf{K}_3]_{I_2'\boldsymbol{k}'S'm',I_2\boldsymbol{k}Sm} \supset \mathcal{K}^{J^P}_{3;\ell'S',\ell S}(k',k) = h_{\ell'S'}(k')\widetilde{\mathcal{K}}^{J^P}_{3;\ell'S',\ell S}(s)h_{\ell S}(k)$$

$$m_\pi^2\widetilde{\mathcal{K}}^{J^P}_{3;\ell'S',\ell S}(s) = \alpha\left({}^{2S'+1}\ell'_J|{}^{2S+1}\ell_J\right) + \beta\left({}^{2S'+1}\ell'_J|{}^{2S+1}\ell_J\right)\left(\frac{s-(3m_\pi)^2}{m_\pi^2}\right) + \cdots$$

$$h_{\ell S}(k) = 1 + \gamma_{q_k^\star}\left({}^{2S+1}\ell_J\right)\frac{q_k^{\star 2}}{m_\pi^2} + \cdots$$

Jackura PRD [2208.10587]
Briceño,Costa,Jackura PRD [2409.15577]

$\mathbf{K}_3$ Matrix Parameterizations: I_3 = 3

Building Blocks

$$\mathbf{K}_3 = \sum_m \eta_3^{(m)} \boldsymbol{\kappa}_3^{(m)}$$

Fit Params

- 3π analog of threshold expansion
- Sub-channel momenta dependence neglected in I_3 = 3

Isotropic Approximation

- Neglect dependence on sub-channel momenta
- Good approx for I_3 = 3 (S-wave)
- $\mathbf{K}_3$ rank-1 ⇒ Project $\boldsymbol{F}_3$ to subspace

$$[\mathbf{K}_3]_{I_2'\boldsymbol{k}'S'm',I_2\boldsymbol{k}Sm} \supset \mathcal{K}^{J^P}_{3;\ell'S',\ell S}(k',k) = h_{\ell'S'}(k')\widetilde{\mathcal{K}}^{J^P}_{3;\ell'S',\ell S}(s)h_{\ell S}(k)$$

$$m_\pi^2\widetilde{\mathcal{K}}^{J^P}_{3;\ell'S',\ell S}(s) = \alpha\left({}^{2S'+1}\ell'_J|{}^{2S+1}\ell_J\right) + \beta\left({}^{2S'+1}\ell'_J|{}^{2S+1}\ell_J\right)\left(\frac{s-(3m_\pi)^2}{m_\pi^2}\right) + \cdots$$

$$h_{\ell S}(k) = 1 + \xcancel{\gamma_{q_k^\star}\left({}^{2S+1}\ell_J\right)\frac{q_k^{\star 2}}{m_\pi^2}} + \cdots$$

Hansen,Sharpe PRD [1408.5933]

Bayesian Model Averaging

- Asymptotically unbiased model probabilities

$$\mathrm{pr}(M|D) \propto \mathrm{pr}(M) \exp\left[-\frac{1}{2}\underbrace{\left(\chi^2_{\mathrm{data}}(\eta^*) + 2N_{\mathrm{params}} - 2N_{\mathrm{data}}\right)}_{\text{Bayesian Akaike Information Criterion (BAIC)}}\right]$$

- Posterior mean: $$\langle f(\eta)\rangle = \sum_{\mu}^{N_{\mathrm{models}}} f(\eta^*_\mu)\mathrm{pr}(M_\mu|D)$$

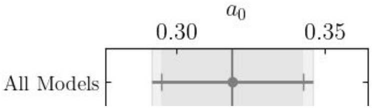
- Posterior variance:

$$\sigma^2_{\langle f(\eta)\rangle} = \underbrace{\sum_\mu \sigma^2_{f(\eta),\mu}\mathrm{pr}(M_\mu|D)}_{\text{Statistical}} + \underbrace{\sum_\mu f(\eta^*_\mu)\mathrm{pr}(M_\mu|D) - \langle f(\eta)\rangle^2}_{\text{Systematic}}$$

Jay, Neil PRD [2008.01069]
Neil, Sitison PRD,E [2208.14983,2305.19417]

Preliminary K-Matrix Fits

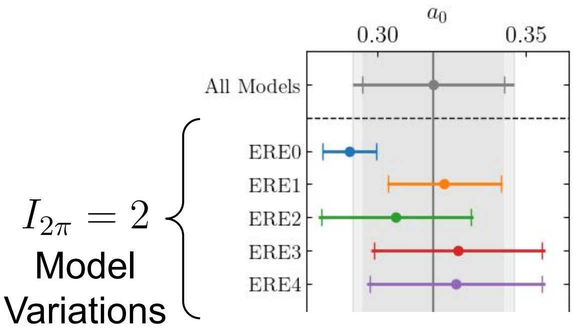
Preliminary Model Average Results



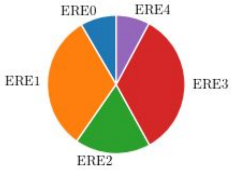
- Average of 60 models with 2π and 3π fit variations
- E.g., Scattering length a_0 (I_2 = 2, S-wave)

$$\sigma^2_{\langle f(\eta)\rangle} = \underbrace{\sum_\mu \sigma^2_{f(\eta),\mu} \mathrm{pr}(M_\mu|D)}_{\substack{\text{Statistical} \\ \text{(Inner errorbar)}}} + \underbrace{\sum_\mu f(\eta^*_\mu)\mathrm{pr}(M_\mu|D) - \langle f(\eta)\rangle^2}_{\substack{\text{Systematic} \\ \text{(Outer errorbar)}}}$$

Preliminary Model Average Results



Subset Probabilities



$$\mathrm{pr}(S|D) = \sum_{M_\mu \in \mathrm{S}} \mathrm{pr}(M_\mu|D)$$

- Average of 60 models with 2π and 3π fit variations
- E.g., Scattering length a_0 (I_2 = 2, S-wave)

- Can fix one model choice (e.g., ERE order) and average over other models
- 15 fits per subset in this example

- Spread of subsets corresponds to systematic (according to subset probabilities)

Subset averages:
Lahert, *et al*. [2301.08274]

Preliminary Model Average Results

$I_{2\pi} = 2$ Parameter

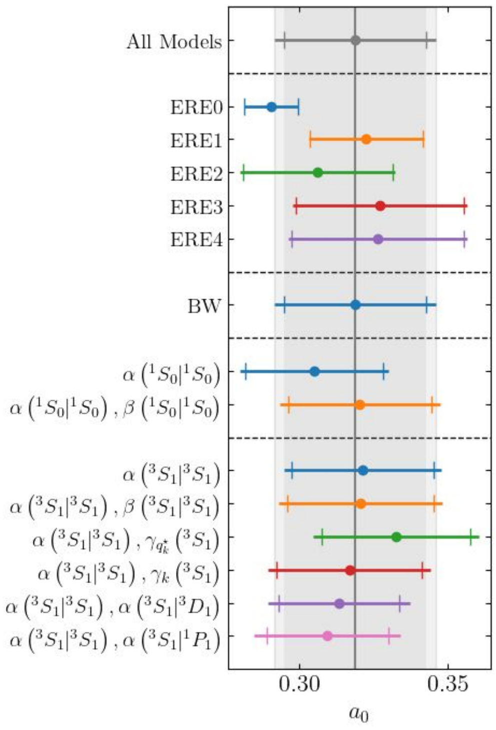
$I_{2\pi} = 2$

$I_{2\pi} = 1$

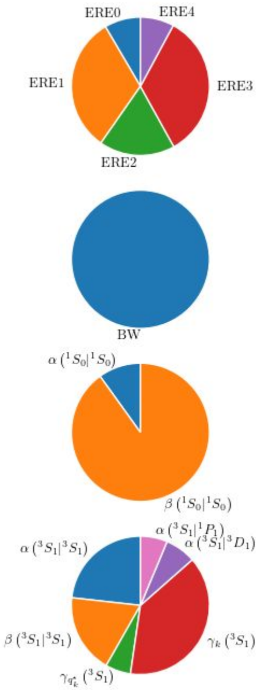
$I_{3\pi} = 3$

$I_{3\pi} = 2$

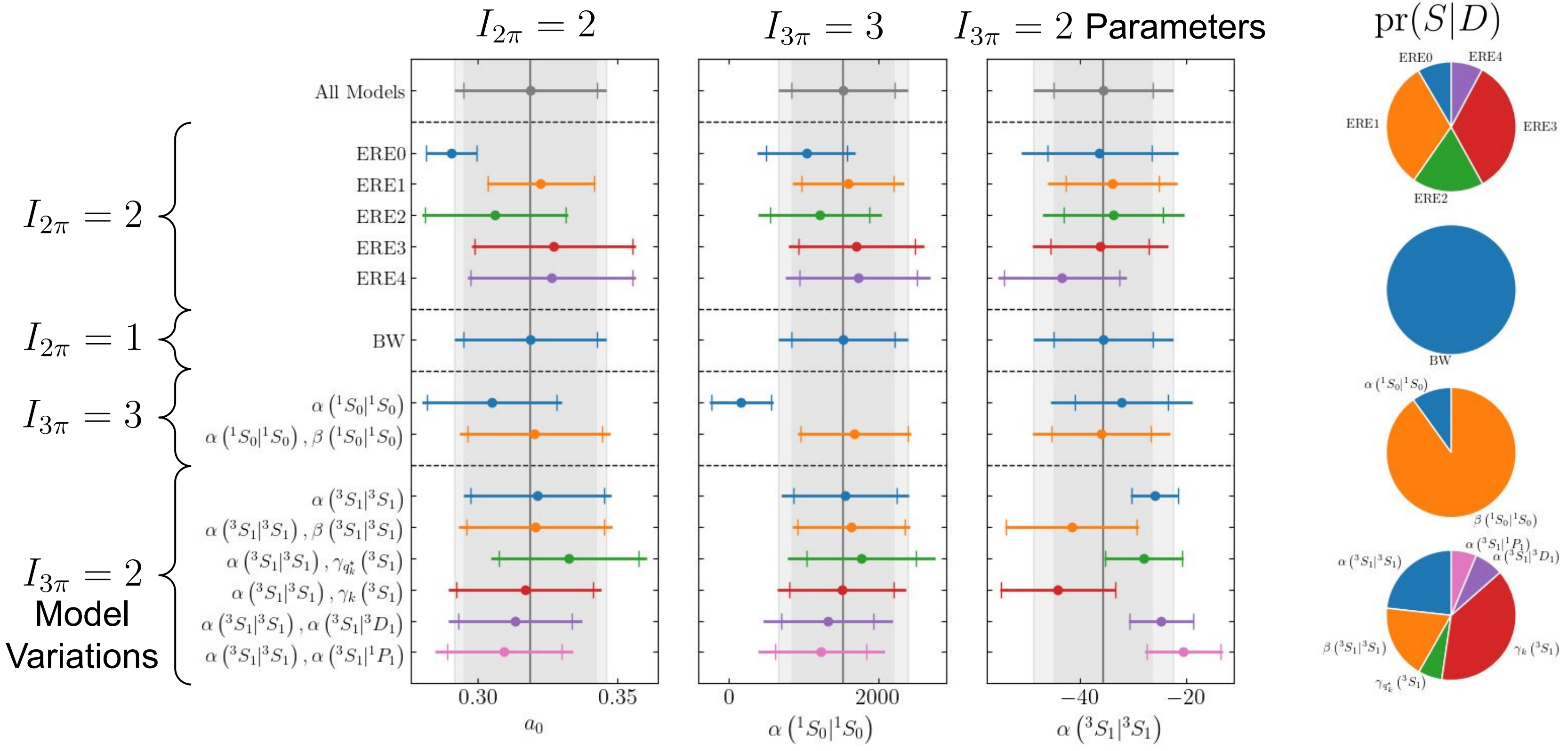
Model Variations



$\mathrm{pr}(S|D)$



Preliminary Model Average Results

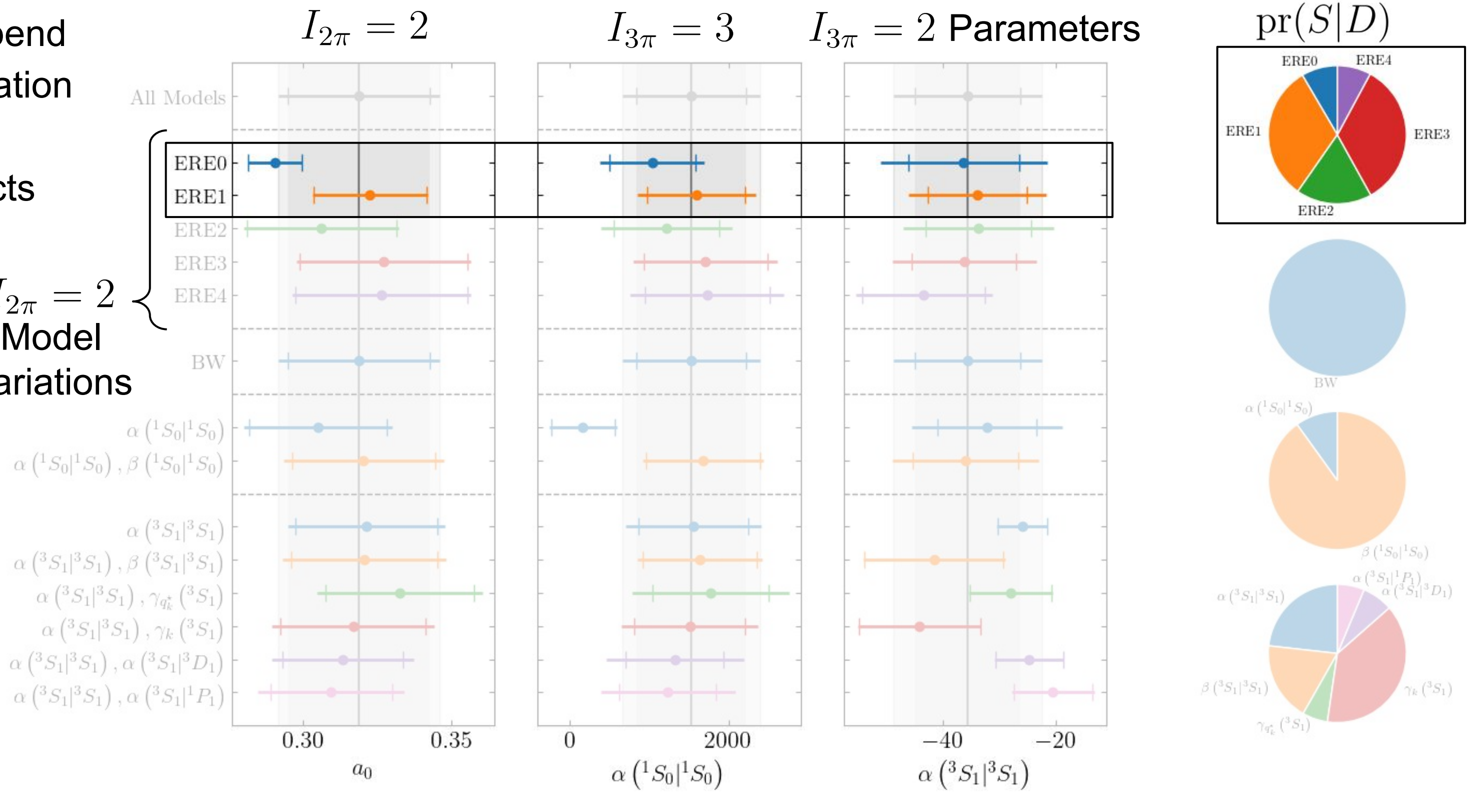


Preliminary Model Average Results

3π parameters depend on 2π parameterization

⇒ Fixing $\mathcal{K}_2$ neglects this effect

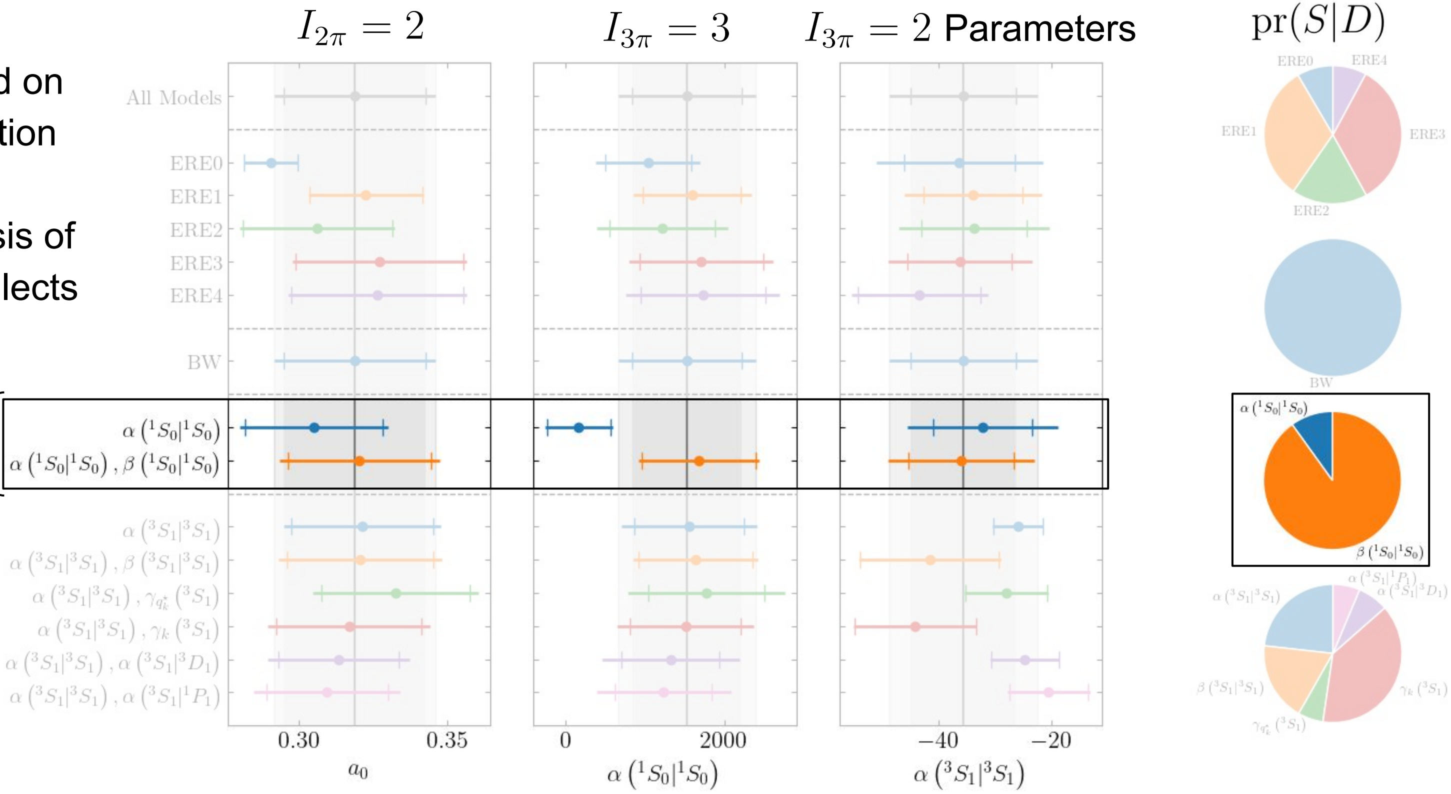
$I_{2\pi} = 2$
Model Variations



Preliminary Model Average Results

I_2 = 2 and I_3 = 2 parameters depend on I_3 = 3 parameterization

⇒ Separate analysis of 3π channels neglects this effect



$I_{3\pi} = 3$ Model Variations

Preliminary Scattering Amplitudes

Example Scattering Amplitudes

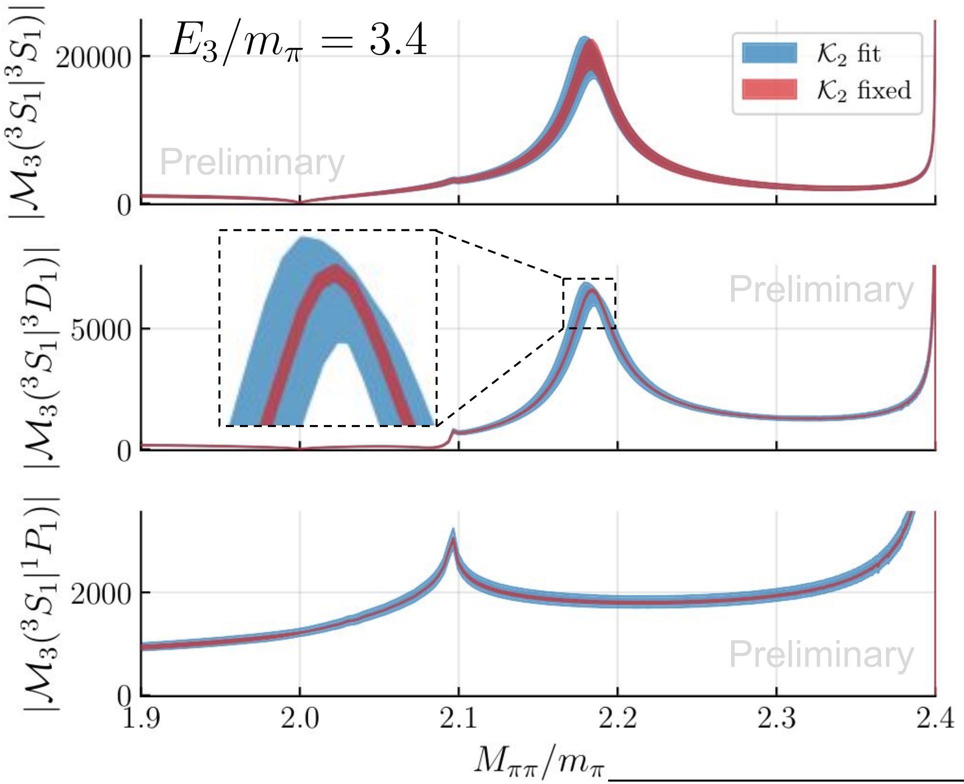
$$\mathcal{K}_3 \xrightarrow[\text{Eqns}]{\text{Integral}} \mathcal{M}_3$$

$$\begin{tikzcd}
\mathcal{K}_3 \arrow[r, "\text{Integral}", "\text{Eqns}"'] & \mathcal{M}_3 \\
\text{Scheme Dep} \arrow[u] & \text{Scheme Indep} \arrow[u]
\end{tikzcd}$$

- Single example fit w/ params

$$\eta_2 = \{a_0, m_{R,1}, g_1\}\,, \quad \eta_3 = \alpha({}^3S_1|{}^3S_1)$$

- Comparison of fixing or fitting $\mathcal{K}_2$
- Similar conclusions about correlations, etc. as for K-matrices



* Unsymmetrized amplitudes

Briceño,Costa,Jackura PRD [2409.15577]

Summary

Summary

- For precision multi-particle physics, all statistical and systematic effects must be understand
- Generalization of previous 3π analyses in I_3 = 3 and I_3 = 2 channels presented
- Correlations between several 2π and 3π isospin channels considered, propagating statistical uncertainties and quantifying fit systematics
- Preliminary analysis shows correlations important between 2π and 3π channels and between different 3π channels
- Additional parameterizations and amplitude analysis ongoing

Backup

More General QC Building Blocks

- Not all parameterizations are linear combos, e.g, Adler zero

$$\left(\frac{q^\star}{m_\pi}\right)\cot\delta_0^{\mathrm{AZ}}(q_k^\star) = \frac{1}{1-(2m_\pi^2/E_2^{\star 2})\, z_0^2}\sum_n \eta_2^{(n)}\, \frac{m_\pi}{E_2^*}\left(\frac{q^{\star 2}}{m_\pi^2}\right)^n$$

- Generalized ansätze

$$\mathbf{K}_3 = f_3(\eta_3)\sum_m \eta_3^{(m)}\,\boldsymbol{\kappa}_3^{(m)} \qquad \mathbf{K}_2^{-1} = f_2(\eta_2)\sum_n \eta_2^{(n)}\,\boldsymbol{\kappa}_2^{(n)}$$

- Fixed η_2:

$$\mathrm{QC}_3 = \det_{I_2\boldsymbol{k}Sm}\left[\mathbf{F}_3^{-1}(\eta_2) + f_3(\eta_3)\sum_m \eta_3^{(m)}\,\boldsymbol{\kappa}_3^{(m)}\right]$$

- Variable η_2:

$$\mathrm{QC}_3 = \det_{I_2\boldsymbol{k}Sm}\left[L^3\left((\mathbf{F}+\mathbf{G})^{-1} + f_2^{-1}(\eta_2)\left(\sum_n \eta_2^{(n)}\,\boldsymbol{\kappa}_2^{(n)}\right)^{-1}\right) + f_3(\eta_3)\sum_m \eta_3^{(m)}\,\boldsymbol{\kappa}_3^{(m)}\right]$$

Building Block Interpolation

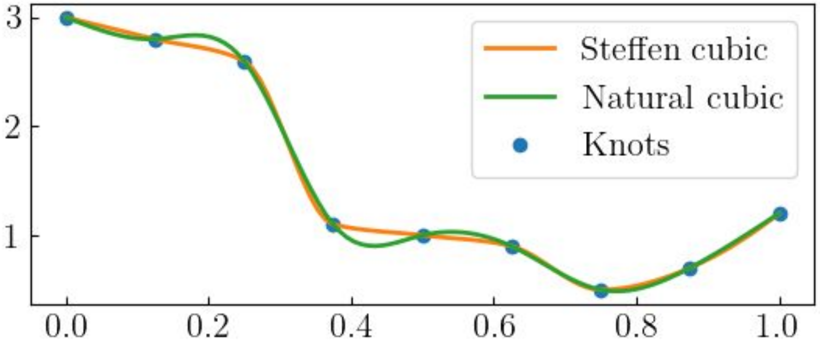
- Building-block approach requires interpolation in QC root finding

Spline vs Linear

- Spline has smaller interpolation error
    - Allows coarser E spacing
    - Faster and/or more accurate
- Spline gives better estimate of derivatives
    - More accurate Jacobians in linear error propagation
    - Faster convergence of Levenberg-Marquardt–type minimizers in fits (better adaptive step size)

Steffen Spline

- Monotonic between knots



- Monotonicity mitigates spurious zeros in QC's
- Cross-checked with linear interpolation

Gaussian Error Propagation Implementation

Linear Approx

$$f_i(\boldsymbol{p}) \approx \bar{f}_i + \overbrace{\left(\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{p}}\right)_{ij}}^{J_{ij}} (p_j - \bar{p}_j)$$

$$\implies \Sigma_{\boldsymbol{f}} \approx J \Sigma_{\boldsymbol{p}} J^T$$

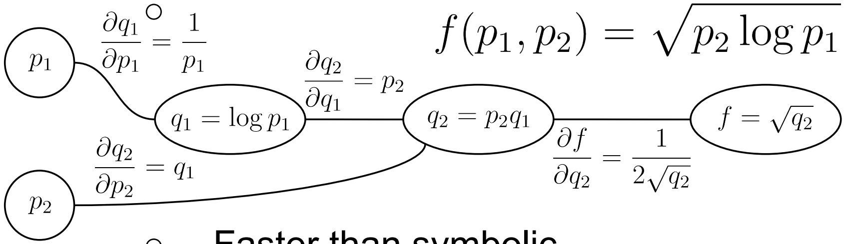
Cross-Checks

- Sampling-based error propagation through QC’s agree
- Hessian corrections to linalg operations small
- Two-sided fit errors match symmetric

⇒ Linear approx seems reasonable so far

Implementation

- Auto-differentiation (`gvar`) tracks jacobians



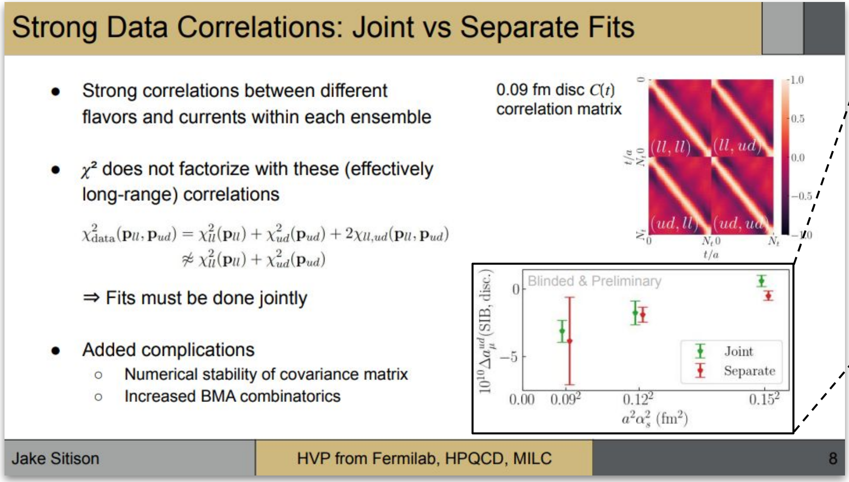
$$f(p_1, p_2) = \sqrt{p_2 \log p_1}$$

  - Faster than symbolic
  - More accurate than finite difference
- Wrapper written to extend to complex numbers
  - Needed for subduction to definite FV irreps
  - Cross-checked with `pyerrors`

Other Example of Bias in Separate Fits

From my talk at Lattice 2025:

Toward sub-percent precision in muon g − 2:
Isospin breaking corrections to HVP



Strong Data Correlations: Joint vs Separate Fits

- Strong correlations between different flavors and currents within each ensemble
- χ^2 does not factorize with these (effectively long-range) correlations

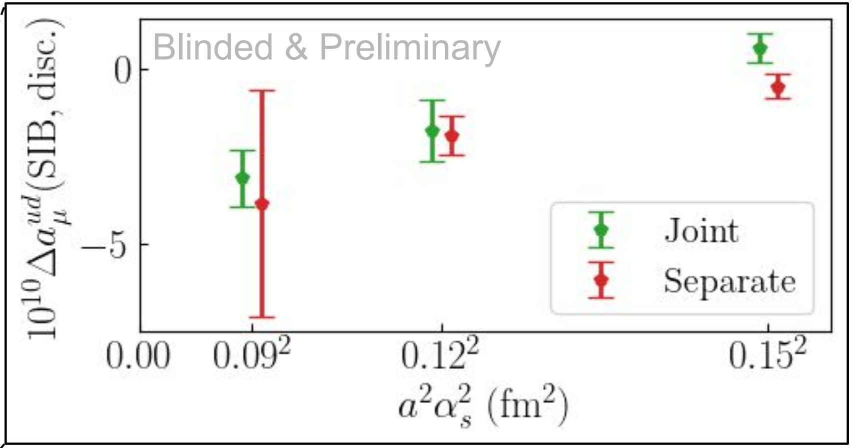
$$\chi^2_{\text{data}}(\mathbf{p}_{ll}, \mathbf{p}_{ud}) = \chi^2_{ll}(\mathbf{p}_{ll}) + \chi^2_{ud}(\mathbf{p}_{ud}) + 2\chi_{ll,ud}(\mathbf{p}_{ll}, \mathbf{p}_{ud})$$
$$\not\approx \chi^2_{ll}(\mathbf{p}_{ll}) + \chi^2_{ud}(\mathbf{p}_{ud})$$

⇒ Fits must be done jointly

- Added complications
  - Numerical stability of covariance matrix
  - Increased BMA combinatorics

0.09 fm disc $C(t)$ correlation matrix

Jake Sitison | HVP from Fermilab, HPQCD, MILC | 8



- Independent fits can over- or under-estimate uncertainties or introduce bias
- Difficult to predict behavior *a priori*