Real Time GEVP on Quantum Hardware

- **Valery Simonyan**

Paulo Bedaque, Gregory Ridgeway

University of Maryland, College Park

Extracting low-lying spectra in real time

- Want to know the gap? In Euclidean time just evolve to isolate

$$\langle 0|O(\tau)O^\dagger(0)|0\rangle = \sum_n |O_{0n}|^2 e^{-(E_n-E_0)\tau} \to |O_{01}|^2 e^{-(E_1-E_0)\tau}$$

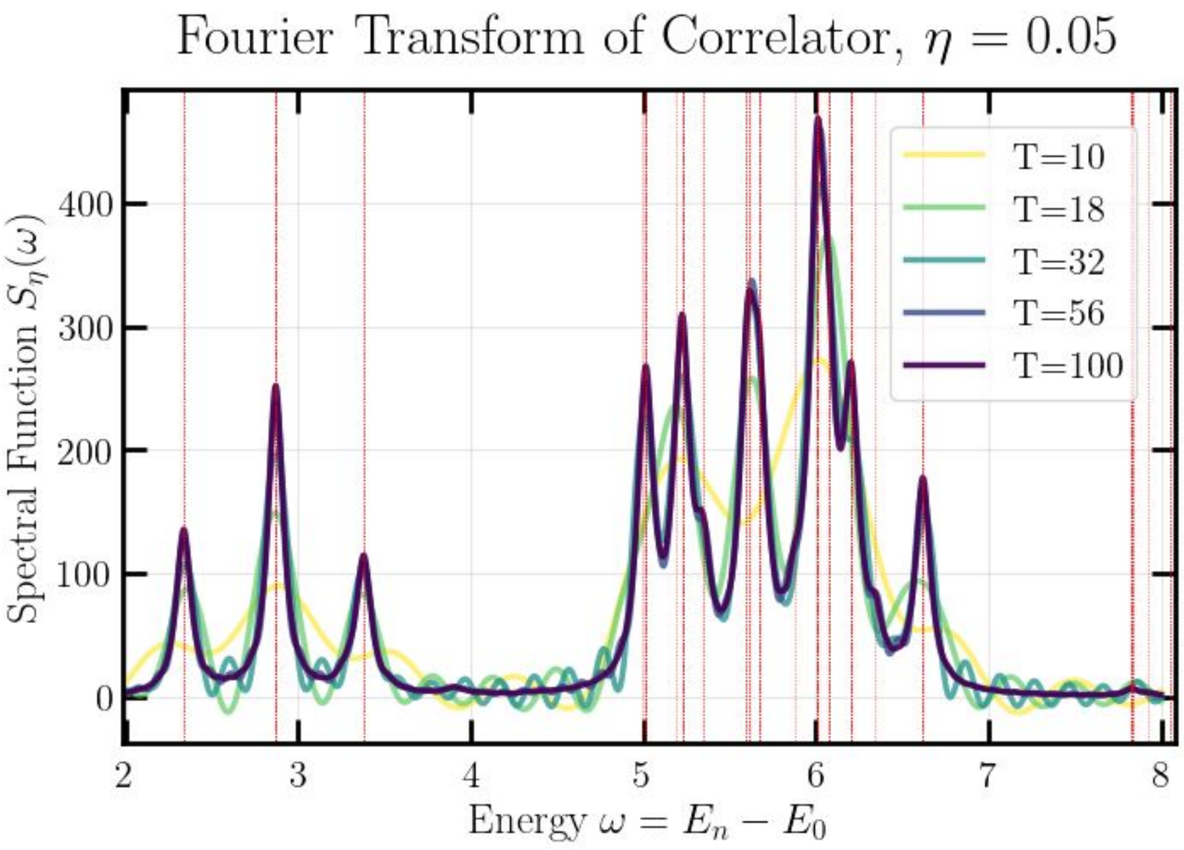
- In real time (on a quantum computer) however…

$$\langle 0|O(t)O^\dagger(0)|0\rangle = \sum_n |O_{0n}|^2 e^{-i(E_n-E_0)t} \to \sum_n |O_{0n}|^2 e^{-i(E_n-E_0)t}$$

- The solution:

$$C_{ij}(t) = \langle 0|O_i(t)O_j^\dagger(0)|0\rangle = \sum_n (O_i)_{0n}(O_j)^*_{0n} e^{-i(E_n-E_0)t}$$

Why don't you just Fourier transform?



$$S_\eta(\omega) = 2\,\mathrm{Re}\int_0^\infty dt\, e^{i\omega t - \eta t}\, \mathrm{Tr}\, C(t)$$

- Long time to resolve the peaks
- Noise will destroy the signal
- Deeper circuits are expensive and sensitive to noise
- **~18x** longer time evolution to recover same results that will be shown in a couple minutes, more for larger volume

How to do GEVP (Lüscher-Wolff) in Real Time

- Choose operator set with overlap onto desired state

$$|n\rangle \approx \sum c_i O_i |0\rangle$$

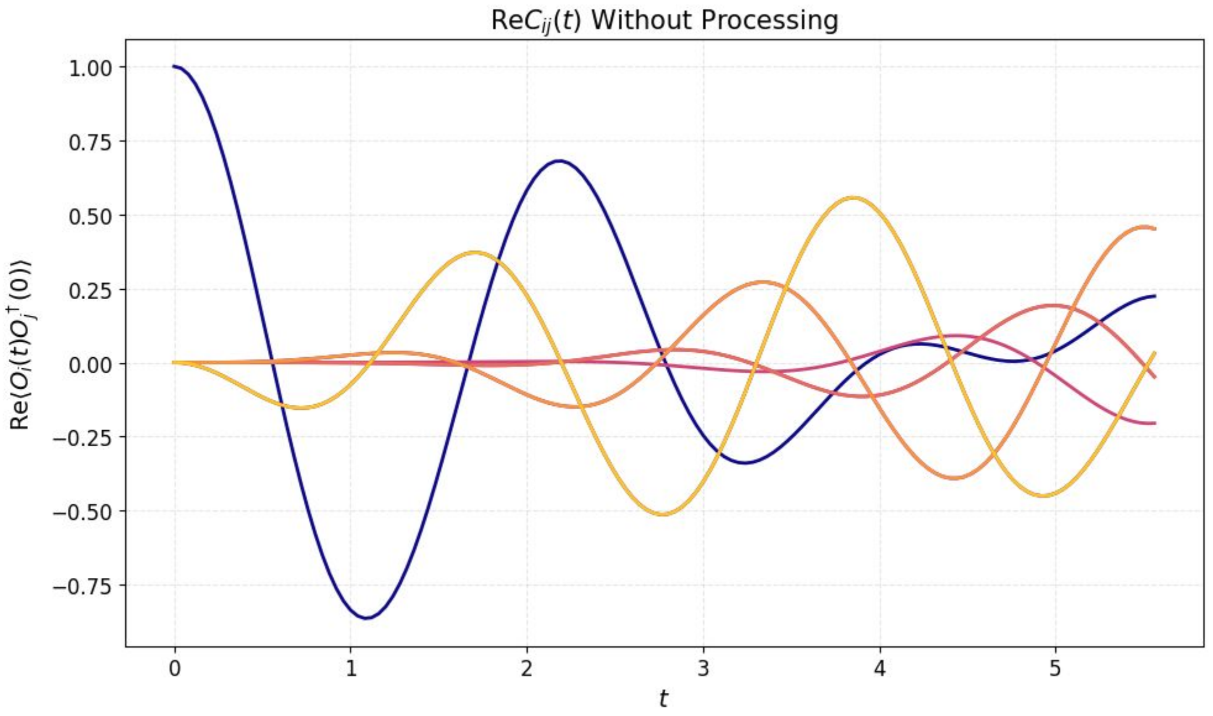
- Evaluate matrix from last slide, find eigenvalues

$$C^{-1}(0)C(t)w_k = (\frac{1}{Q^\dagger}U_{\text{in}}(t)Q^\dagger + \frac{1}{QQ^\dagger}PU_{\text{out}}(t)P^\dagger)w_k = \lambda_k(t)w_k$$

- Get energy, errors die off with 1/t and from dephasing

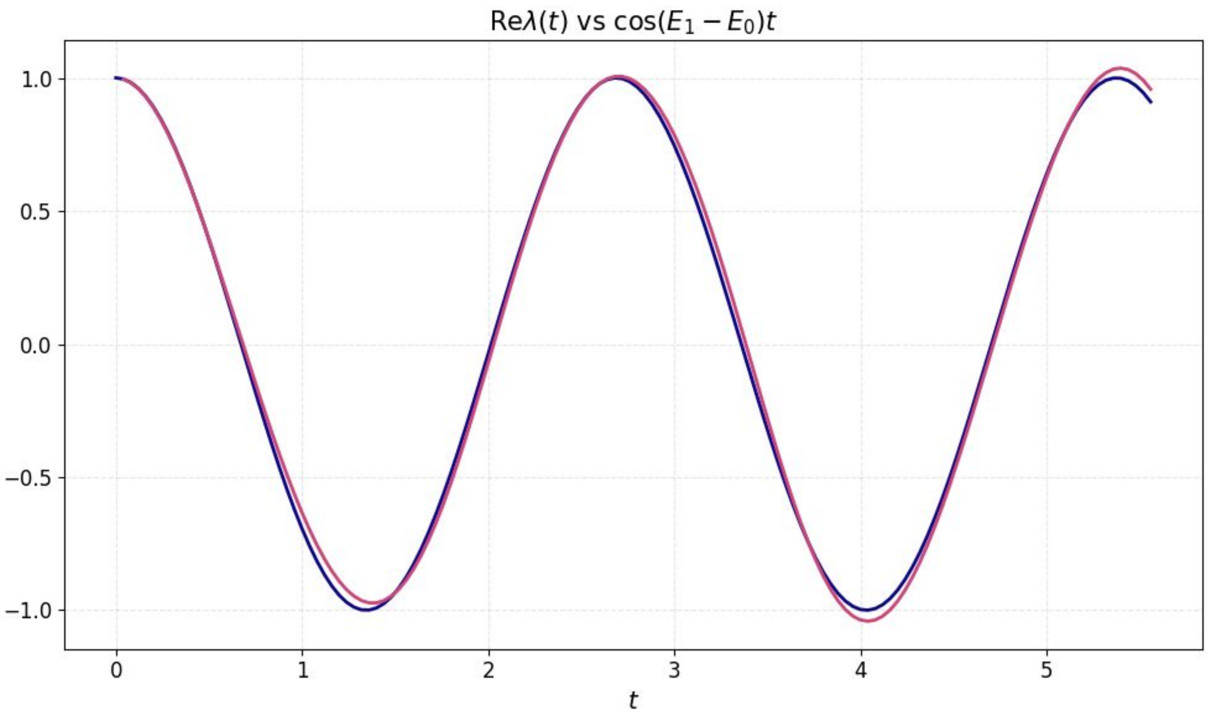
$$E^k_{\text{eff}}(t) = \frac{i}{t}\ln\lambda_k(t) = E_k + \frac{i}{t}\mathcal{O}((Q^{-1}P)^2)$$

Before Applying Lüscher-Wolff



- Each correlator contains a superposition of signals
- Lots of interference
- Energy levels are not cleanly separated

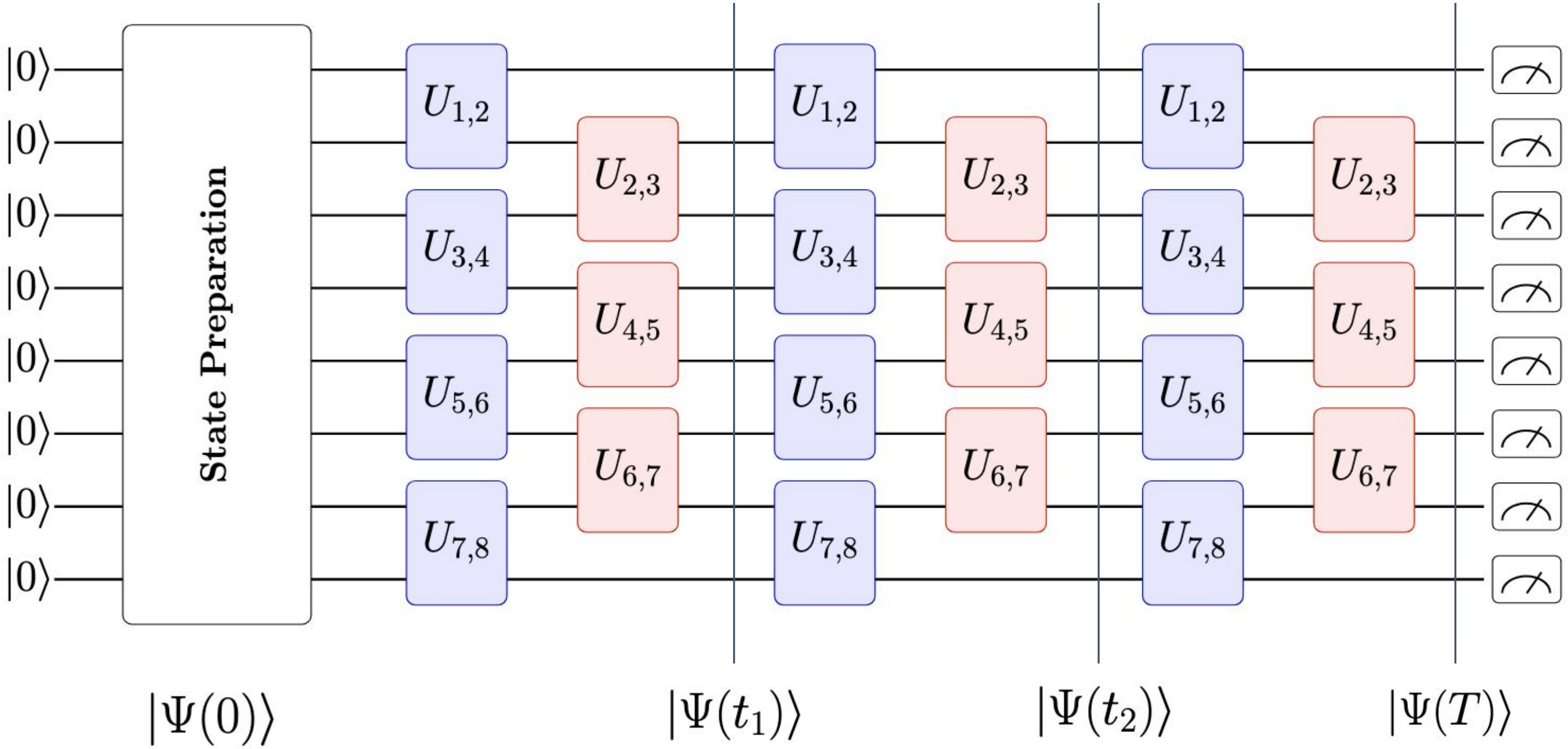
After Applying Lüscher-Wolff



- GEVP takes linear combination of operators isolating individual states
- Get single frequency oscillation, energy can be extracted

Quantum Computing 101

- Encode your system into qubits
- Prepare a state
- Evolve it
- Measure the expectation value of an operator

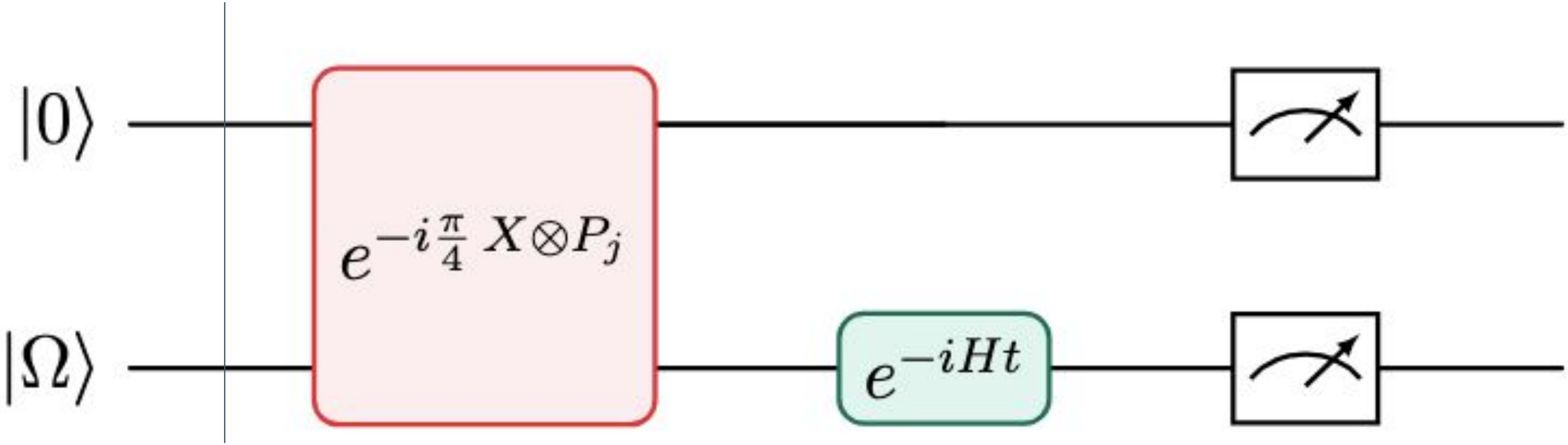


Changes on a Quantum Computer

- Only act on qubits with a unitary operation
- We decompose our operator basis into Pauli strings: $O_i = \sum c_n^{(i)} P_n$
- Directly measure Pauli string correlators: $\langle P_n(t) P_m(0) \rangle$
- Take linear combinations getting correlator matrix:

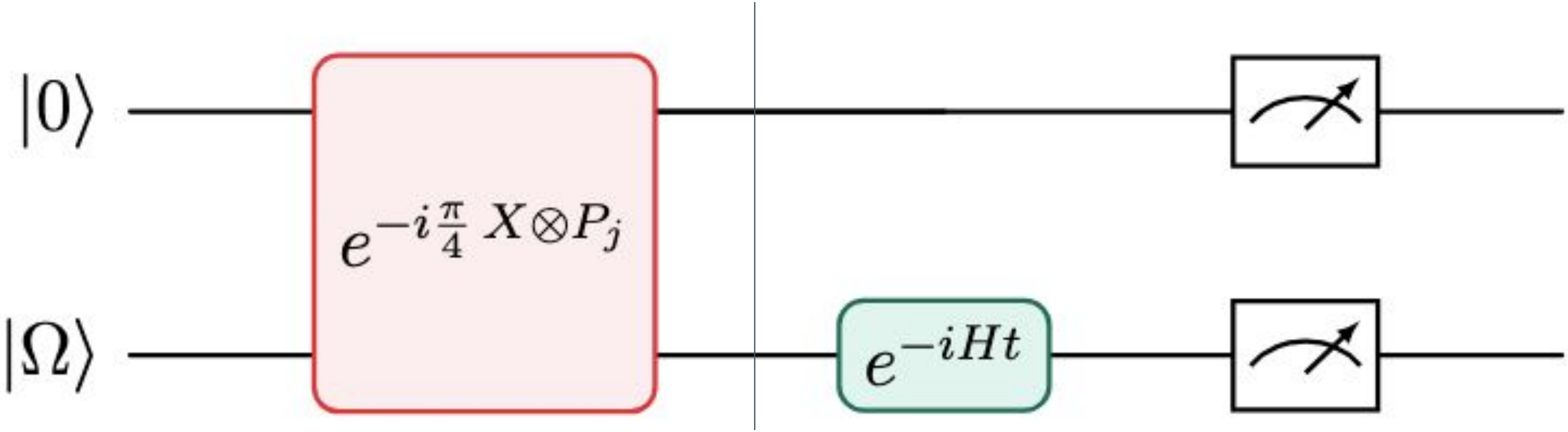
$$C_{ij}(t) = \langle O_i(t) O_j^\dagger(0) \rangle = \sum c_n^{(i)} c_m^{(j)*} \langle P_n(t) P_m(0) \rangle$$

How do you measure $\mathbf{C_{ij}(t) = \langle 0|O_i(t)O_j^\dagger(0)|0\rangle}$?



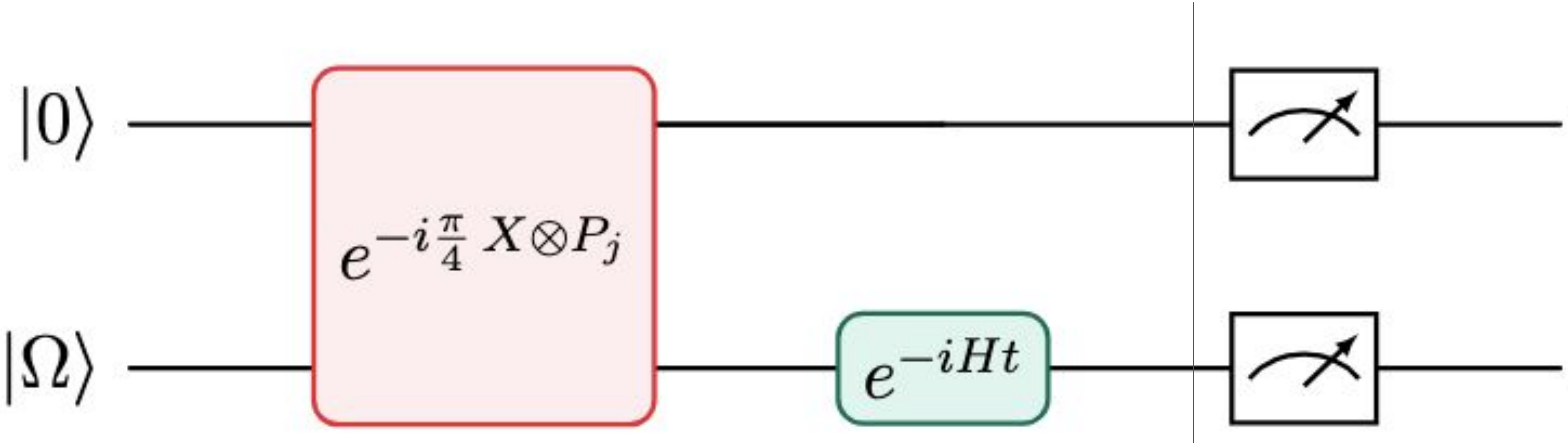
$|0\rangle \otimes |\Omega\rangle$

How do you measure $\mathbf{C_{ij}(t) = \langle 0|O_i(t)O_j^\dagger(0)|0\rangle}$?



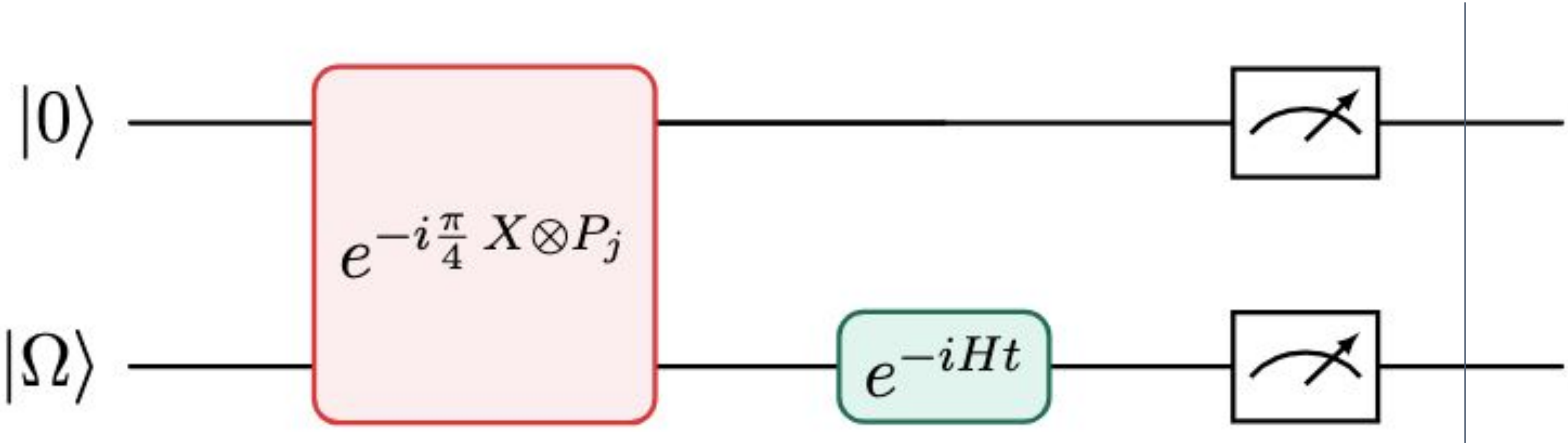
$$\frac{1}{\sqrt{2}}(|0\rangle \otimes |\Omega\rangle - i|1\rangle \otimes P_j|\Omega\rangle)$$

How do you measure $\mathbf{C_{ij}(t) = \langle 0 | O_i(t) O_j^\dagger(0) | 0 \rangle}$?



$$\frac{1}{\sqrt{2}}\left(|0\rangle \otimes e^{-iHt}|\Omega\rangle - i|1\rangle \otimes e^{-iHt} P_j |\Omega\rangle\right)$$

How do you measure $\mathbf{C_{ij}(t) = \langle 0|O_i(t)O_j^\dagger(0)|0\rangle}$?



- Measure interference terms for real and imaginary part of C(t):

$$\langle X \otimes P_i \rangle \qquad \langle Y \otimes P_i \rangle$$

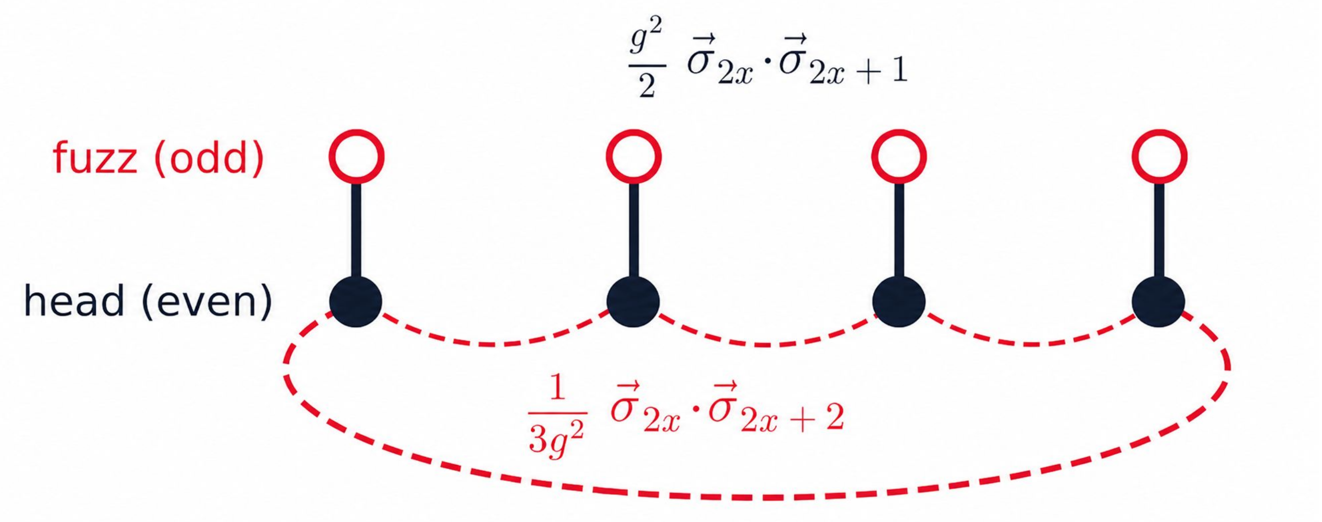
Quantum Algorithm Details

- Naively if we take operator set size M must measure $\mathcal{O}\left(M^2\right)$ circuits
- However may measure multiple $\langle P_i(t)P_j(0)\rangle$ simultaneously if $P_i(t)$ are qubit wise commuting
- Utilizing symmetries of Hamiltonian being studied significantly reduce circuit count further
- Partially trade off long simulation times in Fourier transform with more circuits

A case study: the fuzzy σ-model

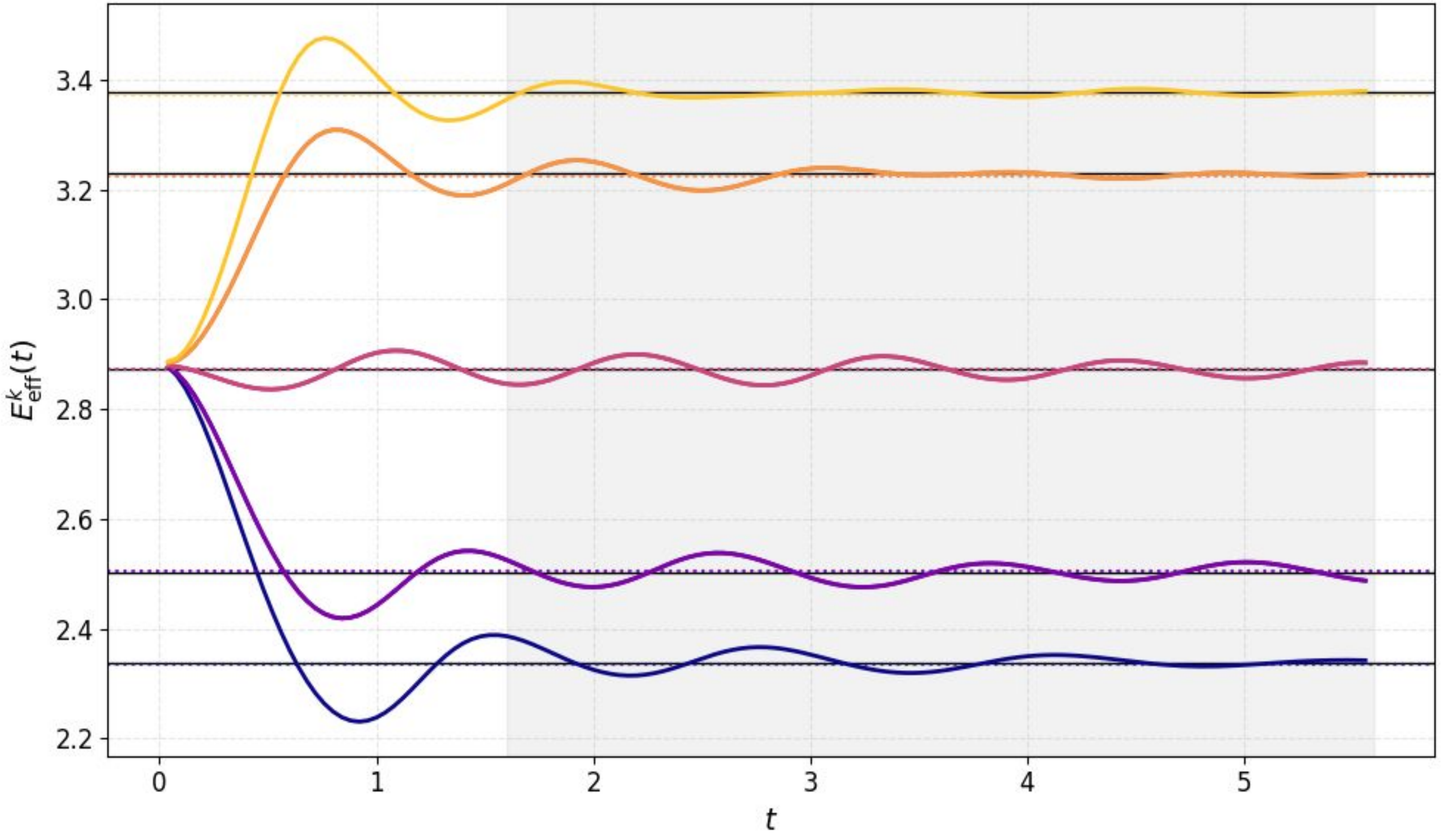
$$H = \frac{g^2}{2}\sum_{x=0}^{L-1} \vec{\sigma}_{2x}\cdot\vec{\sigma}_{2x+1} + \frac{1}{3g^2}\sum_{x=0}^{L-1}\vec{\sigma}_{2x}\cdot\vec{\sigma}_{2x+2}$$

- Small g limit = antiferromagnetic Heisenberg chain
- Large g limit = factorizable antiferromagnetic spin pairs
- Use large g limit, ground state prep is cheaper
- O(3), translation invariance significantly reduce circuit count



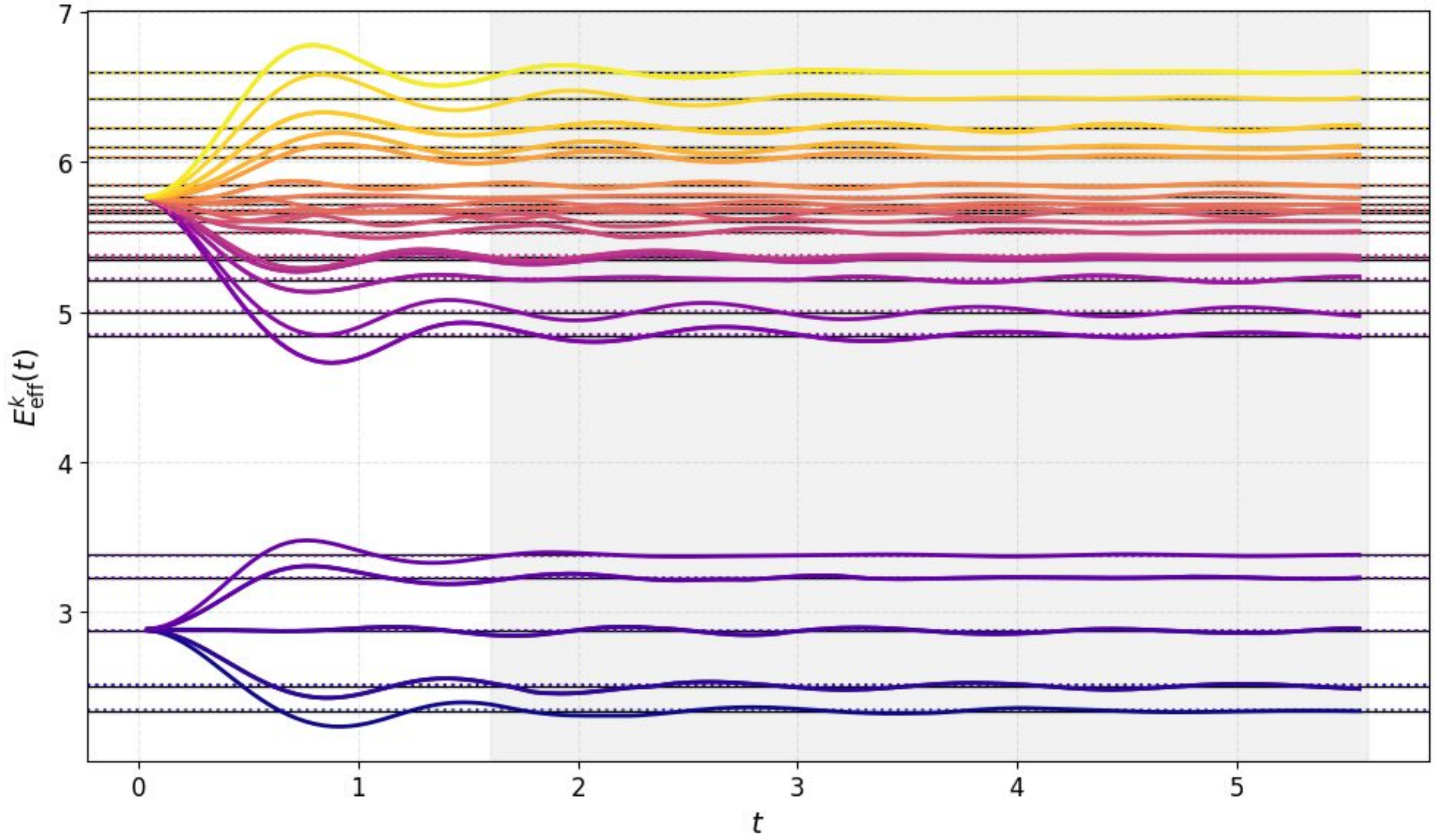
$$|\Omega\rangle = \frac{1}{2^{\frac{L}{2}}}(|01\rangle - |10\rangle)\otimes(|01\rangle - |10\rangle)\otimes \dots \otimes(|01\rangle - |10\rangle)$$

Classical Exact Simulation - L=8 - $\mathbf{P_i \in \{Z_1, Z_3, \ldots\}}$



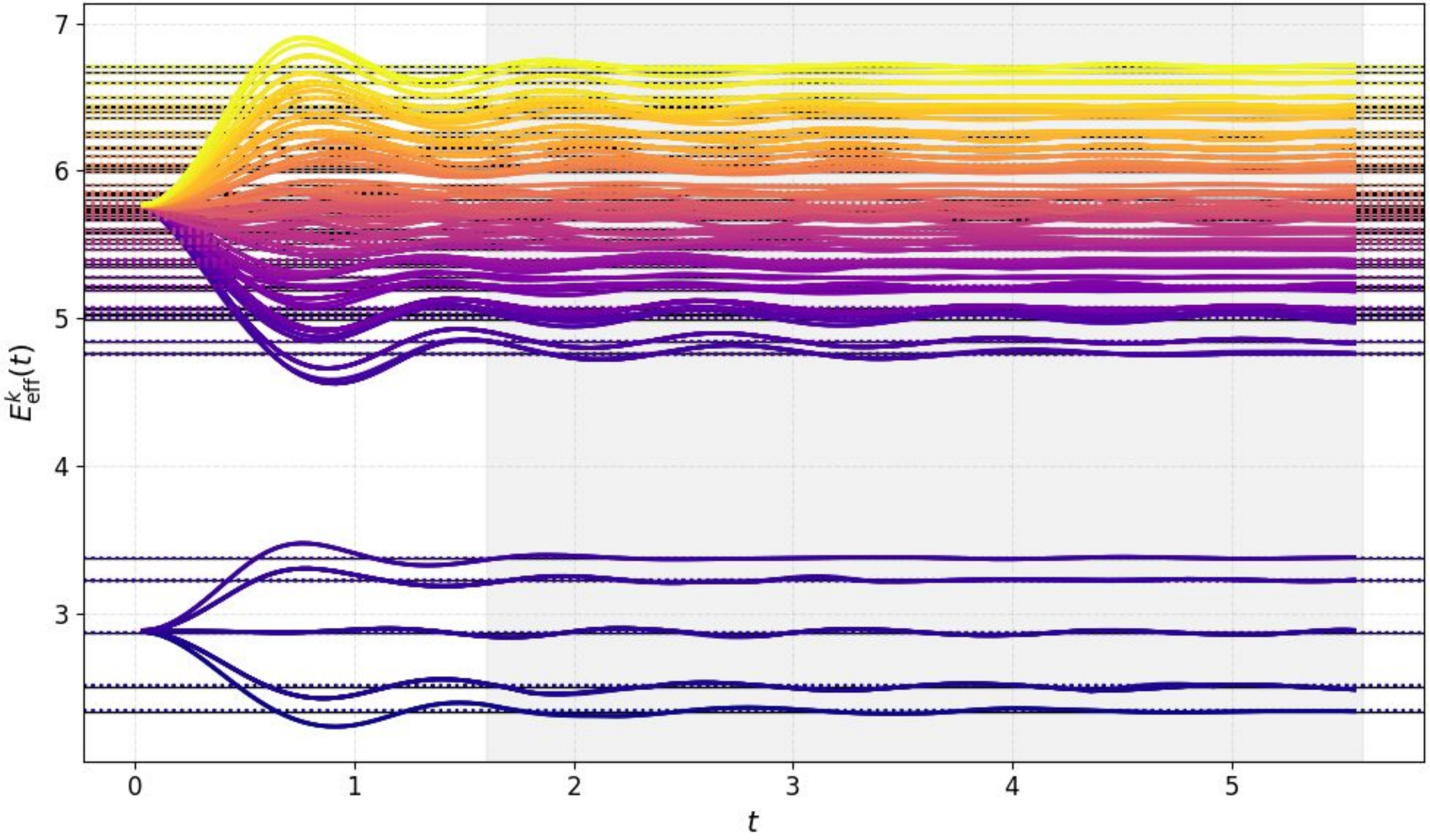
8 operators, plateau at correct energy levels within **0.1 %** of exact diagonalization

Classical Exact Simulation - Linear combinations of 1 and 2 Pauli $\mathbf{P_i}$ with l = 1, m = 0



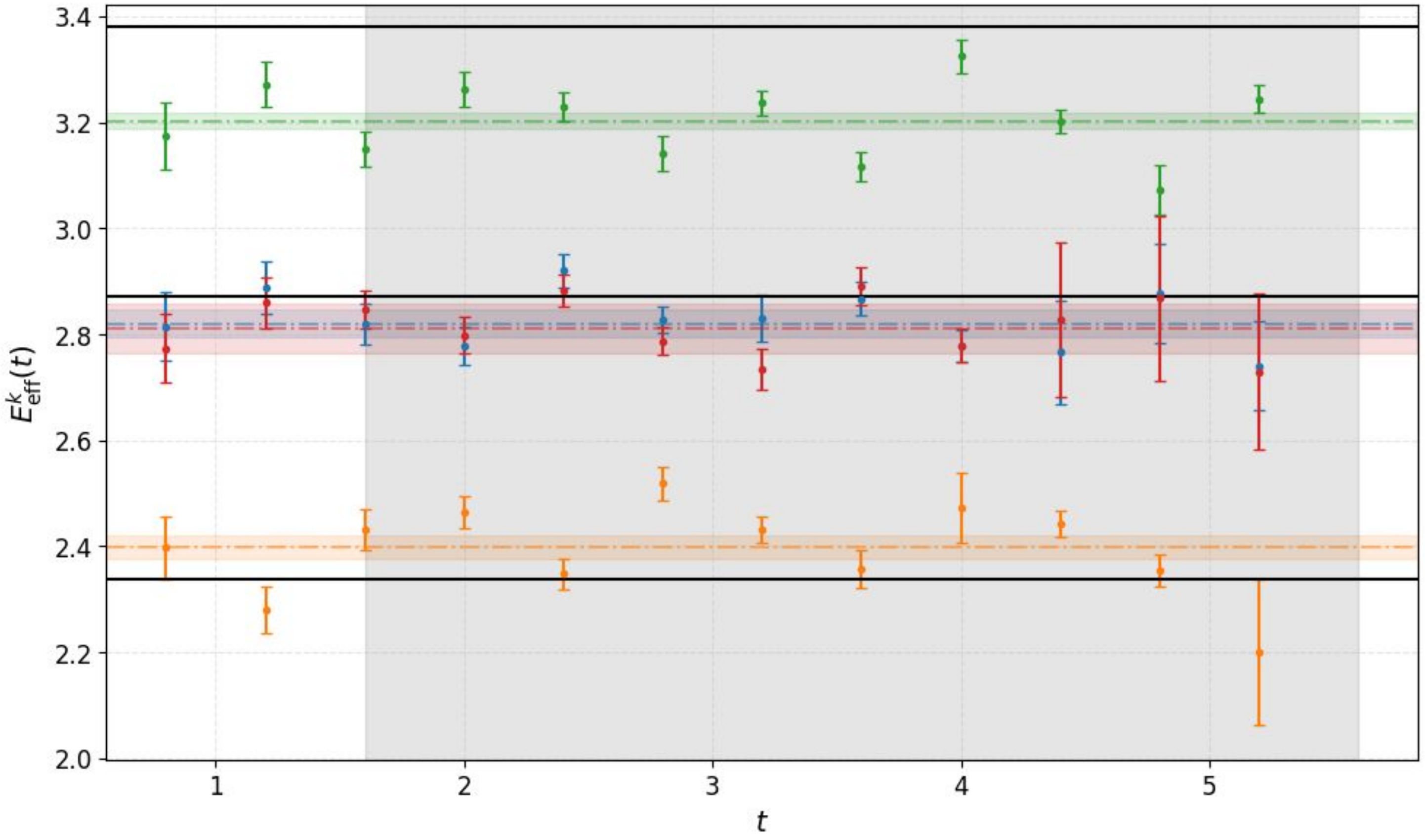
36 operators, plateau at correct energy levels within **0.3 %** of exact diagonalization

Classical Exact Simulation - All 1 and 2 Pauli $\mathbf{P_i}$



276 operators, plateau at correct energy levels within **0.5 %** of exact diagonalization

IonQ Forte Enterprise - L=4 - $\mathbf{P_i} \in \{\mathbf{Z_1}, \mathbf{Z_3}, \ldots\}$



- 14 time steps, dt =0.4
- Estimate fit uncertainties with bootstrap (shaded regions)
- Results within 5.2% of exact diagonalization
- Systematic error from noise and trotterization

In summary

- Systematic improvement in spectrum resolution with operator basis size
- No noise can recover the low-lying spectrum with a truncated operator basis within sub-percent precision.
- Current NISQ hardware can recover low-lying spectra within a couple percent on sufficiently small systems.

Hardware numbers (L = 4, Z basis)

state	E (GEVP)	E (exact)	δ (%)	|Δ|/σ
k = π (single)	2.399(21)	2.337	+2.6	2.9
k = ±π/2 (double)	2.820(26)	2.870	−1.7	1.9
k = ±π/2 (double)	2.810(46)	2.870	−2.1	1.3
k = 0 (single)	3.203(14)	3.380	−5.2	12.6

- Same signed pattern as the noise model — bottom pulled up, top pulled down: the compression signature reproduces on hardware.
- k = 0: smallest statistical error of the four, largest deviation → a systematic, not shot noise. Trotter ($\Delta t = 0.4$) + coherent gate error.