Tensor-polarised structure functions of the ρ meson from lattice QCD using the Feynman–Hellmann method

Nabil Humphrey, James Zanotti, Ross Young

CSSM, Adelaide University

QCDSF Collaboration

Lattice 2026 • University of Maryland

Why tensor-polarised structure functions?

- A spin-1 DIS target has a hadron tensor with four extra structure functions b_1–b_4 with no spin-$\frac{1}{2}$ analogue [Hoodbhoy, Jaffe, Manohar, NPB 312 (1989)]
- Parton model: $b_1(x) = \frac{1}{2}\left[q^0(x) - q^{+1}(x)\right]$, the difference between target helicity states. Vanishes for spin-$\frac{1}{2}$ constituents at rest or in a relative S-wave
- Close–Kumano sum rule $\int_0^1 b_1(x)\,dx = 0$ absent sea tensor polarisation [Close, Kumano, PRD 42 (1990)]

Experiment vs. the lattice

- HERMES: first b_1 on the deuteron, coming out *larger* than conventional-nuclear expectations [PRL 95 (2005)]
- Deuteron b_1 is small (D-state suppressed); JLab & EIC target it
- Lattice reaches any spin-1 target

The spin-1 vector meson

- Tuning mass spans ρ (more relativistic quarks) to J/ψ (less relativistic quarks).
- Expect $b_1(x)$ to range from $O(F_1)$ for the ρ, to significantly suppressed for the J/ψ (and the deuteron).

Deep-inelastic scattering off a spin-1 target

Hadron tensor with polarisation indices α, α': [HJM 1989]

$$W_{\mu\nu}^{\alpha\alpha'} = \underbrace{-\hat{g}_{\mu\nu} F_1^{\alpha\alpha'} + \frac{\hat{p}_\mu \hat{p}_\nu}{\nu} F_2^{\alpha\alpha'}}_{\text{spin-averaged}} + \underbrace{\frac{i\epsilon_{\mu\nu\lambda\sigma} q^\lambda}{\nu} \left[s^\sigma g_1^{\alpha\alpha'} + \frac{\nu\, s^\sigma - (s\cdot q)\, p^\sigma}{\nu} g_2^{\alpha\alpha'} \right]}_{\text{vector pol.}}$$

$$+ \underbrace{-b_1^{\alpha\alpha'} r_{\mu\nu} + \tfrac{1}{6} b_2^{\alpha\alpha'} (s+t+u)_{\mu\nu} + \tfrac{1}{2} b_3^{\alpha\alpha'} (s-u)_{\mu\nu} + \tfrac{1}{2} b_4^{\alpha\alpha'} (s-t)_{\mu\nu}}_{\text{tensor polarised (new for spin-1)}}$$

- $\hat{g}_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu / q^2$, $\hat{p}_\mu = p_\mu - \nu q_\mu / q^2$, $\nu = p \cdot q$
- r, s, t, u: tensors from p^μ, q^μ, E^μ
- b_1, b_2 twist-2; b_3, b_4 twist-4

Compton amplitude, crossing & dispersion relations

Forward Compton amplitude $T_{\mu\nu}(p,q)$ carries the same decomposition with $b_i \to \mathcal{B}_i(\omega, Q^2)$;
$\omega = 2p \cdot q/Q^2$, $\mathrm{Im}\,\mathcal{B}_i = 2\pi b_i$.

- Crossing $T_{\mu\nu}(p,q) = T_{\nu\mu}(p,-q)$: $\mathcal{B}_1$ *even* in ω, $\mathcal{B}_{2,3,4}$ *odd*
- Dispersion relations $\Rightarrow$ for $|\omega| < 1$ a Taylor series in ω with structure-function moments as coefficients

$$\boxed{\mathcal{B}_1(\omega) = 2\omega^2(m_1^{(1)} + \omega^2 m_1^{(2)} + \cdots)} \qquad \boxed{\mathcal{B}_i(\omega) = 4\omega(m_i^{(1)} + \omega^2 m_i^{(2)} + \cdots)}\ (i{=}2,3,4)$$

- $\mathcal{B}_1$ is even in $\omega \Rightarrow$ starts at ω^2; $m_1^{(0)} = 0$ by Close–Kumano

The Feynman–Hellmann method

Perturb the fermion action with a background field [Can et al., PRD 102 (2020)]:

$$S(\lambda) = S + \lambda \int d^4 z \, (e^{i\boldsymbol{q}\cdot\boldsymbol{z}} + e^{-i\boldsymbol{q}\cdot\boldsymbol{z}}) \, \mathcal{J}_\mu(z) \, , \qquad \mathcal{J}_\mu = Z_V \bar{q} \gamma_\mu q \, .$$

The second-order energy shift gives the forward Compton amplitude:

$$\left. \frac{\partial^2 E_\lambda(\boldsymbol{p})}{\partial \lambda^2} \right|_{\lambda=0} = - \frac{T_{\mu\mu}(\boldsymbol{p}, \boldsymbol{q}) + T_{\mu\mu}(\boldsymbol{p}, -\boldsymbol{q})}{2E(\boldsymbol{p})} \, .$$

- No operator insertion needed; Compton tensor from 2-point functions
- Established for nucleon, pion targets [QCDSF/CSSM 2020–2025]
- This work: extension to a *spin-1* target and its *tensor* polarisation

CSSM @ Lattice 2026

Other Feynman–Hellmann contributions from Adelaide:

- **Ian Van Schalkwyk**, *The Electromagnetic Form Factor of the Pion at Large* Q^2
  Talk, Jul 27, 16:10: Hadronic and nuclear spectrum
- **Jordan Mckee**, *Nachtmann moment of the parity-violating structure function from FH*
  Talk, Jul 29, 11:30: Structure of hadrons and nuclei
- **Mischa Batelaan**, *FH determination of* $K \to \pi$ *transition matrix elements*
  Poster: Quark and lepton flavor physics

Isolating the tensor sector: spin-1 Feynman–Hellmann

Measure $T_{\mu\nu}^{\alpha\alpha'}(\boldsymbol{p},\boldsymbol{q})$ for each polarisation $\alpha \in \{x,y,z\}$ and current index μ. Each is a linear combination:

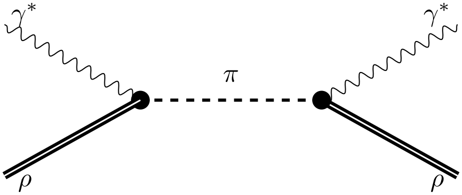
$$T_{\mu\mu}^{\alpha\alpha} = -\hat{g}_{\mu\mu}\mathcal{F}_1 + \frac{2\hat{p}_\mu^2}{Q^2}\frac{\mathcal{F}_2}{\omega} + c_1^{(\alpha,\mu)}\mathcal{B}_1 + c_2^{(\alpha,\mu)}\mathcal{B}_2 + c_3^{(\alpha,\mu)}\mathcal{B}_3 + c_4^{(\alpha,\mu)}\mathcal{B}_4 \,.$$

- Set diagonal: $\alpha{=}\alpha'$, $\mu{=}\nu$
- Stack $\alpha \in \{x,y,z\}$, $\mu \in \{1,2,3,0\}$ $\Rightarrow$ a 12×6 system
- $\mathcal{B}_1$ directly leverages polarisation asymmetry (proportional to $T_{33}^{xx} - T_{33}^{yy}$ at $\boldsymbol{p}=0$)
- $\mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_4$ need $\boldsymbol{p} \neq 0$ ($\Delta = p_x q_y - p_y q_x \neq 0$)

Key kinematic requirement

Vary $\boldsymbol{p}$ (hence ω, with $|\omega| < 1$) to map out $\mathcal{B}_i(\omega)$ and extract the leading moments. Current $\boldsymbol{q}$ in-plane ($q_z = 0,\ q_{x,y} \neq 0$) with $q_x \neq q_y$.

Pion pole contribution



Anomalous $\rho\pi\gamma$ vertex

The problem for Feynman–Hellmann

- Pion can be near on-shell: $E_\pi(\boldsymbol{p} \pm \boldsymbol{q}) - E_\rho(\boldsymbol{p}) \approx 0.1$–$0.25$ for $m_\pi/m_\rho = 0.90$–0.98
- FH relies on Euclidean time suppression of certain time-orderings, which fails when energy gap is small
- This contribution not in OPE for $b_i(x)$

Solution: selection rules

- Levi-Civita structure $\Rightarrow$ 7 of 12 components vanish for in-plane $\boldsymbol{q}$, $\boldsymbol{p}$
- Use only pion-free rows $\Rightarrow$ clean extraction
- A rank-5 coefficient matrix with 6 structure functions prevents per-ω solves, so parameterise $\mathcal{B}_i(\omega) = 4\omega(m_i^{(1)} + \omega^2 m_i^{(2)} + \cdots)$ and solve for $m_i^{(j)}$

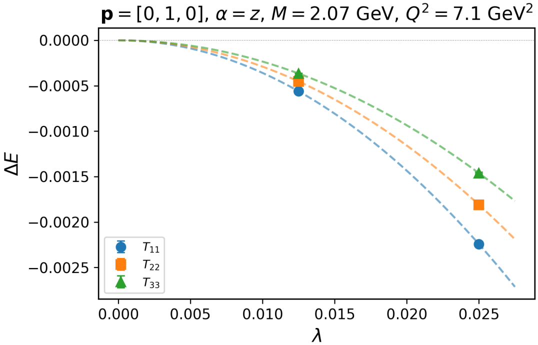
Lattice setup

Lattice	$L^3 \times T = 32^3 \times 64$, $a = 0.074(2)$ fm
Action	$N_f = 2{+}1$ clover, $\beta = 5.50$
m_π	0.467(12) GeV
N_{cfg}	1763
$\boldsymbol{q}$	$[3,1,0]\,(2\pi/L)$, $[4,1,0]\,(2\pi/L)$, $[5,1,0]\,(2\pi/L)$, $[6,2,0]\,(2\pi/L)$
Q^2	$2.8,\ 4.6,\ 7.1,\ 11.0\ \mathrm{GeV}^2$
$\boldsymbol{p}$	12 in-plane momenta per $\boldsymbol{q}$
M	$1.46,\ 2.07,\ 2.61,\ 3.10$ GeV
λ	$\pm 0.0125,\ \pm 0.025$
Z_V	0.8622(84)

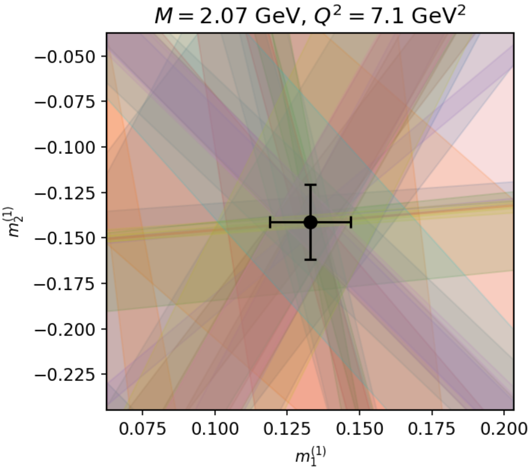
Strategy

- At $\boldsymbol{p}{=}0$ only b_1, $b_2{-}3b_3$ appear; b_4 needs $\boldsymbol{p}{\neq}0$
- $q_x \neq q_y$, in-plane $\Rightarrow$ clean-row analysis
- 12-parameter Bayesian fit

Energy shifts in practice



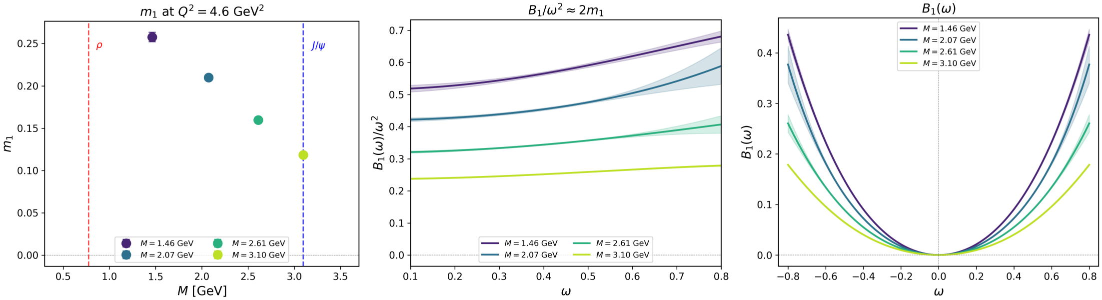
$\Delta E(\lambda)$ for T_{11}, T_{22}, T_{33} at $\boldsymbol{p} = [0, 1, 0]$.
Quadratic fit extracts $T_{\mu\mu}$.



Global moment fit: constraint bands from 84 clean-row measurements across 12 momenta.

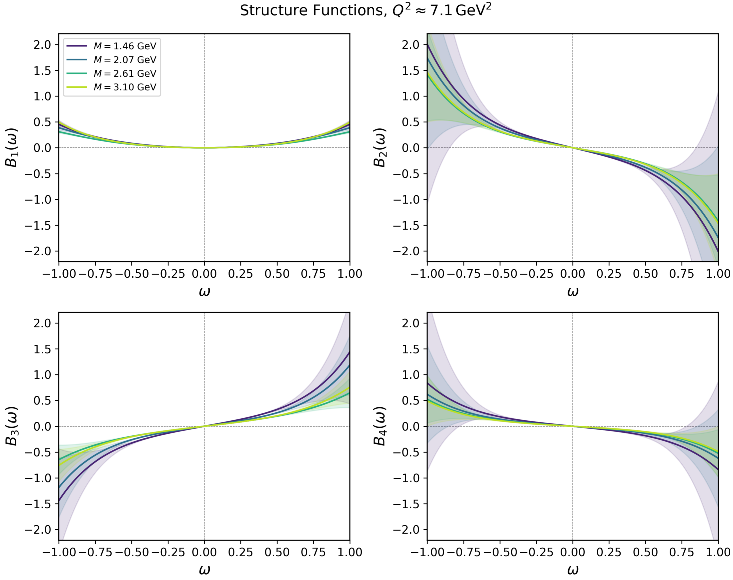
correlator $\to$ effective energy $\to$ λ fit $\to$ $T^{(\alpha)}_{\mu\mu}$ $\to$ clean-row global moment fit.

Tensor-polarised Compton amplitude of spin-1 vector mesons



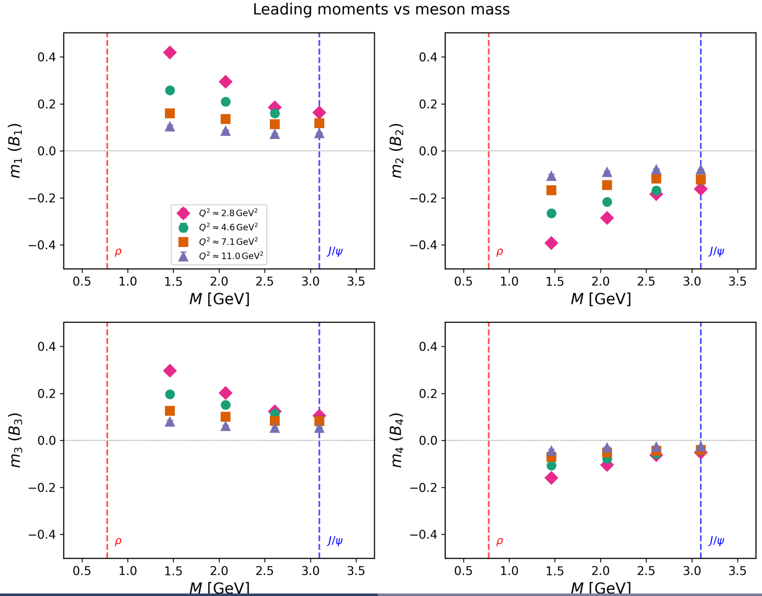
- First lattice determination of $\mathcal{B}_1(\omega, Q^2)$ for spin-1 vector mesons
- $\mathcal{B}_1/\omega^2 \approx 2m_1$ shows mass dependence directly
- At M=2.07 GeV: $m_1 = +0.21(1)$, $\chi^2/\text{dof} = 1.0$

All four tensor structure functions



- $\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_4$ extracted for 4 meson masses at $\boldsymbol{q} = [5, 1, 0]$
- Even-odd structure in ω is manifest.
- OPE convergence via soft Bayesian priors

Mass dependence of the moments

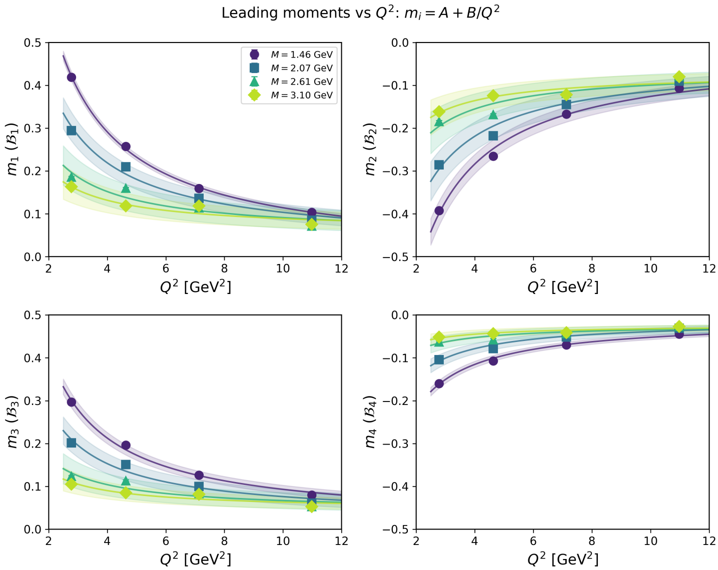


- Four Q^2 values: 2.8, 4.6, 7.1, 11.0 GeV^2
- Consistent mass dependence across Q^2
- $|m_1| \approx |m_2| > |m_3| > |m_4|$
- Hierarchy persists across all Q^2

Observation

Tensor polarisation signal is suppressed for heavy vector mesons.

Q^2 dependence of the moments

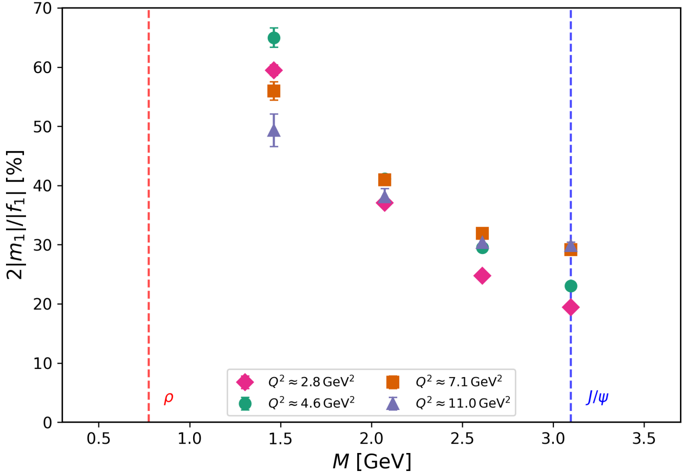


- Moments at four Q^2 values (2.8–11.0 GeV^2)
- OPE-motivated fits: $m_i = A + B/Q^2$
- Steeper Q^2 evolution for light M; limited lever arm to constrain asymptotic value
- All four moments evolve similarly

Observation

Q^2 evolution steepens toward lighter M; OPE form describes data well.

Tensor polarisation relative to spin-averaged



- Ratio of leading moments: $2|m_1|/|f_1|$ (tensor / spin-avg)
- 50–65% for lightest meson
  20–30% near J/ψ
- Mild Q^2 dependence

Key result

Tensor polarisation is comparable to the spin-averaged structure for more relativistic vector mesons, and suppressed at higher masses.

Pathway to other spin-1 targets

- Framework is target-agnostic: any spin-1 interpolator (ρ, J/ψ; deuteron)
- Deuteron is of experimental interest (HERMES, JLab A_{zz}, EIC)
- Same methodology applies; deuteron has no lighter-state contamination issues

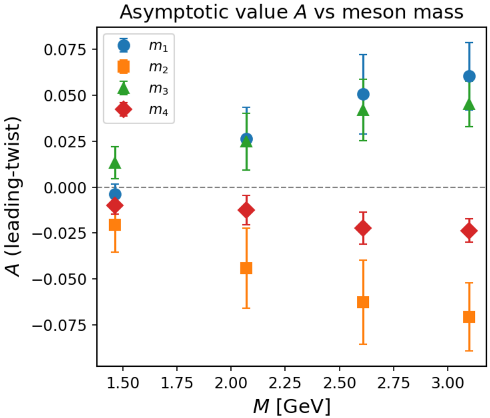
Summary

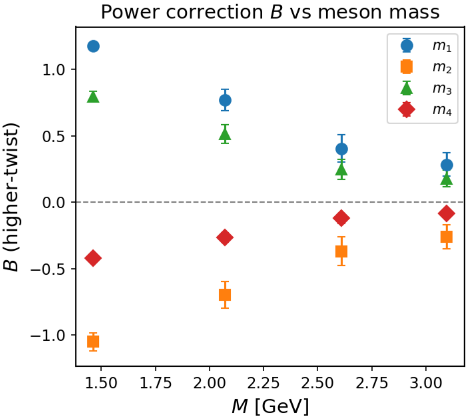
1. **Framework:** tensor structure functions of *any* spin-1 target from the Compton amplitude via Feynman–Hellmann, with dispersive moment extraction
2. **First results:** $\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_4(\omega, Q^2)$ of spin-1 vector mesons across four Q^2 and four meson masses up to J/ψ
3. **Clear signal:** Bayesian moment fits with $\chi^2/\mathrm{dof} \approx 1$; consistent mass dependence across Q^2
4. **Leading moment trend:** $2|m_1|$ is $O(|f_1|)$ for lightest (most-relativistic) vector meson, and suppressed as meson mass increases to the J/ψ region
5. **Next:** Compare deuteron-like state structures

Thank you!

nabil.humphrey@adelaide.edu.au

Backup: Tensor moment OPE fit parameters





- Fits to $m_i = A + B/Q^2$ for each meson mass
- **Left:** Fitted A small for light M; grows with M
- **Right:** Slope $|B|$ largest for light M; decreases with M
- Steeper slope $\Rightarrow$ smaller intercept; limited Q^2 range to disentangle

Backup: Coefficient matrix for $T^{\alpha\alpha}_{\mu\mu}$

Full diagonal Compton tensor ($\mu \in \{0,1,2,3\}$, $\alpha \in \{x,y,z\}$):

$$T^{(\alpha)}_{\mu\mu} = \underbrace{-\hat{g}_{\mu\mu}\mathcal{F}_1 + \frac{2\hat{p}_\mu^2}{Q^2}\frac{\mathcal{F}_2}{\omega}}_{\text{spin-averaged}} + \underbrace{c_1^{(\alpha,\mu)}\mathcal{B}_1 + c_2^{(\alpha,\mu)}\mathcal{B}_2 + c_3^{(\alpha,\mu)}\mathcal{B}_3 + c_4^{(\alpha,\mu)}\mathcal{B}_4}_{\text{tensor-polarised}}$$

Tensor coefficients ($\Phi_\mu = \hat{p}_\mu$, $\xi^{(\alpha)} = (B^{(\alpha)})^2 - \nu^2\kappa/3$, $\sigma^{(\alpha)}_\mu = B^{(\alpha)}\Phi_\mu\hat{E}^{(\alpha)}_\mu$, $\mathcal{E}^{(\alpha)}_\mu = (\hat{E}^{(\alpha)}_\mu)^2$):

$$c_1^{(\alpha,\mu)} = -\frac{\xi^{(\alpha)}\hat{g}_{\mu\mu}}{\nu^2} \qquad c_2^{(\alpha,\mu)} = \frac{M^2\hat{g}_{\mu\mu}}{9\nu} + \frac{1}{3\nu}\left(\frac{\xi^{(\alpha)}\Phi_\mu^2}{\nu^2} + \frac{\sigma^{(\alpha)}_\mu}{\nu} + \mathcal{E}^{(\alpha)}_\mu - \frac{2\Phi_\mu^2}{3}\right)$$

$$c_3^{(\alpha,\mu)} = -\frac{M^2\hat{g}_{\mu\mu}}{3\nu} + \frac{\xi^{(\alpha)}\Phi_\mu^2}{\nu^3} - \frac{\mathcal{E}^{(\alpha)}_\mu}{\nu} + \frac{\Phi_\mu^2}{3\nu} \qquad c_4^{(\alpha,\mu)} = \frac{\xi^{(\alpha)}\Phi_\mu^2}{\nu^3} - \frac{\sigma^{(\alpha)}_\mu}{\nu^2} + \frac{\Phi_\mu^2}{3\nu}$$

Backup: dispersion relations

Compton amplitudes related to structure functions via dispersion integrals:

$$\mathcal{F}_1(\omega, Q^2) = 2\int_0^1 \frac{x\, F_1(x, Q^2)\, dx}{1 - \omega^2 x^2} = 2\sum_{n=0}^{\infty} \omega^{2n} \int_0^1 x^{2n+1} F_1(x)\, dx$$

$$\mathcal{B}_1(\omega, Q^2) = 4\int_0^1 \frac{x\, b_1(x, Q^2)\, dx}{1 - \omega^2 x^2} = 2\omega^2 \left(m_1^{(1)} + \omega^2 m_1^{(2)} + \cdots \right)$$

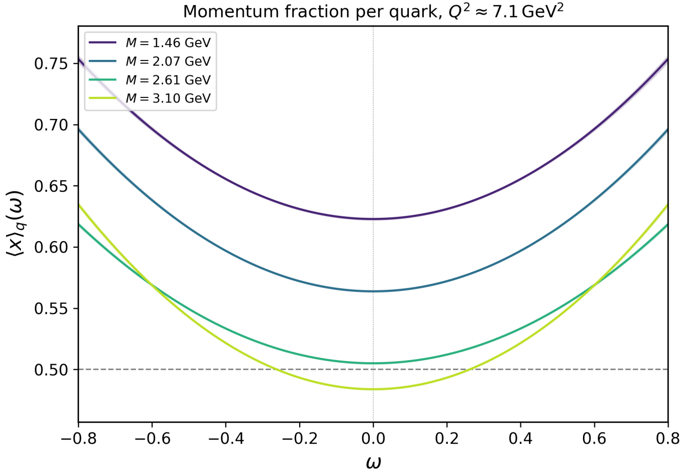
$$\mathcal{B}_{2,3,4}(\omega, Q^2) = 4\int_0^1 \frac{b_i(x, Q^2)\, dx}{1 - \omega^2 x^2} = 4\omega \left(m_i^{(1)} + \omega^2 m_i^{(2)} + \cdots \right)$$

General moment structure:

$$m_1^{(n)} = \int_0^1 x^{2n-1}\, b_1(x)\, dx, \qquad m_i^{(n)} = \int_0^1 x^{2(n-1)}\, b_i(x)\, dx \quad (i = 2, 3, 4)$$

Close–Kumano: $m_1^{(0)} = \int_0^1 b_1(x)\, dx = 0 \Rightarrow \mathcal{B}_1$ starts at ω^2.

Backup: Momentum fraction $\langle x \rangle_q(\omega)$

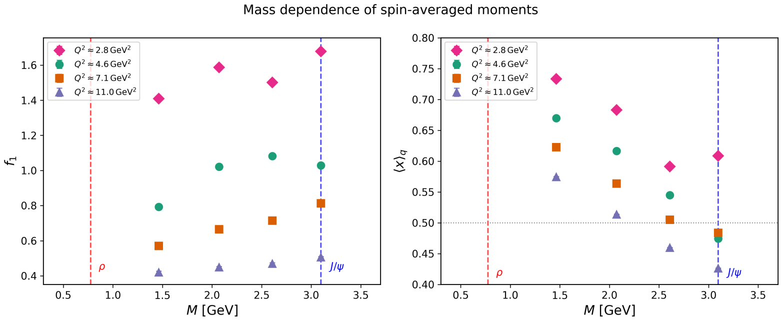


- Momentum fraction per quark from $\mathcal{F}_2$
- Clear mass ordering: lighter quarks carry more momentum
- At $\omega = 0$: $\langle x \rangle_q \approx 0.48$–$0.62$
- Dashed line shows $\langle x \rangle = 0.5$

Observation

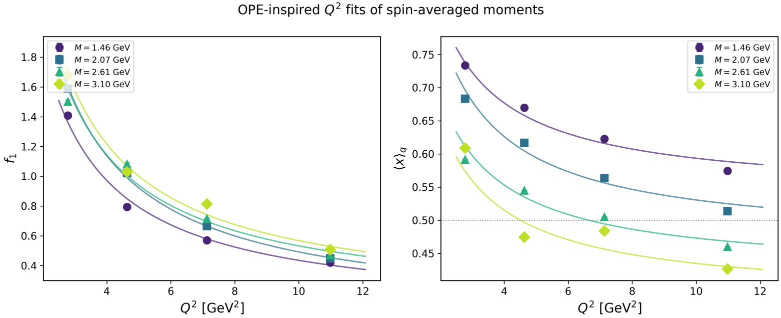
Momentum fraction increases toward lighter (more relativistic) quarks, consistent with valence dominance.

Backup: $\mathcal{F}_1$, $\langle x \rangle_q$: mass dependence



- Meson mass range: $M = 1.4$–2.9 GeV
- Vertical lines: physical ρ (775 MeV), J/ψ (3.1 GeV)
- Heaviest point $\approx J/\psi$
- $\langle x \rangle_q$ increases toward lighter masses

Backup: $\mathcal{F}_1$, $\langle x \rangle_q$: Q^2 evolution (OPE fits)



- OPE fits:
  $f_1^{(0)} = A + B/Q^2$
  $\langle x \rangle_q = A + B/Q^2$
- Clear $1/Q^2$ behaviour
- $\langle x \rangle_q$ decreases with Q^2
- $M = 3.10$ GeV $\approx J/\psi$

Observation

QCD evolution: gluons carry more momentum at higher Q^2. Dashed line shows $\langle x \rangle = 0.5$.

Backup: Why t-suppression fails for the pion pole

Feynman–Hellmann extracts physics from large-t correlation functions:

$$C(t) = \langle 0|\mathcal{O}\, e^{-Ht}\, \mathcal{O}^\dagger|0\rangle = \sum_n |Z_n|^2 e^{-E_n t} \xrightarrow{t\to\infty} |Z_0|^2 e^{-E_0 t}$$

The problem

- Excited states suppressed by $e^{-\Delta E \cdot t}$
- For $\rho \to \pi \to \rho$: $\Delta E = E_\pi - E_\rho$
- With $m_\pi/m_\rho \approx 0.90$–$0.98$:
  $\Delta E \approx 0.1$–0.25 (lattice units)
- Need $t \gtrsim 1/\Delta E \approx 4$–$10$ for suppression
- But signal/noise $\propto e^{-(E_\rho - \frac{3}{2}m_\pi)t}$ degrades!

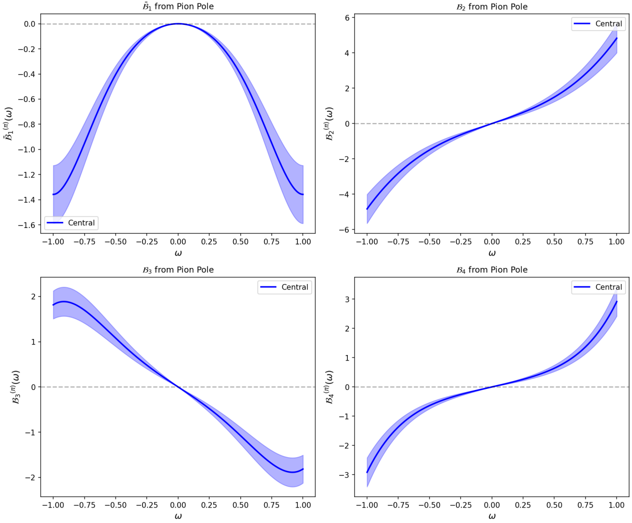
Consequence

Cannot reach t large enough for pion suppression before noise dominates.

The pion intermediate state contaminates the extracted Compton amplitude.

Solution: use tensor components where pion contribution vanishes by symmetry.

Backup: pion pole contribution estimate



$\mathcal{B}_i^{(\pi)}(\omega)$ from pion pole vertex, κ=0.115, $\boldsymbol{q}$=[4, 1, 0]

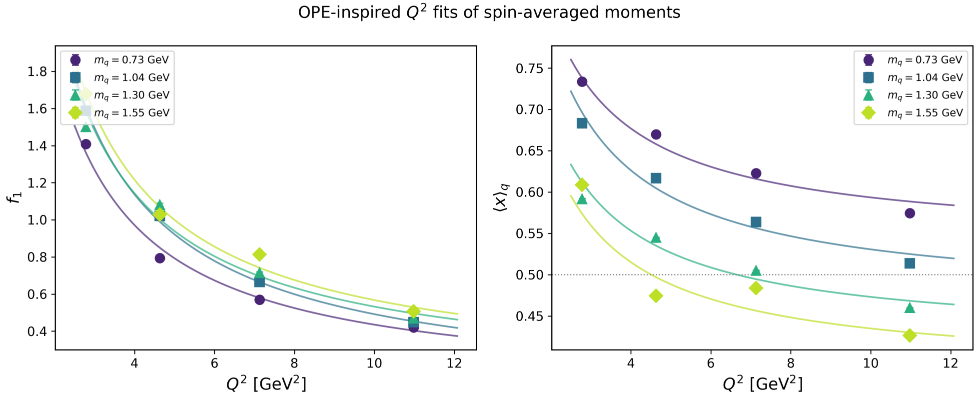
Method: compute $T_{\mu\nu}^{(\pi)}$ from $\rho\pi\gamma$ vertex, propagate through moment fit.

Result: if *not* excluded, pion pole would overwhelm the signal:

	Bias	Signal	Bias/Signal
m_1	$-0.68(12)$	$\sim$0.14	490%
m_2	$+0.60(10)$	$\sim$0.06	1000%
m_3	$-0.45(8)$	$\sim$0.04	1130%
m_4	$+0.24(4)$	$\sim$0.02	1200%

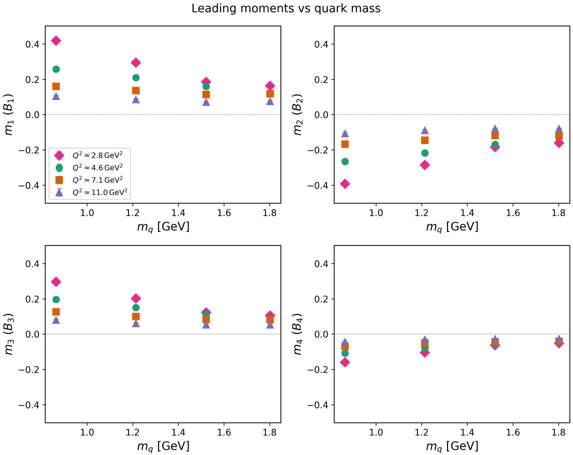
Uncertainties from $g_{\rho\pi\gamma} = 0.55(5)$ GeV^{-1}. Point-like coupling; form factor reduces by $\sim$3–10$\times$.

Backup: $\mathcal{F}_1$, $\langle x \rangle_q$ OPE fits (m_q labels)



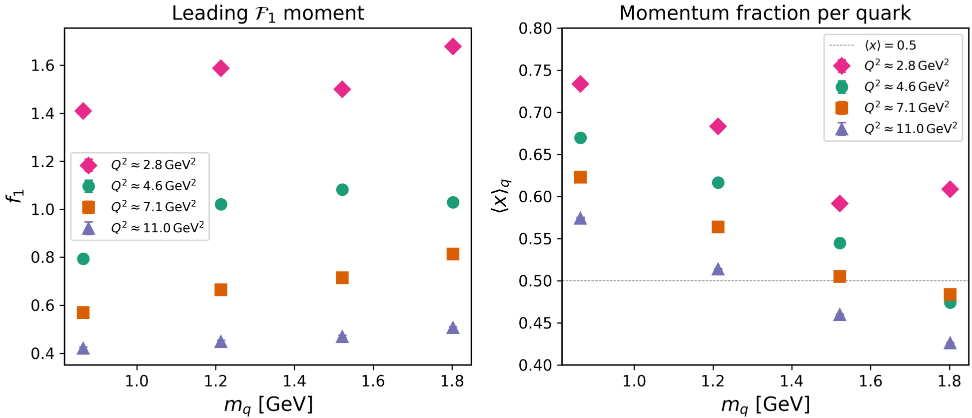
Same data as main slides, labeled by constituent quark mass $m_q = M/2$.

Backup: B-function moments vs m_q



Same data as main slides, plotted against constituent quark mass $m_q = M/2$.

Backup: $\mathcal{F}_1$, $\langle x \rangle_q$ vs m_q



Spin-averaged structure function moments vs constituent quark mass $m_q = M/2$.

Backup: Clean-row moment fit

Per-ω solve is impossible

- 7 clean rows, but 6 structure functions
- Exact rank-5 degeneracy at fixed (ω, Q^2): one null direction, independent of $\boldsymbol{p}$
- Cannot solve for all $\mathcal{B}_i(\omega)$ per momentum

Global moment fit

- Parameterise: $\mathcal{B}_i(\omega) = 4\omega(m_i^{(1)} + \omega^2 m_i^{(2)} + \cdots)$
- Stack 12 in-plane momenta $\times$ 7 clean rows = 84 constraints
- Fit 12 parameters: $m_i^{(1)}, m_i^{(2)}, m_i^{(3)}$ for each of $\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_4$
- Full rank; exactly zero pion-pole bias

Energy gap at our kinematics:

$\boldsymbol{p}$=[1, 1, 0], $\boldsymbol{q}$=[5, 1, 0], crossed term $\boldsymbol{p}-\boldsymbol{q}$

m_π/m_ρ	$E_\pi - E_\rho$	$(E_\pi - E_\rho)^{-1}$
0.98	0.12	8.2
0.97	0.15	6.5
0.95	0.20	5.1
0.90	0.25	4.0

Enhancement in propagator; without clean-row selection, bias is 100–800% of signal.

Backup: mass calibration

Two-point calibration: anchor to physical ρ (775 MeV) and J/ψ (3097 MeV).

Tuning point	M_{lat}	M_{phys}
$\kappa = 0.121$ (ρ)	0.351	0.775 GeV
$\kappa = 0.109$ (J/ψ)	1.350	3.097 GeV

Linear map: $M_{\text{phys}} = 2.32 \times M_{\text{lat}} - 0.04$ GeV

κ	M_{lat}	M [GeV]	Physical analogue
0.109	1.350	3.10	J/ψ
0.112	1.140	2.61	heavy-light
0.115	0.909	2.07	heavy-light
0.118	0.647	1.46	heavy-light
0.121	0.351	0.78	ρ (discarded)

Note: $\kappa = 0.121$ data discarded due to pion contamination at light quark masses.