# Recent Progress on the Hadronic Vacuum Polarization Contribution from RBC/UKQCD

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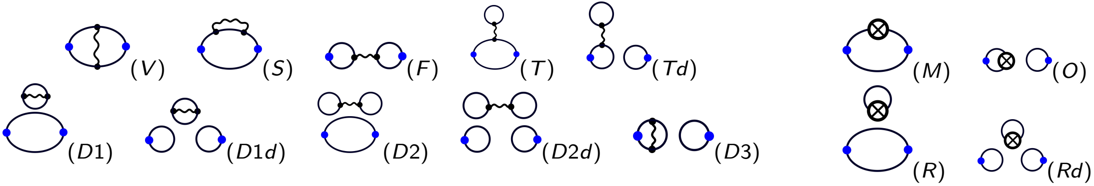
# Topics

1. Update on the isospin breaking corrections to the HVP contribution

2. Reconstructing the HVP above a timelike cut in lattice QCD

# Isospin Breaking Corrections: Status




1. Computed correlators for all QED and SIB diagrams at the physical point on 48I ensemble ($a^{-1} = 1.7312(28)$ GeV, $m_\pi L = 3.9$) using stochastic coordinate sampling

   [Bruno et al., arxiv:2602.24132]

2. Valence mass study for the leading SIB diagram (M)
3. Determination of the quark mass shifts from IB effects in pion and kaon
4. Strange component for diagrams (V), (S) and (F)

# Valence Mass Difference Computation: Diagram (M)

- Computation of diagram (M) from difference of leading order connected diagram (c) for two different valence quark masses on ensemble 48I

$$(c)_m = \frac{1}{3}\sum_{\boldsymbol{x},i}\langle \mathrm{Tr}[D_m^{-1}(0;\boldsymbol{x},t)\gamma_i D_m^{-1}(\boldsymbol{x},t;0)\gamma_i]\rangle_U$$

$$\begin{aligned}(M) &= \frac{1}{2}\frac{d}{dm}(c)_m\Big|_{m=m_l} \\ &\approx \frac{1}{2}\frac{(c)_{m_l}-(c)_{m_1}}{m_l - m_1}\end{aligned}$$

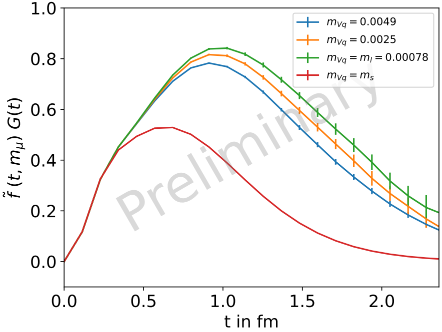



**Fig. 1**: Blinded HVP integrand for different valence quark masses

# Valence Mass Difference Computation: Diagram (M)

Comparison on 48I ensemble between

- Finite difference

$$(M)^{\text{fin. dif.}} = \frac{1}{2}\frac{(c)_{m_l} - (c)_{m_1}}{m_l - m_1}$$

- Scalar insertion

$$(M)^{\text{sca. ins.}} = \frac{1}{3}\sum_{\boldsymbol{x},i}\langle \mathrm{Tr}[D_m^{-1}(0;y)\mathbf{1}D_m^{-1}(y;\boldsymbol{x},t)\gamma_i D_m^{-1}(y,\boldsymbol{x},t;0)\gamma_i]\rangle_U$$

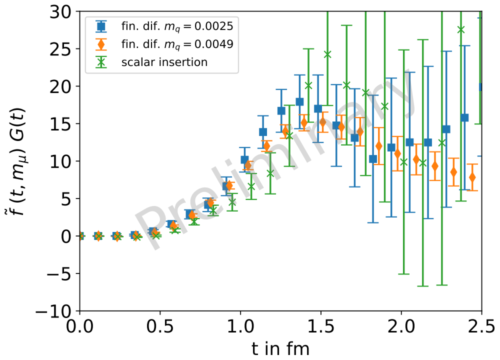



**Fig. 2**: Comparison of blinded HVP integrand

# Quark Mass Shifts

- We determine the quark mass shifts by requiring

$$\begin{aligned} m^0_{\pi^0} + \Delta m_{\pi^0} &= m^{phys}_{\pi^0} \\ m^0_{K^0} + \Delta m_{K^0} &= m^{phys}_{K^0} \\ \Delta_K &= \Delta^{phys}_K \end{aligned}$$

- Expanding the Kaon mass shift in $e^2$, one obtains

$$\Delta^{phys}_K = \Big[ e^2 (Q_d - Q_u) Q_s \Delta m^{(V)}_K + e^2 (Q_d^2 - Q_u^2) \Delta m^{(S)}_K + e^2 (Q_u - Q_d) \sum_f \Delta m^{(T)_f}_K + (m_d - m_u) \Delta m^{(M)}_K \Big] + \Delta m^{FV}_K$$

- To determine $(m_d - m_u)$, apply universal FV correction $\Delta m^{FV}_K$ and obtain $\Delta m^{(V)}_K$, ..., from a fit to the ratio (V)/($c$), ...

# Quark Mass Shifts

- Determine $(\Delta m_d + \Delta m_u)$ from fixing $m^0_{\pi^0} + \Delta m_{\pi^0} = m^{phys}_{\pi^0}$
- And $\Delta m_s$ from $m^0_{K^0} + \Delta m_{K^0} = m^{phys}_{K^0}$
- We obtain for the quark mass ratio on the 48I ensemble

$$\frac{m_u}{m_d} = 0.455 \pm 0.0123$$

- Obtained pion mass splitting on 48I, agrees with earlier determination [Feng, Jin, Riberdy, 10.1103/PhysRevLett.128.052003]

$$M_{\pi^+} - M_{\pi^0} = 4.31 \pm 0.113\,\mathrm{MeV}$$

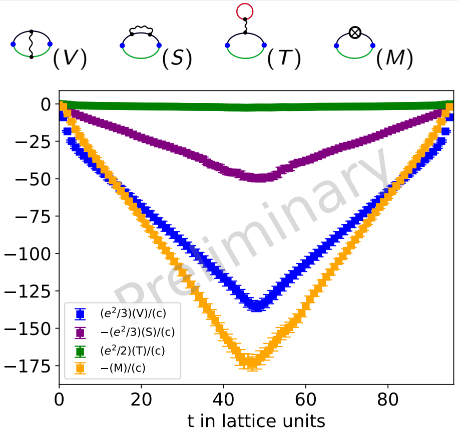



**Fig. 3**: Comparison of the correlator ratios to compute $\Delta_K$

# Strange Components

- Computation of strange components for leading diagrams (V), (S), (F) at the physical point
- Strange connected is however suppressed with charge factor $1/17$ compared to light
- For diagram (F), strange-light, light-strange and strange-strange components are negligible compared to light-light, even without suppression from charge factor

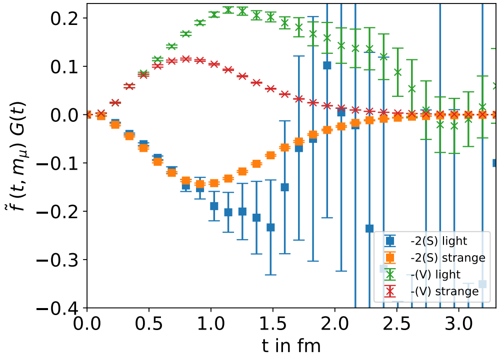



**Fig. 4**: Comparison of light and strange connected contributions.

# Preliminary Results: Leading Contributions

Comparison of partial sum, with blinded TMR kernel on ensemble 48I

- Partial sums for diagrams (V),(M) and (F) reach plateau
- Diagram (S) still noisy $\rightarrow$ QED+QCD Long-distance reconstruction

  [Lehner, JP, Völklein, Phys.Rev.D 113 (2026) 5, 054503]
- Cancellation between (M) and (O) predicted by LO $\chi$PT [Lehner, Meyer, 10.1103/PhysRevD.101.074515]

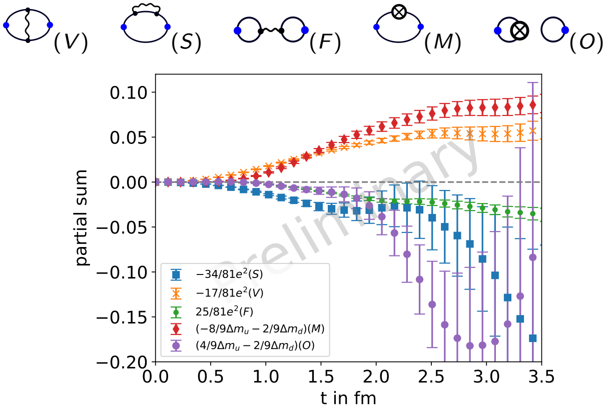



**Fig. 5**: Leading IB diagrams (V), (S), (F), (M), (O)

# Preliminary Results: Subleading contributions

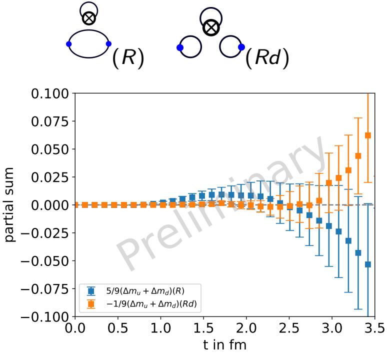



**Fig. 6**: SIB diagrams (R),(Rd)

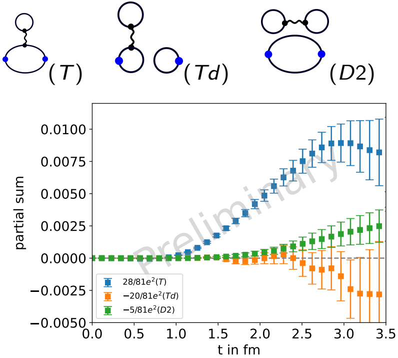
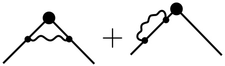



**Fig. 7**: Subleading QED contributions

- Developement for new noise reduction techniques for diagrams (D1), (D1d), (D3) (Talk by Christoph Lehner, Mo, 4:10 PM )

# Isospin Breaking Corrections: Outlook

- Apply noise reduction techniques for diagrams D1, D1d, D3
- Calculation of QED corrections to renormalization factor for local vector current $Z_V$ in RI/SMOM scheme (Talk by Mattia Bruno, Fr, 4:30PM)



$+ \ldots$

  or using PCAC relation, assuming $Z_A = Z_V$ for Domain-Wall fermions
- Additional lattice spacing to constrain cutoff effects
- Additional volume to constrain FV effects, already done for diagram (F)

# Topics

1. Update on the isospin breaking corrections to the HVP contribution

2. Reconstructing the hadronic vacuum polarization above a timelike cut in lattice QCD

# Reconstructing the HVP Above a Timelike Cut

- For a fully inclusive determination of HVP from $\tau$-decay [Bruno et al, 2607.00564 ], it is necessary to estimate the contribution above the $\tau$ mass (Talk by Mattia Bruno, Fr, 4:30PM)

$$a_\mu^{>}(\omega_{\text{cut}}) = \frac{\alpha^2}{3\pi^2}\int_{m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)\theta(\sqrt{s}-\omega_{\text{cut}})$$

- Another window for scrutinizing the difference of the HVP from lattice QCD and dispersive approach

- Can be evaluated in perturbative QCD, but its $\omega_{\text{cut}} = m_\tau$ is just at the threshold where pQCD can be safely applied

# Reconstructing the HVP Above a Timelike Cut: Inverse Problem

$$a_\mu^>(\omega_{\text{cut}}) = \frac{\alpha^2}{3\pi^2}\int_{m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s)\theta(\sqrt{s}-\omega_{\text{cut}})$$

- Can be obtained in a hybrid approach
- From isovector lattice QCD correlator $G^W(t) = \frac{1}{2}(c)(t)$

$$a_\mu^>(\omega_{\text{cut}}) = \sum_n q_n^{\Delta,\lambda} G^W(t_n)$$

- And using the $e^+e^-$ data to estimate systematic uncertainty from reconstruction

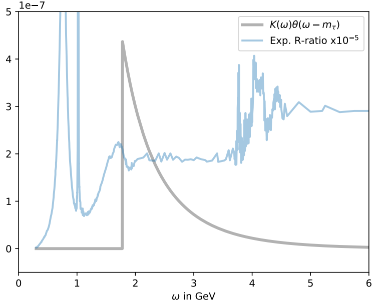



**Fig. 8**: Timelike kernel for $a_\mu^>(\omega_{\text{cut}})$

# Reconstructing the HVP Above a Timelike Cut: Inverse Problem

$$a_\mu^{>}(\omega_{\text{cut}}) = \sum_n q_n^{\Delta,\lambda} G^W(t_n)$$

- To calculate the coefficients $q_n^{\Delta,\lambda}$, we employ a HLT inspired method to approximate the sharp target kernel $T(\omega)$ by a smeared kernel $\widehat{T}^{\Delta,\lambda}(\omega)$.
- $\Delta$ : smearing width,
  $\lambda$: regulator for stat. noise
- As systematic uncertainty, consider

$$\int_{\omega_0}^{\infty} d\omega \left[\widehat{T}^{\Delta,\lambda}(\omega) - T(\omega)\right] R[e^+e^- \to had.](\omega^2)$$

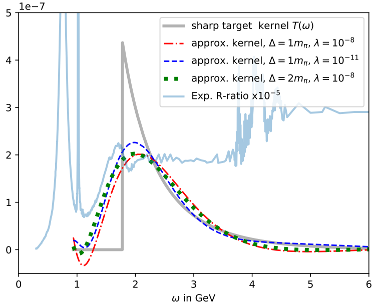



**Fig. 9**: Approximated kernel for $a_\mu^{>}(\omega_{\text{cut}})$

# Reconstructing the HVP Above a Timelike Cut: Pion-state Subtraction

- Balance systematic and statistical uncertainties by tuning $\lambda, \Delta$
- Subtract two-pion states below 1 GeV obtained from GEVP, to increase stability

$$G^{W,sub}(t) = G^{W}(t) - \sum_{n=0}^{N} V_n e^{-E_n t}.$$

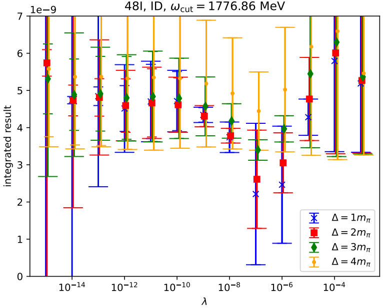



**Fig. 10**: Using $G^{W}(t)$ for reconstruction

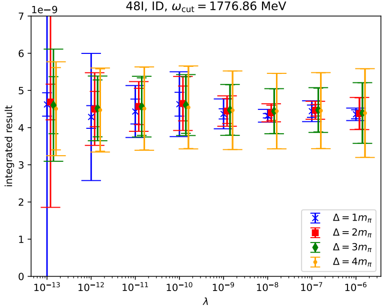



**Fig. 11**: Using $G^{W,sub}(t)$ for reconstruction

# Reconstructing the HVP Above a Timelike Cut: Continuum Extrapolation

- Focus on intermediate-distance window contribution above a timelike cut $a_\mu^{>,ID}(\omega_{\text{cut}} = m_\tau)$ first
- Analysis on 4 physical point ensembles, with 3 different lattice spacings (48I, 64I, 96I, C)
- Continuum extrapolation for local-local and local-conserved discretization using AIC model averaging

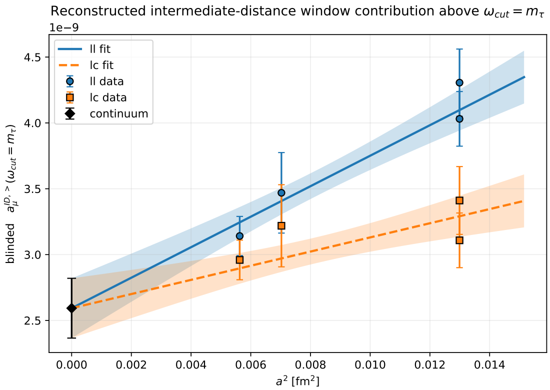



**Fig. 12**: Continuum extrapolation for ID window (blinded)

Backup

# Backup: Isospin breaking corrections: Scheme definition

- Use RM123 approach: Expand around isospin symmetric QCD
- Parameters for $N_f = 1+1+1$: $\boldsymbol{g} = (g_s, m_u, m_d, m_s, \alpha_{\mathrm{QED}})$
- Use physical hadron mass shifts masses to set scale PDG 2024 [PhysRevD.110.030001]

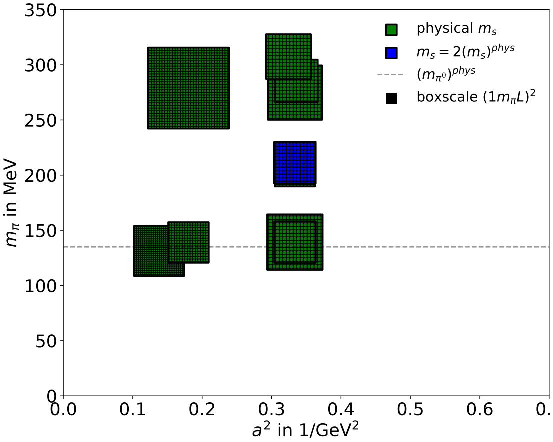
$$\begin{aligned} m_{\pi^0} &= 134.9768(5)\ \mathrm{MeV}, \\ m_{K^0} &= 497.611(13)\ \mathrm{MeV}, \\ m_{K^0} - m_{K^+} &= 3.9340(2)\ \mathrm{MeV}, \\ m_{\Omega} &= 1672.45(29)\ \mathrm{MeV} \end{aligned}$$

- Electromagnetic coupling does not renormalize at leading order

$$(\alpha_{\mathrm{QED}})^{-1} = 137.035999177(21)$$

# Backup: Ensemble Overview

- $N_f = 2 + 1$ Möbius domain wall ensembles
- 4 ensembles at physical pion mass in use so far
- Ensembles up to volumes of $m_\pi L \approx 8$; also new physical pion ensemble with $L^3 = (11\text{fm})^3$
- Generation of new finer physical pion mass ensemble $(128^3 \times 288)$: $a^2$ GeV$^2 = 0.08$ and $m_\pi L = 4.9$



# Backup: Tadpole fields

- For quark-disconnected diagram (d) but also for many QED diagrams, we need estimators of the tadpole field

$$T_x^{(f)} = \langle \psi_f(x) \bar{\psi}_f(x) \rangle$$

with quark flavor $f \in \{u, d, s\}$.

- Employ method of our first quark-disconnected physical pion mass paper [Blum et al., PRL116(2016)232002]:
  - Compute $T^{(ud)} - T^{(s)} = T^{(ud,low)} + T^{(ud,high)} - T^{(s)}$ with $T^{(ud,high)} = T^{(ud)} - T^{(ud,low)}$
  - Use multi-grid Lanczos [Clark, Jung, Lehner, 1710.06884] to compute exact low-mode field $T_x^{(ud,low)}$ over 2000-5000 lowest preconditioned Dirac modes
  - Estimate $T^{(ud,high)} - T^{(s)}$ using a random sparse $Z_2$ grid

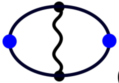
# Backup: Stochastic Coordinate Sampling (SCS)

Lattice QCD implementation using gpt library (http://github.com/lehner/gpt)

- We need to calculate correlators of type

$$\int_{x,y} G_{\nu\rho}(x,y)\langle\pi^+(\boldsymbol{p},t)|\,T\{j_\nu^{em}(y)j_\rho^{em}(x)\}|\pi^-(\boldsymbol{p},0)\rangle_{QCD}$$

$$\supset \int_{x,y,\boldsymbol{z}} G_{\nu\rho}(x,y)\Big\langle \mathrm{Tr}\Big[\gamma_5 S(z,x)\gamma_\rho S(x,(\mathbf{0},t))\gamma_5 S((\mathbf{0},t),y)\gamma_\nu S(y,z)\Big]\Big\rangle_U =$$

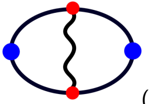


$(V)$

- Challenge: $V^2 \times L^3$ sum has to be computed
- Calculating all-to-all propagators is not feasible

  $\rightarrow$ RBC/UKQCD approach: Evaluate integrals stochastically

# Backup: Stochastic Coordinate Sampling (SCS)



(*V*)

- Propagators are calculated on stochastic subset of points (Introduced in calculation of hadronic light-by-light contribution [Blum et al. , 1510.07100])
- Propagator from $N$ source points to $M$ sink points are saved on disc
- No extra inversion needed to calculate diagram
- Can be contracted with different versions of the photon propagator

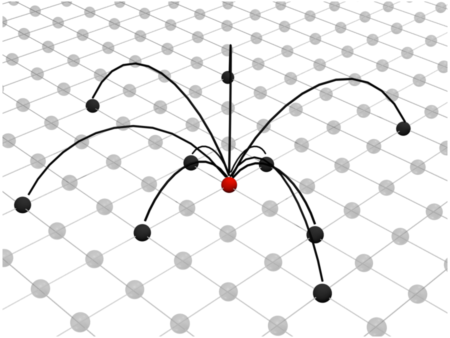


**Fig. 13**: Propagator for (red) source point to several (black) sink points

# Backup: Quark Mass Shifts

- To calculate $(\Delta m_d + \Delta m_u)$, we require

$$\begin{aligned} m_{\pi^0}^{phys} &= m_{\pi^0}^0 + \Big[ - e^2(Q_u^2 + Q_d^2)/2\Delta m_\pi^{(V)} \\ &\quad - e^2(Q_u^2 + Q_d^2)\Delta m_\pi^{(S)} - \Big(\Delta m_d + \Delta m_u\Big)\Delta m_\pi^{(M)} \\ &\quad + (Q_u + Q_d)e^2 \sum_f Q_f \Delta m_\pi^{(T)} + (Q_u - Q_d)^2/2e^2\Delta m_\pi^{(F)}\Big] \end{aligned}$$

- Fix $\Delta m_s$, by

$$\begin{aligned} m_{K^0}^{phys} &= m_{K^0}^0 + \Big[ - \Delta m_d \Delta m_K^{(M)} - Q_d Q_s e^2 \Delta m_K^{(V)} - Q_d^2 e^2 \Delta m_K^{(S)} \\ &\quad e^2 Q_d \sum_f Q_f \Delta m_K^{(T)_f} + e^2(Q_s)\sum_f Q_f \Delta m_K^{(T_s)_f} - Q_s^2 e^2 \Delta m_K^{(S_s)} - \Delta m_s \Delta m_K^{(M)_s}\Big] \end{aligned}$$

# Backup: Quark Mass Shifts

- Obtain the mass shift of a certain diagram, i.e. $\Delta m^{(V)}$ from fitting the ratio $(V)/(c)$ to

$$f(A, \Delta m^{(V)}, m^0) = A + \Delta m^{(V)}(T/2 - t)\tanh(m^0(T/2 - t)). \tag{1}$$

- Use finite volume correction $(P = \pi, K)$

$$\Delta(m_P^{FV}(L))^2 = e^2 m_P^2 \Big[\frac{c_2}{4\pi^2 m_P L} + \frac{c_1}{2\pi(m_P L)^2} - \frac{c_0}{(m_P L)^3}\Big(\frac{\langle r_P^2\rangle m_P^2}{3} + C\Big) + O\Big(\frac{1}{(m_P L)^4}\Big)\Big]$$

# Backup: Reconstructing the HVP Above a Timelike Cut

- Balance systematic and statistical uncertainties by tuning $\lambda, \Delta$

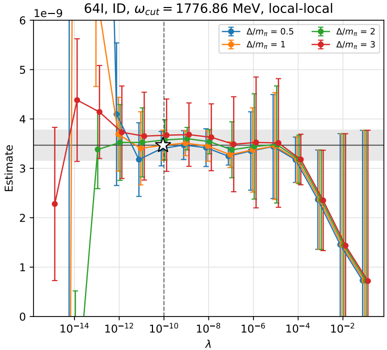



**Fig. 14**: Selection of $\lambda, \Delta$

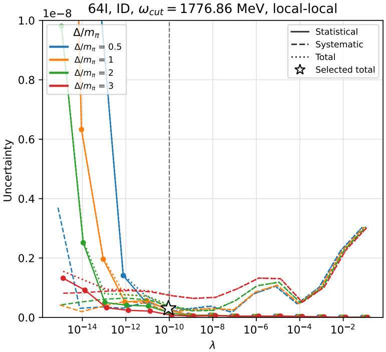



**Fig. 15**: Comparison of systematic and statistical uncertainty