Determination of SU(3) Chiral Perturbation Theory LECs from $N_f = 2 + 1 + 1$ HISQ Ensembles

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Fermilab Lattice and MILC collaborations
Universidad de Granada

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Introduction

Motivation

First row CKM unitarity ($|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$)

- Ratio of Kaon and pion decay constants f_K/f_π
- $K \to \pi \ell \nu_\ell$ form factor $f_+(0)$

Needed for the extraction of $\frac{V_{us}}{V_{ud}}$ and V_{us}

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Needed for the extraction of $\frac{V_{us}}{V_{ud}}$ and V_{us}

ChPT LECs

- Useful for phenomenology
- References (not exhaustive):

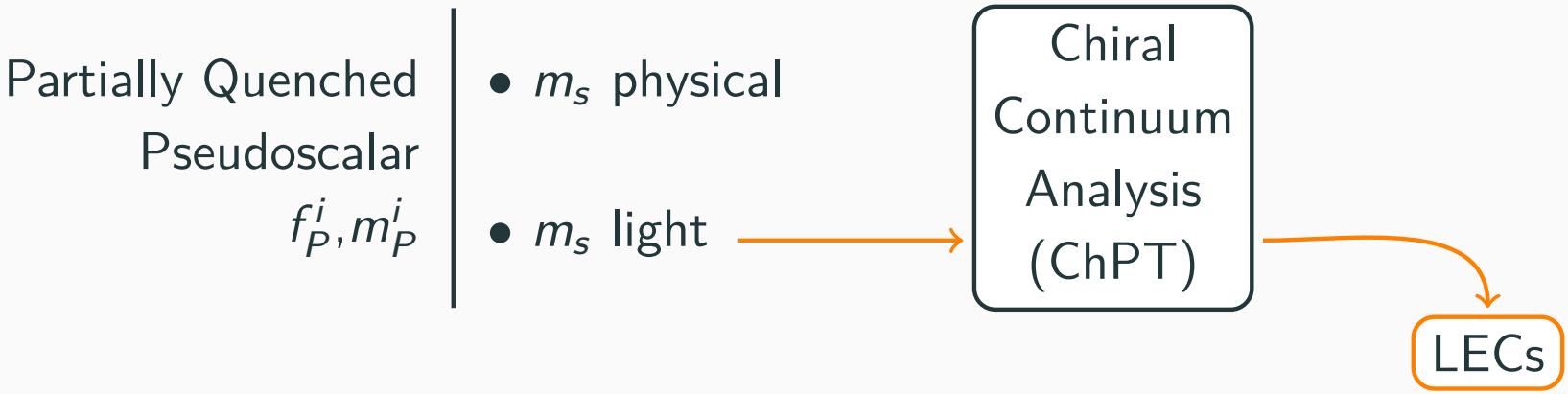
R. J. Hudspith et al. (2605.06560)
G. von Hippel et al. (2503.20689)
A. Bazavov et al. (0903.3598)
H. Fukaya (1012.4052)
G. S. Bali et al. (2201.05591)
M. F. Lutz et al. (2301.06837)
R. J. Dowdall et al. (1303.1670)
J. Liang et al. (2102.05380)

Our Roadmap

Partially Quenched Pseudoscalar f_P^i, m_P^i	• m_s physical • m_s light

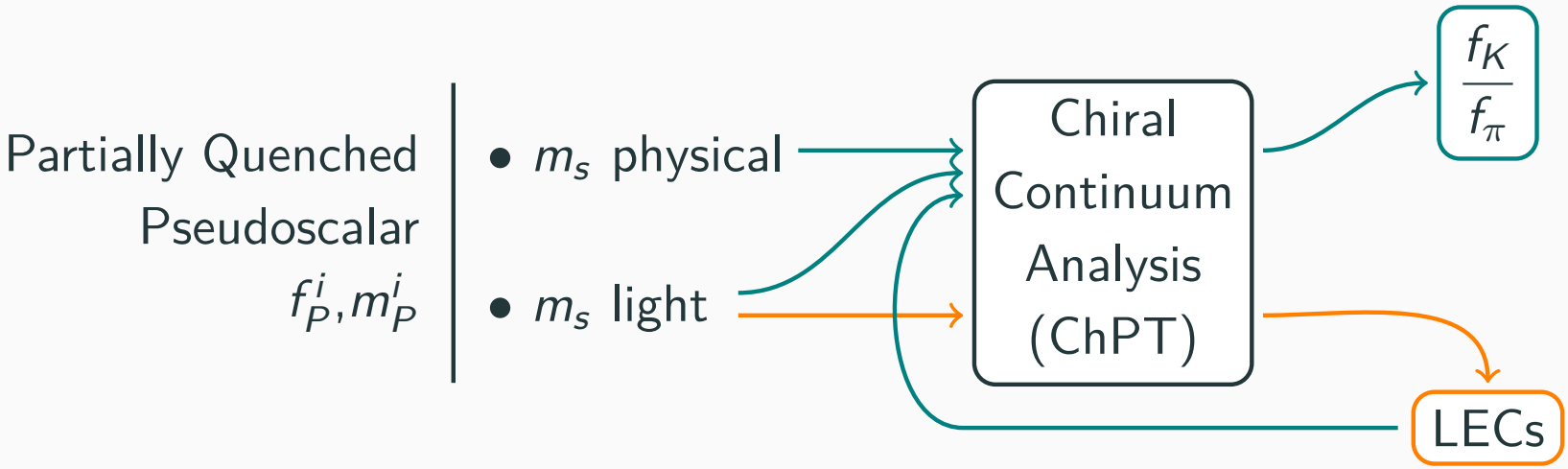
Our Roadmap

- First analysis

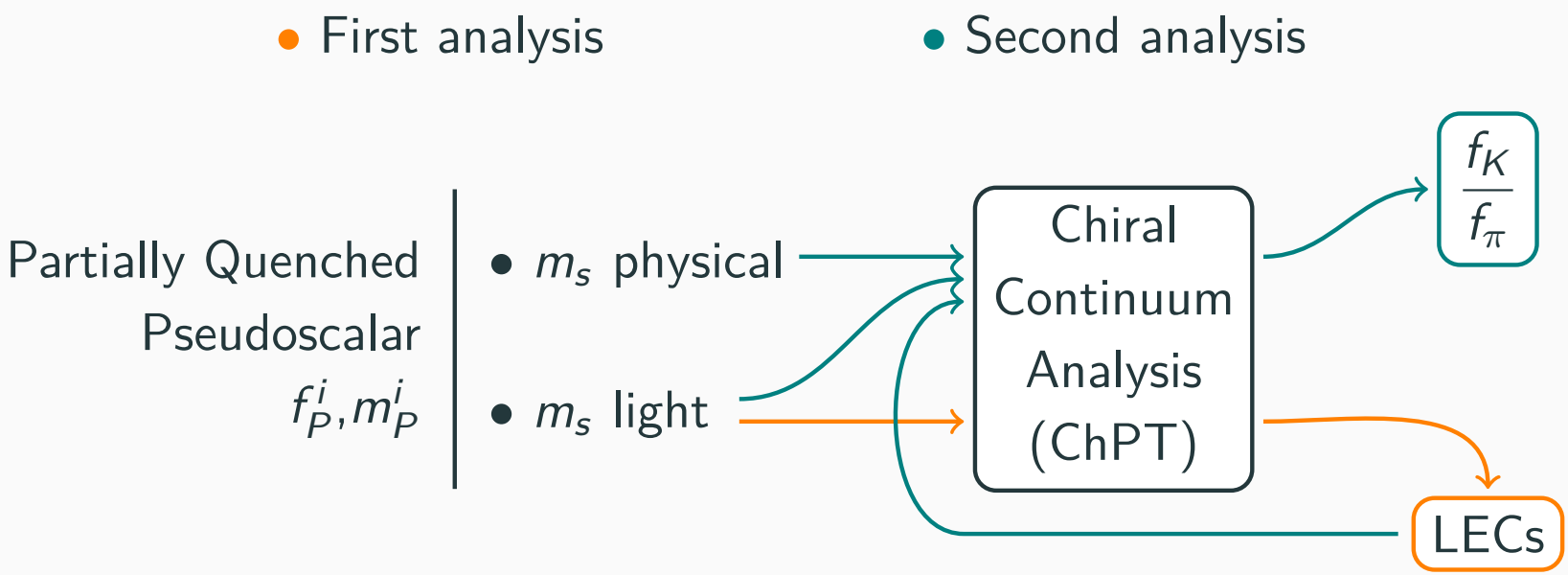


Our Roadmap

- First analysis
- Second analysis



Our Roadmap



Aditionally

- Updated $f_+(0)$ with new LECs
- Study correlation between $f_+(0)$ and LECs

SU(3) Chiral Perturbation Theory

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \cdots$$

Leading Order (LO)

$$\mathcal{L}_2 = \frac{f_0^2}{4} \langle D_\mu \Sigma^\dagger D^\mu \Sigma + 2B_0(\Sigma^\dagger M + M^\dagger \Sigma) \rangle$$

$\mathcal{O}(p^2)$ 2 LECs

Next to Leading Order (NLO)

$$\mathcal{L}_4 = L_1 \langle D_\mu \Sigma^\dagger D^\mu \Sigma \rangle^2 + L_2 \langle D_\mu \Sigma^\dagger D_\nu \Sigma \rangle \langle D^\mu \Sigma^\dagger D^\nu \Sigma \rangle + \cdots$$

$\mathcal{O}(p^4)$ 10 LECs: L_{1-10}

+ Partially Quenched effects: one extra LEC L_0

LECs in our NLO fit-function

NLO fit functions depend on a subset of LECs.

$$f_{xy}^{(\mathrm{NLO})} \supset f_0 \left[1 + \frac{16 B_0}{{f_0}^2} L_4 (2m_l + m_s) + \frac{8 B_0}{{f_0}^2} L_5 (m_x + m_y) \right] + \mathrm{logs}_{\mathrm{NLO}}$$

$$M_{xy}^{2\,(\mathrm{NLO})} \supset B_0 (m_x + m_y) \left[1 + \frac{32 B_0}{{f_0}^2} (2L_6 - L_4)(2m_l + m_s) + \frac{16 B_0}{{f_0}^2} (2L_8 - L_5)(m_x + m_y) \right] + \mathrm{logs}_{\mathrm{NLO}}$$

NNLO fit functions involve an extended (different) subset of LECs.

Observable sensitivity to LECs

Pseudoscalar meson masses and decay constants

- LO: f_0, B_0
- NLO: $L_4, L_5, 2L_6 - L_4, 2L_8 - L_5$.
- NNLO: L_0, L_1, L_2, L_3, L_7

Observable sensitivity to LECs

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Scattering involving π and K

- $L_5, L_{\text{scat}} \propto L_{1-6,8}$

π electromagnetic radius

- L_9

Baryon masses. η' mass

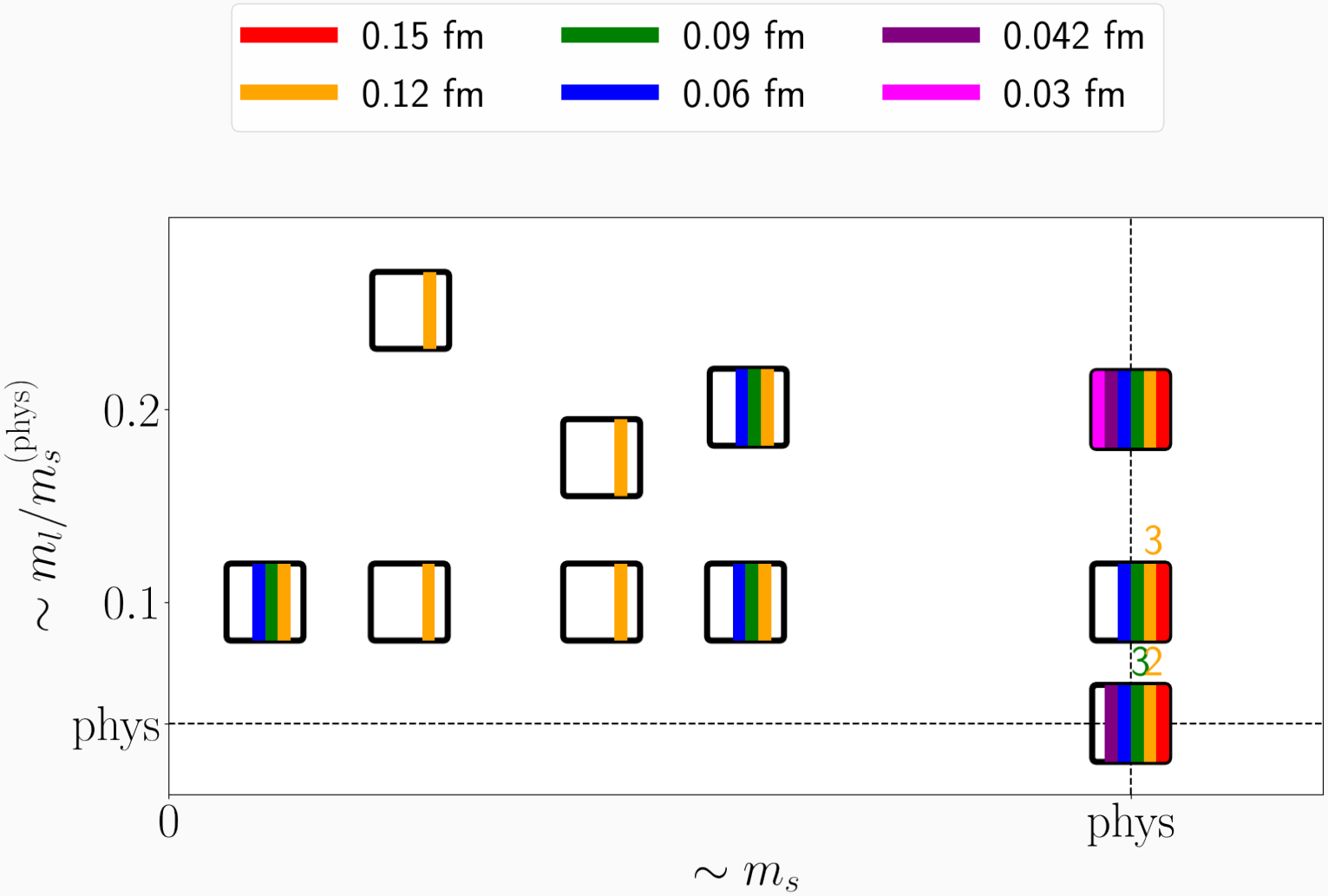
- $2L_6 - L_4, 2L_8 - L_5, L_8 + 3L_7$

$\pi \to e\nu\gamma$ axial form factor

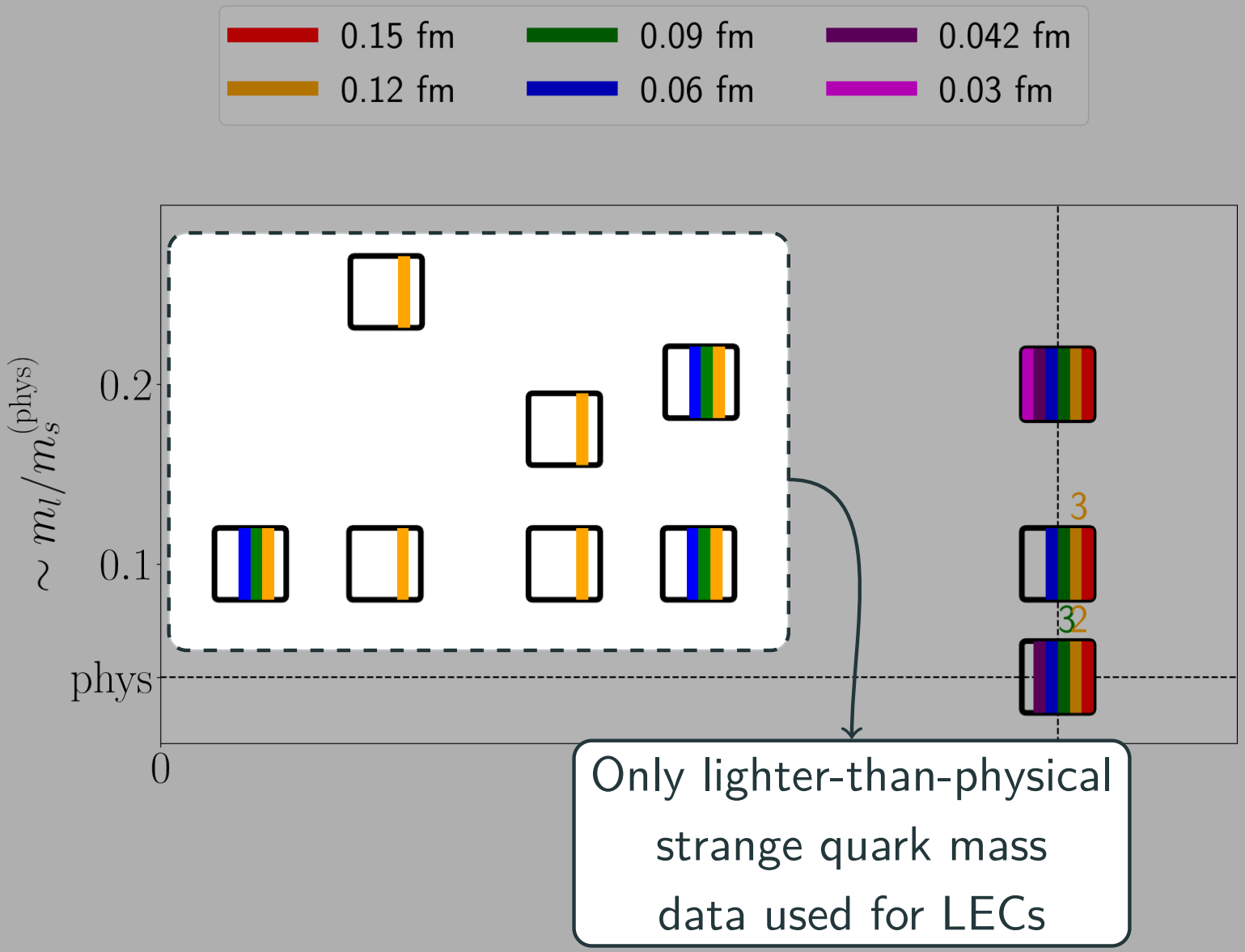
- L_{10}

Data

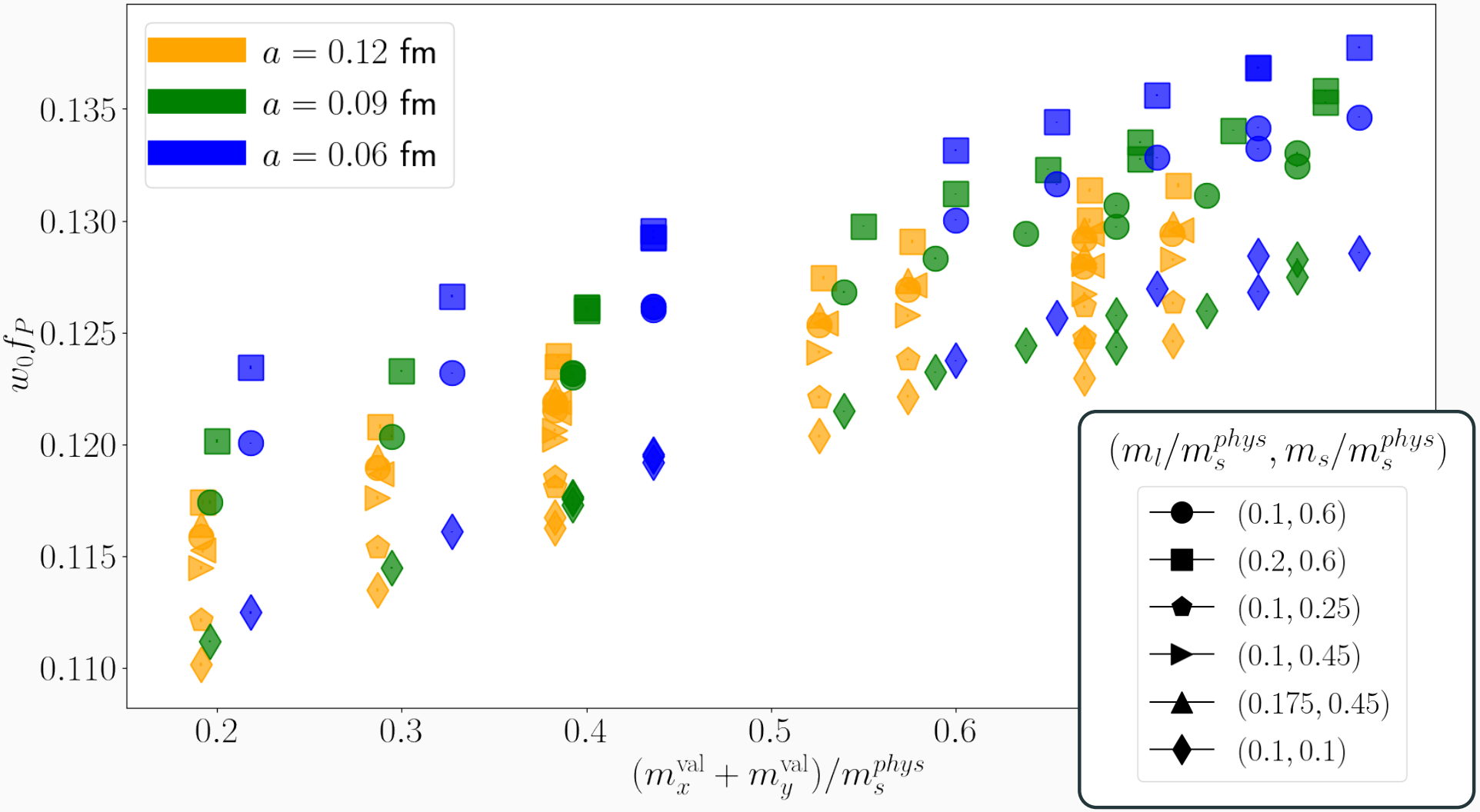
HISQ $N_f = 2 + 1 + 1$ ensembles



HISQ $N_f = 2 + 1 + 1$ ensembles



Pseudoscalar meson masses and decay constants



Chiral-continuum analysis

Chiral-continuum analysis: Fit function

- NLO SU(3) Staggered ChPT (SChPT)

  (Aubin and Bernard hep-lat/0304014 and hep-lat/0306026)

  $\rightarrow$ Two new parameters: hairpins δ_A,δ_V.

- NNLO SU(3) Partially Quenched ChPT (PQChPT)

  (Bijnens, et al. hep-lat/0602003)

- Analytic terms $\mathcal{O}(\alpha_s a^2, \alpha_s a^2 m, m^2)$

  $\rightarrow$ $m \equiv$ sea or valence mass quark masses

Chiral-continuum analysis: Fit parameters

LECs + $\mathcal{O}(\alpha_s a^2)$: 22

LO

$$B_0 + \alpha_s a^2 \delta B_0$$
$$f_0 + \alpha_s a^2 \delta f_0$$

NLO

$$L_4 + \alpha_s a^2 \delta L_4$$
$$L_5 + \alpha_s a^2 \delta L_5$$
$$2L_6 - L_4 + \alpha_s a^2 \delta (2L_6 - L_4)$$
$$2L_8 - L_5 + \alpha_s a^2 \delta (2L_8 - L_5)$$
$$L_0 + \alpha_s a^2 \delta L_0$$
$$L_1 + \alpha_s a^2 \delta L_1$$
$$L_2 + \alpha_s a^2 \delta L_2$$
$$L_3 + \alpha_s a^2 \delta L_3$$
$$L_7 + \alpha_s a^2 \delta L_7$$

SChPT: 2

$\delta_{A,V}$

$\mathcal{O}(a^2)$ correction to loops

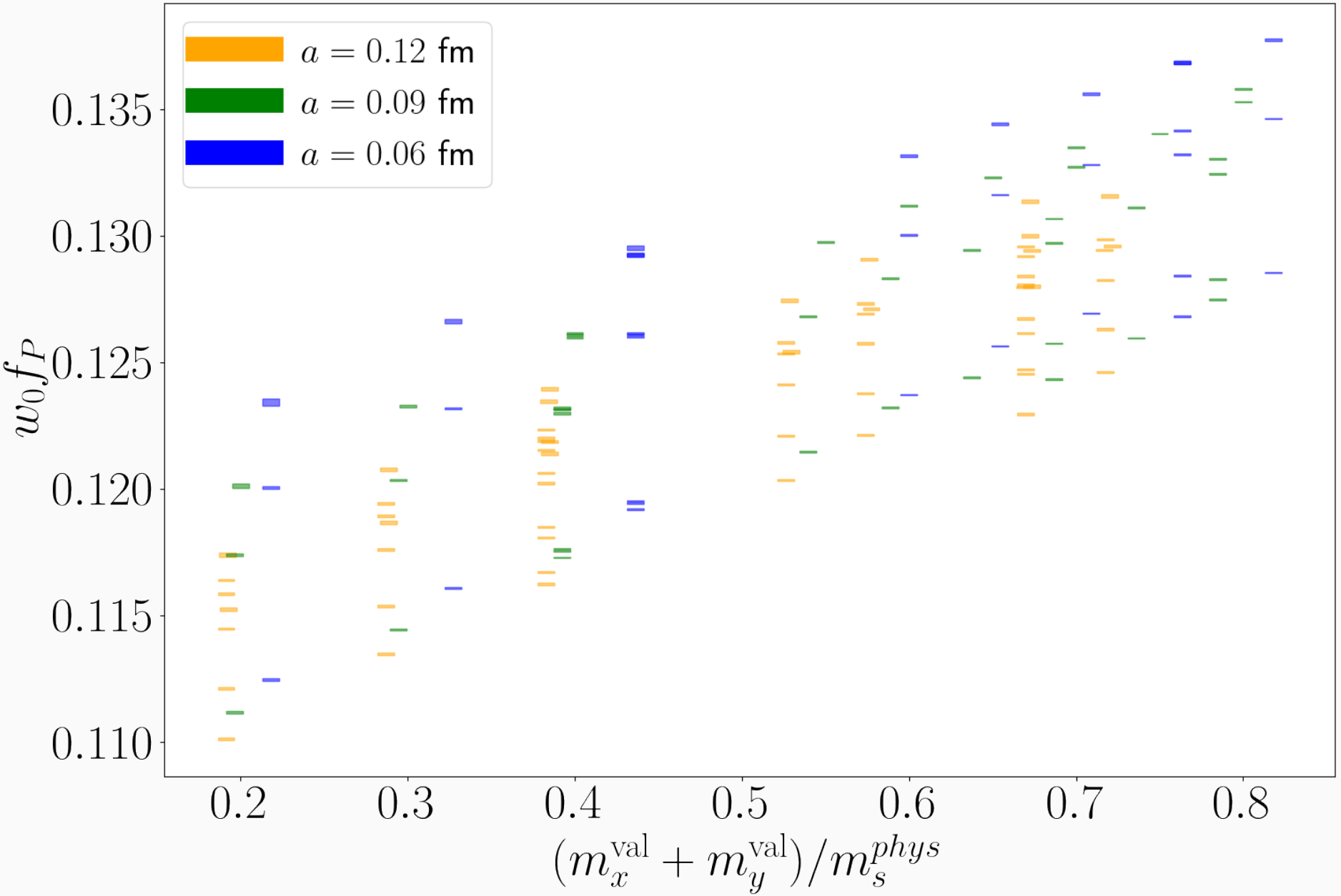
NNLO Analytical Terms: 24

$\mathcal{O}(m^2)$: 11

$\mathcal{O}(\alpha_s a^2 m^2)$: 11

$\mathcal{O}(\alpha_s^2 a^2)$: 2

Example of central fit



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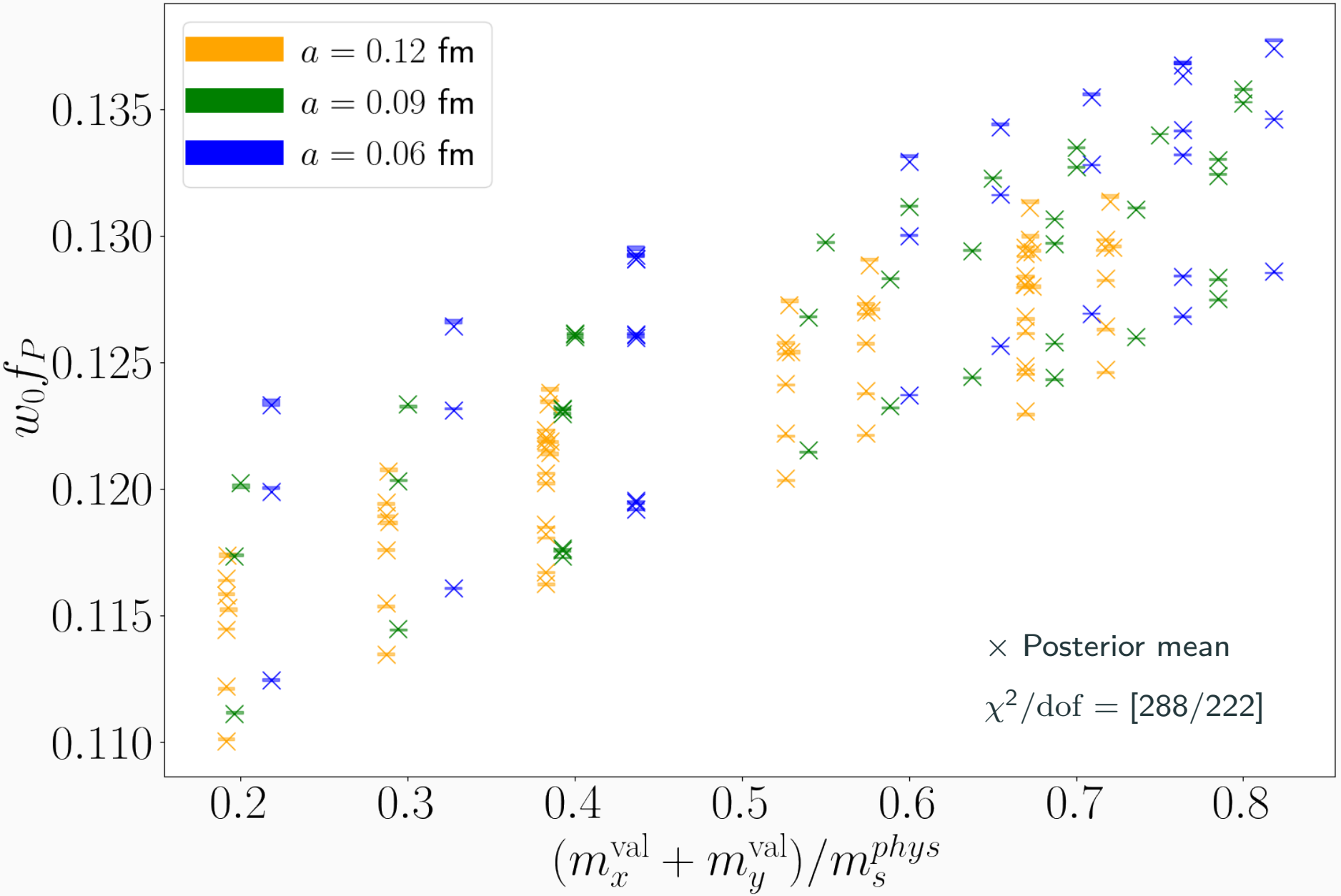
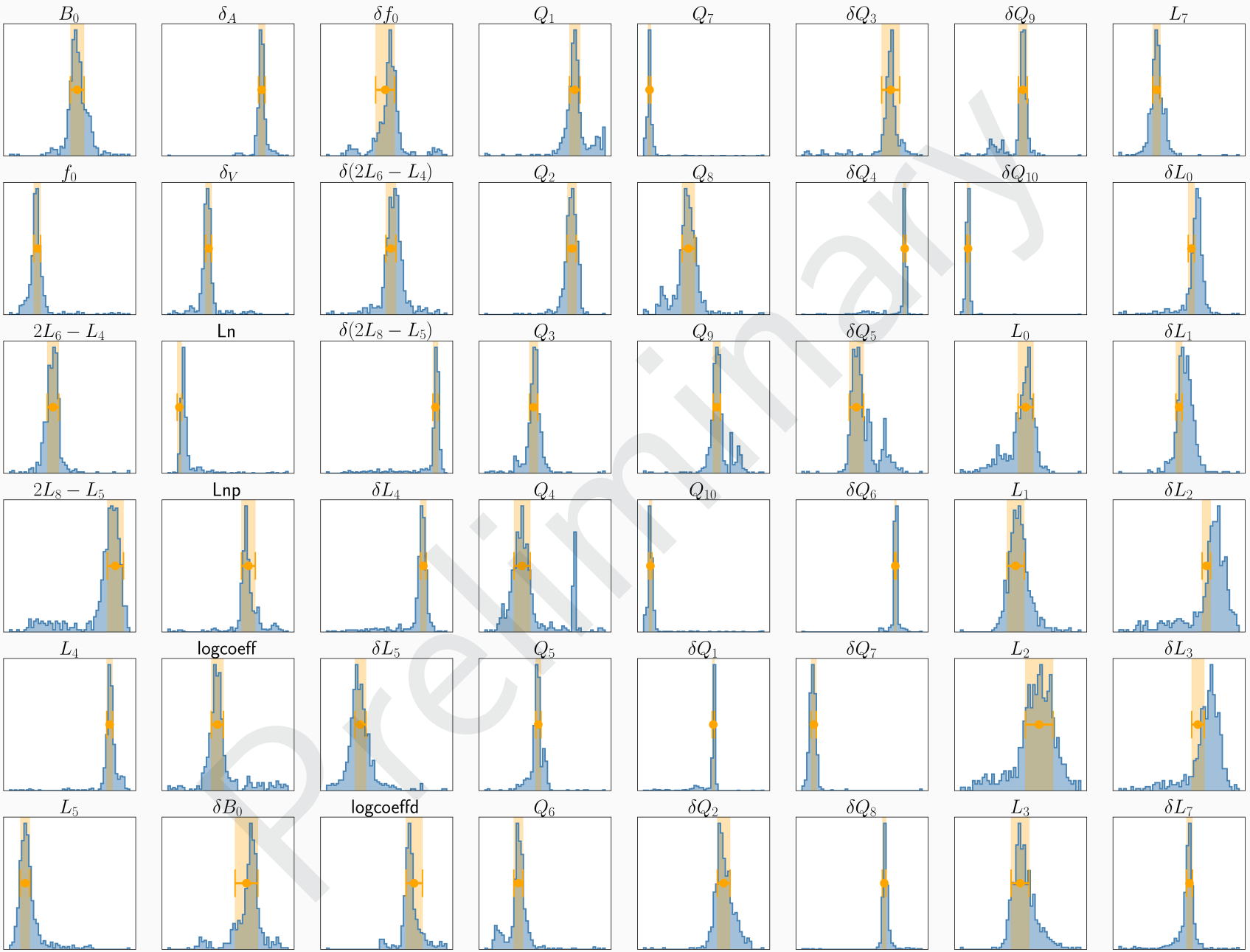
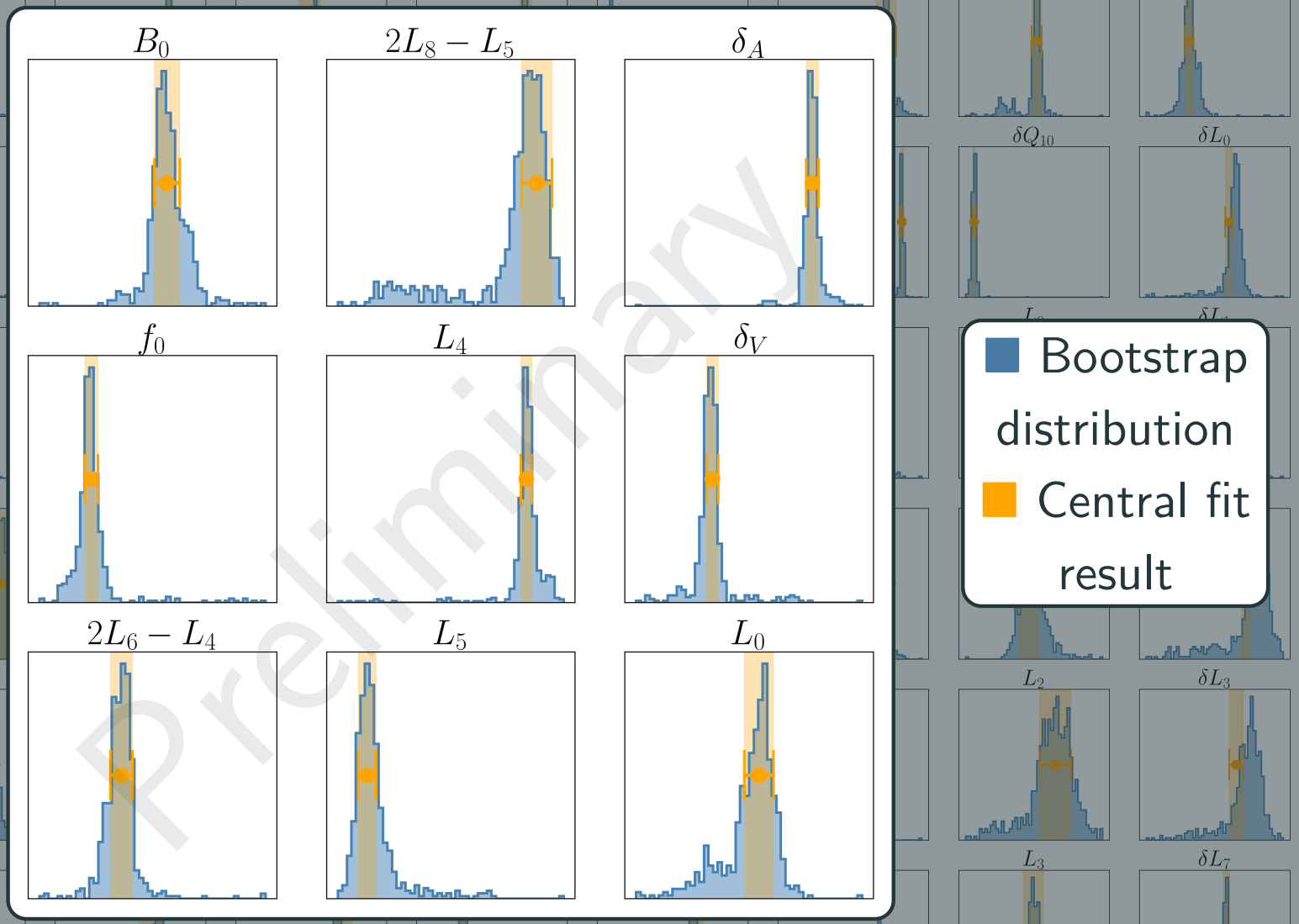


Uncertainty and correlation study

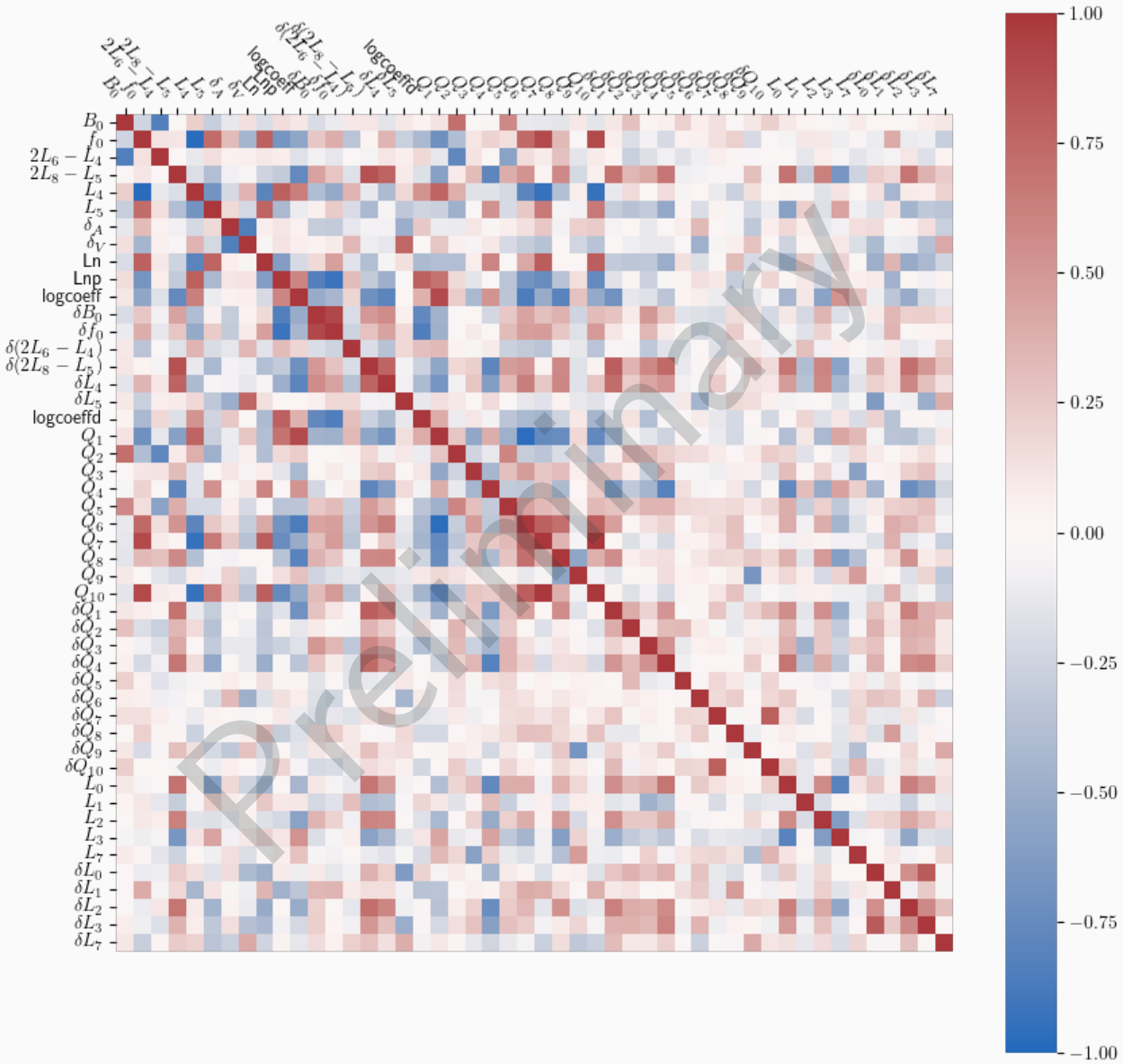
Bootstrap analysis



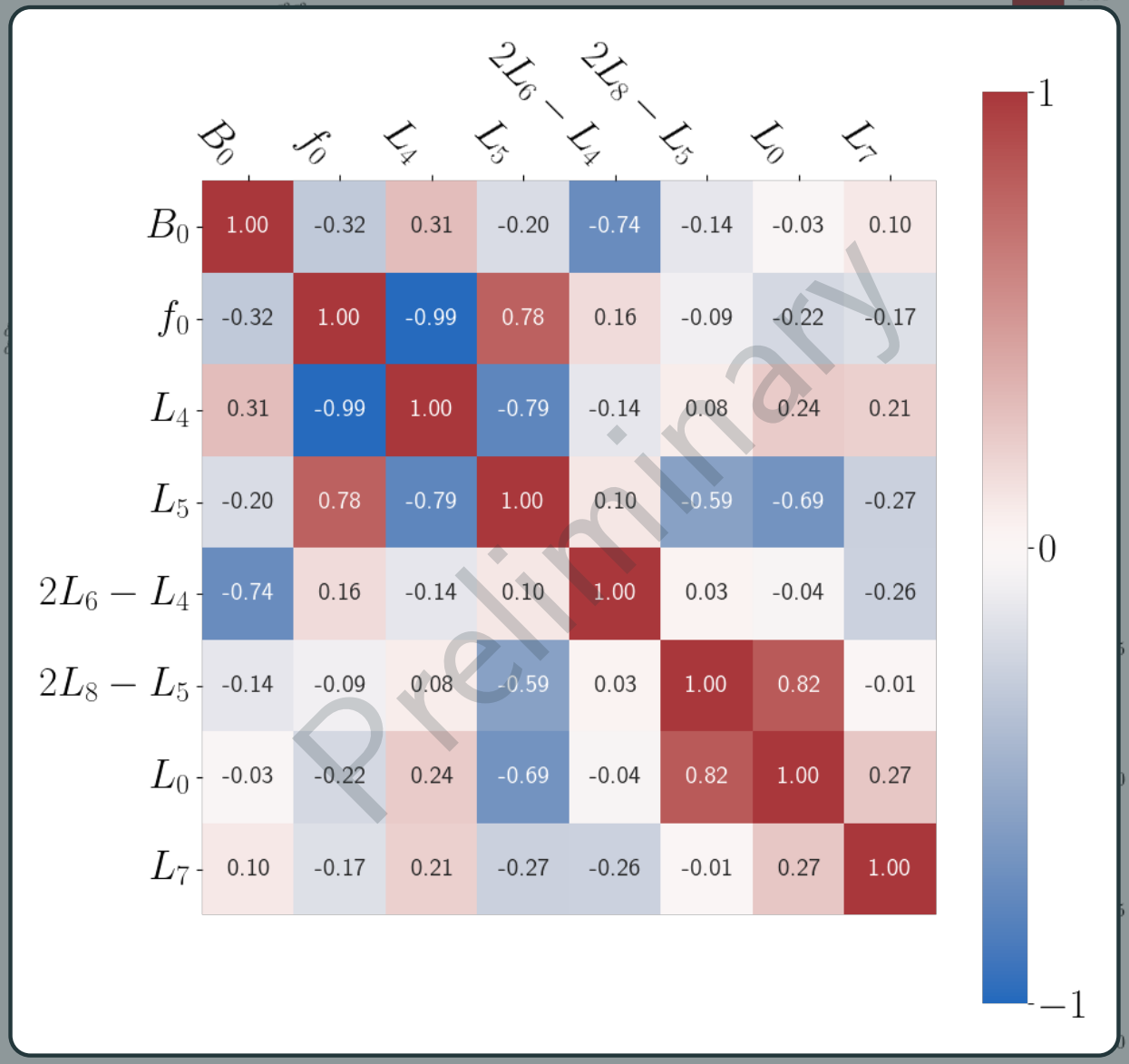
Bootstrap analysis



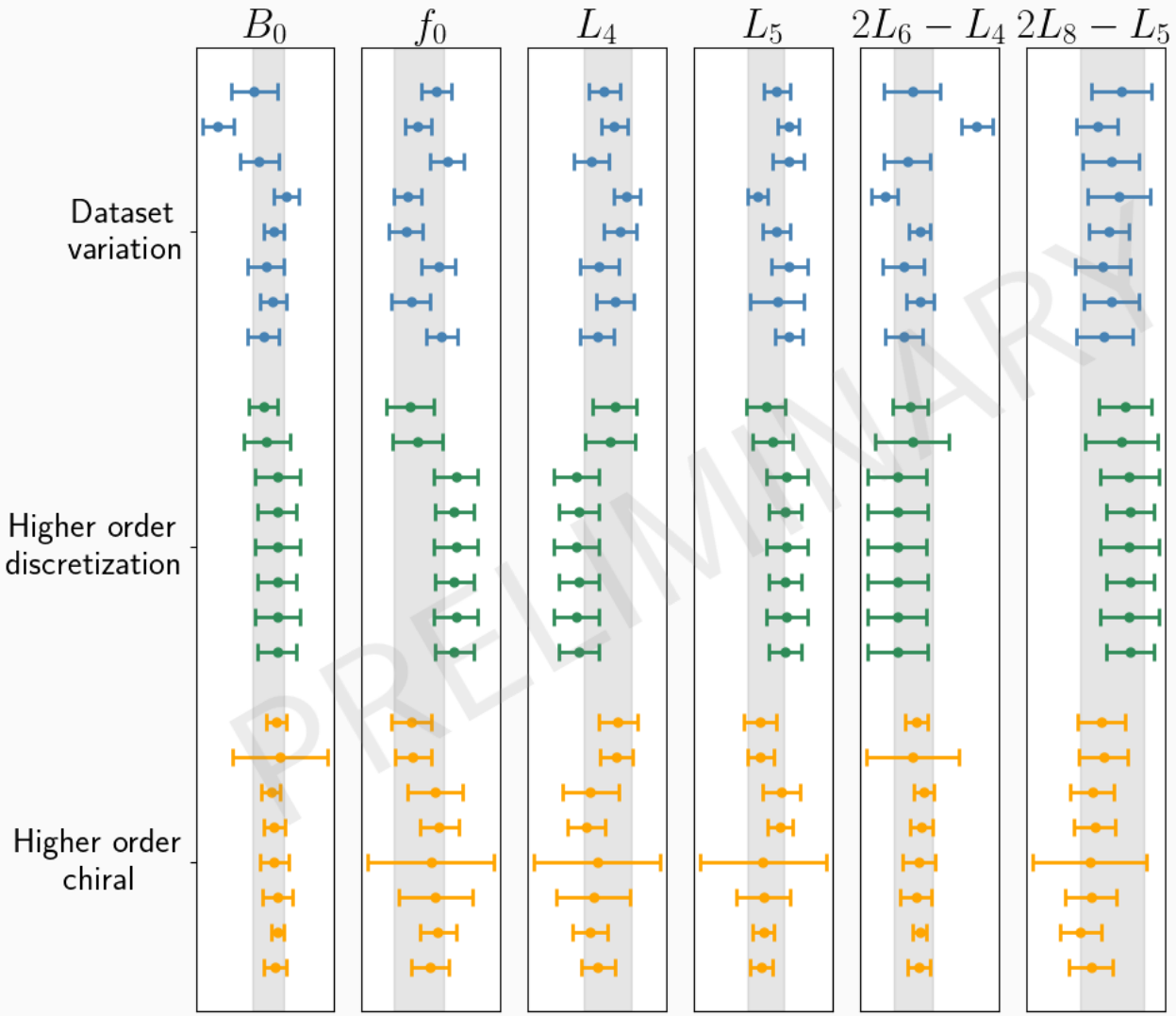
Bootstrap analysis: Correlation matrix



Bootstrap analysis: Correlation matrix



Bayesian Model Average (BMA)



Dataset

Reduce mass datacut
Thinning

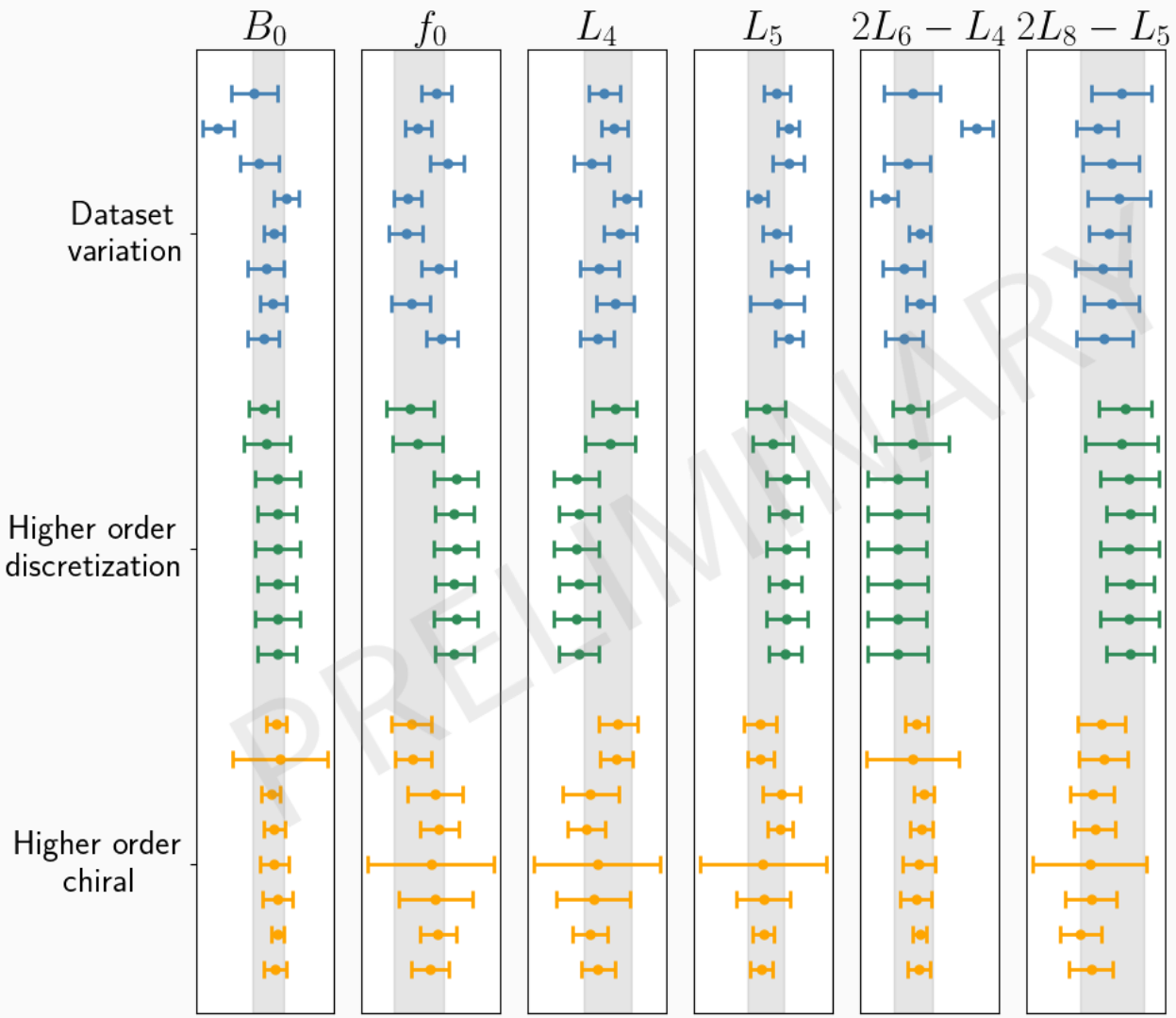
Discretization

$+\mathcal{O}(a_s^2 a^4, a^4)$ to LECs

Chiral

$+\mathcal{O}(m^3)$

Bayesian Model Average (BMA)



Dataset

Reduce mass datacut

Thinning

Discretization

$+\mathcal{O}(a_s^2 a^4, a^4)$ to LECs

Chiral

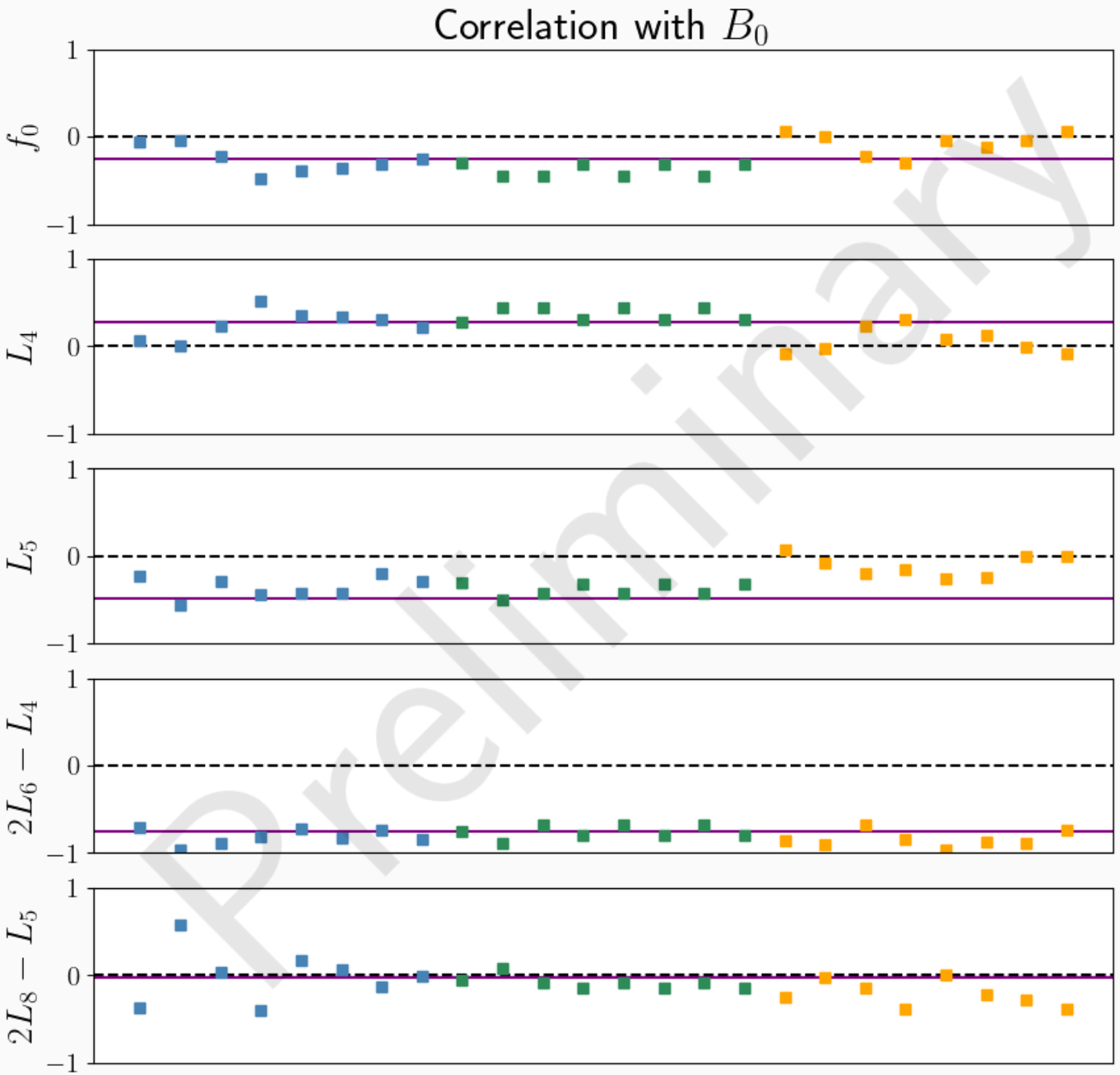
$+\mathcal{O}(m^3)$

BMA

$\text{BAIC} = \chi^2 + 2p - 2N_{\text{dat}}$

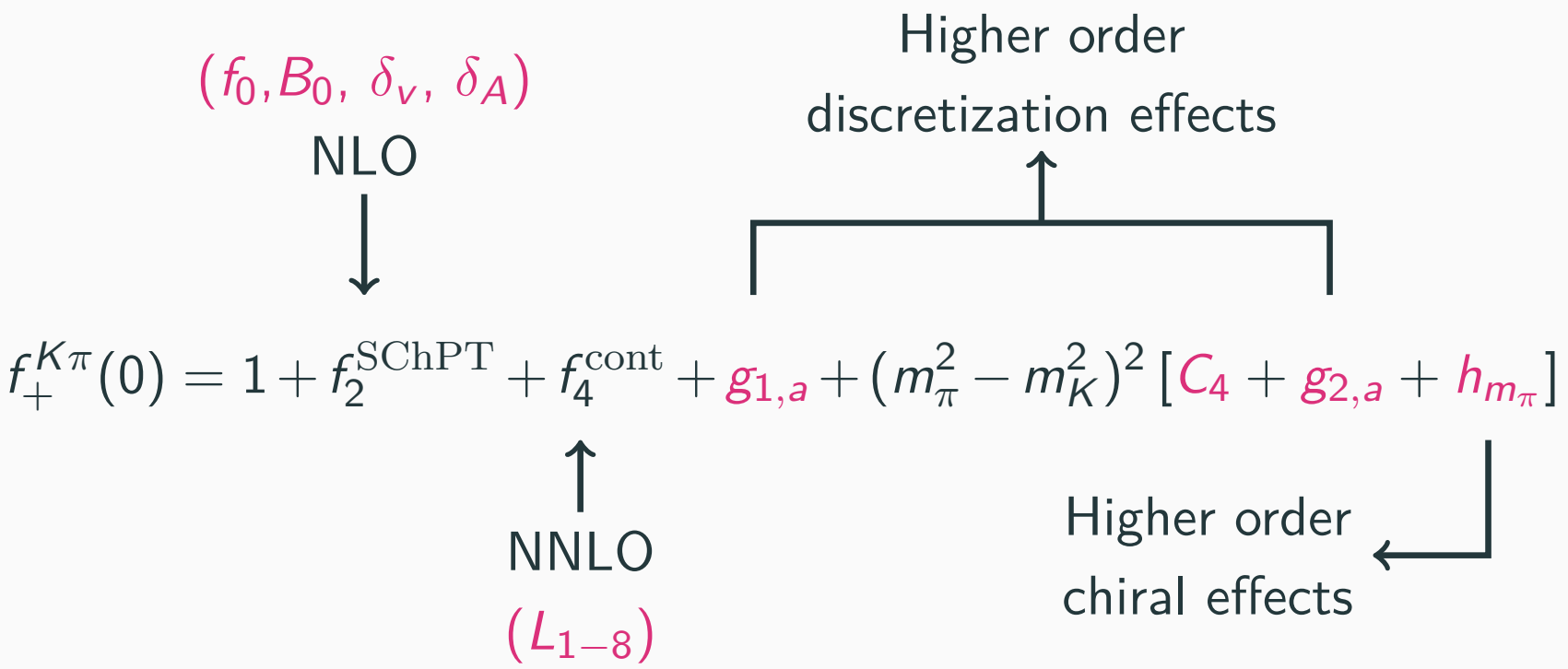
Working on strategy

BMA + Bootstrap: Correlations



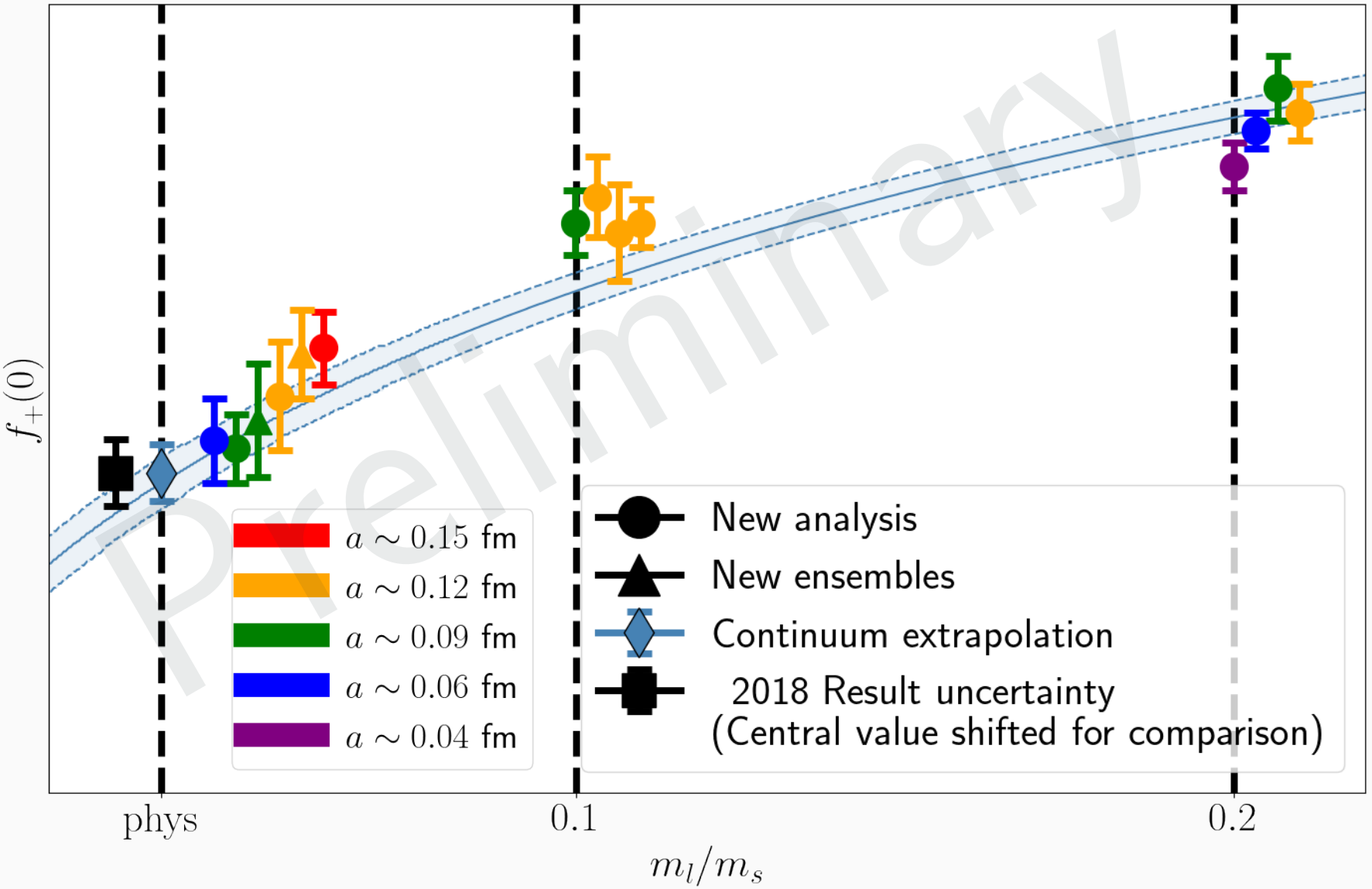
Semileptonic $f_+(0)$ analysis

Staggered Chiral Perturbation Theory (SChPT)

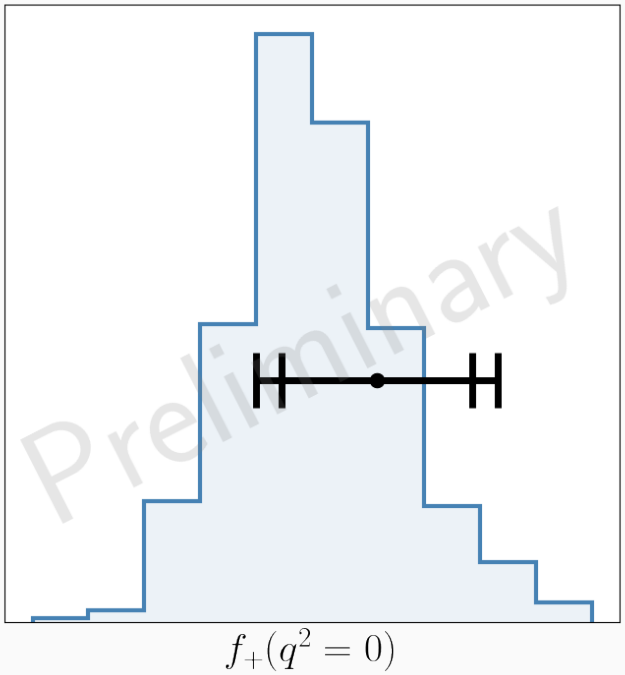


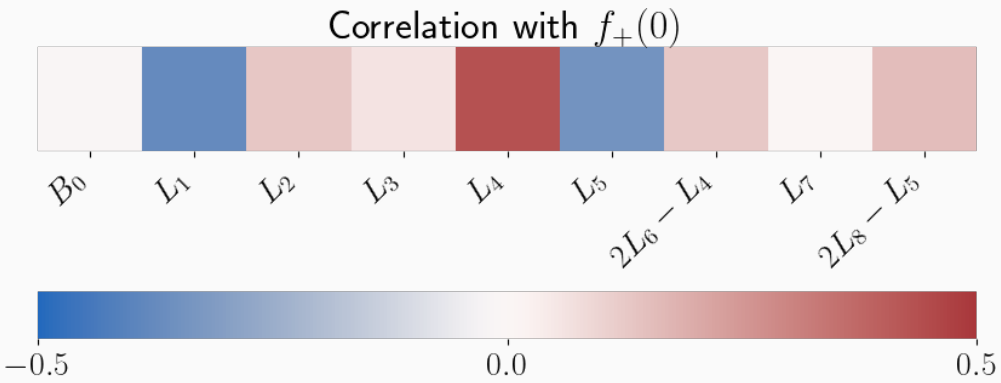
Some Low Energy Constants (LECs) are also present in the decay constants analysis

Semileptonic form factor $f_+(0)$: (RM talk at Lattice'25 2603.02994)



Correlation between $f_+(0)$ and LECs





Conclusions

- New determination of SU(3) ChPT LECs from $N_f = 2 + 1 + 1$ HISQ ensembles.
- Ongoing BMA study to assess systematic uncertainties.
- Updated value of $f_+(0)$ using preliminary value for LECs
- Next steps: compute f_K/f_π

Thanks for your attention!

Analytic terms

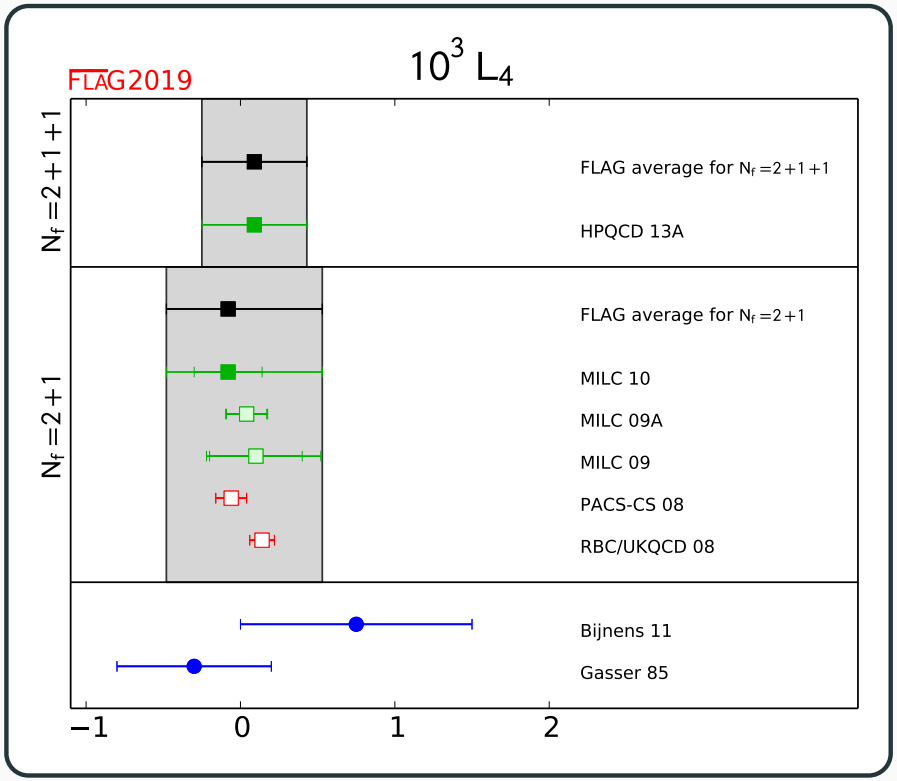
Defining

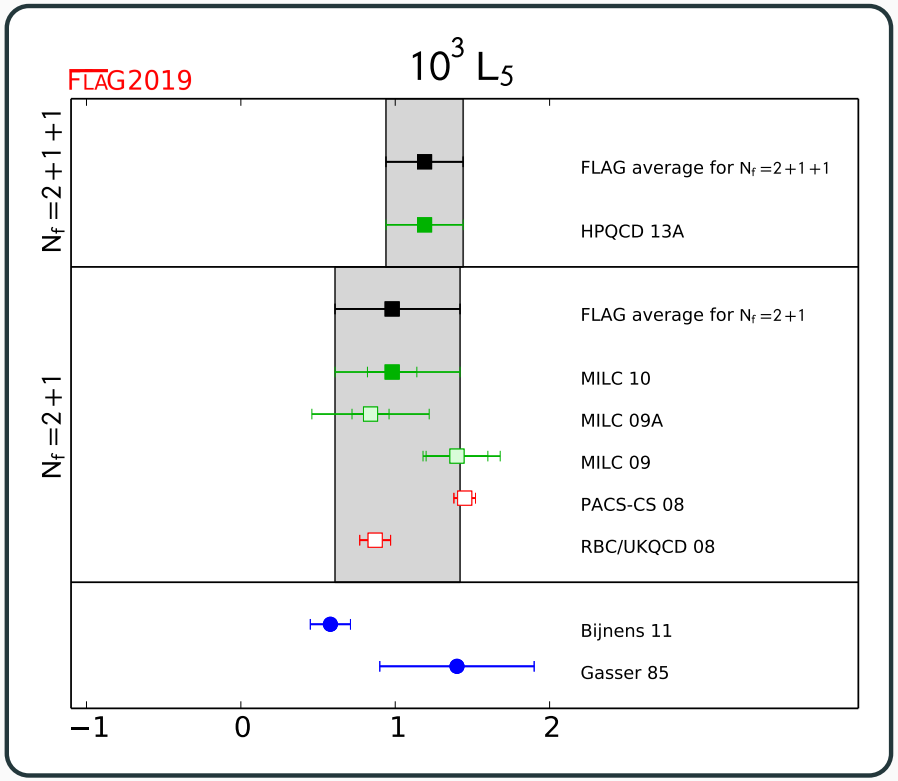
$$\chi_q = \frac{2B_0}{16\pi^2 f_0^2} m_q$$

Terms added to fit-function:

$$\begin{aligned}
&+Q_1(\chi_x + \chi_y)^2 \\
&+Q_2(\chi_x + \chi_y)(2\chi_l + \chi_s) \\
&+Q_3(2\chi_l + \chi_s)^2 \\
&+Q_4(\chi_x - \chi_y)^2 \\
&+Q_5(2\chi_l^2 + \chi_s^2)
\end{aligned}$$

FLAG: L_4 and L_5





FLAG: $2L_6 - L_4$ and $2L_8 - L_5$