Improving hadron creation operators for charmonium, glueballs and baryons

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in collaboration with

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The problems we face

When computing Euclidean temporal correlation functions we have:

Operator problem

- ✓ We use operators with desired quantum numbers...
- × ... they overlap with a tower of excitations we are not interested in.
- × **Large** excited-state contamination at early time separations.

At short time separations:

- ✓ Small(ish) statistical errors...
- × ... no clean resolution of low-lying states.

Noise problem

- ✓ Asymptotics of correlations contain states of interest...
- × ... also dominated by growing statistical noise.
- × Some observables more heavily affected than others.

At large time separations:

- ✓ States of interest dominate...
- × ... large statistical errors make clean extraction extremely hard.

This talk: Tackling the operator problem to partially escape the noise problem.

Nucleon spectroscopy with Distillation profiles

We extend the bases of nucleon operators by including distillation profiles. J. A. Urrea-Niño *et al.*, PRD 106 (2022) 3, 034501, PRD 112, 074502 (2025), PRD 114, 014509 (2026)
Baryon triplets/elementals:

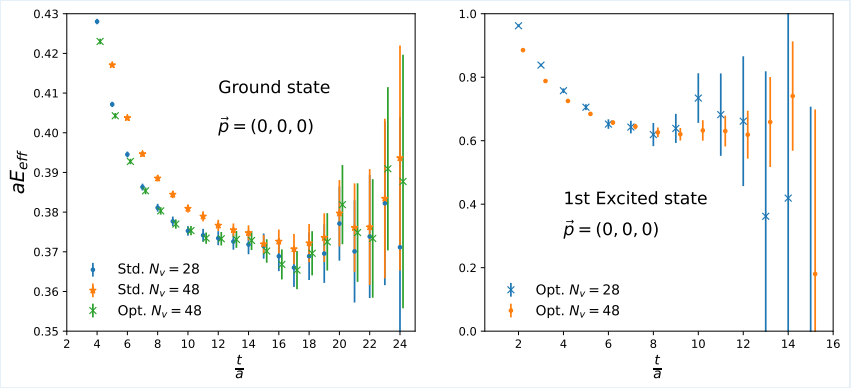
$$\Phi_{ijk}[\vec{p},t] \rightarrow \Phi^{(a)}_{ijk}[\vec{p},t] = g^{(a)}_i \cdot g^{(a)}_j \cdot g^{(a)}_k \cdot \Phi_{ijk}[\vec{p},t]$$

$g^{(a)}_i$: a-th profile function evaluated at i-th eigenvalue.
$a = 1, 2, ..., N_B$: Different profiles, e.g. Gaussians with different widths.

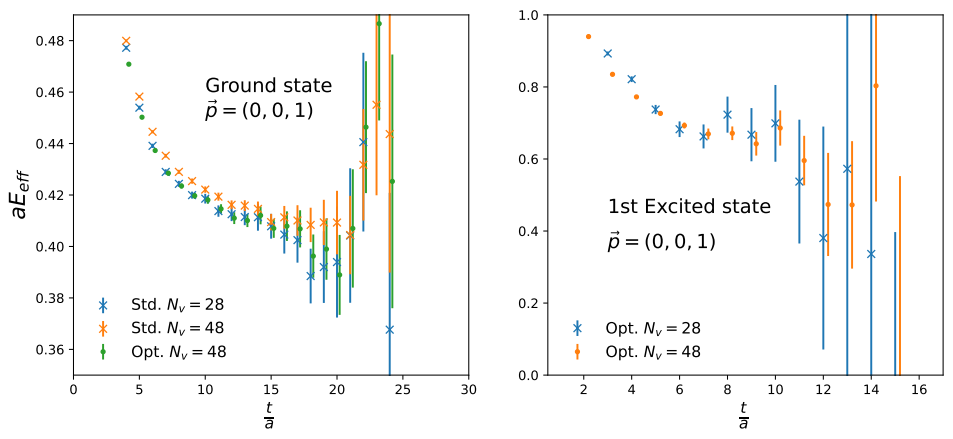
$\rightarrow$ We solve a GEVP with the resulting $N_B \times N_B$ correlation matrix.[2]

- ✓ **Computationally cheap** modification at contraction level.
- ✓ Distillation vectors **used optimally** for each operator/excitation/momentum.
- ✓ **No need to tune** a different number of vectors for each system of interest.
- ✓ Can be **extended** to NN, $N\pi$, etc... calculations.

[2] Work with Fernando Romero-López and the Baryon Scattering Collaboration.

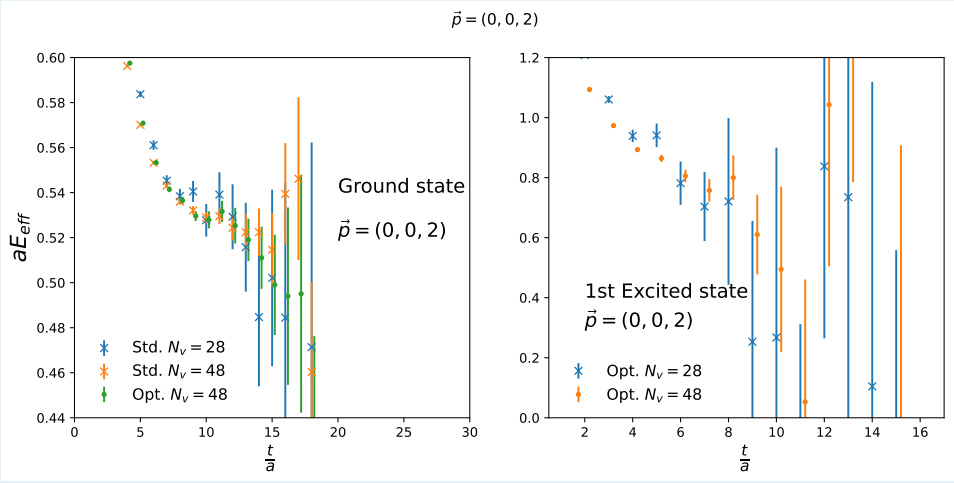


- ✓ Optimal profile (Opt): faster convergence at fixed $N_v = 48$.
- ? Standard distillation (Std): with $N_v = 28$ matches also good. Additional vectors unnecessary?
- ! $N_v = 28$ not enough to resolve radial excitation.
- ✓ Profiles guarantee each vector gets used optimally.
- ✓ **No** need to tune N_v for each case.



✓ Improvement extends to non-zero momentum...

× ... but can saturate → Larger N_v is preferred.



! Large momenta $\rightarrow$ N_v must be larger!

See talk by J. Crawford (Thursday, 3:20 PM) for further applications of distillation profiles for nucleons.

Glueball spectroscopy with improved operators

We use gluonic operators built from $\mathcal{B}_i = \frac{1}{2}\epsilon_{ijk}F_{jk}$ and its gauge-covariant adjoint derivative D_k for a **clearer physical picture** than arbitrary spatial loops.

In Y. Chen *et al.*, hep-lat/0512033 (2005):

- ✓ Subduce from fixed-J^{PC} operators in $SO(3)$.
- ✓ Tensor products $1 \otimes 1 \otimes 1 \otimes ... \otimes 1$ give any J, e.g. $1(\mathcal{B}_i) \otimes 1(\mathcal{B}_i) = 0 \oplus 1 \oplus 2$

In J. A. Urrea-Niño *et al.*, Phys. Rev. D 114, 014509 (2026):

- ✓ Operators "remember" the J from where they were subduced. HadSpec uses this approach for meson and baryon operators.
- ✓ Discretize directly $\mathcal{B}_i$ and D_i via clover form and nearest-neighbors difference. Contrary to using loop shapes for the final operator discretization.
- ✓ Operators built using *opbasis*. See talk by N. Husung (Monday, 5:30 PM) Eur. Phys. J. C 86 (2026) 4, 427
- ✓ Avoids measuring all cubic transformations of the operator to project onto lattice irrep. Unlike for arbitrary spatial loop shapes.
- ✓ Parallel code implementation is straightforward. Unlike for arbitrary spatial loop shapes.

Continuum-like operator examples

For A_1^{++}:

$\mathcal{O}(t) = \sum_{\vec{x}} \epsilon_{ijk} \mathrm{Tr}\left(\mathcal{B}_i(\vec{x},t)\mathcal{B}_j(\vec{x},t)\mathcal{B}_k(\vec{x},t)\right)$

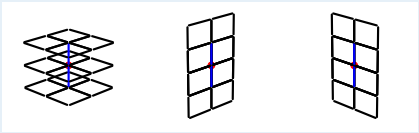
$\mathcal{O}(t) = \sum_{\vec{x}} |\epsilon_{ijk}||\epsilon_{jmn}| \mathrm{Tr}\left(\mathcal{B}_i(\vec{x},t) D_m D_n \mathcal{B}_k(\vec{x},t)\right)$

For A_1^{-+}:

$\mathcal{O}(t) = \sum_{\vec{x}} \epsilon_{ijk} \mathrm{Tr}\left(\mathcal{B}_i(\vec{x},t) D_j \mathcal{B}_k(\vec{x},t)\right)$

$\mathcal{O}(t) = \sum_{\vec{x}} \epsilon_{ijk} \mathrm{Tr}\left(\mathcal{B}_i(\vec{x},t) D_j^2 D_k \mathcal{B}_j(\vec{x},t)\right)$

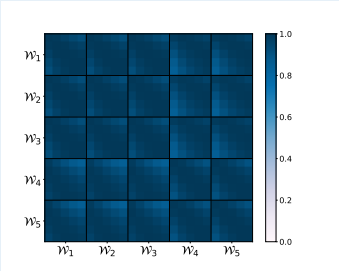
Non-trivial structure and spatial extent



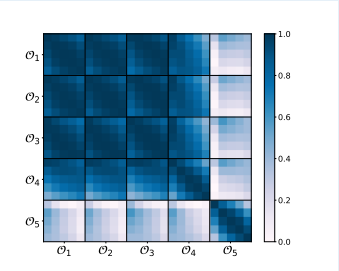
Links involved in $D_k^2 B_i$ for fixed k and all i.

- ✓ Use $4a \times 4a$ cube.
- ✓ Derivative introduces minus signs.

Spatial loops + Smearing



Subduced operators + Smearing



Charmonium multi-hadron operators with distillation profiles

Recent scattering calculations relate to charmonium resonances, e.g. $D\bar{D}$, $D_s\bar{D}_s$, etc... by HadSpec. D. J. Wilson *et al.*, Phys. Rev. D 109, 114503 (2024)
Resolving the finite-volume spectrum requires a large basis of operators, including 2-hadron operators with back-to-back momenta. C. E. Thomas *et al.*, Phys. Rev. D 85 014507 (2012), H. Yan *et al.*, JHEP 10 (2025) 210

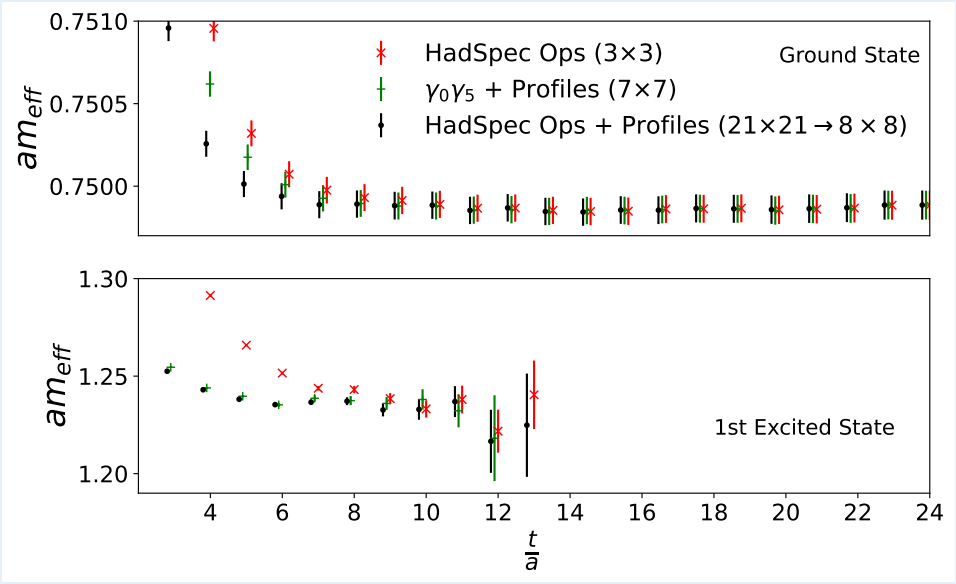
Operators from the literature

- ✓ Large HadSpec basis up to 3-derivatives.
- ✓ Multi-hadron operators! Different relative momenta allowed.
- ✓ Account for flavor symmetry! $\bar{D}D$, charmonium + light meson, etc...

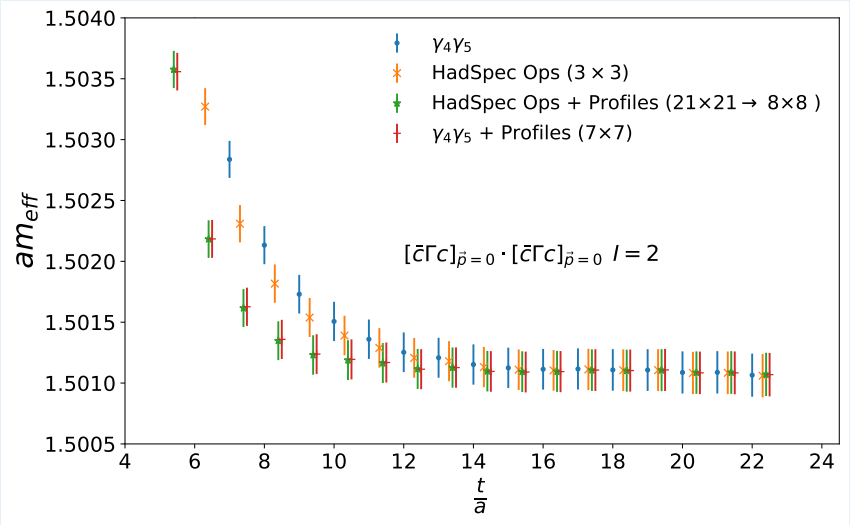
List grows quickly, can we improve without large additional cost?

New improvement

1. Use distillation profiles to improve 1-hadron operators. Technique extends to non-zero momentum too.
2. Use optimal 1-hadron operators to build the multi-hadron operators. Same approach as HadSpec.



$I = 1$ A_1^{-+} charmonium @ $N_f = 2$, $N_v = 200$, $m_\pi \approx 2.2$ GeV, 48×24^3, $a \approx 0.066$ fm.



- **No 2-meson GEVP** → Use only optimized 1-hadron operators.

✓ No optimality guaranteed but improvement is seen!

Conclusions

Baryons

- ✓ Distillation profiles bring **earlier plateaus at very little additional cost**.
- ✓ **No need to fine tune** number of vectors.

Glueballs

- ✓ Subduced-from-continuum operators bring **more physical information**.
- ✓ Systematic construction to **avoid degeneracy from smearing**.
- ✓ **More flexible implementation** than spatial loops, easier access to different irreps.

Charmonium

- ✓ Distillation profiles complement and largely **improve** HadSpec derivative-based basis.
- ✓ Multi-hadron operators can be **optimized at very little additional cost**.

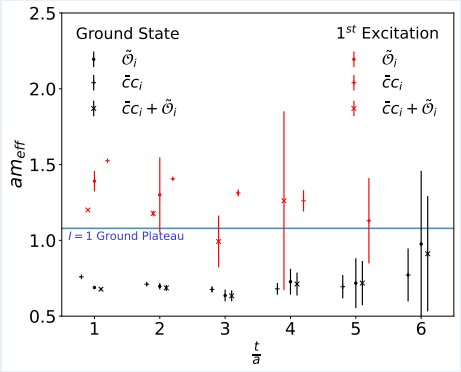
→ Talk by J. Koponen (Thursday, 4:30 PM) for profiles with momentum and 3-point functions.

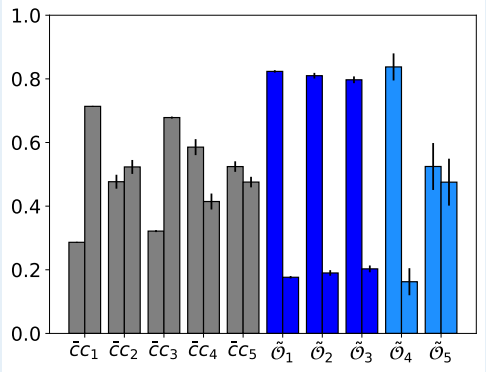
→ My talk (Friday, 5:30 PM) for profiles in charmonium at finite temperature.

Thank you for your attention!

Identifying continuum J in A_1^{++} Urrea-Niño *et al.*, Phys. Rev. D 114, 014509 (2026)

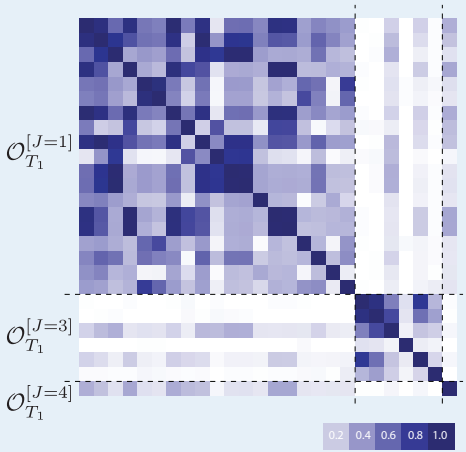
We mix the glueball operators with iso-scalar charmonium ones to study their mixing.





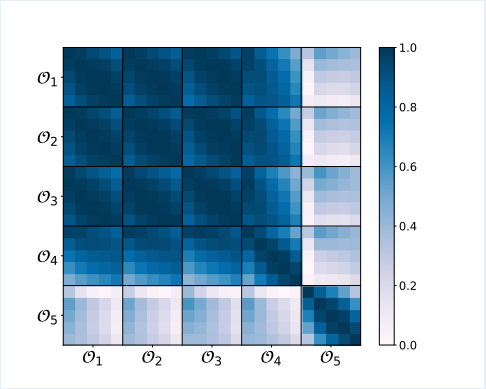
- ✓ Charmonium and glueball operators see common ground state.
- ✓ Mixing GEVP required to access first excitation.
- ✓ $J = 0$ glueball operators favor the ground state.
- ✓ $J > 0$ contributions favor higher excitations not resolved here.

Effects of subduction



Subduction approach for mesons.

J. J. Dudek *et al.*, Phys. Rev. D 82 (2010) 034508

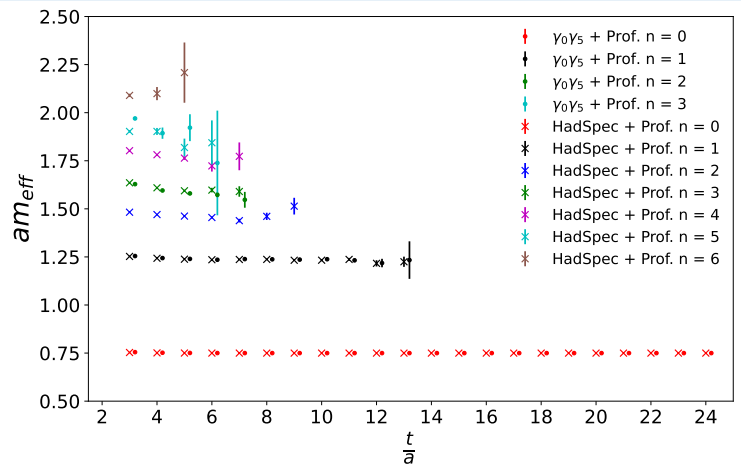


Subduction approach for glueballs.

Y. Chen *et al.*, hep-lat/0512033 (2005)

J. A. Urrea-Niño *et al.*, Phys. Rev. D 114, 014509 (2026)

Profiles **and** derivative operators are needed



- Derivative operators are needed to resolve some states. Possible members of hybrid super-multiplet 1^{+-} $(g) \otimes 1^{--}$ $(\bar{c}c) \rightarrow 0^{-+}$. L. Liu *et al.*, JHEP 07 (2012) 126