Baylor University

Electric & Magnetic Polarizability of Pion from Four-Point Functions on nHYP Ensembles

Sudip Shiwakoti[a], Ben Luke[a] Shayan Nadeem[a], Andrei Alexandru[b], Frank Lee[b], Walter Wilcox[a]

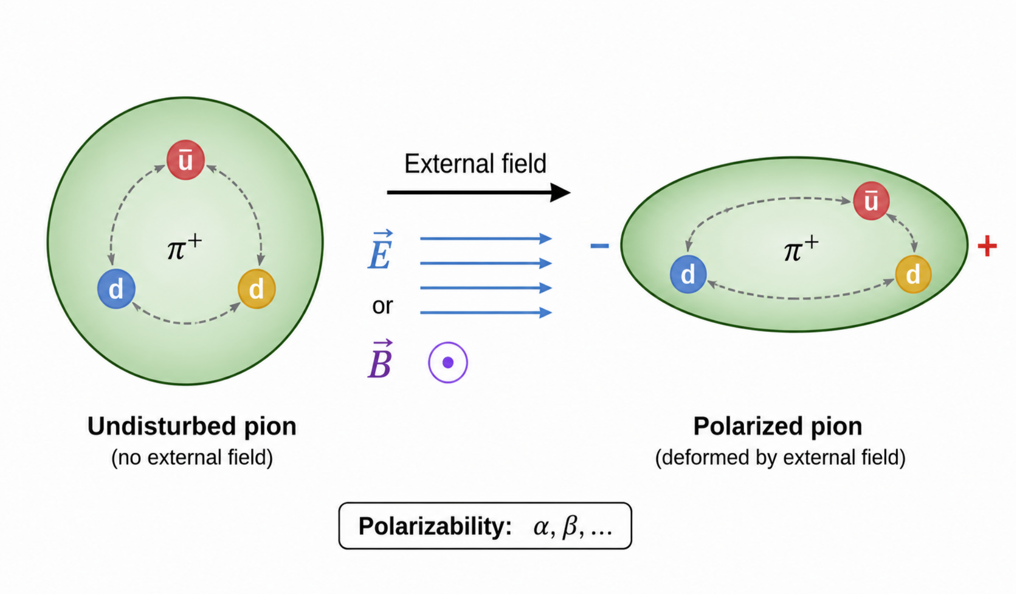
[a] Department of Physics, Baylor University, Waco, TX, USA
[b] Department of Physics, The George Washington University, Washington DC, USA

Introduction

➢ Electromagnetic Polarizability

- Refers to the tendency of matter, in the presence of an electric or magnetic field, to acquire a dipole moment with the strength proportional to the field.

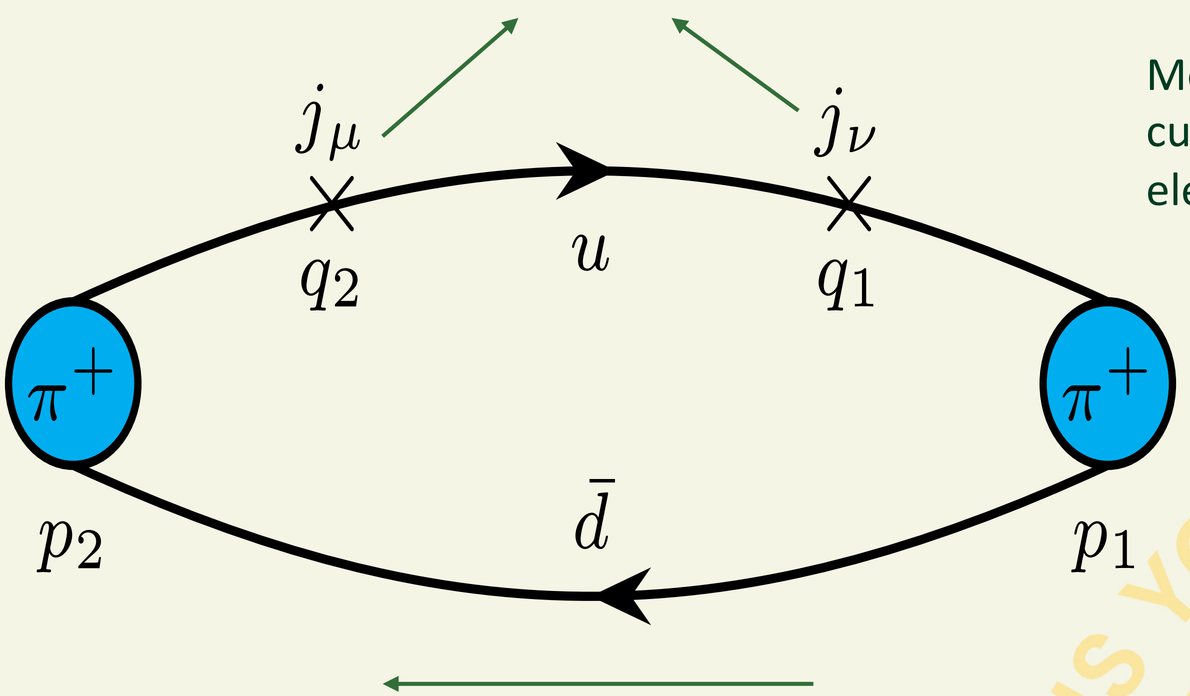
- Reveals information about the internal structure of hadron such as
  - Current Distribution
  - Charge Distribution



➢ We are implementing **Four-Point Function Method** to extract polarizabilities.

Pictorial representation of a four-point function for charged pion

Charged current insertion points



Momentum transferred through two currents allows us to probe the electromagnetic response of the pion

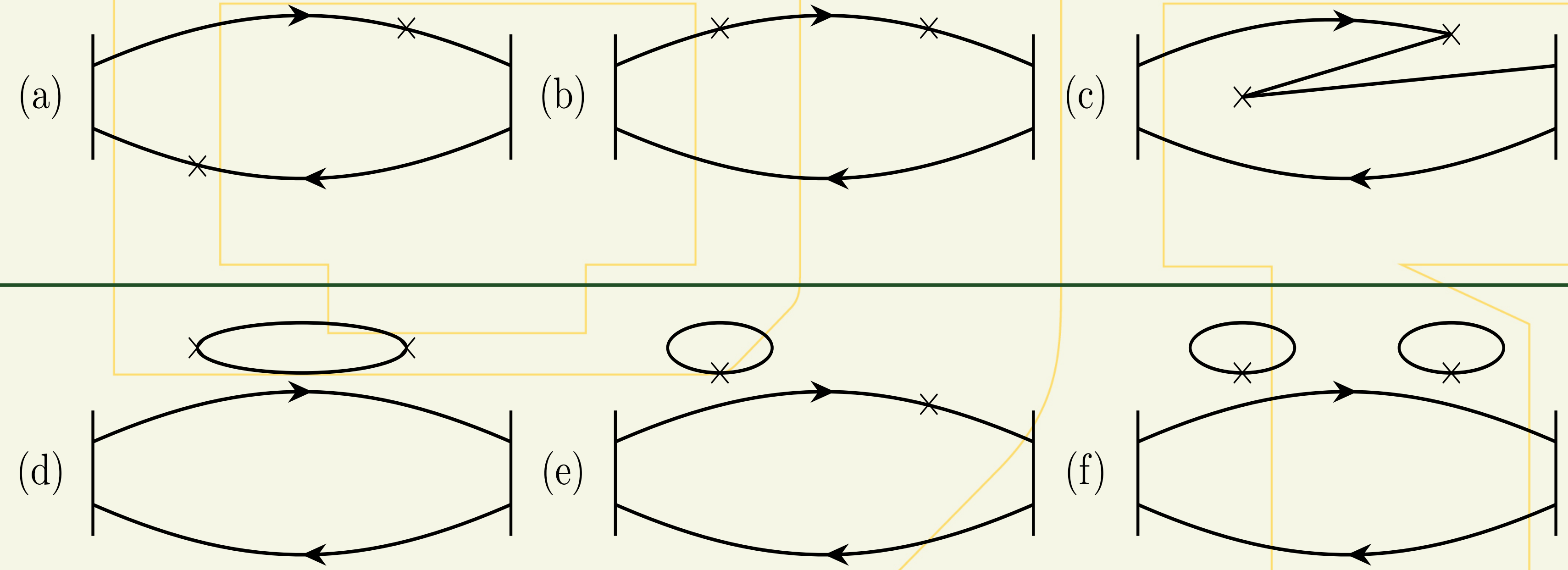
Sink at t_3

Source at t_0

Time flows from right to left

Quark line diagrams contributing to polarizabilities.

Our work is only on these connected diagrams a, b and c



Previous Work and This Work

Previous work

- Quenched Wilson Action
- Mass = 370–1100 MeV
- One primary lattice Volume

Present Work

- nHYP Dynamical Action
- Mass = 220 MeV, 315 MeV
- Three volume at each pion mass

Ensemble Description

Ensemble	Mass (MeV)	$L_x \times L_y \times L_z \times L_t$	a (fm)	T_0	T_1	T_3
1	315	$24^3 \times 48$	0.1210(2)(24)	6	17	41
2	315	$30 \times 24^2 \times 48$		6	17	41
3	315	$48 \times 24^2 \times 48$		6	17	41
4	220	$24^3 \times 64$	0.1215(3)(24)	8	19	55
5	220	$28 \times 24^2 \times 64$		8	19	55
6	220	$32 \times 24^2 \times 64$		8	19	55

Equations used:

Charged Pion

$$\alpha_E^{\pi^+} = \frac{\alpha\langle r_E^2\rangle}{3m_\pi} + \lim_{\mathbf{q}\to 0}\frac{2\alpha}{\mathbf{q}^2}\int dt\,\left[Q_{44}(\mathbf{q},t) - Q_{44}^{\text{elas}}(\mathbf{q},t)\right]$$

$$\beta_M^{\pi^+} = -\frac{\alpha\langle r_E^2\rangle}{3m_\pi} + \lim_{\mathbf{q}\to 0}\frac{2\alpha}{\mathbf{q}^2}\int dt\,\left[Q_{11}(\mathbf{q},t) - Q_{11}(\mathbf{0},t)\right]$$

Neutral Pion

$$\alpha_E^{\pi^0} = \lim_{\mathbf{q}\to 0}\frac{2\alpha}{\mathbf{q}^2}\int dt\,Q_{44}(\mathbf{q},t)$$

$$\beta_M^{\pi^0} = \lim_{\mathbf{q}\to 0}\frac{2\alpha}{\mathbf{q}^2}\int dt\,\left[Q_{11}(\mathbf{q},t) - Q_{11}(\mathbf{0},t)\right]$$

- Elastic charge radius term is absent for neutral pion.

$$Q_{44}^{\text{elas}}(\vec{q},t) = \frac{(E_\pi + m_\pi)^2}{4E_\pi m_\pi} F_\pi^2(\vec{q}^2)\, e^{-a(E_\pi - m_\pi)t}$$

$$\langle r_E^2\rangle = -6\left.\frac{dF_\pi(\vec{q}^2)}{d\vec{q}^2}\right|_{\vec{q}^2\to 0}$$

Normalized four-point functions: Charged pion electric

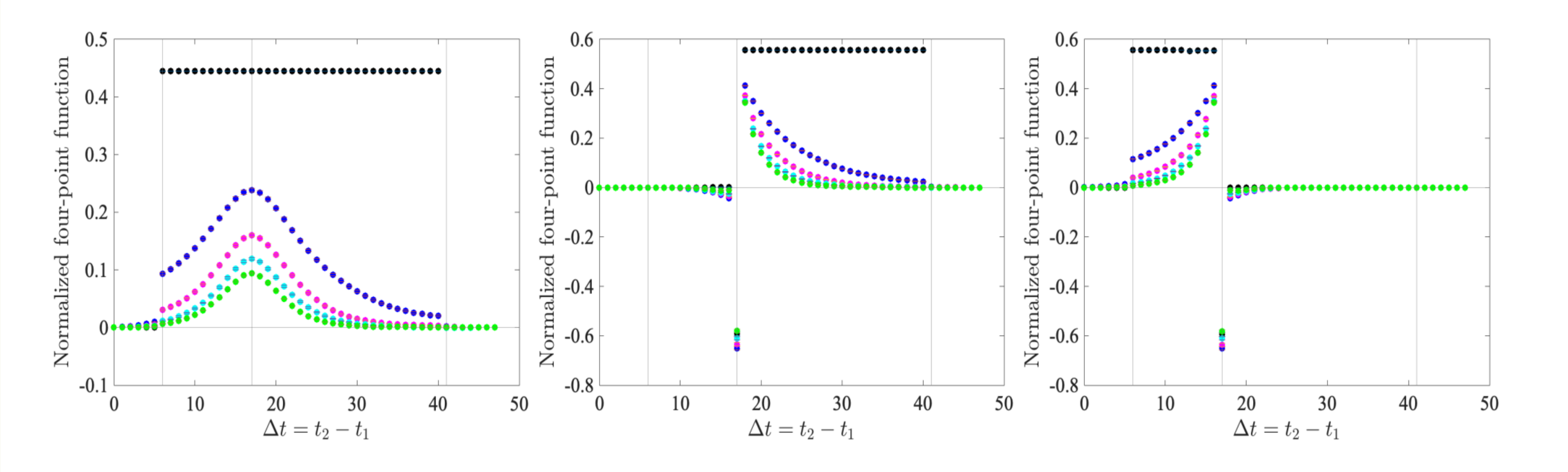


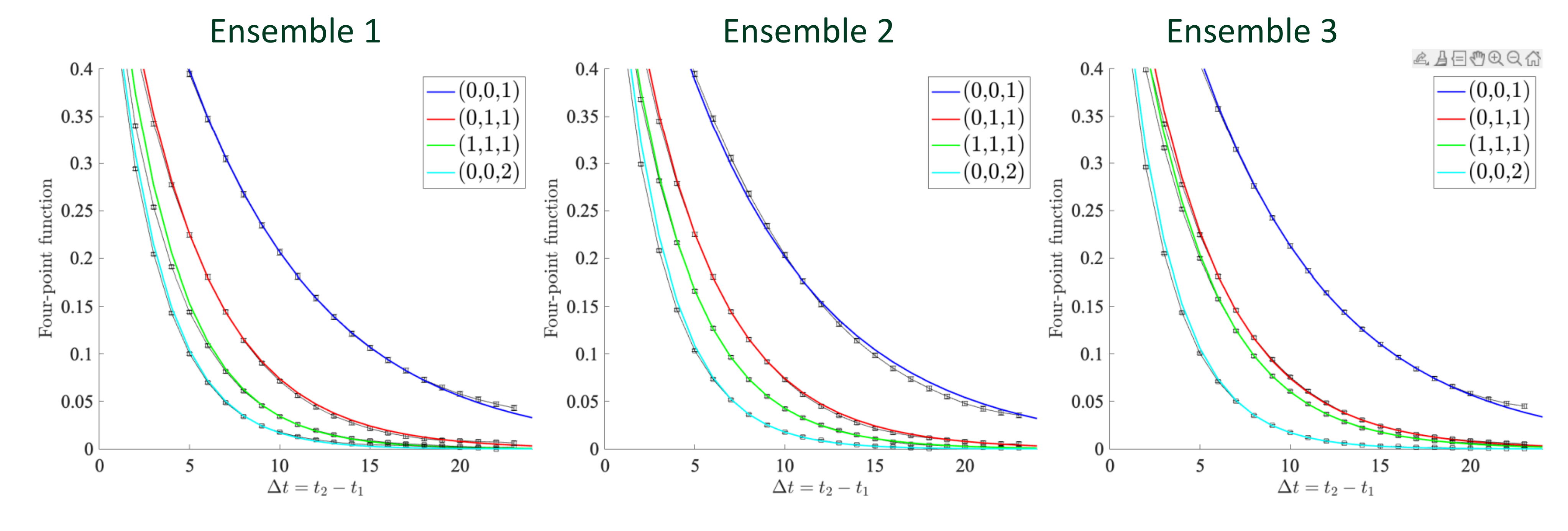
Diagram A

Diagram B

Diagram C

Four-Point Function Fits:

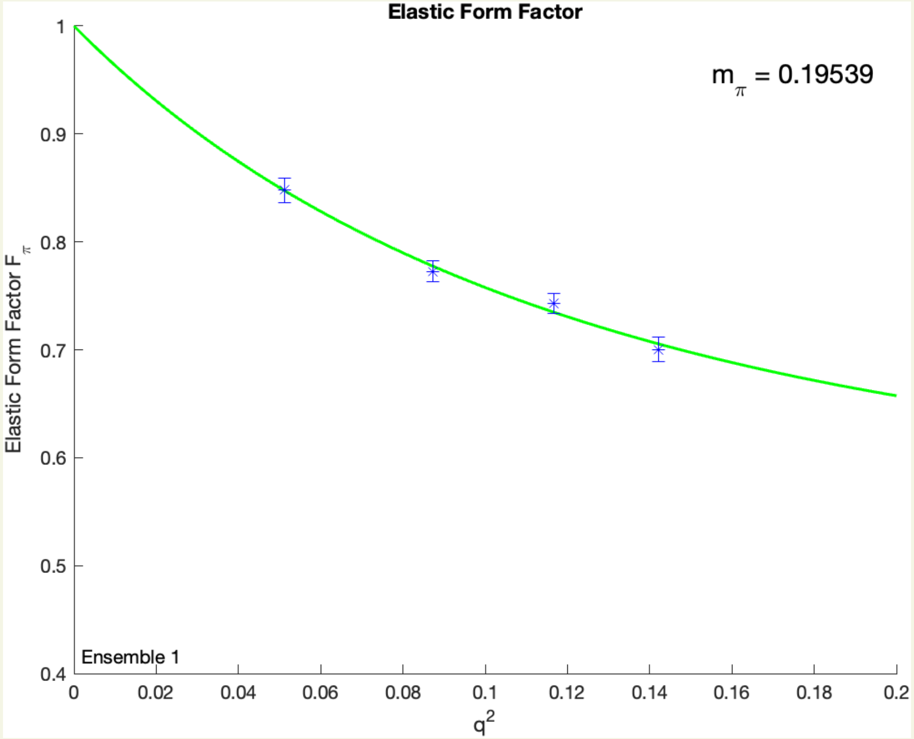
$$Q_{44}^{\text{elas}}(\boldsymbol{q},t) = \frac{(E_\pi + m_\pi)^2}{4E_\pi m_\pi} F_\pi^2(\boldsymbol{q}^2) e^{-a(E_\pi - m_\pi)t}.$$

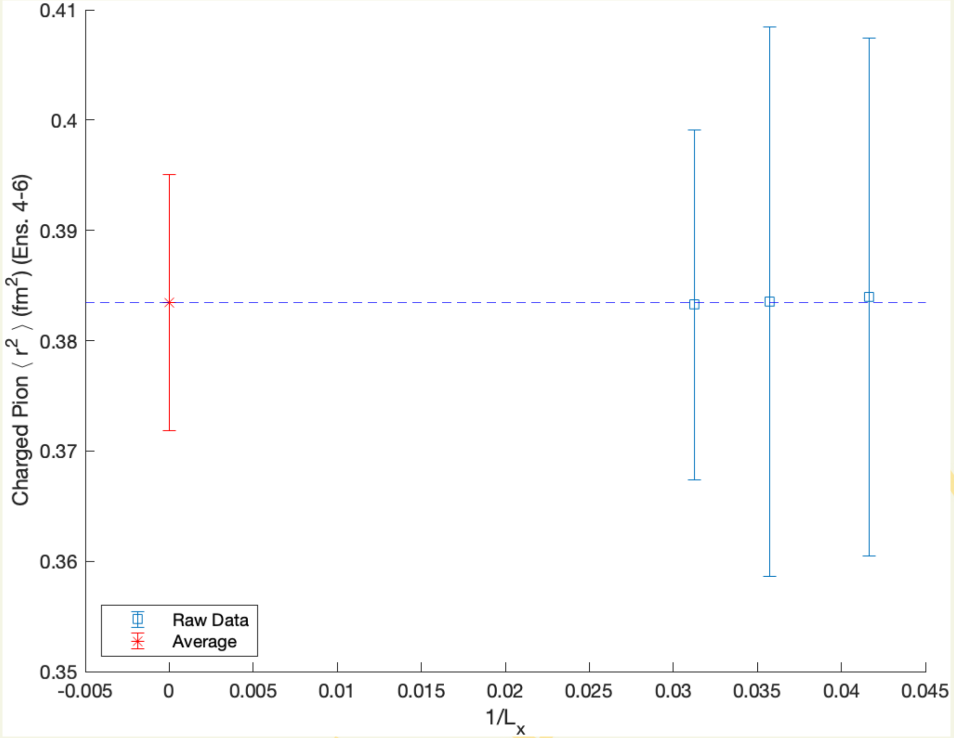


- Sum of A and B at large time separations is fitted according to above eqn.
- We obtain form factors from the 4-point function fits. Then we fit the form factors to the z-expansion.

Charged radius squared:

$$\langle r_E^2 \rangle = -6 \frac{dF_\pi(\vec{q}^2)}{d\vec{q}^2}\bigg|_{\vec{q}^2 \to 0}$$





Form factor fitting using z-expansion.

Charged Pion Electric Polarizability Analysis:

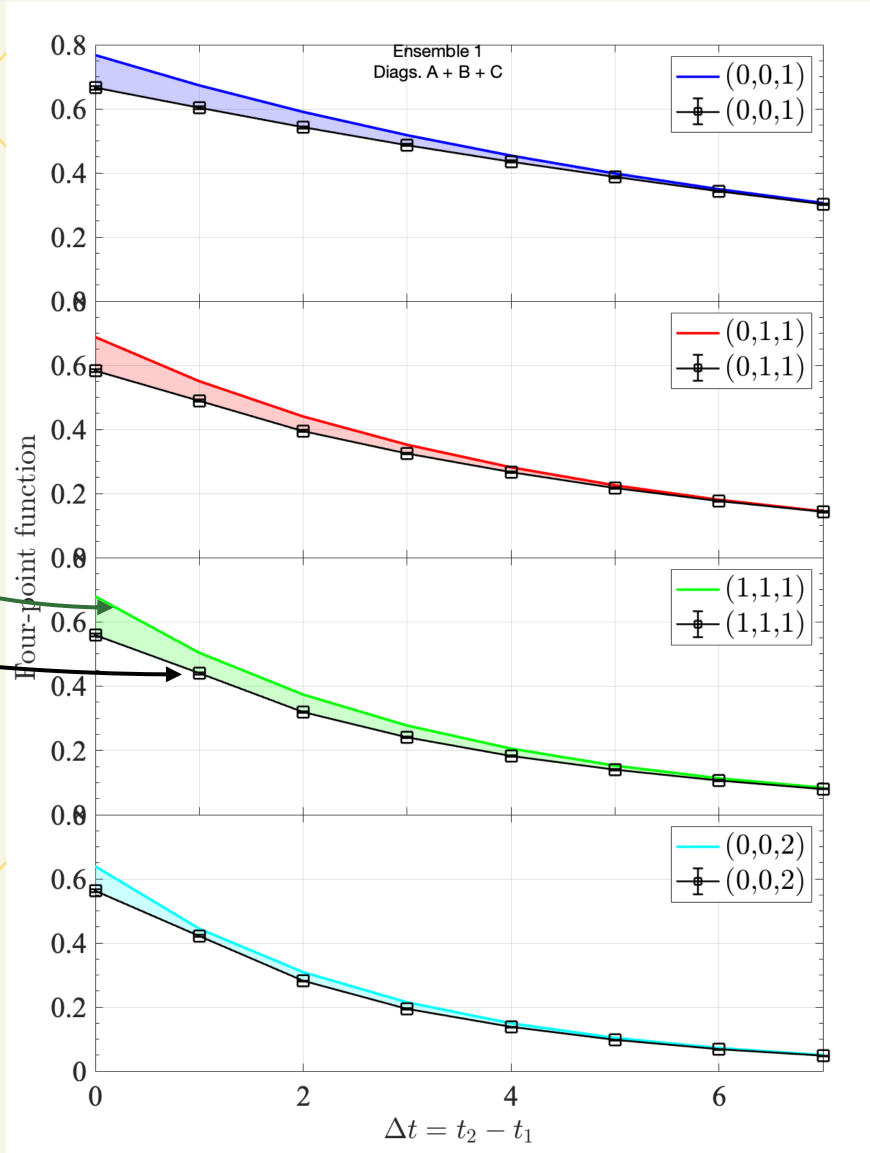
Inelastic Contribution:

Elastic part

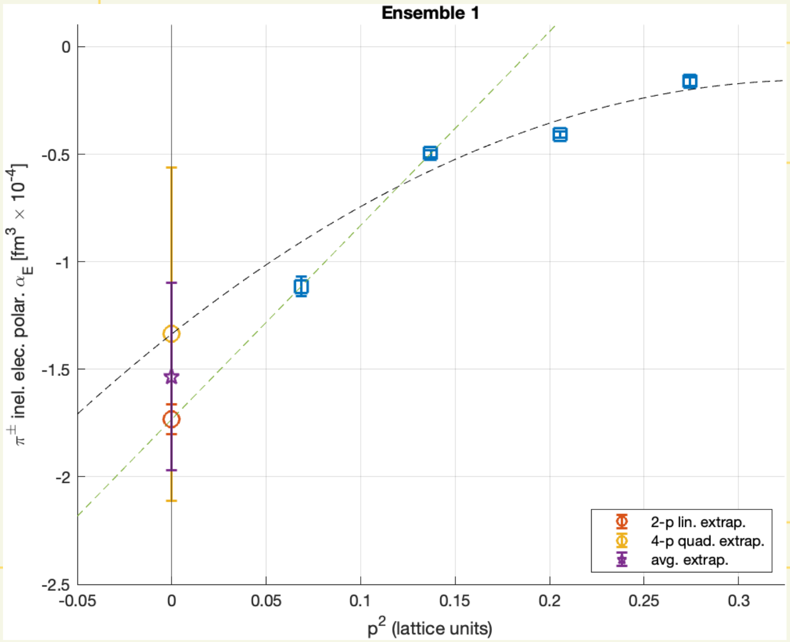
Inelastic part

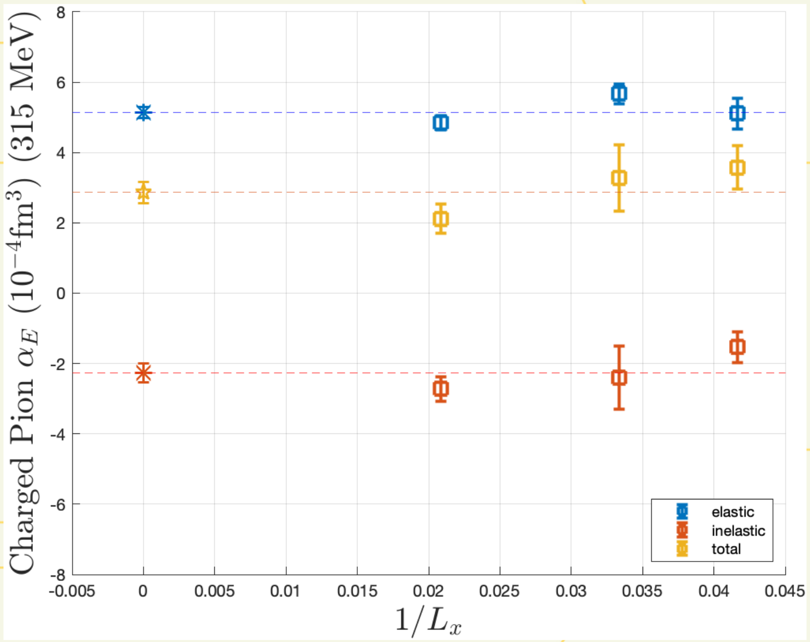
$$\alpha_E^{\pi^+} = \frac{\alpha \langle r_E^2 \rangle}{3 m_\pi} + \lim_{\mathbf{q}\to 0} \frac{2\alpha}{\mathbf{q}^2} \int dt \left[Q_{44}(\mathbf{q}, t) - Q_{44}^{\mathrm{elas}}(\mathbf{q}, t) \right]$$

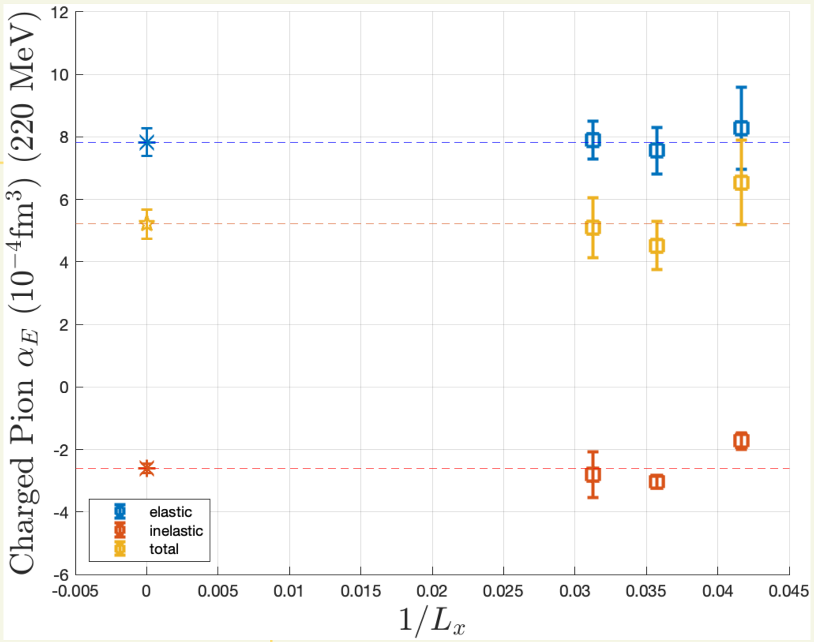
Area between curves is the inelastic contribution.



Charged Pion Electric Polarizability Cont:





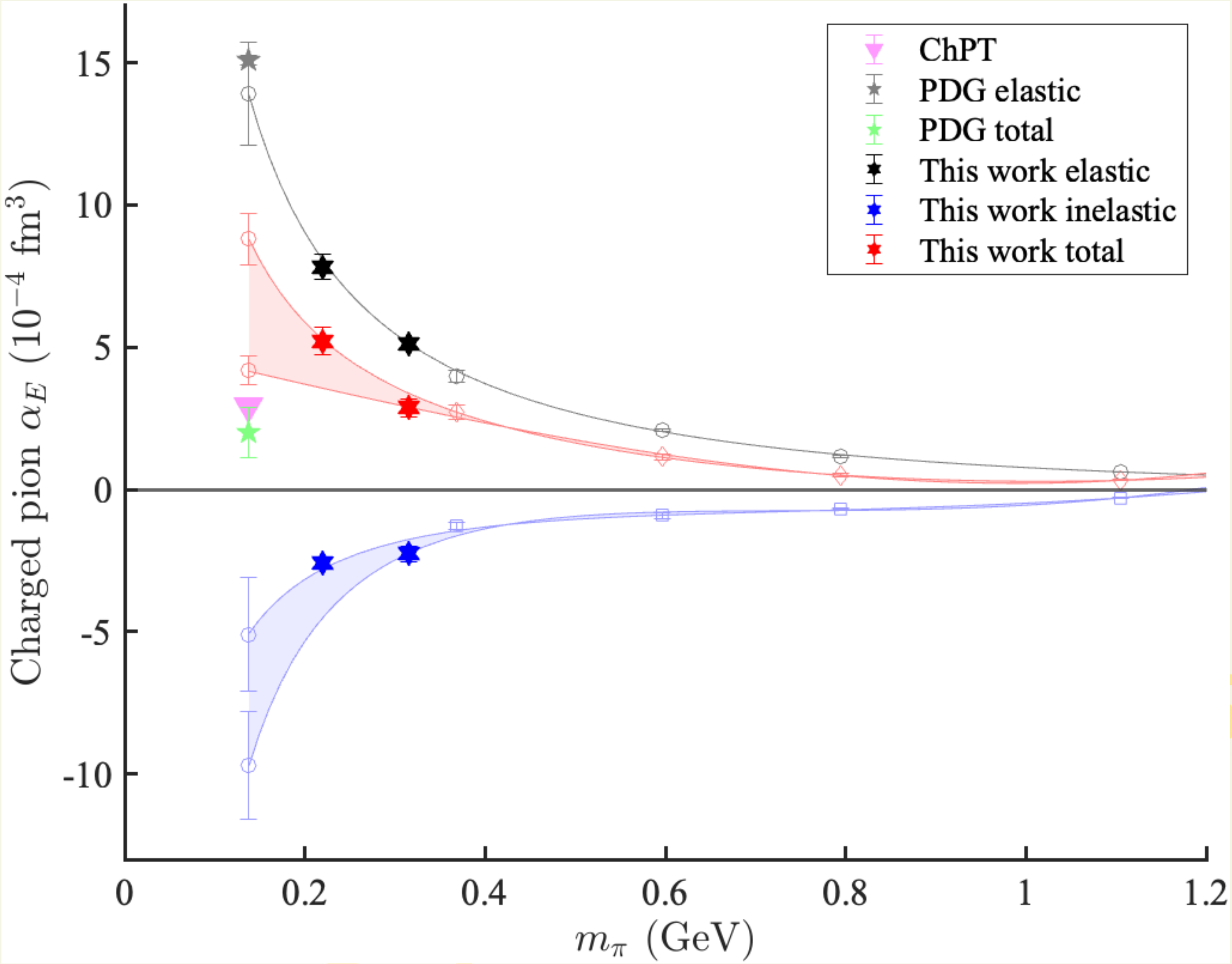


Momentum dependence of inelastic term and extrapolation to q^2=0.

m_π (MeV)	Pion elastic α_E	Pion inelastic α_E	Pion Total α_E
315	5.1316 ± 0.1554	-2.2625 ± 0.2636	2.8691 ± 0.3060
220	7.8229 ± 0.4411	-2.6056 ± 0.1543	5.2174 ± 0.4673

Over lay Plot for Comparison to Previous Work

- Comparison of new data with final result from Lee et al.(2023)
- New data in bold stars
- Result follows the trend of Lee et al.(2023)



Charged Pion Magnetic Polarizability Analysis.

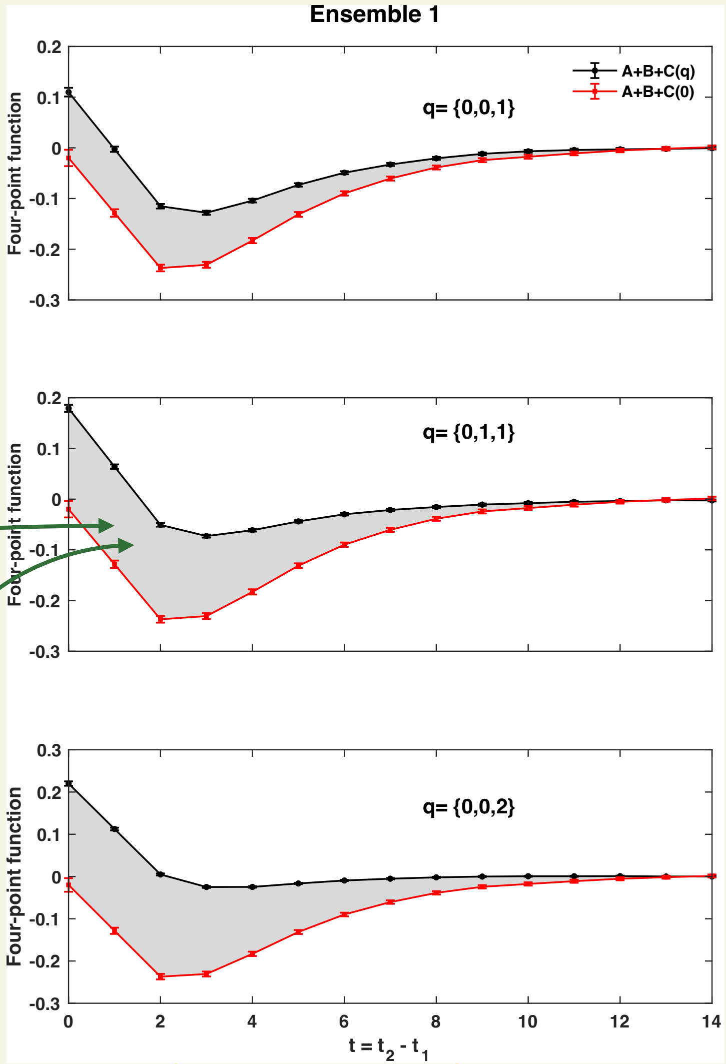
Elastic Contribution

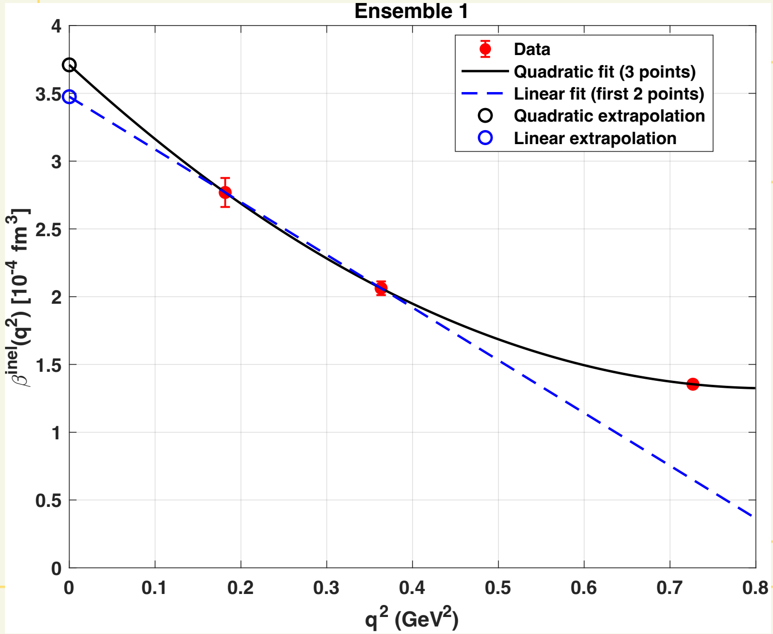
Inelastic Contribution

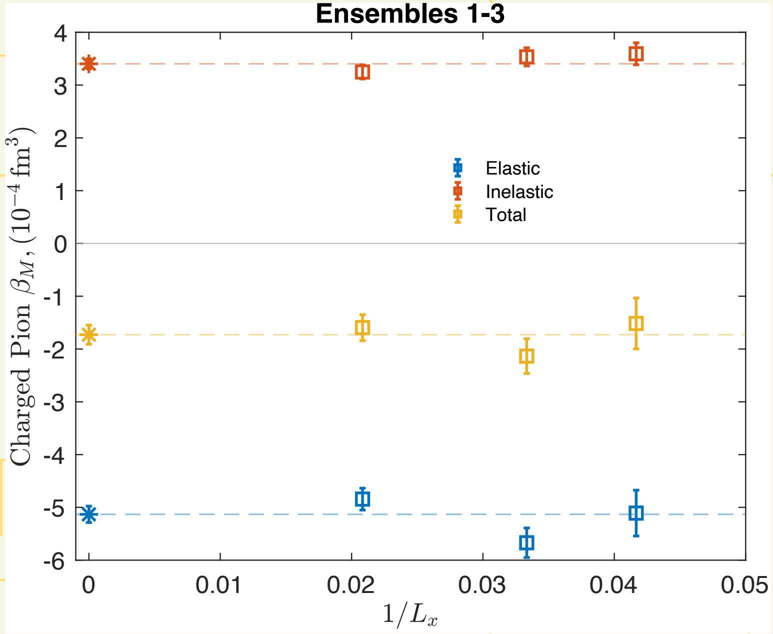
$$\beta_M^{\pi^+} = -\frac{\alpha\langle r_E^2\rangle}{3m_\pi} + \lim_{\mathbf{q}\to 0}\frac{2\alpha}{\mathbf{q}^2}\int dt\,[Q_{11}(\mathbf{q},t) - Q_{11}(\mathbf{0},t)]$$

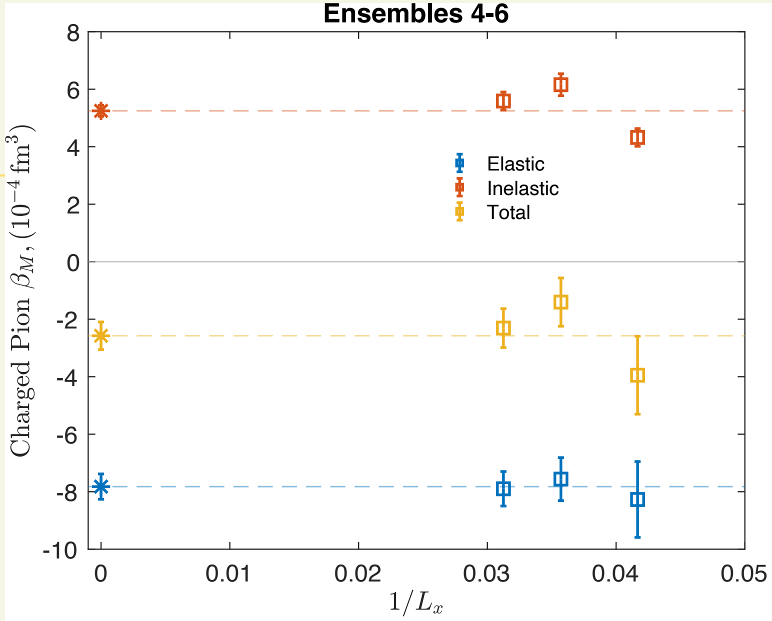
We get from electric polarizability analysis

Area between curves gives the inelastic contribution







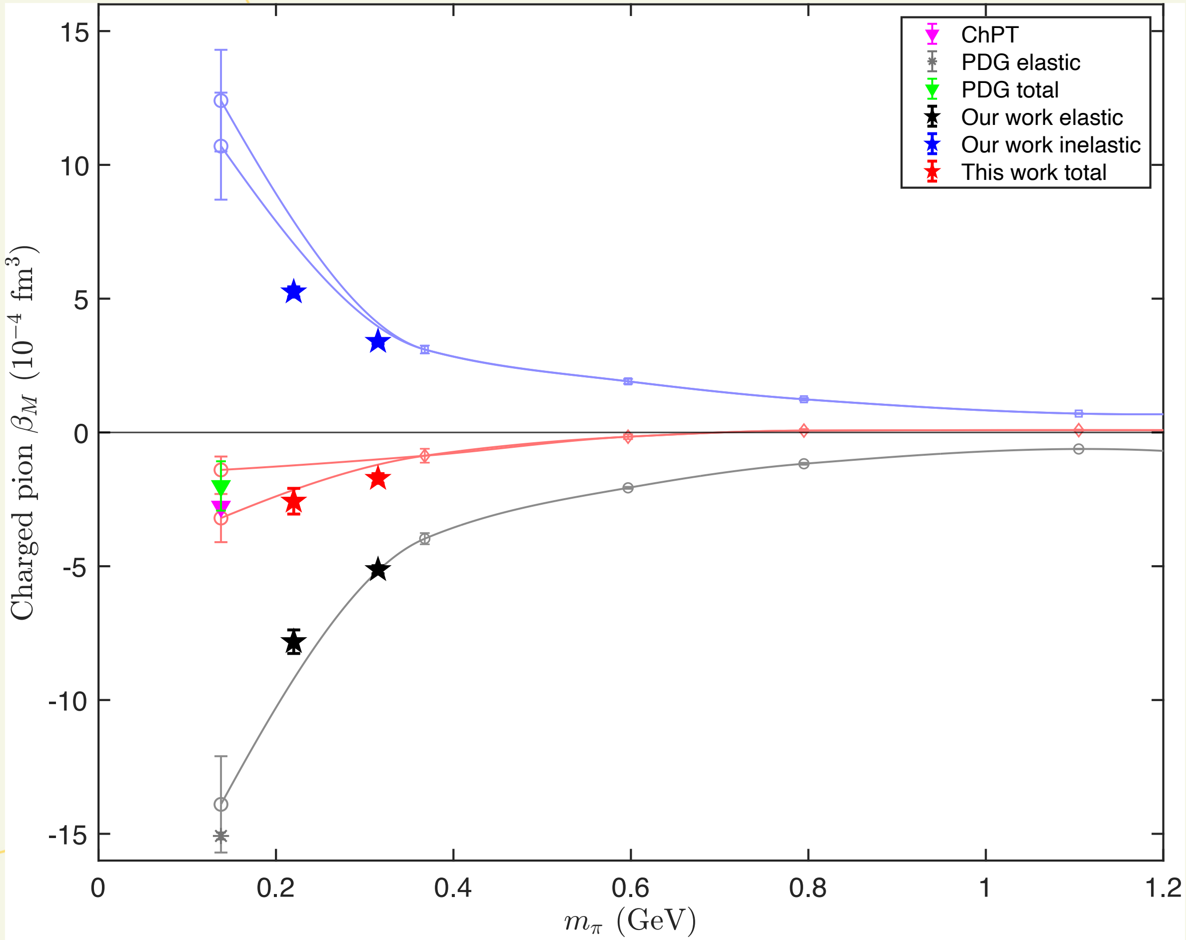


Momentum dependence of inelastic term and extrapolation to q^2=0.

m_π (MeV)	Pion elastic β_M	Pion inelastic β_M	Pion β_M
315	-5.1316 ± 0.1554	3.4028 ± 0.0936	-1.7288 ± 0.1814
220	-7.8229 ± 0.4411	5.2472 ± 0.1905	-2.5757 ± 0.4805

Overlay Plot for Final Comparison

- Comparison of new data with final result from Lee et al.(2023)
- New data in bold stars
- Result follows the trend of Lee et al.(2023)



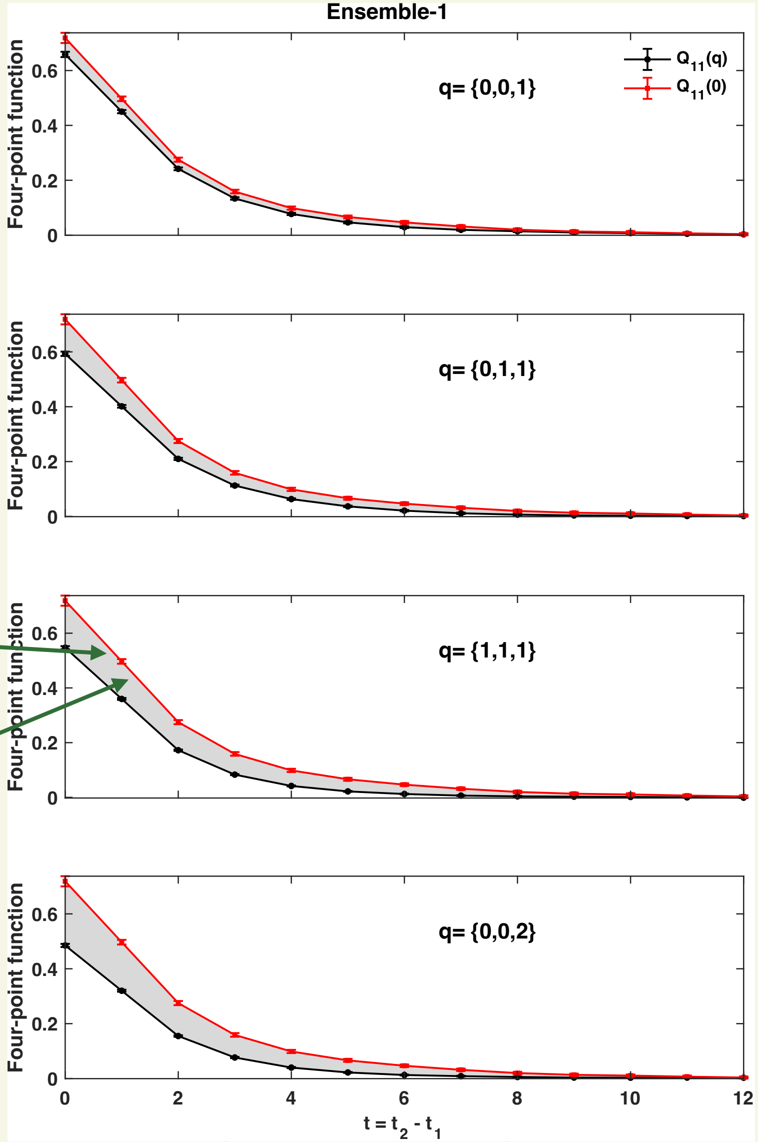
Neutral Pion Magnetic Polarizability Analysis

- For the neutral pion, the elastic charge-radius term is absent.

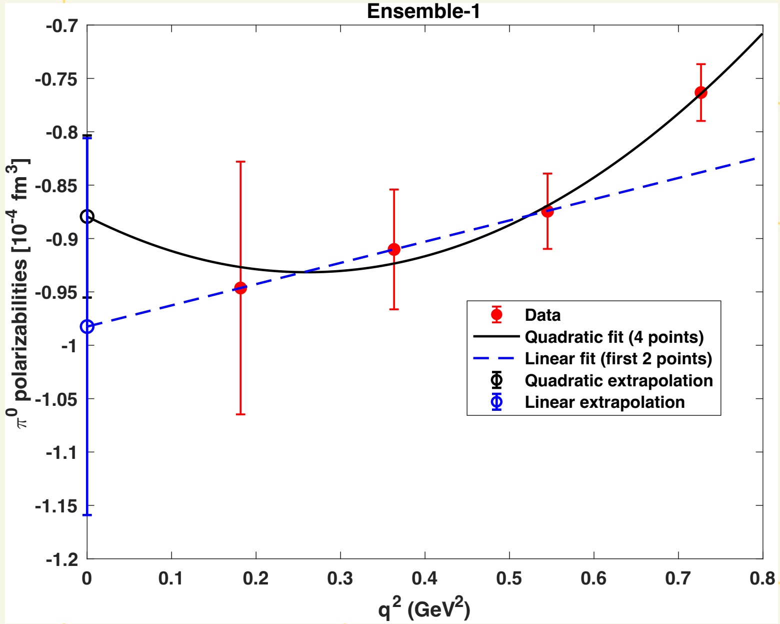
Only inelastic part contributes for the total polarizability.

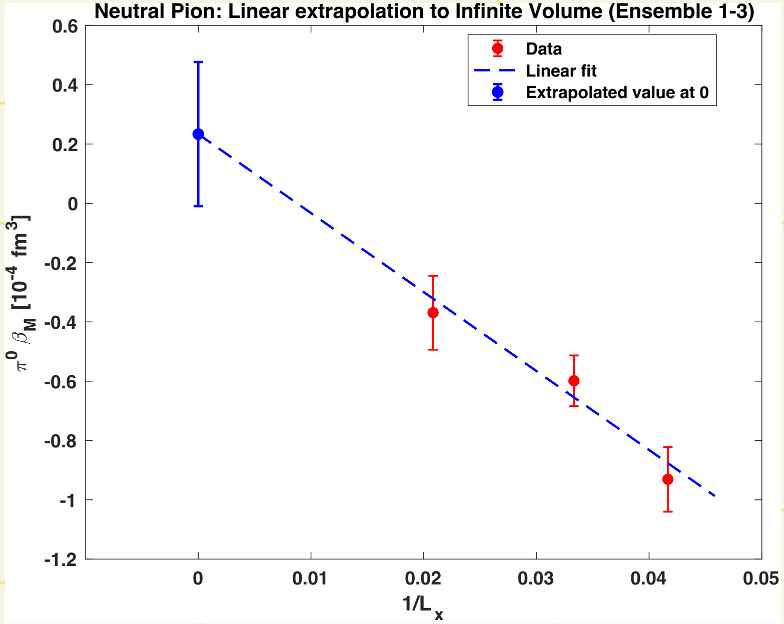
$$\beta_M^{\pi^0} = \lim_{\mathbf{q}\to 0} \frac{2\alpha}{\mathbf{q}^2} \int dt \left[Q_{11}(\mathbf{q}, t) - Q_{11}(\mathbf{0}, t) \right]$$

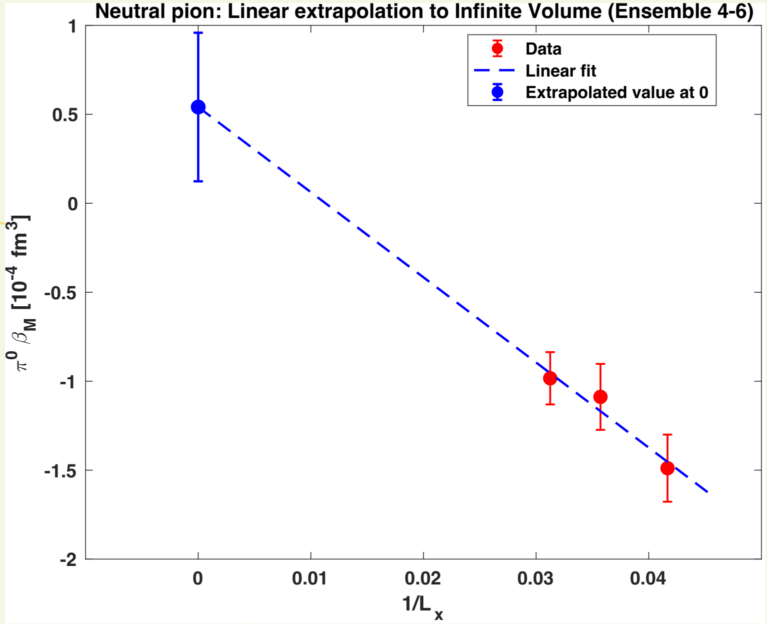
The shaded area gives this integral.



Neutral Pion Analysis cont:



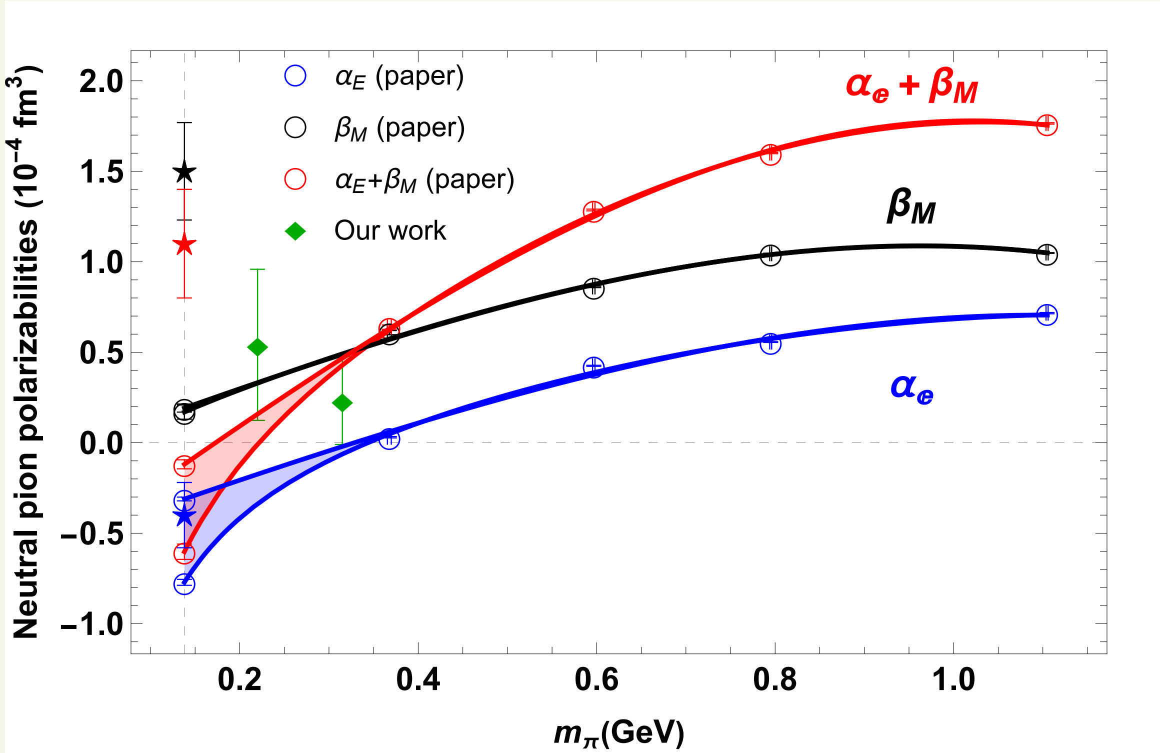


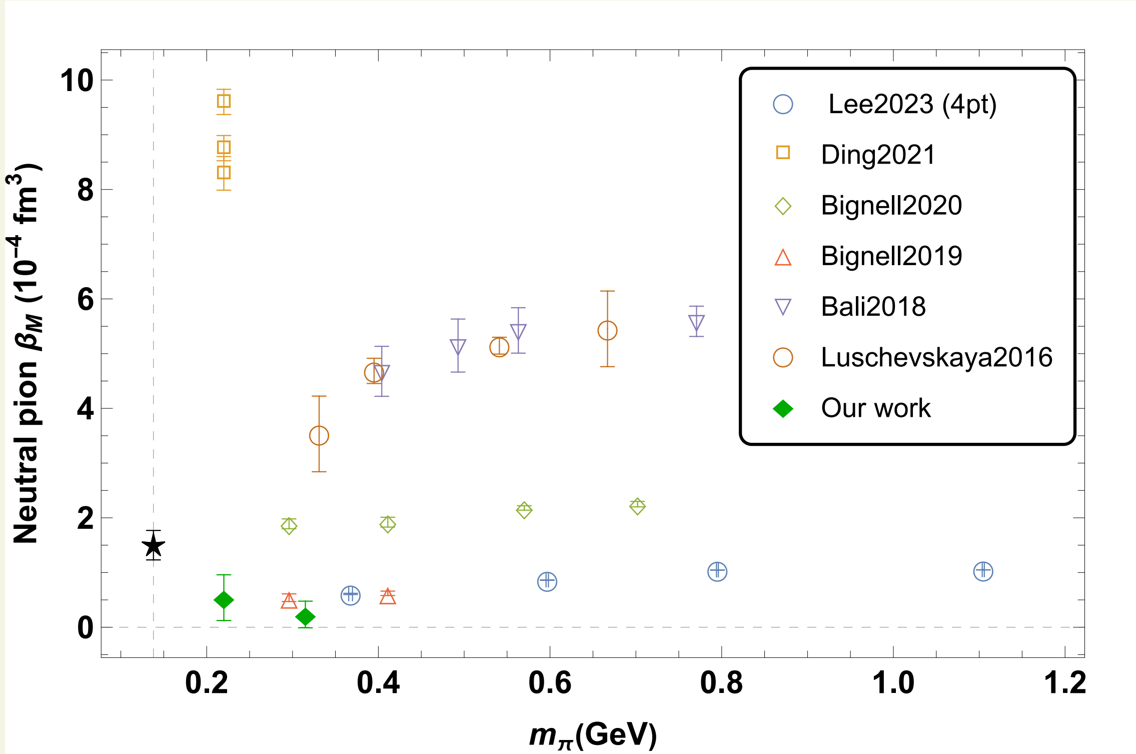


Momentum dependence of inelastic term and extrapolation to q²=0

m_π (MeV)	Neutral Pion β_M
315	0.2333 ± 0.2431
220	0.5410 ± 0.4175

Overlay Plots :





➢ **Result follows the trend of Lee et al.(2023)**

Fangcheng He et al. (2020)
DOI: https://doi.org/10.1103/PhysRevD.102.114509

Conclusions:

- Results from the nHYP ensembles follow the trend observed by Lee et al. (2023).

- A stable infinite-volume extrapolation was not obtained for the charged-pion results.

- Goals:
  - Complete the neutral-pion electric-polarizability calculation.
  - Improve the charged-pion infinite-volume analysis.

THANK YOU!!

QUESTIONS??

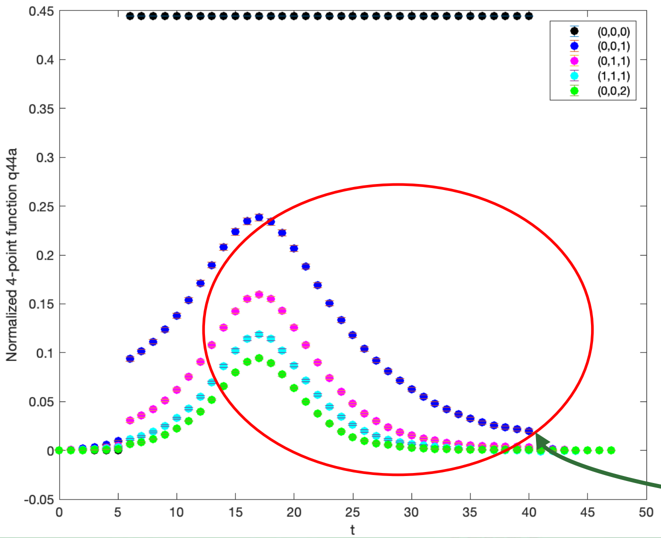
Backup Slides

The Elastic Contribution

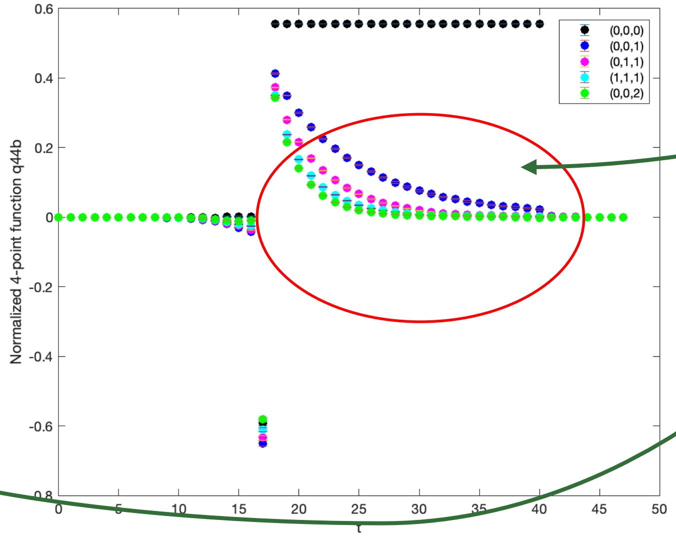
- The *elastic* four-point function is expected to follow an exponential behavior

$$Q_{44}^{\text{elas}}(\vec{q},t) = \frac{(E_\pi + m_\pi)^2}{4E_\pi m_\pi} F_\pi^2(\vec{q}^2)\underbrace{e^{-a(E_\pi - m_\pi)t}}_{\text{Exponential decay}}$$

Diag. A



Diag. B

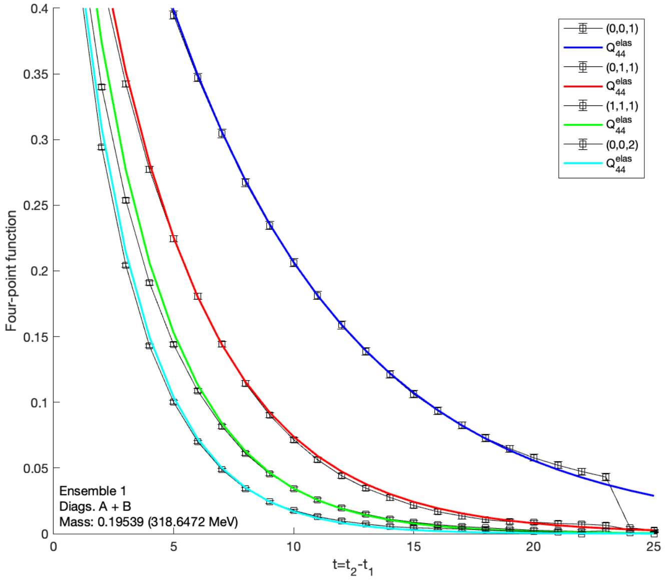


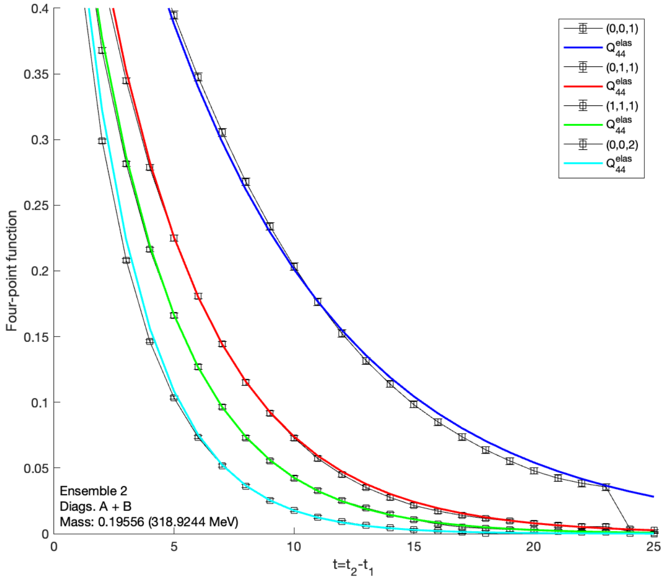
Diagrams A and B are dominated by elastic contributions, so we **add them together and fit at *large times***.

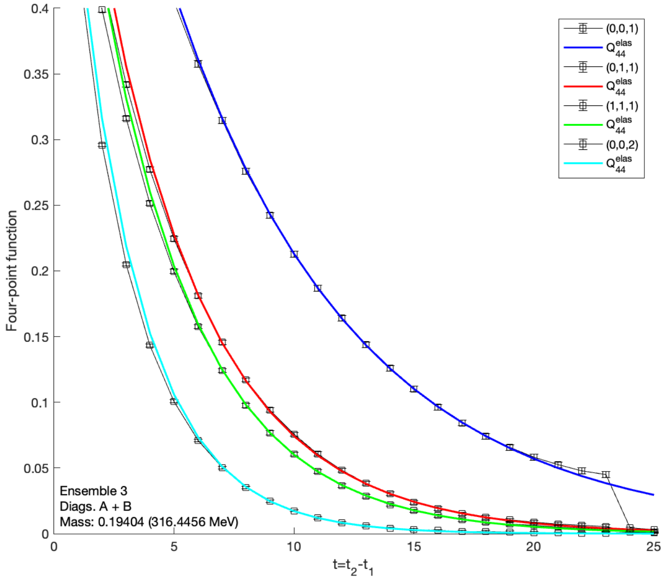
Baylor University

Four-Point Function Fits

- We pick out diagrams A and B (since they demonstrate the elastic contribution), **add and fit**





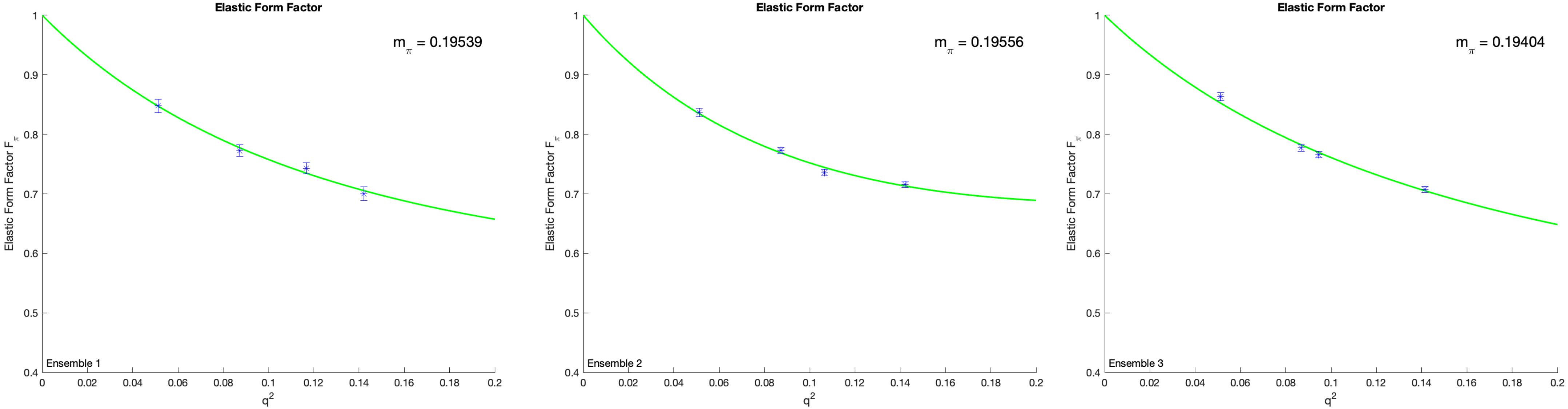


- These are plotted as functions of time separation $t = t_2 - t_1$ between the two currents
- We have three ensembles at mass = 315 MeV and three at mass = 220 MeV.

Baylor University

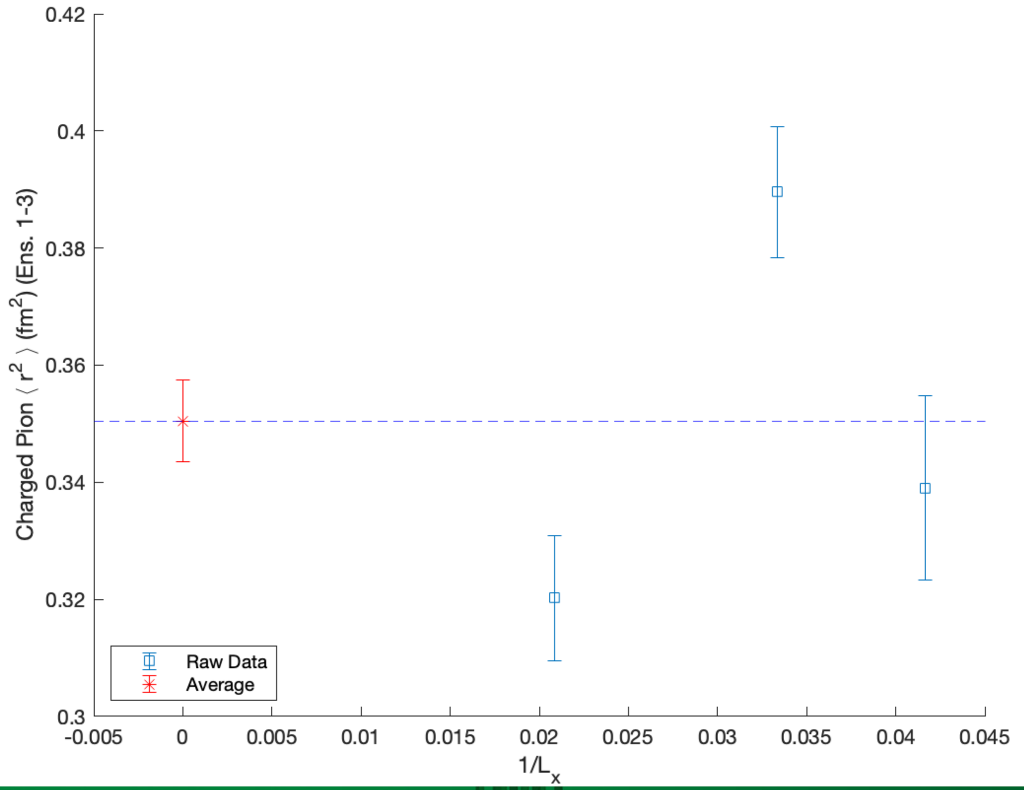
Form Factor

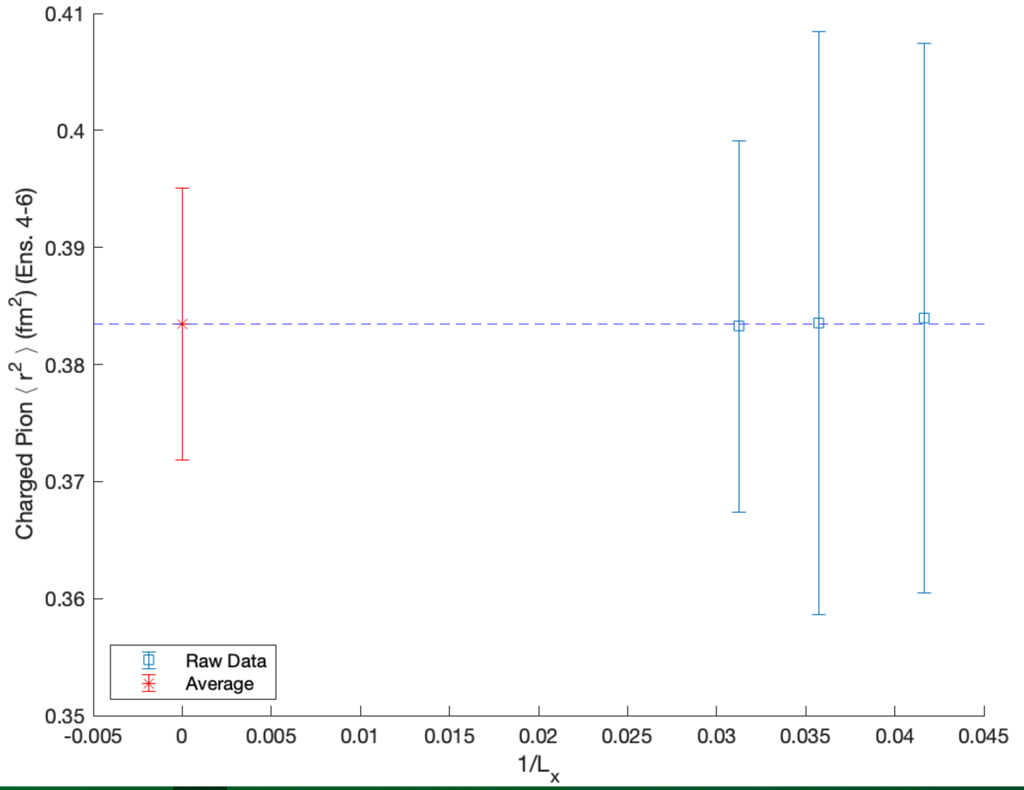
- Recall the form factor F_π from $$Q_{44}^{\text{elas}}(\vec{q},t) = \frac{(E_\pi + m_\pi)^2}{4E_\pi m_\pi} F_\pi^2(\vec{q}^2) e^{-a(E_\pi - m_\pi)t}$$
- We obtain form factors from the 4-point function fits. Then we fit the form factors to the z-expansion.



Charge radius squared

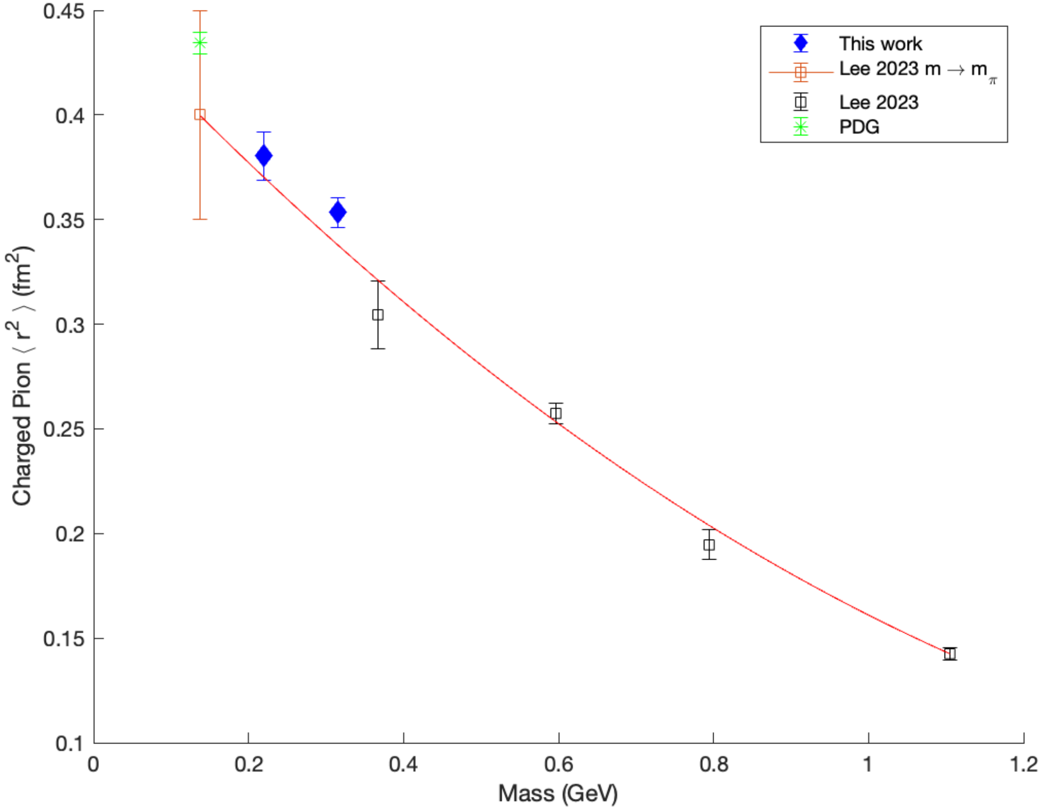
$$\langle r_E^2 \rangle = -6 \frac{dF_\pi(\vec{q}^2)}{d\vec{q}^2}\bigg|_{\vec{q}^2 \to 0}$$

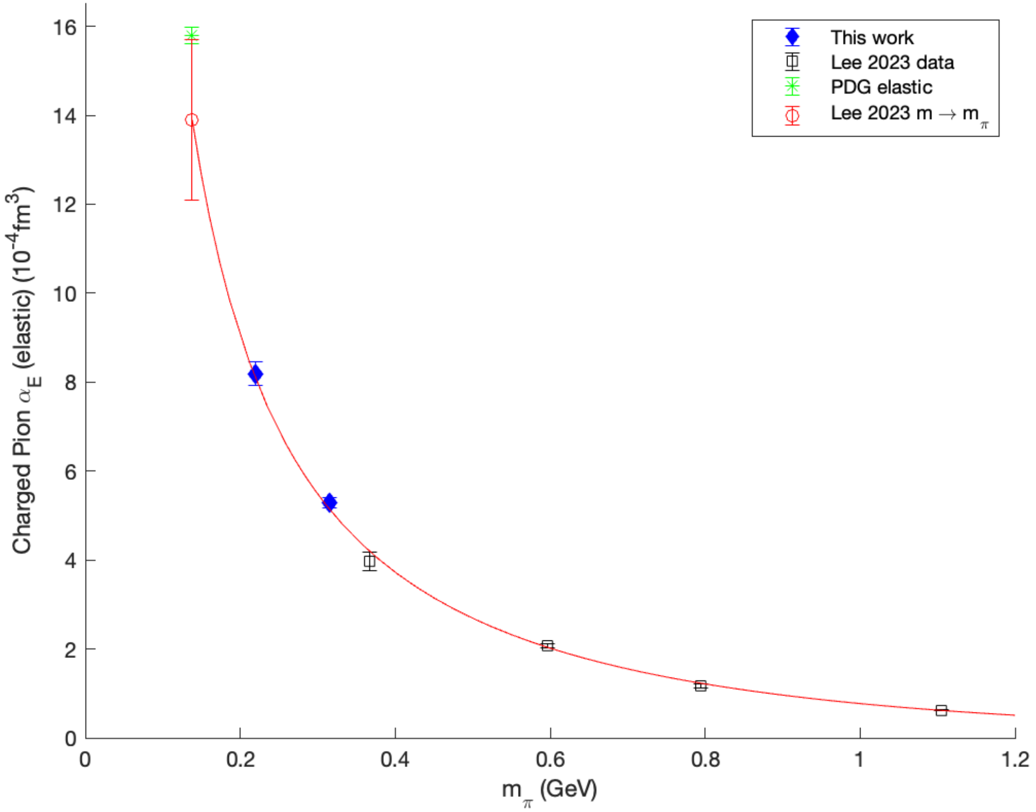




Baylor University

Comparisons: $\langle r^2 \rangle$ and $\alpha_{E,elas}$ with Lee, et al.





Baylor University