Lattice QCD study on $S=+1$ kaon-nucleon interactions and the Θ^+ pentaquark at the physical point: updated analysis

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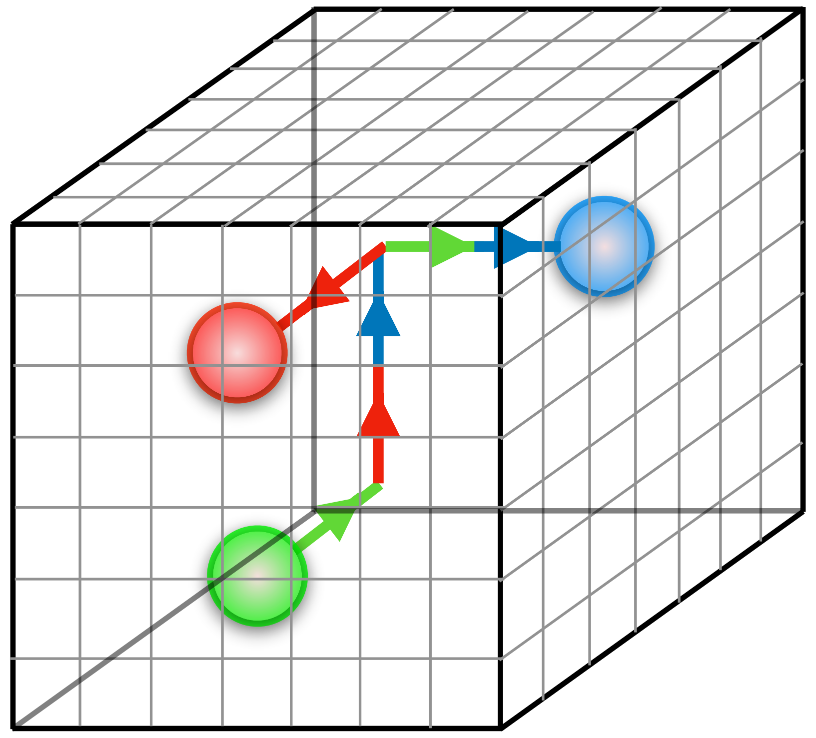
based on

- KM, S. Aoki, T. Doi, Y. Lyu, W. Yamada (HAL QCD), PRD 113, 054506 (2026); erratum: under review at PRD (to be published on arXiv soon)
- KM, S. Aoki, T. Doi, Y. Lyu, W. Yamada (HAL QCD), in preparation

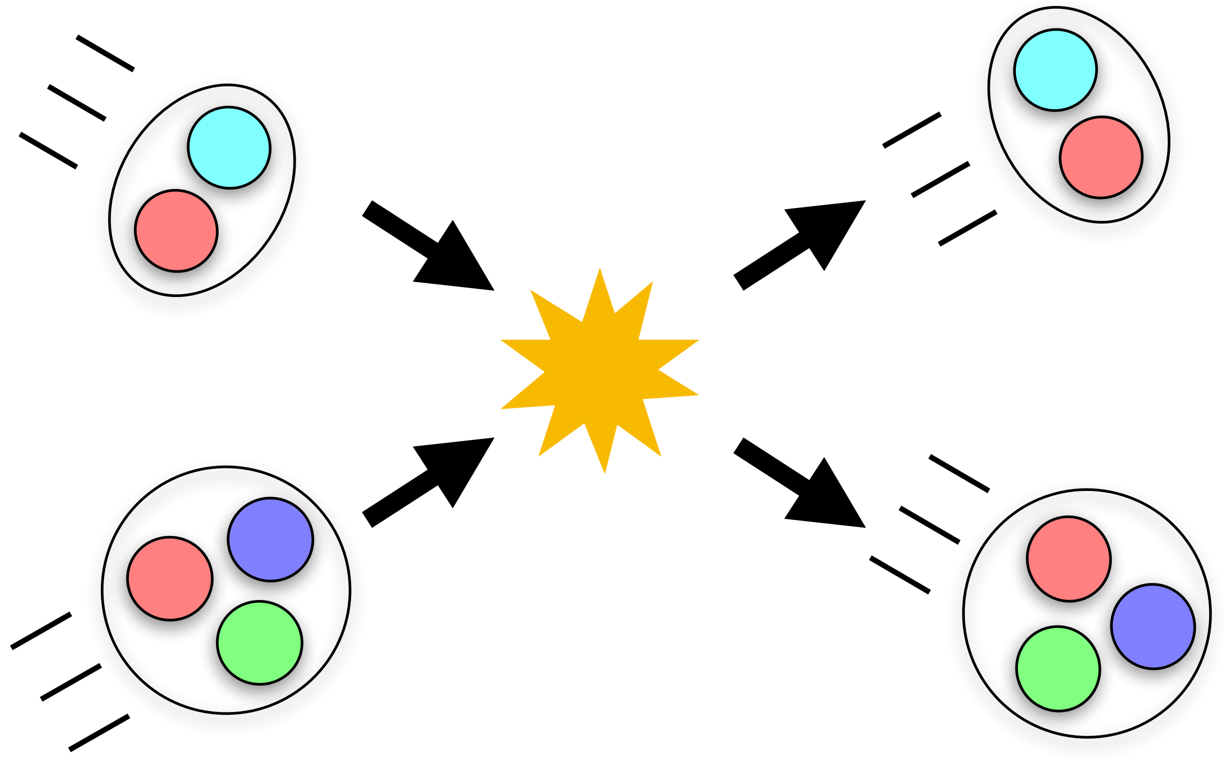
Lattice 2026, University of Maryland, College Park, USA, July 2026

Introduction

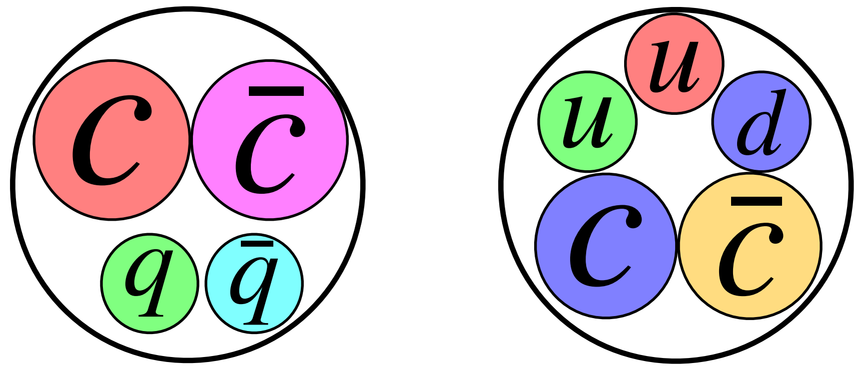
Lattice QCD



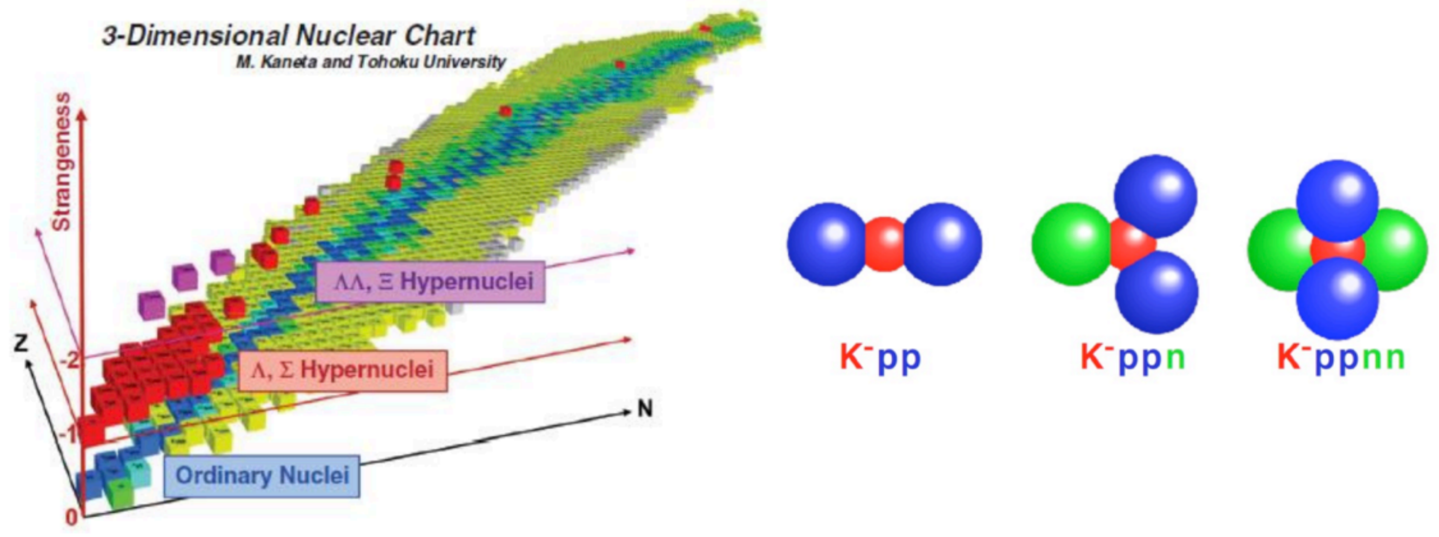
hadron interactions (scatterings)



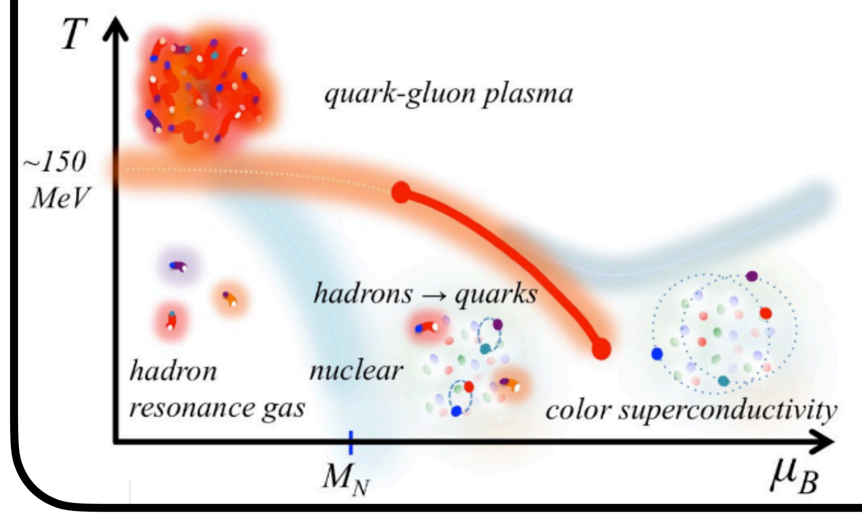
Exotic hadrons

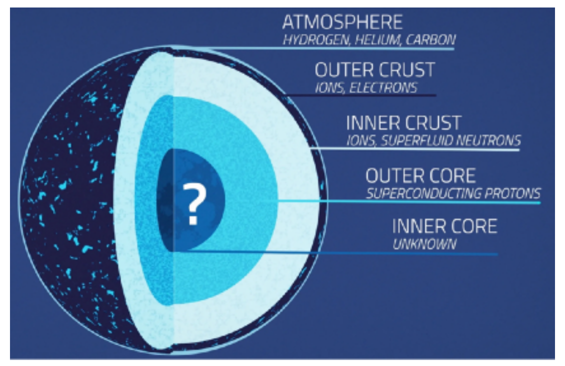


(Exotic) Nuclei



Nuclear matter, Neutron stars





- Finite-volume method [Lüscher, 1991]
- HAL QCD method [Ishii, Aoki, Hatsuda 2007]

(time-dependent) HAL QCD method

[Ishii, Aoki, Hatsuda 2007]
[Ishii et al. 2011]

- R-correlator:

$$R(\mathbf{r},t) = \frac{\langle O_1(\mathbf{r},t)O_2(\mathbf{0},t)\bar{J}(0)\rangle}{\langle O_1(t)\bar{O}_1(0)\rangle\langle O_2(t)\bar{O}_2(0)\rangle} \approx \sum_n C_{\bar{J},n} \underline{\Psi^{W_n}(\mathbf{r})} e^{-(W_n - m_1 + m_2)t}$$

Nambu-Bethe-Salpeter (NBS) wave function

- time-dependent HAL QCD method

(μ: reduced mass)

$$\int d^3r'\, U(\mathbf{r},\mathbf{r}')R(\mathbf{r}',t) \simeq \left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right)R(\mathbf{r},t)$$

$$\approx V(r)\delta^{(3)}(\mathbf{r}-\mathbf{r}')$$

(local (leading-order) approximation)

$$\Rightarrow \quad V(\mathbf{r}) \simeq \frac{1}{R(\mathbf{r},t)}\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right)R(\mathbf{r},t)$$

- **Talks on HAL QCD studies**
  - Optimized operator (S. Aoki, 3:20 p.m. Aug 27)
  - Hyperon forces (T. Doi, 2:00 p.m. today)
  - $N\Lambda, N\Sigma$ (K. Murase, 2:40 p.m. today)
  - $\bar{D}N$ (W. Yamada, 3:20 p.m. today)
  - KN (KM, this talk)

Motivation for KN interactions

- (S=+1) kaon-nucleon (KN) interaction: most simple meson-baryon system
  - no coupled channels or (conventional) resonances
    ➡ reasonable computational cost
  - rich scattering experimental data since 1960s (at low momenta)

good testing ground for
theory (LQCD) vs. experiment

- provides an input for partial restoration of chiral symmetry

$$\frac{\langle \bar{u}u + \bar{s}s \rangle^*}{\langle \bar{u}u + \bar{s}s \rangle_0} = 1 + \rho \frac{\langle N|\bar{u}u + \bar{s}s|N\rangle}{\langle 0|\bar{u}u + \bar{s}s|0\rangle} + \mathcal{O}(\rho^{n>1})$$

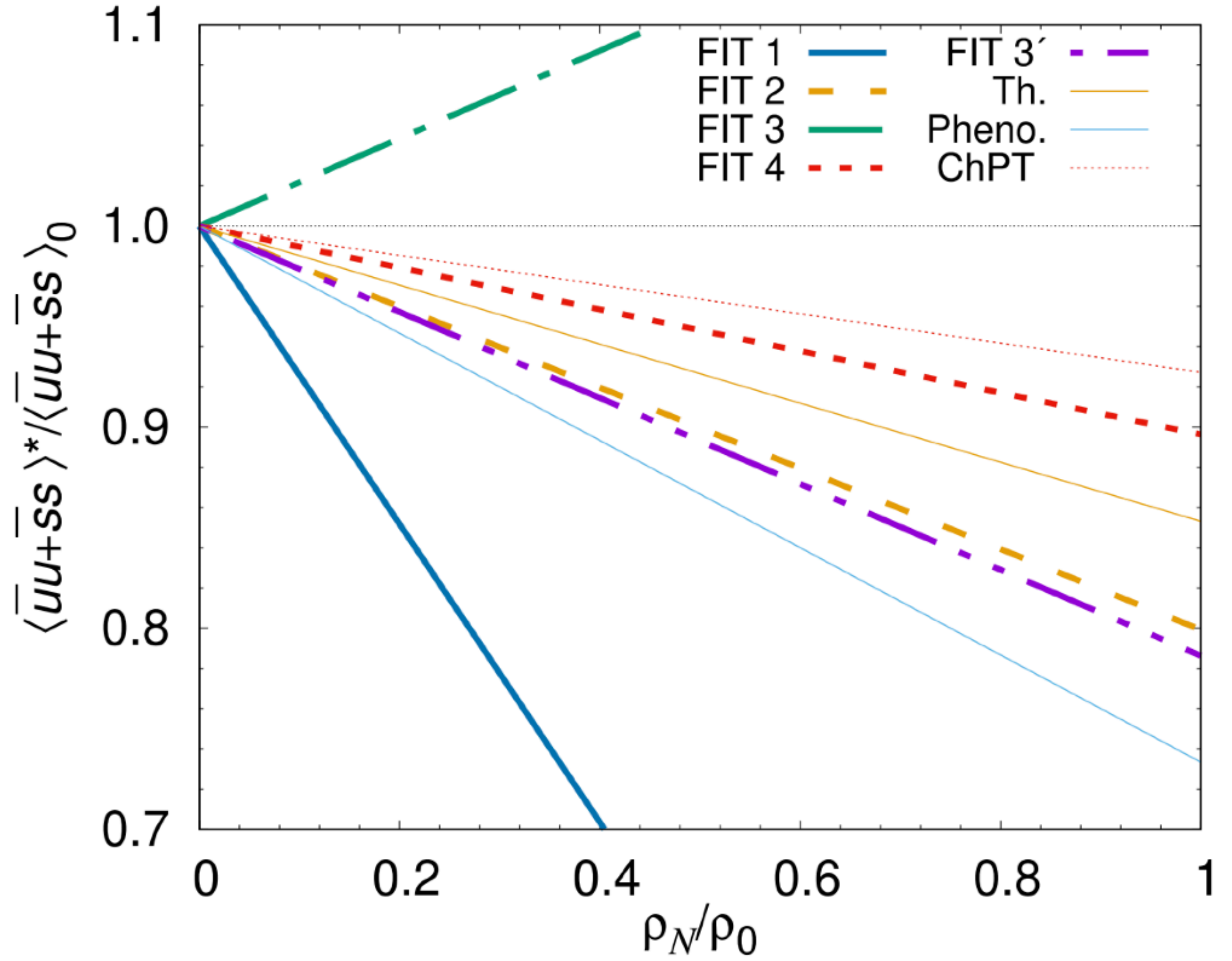
$$= 1 + \frac{\rho}{m_K} \frac{T_{KN}(q=0)}{2m_N} + \mathcal{O}(\rho^{n>1})$$

KN scattering amplitude (in soft limit)

[Iizawa, Jido, Hübsch, PTEP (2024)]

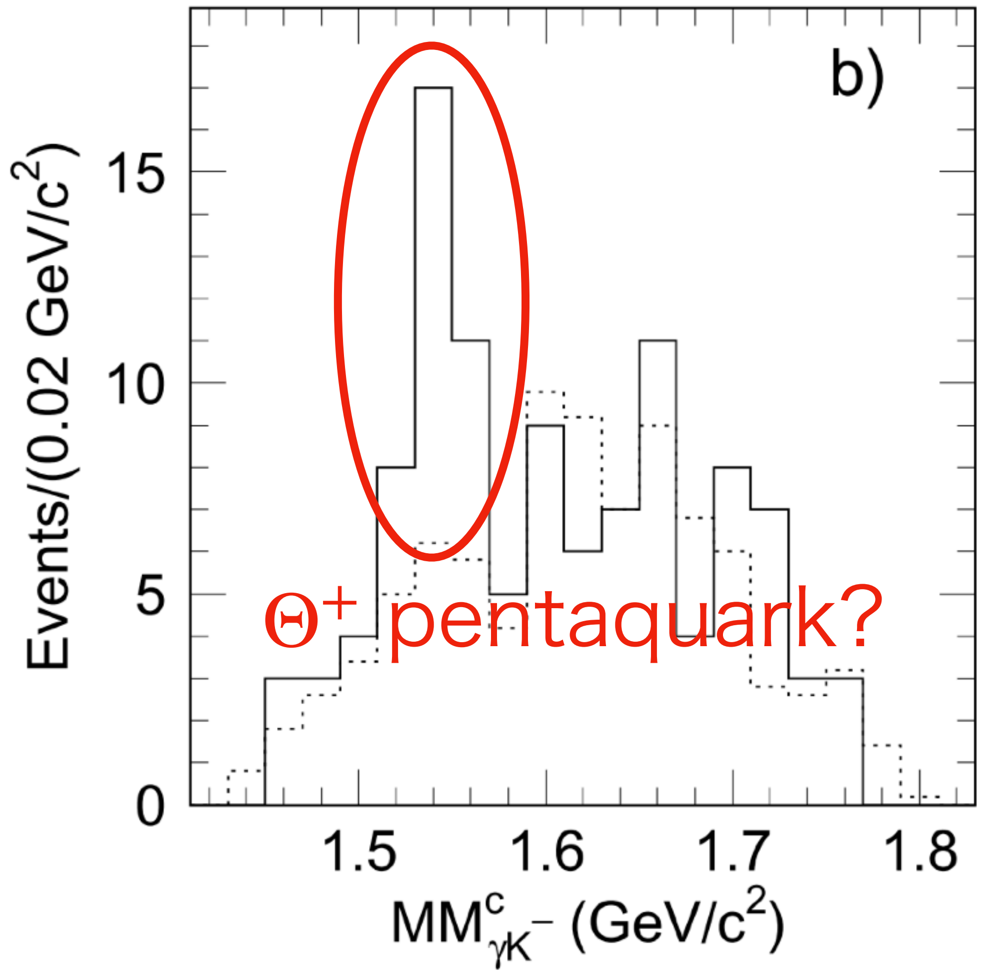
Missing profiles in understanding KN

- lack of scattering data near threshold
  - difficulty near threshold
    ➡ **large ambiguity** (especially for I=0)



[Iizawa, Jido, Hübsch, PTEP (2024)]

- existence of Θ^+ ($uudd\bar{s}$) pentaquark [LEPS, PRL (2003)]
  - most subsequent experiments gave negative results
  - **no definitive conclusion from theoretical side**



[LEPS, PRL (2003)]

LQCD compensates KN's missing profile

Calculation strategy

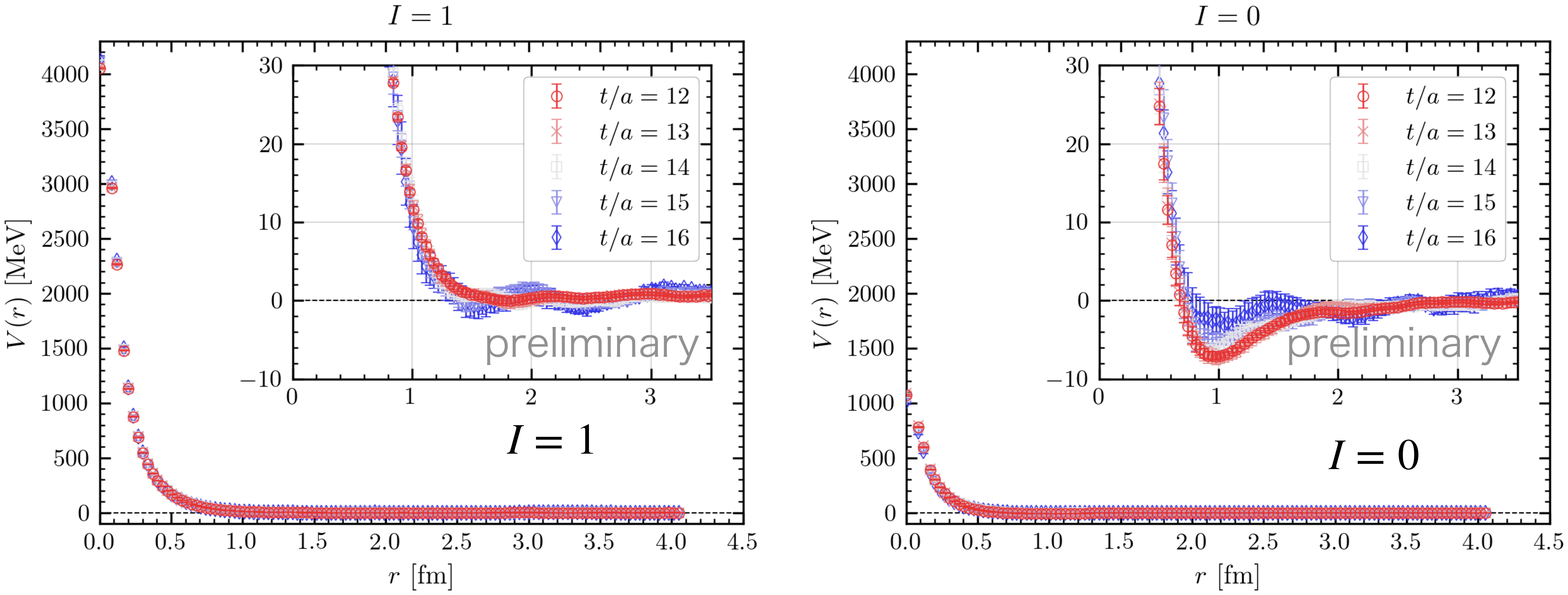
- $N_f = 2+1$ gauge configurations ($m_u = m_d < m_s$) at **the physical point**

  [HAL QCD, PRD (2024)]

  - clover fermion + Iwasaki gauge w/ stout smearing
  - $V = (96a)^4 \approx (8.1\ \mathrm{fm})^4$, $a = 0.084372(54)(^{+109}_{-6})\ \mathrm{fm}$
  - $m_\pi \approx 137\ \mathrm{MeV}$, $m_K \approx 502\ \mathrm{MeV}$, $m_N \approx 940\ \mathrm{MeV}$

- extract (approximated) **S-wave component of wave functions**

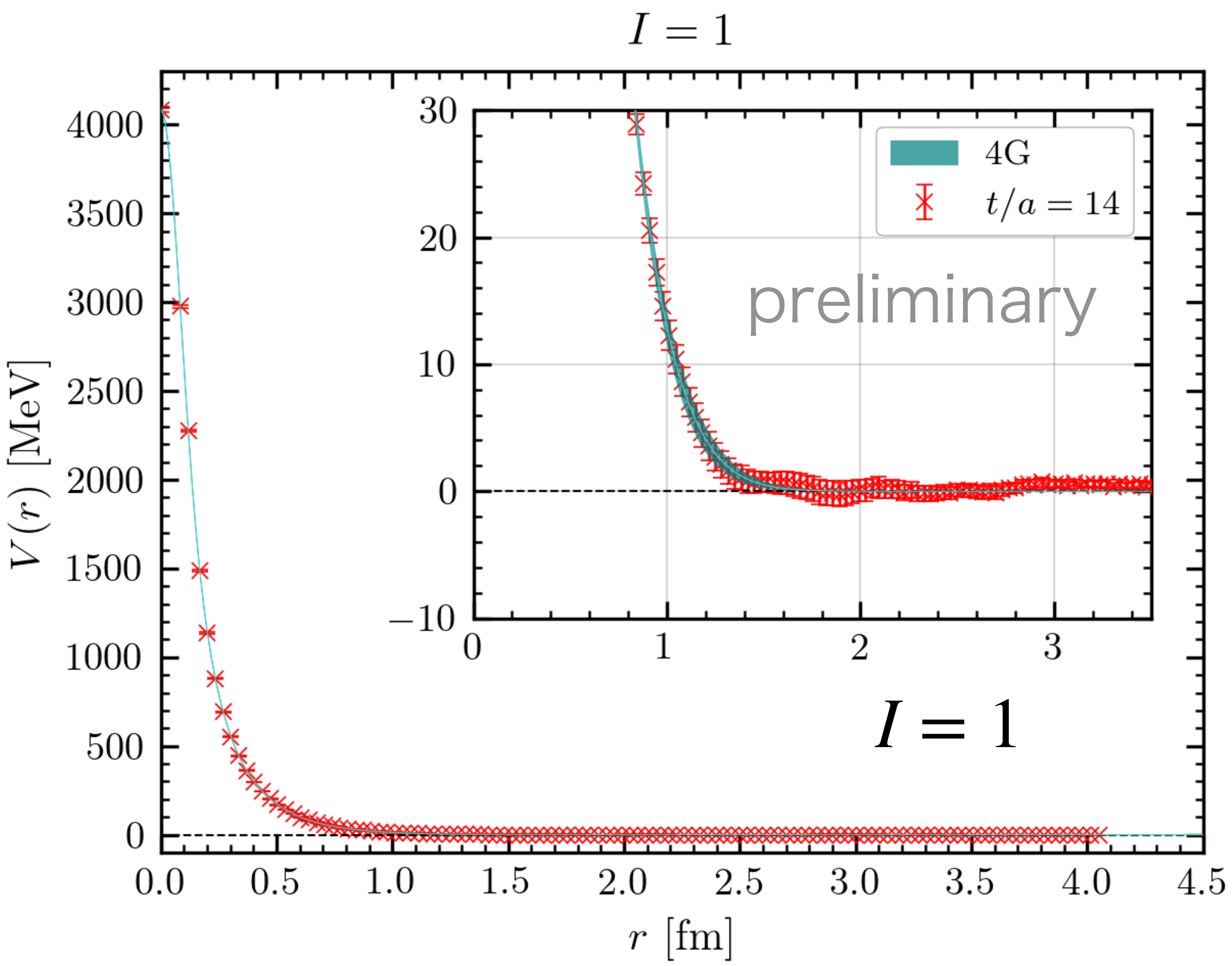
Potentials

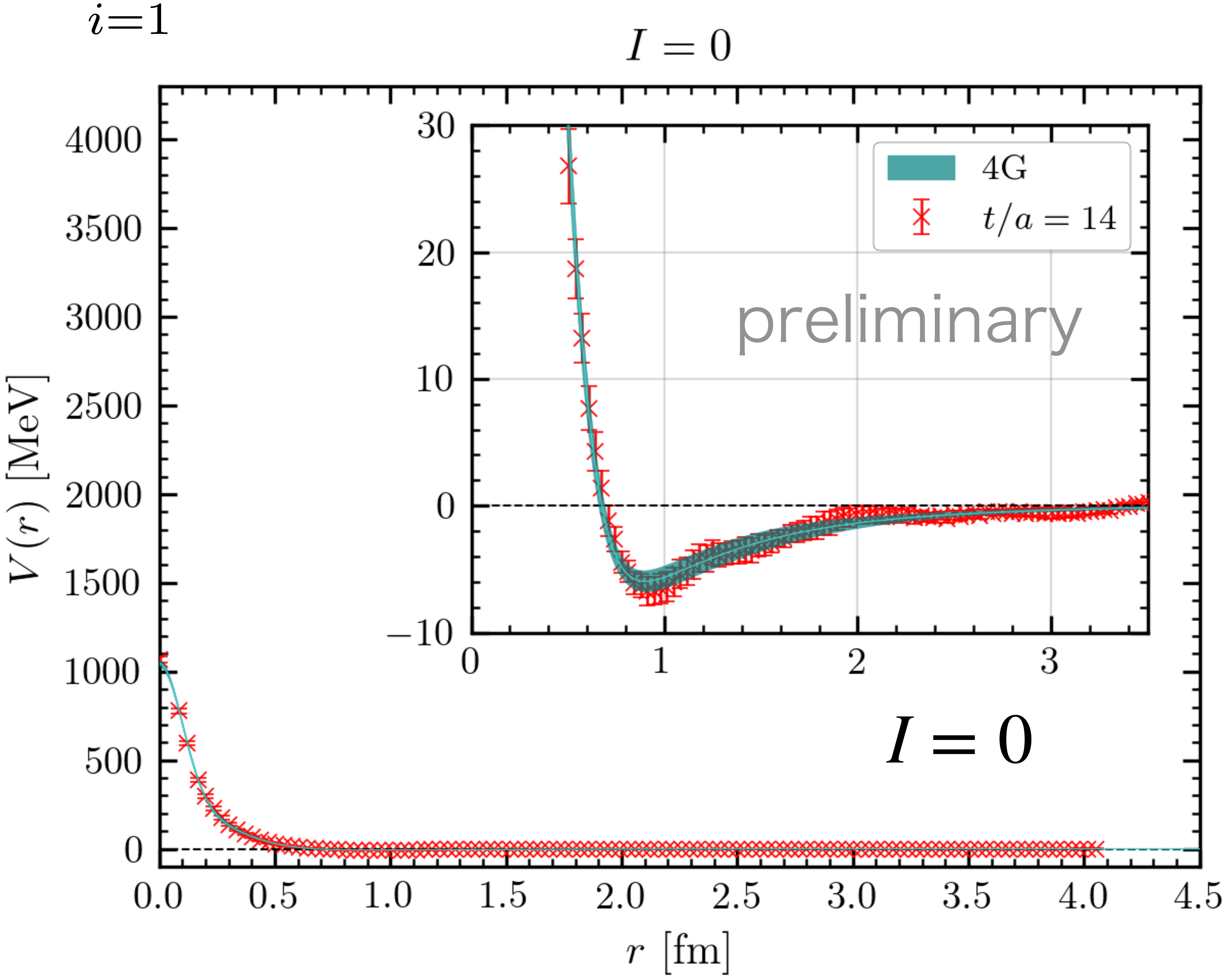


- repulsive core in both channel
- $I = 1$: completely repulsive, $I = 0$: small attractive pocket

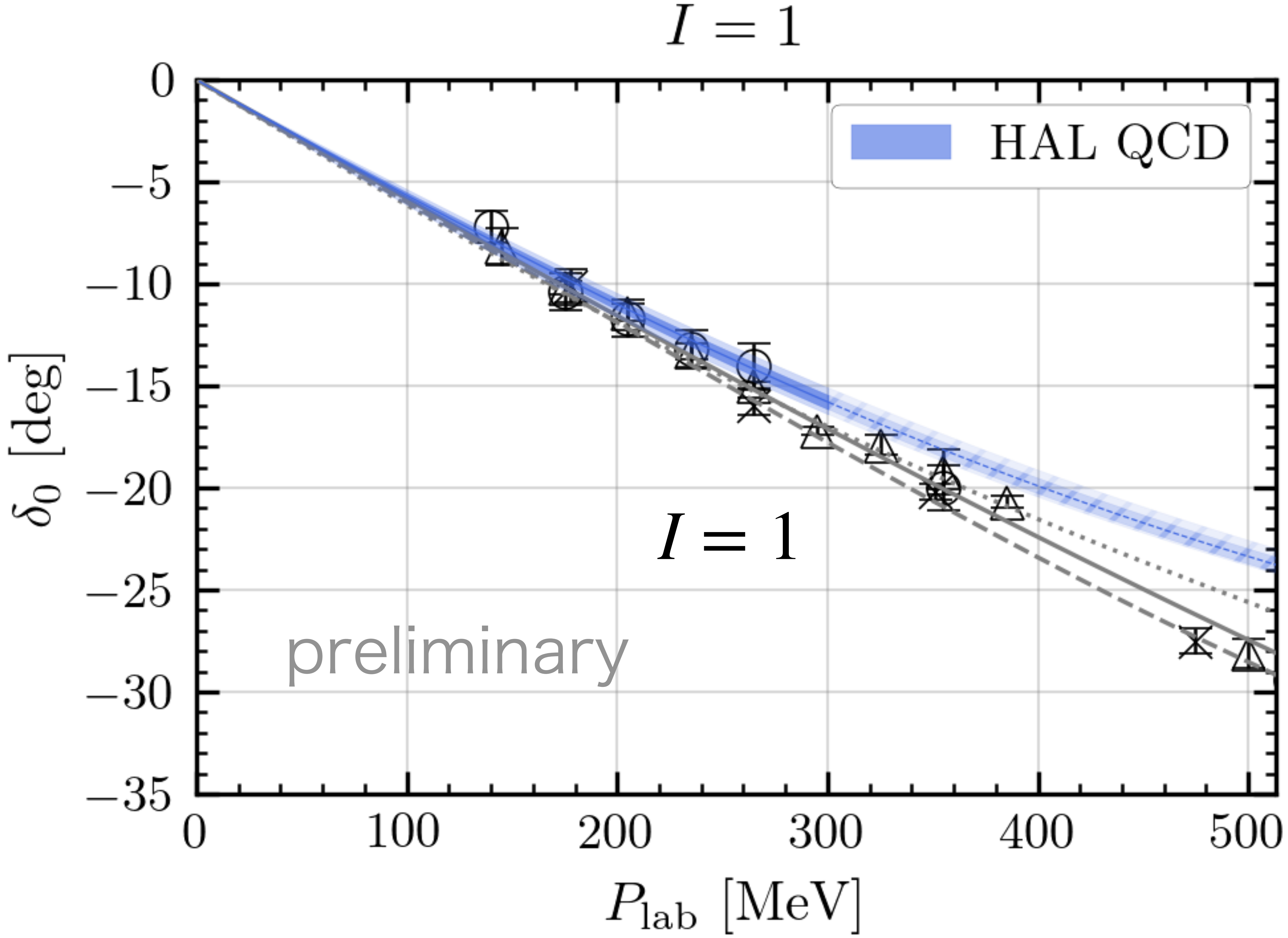
Fittings

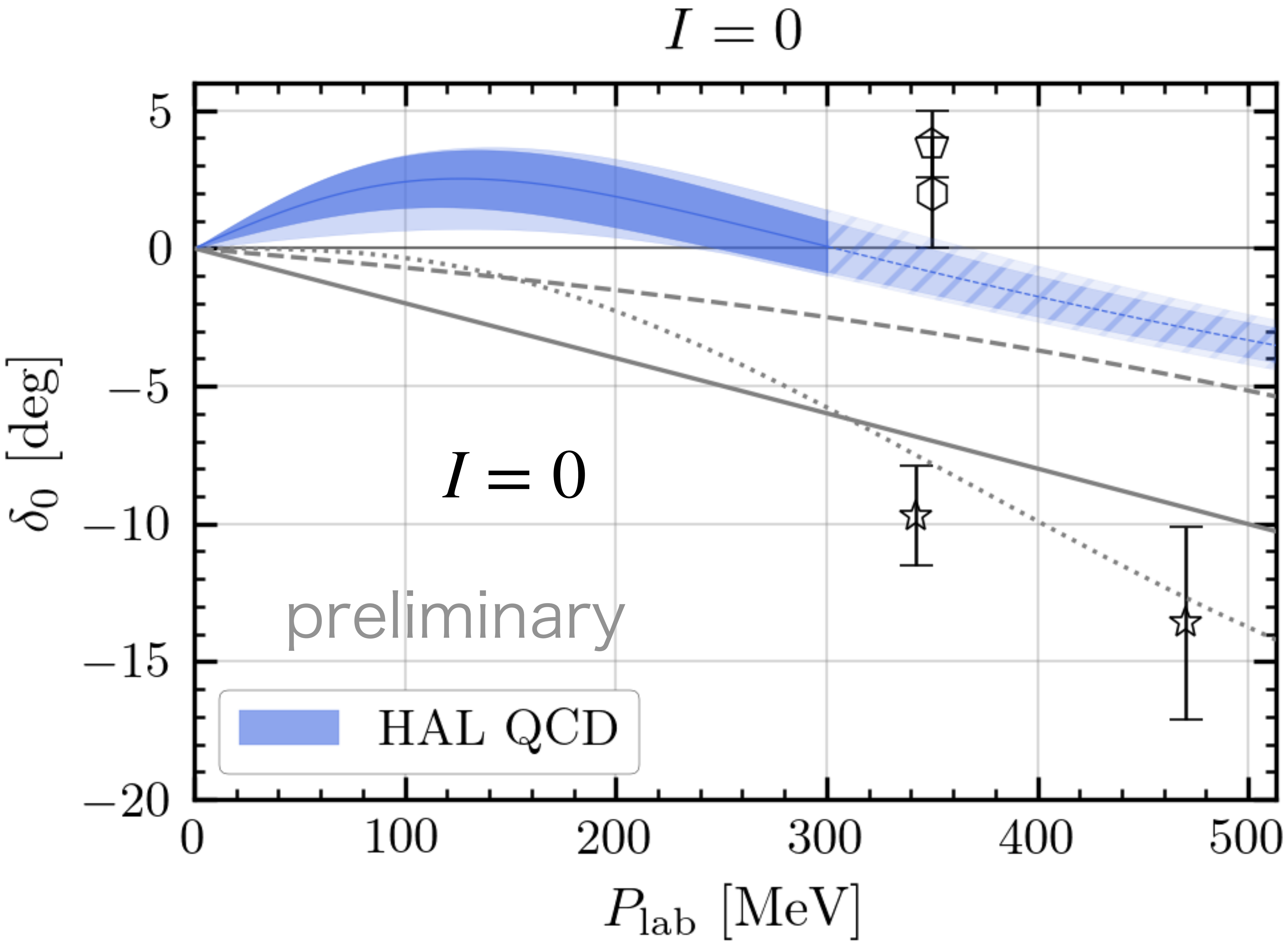
$$V_I^{4\mathrm{G}}(r) = \sum_{i=1}^{4} a_i^I e^{-\left(\frac{r}{b_i^I}\right)^2}$$





Phase shifts



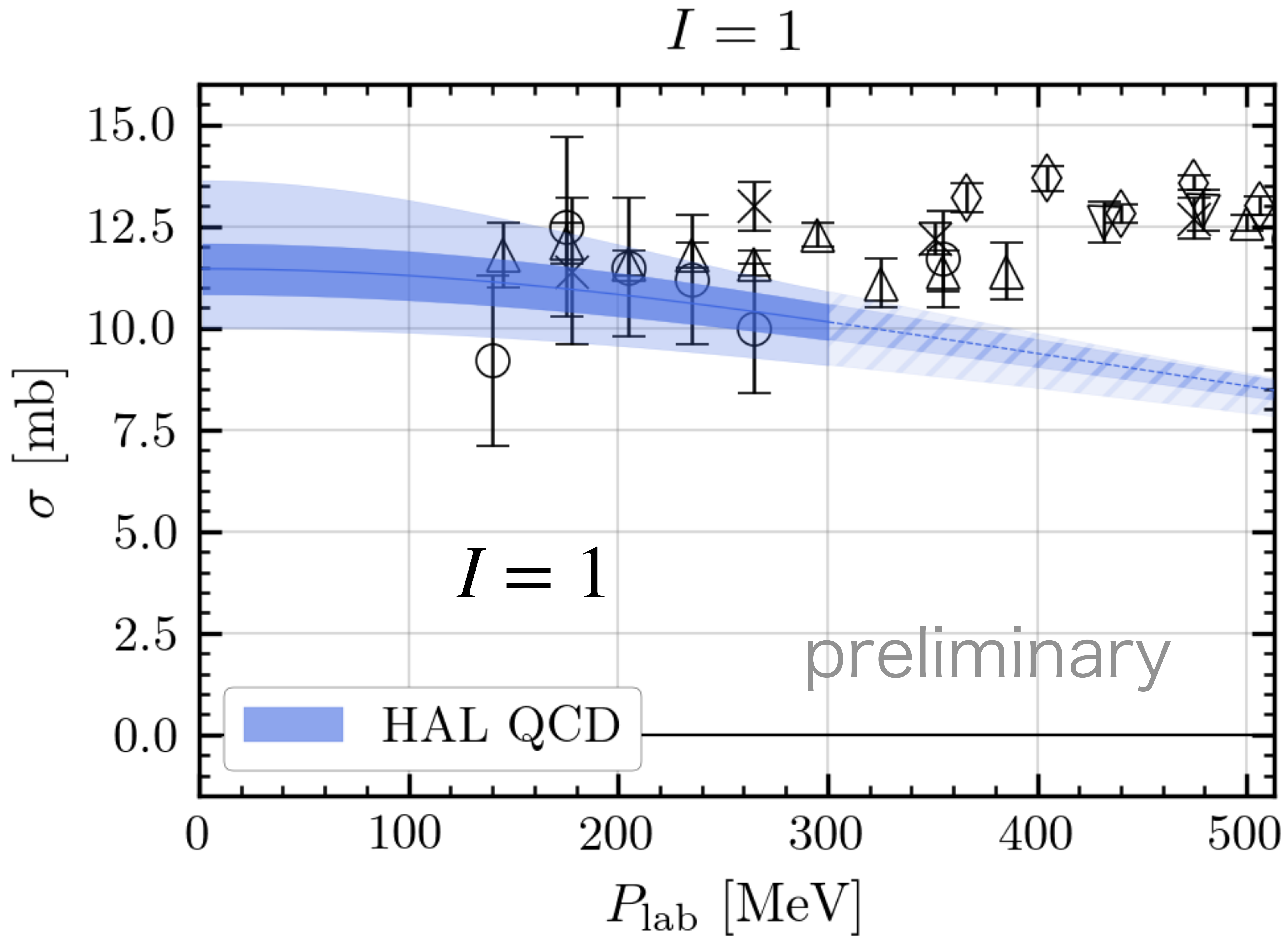


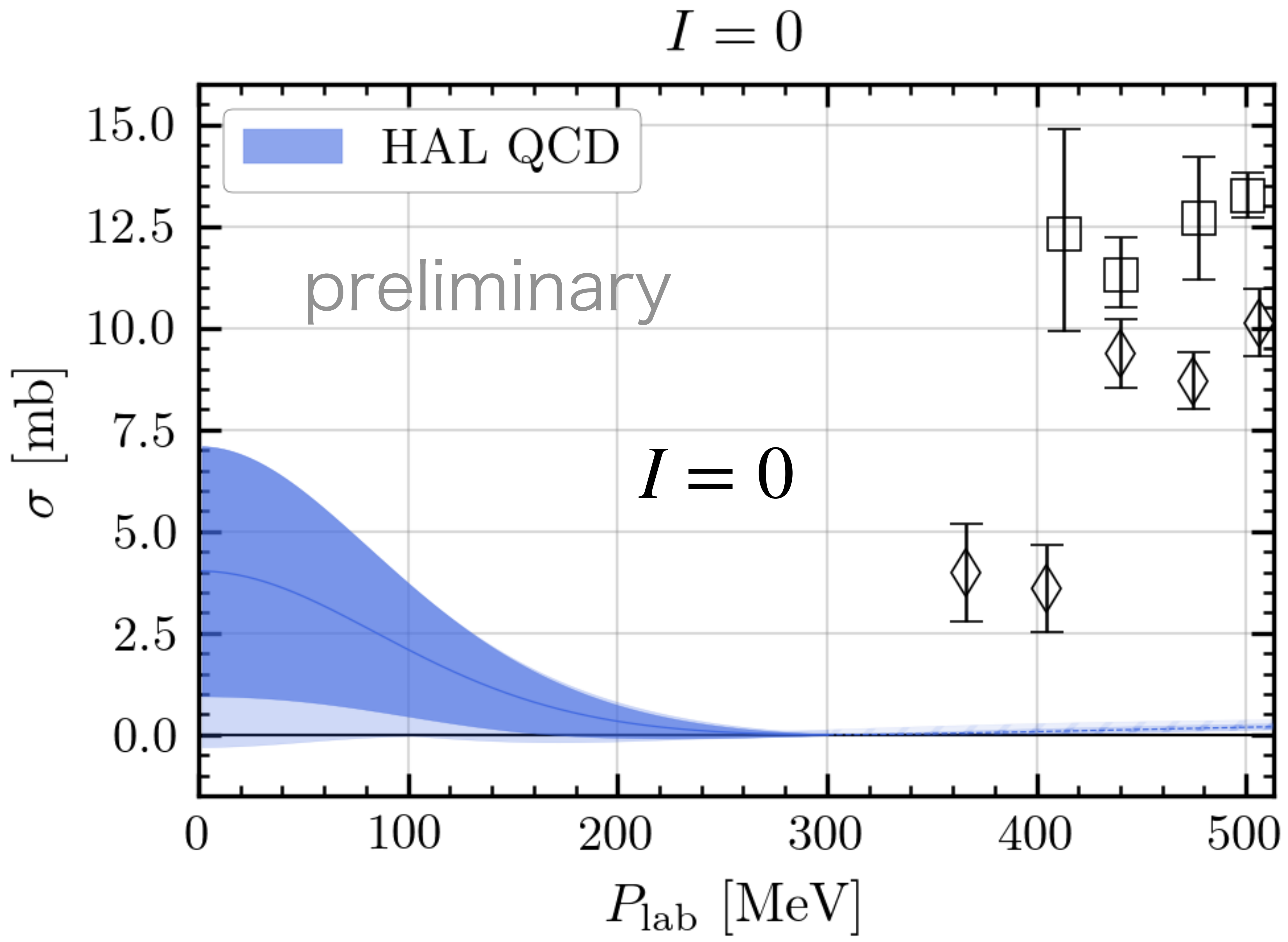
- **light bands: syst. err. including time and fit-func. dependences**

- black: experimental results
- gray curves: partial-wave analysis (fitting with experimental data)

- no bound states or resonances ➡ no Θ^+ pentaquark in S-wave KN

S-wave total cross sections





- **light bands: systematic err. including time and fit-func. dependences**

- bands: our results (only S-wave)
- black: experimental results (**all partial waves**)

- $I = 0$: deviated from experiments at high momentum region

➡ large P-wave contribution? [Slater et al., PRL (1961); Stenger et al., PR (1964); Aoki, Jido, PTEP (2019)]

Higher-partial waves

- extract P- and D-wave phase shifts
  from the S-wave potential $V_{l,s}(r) = V_{l=0}(r)$ (assumption)

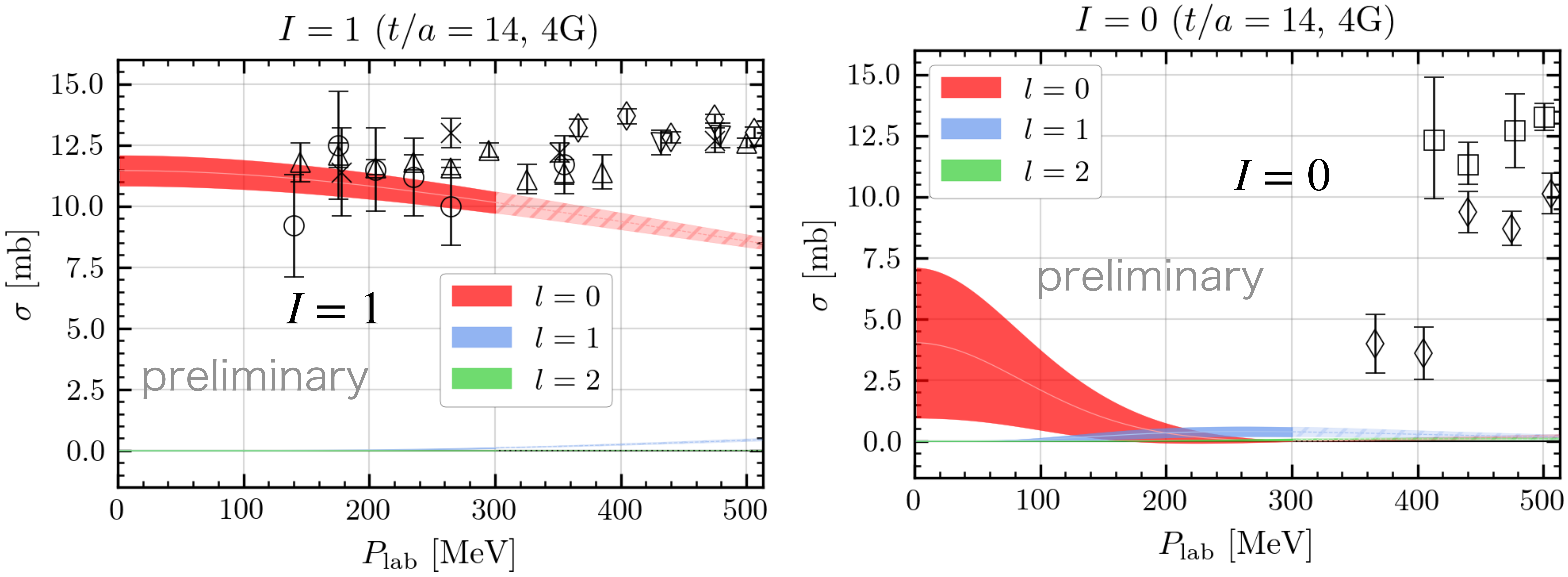
- in other words,
  - $V_{\mathrm{cent}}(r) = V_{l=0}(r)$,
  - $(\mathbf{L}\cdot\mathbf{S})V_{LS}(r) = (\mathbf{L}^2)V_{L^2}(r) = \cdots = 0$

depending on l and s

- Note:
  $(\mathbf{L}\cdot\mathbf{S})V_{LS}(r) = 0$ ➡ two states with $j = l \pm 1/2$ are degenerated

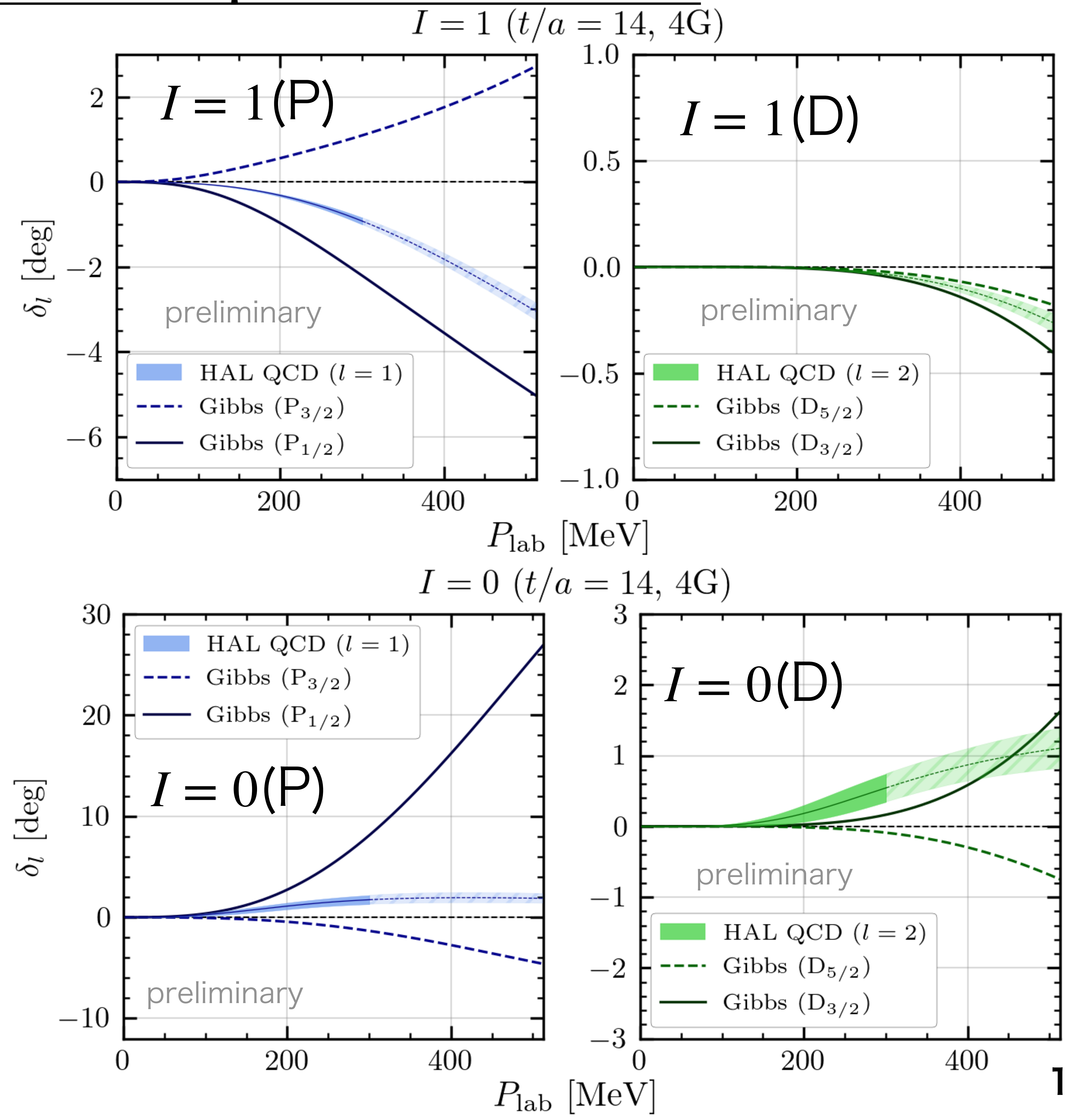
- used the fit results for 4G

HPW: total cross sections



- $I = 1$: S-wave dominates
- $I = 0$: P-wave takes over above $P_{\mathrm{lab}} \sim 300\ \mathrm{MeV}$, but does not reproduce exp. data at high momenta

HPW: phase shifts



- "Gibbs": partial-wave analysis (**PWA**) (fitting with experimental data) [Gibbs Arceo, 2007]

- similar to PWA for $j=l-1/2$
- suggest much larger attraction in $P_{1/2}$ for $I=0$ to reproduce exp. data
  ➡ resonance?
- LS splitting in opposite direction between two isospins
  ➡ isospin-dependent LS force?

  $(\tau_K \cdot \tau_N)(\mathbf{L}\cdot\mathbf{S})V_{LS}^{\tau}(r)$, $V_{LS}^{\tau}(r) < 0$?

Summary

- study S-wave KN interactions in HAL QCD at the physical point ($m_\pi \approx 137$ MeV)
- potentials: repulsive core in both channels, with a small attractive pocket for I=0
- no resonance or bound state ➔ no Θ^+ pentaquark in S-wave KN
- phase shifts & cross sections for I=1 agrees with data at low mom.
- for I=0 suggests P-wave dominance
- higher partial waves: deviation from PWA suggests a sizable isospin-dependent spin-orbit (LS) force and larger attraction in $P_{1/2}$
- future: NLO potential, different lattice spacing, extracting LS-force from P-wave calc.

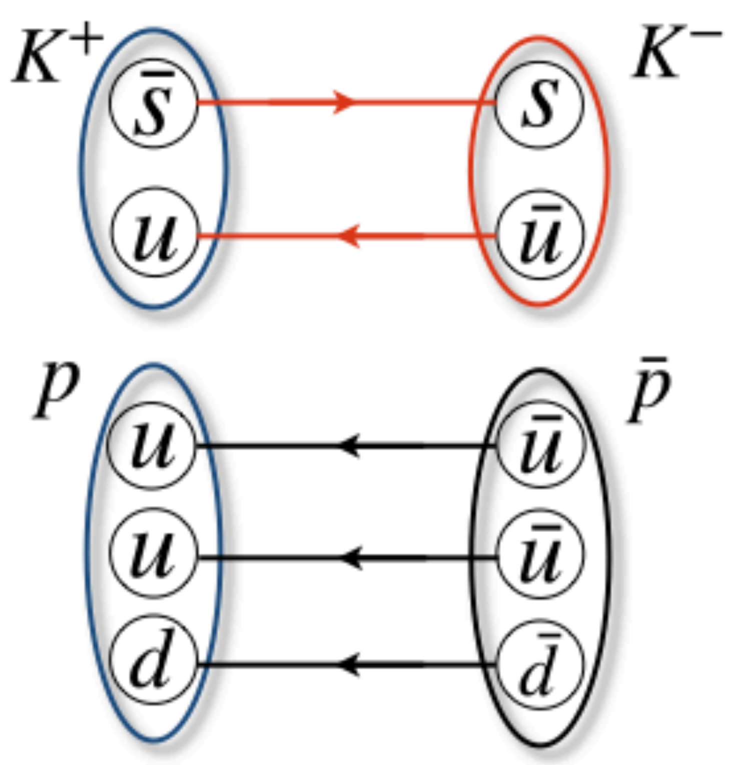
Backups

Details of numerical setups

- KN correlation functions

$$\langle (NK)^{I=1}_{I_z=0}\overline{pK^0}\rangle = \frac{1}{\sqrt{2}}(\langle pK^0\overline{pK^0}\rangle + \langle nK^+\overline{pK^0}\rangle)$$

$$\langle (NK)^{I=0}\overline{pK^0}\rangle = \frac{1}{\sqrt{2}}(\langle pK^0\overline{pK^0}\rangle - \langle nK^+\overline{pK^0}\rangle)$$



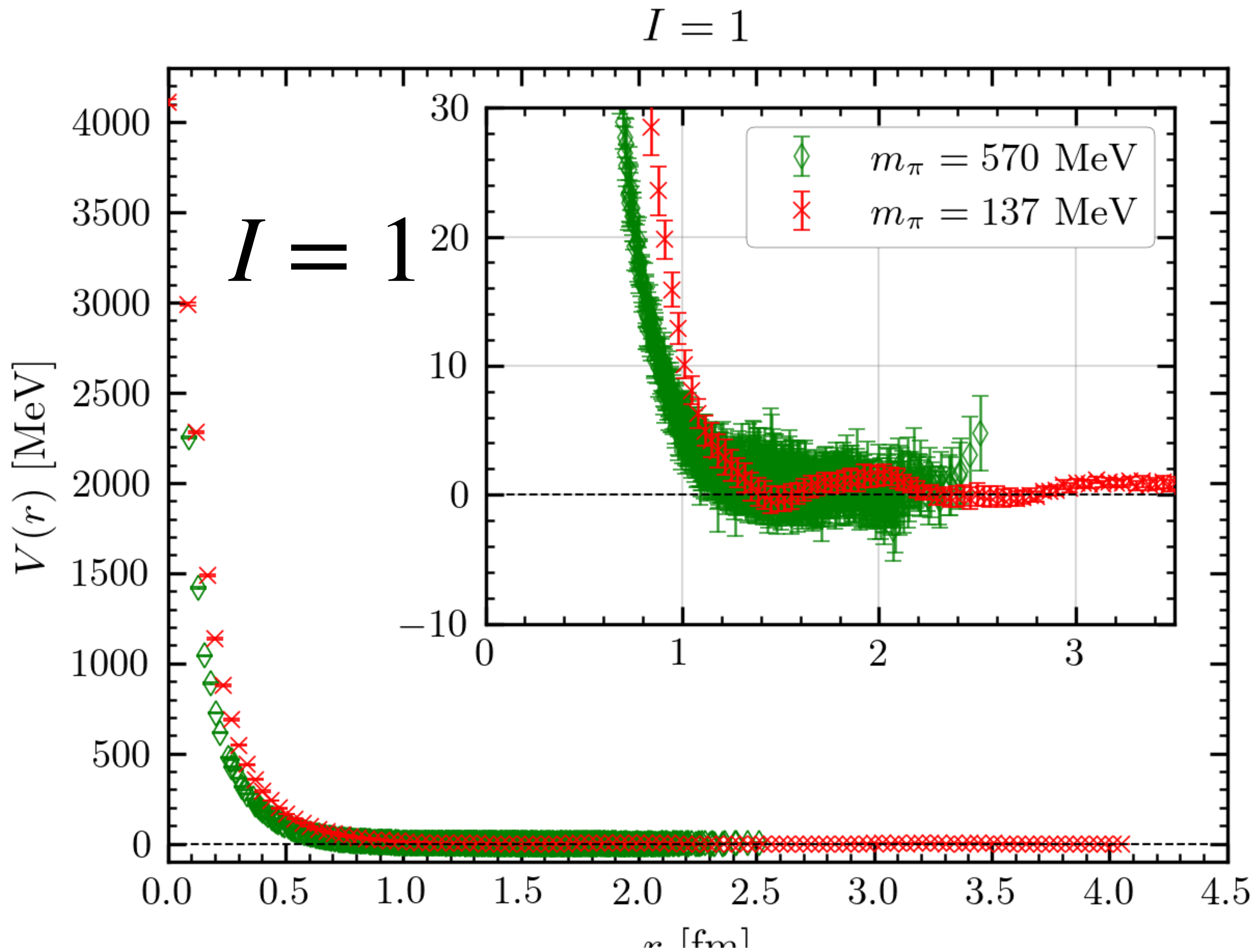
- statistics: 360 confs. x 96 sources x 4 rotations

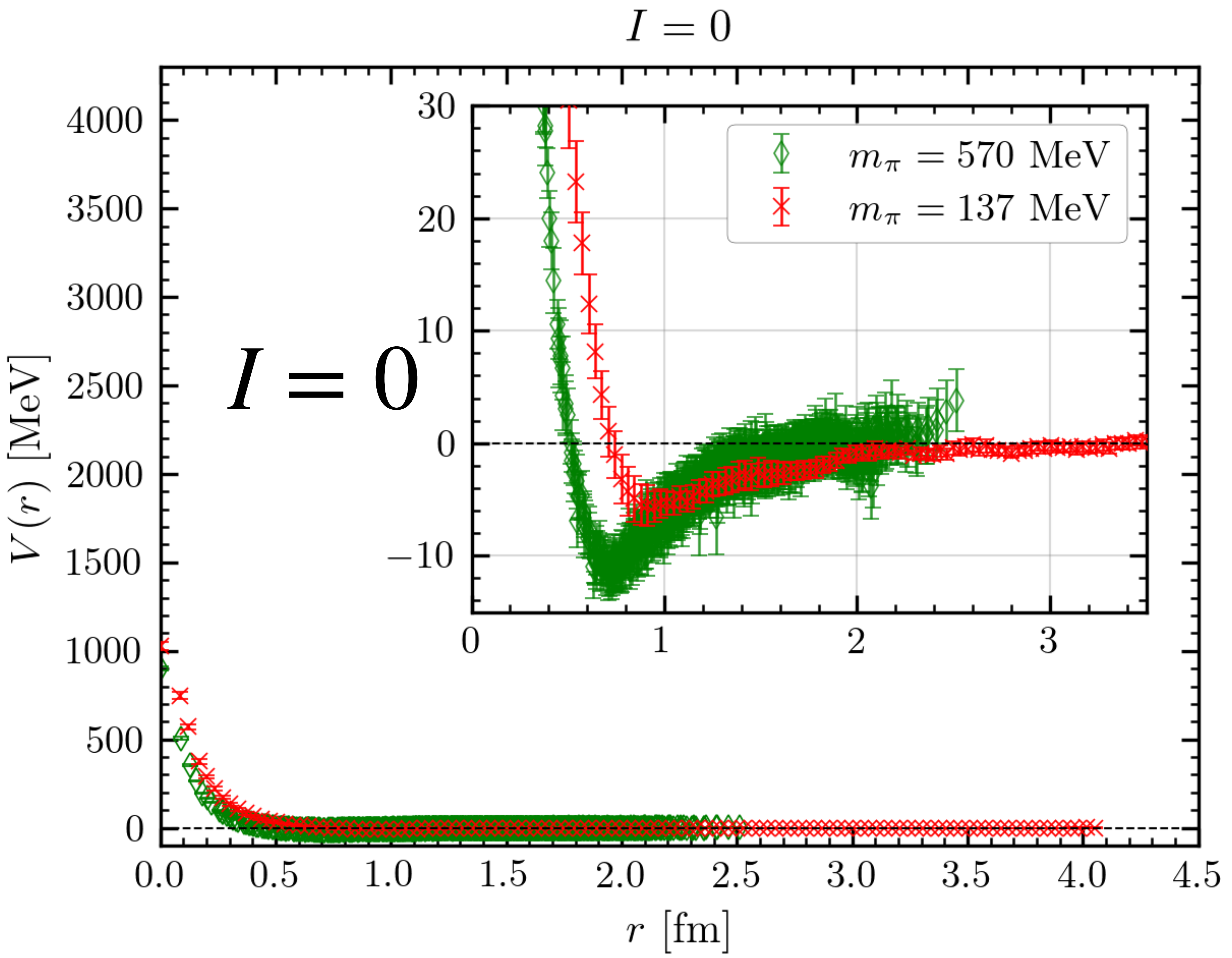
- jackknife with binsize = 40 confs.

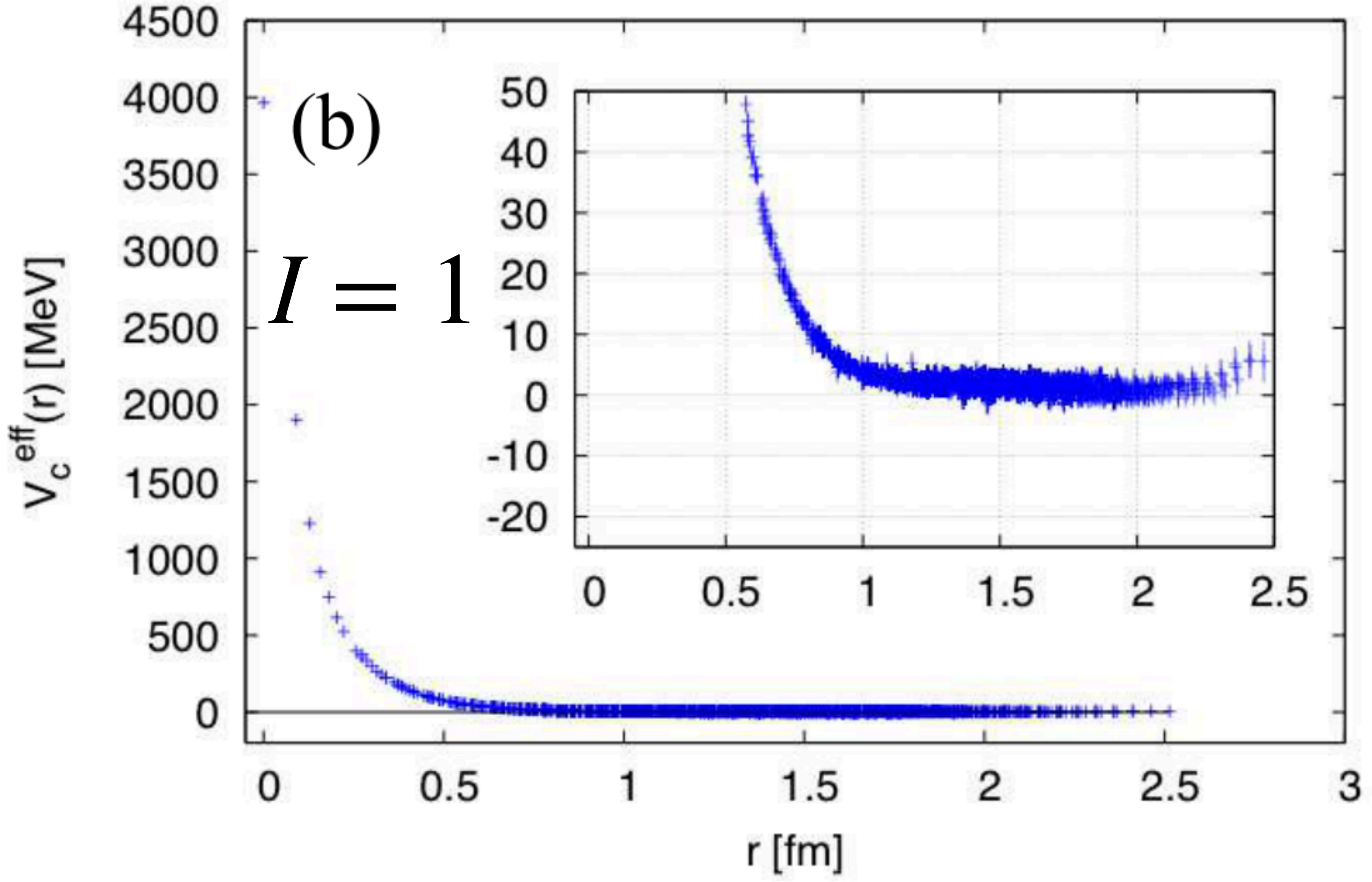
- taking spin average after A_1 projection

- use Misner method w/ $n_r = 4, l_{\max} = 2, \Delta = 1.0a$, except that use the original data where the Misner method does not work ($r/a = 0, 1, \sqrt{2}$)

Comparison to results w/ heavier quark masses







blue: $m_\pi \approx 705$ MeV, $m_K \approx 793$ MeV

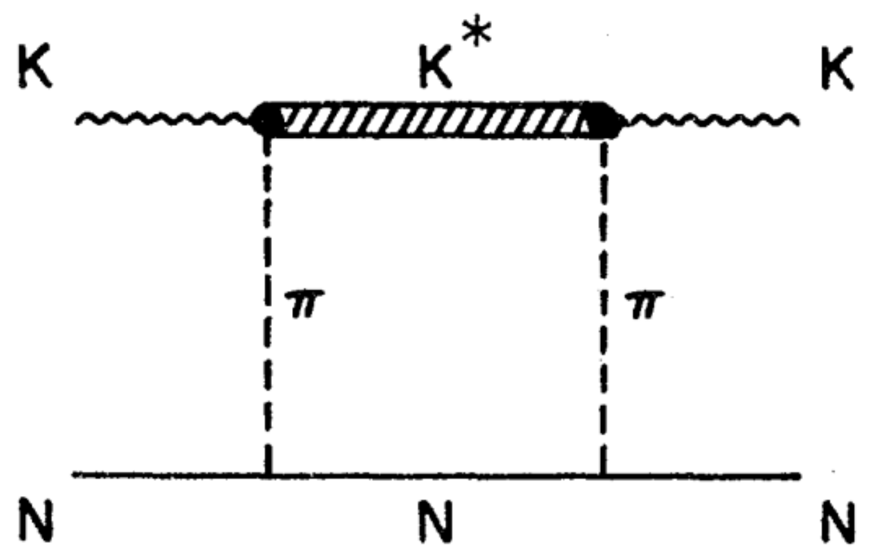
[Ikeda, PoS (2012)]

green: $m_\pi \approx 570$ MeV, $m_K \approx 713$ MeV

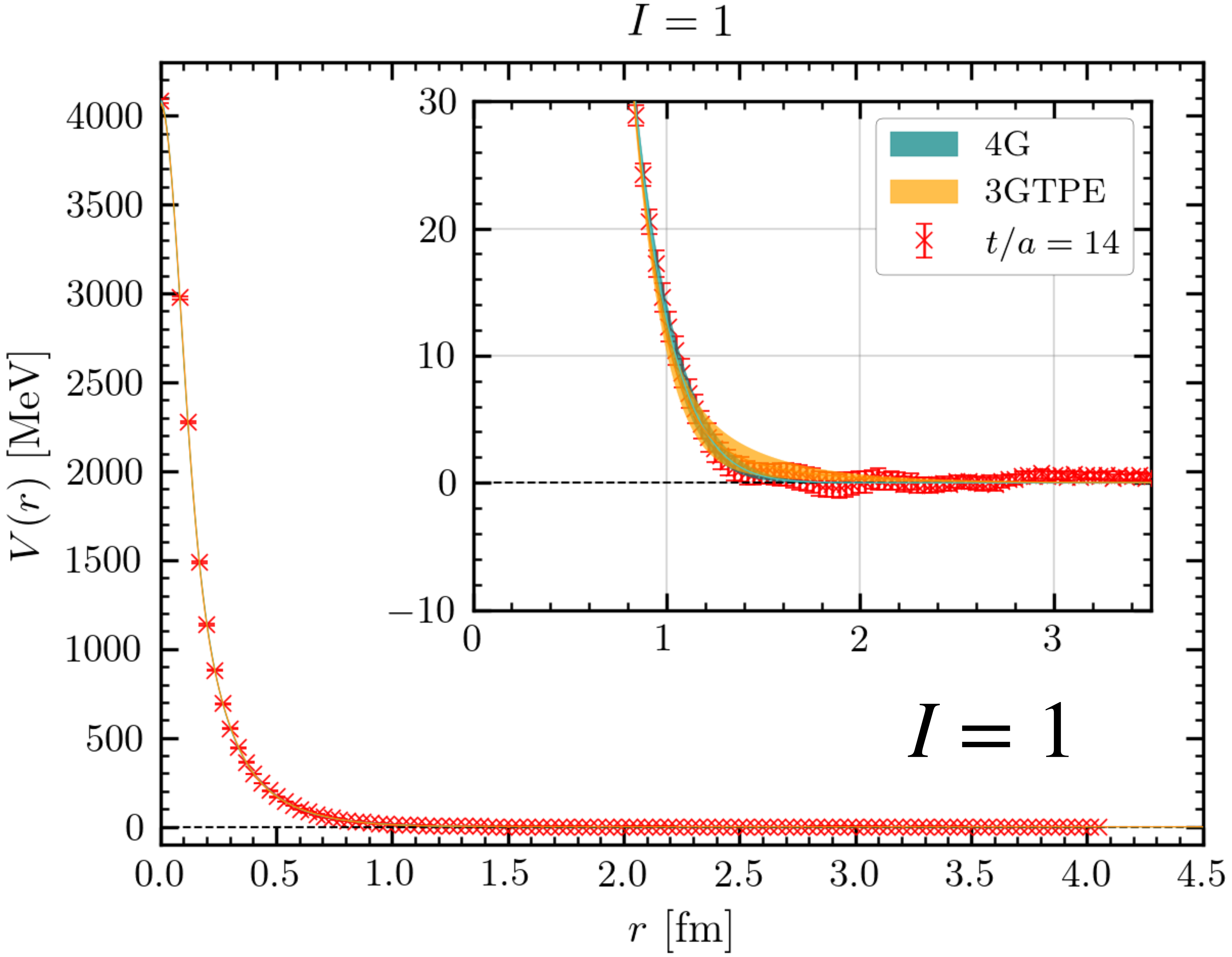
[KM, Akahoshi, Aoki, PTEP (2020)]

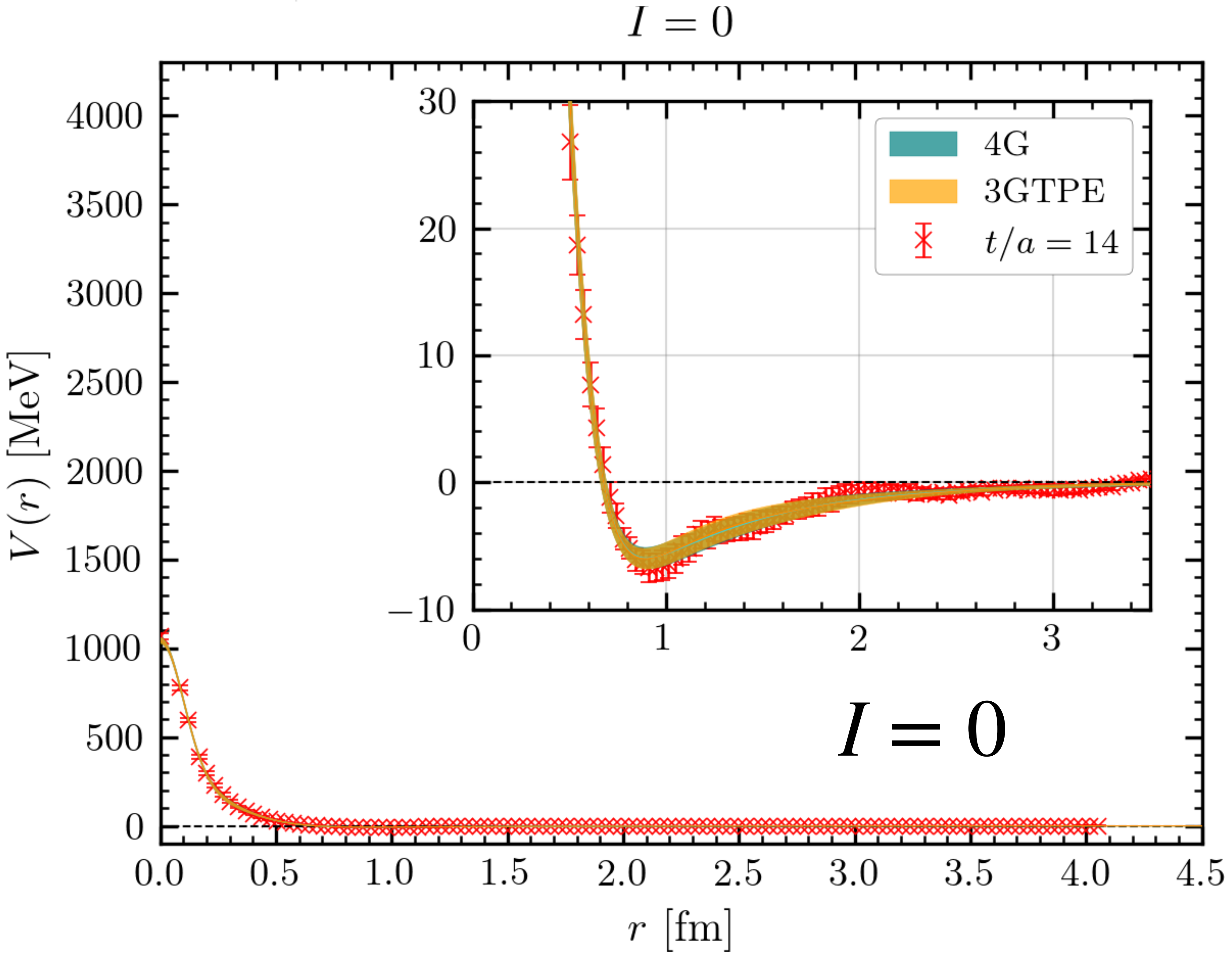
Fittings for all functions (1/2)

$$V_I^{3\mathrm{GTPE}}(r) = \sum_{i=1}^{3} c_i^I e^{-\frac{r^2}{(d_i^I)^2}} + \alpha^I \left[(1 - e^{-\beta r^2}) \frac{e^{-m_\pi r}}{r} \right]^2$$



[Aaron, Amado, Silbar, PRL (1971)]





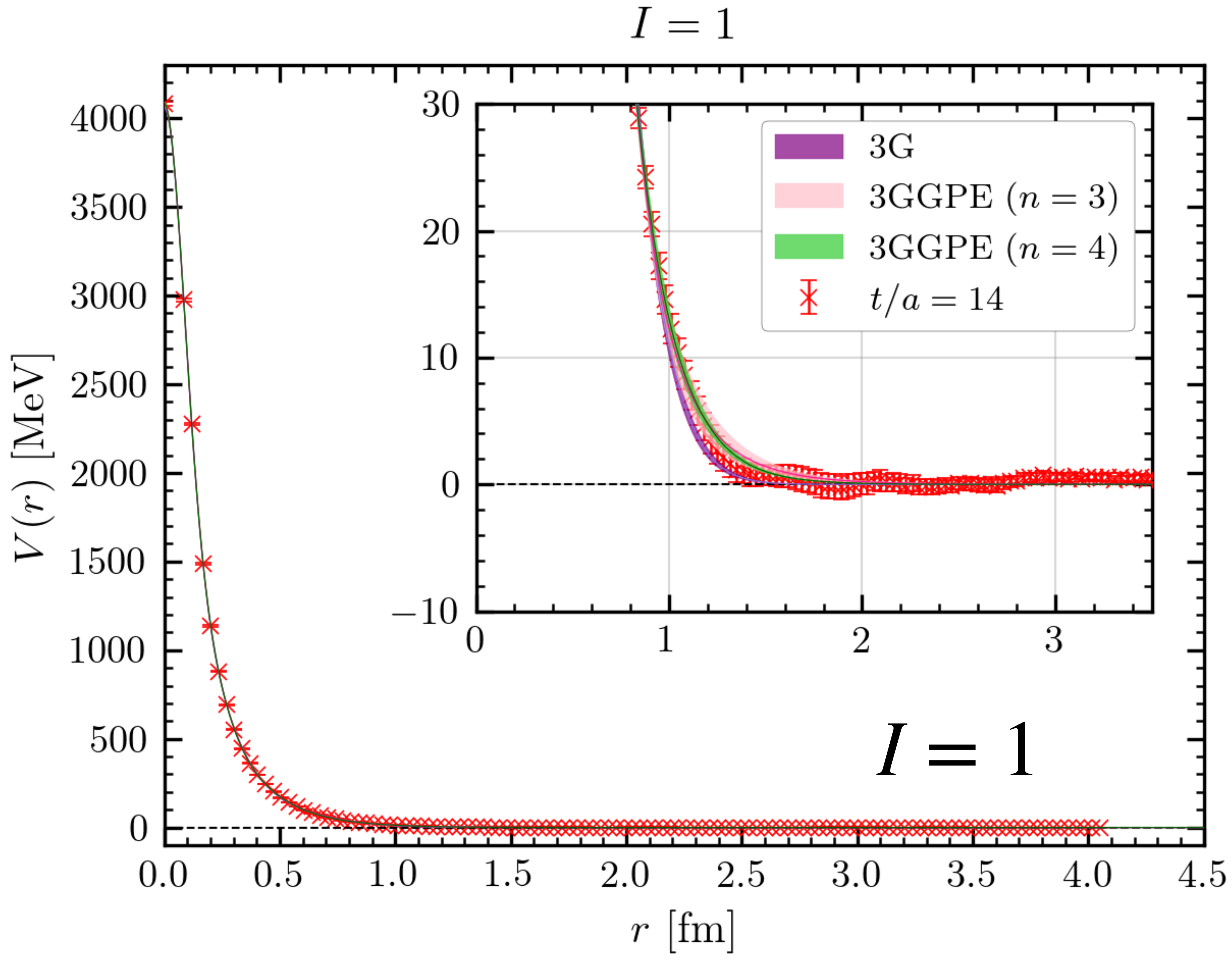
- 3GTPE for I=0: the attraction at long distances is reproduced by TPE + one Gaussian at t=14 but only TPE at t=13

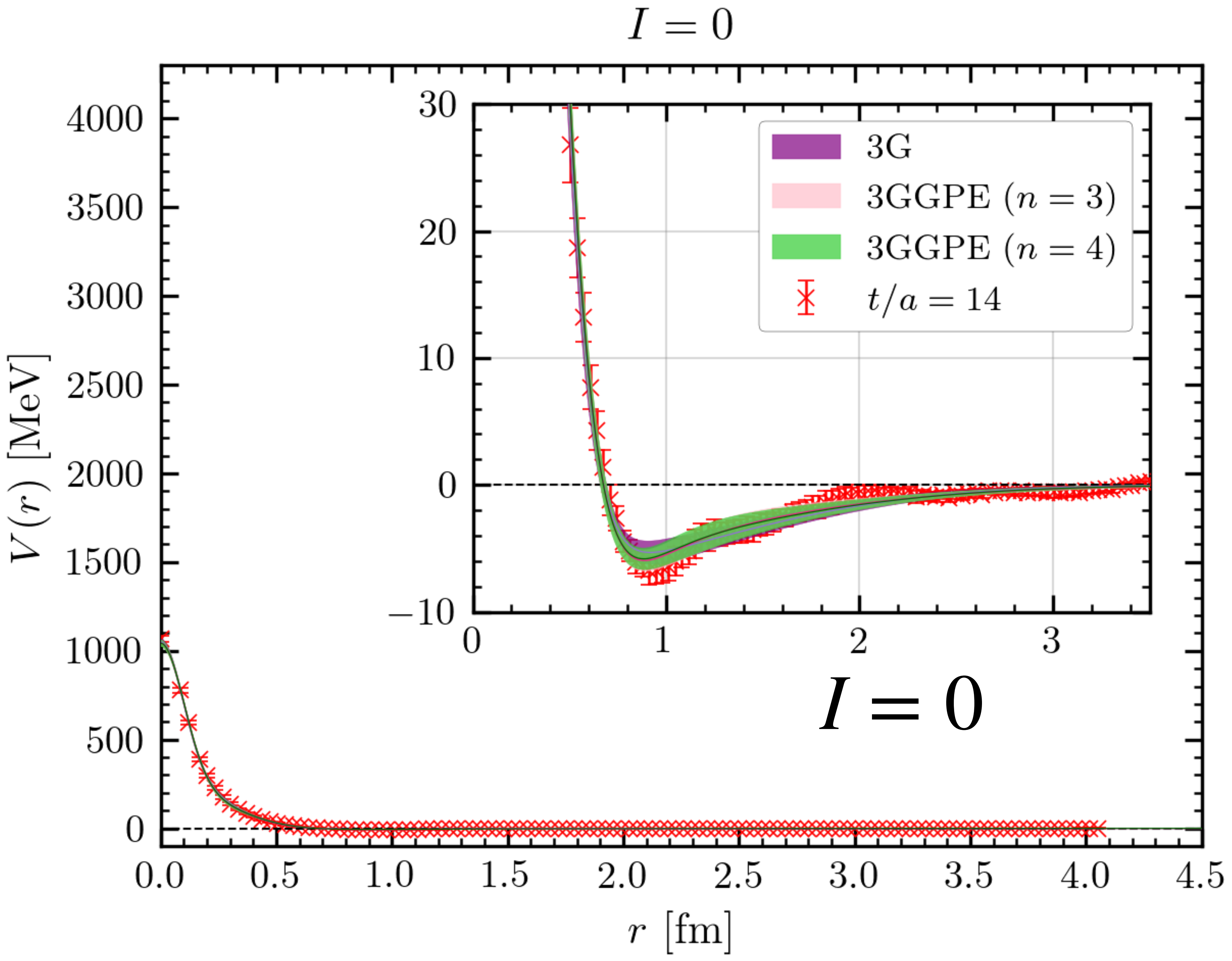
➡ no definitive conclusion on the coupling of TPE

Fittings for all functions (2/2)

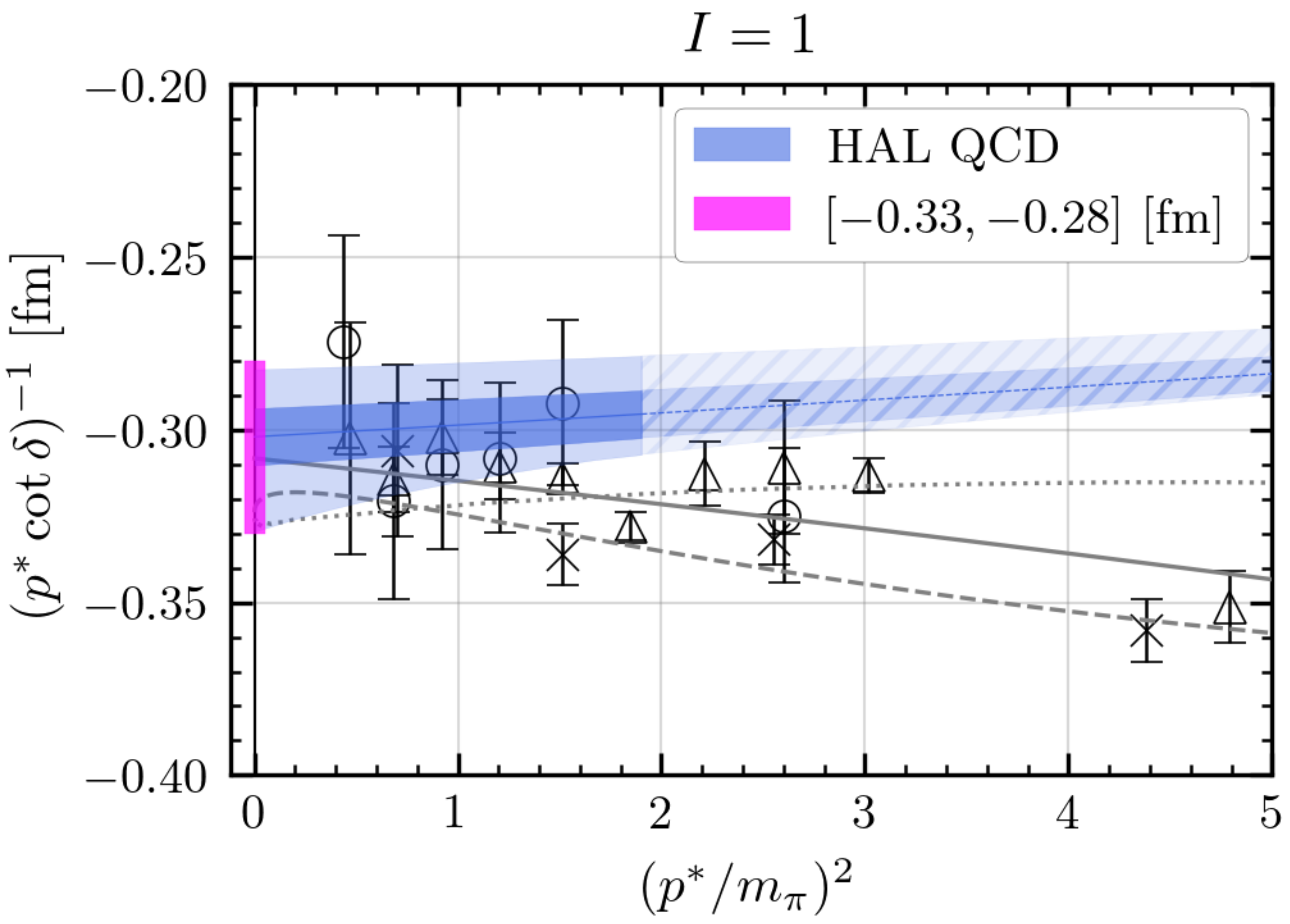
$$V_I^{3G}(r) = \sum_{i=1}^{3} a_i^I e^{-\frac{r^2}{(b_i^I)^2}}$$

$$V_{I,n}^{3GGPE}(r) = \sum_{i=1}^{3} c_i^I e^{-\frac{r^2}{(d_i^I)^2}} + \alpha^I \left[(1 - e^{-\beta r^2}) \frac{e^{-m_\pi r}}{r}\right]^n$$

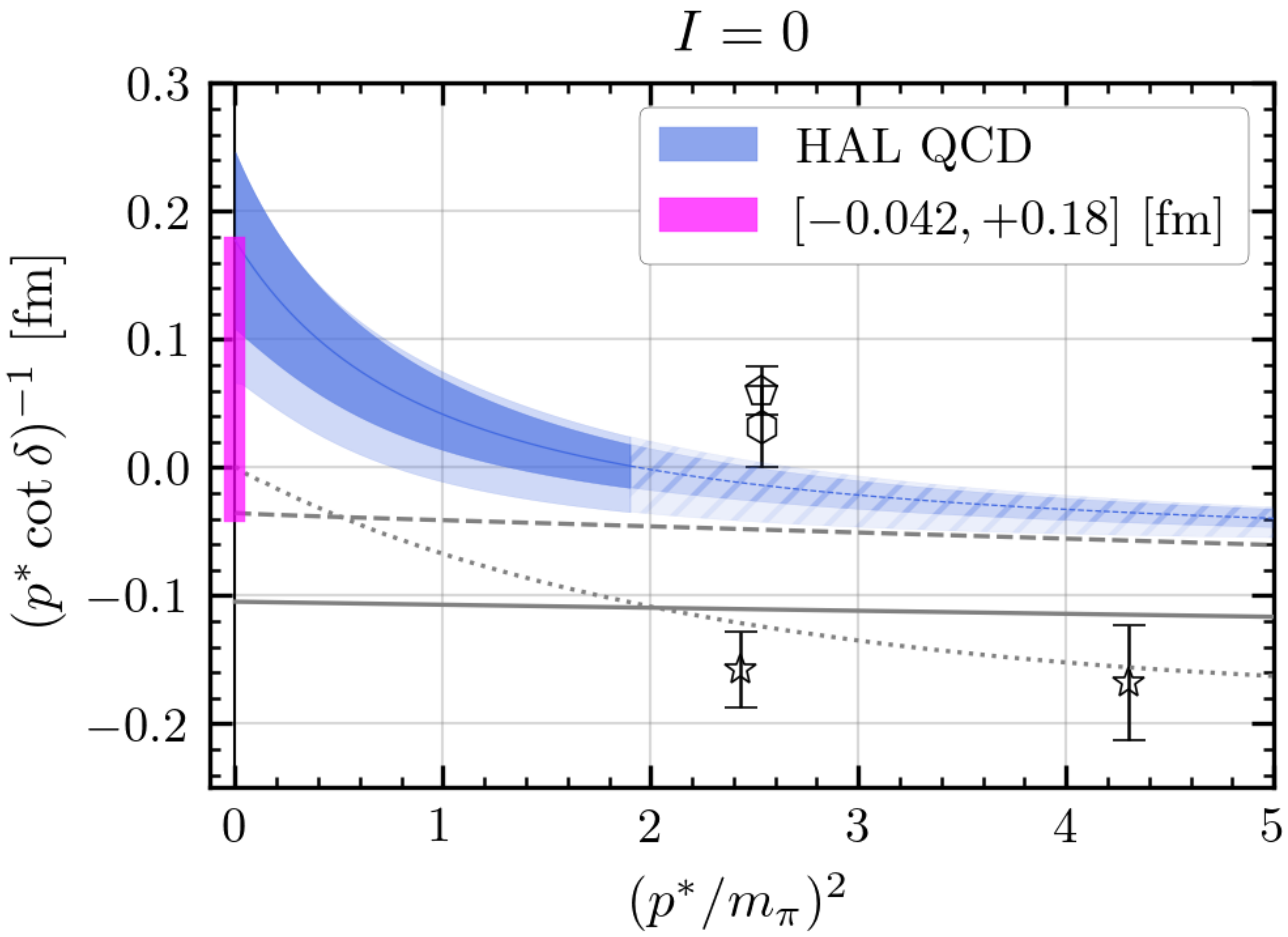




$(k \cot \delta)^{-1}$



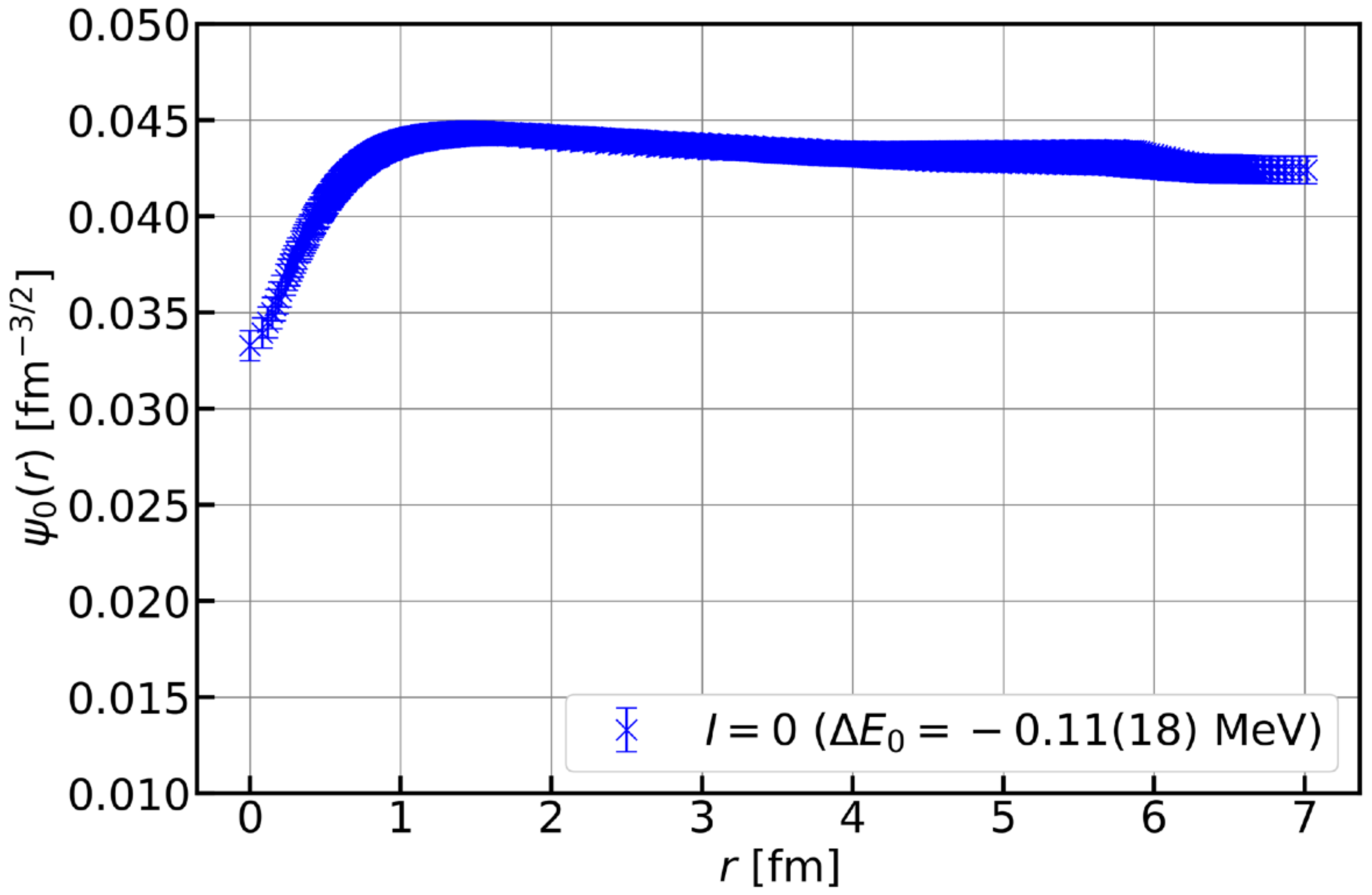
$I = 1$
HAL QCD
$[-0.33, -0.28]$ [fm]
$(p^* \cot \delta)^{-1}$ [fm]
$(p^*/m_\pi)^2$

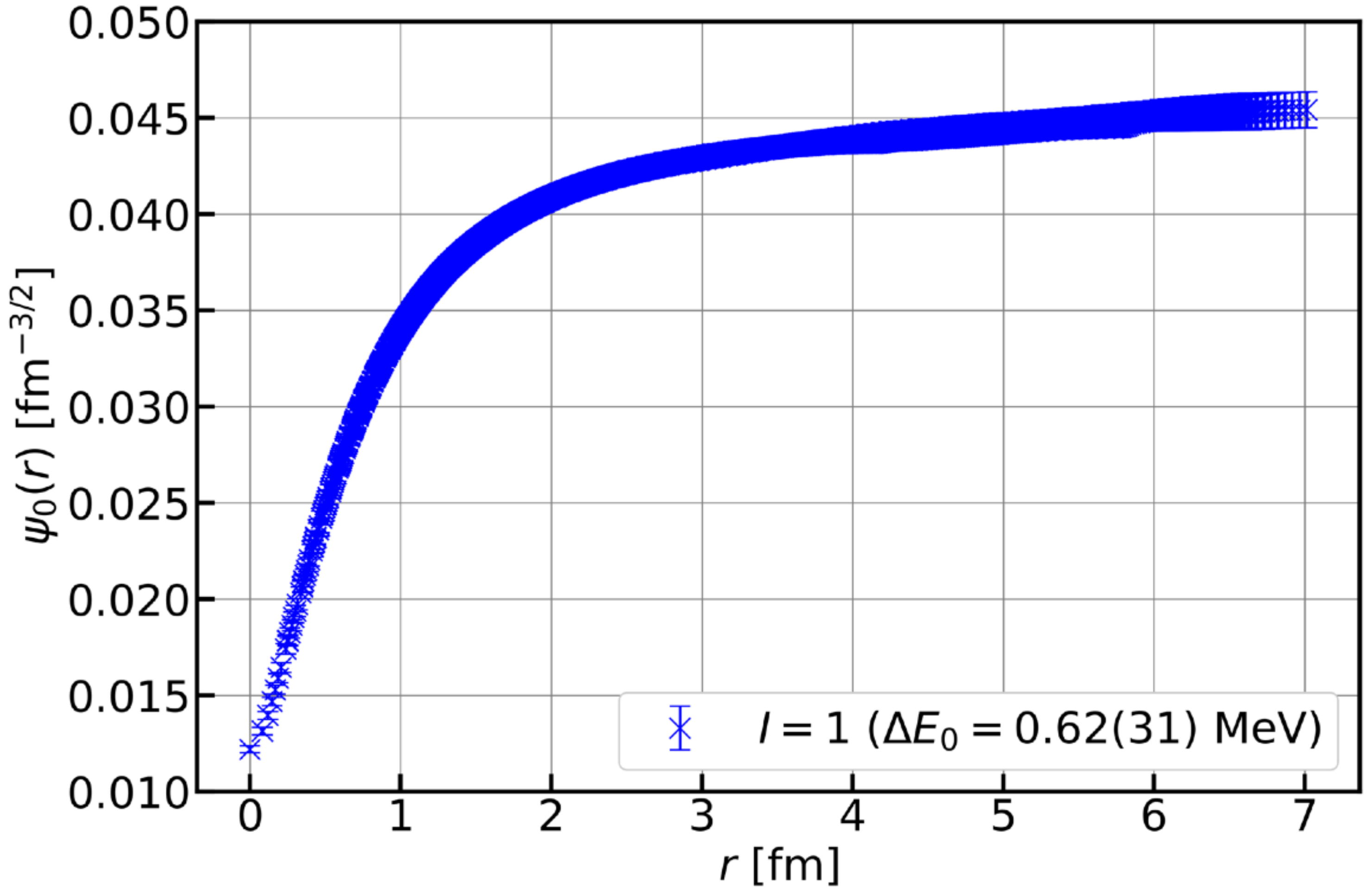


$I = 0$
HAL QCD
$[-0.042, +0.18]$ [fm]
$(p^* \cot \delta)^{-1}$ [fm]
$(p^*/m_\pi)^2$

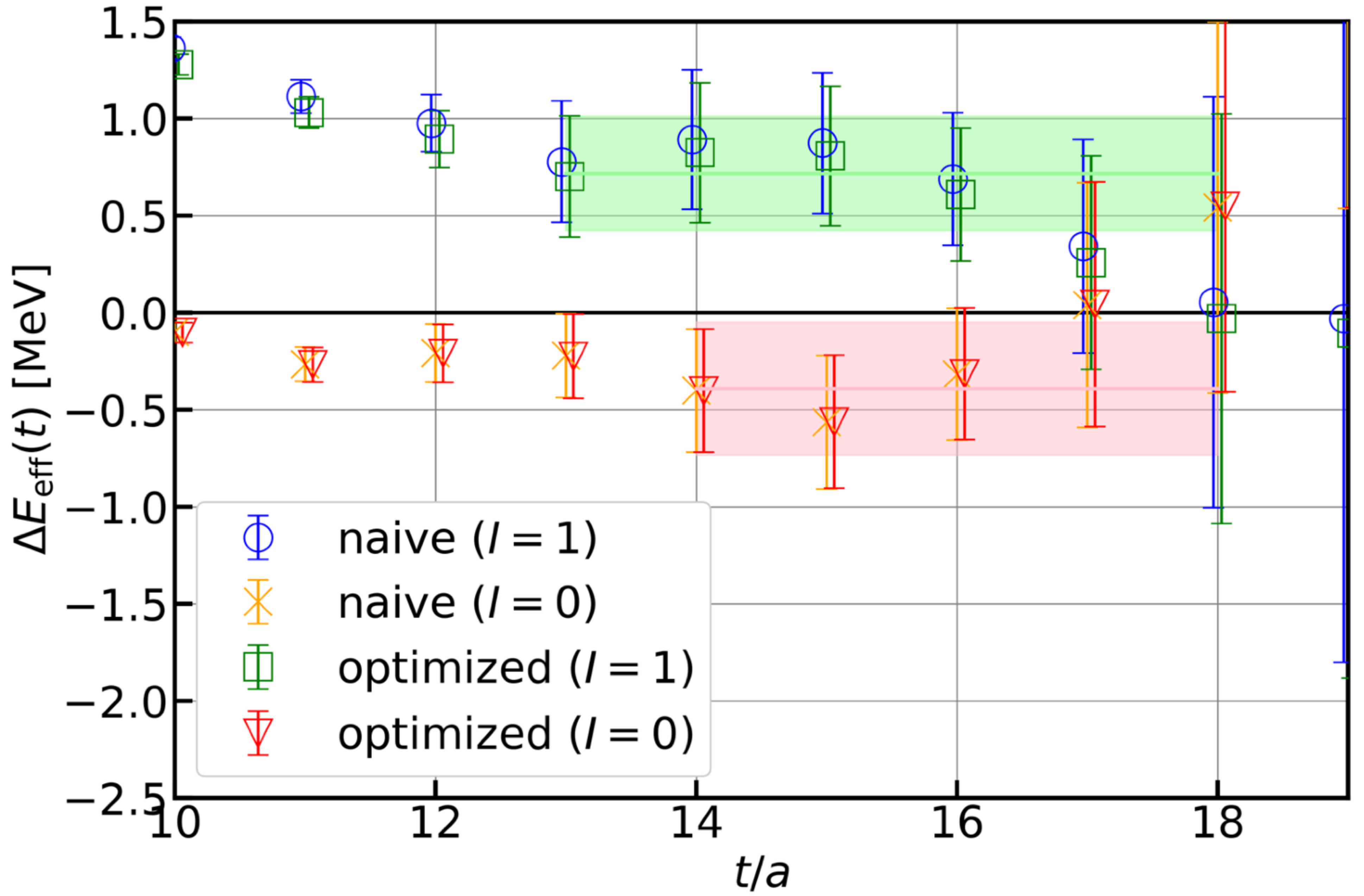
ground-state energy on finite volume

$$R_0^I(t) = \frac{\sum_\alpha \langle \left[\sum_{\mathbf{r}} \Psi_0^I(\mathbf{r}) (KN)_\alpha^{(I)}(\mathbf{r}, t+t_0) \right] \bar{K}^0(t_0) \bar{p}_\alpha(t_0) \rangle}{C_K(t) C_N(t)}.$$





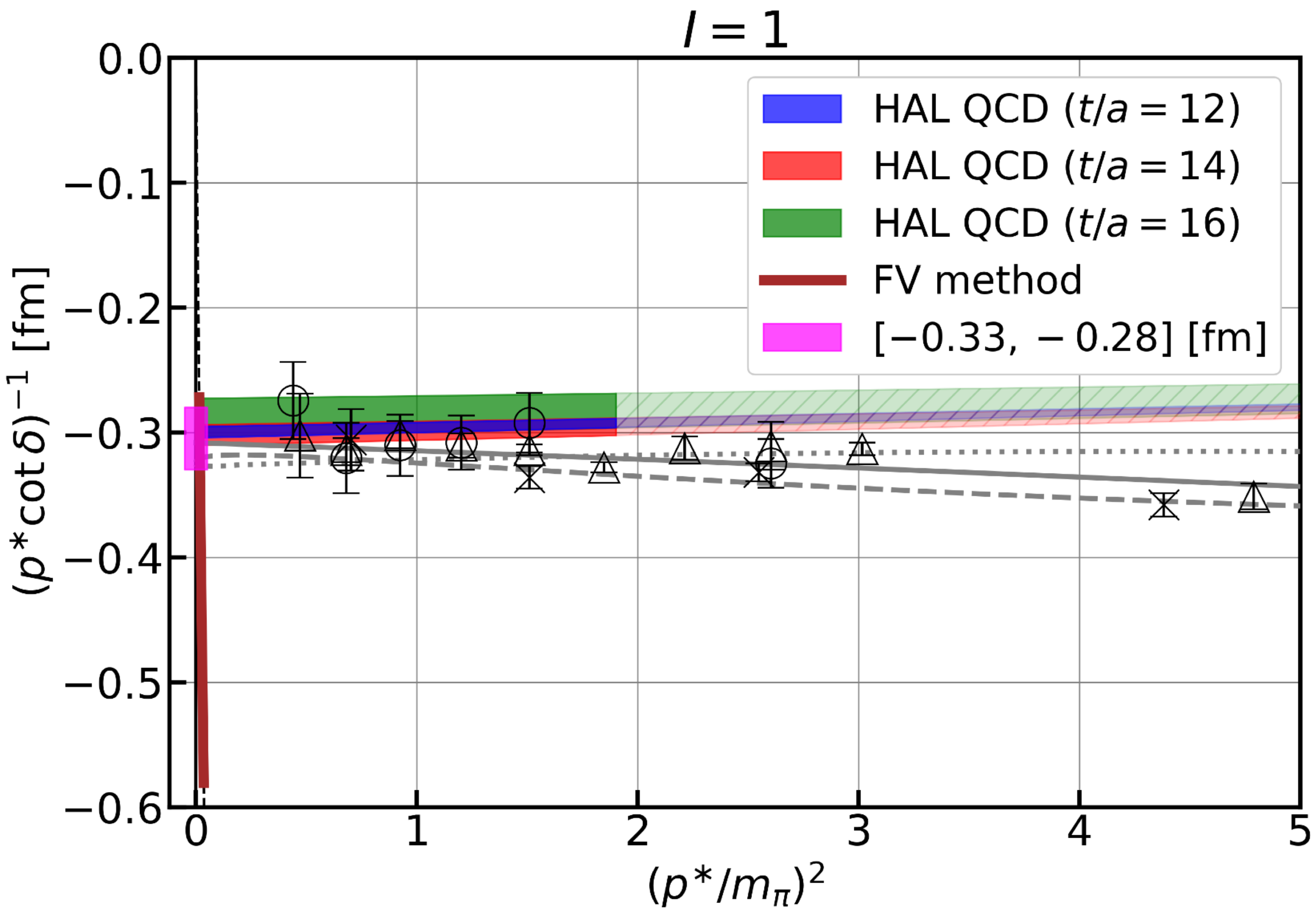
- E_eff

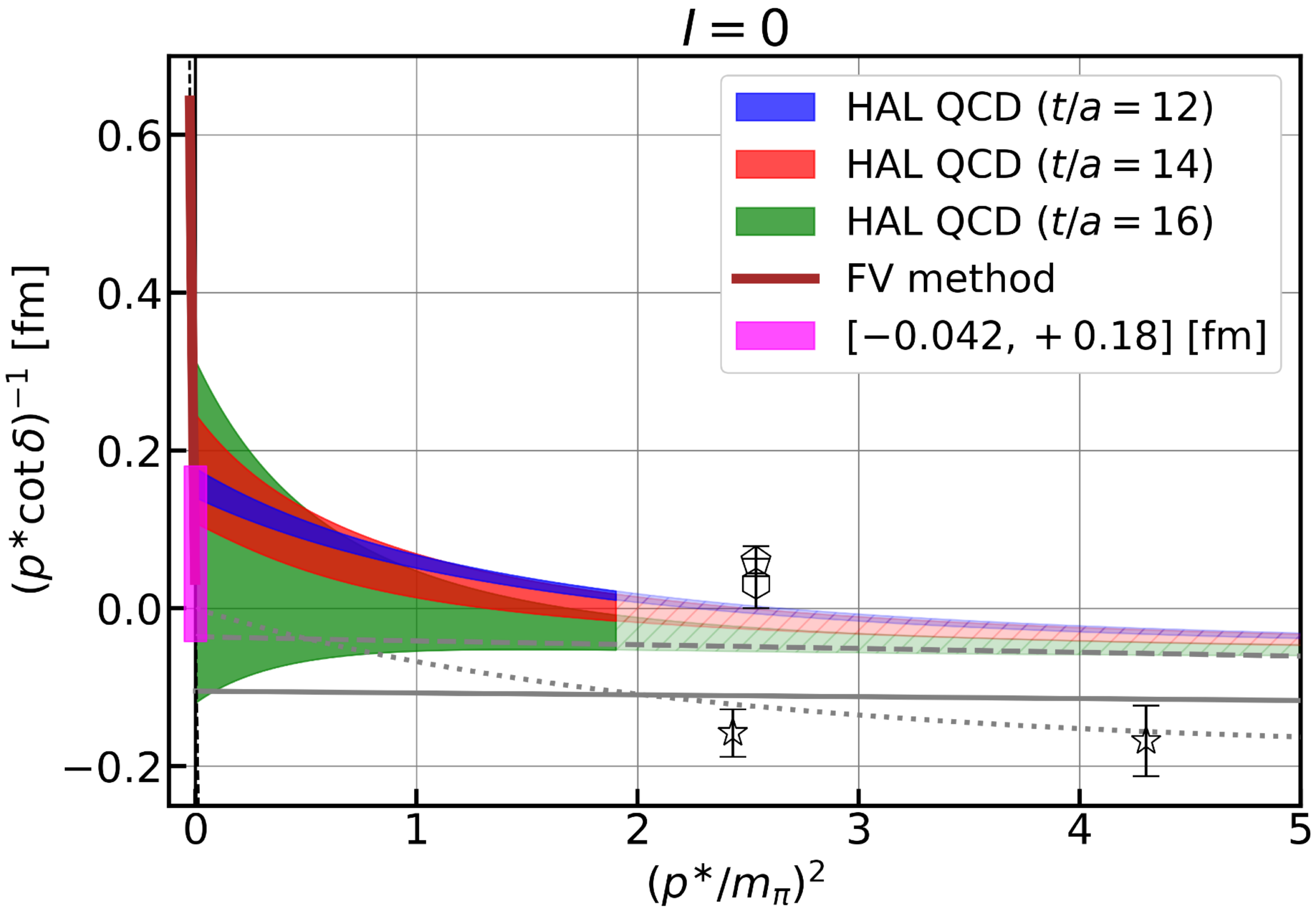


$$E_{\mathrm{gs}}^{I=1} - m_N - m_K = +0.716(295)\ \mathrm{MeV},$$

$$E_{\mathrm{gs}}^{I=0} - m_N - m_K = -0.392(342)\ \mathrm{MeV},$$

$(k \cot \delta)^{-1}$ from FV spectrum





- consistent with HAL QCD

References: phase shift data & PWA

- Experimental data (S-wave phase shifts)
  - circles ○ (I=1) : S. Goldhaber et al., Phys. Rev. Lett. 9, 135 (1962)
  - crosses × (I=1): R. A. Burnstein et al., Phys. Rev. D 10, 2767 (1974)
  - triangles △ (I=1): W. Cameron et al., Nucl. Phys. B 78, 93 (1974)
  - pentagon & hexagon (I=0): V. J. Stenger et al., Phys. Rev. 134, B1111 (1964)
  - stars ☆ (I=0): R. G. Glasser et al., Phys. Rev. D 15, 1200 (1977)
- Partial-wave analysis (gray curves; energy-dependent fits to exp. data)
  - dashed line : B. R. Martin, Nucl. Phys. B 94, 413 (1975)
  - dotted line : J. S. Hyslop et al., Phys. Rev. D 46, 961 (1992)
    (SAID: https://gwdac.phys.gwu.edu/)
  - solid line : W. R. Gibbs and R. Arceo, Phys. Rev. C 75, 035204 (2007)

Cross sections from PWA