ETH zürich
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D-PHYS
HPCP

Renormalization of vector and axial-vector currents for three flavor exponential-clover improved Wilson fermions

J. T. Kuhlmann, P. Fritzsch, J. Heitger

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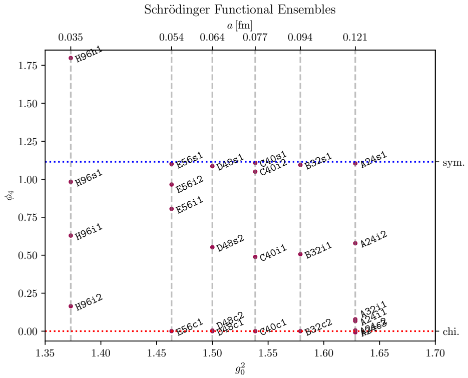
Exponential Wilson Clover Fermions

Small reminder

$$D_{\mathrm{ee}} + D_{\mathrm{oo}} = (4 + m_0) + a c_{\mathrm{sw}} \frac{\mathrm{i}}{4} \sigma_{\mu\nu} \hat{F}_{\mu\nu}$$

$$\rightarrow D_{\mathrm{ee}}^{\exp} + D_{\mathrm{oo}}^{\exp} = M_0 \exp\left\{ \frac{a c_{\mathrm{sw}}}{4 + m_0} \frac{\mathrm{i}}{4} \sigma_{\mu\nu} \hat{F}_{\mu\nu} \right\}$$

- Made for numerical stability **[1911.04533]**
- Goal: QCD matrix elements on OpenLat
- Approach:
  - Schrödinger functional
  - Ability to work close to the chiral limit
  - Intermediate volumes ($3\,\mathrm{fm}$ box)
  - Aligned with OpenLat ensembles
- $N_{\mathrm{f}} = 3$ mass degenerate $\Rightarrow \mathrm{Tr}\{M_{\mathrm{q}}\} = 3 m_{\mathrm{q}}^{\mathrm{sea}}$



General strategy

- Evaluate Ward identities in the Schrödinger functional
- For Z_V: **[hep-lat/9704001, hep-lat/9710071, 1711.03924, 2510.06869]**
  - Vector rotations $\psi(x) \to \psi'(x) = e^{-\mathrm{i}\alpha(x)\frac{\lambda^c}{2}}\psi(x)$
  - Variation: $\delta_\mathrm{V}\langle OO'\rangle$
  - Resulting condition to evaluate on the lattice:

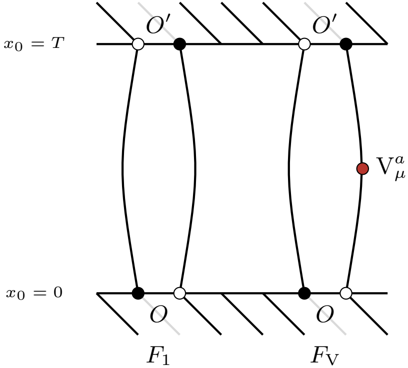
$$\hat{Z}_\mathrm{V} = F_1/F_\mathrm{V} + \mathrm{O}(a^2)$$

- For Z_A: **[hep-lat/9611015, hep-lat/0505026, Della Morte *et al.* (2009), 1604.05827]**
  - Axial rotations $\psi(x) \to \psi'(x) = e^{-\mathrm{i}\alpha(x)\gamma_5\frac{\lambda^c}{2}}\psi(x)$
  - Variation: $\delta_\mathrm{A}\langle AOO'\rangle$
  - Resulting equation to evaluate on the lattice:

$$Z_\mathrm{A} = \sqrt{\frac{F_1}{4m_\mathrm{I}(\tilde{F}_\mathrm{PA})_\mathrm{I} - 2(F_\mathrm{AA})_\mathrm{I}}} + \mathrm{O}(a^2)$$

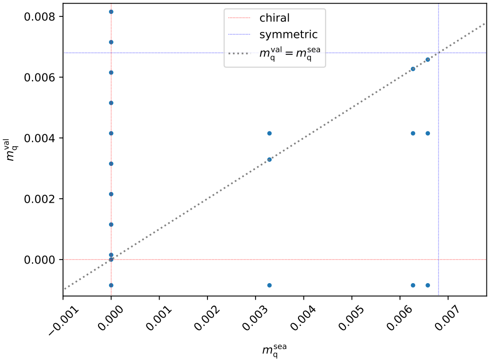
Renormalization of the vector channel

- $\hat{Z}_{\rm V} = Z_{\rm V}\left(1 + b_{\rm V}\ am_{\rm q}^{\rm val} + \bar{b}_{\rm V}\ a{\rm Tr}\{M_{\rm q}^{\rm sea}\}\right) = F_1/F_{\rm V} + {\rm O}(a^2)$
  with $F_1 = \langle O'O\rangle$ and $F_{\rm V} = \langle O'V_0O\rangle$ and $O = \bar{\zeta}\zeta$
- Holds at $m_{\rm q} \neq 0$
- Close to the chiral point, gives access to $Z_{\rm V}$, $b_{\rm V}$, $\bar{b}_{\rm V}$
- Multiple strategies possible **[hep-lat/9704001, 1805.07401]**
- **Here: linear fits** close to the chiral limit

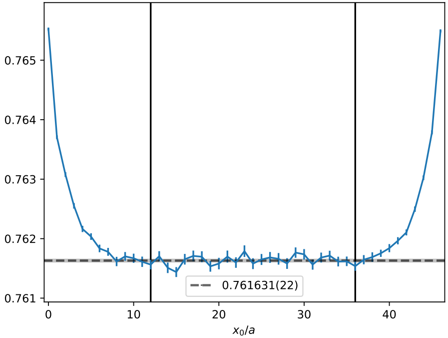


Dataset
Renormalization factors

data points at $\beta = 3.90$

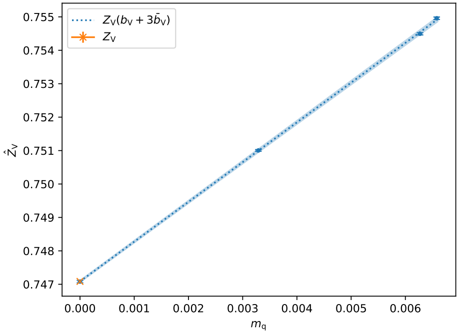


$\hat{Z}_V(\beta = 4.0, m_q^{val} = m_q^{sea} \approx 0; x_0/a)$



Results
Vector channel

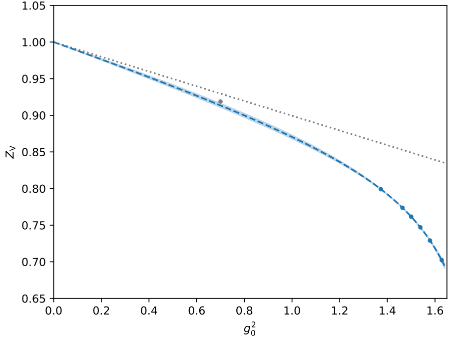
Interpolation of $Z_\mathrm{V}(\beta = 3.90, m_\mathrm{q})$



$$\hat{Z}_\mathrm{V} = Z_\mathrm{V} + Z_\mathrm{V}(b_\mathrm{V} + 3\bar{b}_\mathrm{V})\, am_\mathrm{q}$$

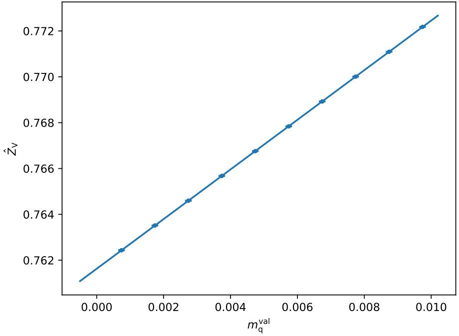
with $m_\mathrm{q} = m_\mathrm{q}^\mathrm{val} = m_\mathrm{q}^\mathrm{sea}$

Interpolation of $Z_\mathrm{V}(g_0^2)$

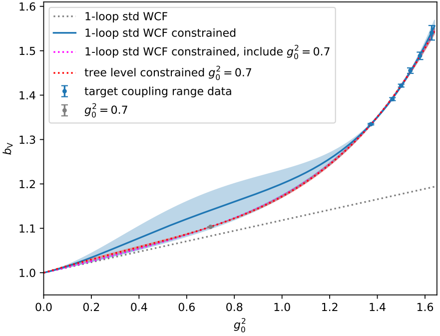


$$Z_\mathrm{V}(g_0^2) = 1 - 0.075427\, \frac{4}{3} g_0^2 \left(\frac{1 + a_1 g_0^2 + a_2 g_0^4}{1 + b_1 g_0^2} \right)$$

Results
Vector channel



Interpolation to access $b_V(\beta = 4.0)$



$$b_V = 1 + 0.0886(4/3)g_0^2 + a_1 g_0^4 + a_2 g_0^6 + a_3 g_0^8$$

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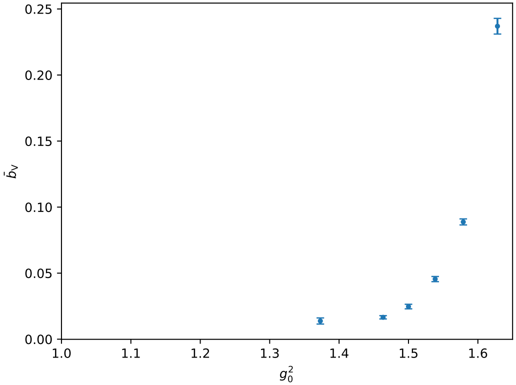
Results
Vector channel



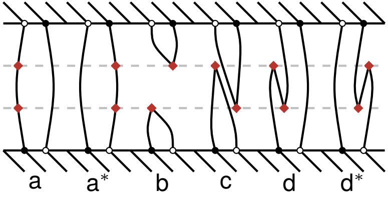
Evaluate: $\bar{b}_{\rm V} = \frac{(\partial_{m_{\rm q}} \hat{Z}_{\rm V}/Z_{\rm V}) - b_{\rm V}}{3}$

Renormalization of the axial-vector channel

- Z_A needs the evaluation of **4-point functions**
- Contact terms $\propto am_\mathrm{q} \Rightarrow$ consider only $m_\mathrm{q} = 0$

$$Z_\mathrm{A} = \sqrt{\frac{F_1}{4m_\mathrm{I}(\tilde{F}_\mathrm{PA})_\mathrm{I} - 2(F_\mathrm{AA})_\mathrm{I}}} + \mathrm{O}(a^2)$$

where $F_\mathrm{XY} = \langle O' X(x_0) Y(y_0) O \rangle$, $\tilde{F}_\mathrm{XY} = \sum_{x_0}^{y_0} F_\mathrm{XY}$
and O, O' SF boundary sources **[hep-lat/0505026]**



- $Z_\mathrm{A}^\mathrm{full}$ and $Z_\mathrm{A}^\mathrm{conn}$ are both legitimate estimators
- Only correct to $\mathrm{O}(a^2)$ if A_μ^a is improved

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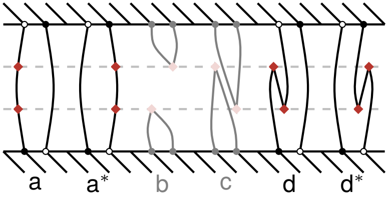

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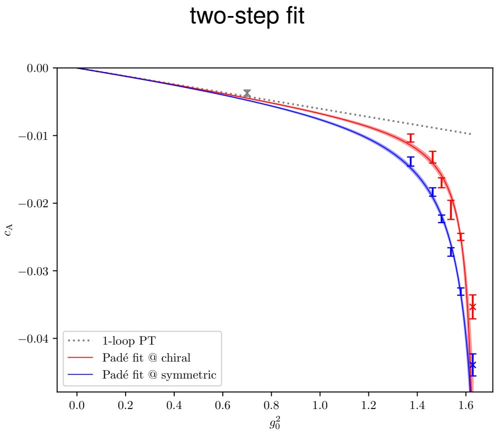
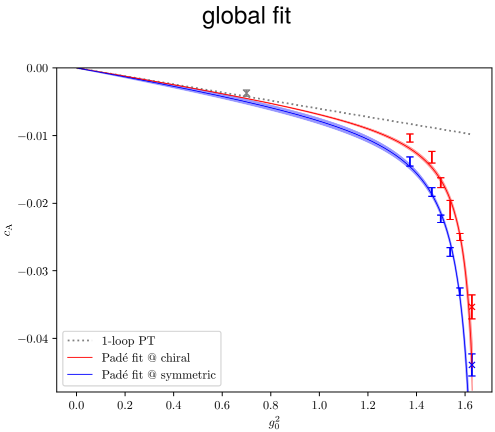
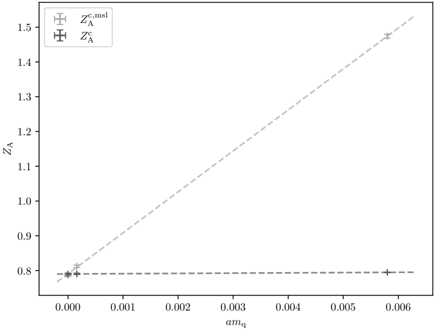
Status of c_A

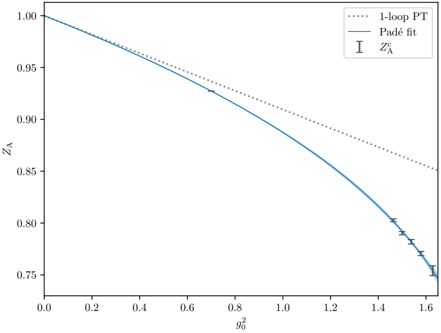




closer look from perturbation theory **[J. Falceto, Fri, 15:00]**

Results
Axial-vector channel





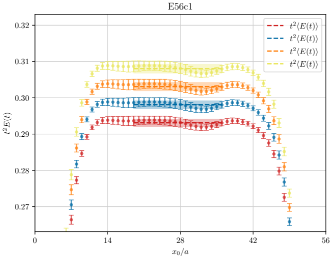
Outlook

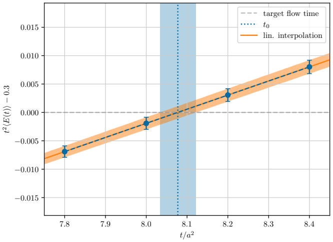
- Focus on finalizing c_{A}
- We have seen: Z_{V}, b_{V}, $\bar{b}_{\mathrm{V}}$ are under investigation
  - Still more data points/statistics
  - last production runs
- Z_{A} viable, final analysis awaits c_{A}
- c_{V} in progress

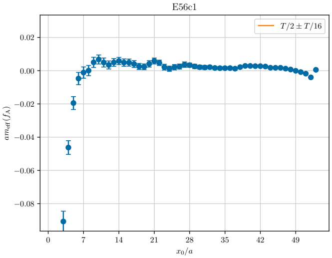
Backup

Determination of the chiral limit | Evaluation of ϕ_4

$$\phi_4 = \frac{3}{2} 8 t_0 m_{\text{eff}}^2$$

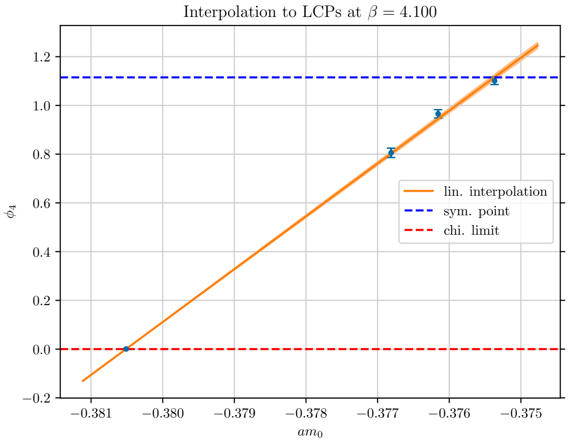






Backup

Determination of the chiral limit | Extrapolate to $m_{\mathrm{q}} = 0$



Backup

Interpolations of c_A