Nucleon momentum fraction & spin at physical pion mass and in the continuum

University of Cyprus

The Cyprus Institute

Constantia Alexandrou

with

S. Bacchio, J. Finkenrath, C. Iona, G. Koutsou, C. Kummer, Y. Li, B. Prasad, G. Spanoudes

Extended Twisted Mass Collaboration

LATTICE 2026

University of Maryland, July 26 - Aug.1, 2026

AQTIVATE

European Joint Doctorate, grant agreement No. 101072344

RESEARCH & INNOVATION FOUNDATION

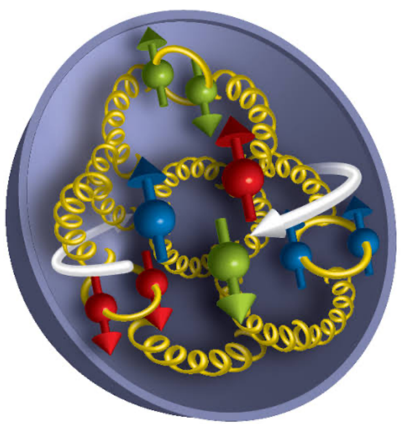
EXCELLENCE/0524/0459

Motivation

- **EMC in a pioneering experiment at CERN revealed that valence quarks in the proton contribute less than half the spin of the proton —> proton spin crisis**

Physics Letters B 206, 364 (1988)

- **We know that the proton is not just valence quarks it is but much more complicated**

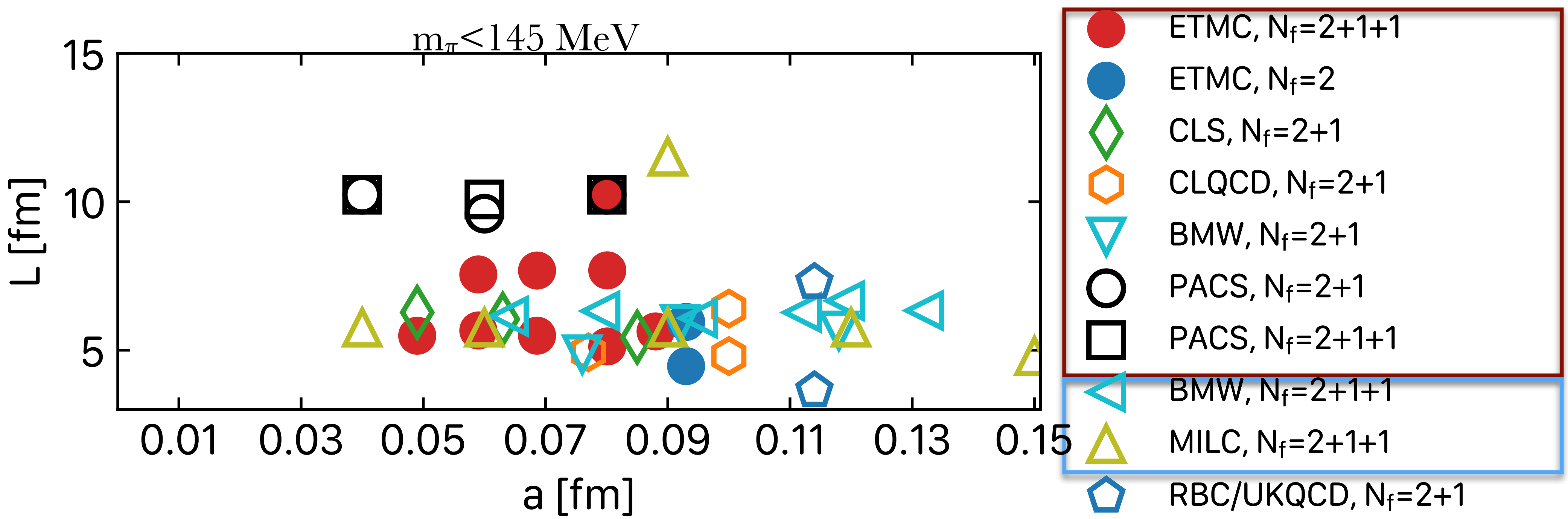


- **But what is the exact fraction carried by each parton - valence and sea quarks and gluons?**

ETMC: C.A. *et al.*, arXiv: 2607.20337

ETMC: C.A. *et al.*, arXiv: 2607.21230

ETMC gauge ensembles



Wilson-type

Staggered

Domain wall

Ensemble	$(\frac{L}{a})^3 \times (\frac{T}{a})$	a [fm]	m_π [MeV]	$m_\pi L$
B64	$64^3 \times 128$	0.0795(1)	140.2(2)	3.62
C80	$80^3 \times 160$	0.0682(1)	136.7(2)	3.78
D96	$96^3 \times 192$	0.0568(1)	140.8(2)	3.90
E112	$112^3 \times 224$	0.0489(1)	136.5(2)	3.79

Lattice spacings determined in isoQCD, corresponding to the Edinburgh/FLAG consensus

ETMC: Phys.Rev.D 111 (2025) 5, 054502 2411.08852 [hep-lat]

Computation of lower Mellin moments of GPDs

- **Lower moments** are readily accessible in lattice QCD from matrix elements of local operators with computations in early 90s
- To leading order we have the twist-2 operators

Spin-1/2

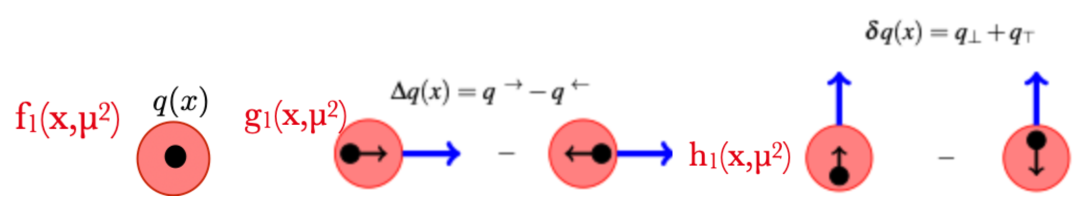
$$\mathcal{O}_V^{\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\gamma^{\{\mu_1} i\overleftrightarrow{D}^{\mu_2}\ldots i\overleftrightarrow{D}^{\mu_n\}}\psi \xrightarrow{unpolarized} \langle x^{n-1}\rangle_q = \int_0^1 dx\, x^{n-1}\left[q(x) - (-1)^{n-1}\bar{q}(x)\right]$$

$$\mathcal{O}_A^{\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\gamma_5\gamma^{\{\mu_1} i\overleftrightarrow{D}^{\mu_2}\ldots i\overleftrightarrow{D}^{\mu_n\}}\psi \xrightarrow{helicity} \langle x^{n-1}\rangle_{\Delta q} = \int_0^1 dx\, x^{n-1}\left[\Delta q(x) + (-1)^{n-1}\Delta\bar{q}(x)\right]$$

$$\mathcal{O}_T^{\rho\mu_1\mu_2\ldots\mu_n} = \bar{\psi}i\sigma^{\rho\{\mu_1} i\overleftrightarrow{D}^{\mu_2}\ldots i\overleftrightarrow{D}^{\mu_n\}}\psi \xrightarrow{transversity} \langle x^{n-1}\rangle_{\delta q} = \int_0^1 dx\, x^{n-1}\left[\delta q(x) - (-1)^{n-1}\delta\bar{q}(x)\right]$$

$$q = q_\downarrow + q_\uparrow, \quad \Delta q = q_\downarrow - q_\uparrow, \quad \delta q = q_\top + q_\perp$$

Made traceless and {} denotes symmetrization



Pol. \ Γ	γ^+	$\gamma^+\gamma_5$	$i\sigma^{+j}\gamma_5$
U	H		E_T
L		$\tilde{H}$	$\tilde{E}_T$
T	E	$\tilde{E}$	H_T, $\tilde{H}_T$

Nucleon first and second moments

n=1 —> axial charge g_A

- Helicity gives intrinsic quark spin: $\Delta\Sigma_{q_+}(\mu^2) = \int_0^1 dx\, \left[\Delta q(x,\mu^2) + \Delta\bar{q}(x,\mu^2)\right] = g_A^q$

n=2 —> unpolarized generalized form factors factors (GFFs)

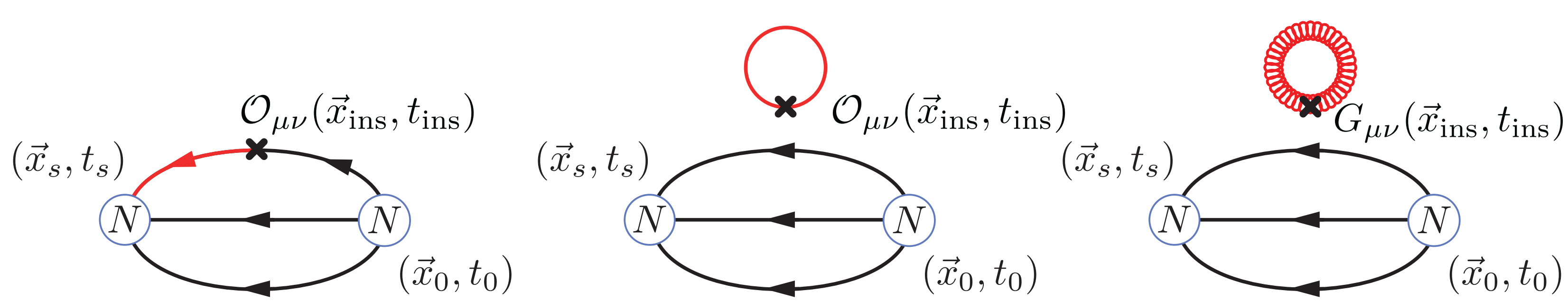
$$N(p',s')|T^{\mu\nu}_{q,g}|N(p,s) = \bar{u}_N(p',s')\left[\frac{i}{2}A^{q,g}_{20}(Q^2)\,\gamma^{\{\mu}P^{\nu\}} + B^{q,g}_{20}(Q^2)\,\frac{P^{\{\mu}\sigma^{\nu\}\rho}q_\rho}{4m_N} + C^{q,g}_{20}(Q^2)\,\frac{q^{\{\mu}q^{\nu\}}}{m_N}\right]u_N(p,s),$$

$$T_q^{\mu\nu} = \bar{\psi}i\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\psi,\quad T_g^{\mu\nu} = F^{\rho\{\mu}F^{\nu\}}_{\rho} \longleftarrow$$ symmetrization over indices and subtraction of the trace

- Moments: $\langle x\rangle_{q,g} = A^{q,g}_{20}(0)$

- Spin: $J_{q,g} = \frac{1}{2}[A^{q,g}_{20}(0) + B^{q,g}_{20}(0)]$

- Spin and momentum sums: $\sum_q\left[\frac{1}{2}\Delta\Sigma_q + L_q\right] + J_g = \frac{1}{2}, \quad \sum_q\langle x\rangle_q + \langle x\rangle_g = 1$



Extraction of matrix elements - connected

- Statistics for two-point and connected three-point functions

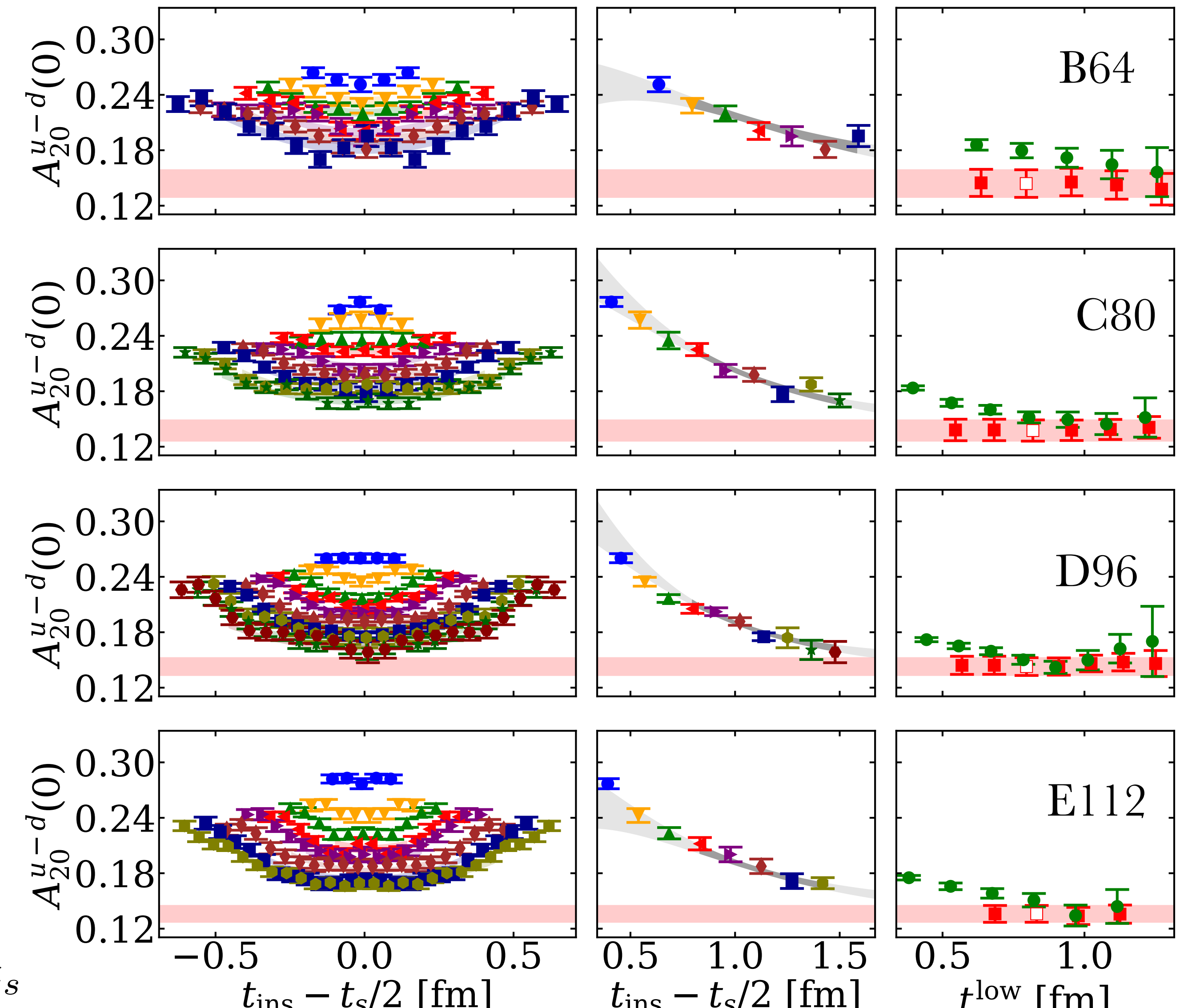
Ensemble	$N_{\rm conf}$	$n_{\rm src}^{\rm 2pt}$	t_s/a	t_s [fm]	$n_{\rm src}^{\rm conn}$
B64	725	349	8, 10, 12, 14 16, 18, 20	0.64, 0.80, 0.96, 1.12 1.28, 1.44, 1.60	1, 2, 5, 10 32, 112, 128
C80	400	650	6, 8, 10, 12, 14 16, 18, 20, 22	0.41, 0.55, 0.69, 0.82, 0.96 1.10, 1.24, 1.37, 1.51	1, 2, 4, 10, 22 48, 45, 116, 246
D96	493	368	8, 10, 12, 14, 16 18, 20, 22, 24, 26	0.46, 0.57, 0.68, 0.80, 0.91 1.03, 1.14, 1.25, 1.37, 1.48	1, 2, 4, 8, 16 32, 64, 16, 32, 64
E112	500	311	8, 11, 14, 17 20, 23, 26, 29	0.39, 0.54, 0.69, 0.83 0.98, 1.13, 1.27, 1.42	1, 2, 4, 8 16, 32, 64, 128

$$R^{\mu\nu}(\Gamma_\alpha,\vec{p}',\vec{p};t_s,t_{\rm ins}) = \frac{C_{\rm 3pt}^{\mu\nu}(\Gamma_\alpha,\vec{p}',\vec{p};t_s,t_{\rm ins})}{\sqrt{C_{\rm 2pt}(\vec{p};t_s)C_{\rm 2pt}(\vec{p};t_s)}} \times e^{-\left(E_N(\vec{p}')-E_N(\vec{p})\right)(t_{\rm ins}-t_s/2)}$$

$$\xrightarrow{t_s\to\infty} \Pi^{\mu\nu}(\Gamma_\alpha,\vec{p}',\vec{p})$$

$$\Pi^{44}(\Gamma_0;\vec{0},\vec{0}) = -\frac{3m_N}{4}\langle x\rangle$$

$$\Pi^{4i}(\Gamma_0;p_j,p_j) = ip_j\langle x\rangle$$



- Use two-state fits to the ratio

- Check with summation method:

$$\sum_{t_{\rm ins}=a}^{t_s-a} R^{\mu\nu}(\Gamma_\alpha,\vec{p}',\vec{p};t_s,t_{\rm ins}) \xrightarrow{t_s\to\infty} c_0+\Pi^{\mu\nu}(\Gamma_\alpha,\vec{p}',\vec{p})t_s$$

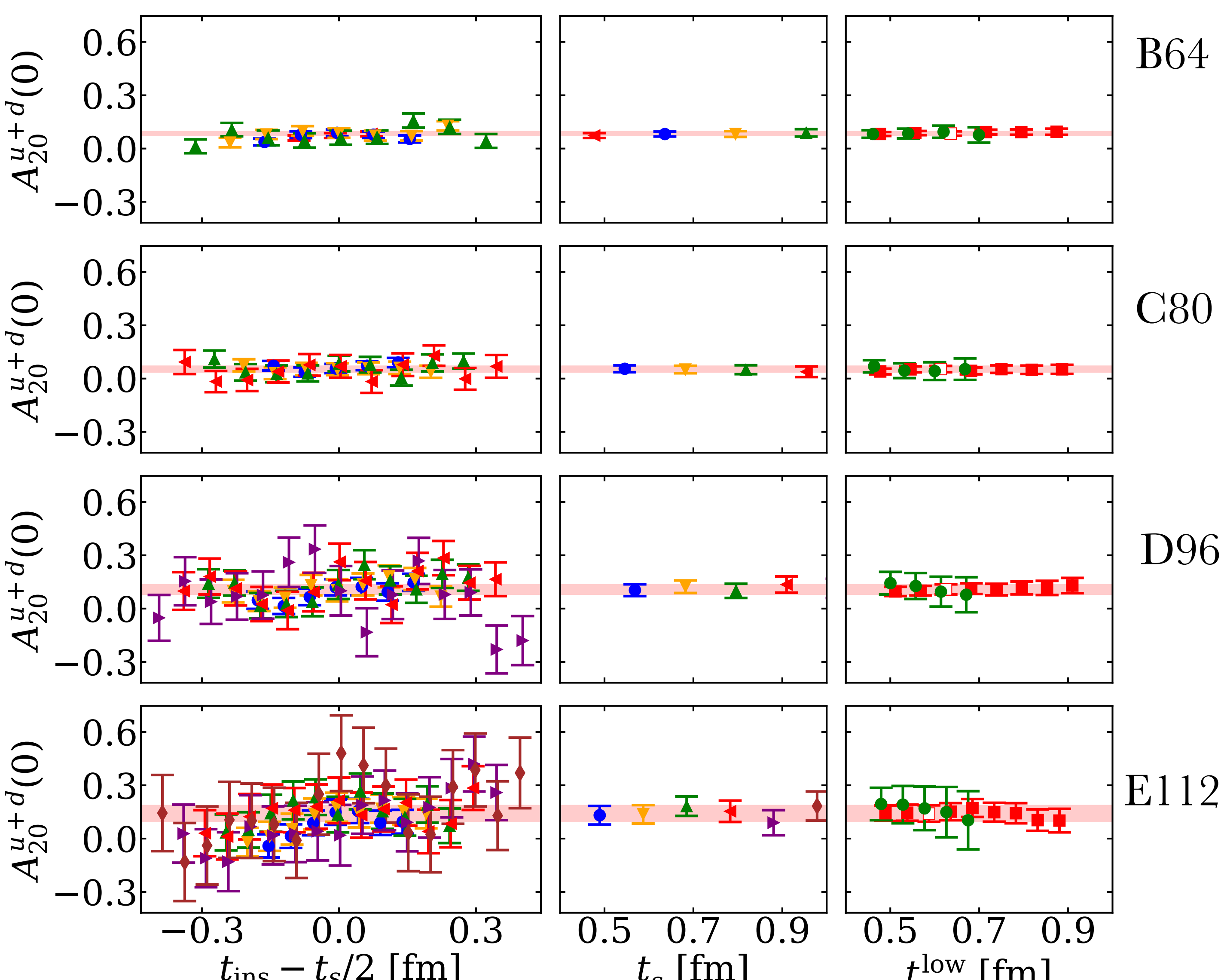
Extraction of matrix elements - disconnected

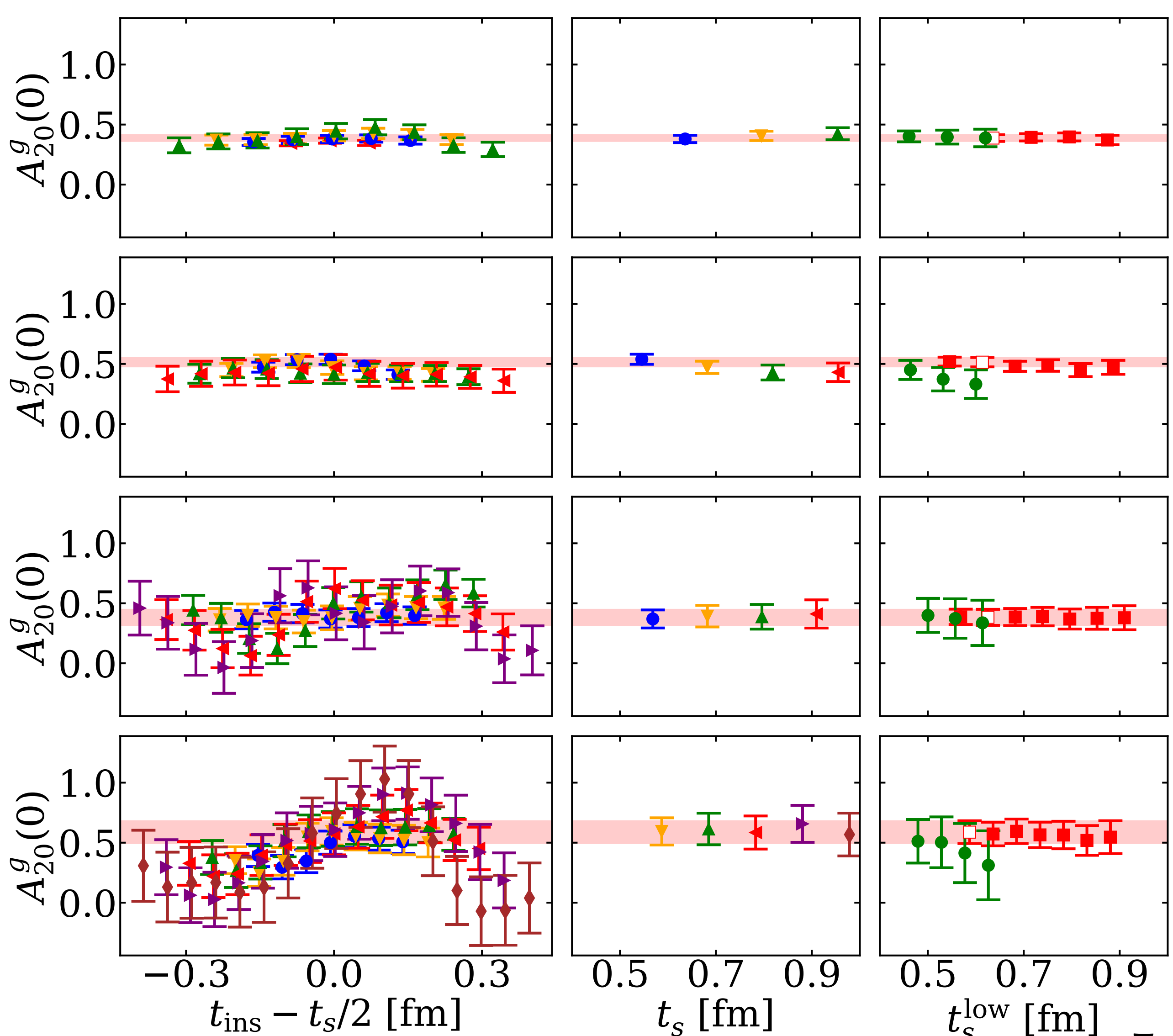
- Statistics for quark-disconnected loops

Flavor	B64 $N_{\rm def}$	B64 $n_{\rm vec}$	C80 $N_{\rm def}$	C80 $n_{\rm vec}$	D96 $N_{\rm def}$	D96 $n_{\rm vec}$	E112 $N_{\rm def}$	E112 $n_{\rm vec}$
u & d	200	$12 \times 512 \times 1$	450	$12 \times 512 \times 1$	0	$12 \times 512 \times 8$	530	$12 \times 512 \times 1$
s	0	$12 \times 512 \times 2$	0	$12 \times 512 \times 4$	0	$12 \times 512 \times 4$	0	$12 \times 512 \times 2$
c	0	$12 \times 32 \times 12$	0	$12 \times 512 \times 1$	0	$12 \times 512 \times 1$	0	$12 \times 512 \times 1$

- Use one-state fits (plateau and summation)

Gluon insertion is stout-smeared





Renormalization - nonsinglet

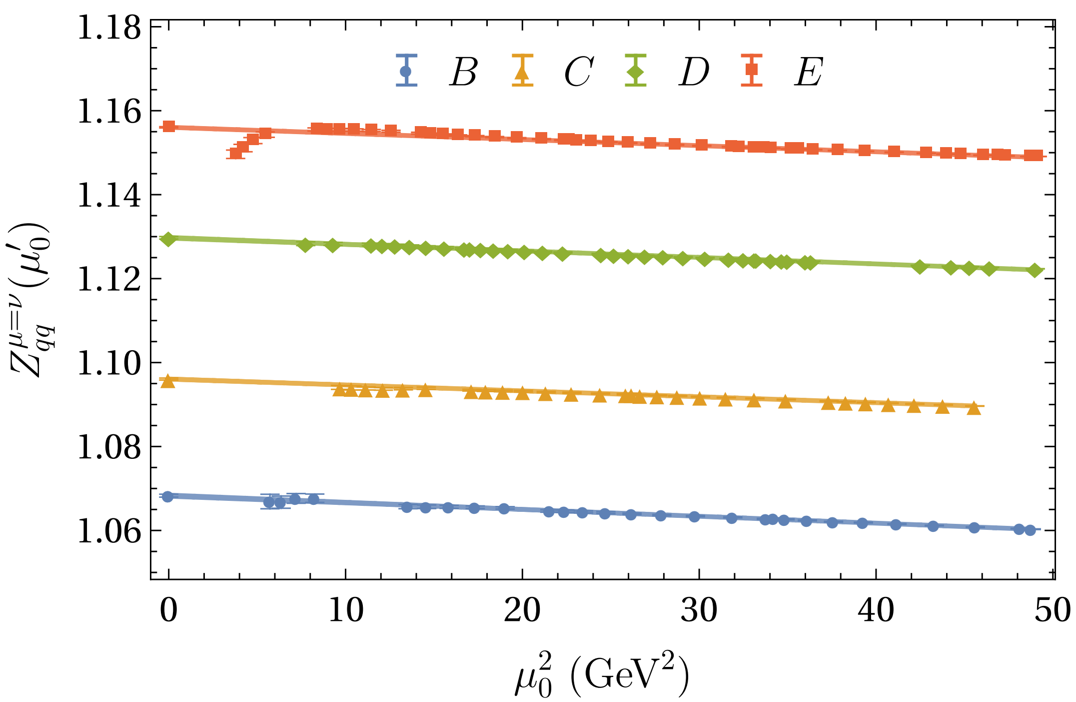
- RI'-MOM for all contributions
- For the three flavor nonsinglets the renormalization is multiplicative

$$\left(Z_{qq}^{\mu\neq\nu}\right)^{\overline{\rm MS}}(\bar\mu) = C_{qq}^{\overline{\rm MS},{\rm RI}'}(\bar\mu,\mu_0)\left(Z_{qq}^{\mu\neq\nu}\right)^{{\rm RI}'}(\mu_0),$$
$$\left(Z_{qq}^{\mu=\nu}\right)^{\overline{\rm MS}}(\bar\mu) = C_{qq}^{\overline{\rm MS},{\rm RI}'}(\bar\mu,\mu_0)\left(Z_{qq}^{\mu=\nu}\right)^{{\rm RI}'}(\mu_0)$$

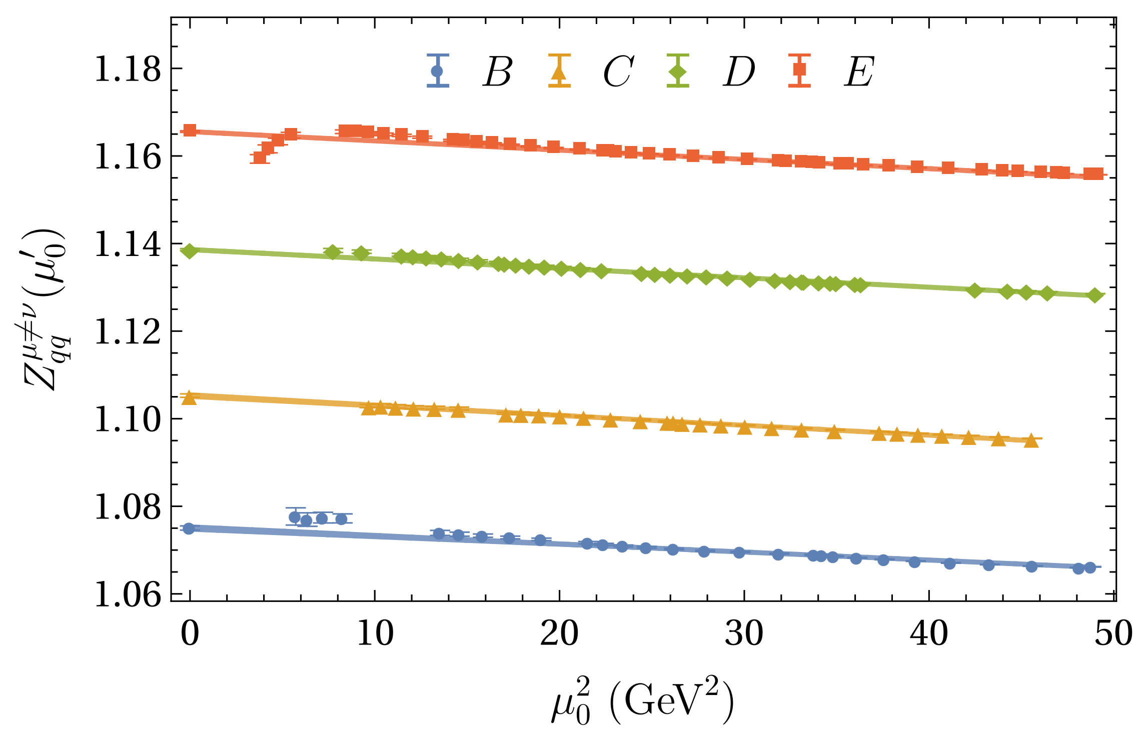
Matching coefficients computed to three-loops

G. Panagopoulos, H. Panagopoulos, and G. Spanoudes, Phys. Rev. D 103, 014515 (2021), arXiv:2010.02062

- To improve convergence, first evolve to an intermediate scale $\mu_0' >> \bar\mu$; then converted to MS at the same scale μ_0'; and then evolve from μ_0' to $\bar\mu$



RI'-MOM at
$(\mu_0')^2 = 21\ {\rm GeV}^2$



Renormalization - singlet & gluon

- RI'-MOM for all contributions
- For the quark singlet u+d+s+c, there is mixing with the gluon operator
- We compute the mixing matrix non-perturbatively for $\mu \neq \nu$

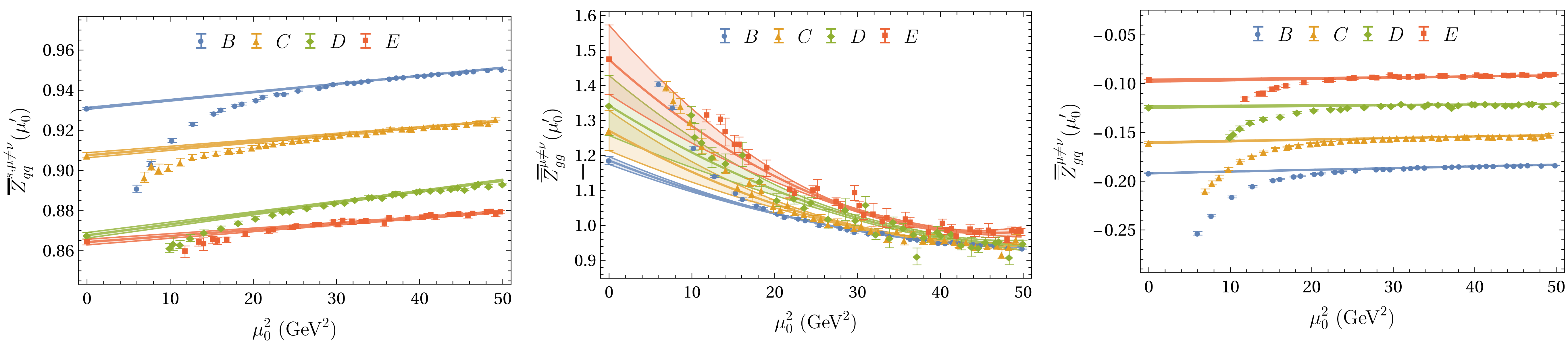
$$\begin{pmatrix} (T^{\mu\nu}_{q^s})^{\mathrm{R}} \\ (T^{\mu\nu}_{g})^{\mathrm{R}} \end{pmatrix} = \begin{pmatrix} Z^s_{qq} & Z_{qg} \\ Z_{gq} & Z_{gg} \end{pmatrix} \begin{pmatrix} T^{\mu\nu}_{q^s} \\ T^{\mu\nu}_{g} \end{pmatrix}$$

$$(\overline{Z}^{\mu\neq\nu}_{ij})^{\overline{\mathrm{MS}}}(\bar{\mu}) = (\overline{Z}^{\mu\neq\nu}_{ik})^{\mathrm{RI}'}(\mu_0)\overline{C}^{\mathrm{RI}',\overline{\mathrm{MS}}}_{kj}(\bar{\mu},\mu_0),$$

Matching coefficients computed to two-loops

G. Panagopoulos, H. Panagopoulos, and G. Spanoudes, Phys. Rev. D 103, 014515 (2021), arXiv:2010.02062

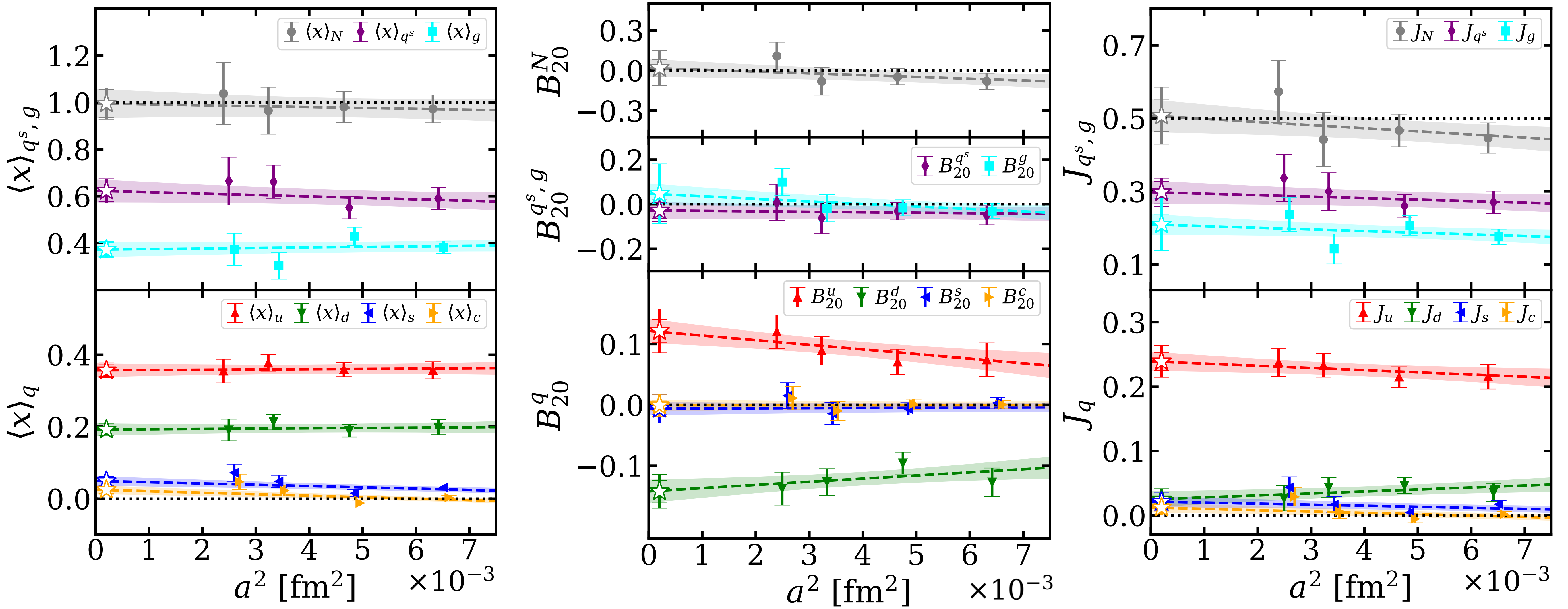
- To improve convergence, first evolve to an intermediate scale $\mu_0' >> \bar{\mu}$; then converted to MS at the same scale μ_0'; and then evolve from μ_0' to $\bar{\mu}$



RI'-MOM at $(\mu_0')^2 = 21\ \mathrm{GeV}^2$

Continuum limit

✵ Continuum limit: Perform constant and linear in a^2 fits and combine using Akaike information criterion

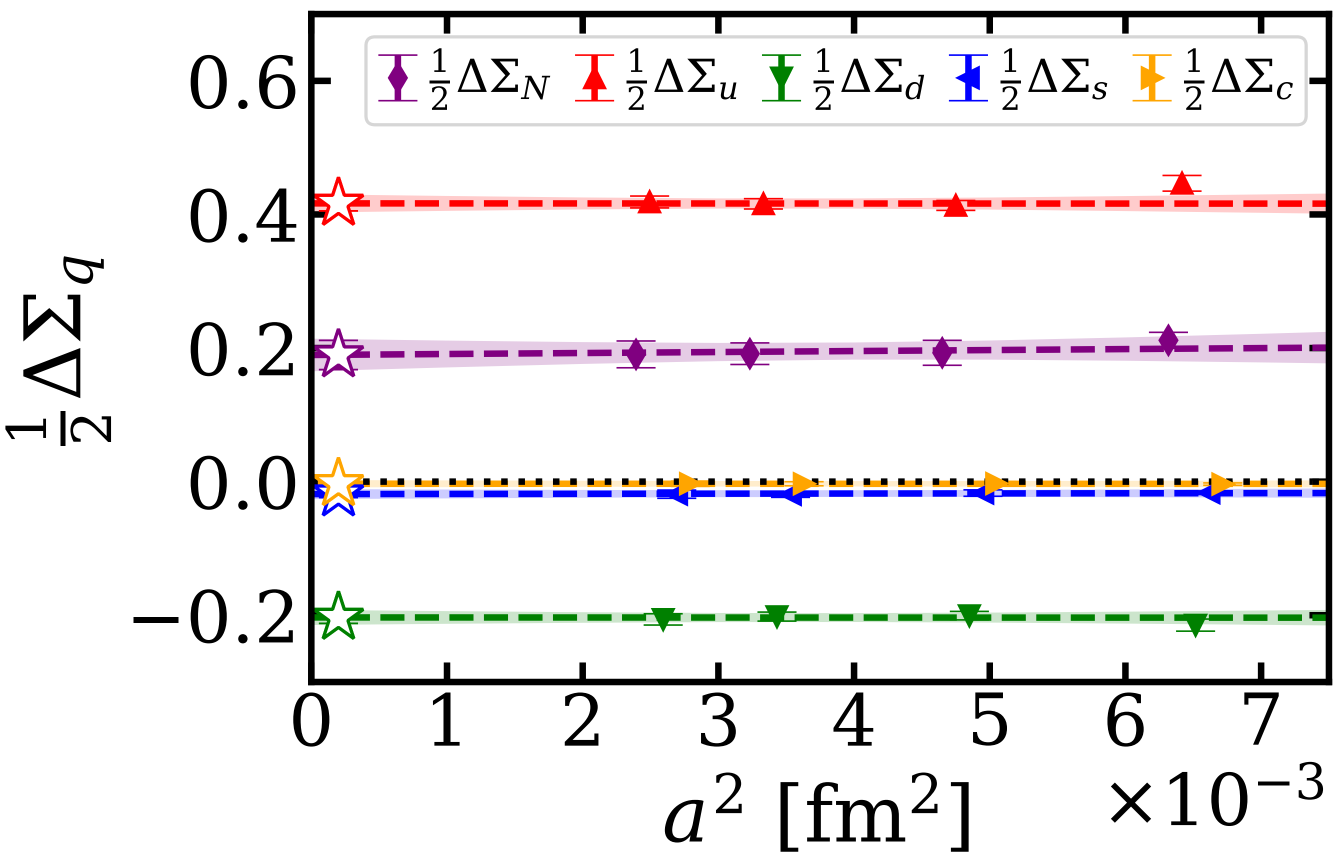


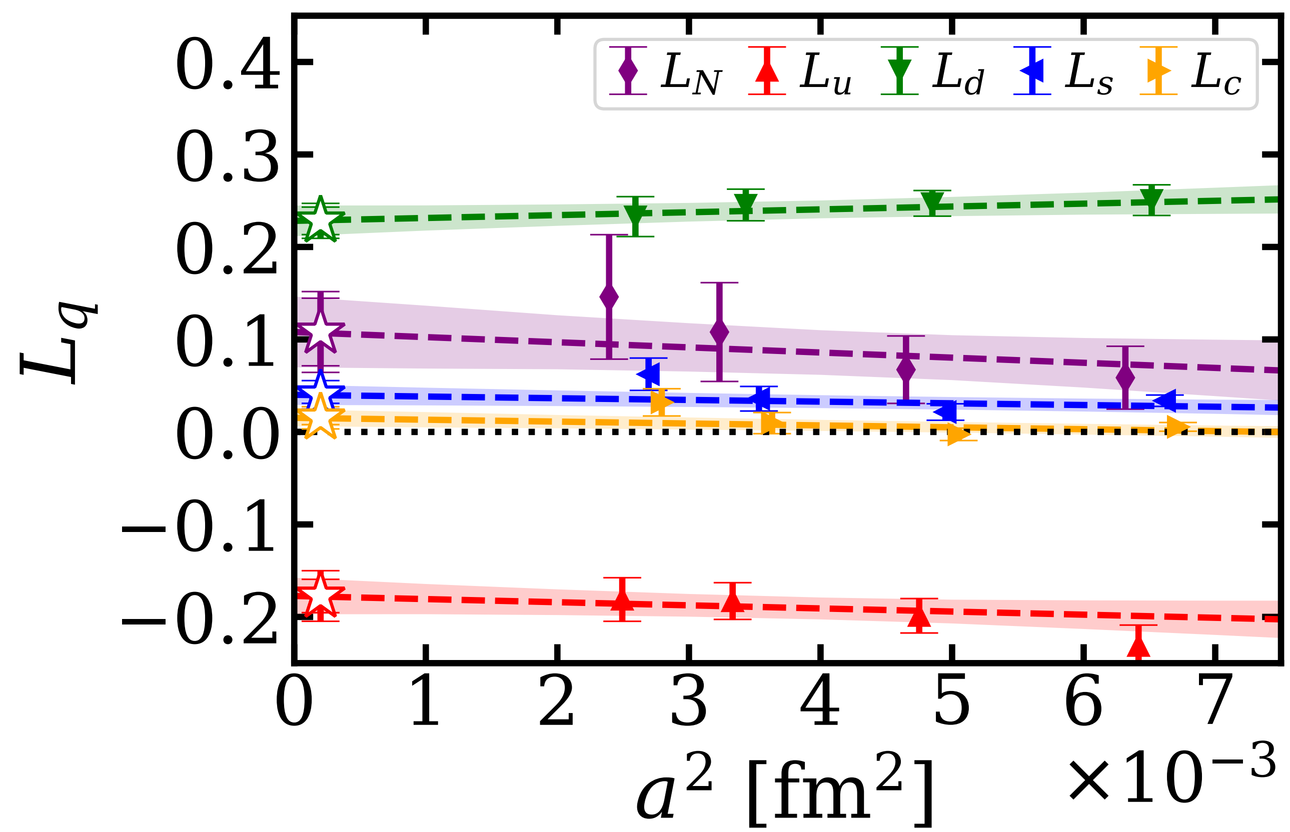
✵ Further systematics are computed by changing the excited state analysis for connected and disconnected, using different upper value of Q^2 in the dipole fits to extrapolate to the forward limit and for the different stout smearing on the gluon

✵ These are small except for the gluon that double the total final error

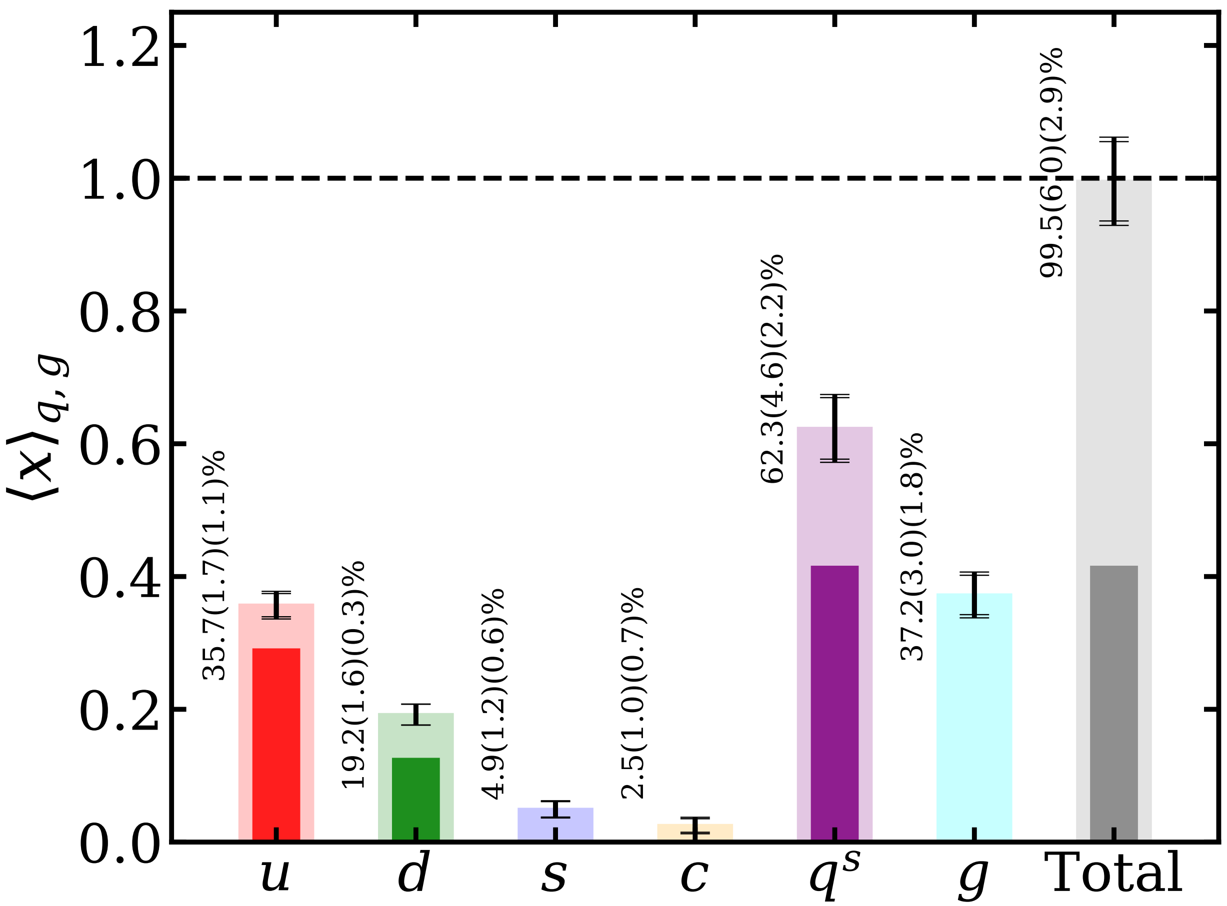
Intrinsic spin and orbital angular momentum

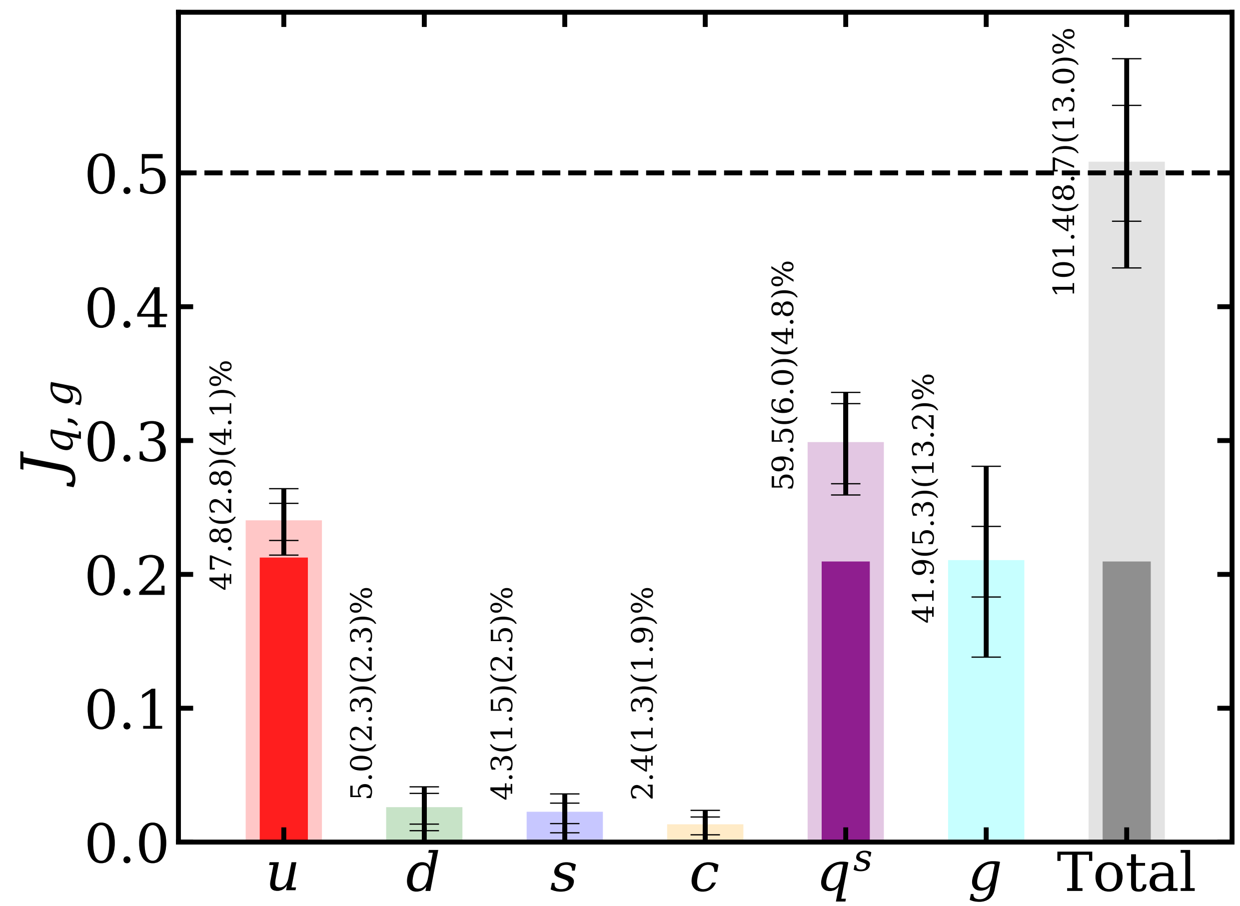
- The helicity moment n=1: $\Delta\Sigma_{q_+}(\mu^2) = \int_0^1 dx\,\left[\Delta q(x,\mu^2) + \Delta\bar{q}(x,\mu^2)\right] = g_A^q$
- The quark intrinsic spin $\frac{1}{2}\Delta\Sigma_q = \frac{1}{2} g_A^q$ is extracted using the same ensembles by evaluating g_A
- The quark orbital angular momentum is then extracted from $L_q = J_q - \frac{1}{2}\Delta\Sigma_q$





Results for <x> and J

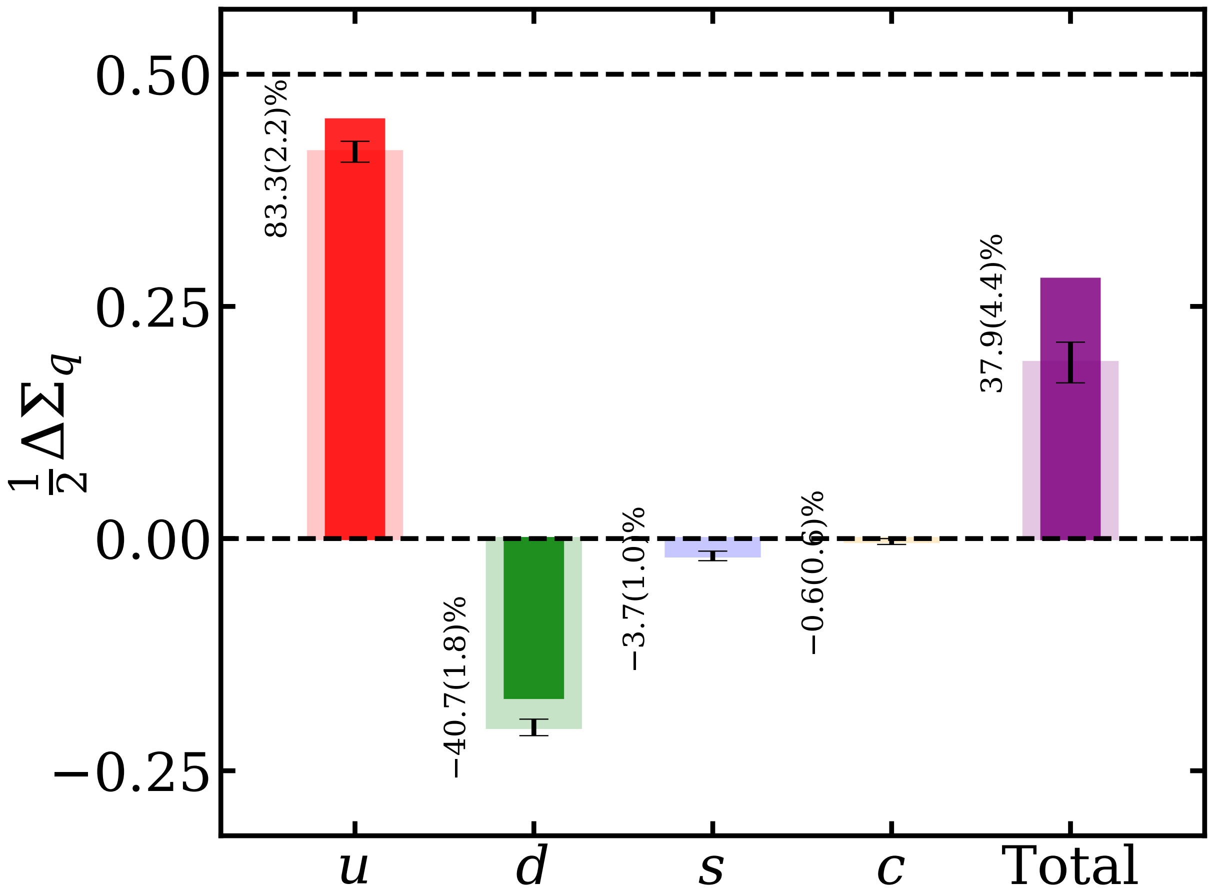


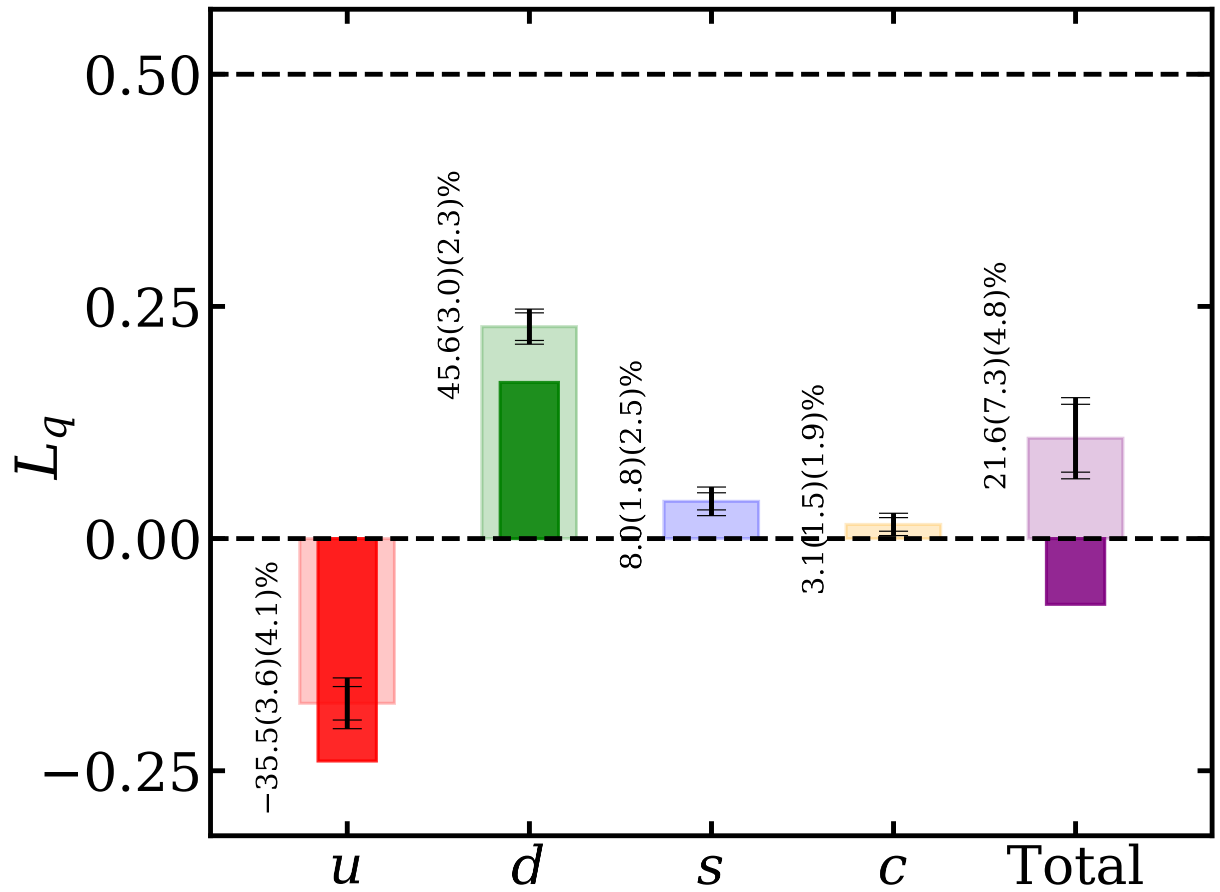


- Momentum and spin sums satisfied

- About 35-40 % from gluons, 10 % from sea quarks confirming what EMC measured

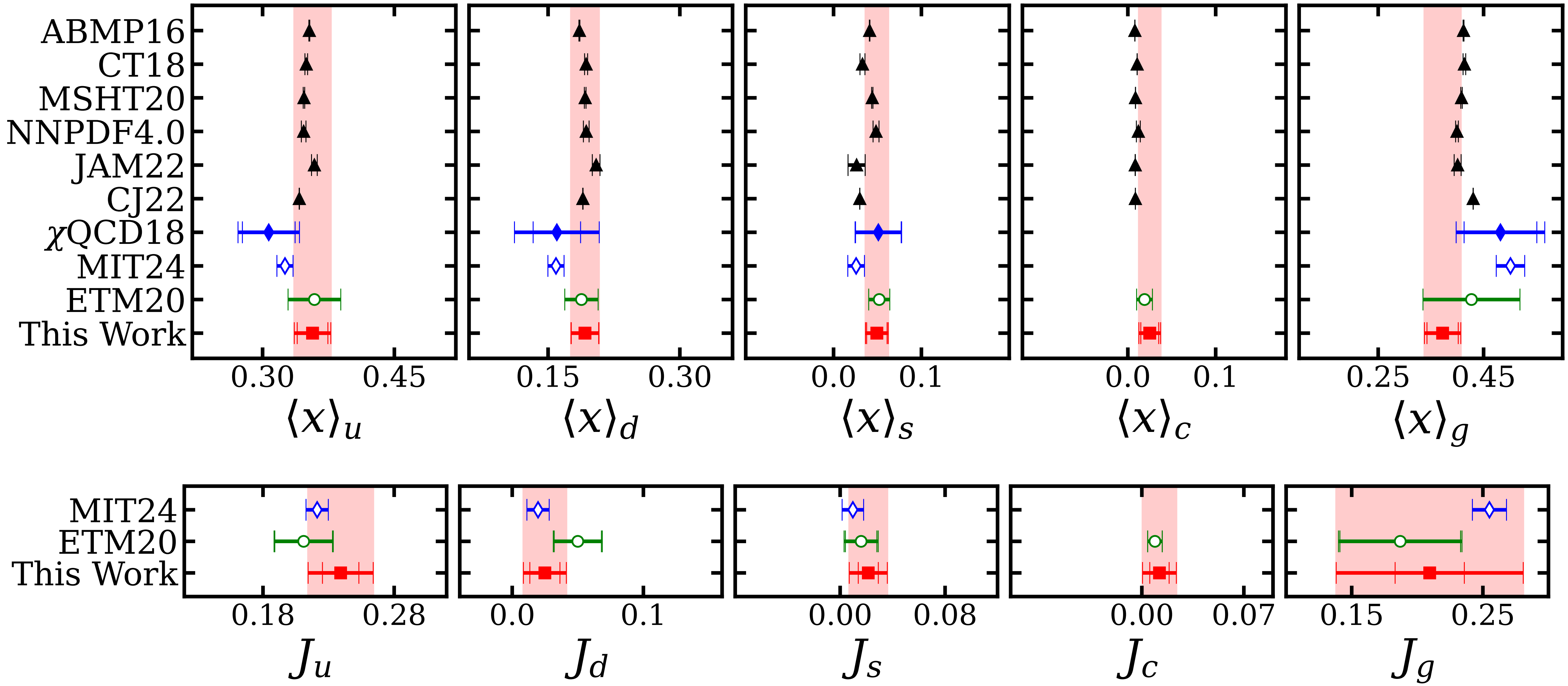
Results on quark intrinsic spin and orbital angular momentum





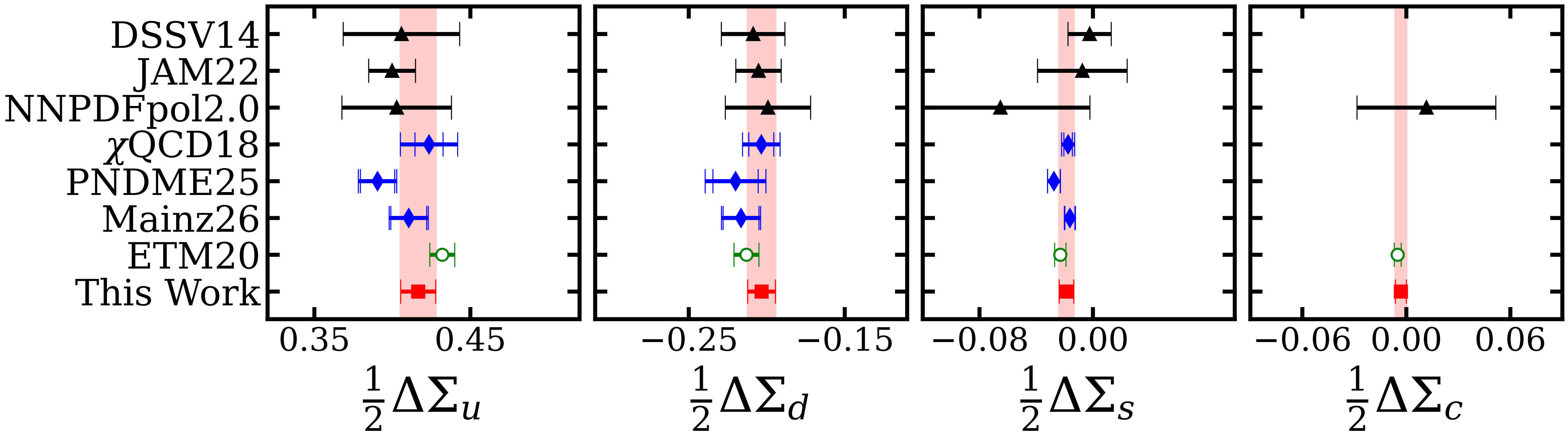
- Total intrinsic quark spin amounts for less than half the proton spin in agreement with the results of the EMC experiment in the late 1980s
- The u-quark contribution largely cancels the one from the d-quark giving a small total quark orbital angular of about 20%
- The s-quark contribution is non-zero in both cases

Comparison of results on <x> and J



❋ Momentum fractions in good agreement with phenomenological analyses

Comparison of results on intrinsic spin



- Lattice QCD determination yields results in agreement to phenomenology but with higher precision

Conclusions

- A full decomposition of the momentum fraction and spin of the nucleon is carried out for the first time using $N_f=2+1+1$ gauge ensembles simulated at physical values of quark masses
  - About 40% comes from the gluon contribution and about 10% from sea quarks
  - Momentum and spin sums are satisfied when all components are added
- The intrinsic spin and orbital angular momentum are extracted with the former providing only about 40% of the proton spin in agreement with the results of the EMC experiment
- The quark orbital angular momentum largely cancels between the d- and u-quarks resulting in a small value
- For all quantities, we find evidence for a small but statistically non-zero s-quark contribution; Interestingly, we find a small contribution to the momentum fraction from the c-quark pointing possibly to charm quark effects in the nucleon
- Our continuum extrapolated results compare well with phenomenological analyses, where for the intrinsic spin the lattice results are more precise

The project EXCELLENCE/0524/0459 is implemented under the programme of social cohesion "THALIA 2021-2027" co-funded by the European Union, through Research and Innovation Foundation.

Co-funded by the European Union

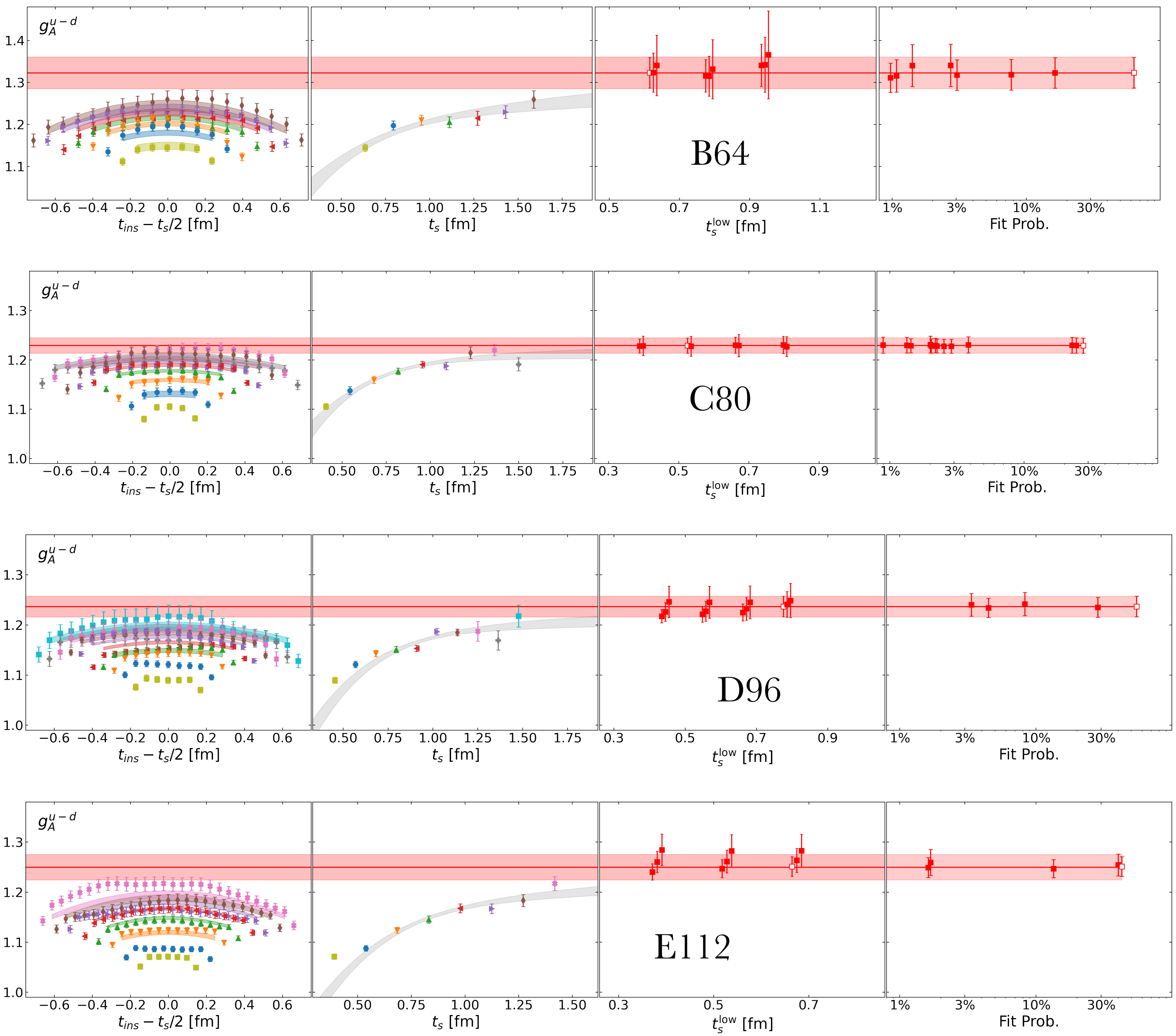
Republic of Cyprus

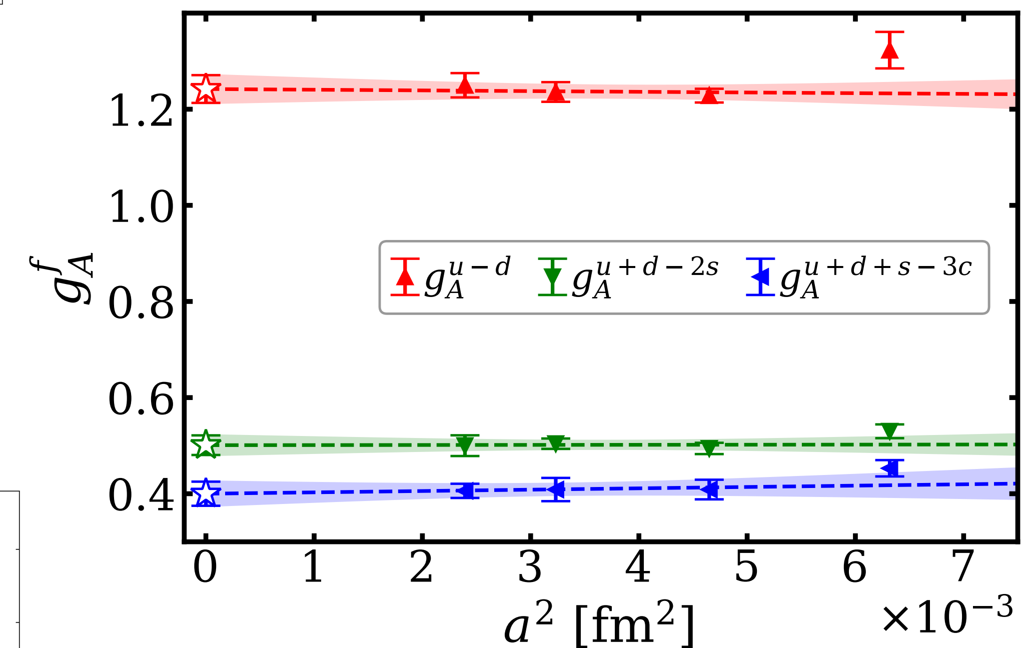
RESEARCH & INNOVATION FOUNDATION

Backup slides

Determination of isovector axial charge

❋ Fit simultaneously the three-point function of A_j & A_0 currents sharing the first excited state energy





Dependence of gluon matrix element on stout-smearing

- The stout smearing dependence on the flavor singlet due to the mixing is negligible
- Different stout smearing lead to the same value at the continuum limit