# Determination of Roper resonance with two- and three-body unitarity

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*7/31/2026 14:40*
*University of Maryland, College Park*
*The 43rd International Symposium on Lattice Field Theory (Lattice 2026)*

THE GEORGE WASHINGTON UNIVERSITY
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The George Washington University, CCAS , Department of Physics

Supported by NSF Grant No. 2310036

# The Roper Resonance

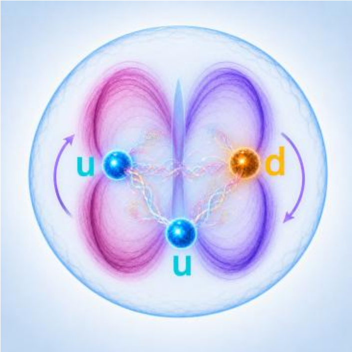
**Quantum Numbers:** J, L, S, I, P, C, ...

"$L_{2J\,2I}$ (wave) channel"

$L=1,\ J=1/2,\ I=1/2 \rightarrow P_{11}$ $\quad N(****)\ J^P$: the nucleon N(940)$1/2^+$

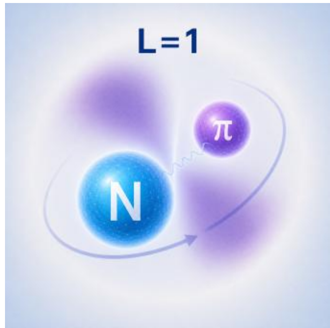
## Inverted-mass problem

- Quark model cannot explain the energy level

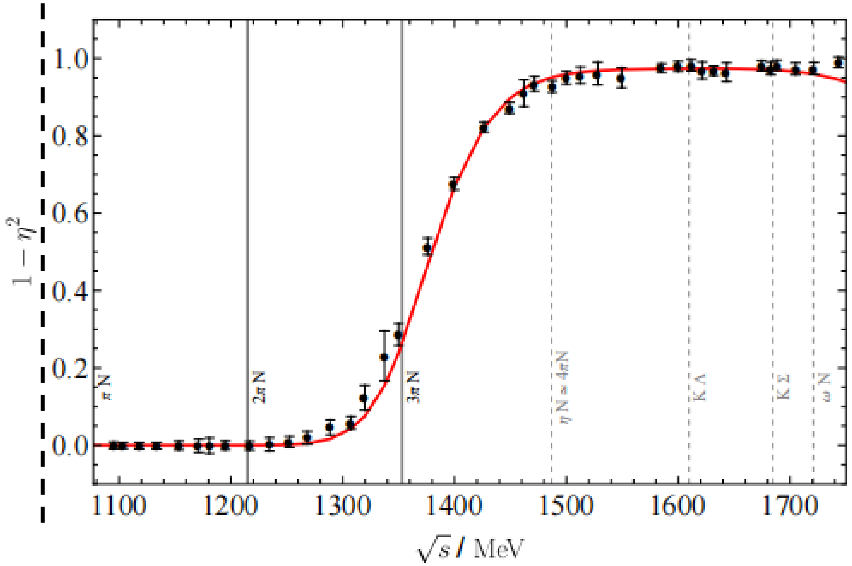



| $\frac{1}{2}^+$ | $\frac{1}{2}^-$ |
|---|---|
| $N(939)$ | $N(1535)$ |
| $N(1440)$ | $N(1650)$ |

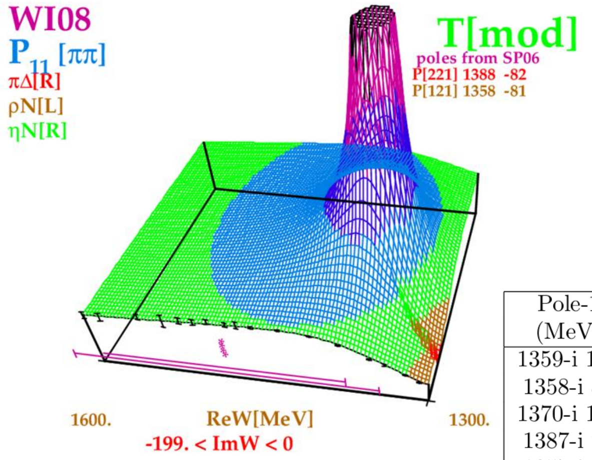
## Strong three-body effect

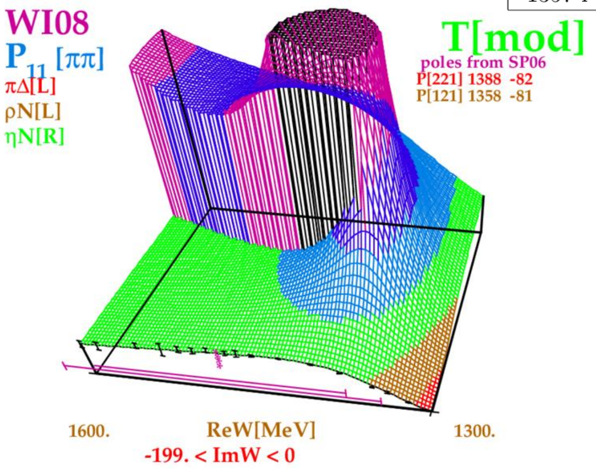

$$P = (-1)(-1)(+1)\cdot(-1)^0 = +1$$
$$P = (-1)(+1)\cdot(-1)^1 = +1$$




## Two pole structure



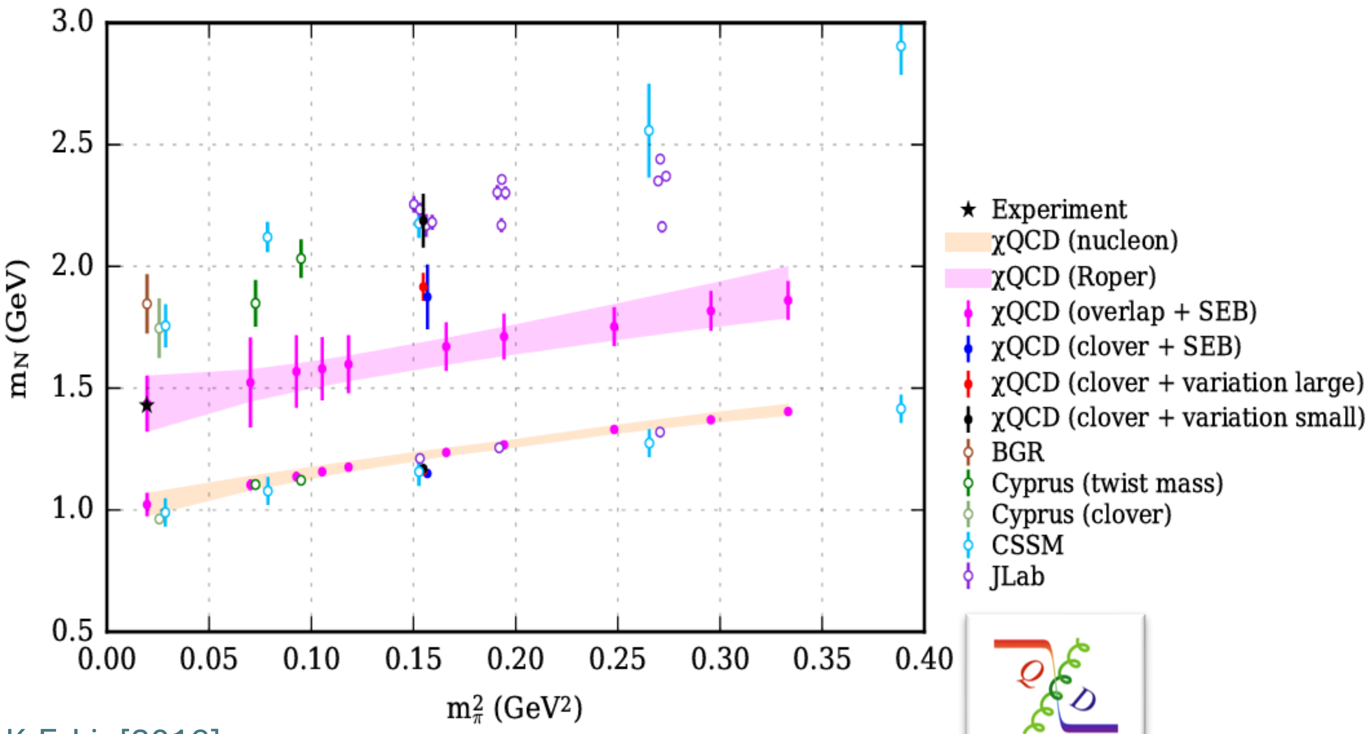

| Pole-1 (MeV) | Pole-2 (MeV) | Reference |
|---|---|---|
| 1359-i 100 | 1410-i 80 | [10] |
| 1358-i 80 | 1388-i 82 | [11] |
| 1370-i 114 | 1360-i 120 | [15] |
| 1387-i 73 | 1397-i 71 | [13] |
| 1357-i 76 | 1364-i 105 | [14] |




I. Strakovsky[2026]

# The Roper Resonance

**Lattice Calculations**




K-F. Liu[2016]

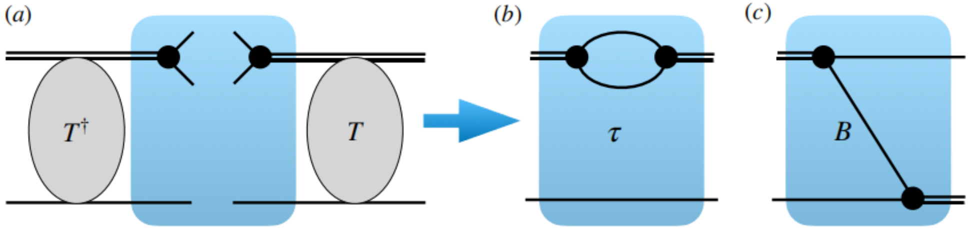
- No Roper energy eigenstate found near $m_R \sim 1.43$ GeV with only $\pi N$ elastic scattering
  C.B.Lang, *et al.* [2017]

  - $\pi\pi N$ Three-body effect
  - Coupled-channel scattering

**B. Scenarios with coupled $N\pi - N\sigma - \Delta\pi$ scattering**

Our analysis does not show the resonance related level. One reason could be that the Roper resonance is a truly coupled channel phenomenon and one has to include further interpolators like $\Delta\pi$, $N\rho$ and an explicit $N\pi\pi$ three hadron interpolator. The scattering of $N\pi - N\sigma - \Delta\pi$

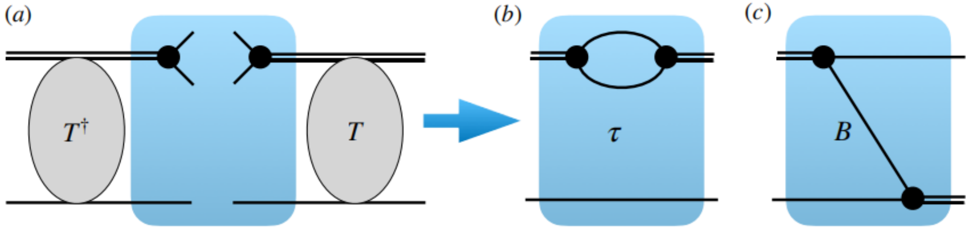
# Two-plus-three Body Unitary Approach $SS^{\dagger} = 1$

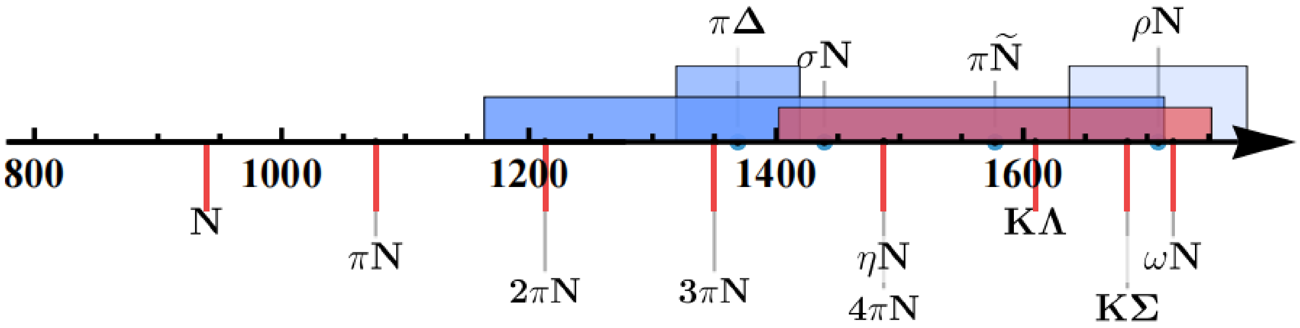



$S = 1 - 2i \cdot \rho T$

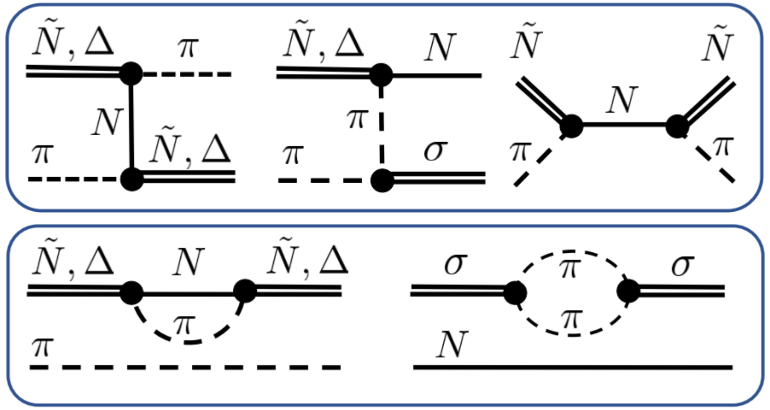
$(T^{\dagger} - T) = 2i \cdot \rho T T^{\dagger}$

# Two-plus-three Body Unitary Approach $SS^\dagger = 1$




$$S = 1 - 2i \cdot \rho T \qquad (T^\dagger - T) = 2i \cdot \rho T T^\dagger$$



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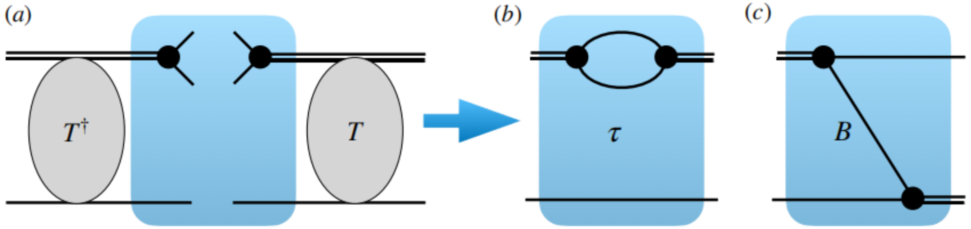
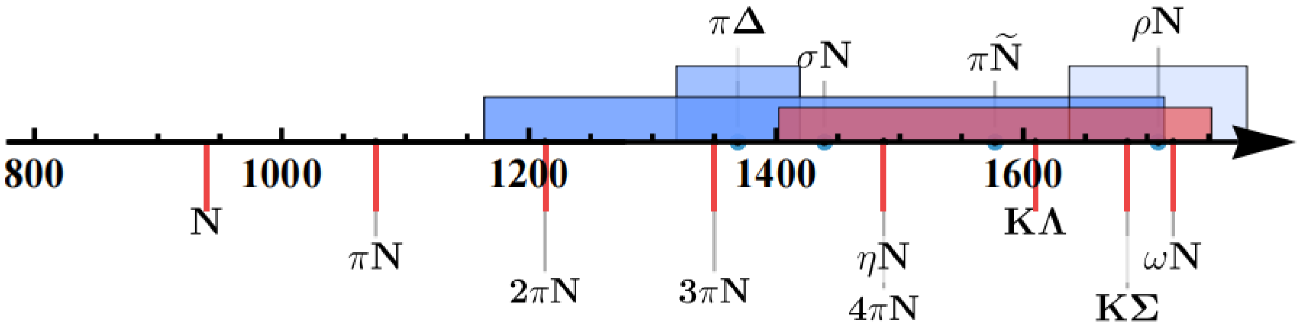
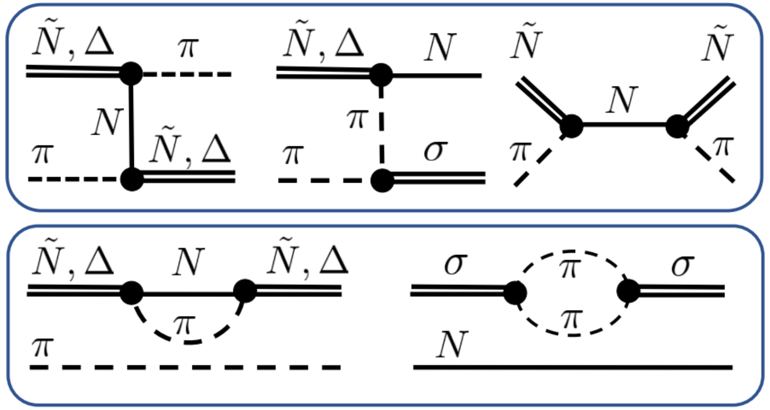


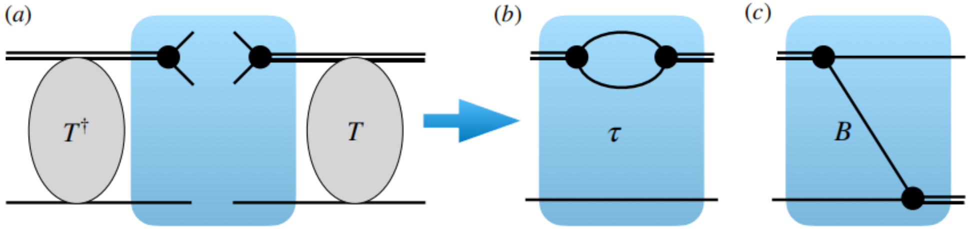




$$S = 1 - 2i \cdot \rho T \qquad (T^\dagger - T) = 2i \cdot \rho T T^\dagger$$

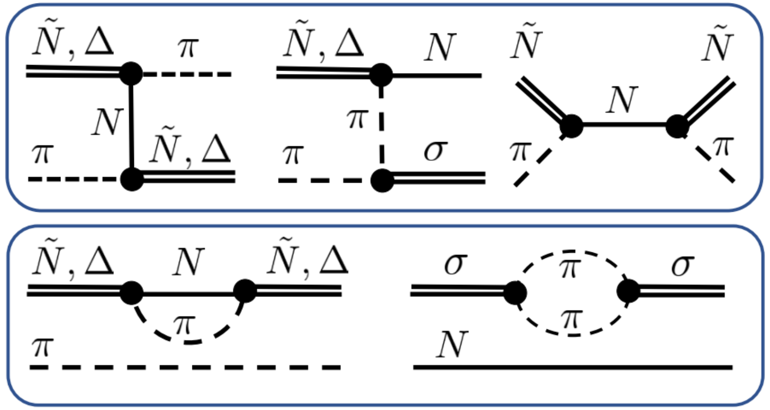



$$\langle k'|\, T^{JLS}_{\mu\nu} |k\rangle = \langle k'|\, V^{JLS}_{\mu\nu} |k\rangle + \sum_\rho \int q^2 dq \,\langle k'|\, V^{JLS}_{\mu\rho}\, |q\rangle\, \tau_\rho(q)\, \langle q|\, T^{JLS}_{\rho\nu} |k\rangle$$

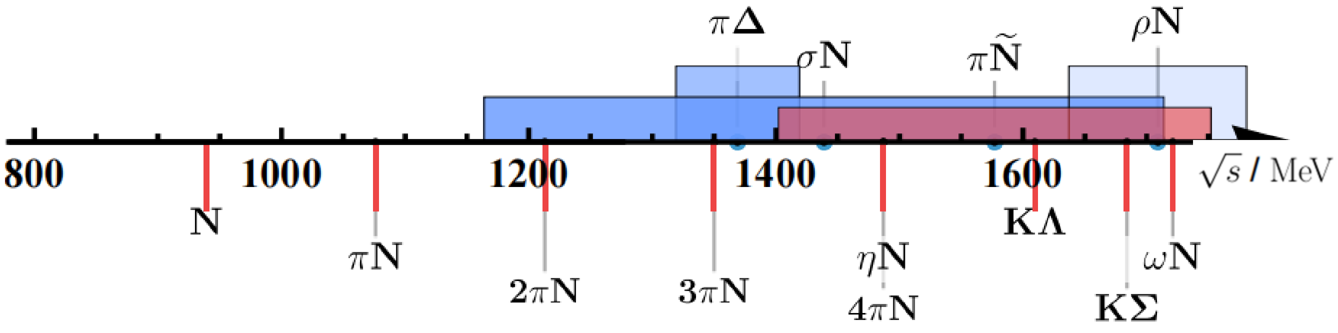
# Two-plus-three Body Unitary Approach $SS^\dagger = 1$




$$S = 1 - 2i \cdot \rho T \qquad (T^\dagger - T) = 2i \cdot \rho T T^\dagger$$

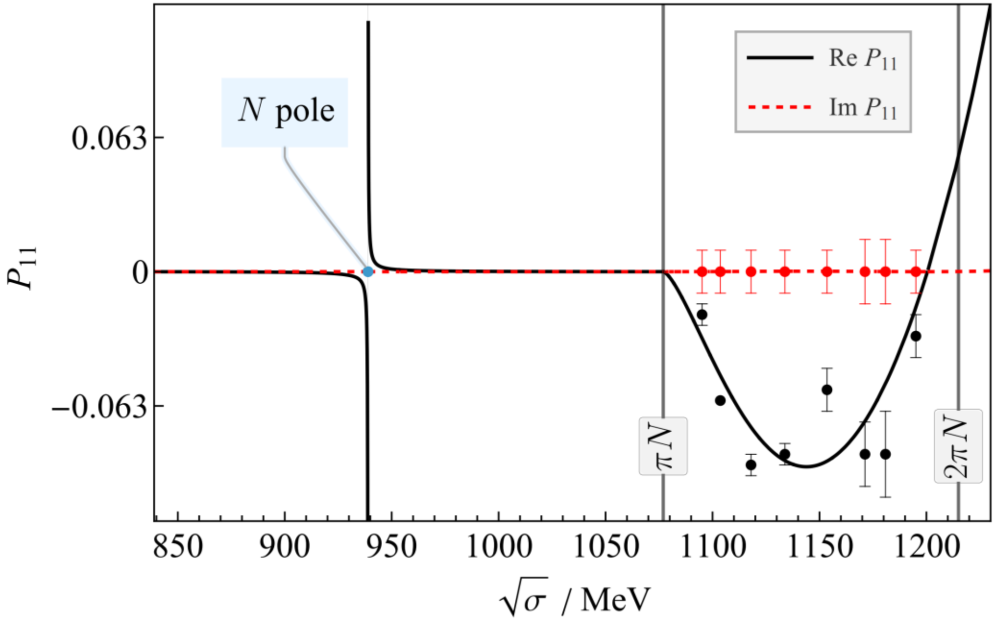



$$\langle k'|\, T^{JLS}_{\mu\nu} |k\rangle = \langle k'|\, V^{JLS}_{\mu\nu} |k\rangle + \sum_\rho \int q^2 dq\, \langle k'|\, V^{JLS}_{\mu\rho}\, |q\rangle\, \tau_\rho(q)\, \langle q|\, T^{JLS}_{\rho\nu} |k\rangle$$




- **Nucleon $N$ as its own isobar $\tilde{N}$**

  No explicit inclusion of $\pi N$ channel, included automatically by $\pi\tilde{N}$



# Input isobars

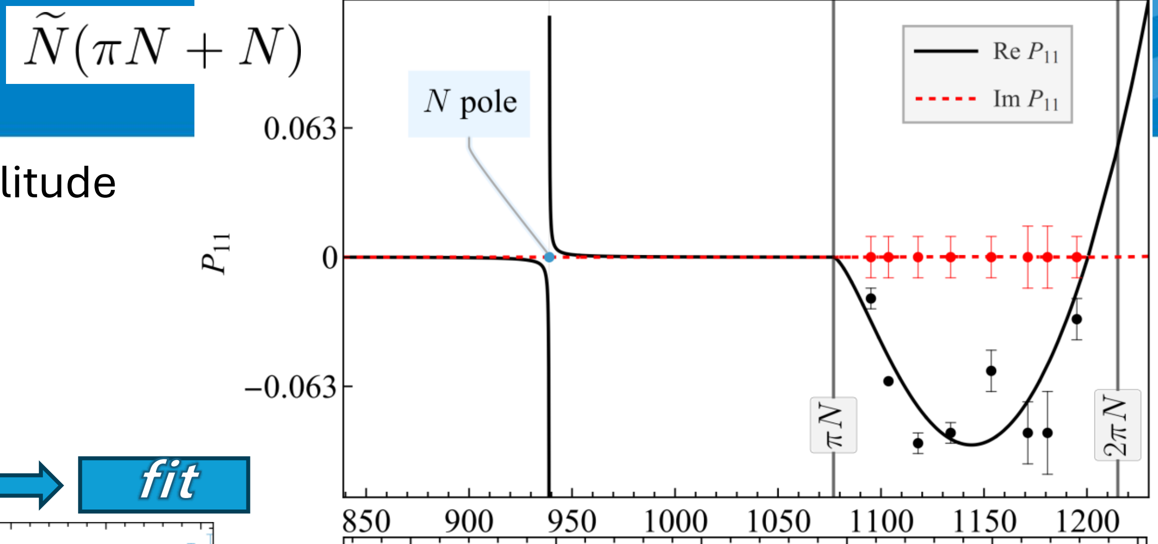
- Isobars are constructed as full two-body PW amplitude

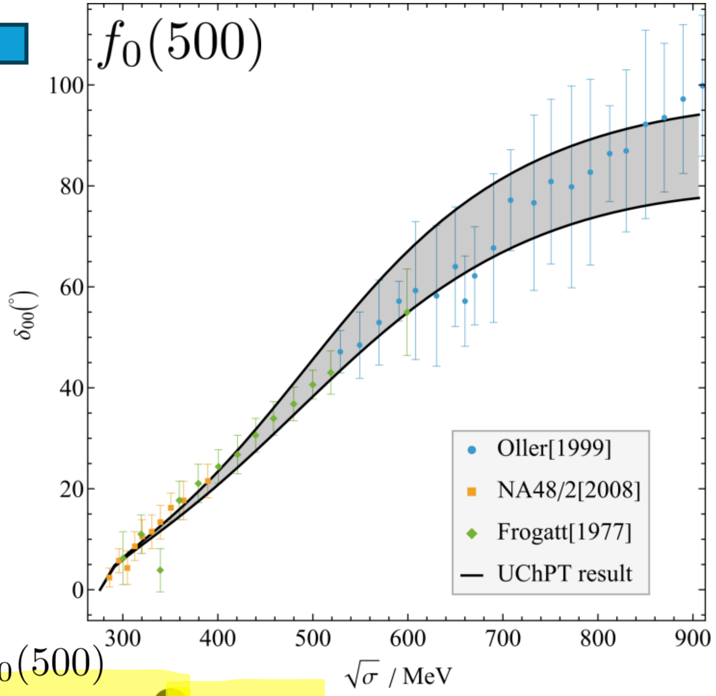
$$V_{\widetilde{N}} = \gamma^a \left[ \frac{2m_N}{\sigma - m_N^2} + \frac{2m_{\widetilde{N}}}{\sigma - m_{\widetilde{N}}^2} + c_{\widetilde{N}}(\sigma) \right] \gamma^c$$

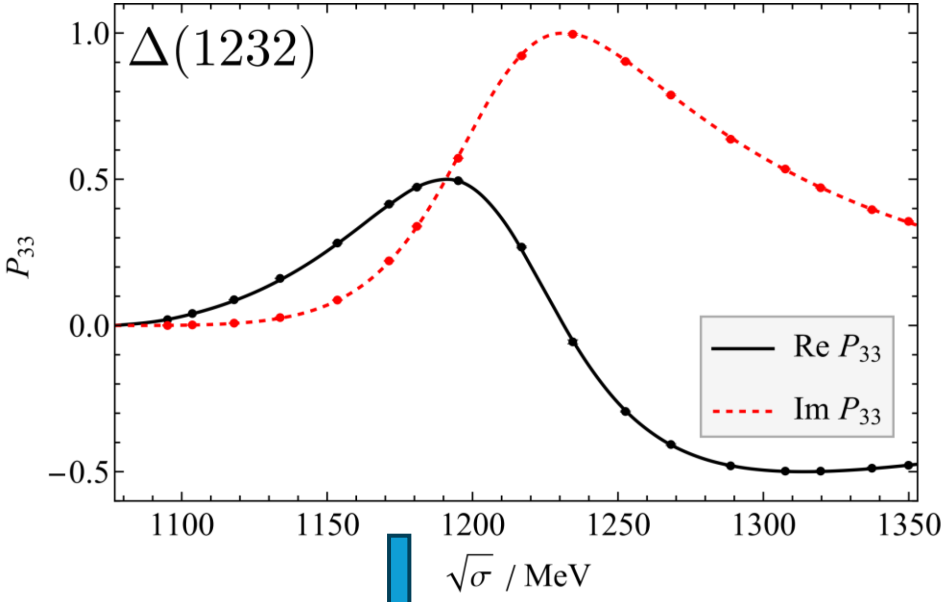
$$V_\Delta = \gamma_\Delta^a \left[ \frac{2m_\Delta}{\sigma - m_\Delta^2} + c_\Delta(\sigma) \right] \gamma_\Delta^c$$

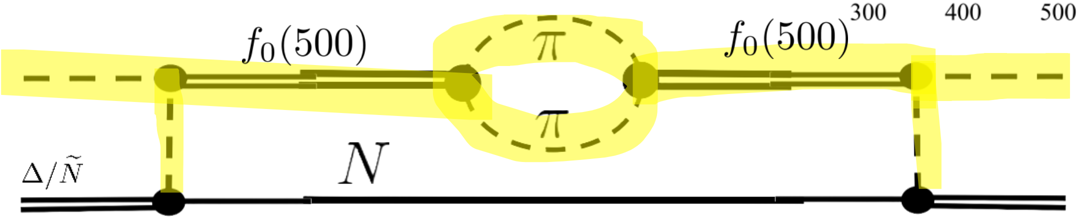
**LSE** → **Renormalization(P11)** → **fit**

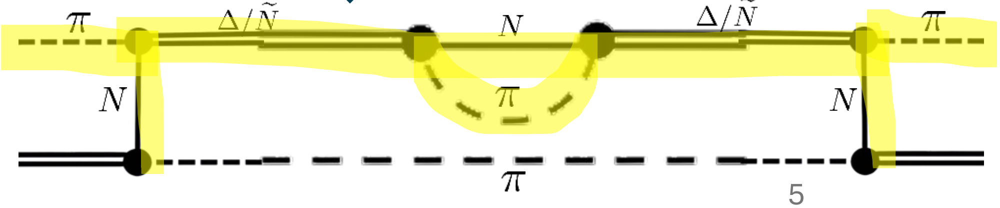
**UChPT**



# Coupled channel space

$$V = \begin{pmatrix} [\pi\widetilde{N}(P_{11}) \to \pi\widetilde{N}(P_{11})] & [\pi\widetilde{N}(P_{11}) \to \pi\Delta(P_{11})] & [\pi\widetilde{N}(P_{11}) \to \sigma N(S_{11})] \\ [\pi\Delta(P_{11}) \to \pi\widetilde{N}(P_{11})] & [\pi\Delta(P_{11}) \to \pi\Delta(P_{11})] & [\pi\Delta(P_{11}) \to \sigma N(S_{11})] \\ [\sigma N(S_{11}) \to \pi\widetilde{N}(P_{11})] & [\sigma N(S_{11}) \to \pi\Delta(P_{11})] & [\sigma N(S_{11}) \to \sigma N(S_{11})] \end{pmatrix}$$

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s-channel, u-channel

# Coupled channel space

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s-channel, u-channel

u-channel nucleon exchange

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s-channel, u-channel

u-channel nucleon exchange

t-channel pion exchange

# Coupled channel space

$$V = \begin{pmatrix} [\pi\widetilde{N}(P_{11}) \to \pi\widetilde{N}(P_{11})] & [\pi\widetilde{N}(P_{11}) \to \pi\Delta(P_{11})] & [\pi\widetilde{N}(P_{11}) \to \sigma N(S_{11})] \\ [\pi\Delta(P_{11}) \to \pi\widetilde{N}(P_{11})] & [\pi\Delta(P_{11}) \to \pi\Delta(P_{11})] & [\pi\Delta(P_{11}) \to \sigma N(S_{11})] \\ [\sigma N(S_{11}) \to \pi\widetilde{N}(P_{11})] & [\sigma N(S_{11}) \to \pi\Delta(P_{11})] & [\sigma N(S_{11}) \to \sigma N(S_{11})] \end{pmatrix}$$

s-channel, u-channel

u-channel nucleon exchange

t-channel pion exchange

contact interactions and Roper pole term

# Coupled channel space

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s-channel, u-channel

u-channel nucleon exchange

t-channel pion exchange

contact interactions and Roper pole term

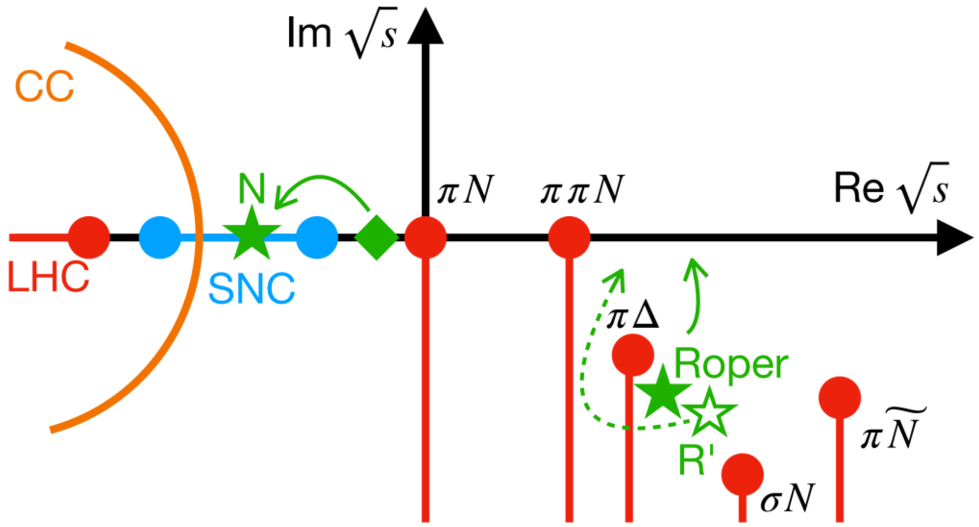
# Minimize model dependence

$$\sum_\rho \int_0^{\Lambda} q^2 dq \, \langle k'| V^{JLS}_{\mu\rho} |q\rangle \, \tau_\rho(q) \, \langle q| T^{JLS}_{\rho\nu} |k\rangle$$

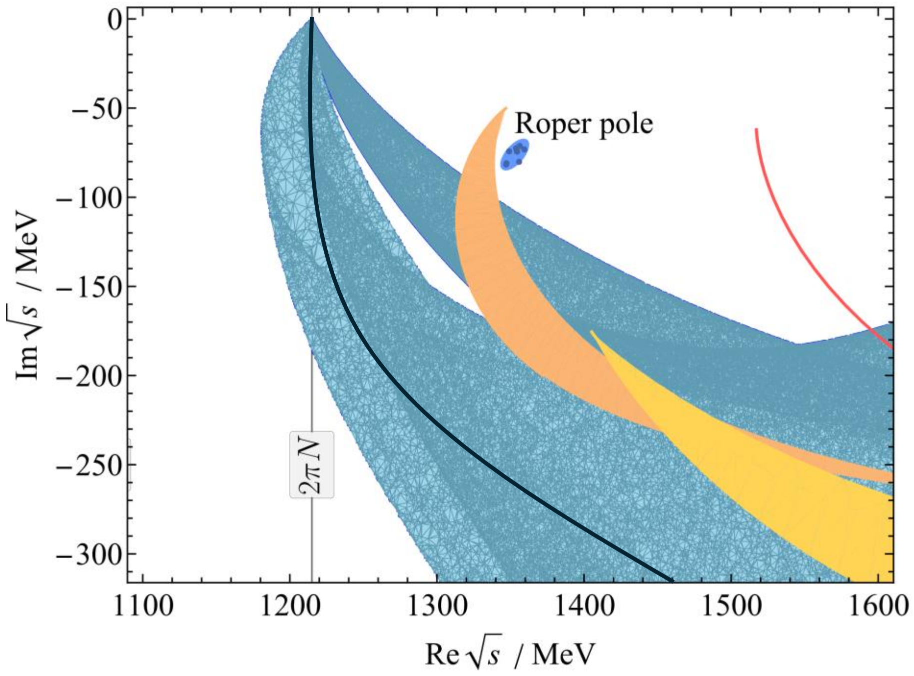
| | cutoff / MeV | | | fit range / MeV | | | $\sqrt{s_\Delta}$ / MeV | $\sqrt{s_\sigma}$ / MeV |
|---|---|---|---|---|---|---|---|---|
| fit 1 | 950 | | | 1350 | 1621 | 1767 | $1208 - 50i$ | $466 - 176i$ |
| fit 2 | 1060 | | | 1350 | 1621 | 1767 | $1208 - 50i$ | $466 - 176i$ |
| fit 3 | 1060 | 1060 | 960 | 1350 | 1621 | 1767 | $1208 - 50i$ | $466 - 176i$ |
| fit 4 | 1060 | 1060 | 960 | 1350 | 1621 | 1767 | $1208 - 50i$ | $466 - 176i$ |
| fit 5 | 1200 | | | 1350 | 1621 | 1767 | $1208 - 50i$ | $466 - 176i$ |
| fit 6 | 1060 | | | 1650 | | | $1209 - 50i$ | $466 - 176i$ |
| fit 7 | 1200 | | | 1650 | | | $1209 - 50i$ | $466 - 176i$ |
| fit 8 | 1060 | 1060 | 960 | 1650 | | | $1208 - 50i$ | $466 - 216i$ |

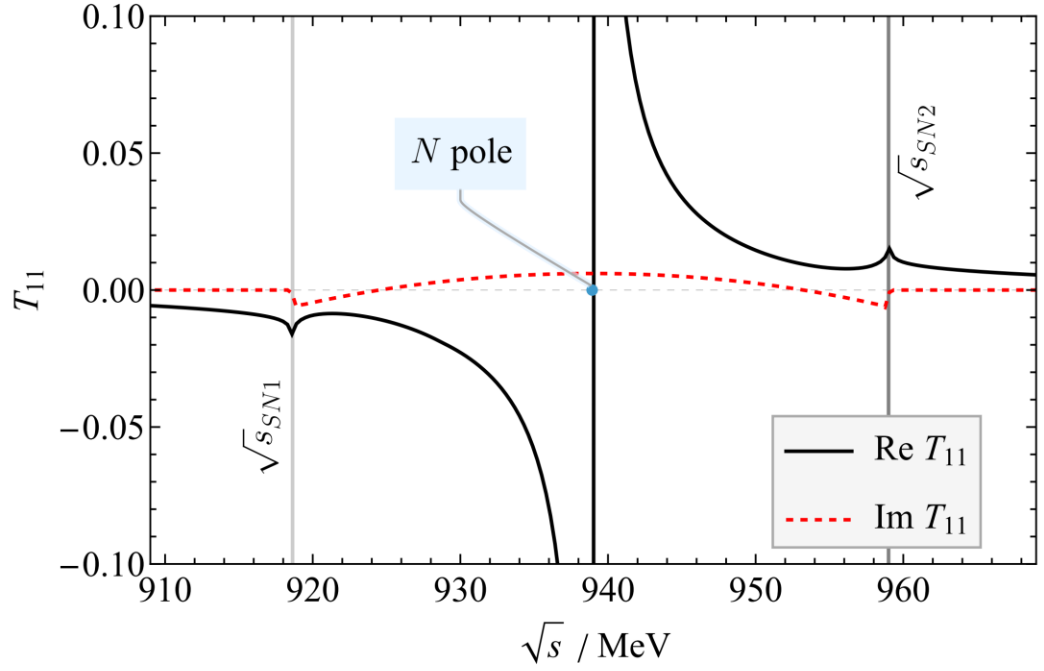
# Analytical structure

- ***Below threshold:***
  - Left hand cut from u-channel
  - Short nucleon cut from projecting the u-channel to PW
  - Nucleon pole
  - Circular cut from the on-shell pion t-channel exchange

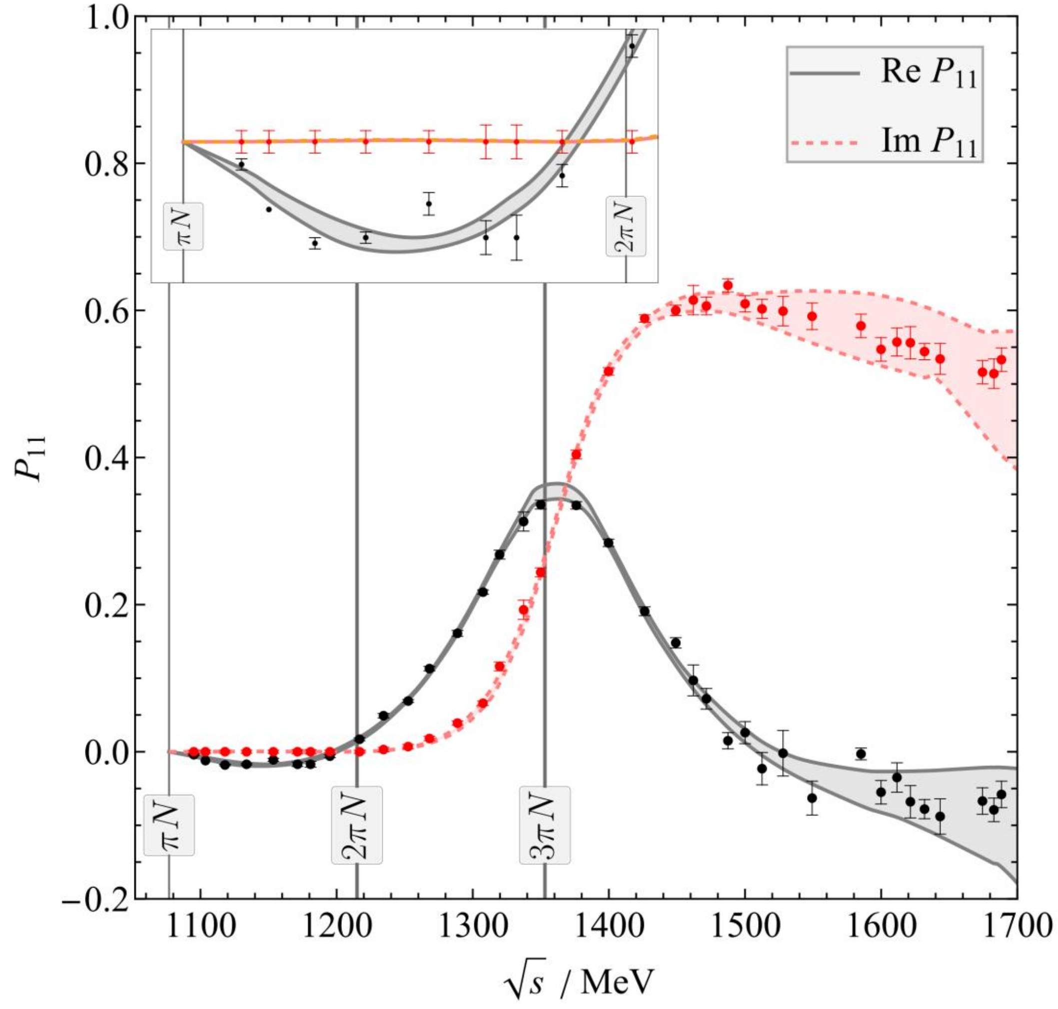


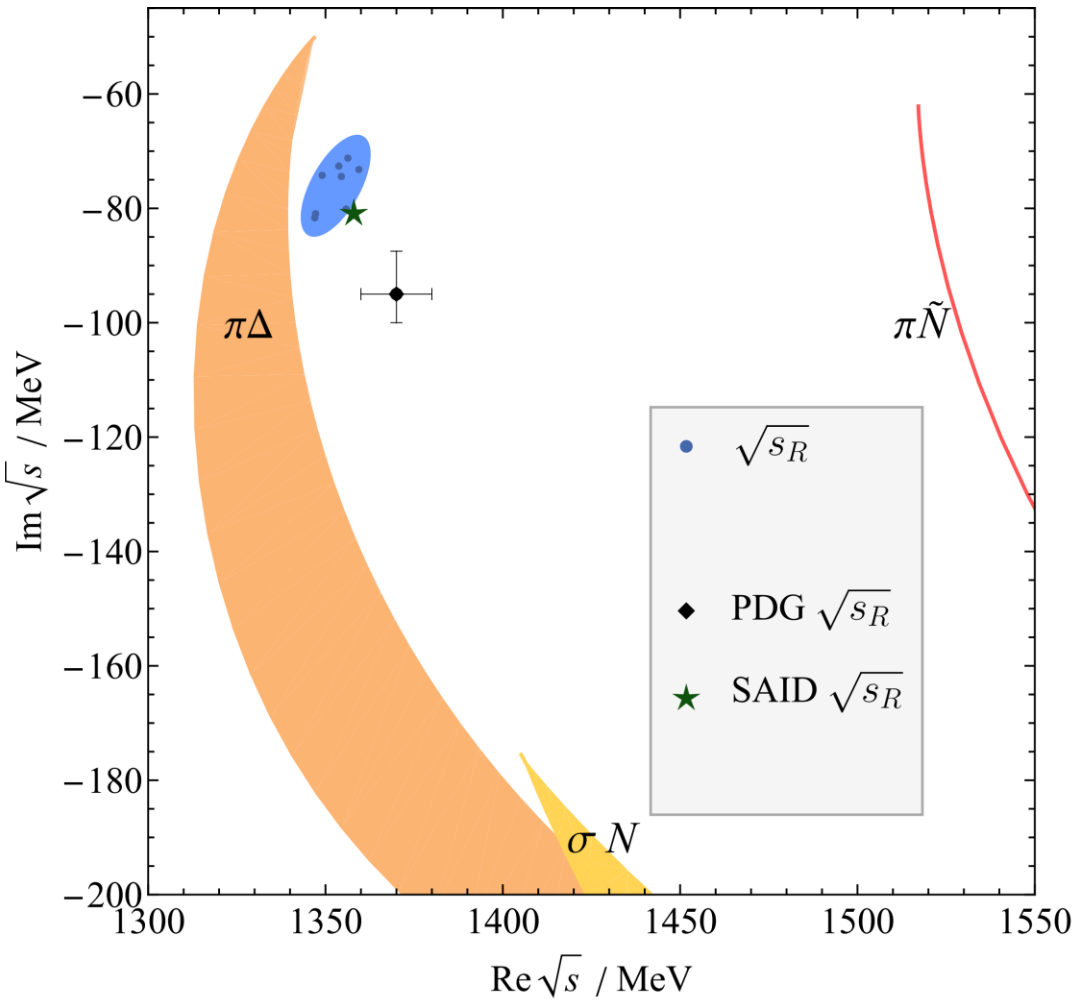

- ***Above threshold:***
  - Complex threshold introduced by the resonance isobars



# Fitted $\pi\pi N$ amplitude and the poles



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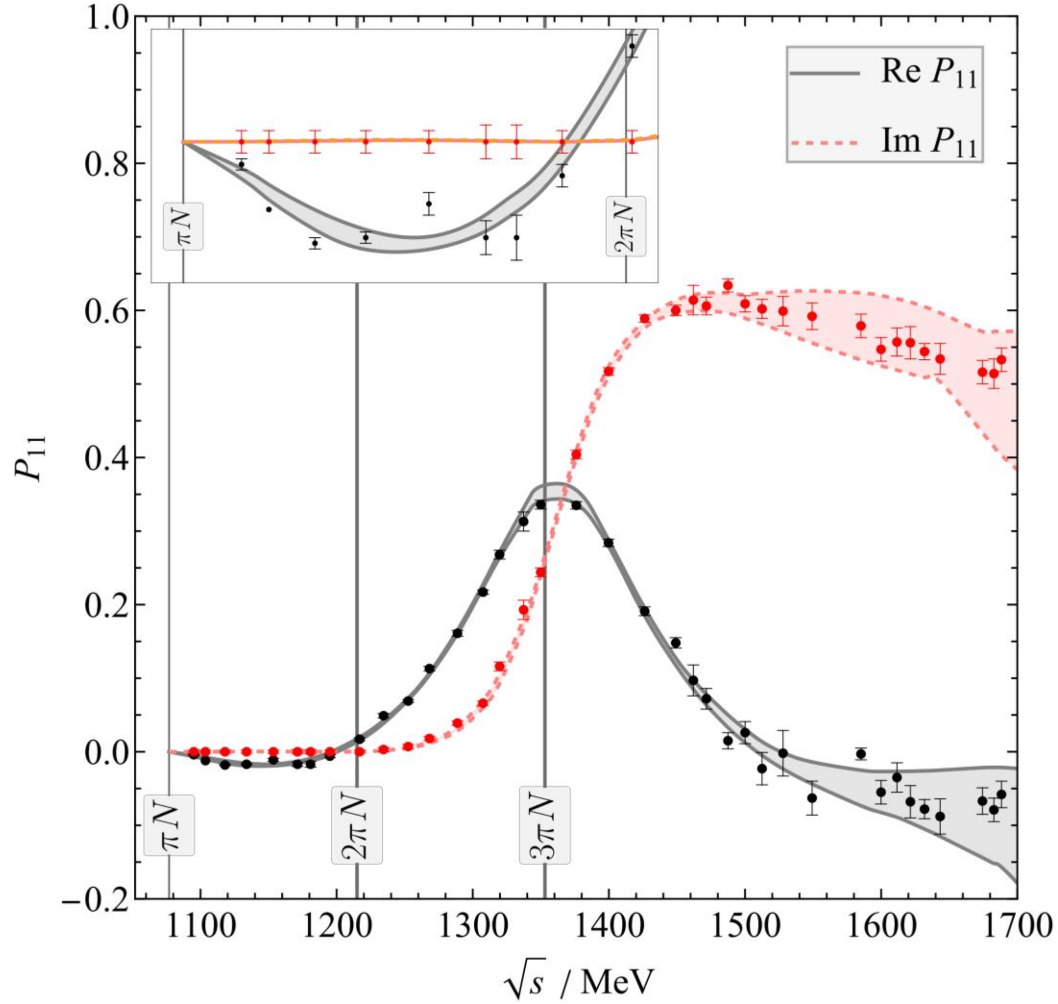
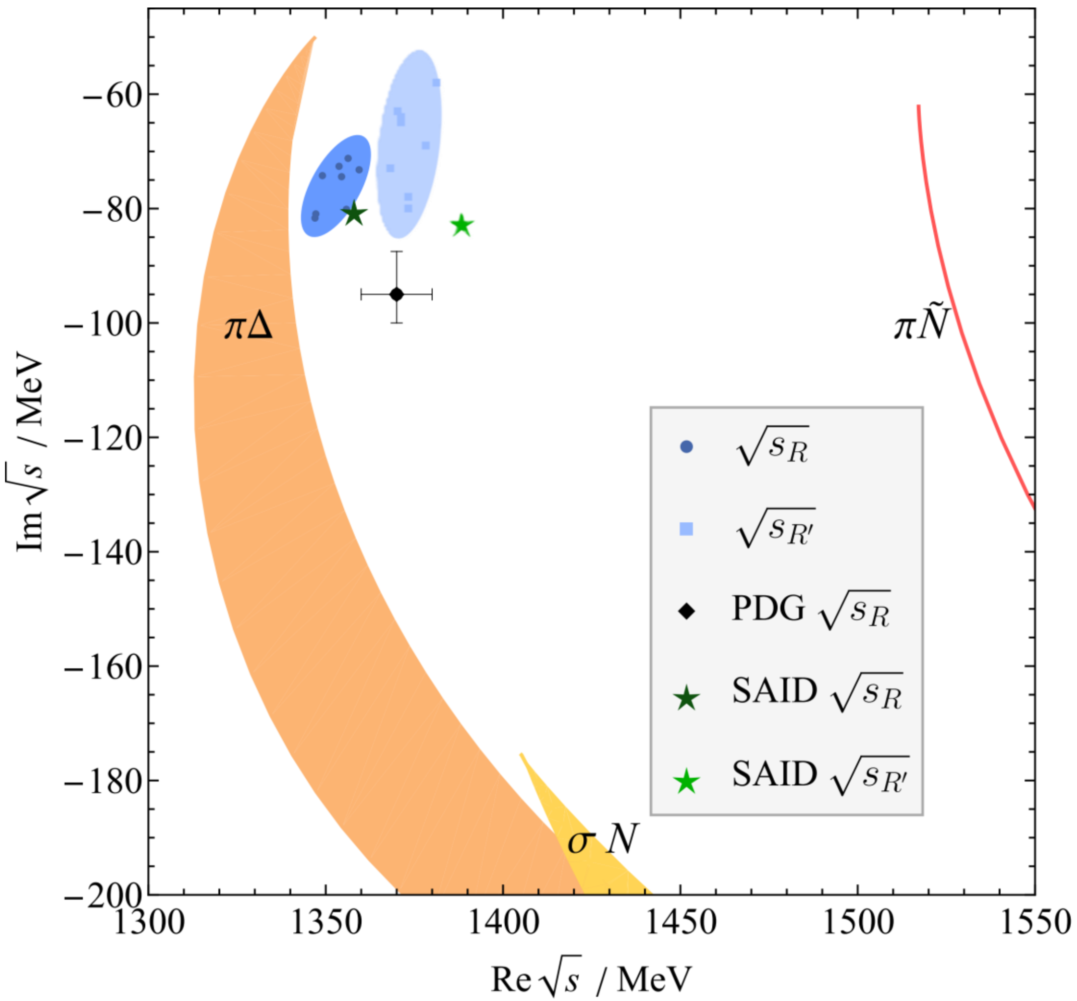
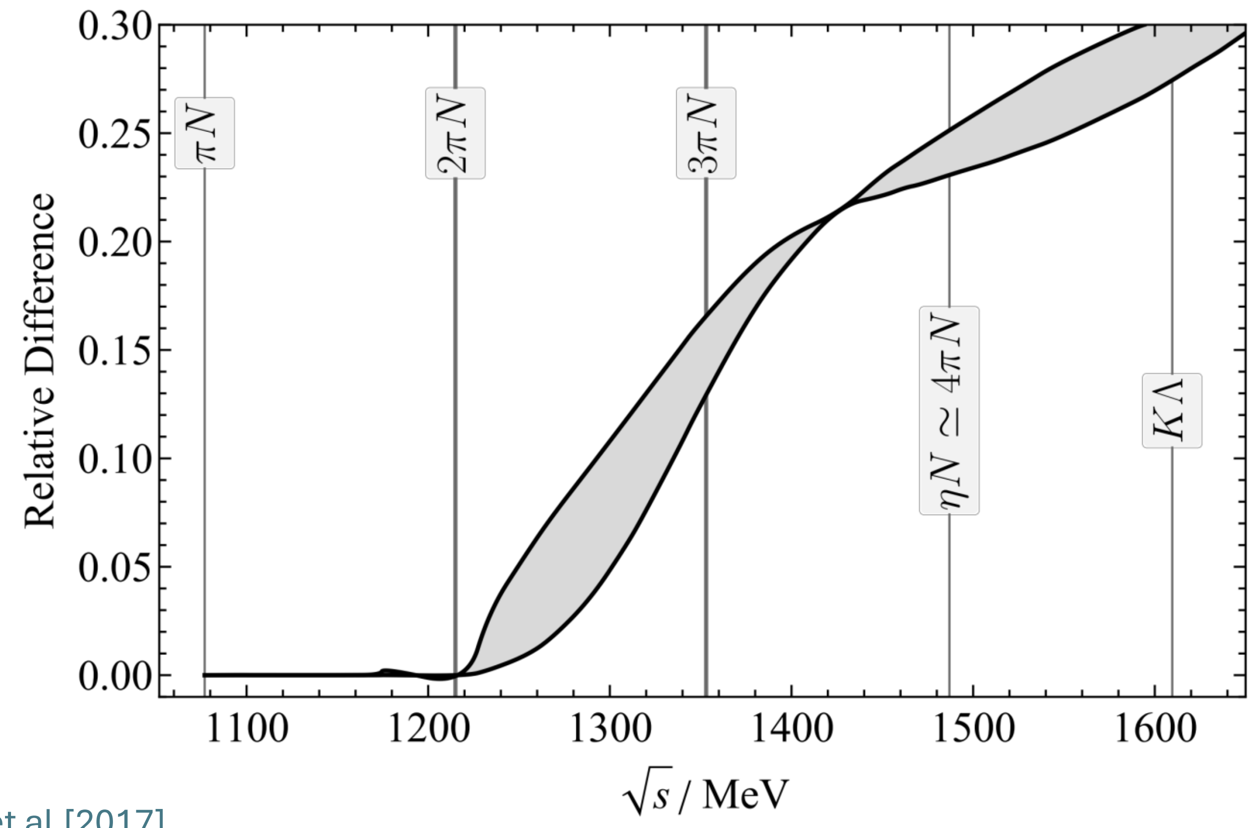


# Two- and three-body unitarity $SS^\dagger = 1$



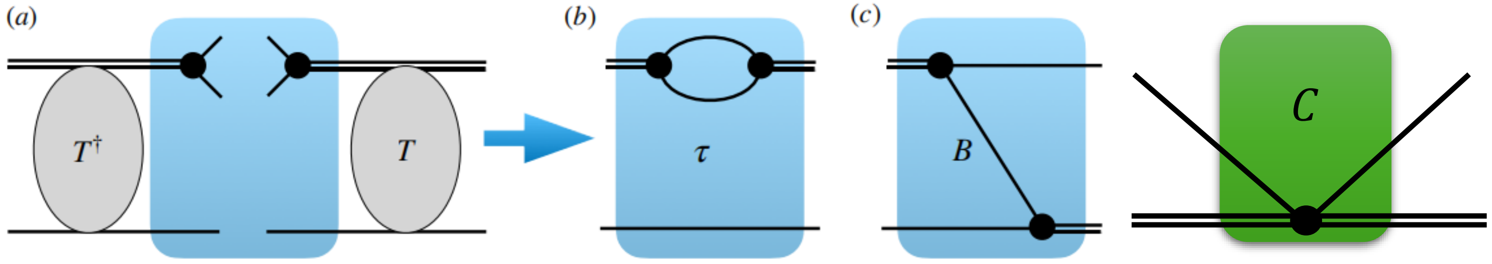

$$S = 1 - 2i \cdot \rho T$$

- Unitary condition:

$$\left(T^\dagger - T\right) = 2i \cdot \rho T T^\dagger \rightarrow \mathrm{Im}\left(T\right)^{-1} = \rho$$

- In the two-body region, fulfilled by $\rho(\sqrt{s}) = \frac{m_N q_{\text{on}}}{16\pi^2\sqrt{s}}$

- In the three-body region, fulfilled by [Three-body unitarity with isobars, M.Mai, et al.[2017]]




M. Doering, *et al.*[2025]

Quantization Condition for Lattice analysis

M.Mai, and M. Doering[2017]

# Summary and plans

- We construct the three-body amplitude from two-body input $+$ three-body unitarity with spectator-isobars in S- and P- waves;
- Two-body sub-amplitudes are described, including the $\pi\pi$-scattering and $\pi N$-scattering with $\Delta$ and $\widetilde{N}$ resonance;
- The coupled-channel amplitude is fitted to the SAID analysis (P11). The Roper pole and the hidden pole are determined.

(paper in preparation) → Best model to date

- The formalism is ready to be utilized for finite volume spectrum and the interpretation of lattice spectrum with

Finite Volume Unitarity (FVU).

# Thank you!

7/31/2026

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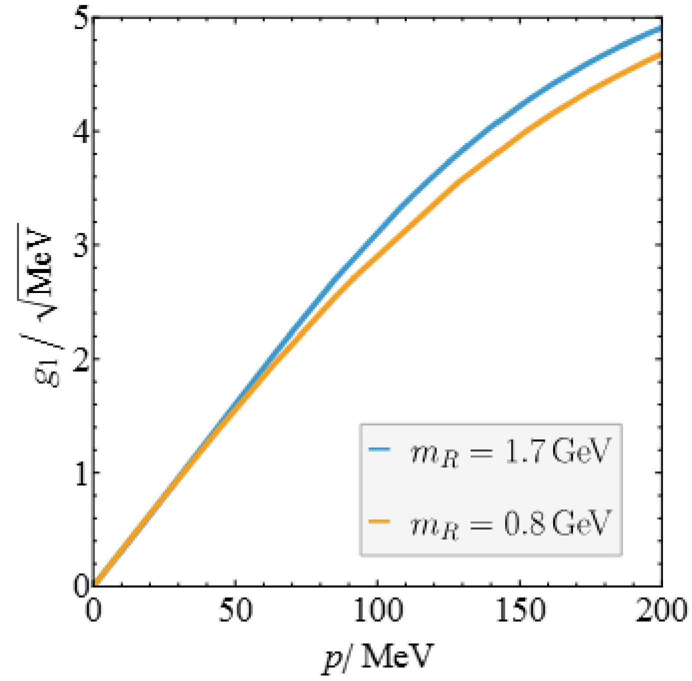
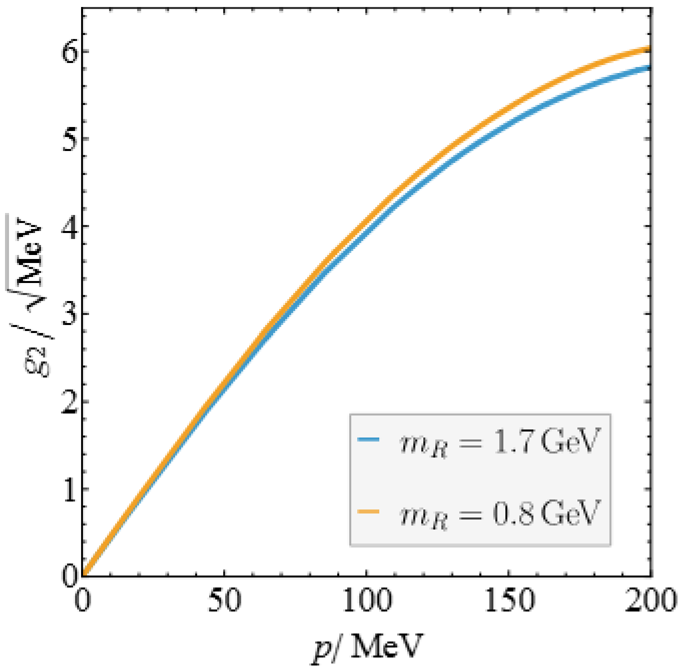
**Table 2.** Results for different fits, compared with the SAID analysis. The first column shows the fitted bare mass for the Roper resonance. The second and third columns show the $\chi^2$ for the fitted curve up to $3\pi N$ threshold and up to 1650 MeV; in parentheses is the $\chi^2$ per data point. Further discussion of degrees of freedom can be found in the text. The last two columns show the pole positions $\sqrt{s_R}$ of the Roper and the hidden pole $\sqrt{s_{R'}}$, respectively.

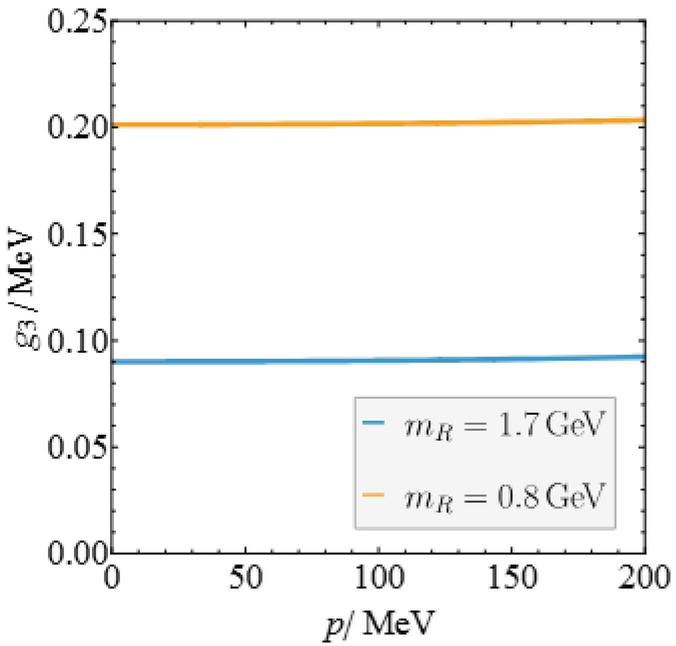
| | $m_R$/GeV | $\chi^2_{3b}$ | $\chi^2_R$ | $\sqrt{s_R}$/MeV | $\sqrt{s_{R'}}$/MeV |
|---|---|---|---|---|---|
| fit 1 | 1.6 | 55.5 | 128.6(4.0) | $1354-73i$ | $1371-65i$ |
| fit 2 | 0.6 | 30.7 | 60.2(1.9) | $1347-81i$ | $1373-78i$ |
| fit 3 | 1.7 | 44.7 | 129.3(4.0) | $1349-74i$ | $1368-73i$ |
| fit 4 | 0.8 | 44.1 | 77.9(2.4) | $1356-80i$ | $1378-69i$ |
| fit 5 | 1.7 | 55.6 | 115.7(3.6) | $1356-71i$ | $1370-63i$ |
| fit 6 | 0.6 | 30.0 | 57.3(1.8) | $1347-82i$ | $1373-80i$ |
| fit 7 | 1.8 | 51.1 | 86(2.7) | $1354-74i$ | $1371-64i$ |
| fit 8 | 0.8 | 51.8 | 96(3.0) | $1359-73i$ | $1381-58i$ |
| SAID analysis [36] | – | 55.7 | 88.1(2.8) | $1358-80i$ | $1388-82i$ |

# Roper coupling residue to each channels

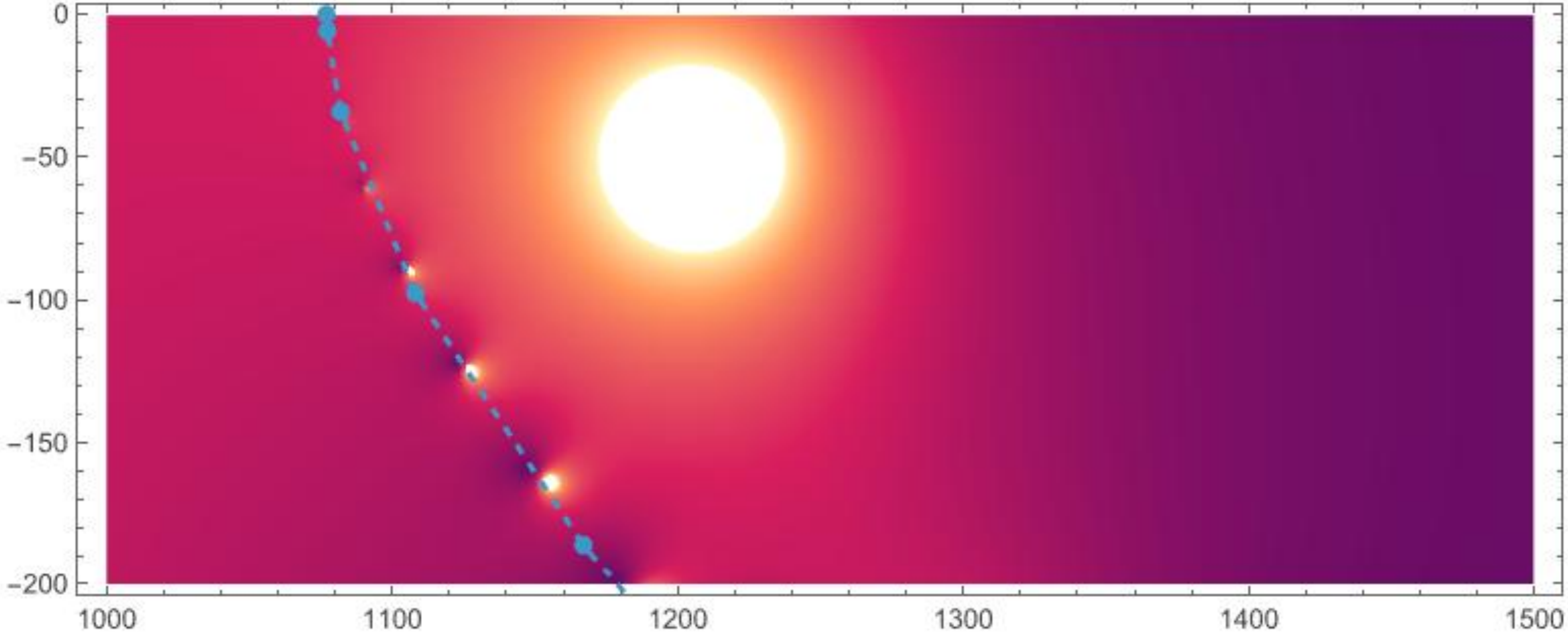
$$T_{ij}(p_3, p_1; \sqrt{s}) = \frac{g_j(p_3)g_i(p_1)}{\sqrt{s} - \sqrt{s_R}}, \quad i, j \in \{1, 2, 3\},$$



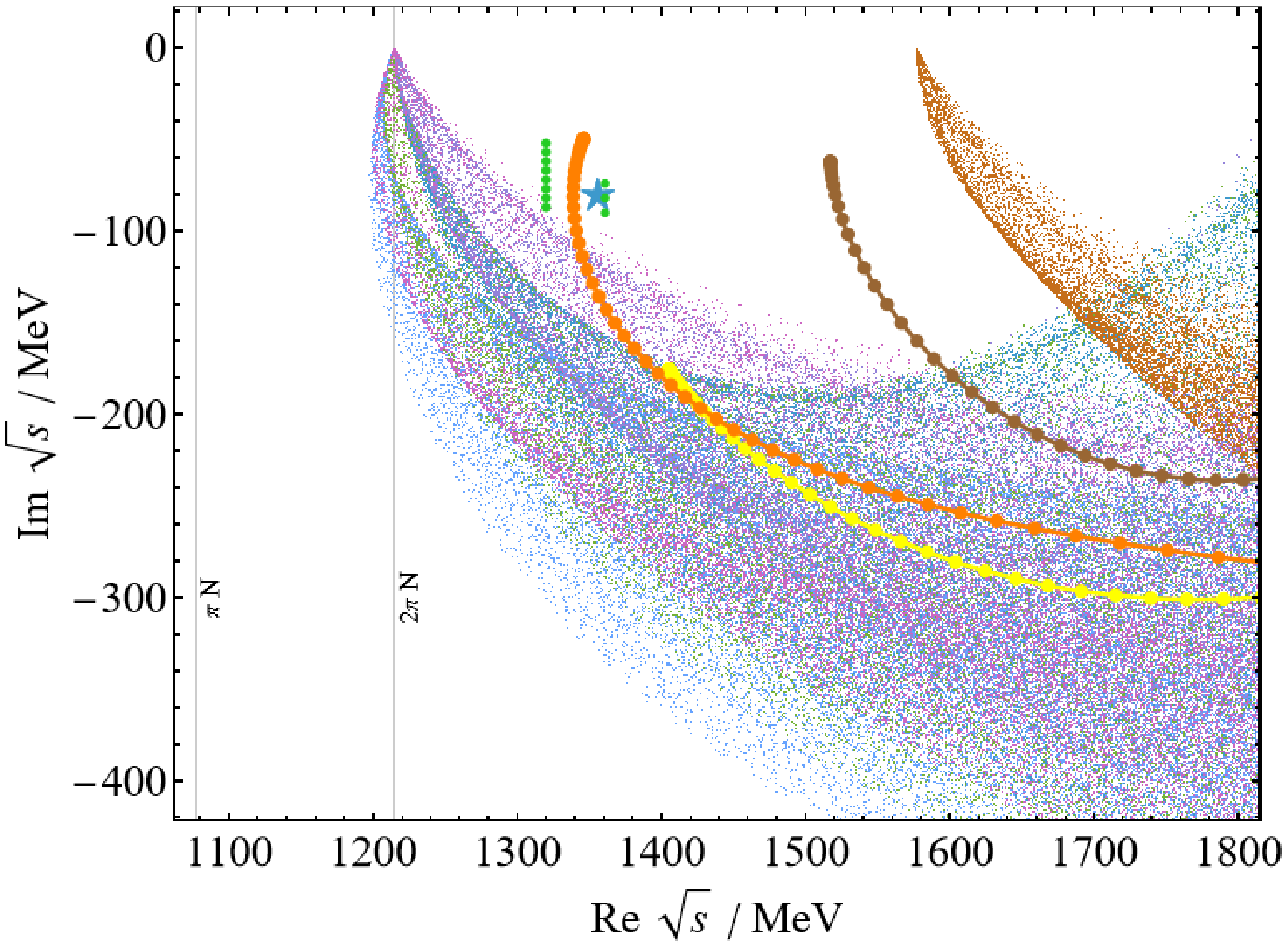



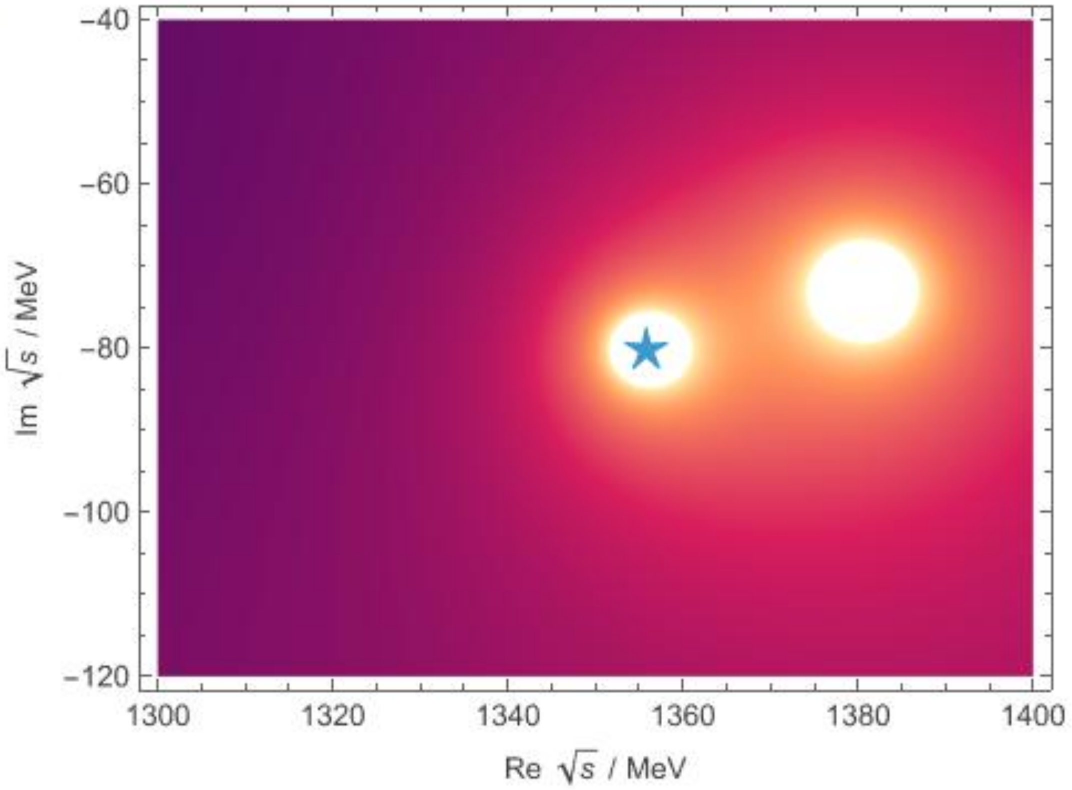



# P33 pole position and near by piN branch cut



# P11 pole position and hidden pole with samples




Preliminary Result

# P11 pole position and near by piDelta branch cut