Quantum Complexity in Simulations of Scattering

Michael Hite

(work in progress)

THE UNIVERSITY OF ARIZONA

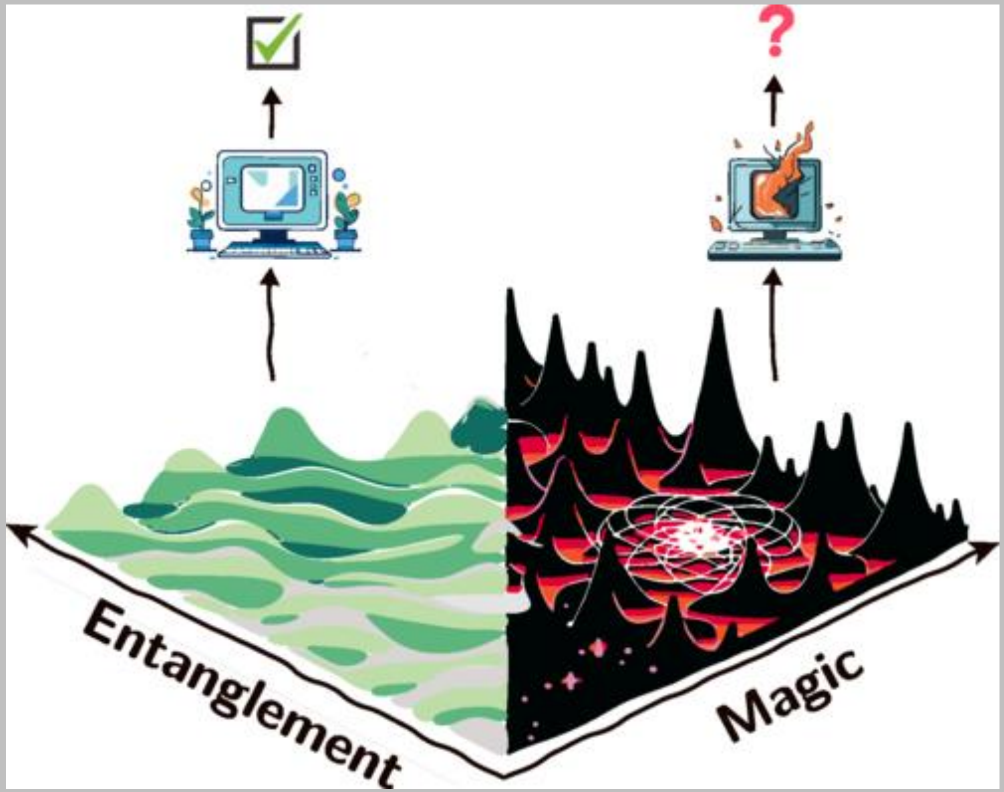
LATTICE 2026
UNIVERSITY OF MARYLAND, COLLEGE PARK

Motivation

- **Question.** Where (and when) is the complexity *over the entirety* of position space scattering simulations?
- Map the complexity in magic-entanglement phase space.

Related works:

- Robin and Savage. arXiv:2510.23426 (2025).
- Greininger et al. arXiv:2601.08825 (2026).
- Zemlevskiy. arXiv:2603.15621 (2026).
- Jha et al. arXiv:2606.09971 (2026).
- Hite and Siwach. arXiv:2606.14642 (2026).



PRX Quantum **6**, 020324

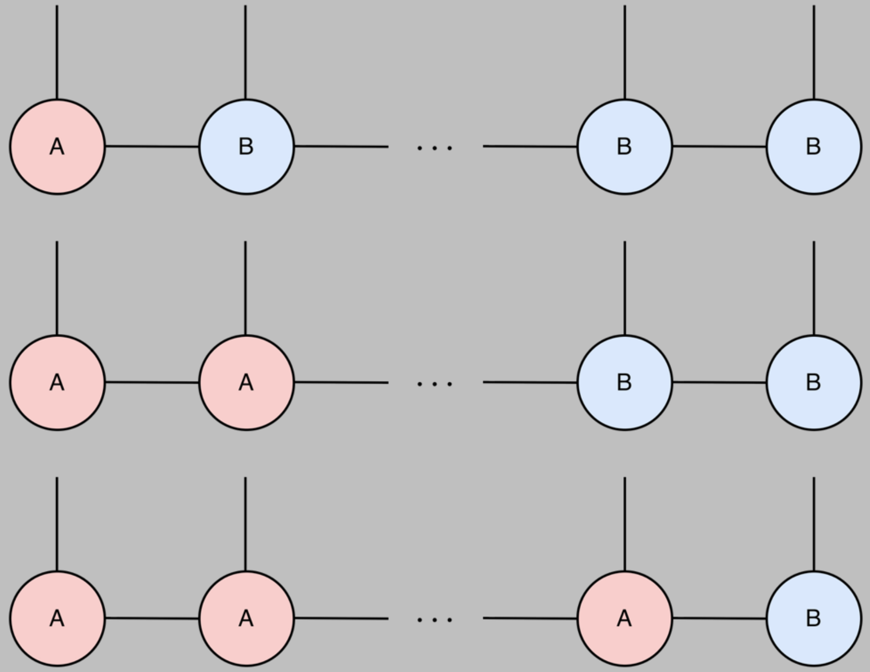
Quantum Complexity

Bipartite Entanglement Entropy

- Definition:

$$S(\rho_A) = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$$

Traditional measure of quantum complexity.



Stabilizerness

- Stabilizer state – Generated by the Clifford set $\{H, S, CNOT\}$.
- $\mathcal{O}(N)$ computational complexity* (Gottesman-Knill**).

$$\left(\begin{array}{ccc|ccc|c} x_{11} & \cdots & x_{1N} & z_{11} & \cdots & z_{1N} & r_1 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ x_{N1} & \cdots & x_{NN} & z_{N1} & \cdots & z_{NN} & r_N \\ \hline x_{(N+1)1} & \cdots & x_{(N+1)N} & z_{(N+1)1} & \cdots & z_{(N+1)N} & r_{N+1} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ x_{(2N)1} & \cdots & x_{(2N)N} & z_{(2N)1} & \cdots & z_{(2N)N} & r_{2N} \end{array}\right)$$

*Aaronson and Gottesman. PRA 70, 052328 (2004).
**Gottesman. arXiv:quant-ph/9807006 (1998).

Magic (Nonstabilizerness)

- Magic – Deviation from a stabilizer state.
- Generated by $\{H, S, CNOT\} + \{T\}$.
- Measure:

$$M_2(|\psi\rangle) = -\log_2\left(\sum_{P\in\mathcal{P}_N} \frac{|\langle\psi|P|\psi\rangle|^4}{2^N}\right)$$

  - Computationally expensive ($|\mathcal{P}_N| = 4^N$).
  - Basis dependent.

Bipartite Non-local magic

$$\mathcal{M}_2\left(\{\lambda_i\}\right) = -\log\left(\sum_{i_1,i_2,i_3,i_4=0}^{r-1}\sqrt{\lambda_{i_1}\lambda_{i_2}\lambda_{i_3}\lambda_{i_4}\lambda_{i_1\wedge i_2\wedge i_3}}\right.$$
$$\left.\times\sqrt{\lambda_{i_1\wedge i_2\wedge i_4}\lambda_{i_1\wedge i_3\wedge i_4}\lambda_{i_2\wedge i_3\wedge i_4}}\right),$$

Singular values of ρ_A.

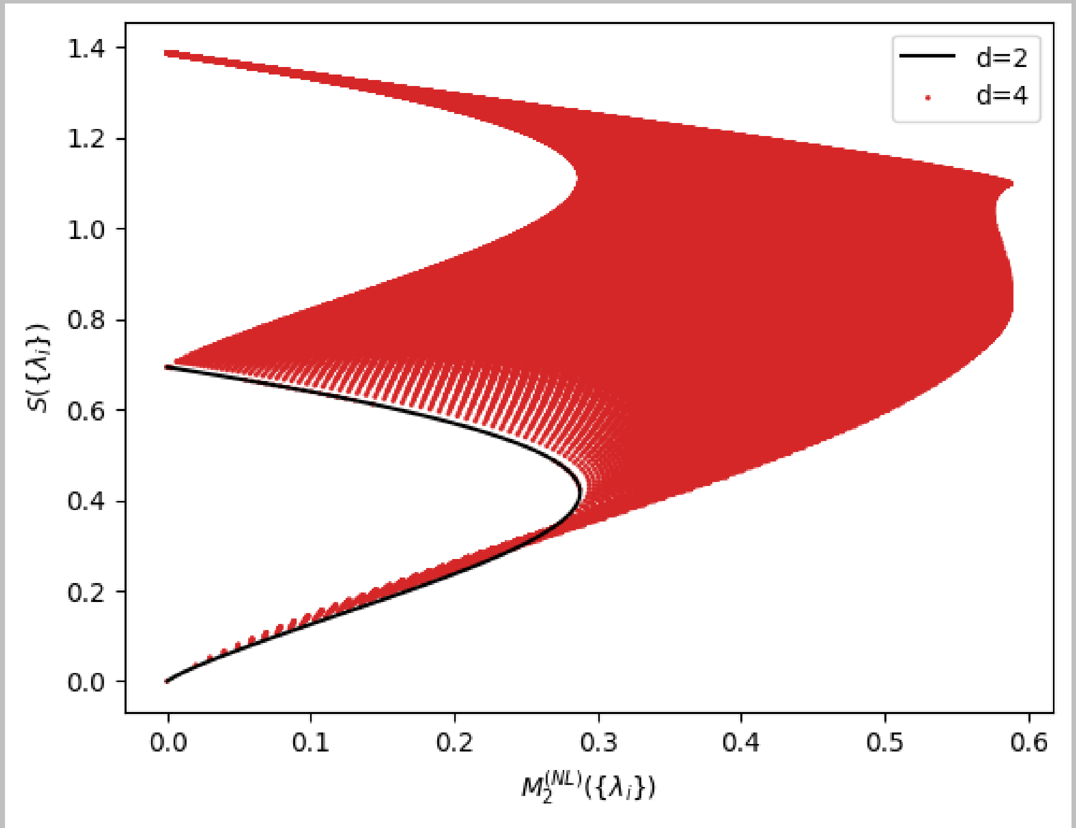
Magic left in the correlations between A and B when basis contributions are minimized*.

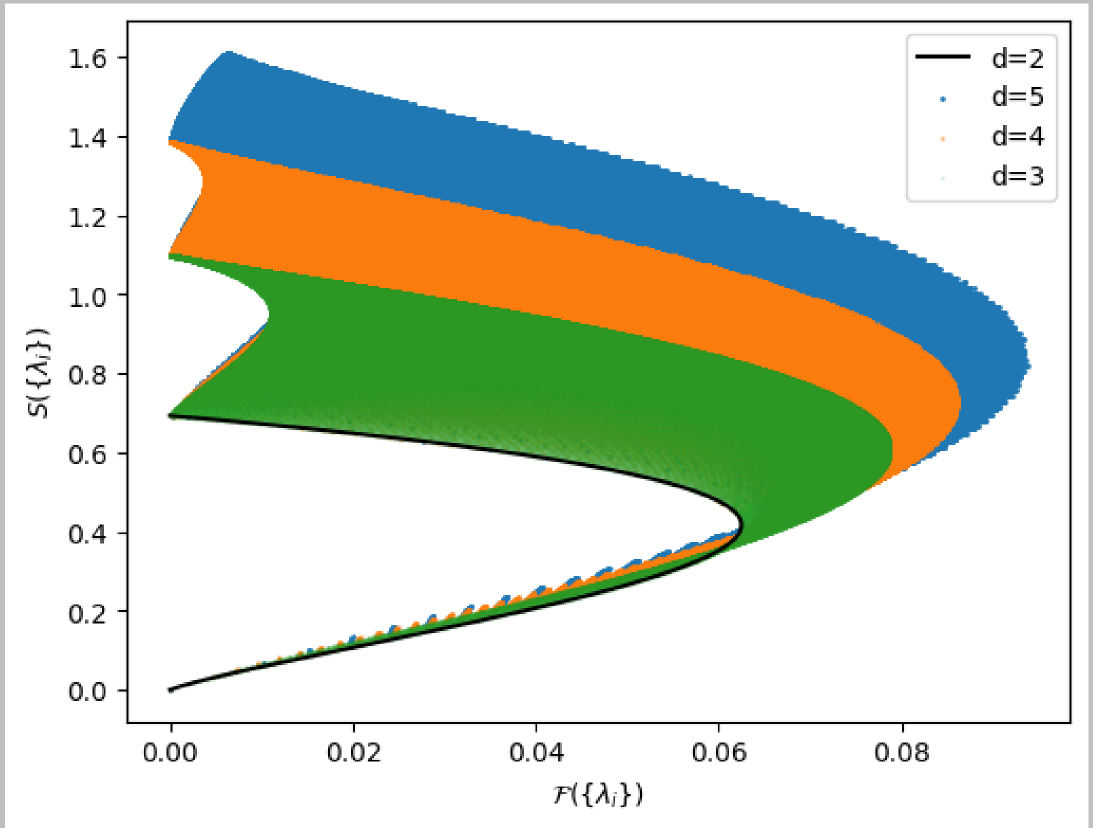
- Basis independent (~ Entanglement Entropy)
- Lower bounded by **antiflatness**

$$\mathcal{F} = \mathrm{Tr}\left(\rho_A^3\right) - \mathrm{Tr}\left(\rho_A^2\right)^2$$

*Cao et al. PRX QUANTUM 6, 040375 (2025)

Magic vs. Entanglement Entropy





Model

X(Y)Z Heisenberg Model

- Hamiltonian (PBC):

$$\hat{H} = -\sum_{j} \left(J \hat{X}_j \hat{X}_{j+1} + \hat{Z}_j \right) - g \sum_{j} \hat{Z}_j \hat{Z}_{j+1}$$

X(Y)Z Heisenberg Model

- Hamiltonian (PBC):

$$\hat{H} = \boxed{-\sum_j \left(J \hat{X}_j \hat{X}_{j+1} + \hat{Z}_j \right)} - g \sum_j \hat{Z}_j \hat{Z}_{j+1}$$

Ising Model in Particle Basis

X(Y)Z Heisenberg Model

Four-fermion Interaction $(g \to -1/2)$

$$4\sum_j \hat{c}_j^\dagger \hat{c}_j \hat{c}_{j+1}^\dagger \hat{c}_{j+1}$$

- Hamiltonian (PBC):

$$\hat{H} = -\sum_j \left(J\hat{X}_j\hat{X}_{j+1} + \hat{Z}_j \right) - g\sum_j \hat{Z}_j\hat{Z}_{j+1}$$

Ising Model in Particle Basis

X(Y)Z Heisenberg Model

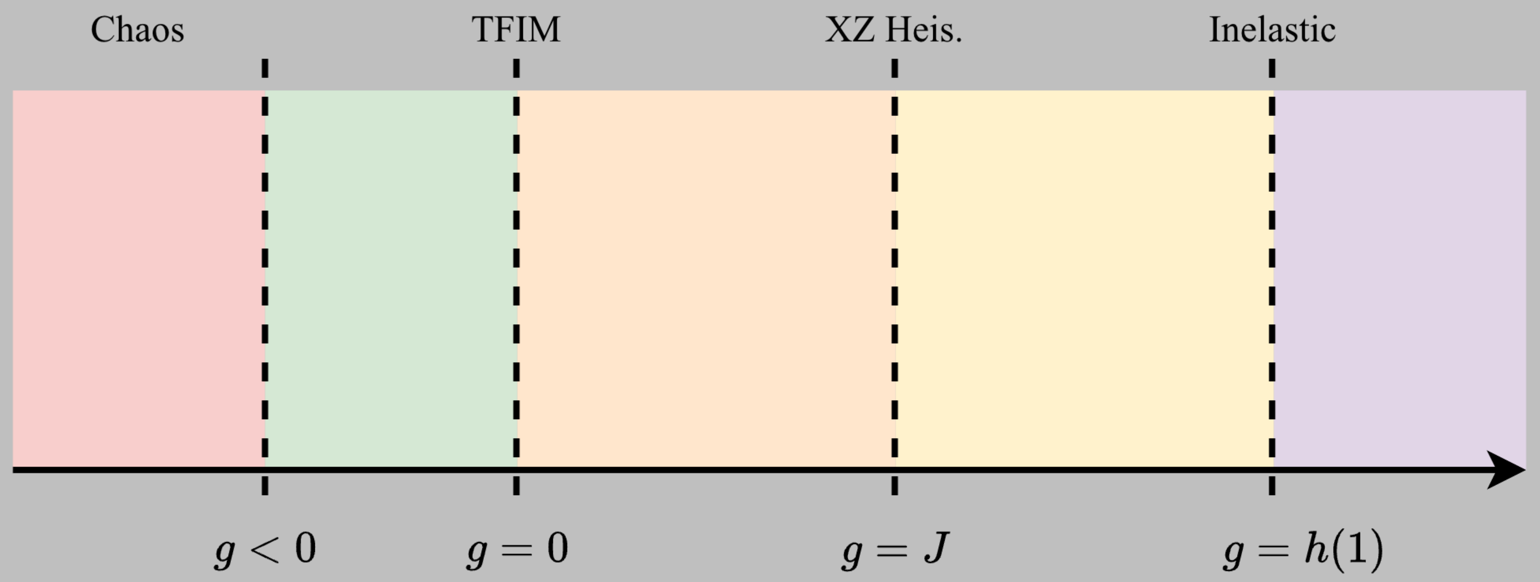
Four-fermion Interaction $(g \to -1/2)$

$$4\sum_j \hat{c}_j^\dagger \hat{c}_j \hat{c}_{j+1}^\dagger \hat{c}_{j+1}$$

- Hamiltonian (PBC):

$$\hat{H} = -\sum_j \left(J\hat{X}_j\hat{X}_{j+1} + \hat{Z}_j \right) - g\sum_j \hat{Z}_j\hat{Z}_{j+1}$$

Ising Model in Particle Basis



GWP State Preparation

$$\boxed{\mathcal{G}^{k_A,x_A}\mathcal{G}^{k_B,x_B}|\Omega\rangle}$$

- Free ground state is Ising limit:

$$\mathcal{G}^{k,x} = \frac{1}{\sqrt{N}}\sum_{x'}\phi^{k,x}_{x'}\hat{c}^{\dagger}_{x'}$$

$$\phi^{k,x}_{x'} = \sum_{k'} e^{-ik'(x'-x)}e^{-|k'-k|^2/4\sigma_k^2}\sec\phi_{k'}$$

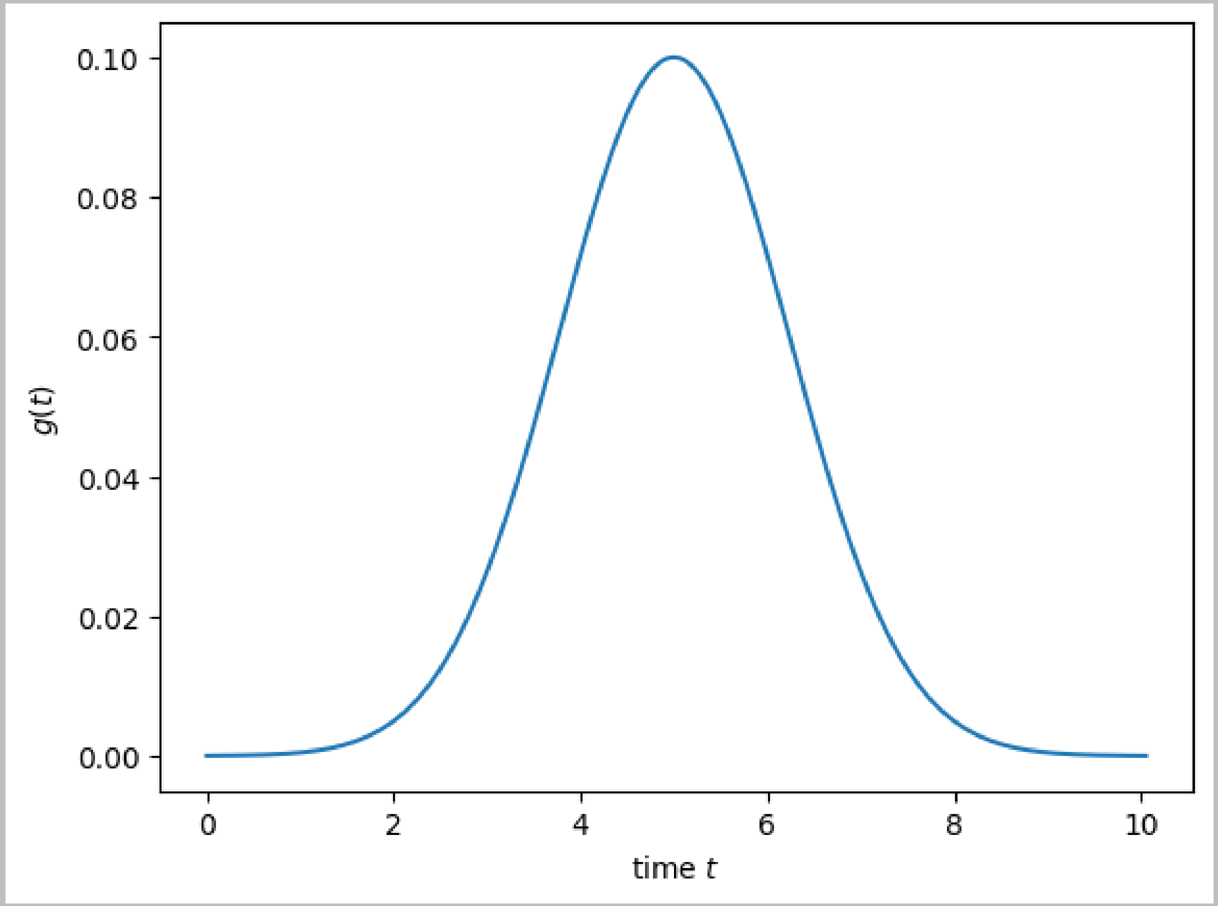
$$\tan\phi_k = \frac{J\sin k}{1-J\cos k} \qquad |k\rangle = \xi^{\dagger}_k|\Omega\rangle = \sec\phi_k\eta^{\dagger}_k|\Omega\rangle = \sum_j e^{ikj}\sec\phi_k c^{\dagger}_j|\Omega\rangle$$

Time Evolution

- Slow ramp up/down.
- Time dependent coupling:

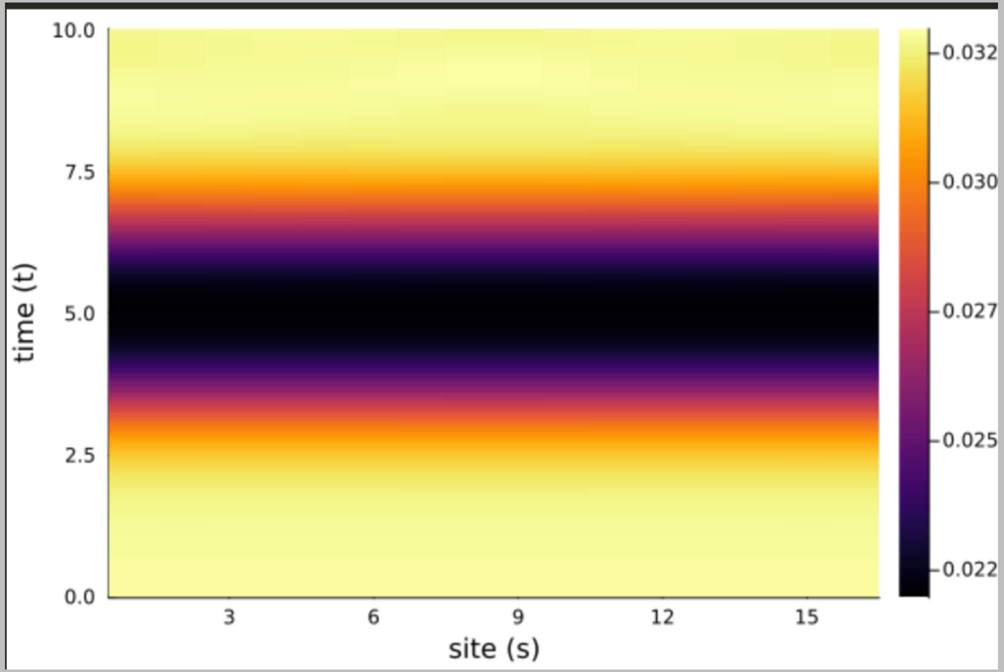
$$g \to g(t) = g e^{-(t-t_{1/2})^2/\sigma}$$

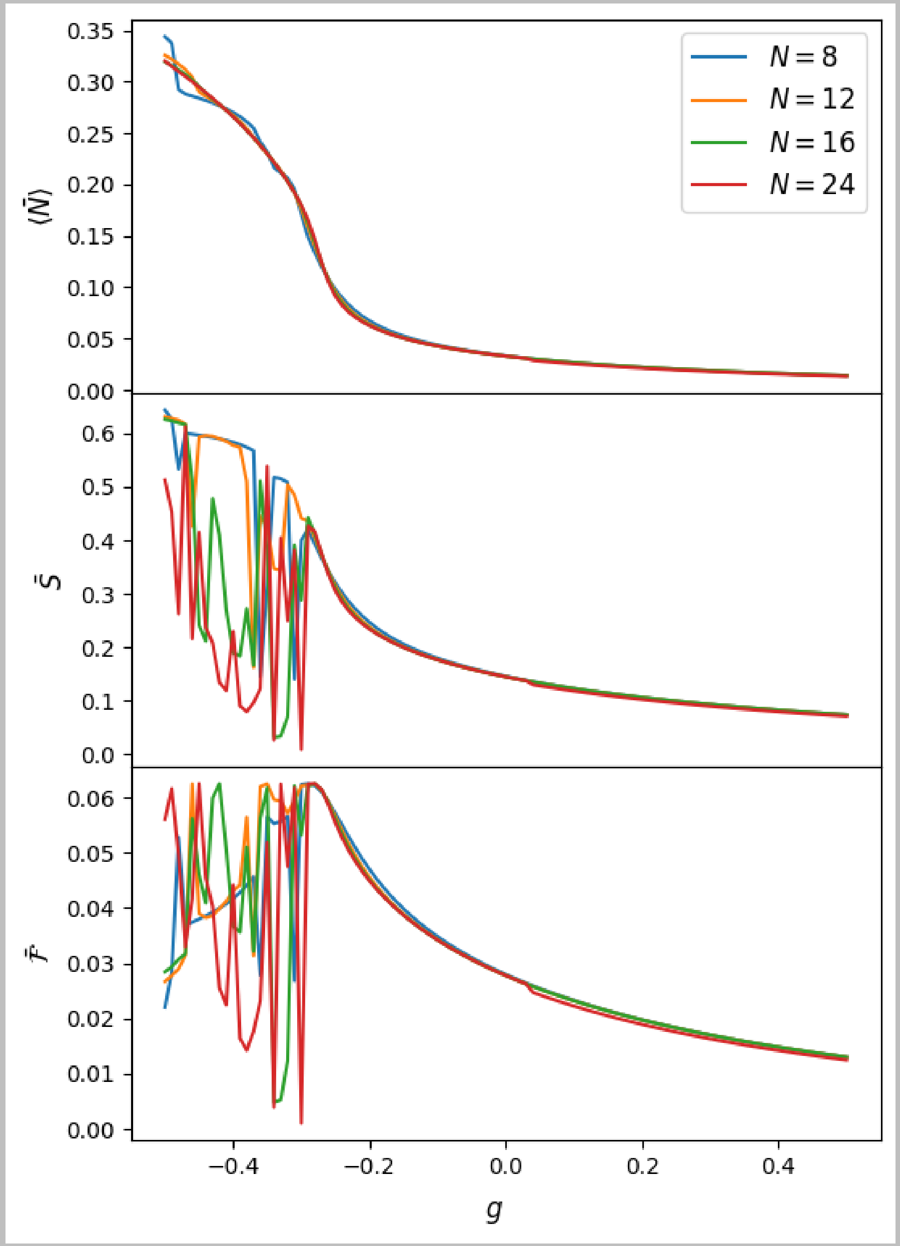
- Method: MPS Time evolution with TDVP.



Ground state behavior

- Average per site entanglement entropy and antiflatness.
- Chaotic behavior ensues for $g \lesssim= 0.2$.





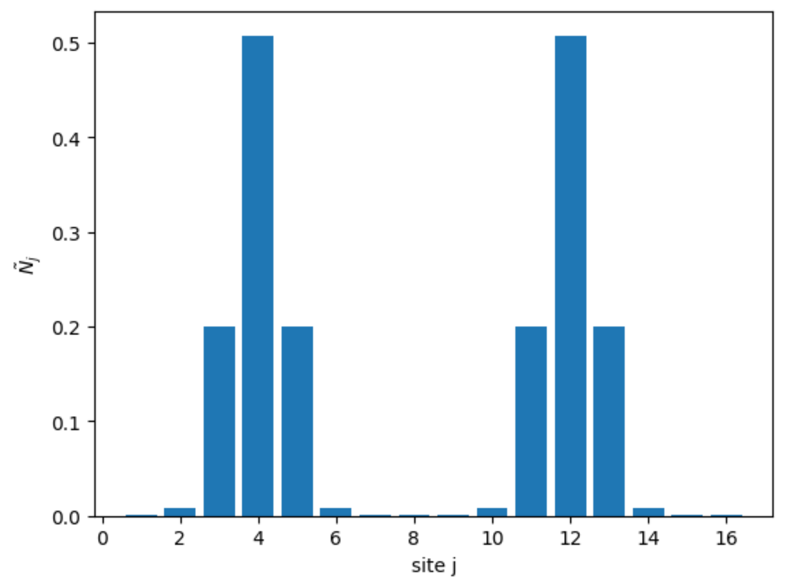
Results

On Site Energy

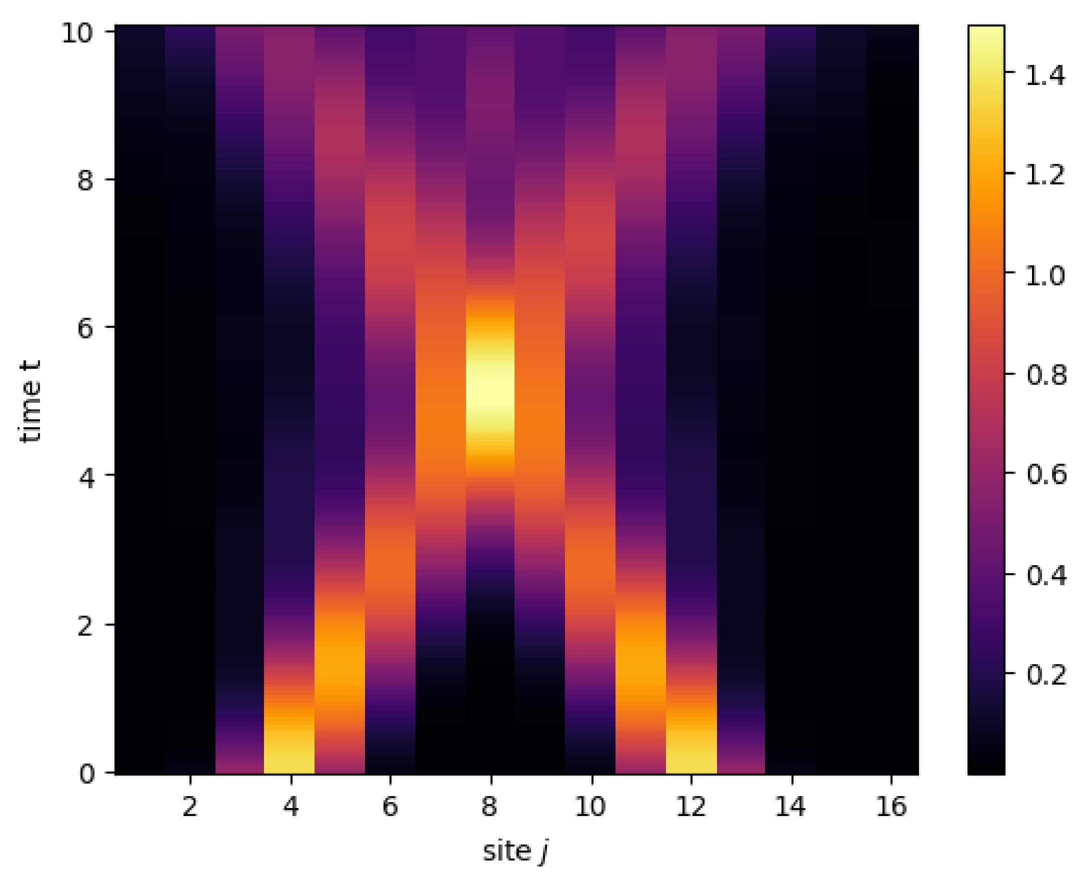
$$\tilde{E}_j(t) = E_j(t) - E_j^{(\Omega)}(t)$$

Parameters:

- $N = 16, \lambda = 0.5$
- $\bar{k} = \pm\frac{7\pi}{16}, \sigma_k = 0.75$
- $\Delta t = 0.01$



$$\tilde{N}_j = N_j - N_j^{(\Omega)}$$

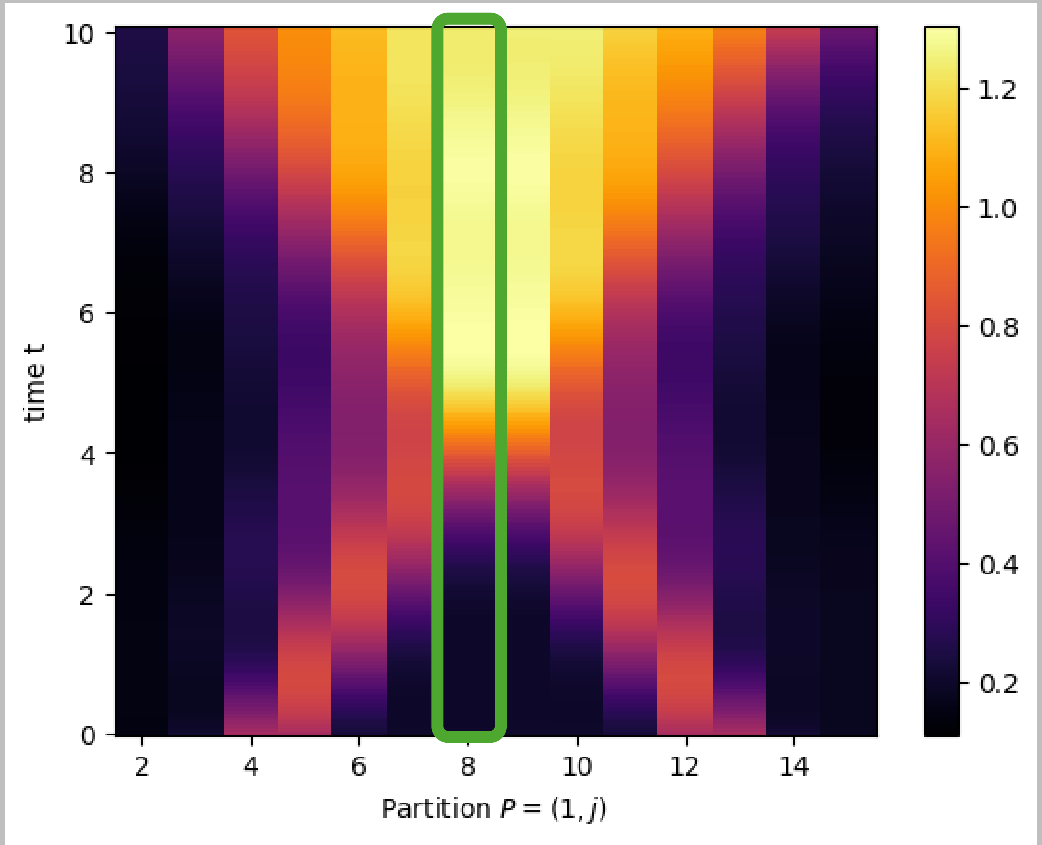


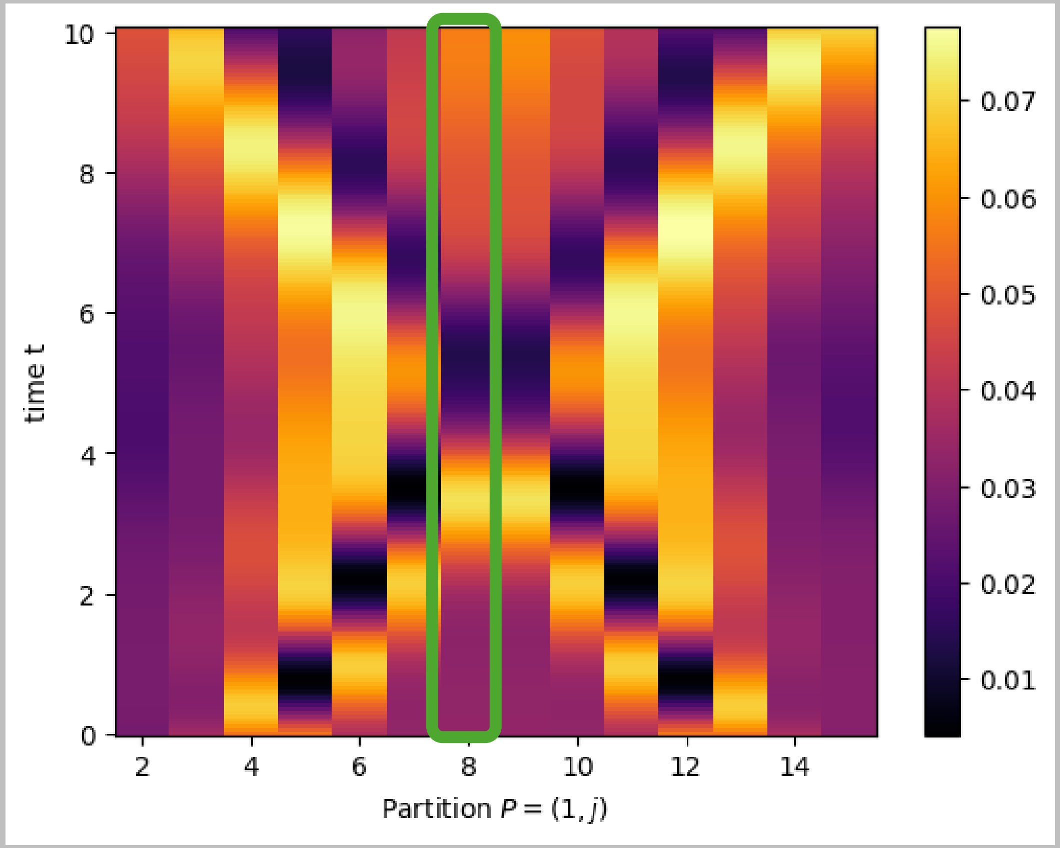
$$g = 0.25$$

Entanglement Entropy

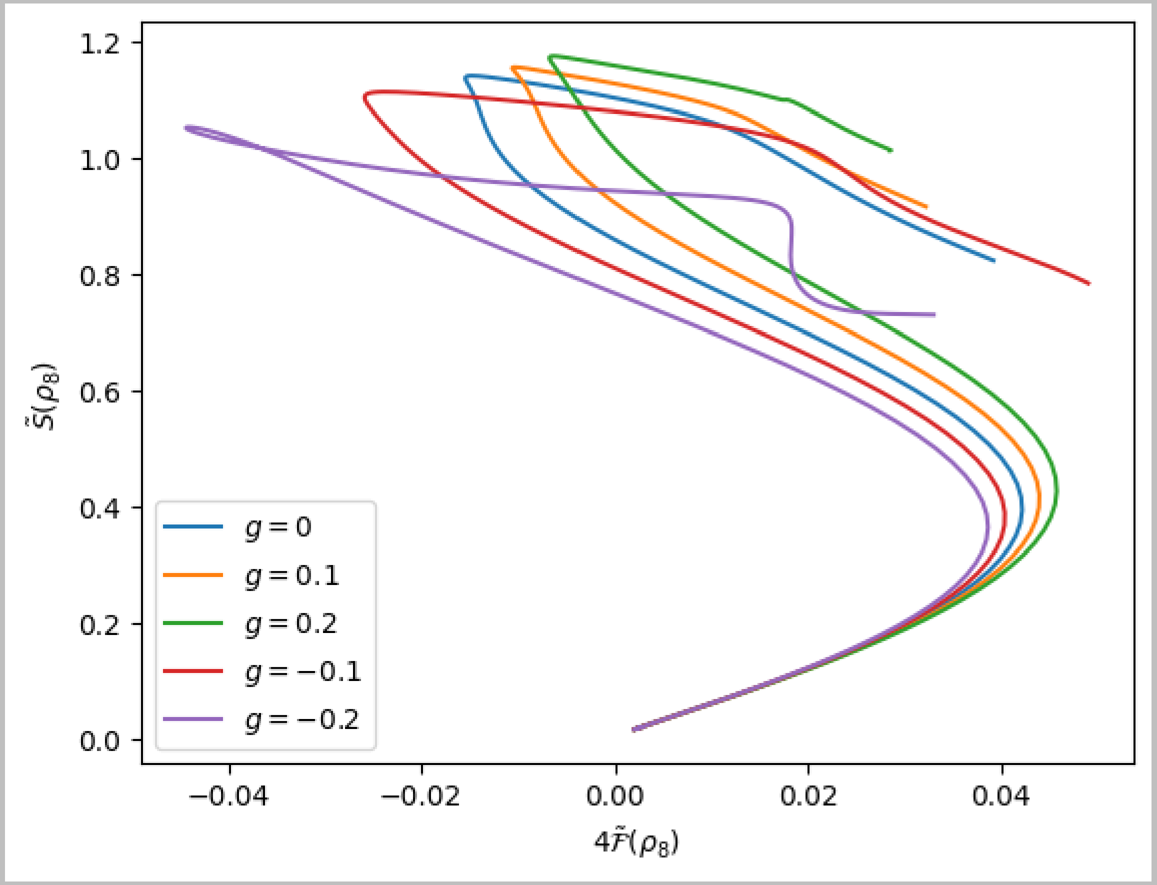
Antiflatness

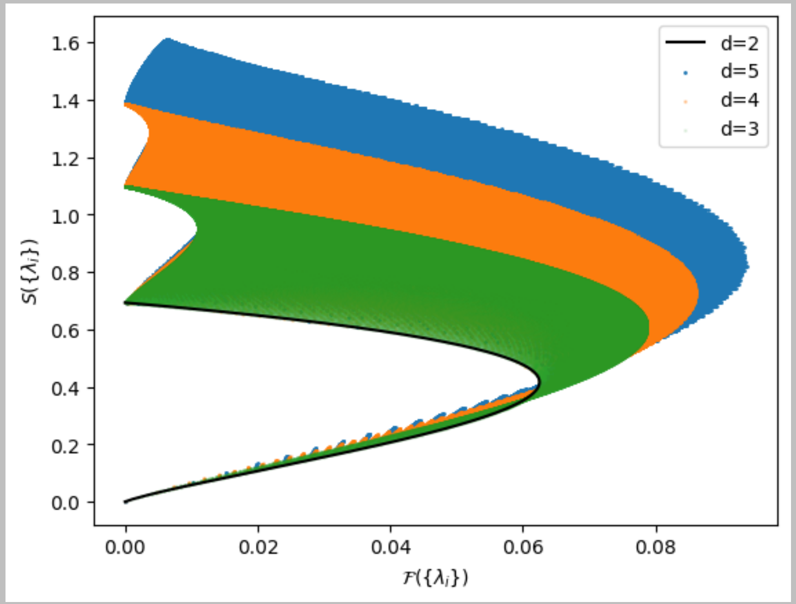
$g = 0.25$





Quantum Complexity (Left↔Right Half)





Maximum overlap corresponds to a low magic regime.

$$\tilde{\mathcal{O}} = \mathcal{O}(|\psi(t)\rangle) - \mathcal{O}(|\Omega(t)\rangle)$$

Conclusion

- We mapped the quantum complexity in position space simulations of 1+1D scattering.
- Maximum overlap corresponds to high entanglement/low non-local magic.

Future Work

- Include 2-Stabilizer Rényi Entropy as a measure of total magic.
- Larger N.
- Approach criticality.
- Extend to fermion/anti-fermion and multi-flavor models.

Questions

michaelhite@arizona.edu

THE UNIVERSITY OF ARIZONA

LATTICE 2026

UNIVERSITY OF MARYLAND, COLLEGE PARK