Aspects of a high-precision DWF physics program

Christoph Lehner

University of Regensburg

July 27, 2026, Lattice 26

The RBC & UKQCD collaborations

BNL & RBRC
Peter Boyle
Taku Izubuchi
Chulwoo Jung
Christopher Kelly
Patrick Oare
Amarjit Soni (emeritus)
Xin-Yu Tuo
Tianle Wang
Shuhei Yamamoto

CERN
Matteo Di Carlo
Felix Erben
Andreas Jüttner (Southampton)

Columbia University
Ryan Abbott
Norman Christ
Jonah Eick
Rajiv Eranki
Sarah Fields
Ceran Hu
Yikai Huo
Erik Lundstrum
Robert Mawhinney

University of Connecticut
Thomas Blum
Jonas Hildebrand
Luchang Jin
Vaishakhi Moningi
Anton Shcherbakov
Douglas Stewart
Joshua Swain

DESY Zeuthen
Raoul Hodgson

University of Edinburgh
Matthew Black
Luigi Del Debbio
Vera Gülpers
Maxwell T. Hansen
Nils Hermansson Truedsson
Ryan Hill
Rajnandini Mukherjee
Antonin Portelli
Teseo San Jose

Johannes Gutenberg University of Mainz
Alessandro Barone

KEK
Shigemi Ohta (emeritus)

Liverpool Hope
Nicolas Garron (Liverpool)

University of Liverpool
Tobias Tsang
Aurelia Zuchanke

Massachusetts Institute of Technology
En-Hung Chao

University of Milano Bicocca
Francesca A. Bresciani
Mattia Bruno
Gabriele Morandi

Peking University
Xu Feng

University of Regensburg
Daniel Knüttel
Christoph Lehner
Julian Parrino
Andreas Völklein
Haoyu Wang

RIKEN R-CCS
Masaaki Tomii

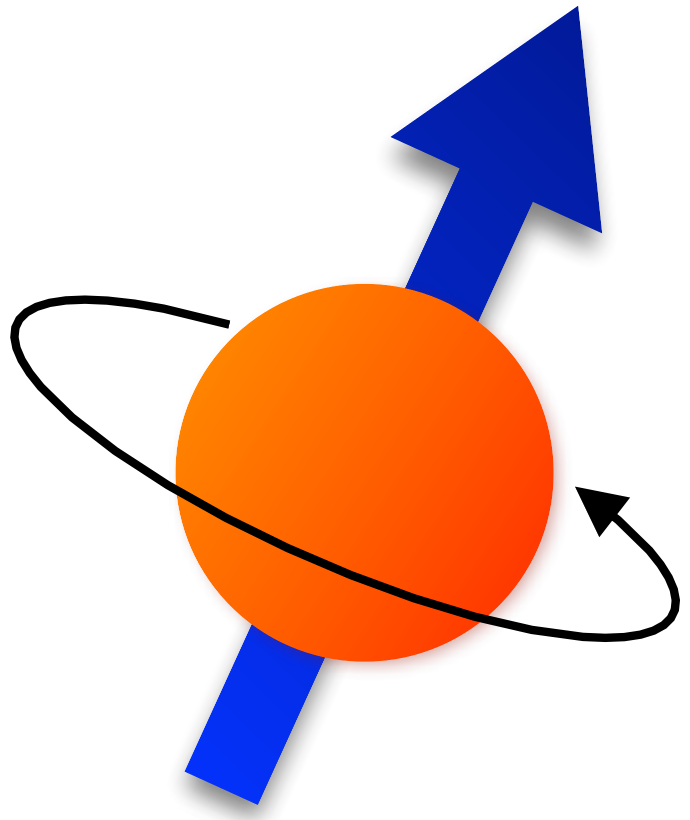
University of Siegen
Oliver Witzel

University of Southampton
Bipasha Chakraborty
Ahmed Elgaziari
Jonathan Flynn
Ollie Heald
Joe McKeon
Chris Sachrajda

Stony Brook University
Sergey Syritsyn (RBRC)

62 members

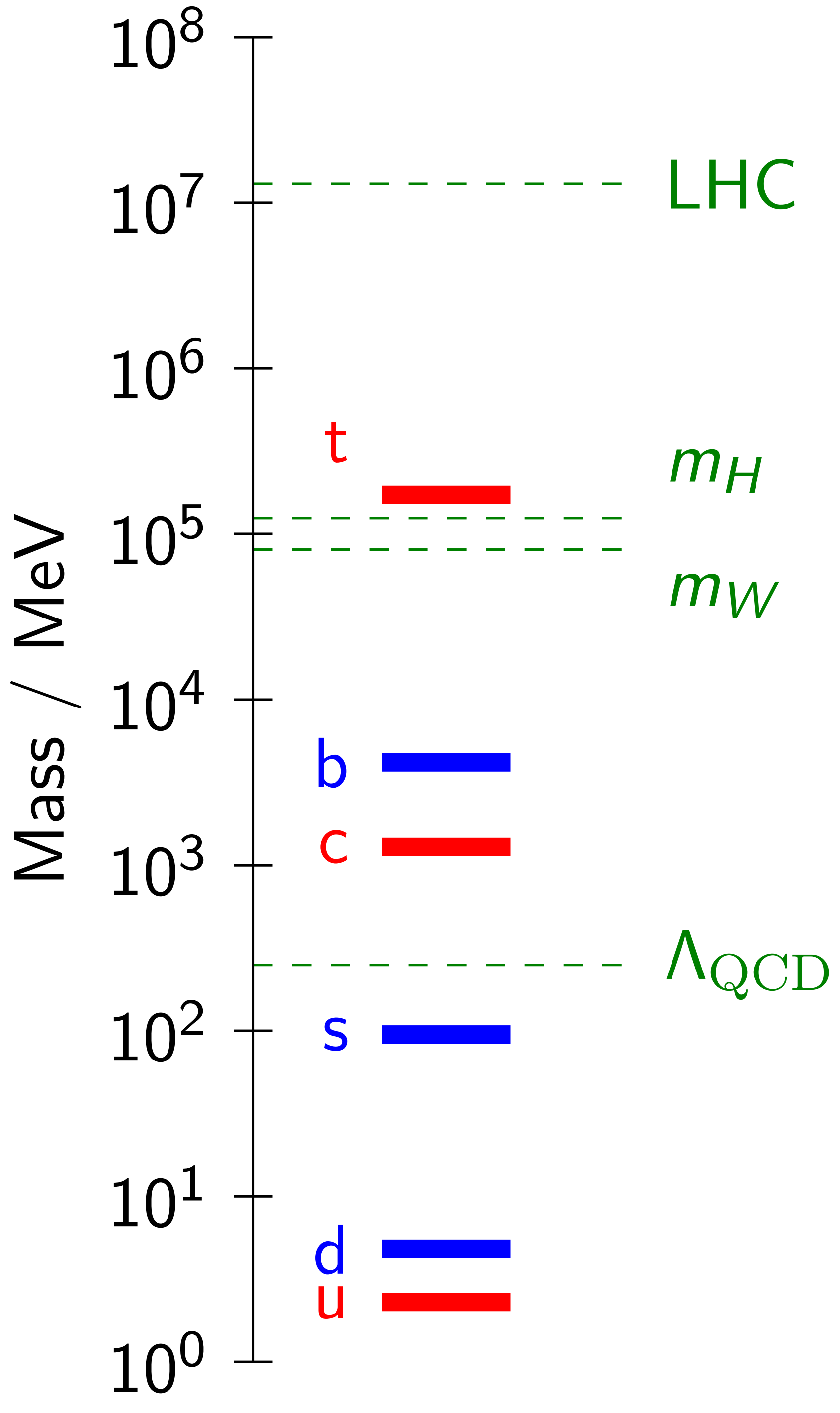
Three major target areas



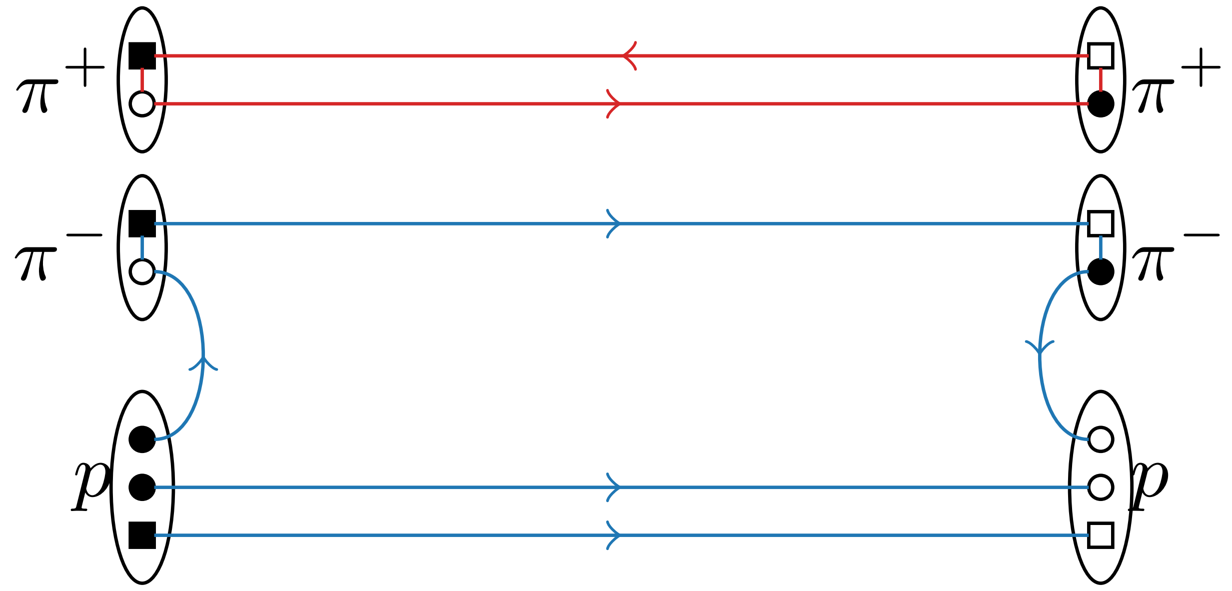
Muon g-2, match FNAL E989

see later in this session: J. Parrino

see Friday: M. Bruno



Quark flavor physics



Nucleons and their resonances

see later in this session: J. Hildebrand, M. Tomii

Common challenges

$$\lim_{a \to 0}$$

Critical slowdown of
Markov Update

$$m_u \neq m_d$$

$$\alpha \neq 0$$

QED / Isospin-breaking

$$C_{ij} = \langle O_i^\dagger O_j \rangle$$

Variational approach

This talk: improvements to all three

Towards the continuum limit

- HMC/SMD is a great algorithm, takes care of exponential volume scaling
- Can quantify autocorrelation via a diffusion process that scales naively as a^{-2} and a tunneling process that scales exponentially in the lattice spacing

Addressing the diffusion aspect:

- Applying a field transformation, U = f(V), and integrating in V instead of U can speed up the diffusive process by slowing down the movement of high-frequency modes and rescaling the trajectory length (FTHMC)

see arXiv:2502.05452, see also talk by S. Yamamoto (Fr)

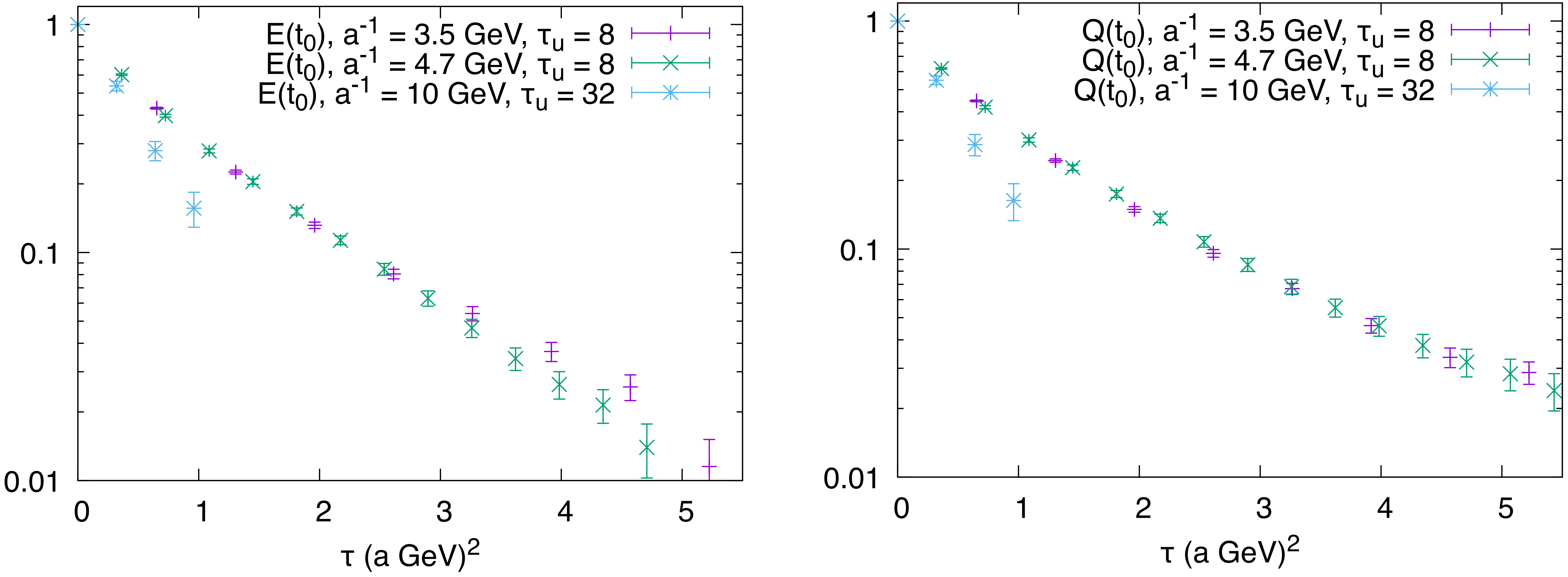
Addressing the tunneling aspect:

- Add bias potential V(Q) to the action with appropriately smeared Q to re-gain control over global topological sectors. Effect of bias potential can then be removed at analysis stage. (Parallel tempered metadynamics is only one option to do so.)

for bias potential, see arXiv:2307.04742

Diffusive scaling

- Let's look at FTHMC with stout smearing ($\rho = 0.12$), Iwasaki gauge, 2+1+1f Mobius DWF, physical m_s and m_c and $m_u = m_d = m_s$; dimensionless trajectory length is τ_u
- FTHMC and large τ_u approximately 3x faster than regular HMC, effects from FTHMC and large τ_u compound
- Autocorrelation (with stochastically local measure):

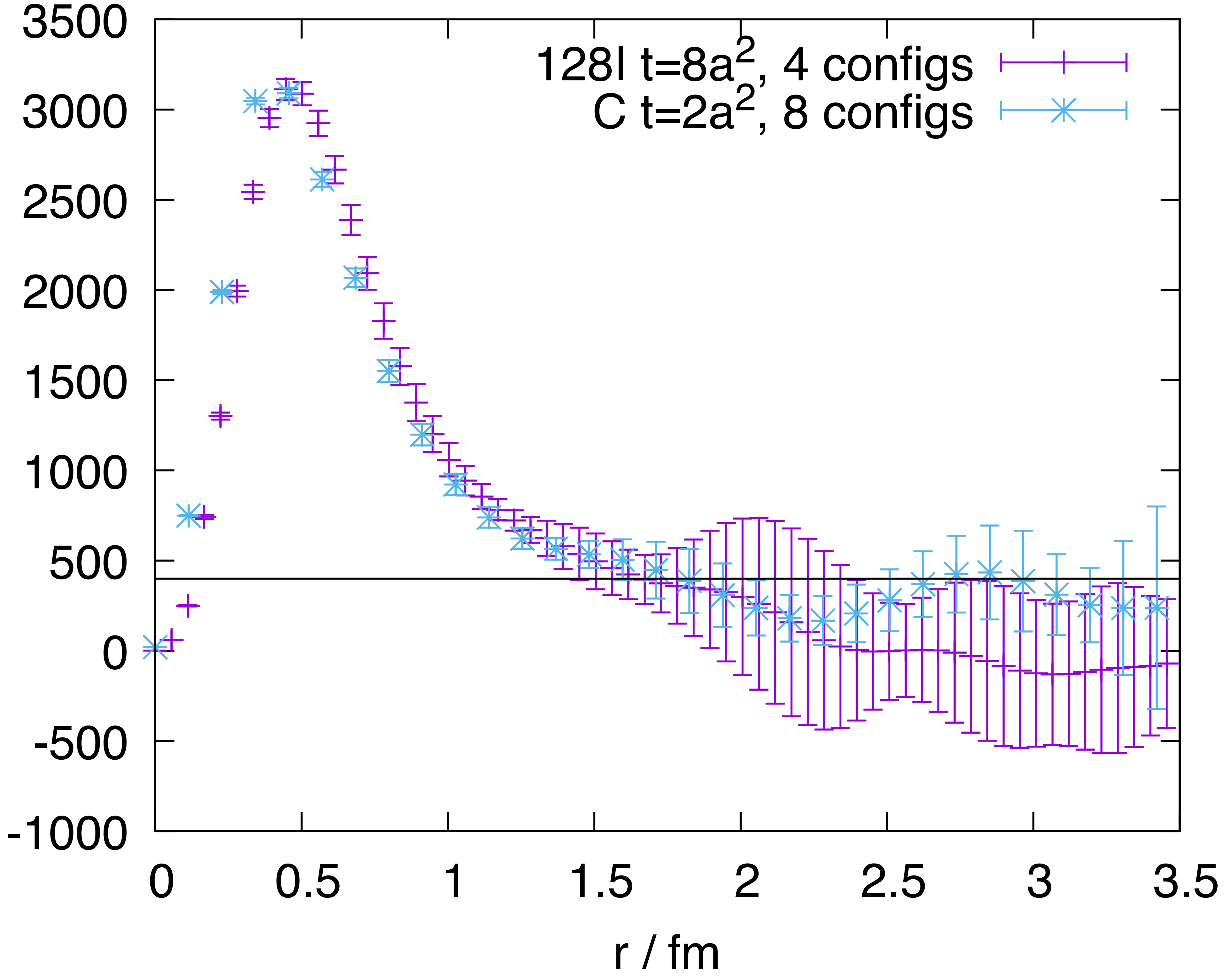


- With very large τ_u simulation at 10 GeV scales better than a^{-2}

Tunneling

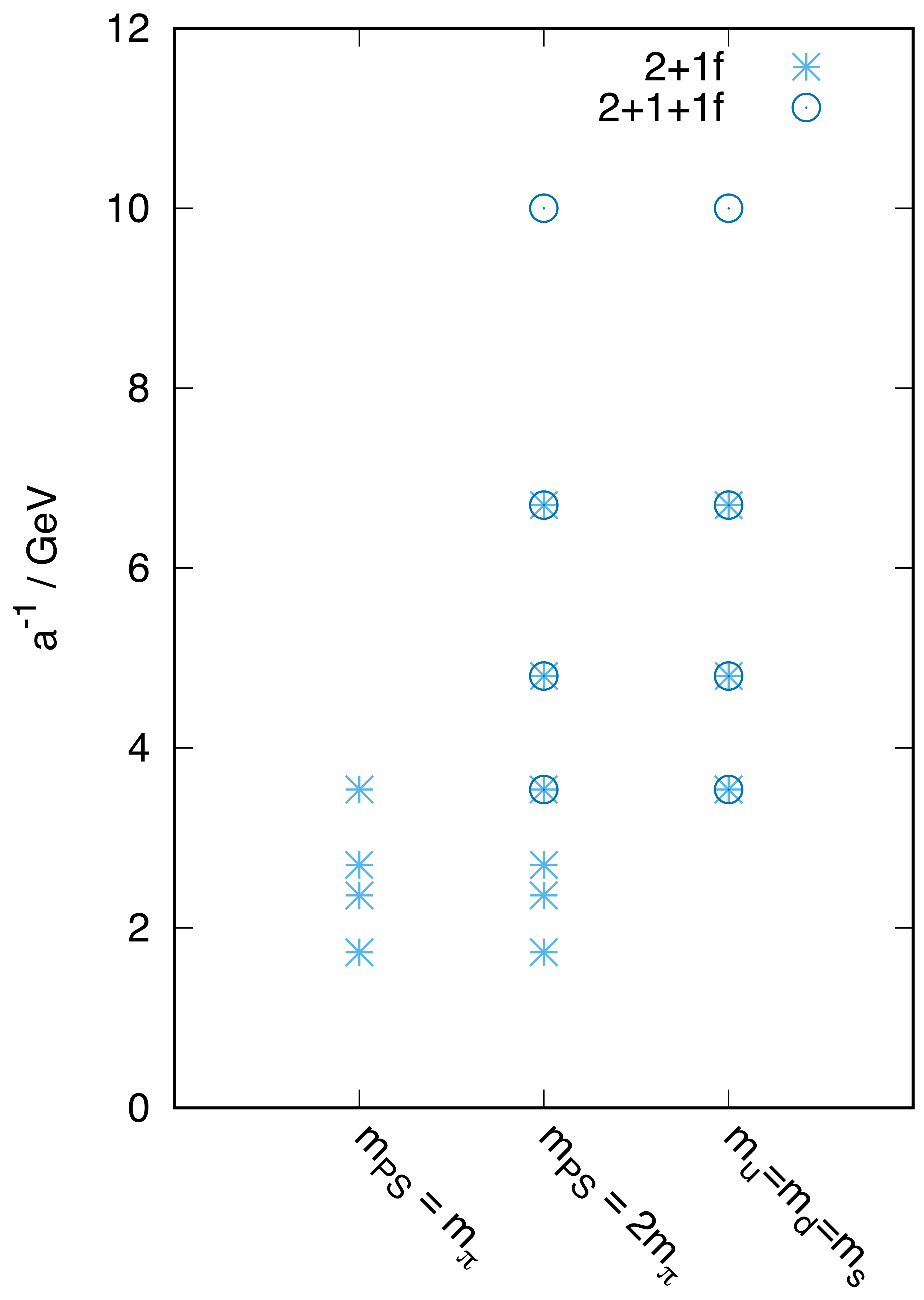
- Topological tunneling needs separate attention, problem is mild at large physical volumes
- We take control again of topological sectors by adding a bias potential V(Q) with GPT's automatic differentiation features; then perform separate streams in fixed sectors and combine them with locally determined susceptibility:

$$\langle Q^2 \rangle = \lim_{r\to\infty} \langle Q^2 \rangle_r \quad \text{with} \quad \langle Q^2 \rangle_r = \sum_{x,|\delta|<r} \langle Q(x) Q(x+\delta) \rangle$$



- Plot shows $\langle Q^2 \rangle_r$ for two ensembles after adjusting for their
  volume and lattice spacing
- C is a $64^3 \times 128$ physical pion mass ensemble at $a^{-1} = 1.73\ \mathrm{GeV}$
- 128I is a $128^3 \times 288$ physical pion mass ensemble at $a^{-1} = 3.54\ \mathrm{GeV}$

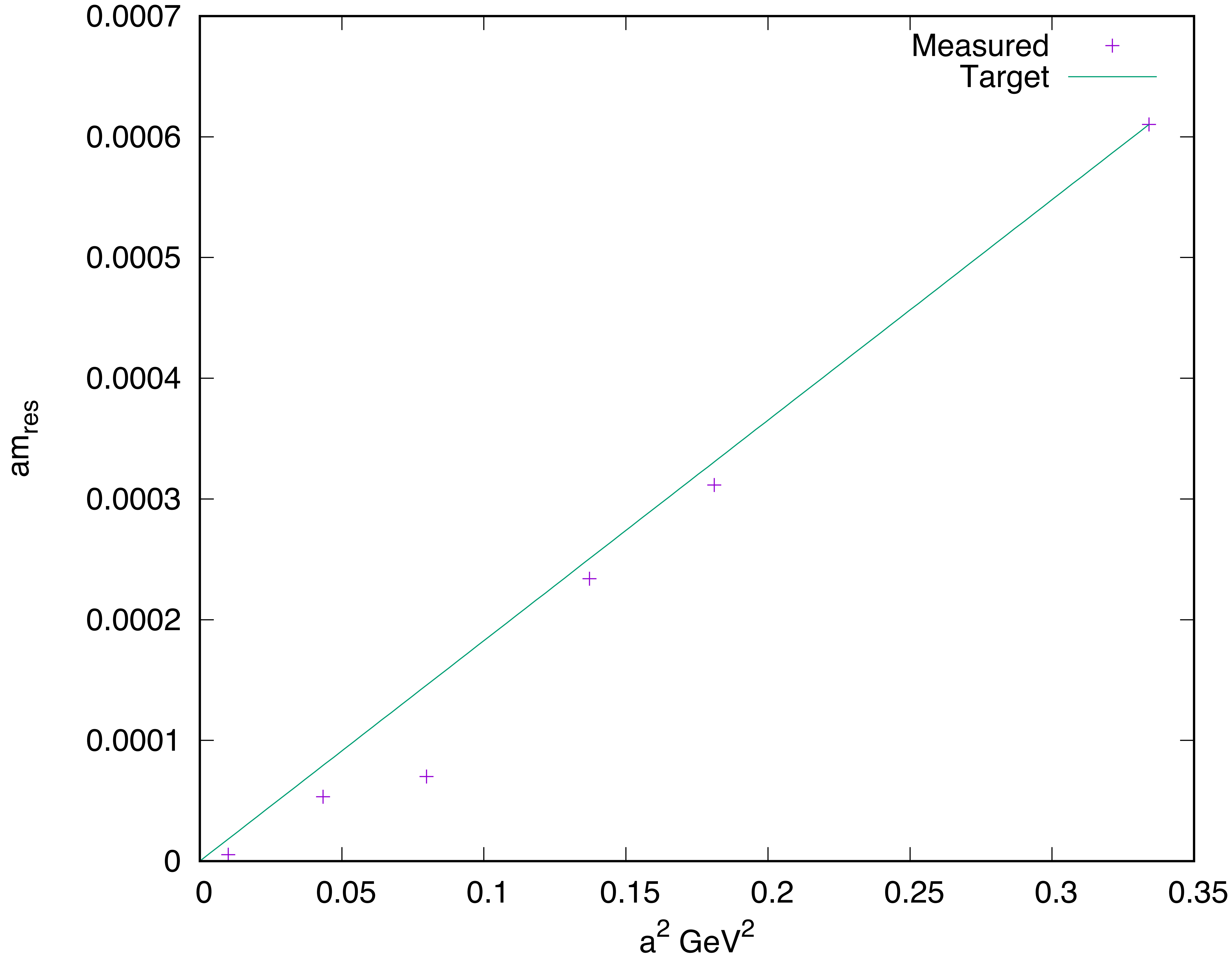
New 2+1 and 2+1+1 flavor ensembles



- Unitary charm for 2+1+1 flavor (M_5=1.8)
- Relativistic bottom possible for $a^{-1} > 5.5\ \mathrm{GeV}$
- First physics application to short-distance HVP window and τ decays
- New heavy quark physics program built on these new ensembles
- Large window of scales for factorization studies (including PDFs), perturbative QCD

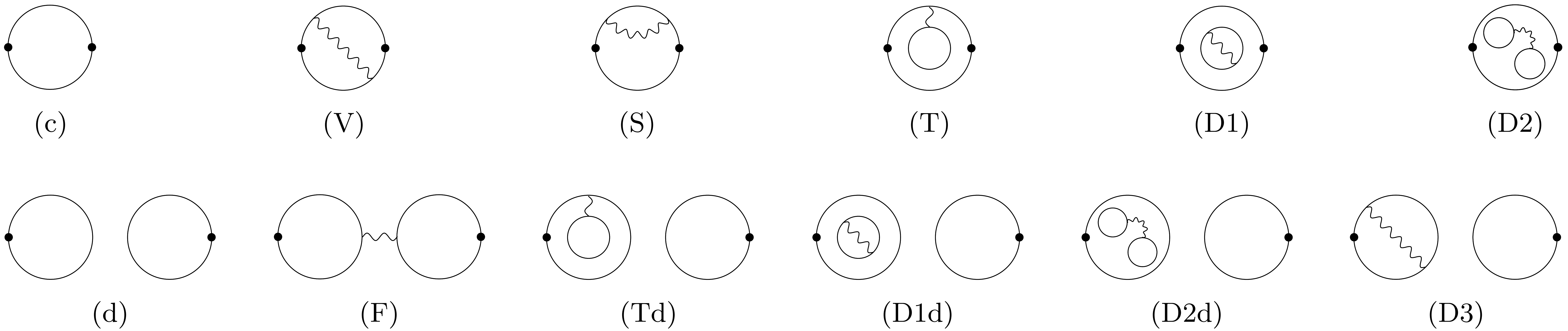
data produced at Aurora (< 10 GeV) and JUPITER = 10 GeV using GPT (production code at https://github.com/lehner/gpt)

Residual chiral symmetry breaking



- Tuned such that linear discretization effects are always negligible compared to quadratic ones
- Continuum effective theory of corresponding Overlap applies

Noisy QED diagrams



- Needed for muon g-2 HVP at desired precision, long-distance component not yet well studied
- Burger diagrams (D1) and (D1d) are a particular challenge (reweighting) that universally applies to all quantities at sufficiently high precision
- See Julian Parrino's talk for an update on all apart from (D1) and (D1d) and (D3)

1

Long-distance reconstruction of QED corrections to the hadronic vacuum polarization for the muon g-2

C. Lehner,[1,*] J. Parrino,[1] and A. Völklein[1]

[1] *Fakultät für Physik, Universität Regensburg, Universitätsstraße 31, 93040 Regensburg, Germany*

(Dated: September 1, 2025)

The long-distance contribution of QED corrections to the hadronic vacuum polarization is particularly challenging to compute in lattice QCD+QED. Currently, it is one of the limiting factors towards matching the precision of the recent result by the Fermilab E989 experiment for the muon g-2. In this work, we present a method for obtaining high-precision results for this contribution by reconstructing exclusive finite-volume state contributions. We find relations between the pion-photon contributions of individual diagrams and demonstrate the reconstruction method with lattice QCD+QED data at a single lattice spacing of $a^{-1} \approx 1.73$ GeV and $m_\pi \approx 275$ MeV.

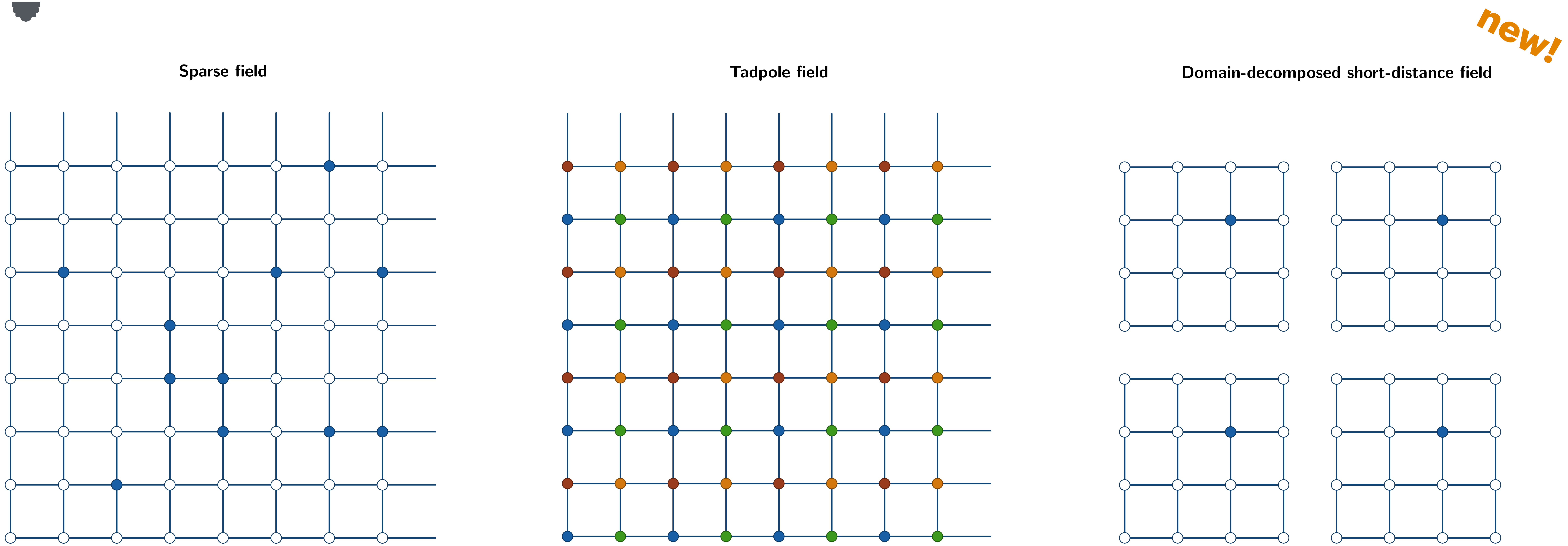
PACS numbers: 12.38.Gc

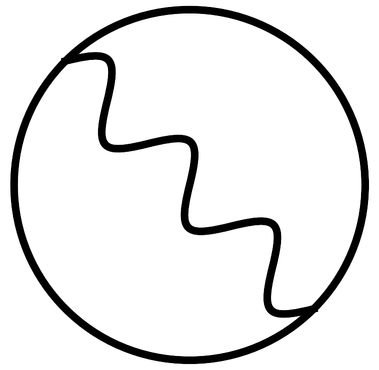
Phys. Rev. D 113 (2026) 5, 054503, arXiv:2508.21685

2

See C. Lehner talk at CERN on July 8, 2026 on how to simulate dynamical DWF QCD+QED with local vector currents (https://indico.cern.ch/event/1616325/)

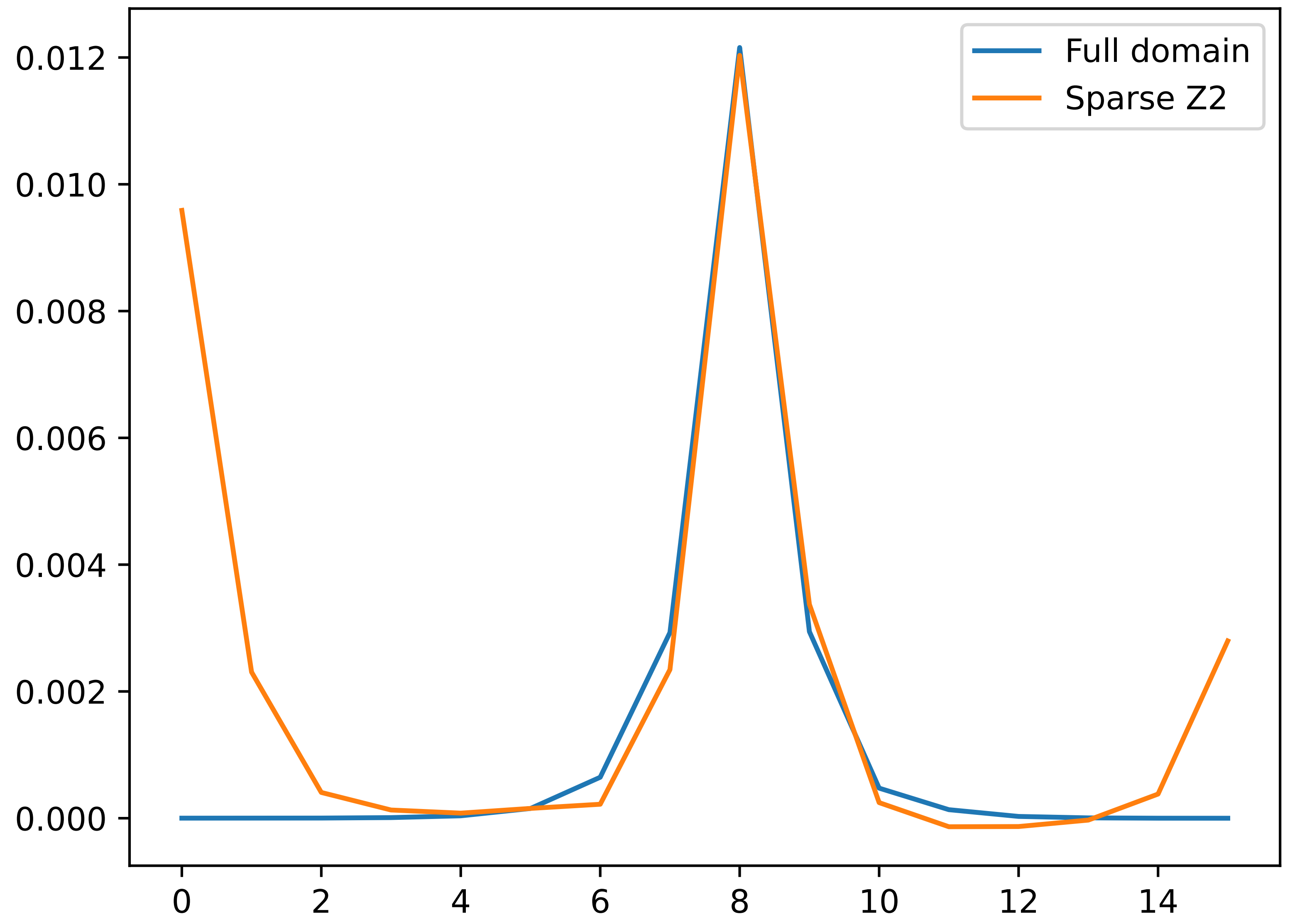
Domain decomposed short-distance fields

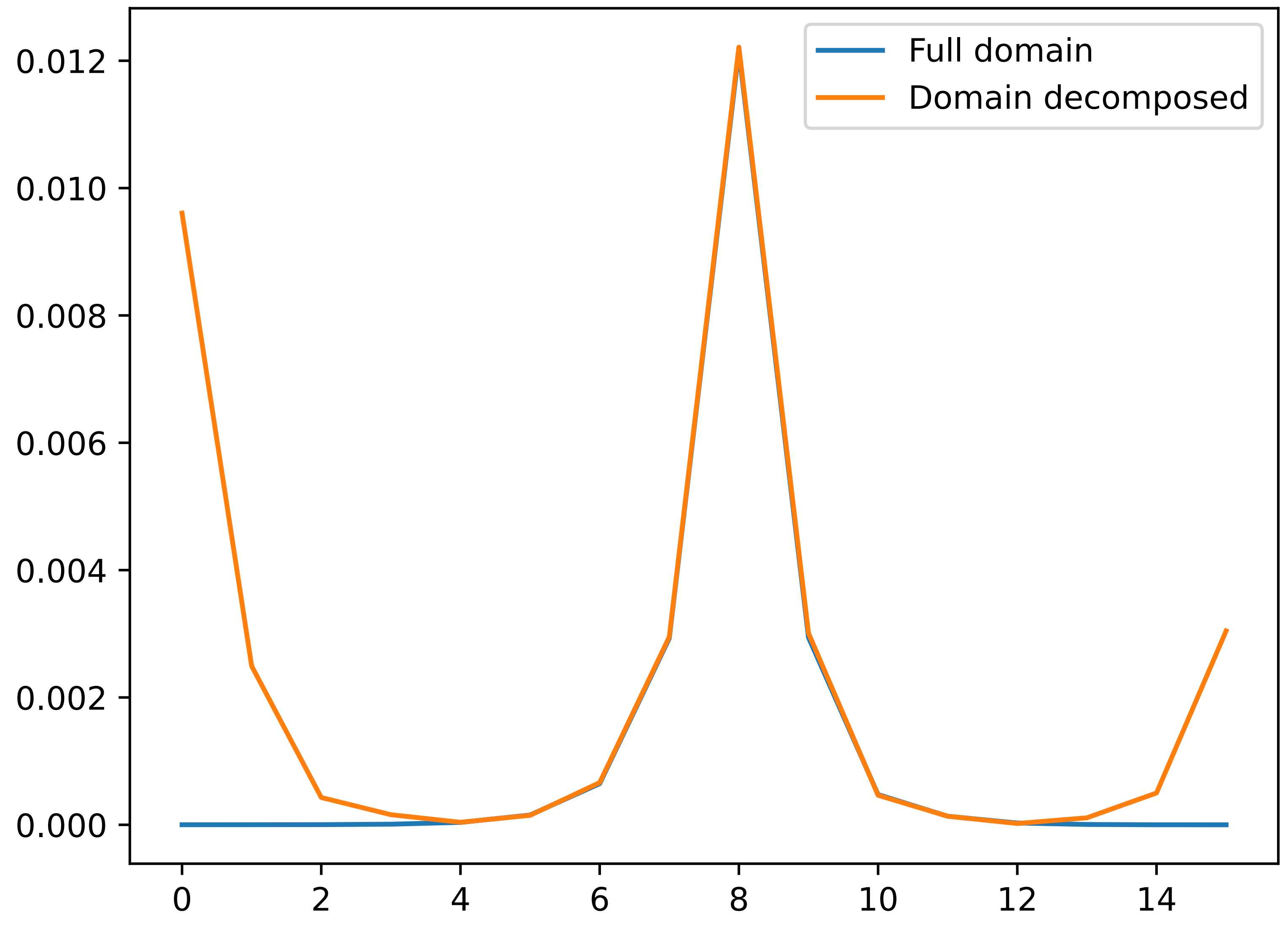




Idea: two vertices never far from each other, so quark propagators should not depend on gauge fields far away. Setup small (8^4) Dirichlet subvolumes that can all be solved at same time. For a $64^3 \times 128$ ensemble get 8192 source positions for cost of one propagator.

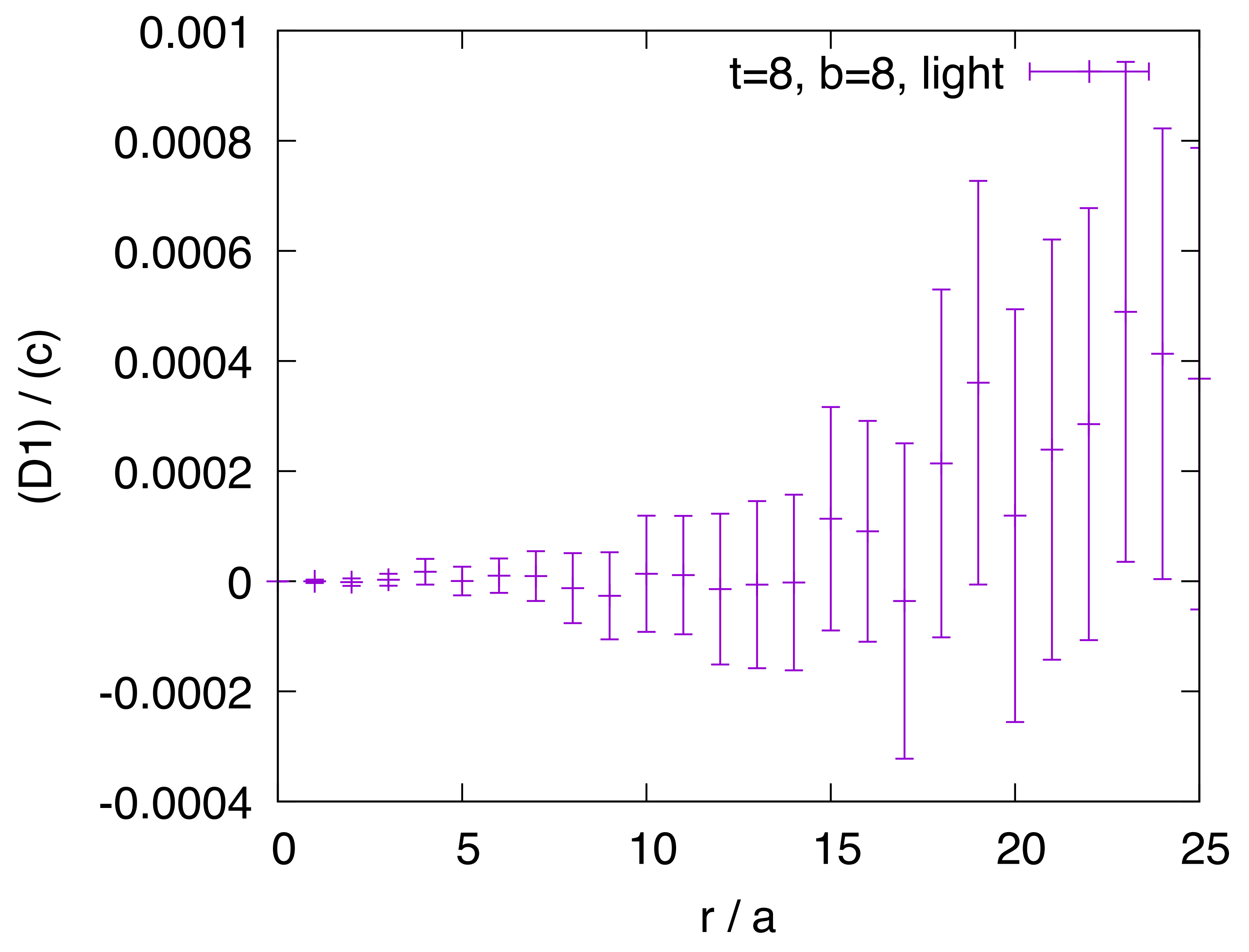
Why are domain decomposed Dirichlet boxes better than a sparse Z2 grid?

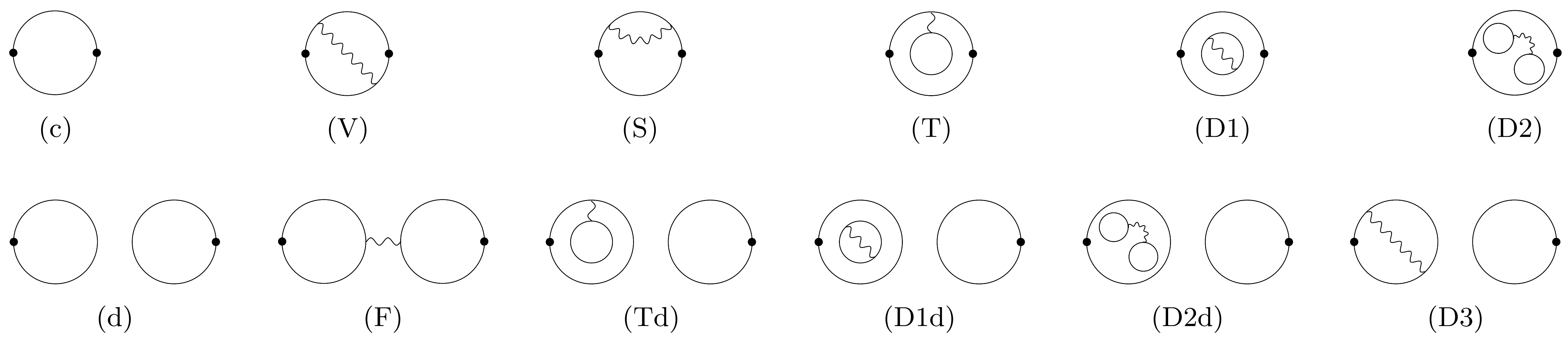




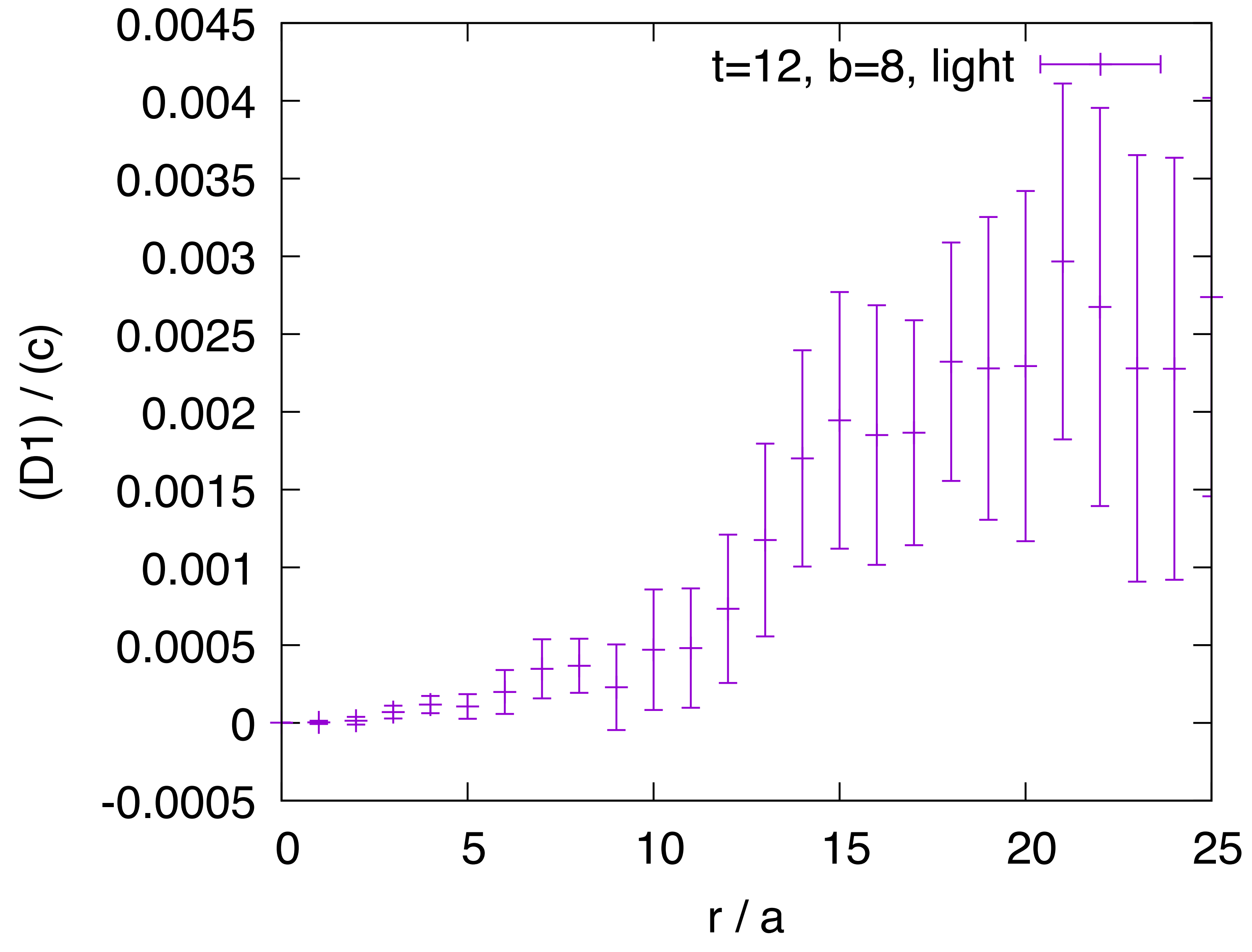
Spurious Z2 sources spoil crucial short-distance quark propagators! Domain decomposed short-distance fields do not have this issue.

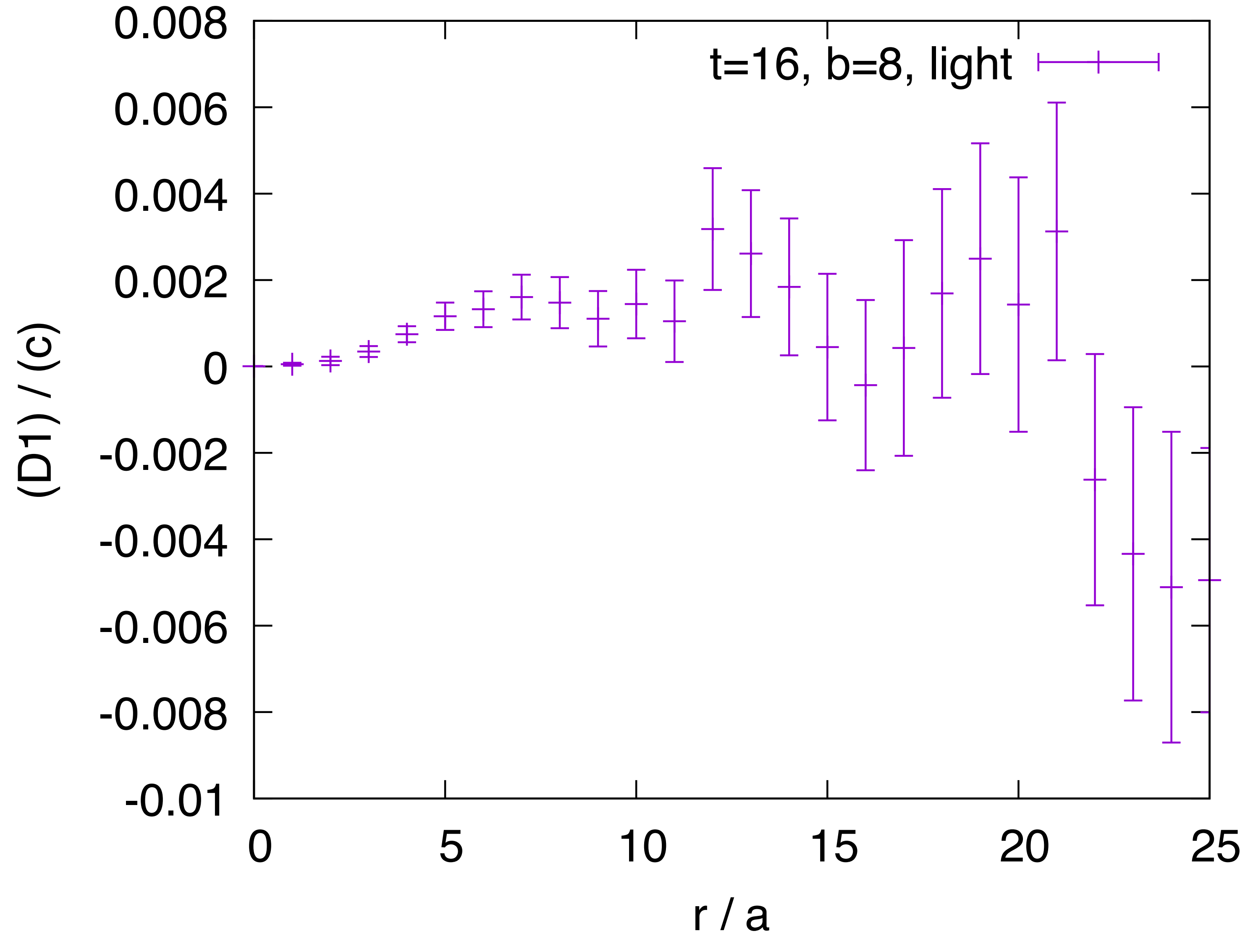
First results of (D1) with new method on large-volume physical pion mass ensemble (C)





Time separation between external vector currents (t), Dirichlet box size (b^4), distance between one external current and one burger current (r)





Improved variational approach

So far, we fared well with exact distillation based on a particularly smeared-gauge-field Laplace operator's eigenmodes:

PHYSICAL REVIEW LETTERS **134,** 201901 (2025)

Editors' Suggestion

Long-Distance Window of the Hadronic Vacuum Polarization for the Muon $g-2$

T. Blum,[1] P. A. Boyle,[2,3] M. Bruno,[4,5] B. Chakraborty,[6] F. Erben,[7] V. Gülpers,[3] A. Hackl,[8] N. Hermansson-Truedsson,[3] R. C. Hill,[3] T. Izubuchi,[2,9] L. Jin,[1] C. Jung,[2] C. Lehner,[8,*] J. McKeon,[6] A. S. Meyer,[10] M. Tomii,[1,9] J. T. Tsang,[7] and X.-Y. Tuo[2]

(RBC and UKQCD Collaborations)

[1]*Physics Department, University of Connecticut, Storrs, Connecticut 06269-3046, USA*
[2]*Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA*
[3]*School of Physics and Astronomy, The University of Edinburgh, Edinburgh EH9 3FD, United Kingdom*
[4]*Dipartimento di Fisica, Università di Milano-Bicocca, Piazza della Scienza 3, I-20126 Milano, Italy*
[5]*INFN, Sezione di Milano-Bicocca, Piazza della Scienza 3, I-20126 Milano, Italy*
[6]*School of Physics and Astronomy, University of Southampton, Southampton SO17 1BJ, United Kingdom*
[7]*CERN, Theoretical Physics Department, Geneva, Switzerland*
[8]*Fakultät für Physik, Universität Regensburg, Universitätsstraße 31, 93040 Regensburg, Germany*
[9]*RIKEN-BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA*
[10]*Nuclear and Chemical Sciences Division, Lawrence Livermore National Laboratory, Livermore, California 94550, USA*

(Received 31 October 2024; revised 26 March 2025; accepted 22 April 2025; published 19 May 2025)

We provide the first *ab initio* calculation of the Euclidean long-distance window of the isospin-symmetric light-quark connected contribution to the hadronic vacuum polarization for the muon $g-2$ and find $a_\mu^{\mathrm{LD,iso,conn,ud}} = 411.4(4.3)(2.4)\times 10^{-10}$. We also provide the currently most precise calculation of the total isospin-symmetric light-quark connected contribution, $a_\mu^{\mathrm{iso,conn,ud}} = 666.2(4.3)(2.5)\times 10^{-10}$ in the BMW20 world, which is more than 4σ larger compared to the data-driven estimates of Boito *et al.* [Phys. Rev. D **107**, 074001 (2023)] and 1.7σ larger compared to the lattice QCD result of BMW20.

DOI: 10.1103/PhysRevLett.134.201901

PHYSICAL REVIEW D **113,** 114502 (2026)

Spectral analysis for nucleon-pion and nucleon-pion-pion states in both parity sectors using distillation with domain-wall fermions

A. Hackl* and C. Lehner

Fakultät für Physik, Universität Regensburg, Universitätsstraße 31, 93040 Regensburg, Germany

(Received 29 October 2025; accepted 17 April 2026; published 2 June 2026)

We present a study using the distillation method to analyze the spectra of nucleon, nucleon-pion, and nucleon-pion-pion states in the positive-parity sector, as well as nucleon and nucleon-pion states in the negative-parity sector. The study uses seven domain-wall fermion ensembles with varying pion masses ($m_\pi = 139$–279 MeV), lattice spacings ($a^{-1} = 1.730$ GeV and $a^{-1} = 2.359$ GeV) and volumes ($m_\pi L = 3.8$–7.5). To address the large number of contractions in this project, we implemented an algorithm to automate the contraction of nucleon-pion correlation functions that contain an arbitrary number of pions. In the positive-parity sector, we extrapolate the nucleon mass to the physical point. This study demonstrates the effectiveness of the distillation method for baryonic quantities with a focus on multihadronic states and establishes a foundation for future work.

DOI: 10.1103/lmtl-c92f

But points to optimize:

- more efficient creation of operators with a smaller smearing radius
- Laplace eigenmodes not translationally covariant, so distillation matrix space not optimized for well-defined momentum operators, this becomes important for baryons where tensor contractions are no longer of matrix-matrix and trace kind

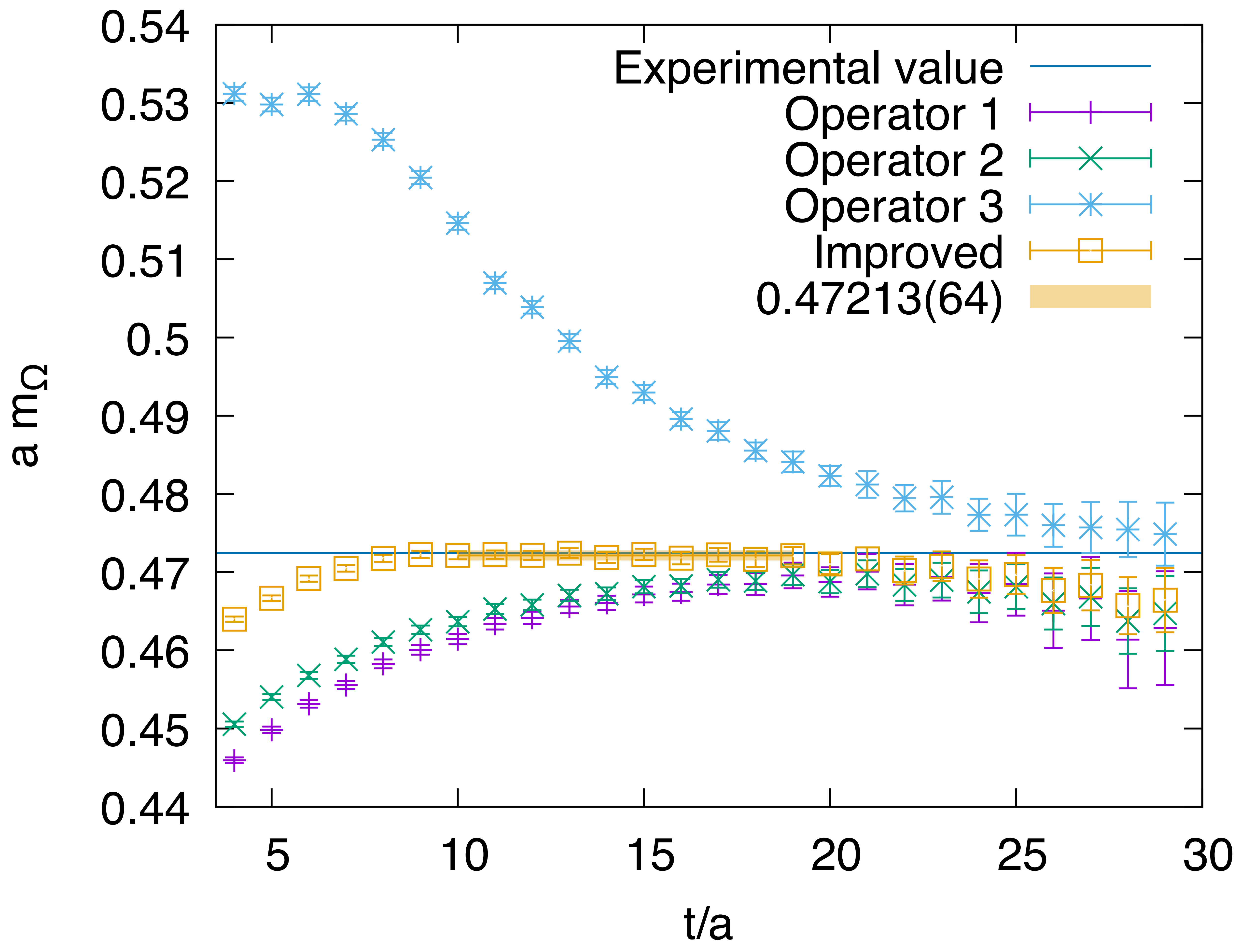
Improved (transl. cov.) variational approach

New idea:

- Translationally covariant Coulomb-Gauge-Fixed momentum sources with fixed spatial momentum
- Employ similar smearing before gauge fixing that we had used for exact distillation
- Straightforward to localize operators in particular dimension by only picking subset of all momentum modes
- Drastically cuts the relevant subspace which reduces contraction costs

First applications:

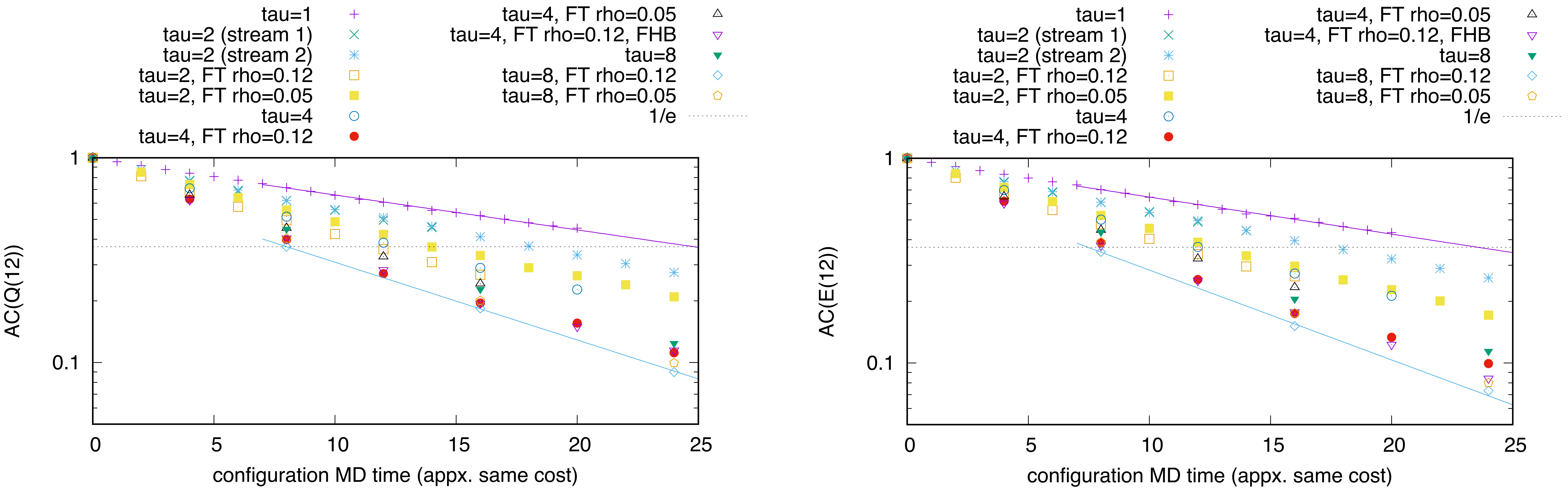
- Omega baryon mass on new 128I physical pion mass ensemble at 3.54 GeV cutoff

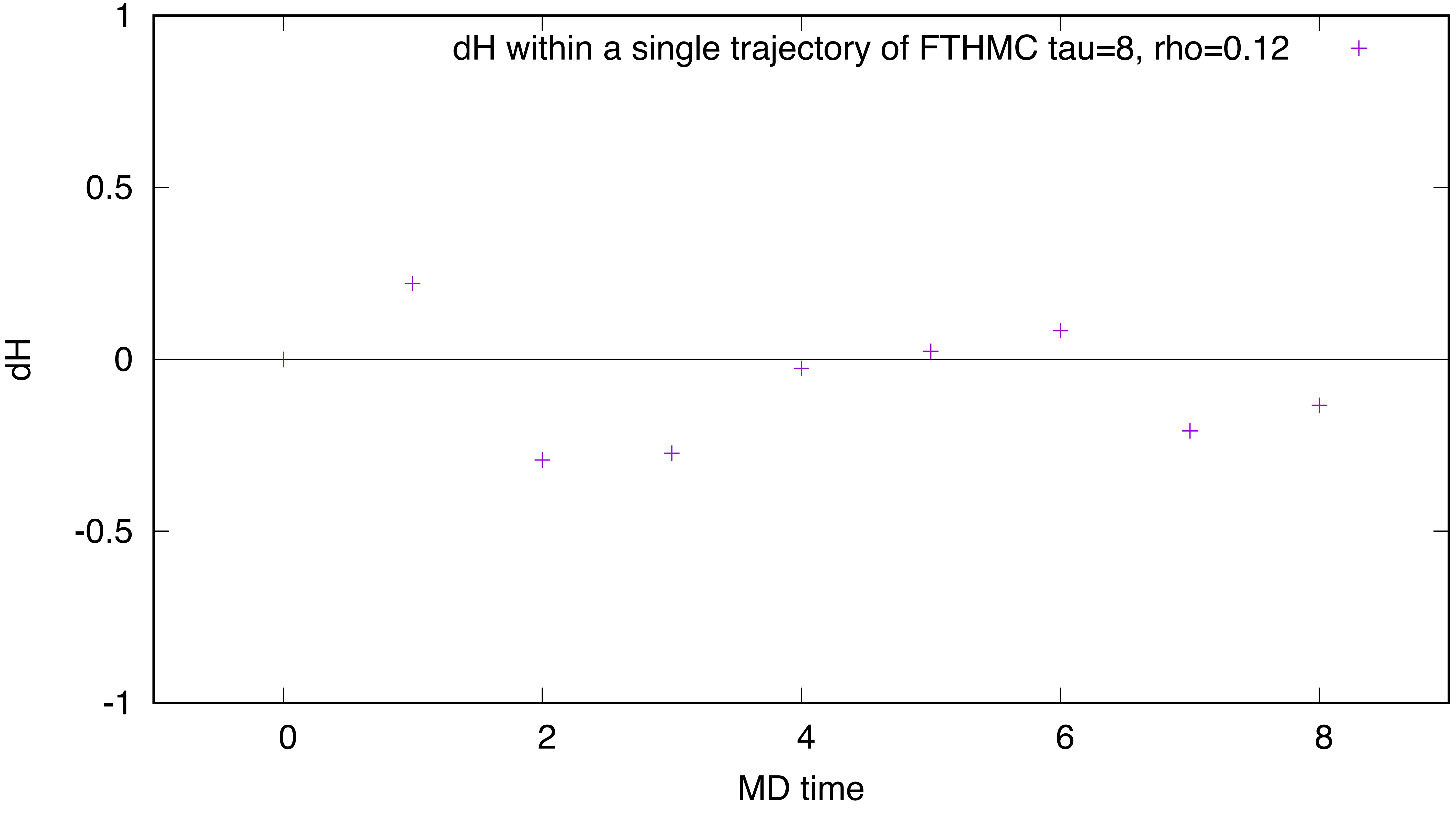


Summary

- Setting up a single consistent data set that supports a diverse high-precision QCD program with DWF is challenging.
- With new ultrafine ensembles up to 10 GeV cutoff, new sampling techniques, and a new optimized variational approach we are ready for the next precision frontier.

Backup slides


tau=1
tau=2 (stream 1)
tau=2 (stream 2)
tau=2, FT rho=0.12
tau=2, FT rho=0.05
tau=4
tau=4, FT rho=0.12
tau=4, FT rho=0.05
tau=4, FT rho=0.12, FHB
tau=8
tau=8, FT rho=0.12
tau=8, FT rho=0.05
1/e
AC(Q(12))
AC(E(12))
1
0.1
0
5
10
15
20
25
configuration MD time (appx. same cost)


dH within a single trajectory of FTHMC tau=8, rho=0.12
dH
MD time
1
0.5
0
-0.5
-1
0
2
4
6
8