Nonperturbative Hadronic Structure of the Photon

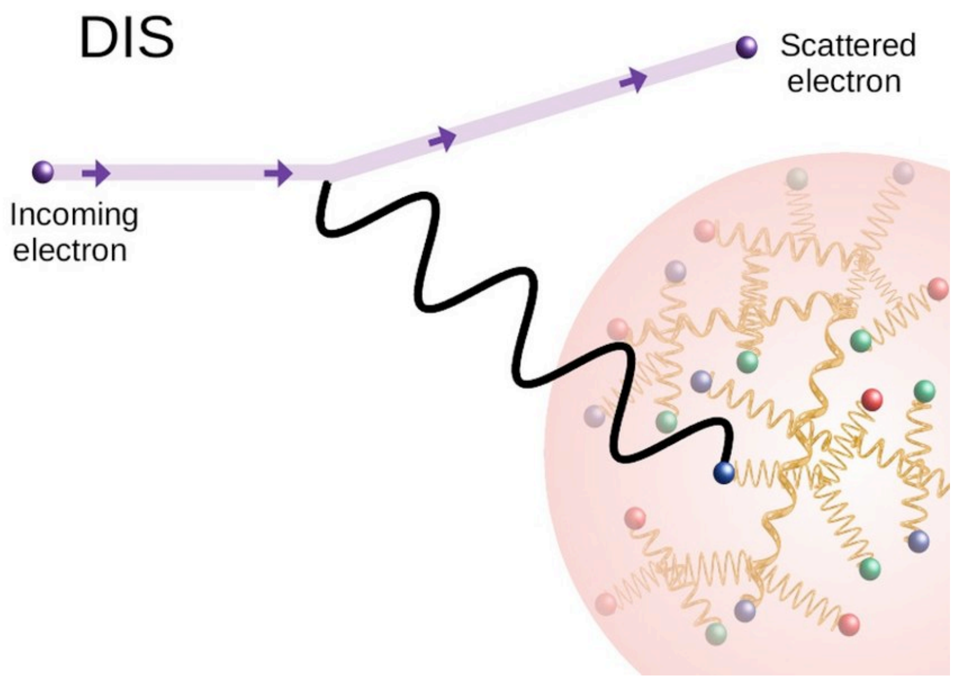
Raza Sabbir Sufian

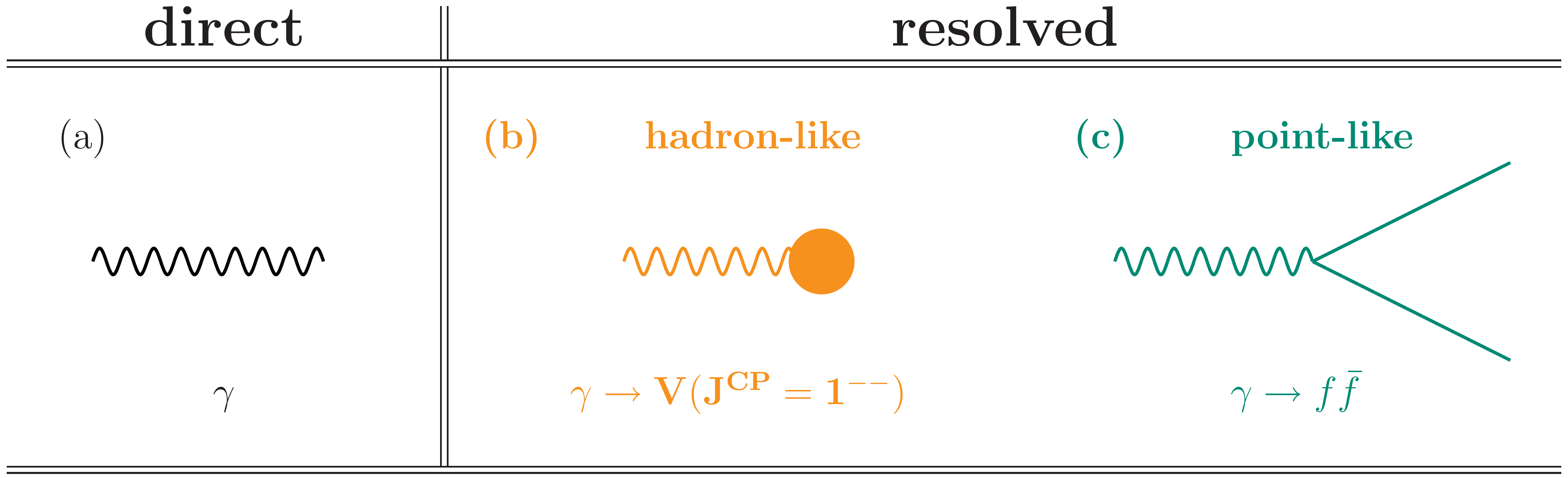
In collaboration with Taku Izubuchi, Luchang Jin, Tanjib Khan, Masaaki Tomii

Funding acknowledgment:

Hadronic structure of the “resolved” photon from LQCD

- QCD allows photon to exhibit a hadronic structure due to quantum fluctuations





- $d\sigma = d\sigma_{\mathrm{direct}} + d\sigma_{\mathrm{resolved}}$

- Well controlled parametrization when photon appears as an active, partonic constituent of the proton.
  - Not so for “resolved” photon’s parton contents

QCD factorization

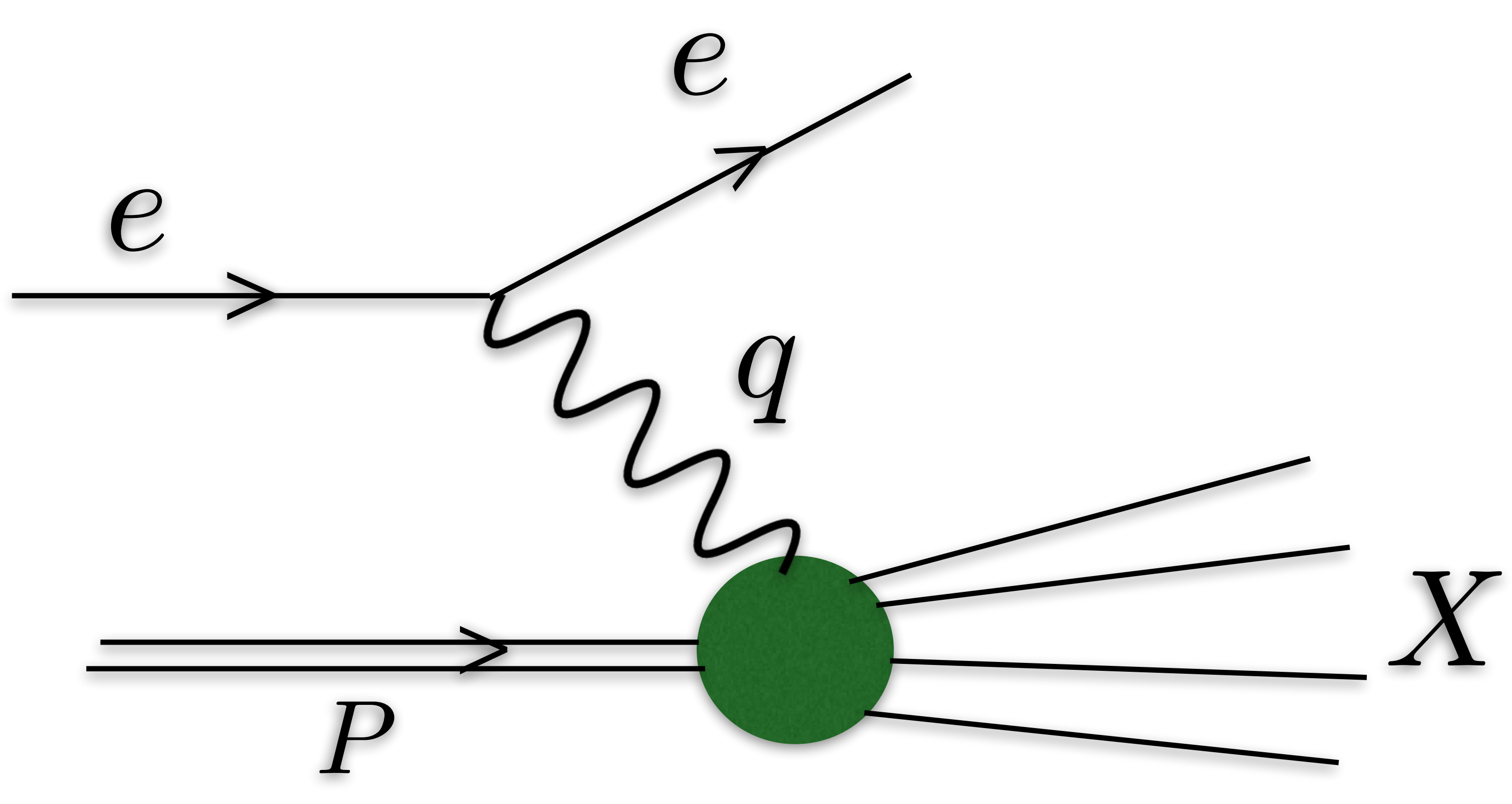
- QCD factorization is the cornerstone that connects hadronic scattering measurements into quantitative information on the quark and gluon structure of matter.

$$\boxed{\sigma^{DIS}(x, Q^2, \sqrt{s})} = \sum_{\alpha=q,\bar{q},g} K_\alpha\left(x, \frac{Q^2}{\mu^2}, \sqrt{s}\right) \otimes f_\alpha(x, \mu^2) + \text{Power Corrections}$$

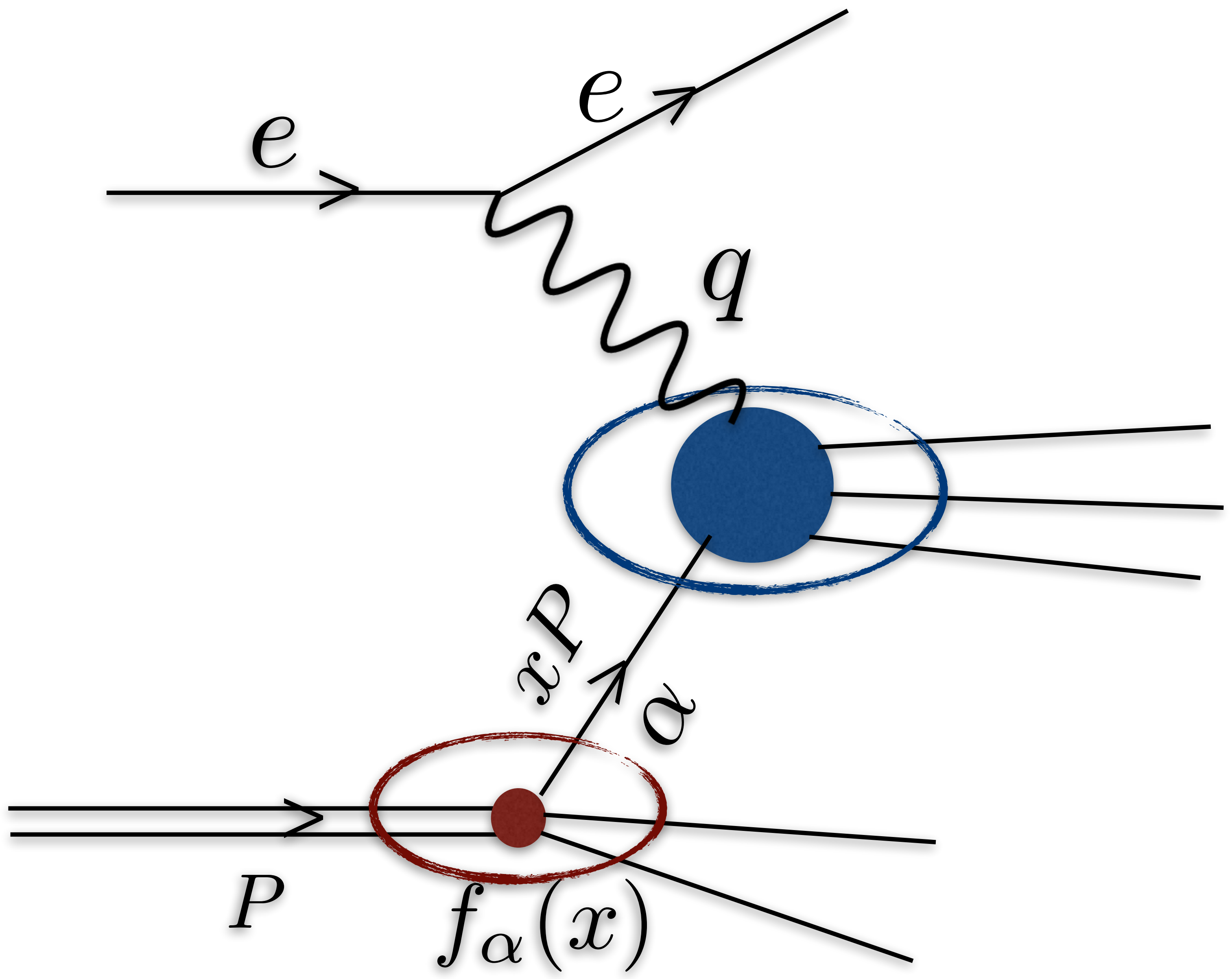
Experimental cross sections
(Lattice QCD matrix elements)

Short-distance physics (pen and paper)

Nonperturbative long-distance physics



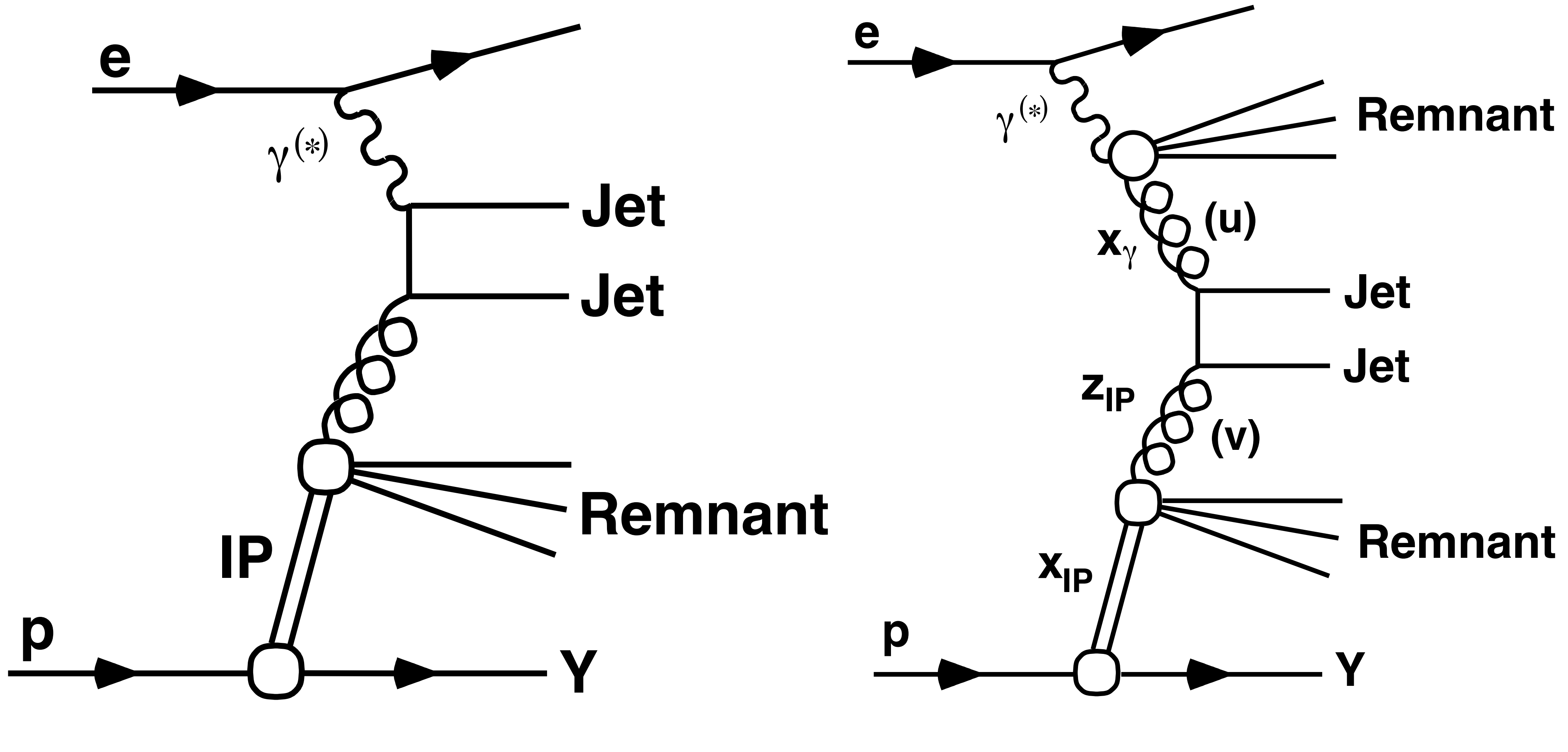
Scattering Process



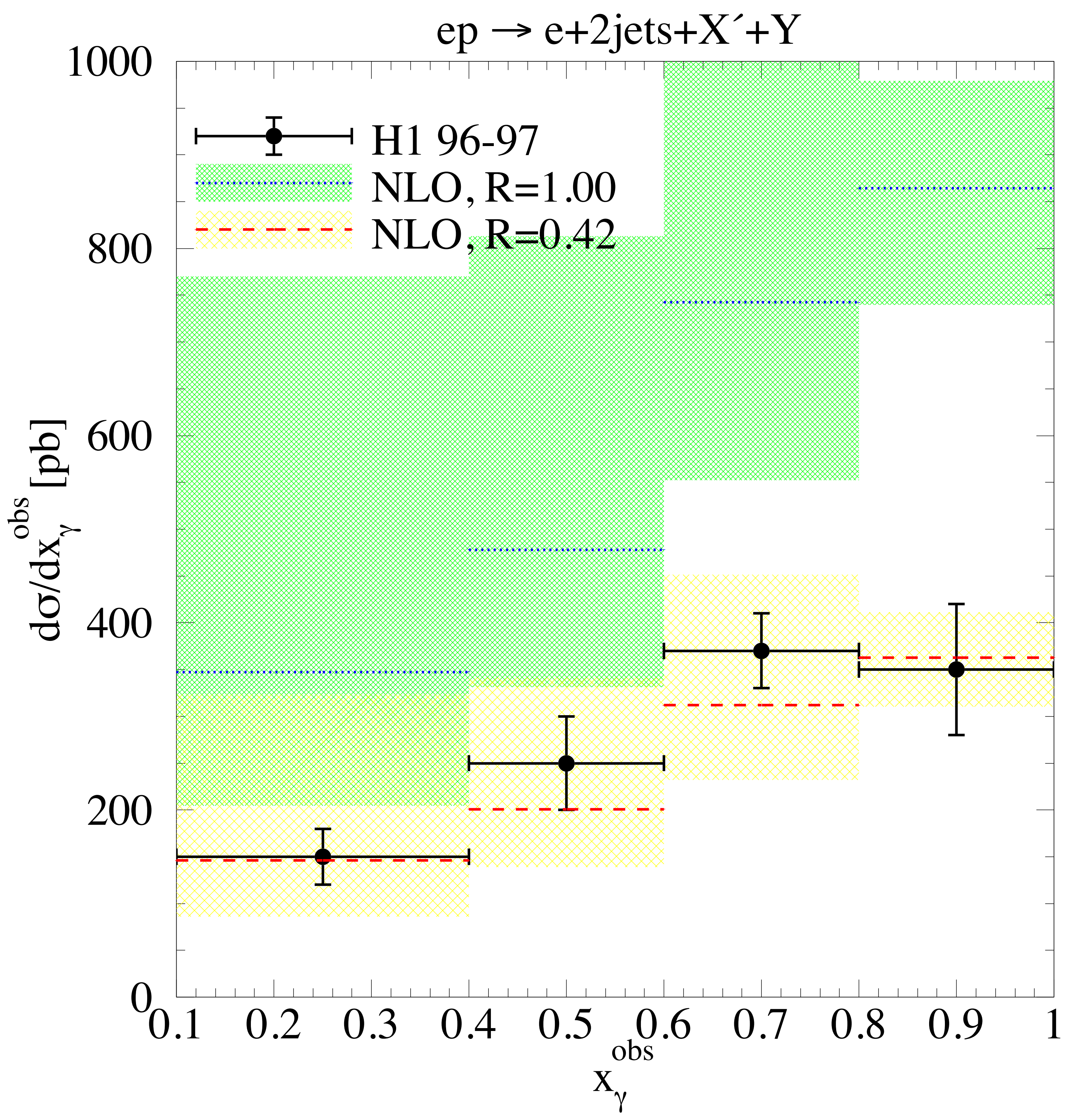
Parton Picture

Hadronic structure of the "resolved" photon from LQCD

- Test of QCD factorization "breaking" (or lack of understanding of resolved photon structure)



Differential cross sections for dijet photo production measured by H1 and ZEUS [Phys. Lett. 2008]



- Critically important for EIC and constraining gluon PDF [Chu, Aschenauer, Lee 2017]

- Experiments are only able to probe the structure of quasi-real or virtual photons, extrapolating to real-photon structure involves significant complications and systematic uncertainties

Hadronic structure of the "resolved" photon from LQCD

- LQCD calculation of moment

X. Ji, C. Jung [PRL 2001, PRD 2001]

- Using LSZ reduction formula
  - $$\langle\gamma(k\lambda')|\hat{O}(0)|\gamma(k\lambda)\rangle = -\lim_{k'^\mu\to k^\mu}\int d^4x d^4y e^{-ik'(y-x)} k'^2 \langle 0|T\epsilon^*(\lambda')\cdot A(x)\ \hat{O}(0)\ \epsilon(\lambda)\cdot A(y)|0\rangle k'^2$$

- Evaluate electromagnetic part of the Green's function in the lowest-order perturbation theory and write the matrix element

$$\langle\gamma(k\lambda')|\hat{O}(0)|\gamma(k\lambda)\rangle = -e^2\int d^4x d^4y e^{-ik(y-x)}\epsilon^{*\mu}(\lambda')\langle 0|TJ_\mu(x)\hat{O}(0)J_\nu(y)|0\rangle\epsilon^\nu(\lambda)$$

  - Minkowski space correlation function

- Our goal is to calculate quasi-PDF matrix elements associated with the photon's hadronic structure
  - Quasi-PDF formalism

X. Ji [PRL 2013]

Hadronic structure of the “resolved” photon from LQCD

- Quasi-PDF hadronic matrix element with a photon state

$$h_f(z) = \langle \gamma(\vec{p}) | O_f(z,0) \exp\Big(-iq_f e \int_0^z A_\mu(t) dt_\mu\Big) | \gamma(\vec{p}) \rangle$$

$$O_f(z,0) = Z(z)\, \bar{q}_f(z)\, \gamma_t\, U(z,0)\, q_f(0)$$

- Including both QCD and QED

  - $$h_f(z,p_z) \to \sum_{\lambda=x,y} \sum_{\vec{w},\vec{v}} e^{i\vec{p}\cdot(\vec{w}-\vec{v})} \langle T A_\lambda(T,\vec{v}) O(z,0) \exp\Big(-iq_f e \int_0^z A_\mu(t) dt_\mu\Big) A_\lambda(-T,\vec{w}) \exp\Big(-ie \int A_\mu(x) J_\mu(x) d^4x\Big) \rangle$$

- After integrating out QED part:

$$h(z) = \frac{1}{2}(-ie)^2 \int d^4x \int d^4y \sum_{\lambda=x,y} (e^{|\vec{p}|x_t} e^{-i\vec{p}\cdot\vec{x}} - 1)(e^{-|\vec{p}|y_t} e^{i\vec{p}\cdot\vec{y}} - 1) \langle T J_\lambda(x) O_u(z,0) J_\lambda(y) \rangle_{\mathrm{QCD}}$$

  - Calculable on the lattice

Lattice ensembles

- RBC/UKQCD domain wall fermion

Ensemble-1: $16^3 \times 32\,(a \approx 0.12\,\mathrm{fm})$

Ensemble-2: $32^3 \times 64\,(a \approx 0.063\,\mathrm{fm})$

Ensemble-3: $32^3 \times 64\ (a \approx 0.088\ \mathrm{fm})$

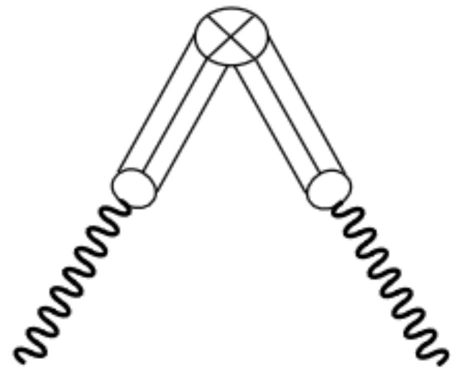
- Light- and strange-quark propagators already generated by the RBC and UKQCD collaborations with O(1,000) randomly chosen point sources per configuration

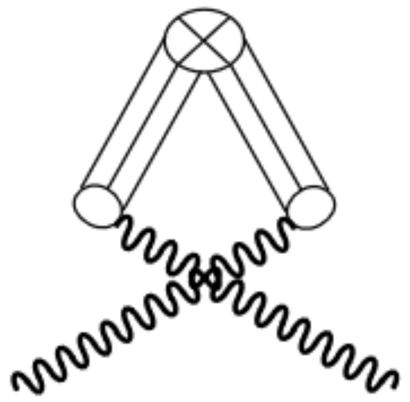
- ~50 configurations per ensemble

- $m_\pi \approx 450 - 330\mathrm{MeV}$

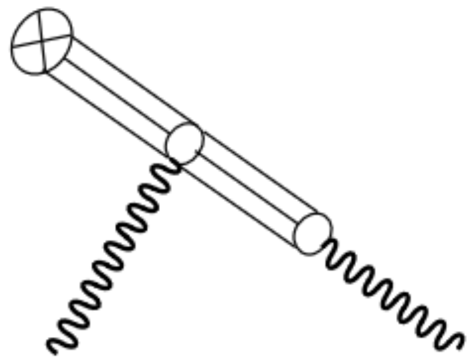
Qlattice: https://github.com/jinluchang/Qlattice

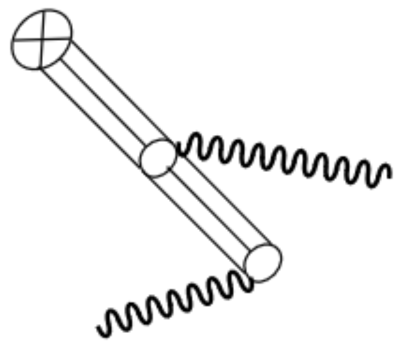
- Different diagrams


a)

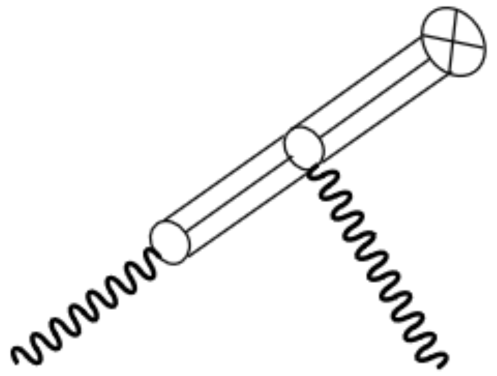

b)

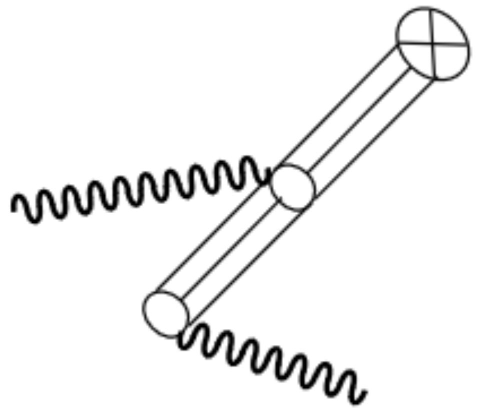
- The incoming photon converts through J_1 into a QCD eigenstate $|n\rangle$.
  The operator acts between hadronic states and the second current converts the hadronic state back into the outgoing photon.


c)


d)

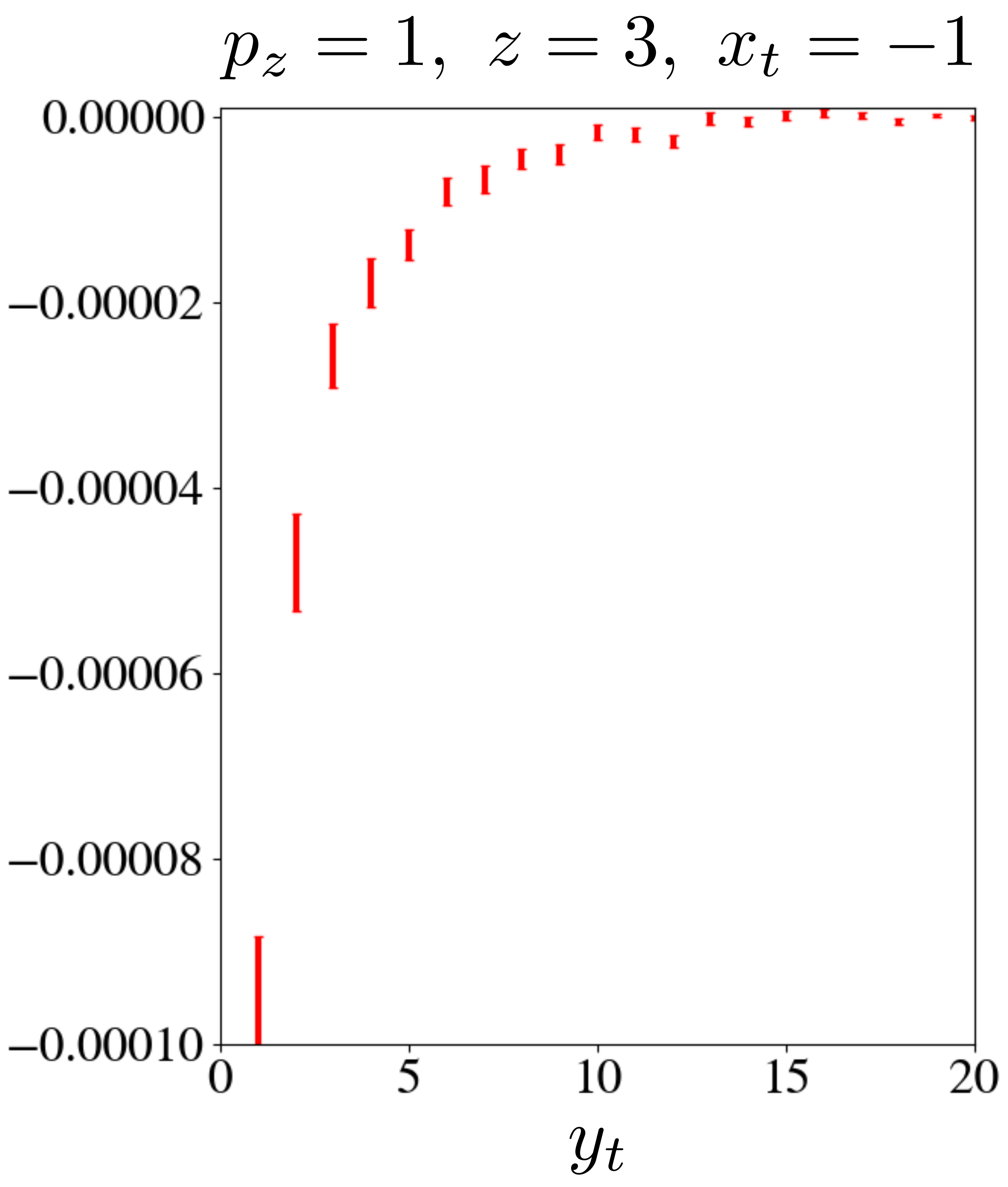
- The operator first acts on the vacuum sector. Subsequent electromagnetic interactions generate and then remove the hadronic component associated with the photon.

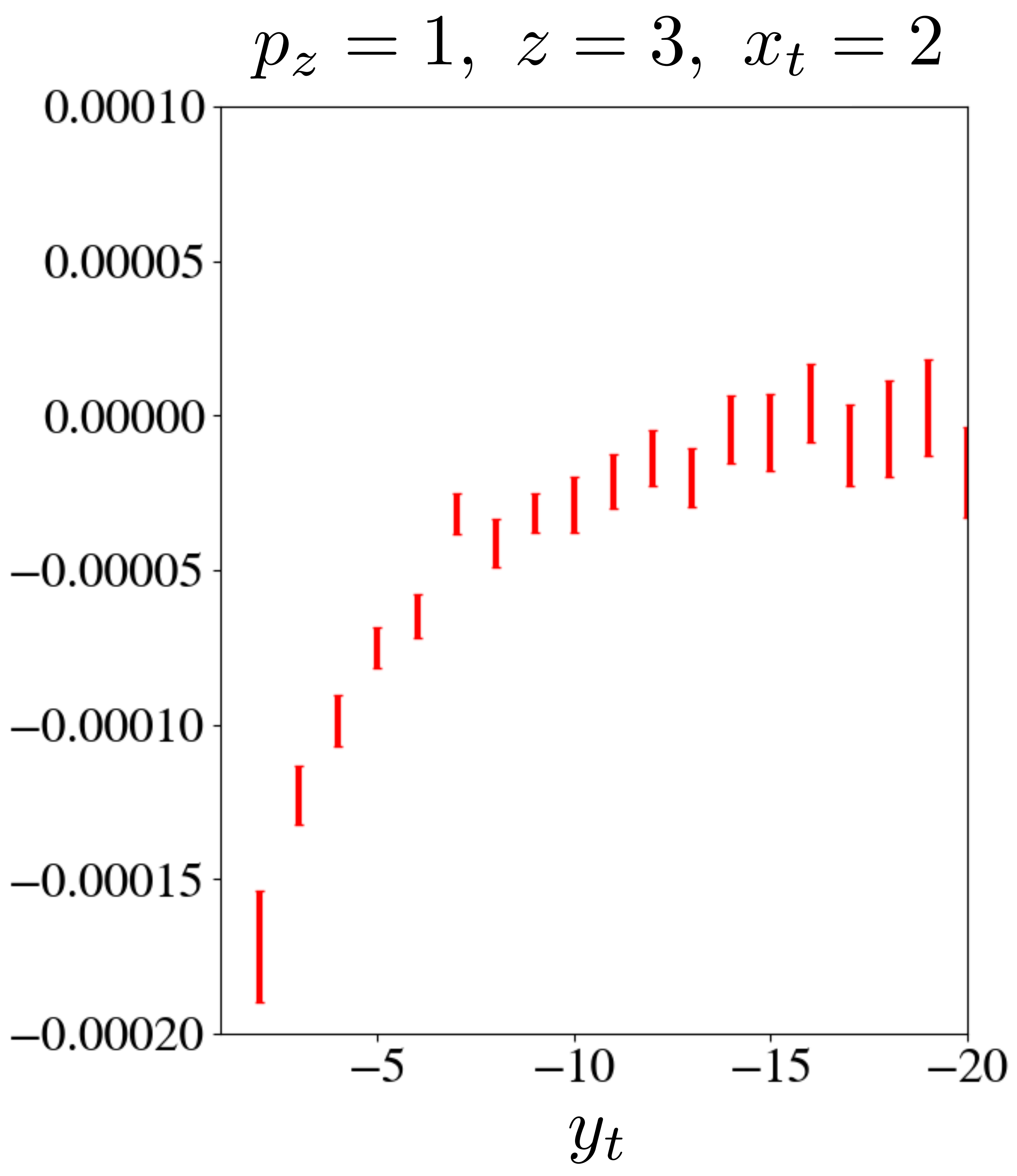

e)


f)

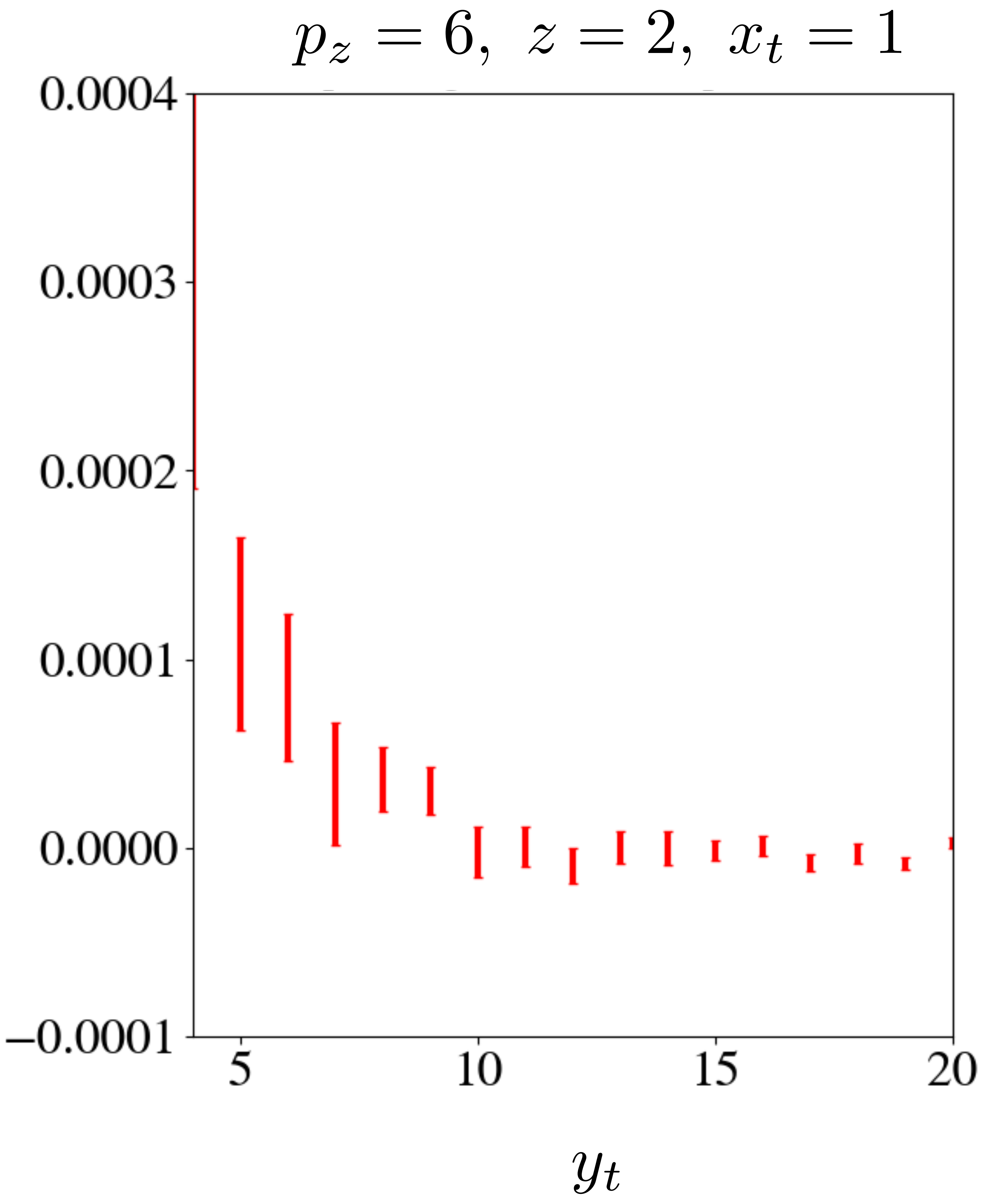
- The incoming photon interacts twice before the operator insertion. At the time when the operator acts, the state can contain a QCD excitation and/or a photon-hadron mixture generated by the two electromagnetic interactions.

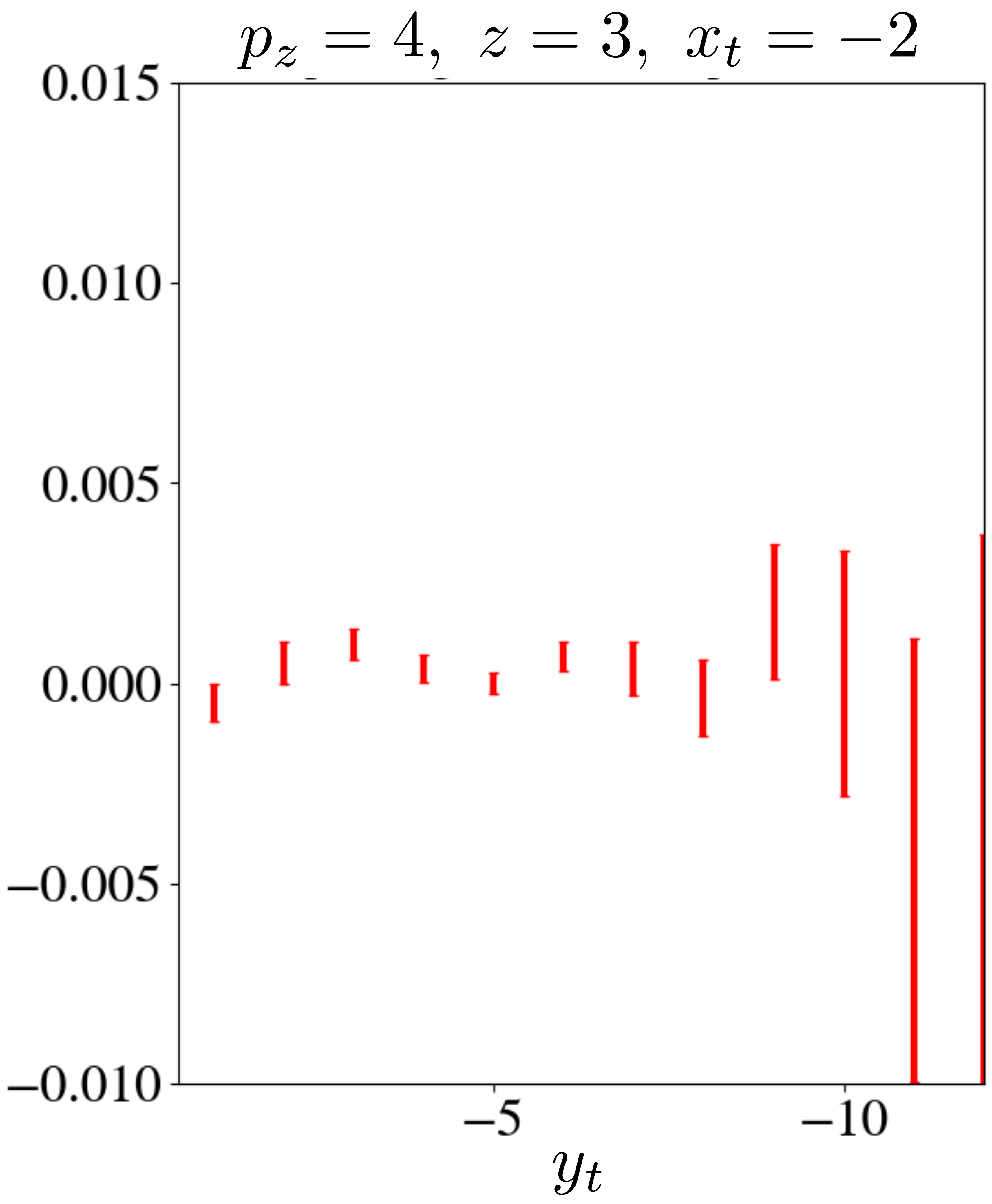
- Ensemble-3: $32^3 \times 64 \ (a \approx 0.088 \text{ fm})$





Preliminary matrix elements

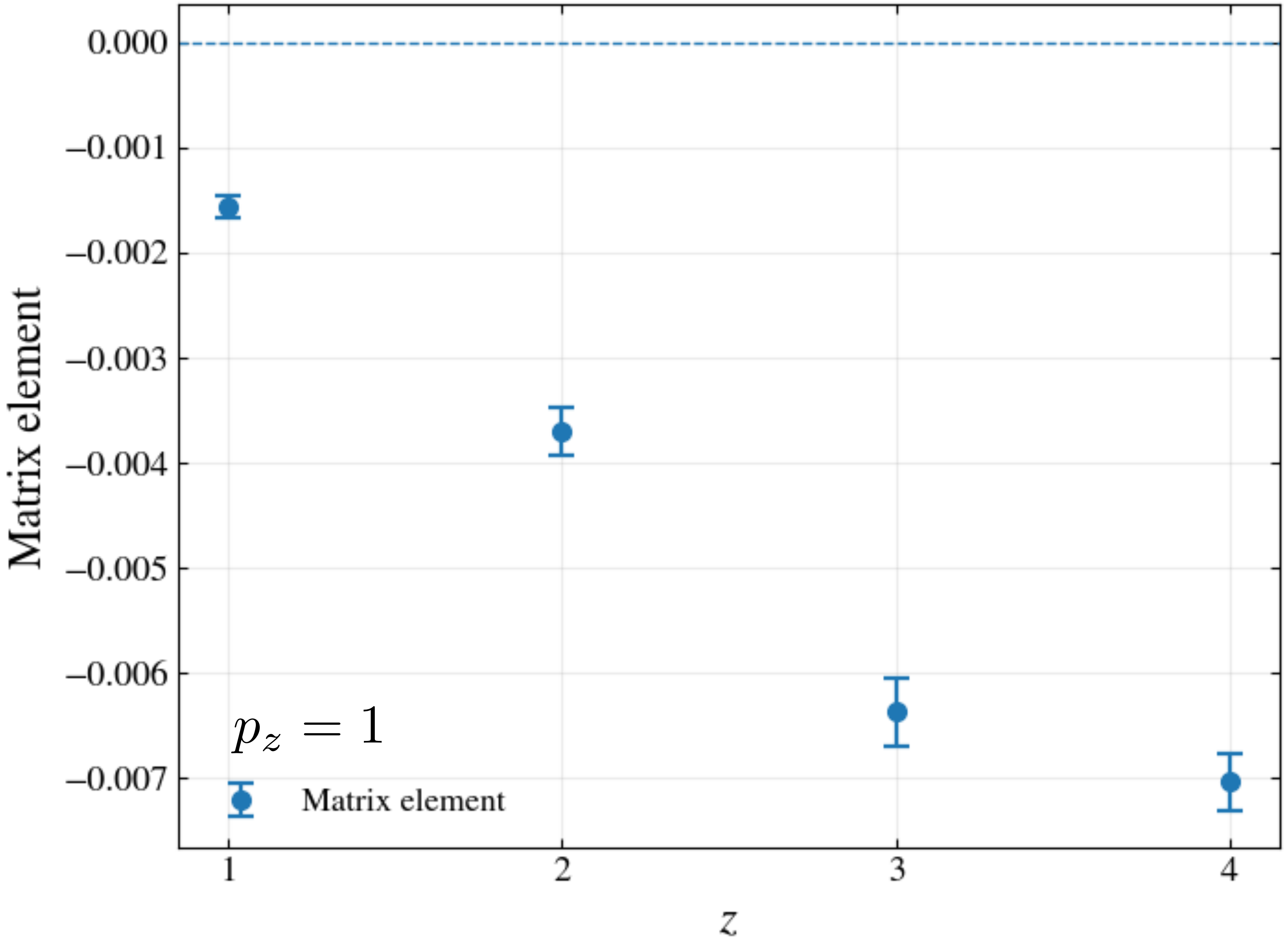




Toward hadronic structure of the “resolved” photon from LQCD

- Recall

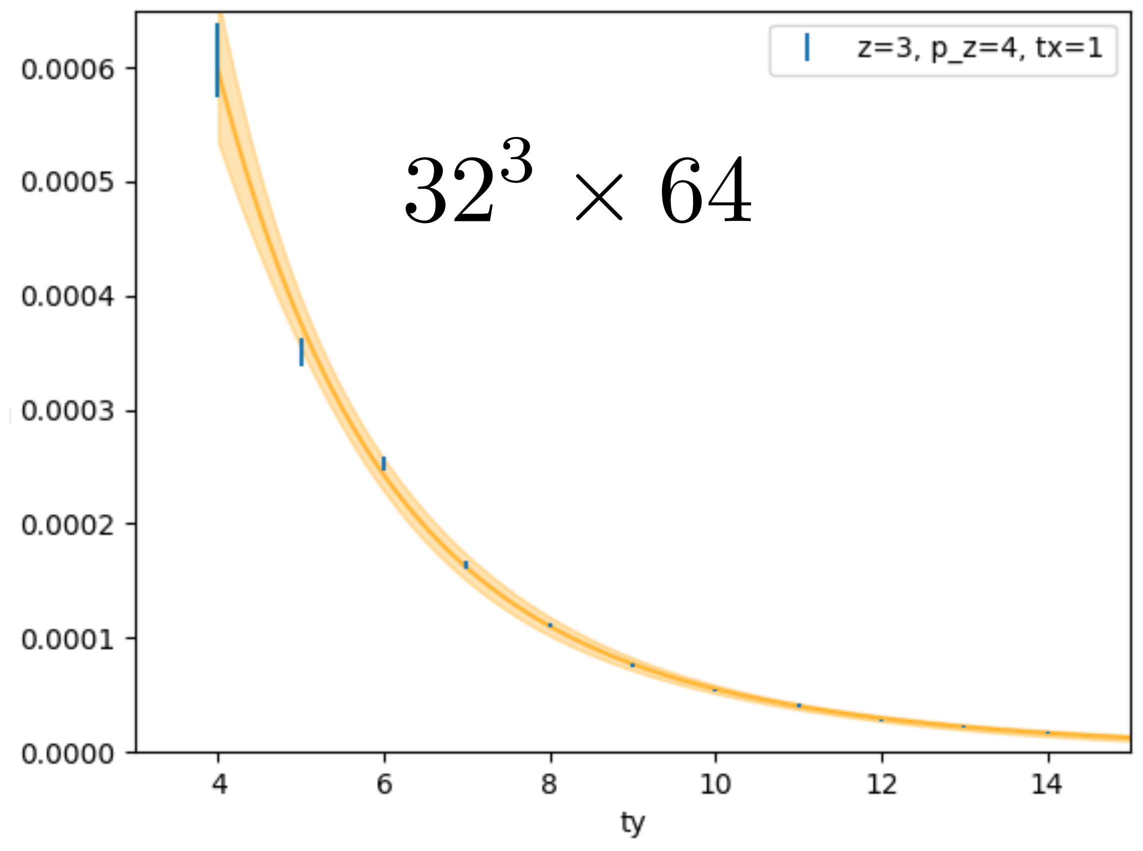
$$h(z) = \frac{1}{2}(-ie)^2 \int d^4x \int d^4y \sum_{\lambda=x,y} (e^{|\vec{p}|x_t} e^{-i\vec{p}\cdot\vec{x}} - 1)(e^{-|\vec{p}|y_t} e^{i\vec{p}\cdot\vec{y}} - 1)\langle T J_\lambda(x) O_u(z,0) J_\lambda(y)\rangle_{\text{QCD}}$$

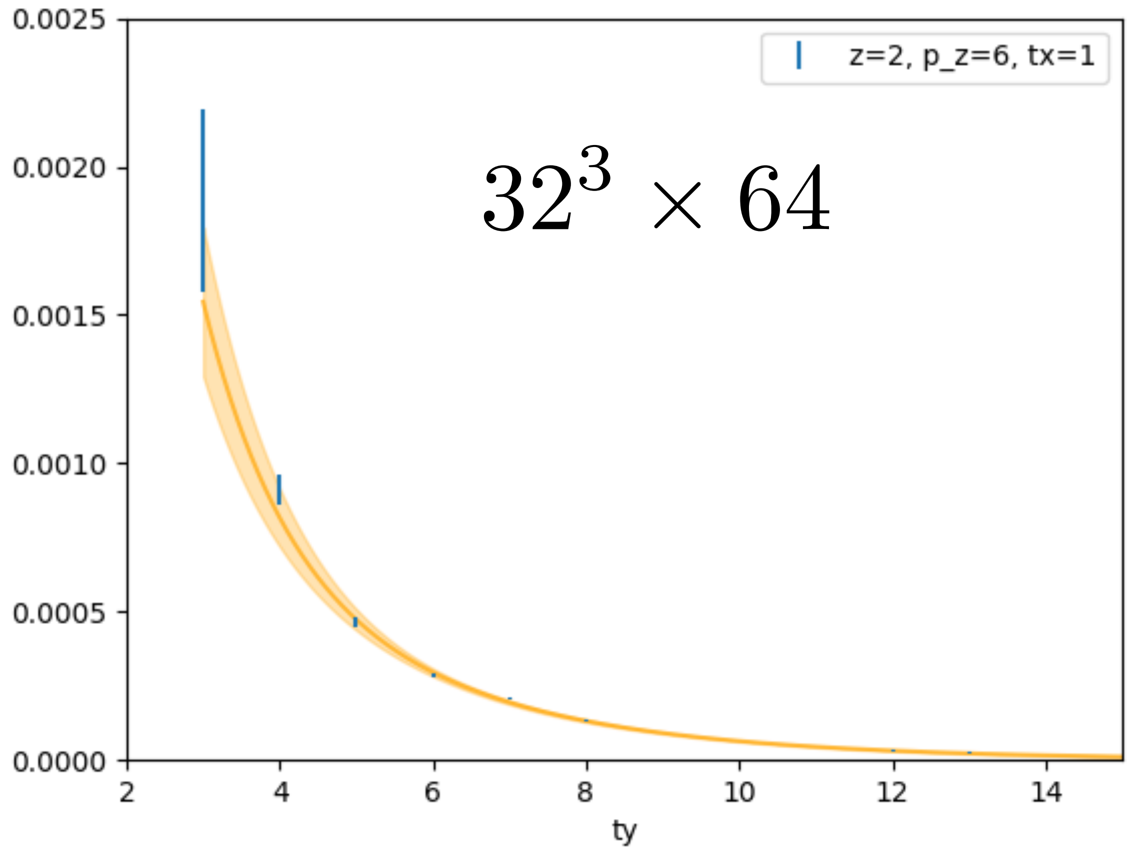


In progress

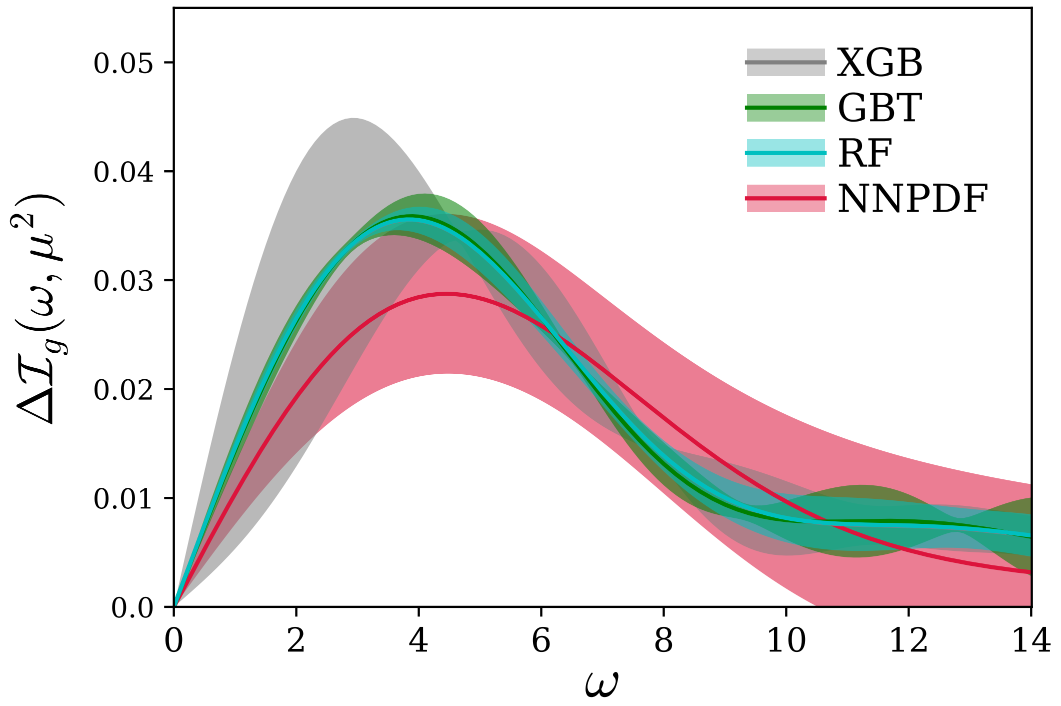
- One can do better
- Preliminary fit to matrix elements:

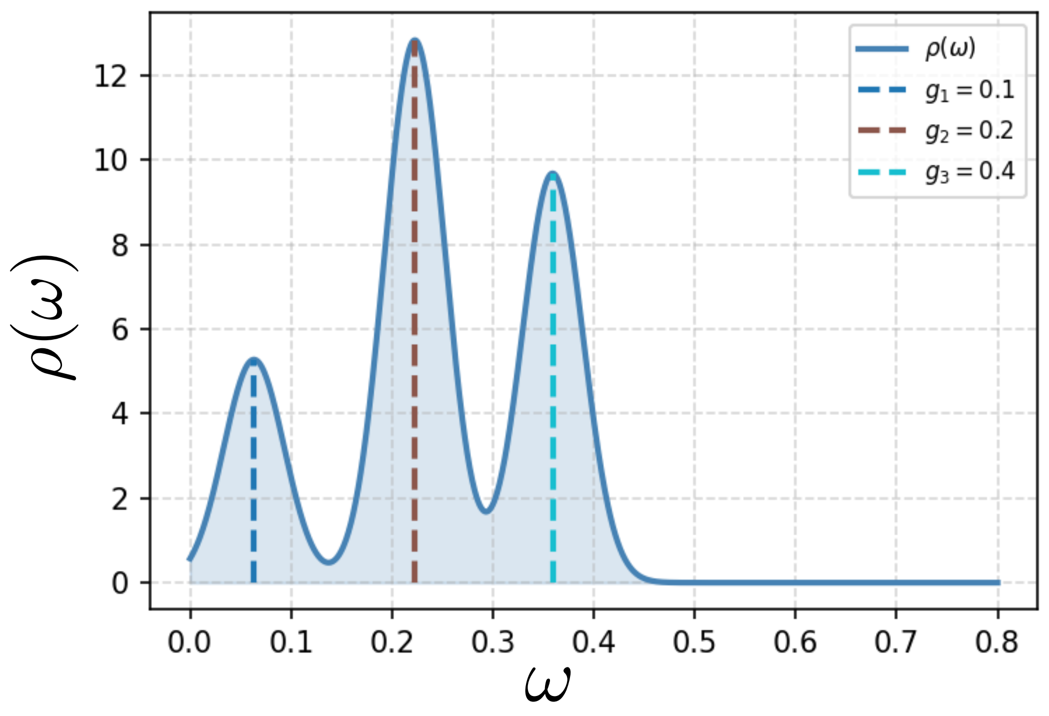
$$32^3 \times 64\,(a \approx 0.063\,\mathrm{fm})$$





- Sum over fitted matrix elements
- Renormalization: Ratio-renormalization [Radyushkin 2017] & self renormalization [Ji, et. al. NPB 2021]
- Perturbative matching
- PDF extraction using generative ML (**earlier/ongoing** examples: gluon helicity PDF, reconstruction of spectral function from hadronic tensor)





arXiv[2409.17234]
arXiv [2311.04206]
In preparation [2026]

Outlook

- First LQCD determination of the resolved photon's hadronic structure is in progress
  - QCD allows photon to exhibit a hadronic structure due to quantum fluctuations
  - EIC will have the unique opportunity to explore (un)polarized photon's structure (essential but hard!)
  - Can we investigate it on the lattice? - Yes, clear advantage over phenomenology and experiment
  - Preliminary data indicates existence of nonzero PDF

Thank you!

$$h_f(z) = 2|\vec{p}|L^3 \frac{\sum_{\lambda=x,y}\sum_{\vec{w},\vec{v}} e^{i\vec{p}\cdot(\vec{w}-\vec{v})} \langle 0|\, T A_\lambda(T,\vec{v})\, O_f(z,0)\exp\Big(-iq_f e\int_0^z A_\mu(t)dt_\mu\Big) A_\lambda(-T,\vec{w})\,|0\rangle_{\mathrm{QCD+QED}}}{\sum_{\lambda=x,y}\sum_{\vec{w},\vec{v}} e^{i\vec{p}\cdot(\vec{w}-\vec{v})} \langle 0|\, T A_\lambda(T,\vec{v}) A_\lambda(-T,\vec{w})\,|0\rangle}$$

$$d\sigma(e + A \;\rightarrow\; e' + 2\,\mathrm{jets} + X) = \sum_{,} \int dy \int dx_\gamma \int dx_A f_{\gamma/e}(y) \times \;\; f_{a/\gamma}(x_\gamma,\mu^2) f_{b/A}(x_A,\mu^2) d\hat{\sigma}(ab \rightarrow \mathrm{jets})$$

where $f_{\gamma/e}(y)$ is the photon flux of the electron with y being the momentum fraction carried by the photon; $f_{a/\gamma}(x_\gamma,\mu^2)$ are the photon PDFs in the resolved-photon case, which depend on the parton-in-photon momentum fraction x_γ and the scale μ; $f_{b/A}(x_A,\mu^2)$ are nuclear PDFs depending on the parton momentum fraction x_A and the scale μ; $d\hat{\sigma}(ab \rightarrow \mathrm{jets})$ is the cross section of hard scattering of partons a and b into jets.