The Physics of Phase Transitions is Universal, Numerical Simulations aren't

[Hands & JO *PRB* **accepted** (2026)]

Simon Hands, **Johann Ostmeyer**

Helmholtz-Institut für Strahlen- und Kernphysik, University of Bonn, Germany

University of Maryland, College Park, July 28, 2026

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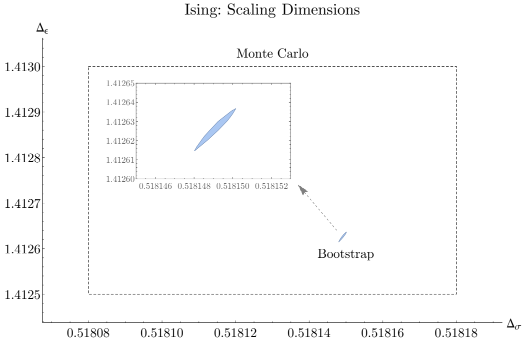
The 3D Ising model

[Hasenbusch *PRB* **17** (2010); Kos + *JHEP* **08** (2016); Chang + *JHEP* **03** (2025)]



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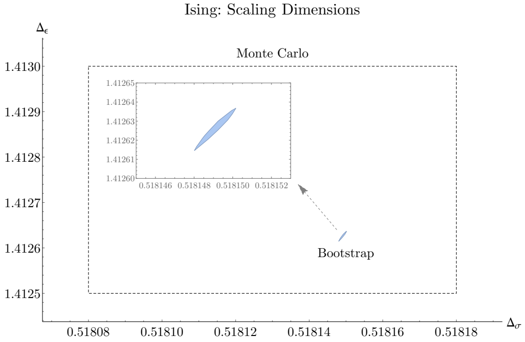

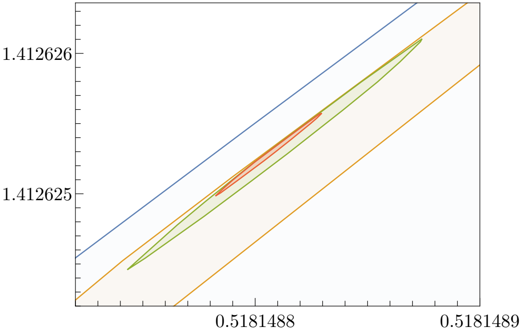

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[motivation by Michael Scherer (talk @ Perimeter Institute, 2022) and Igor Herbut]

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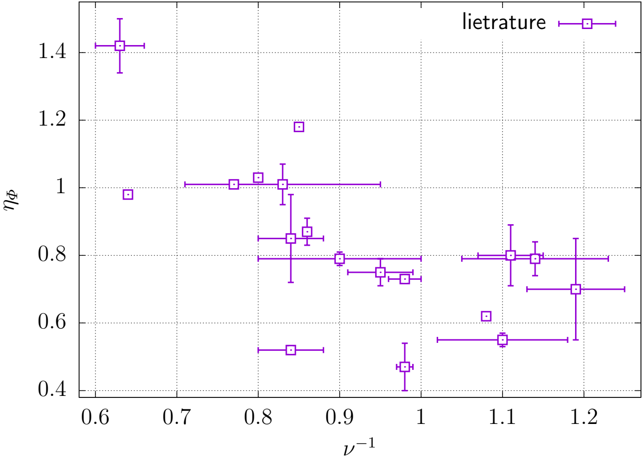

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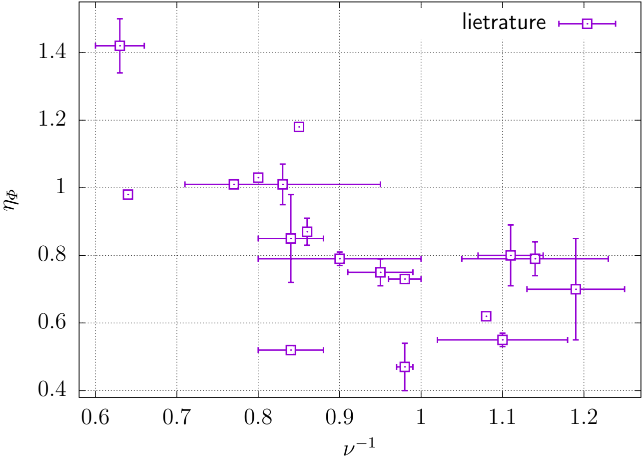

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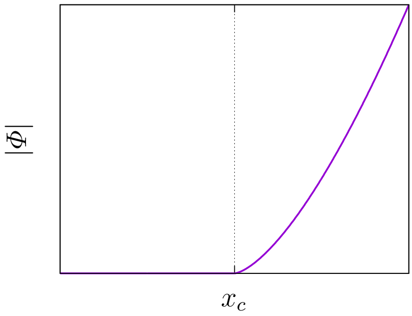

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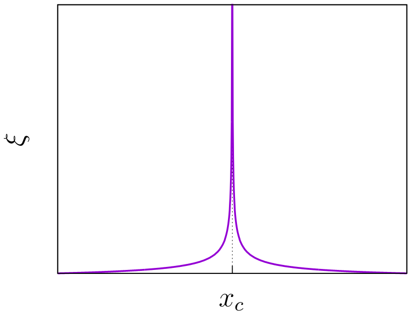

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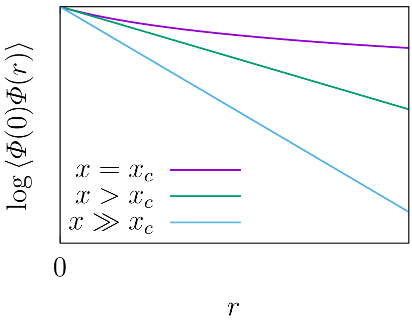

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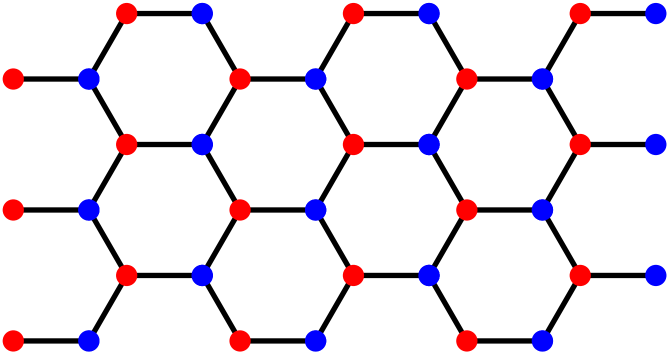

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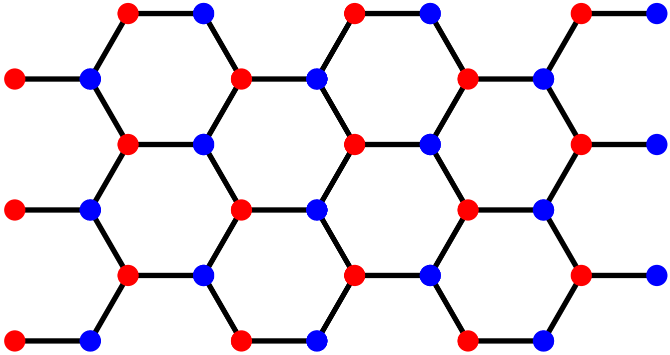

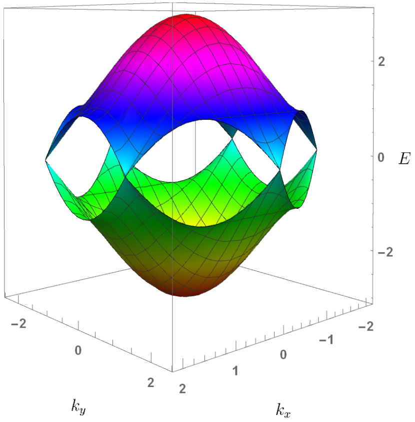

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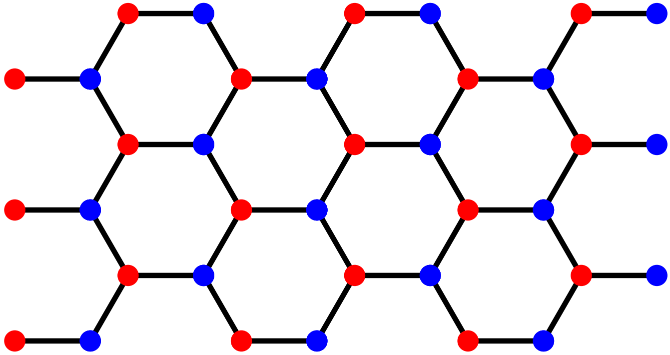
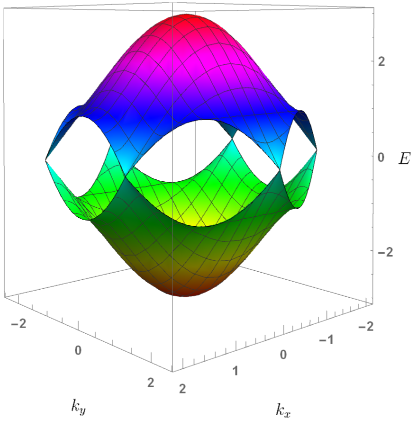
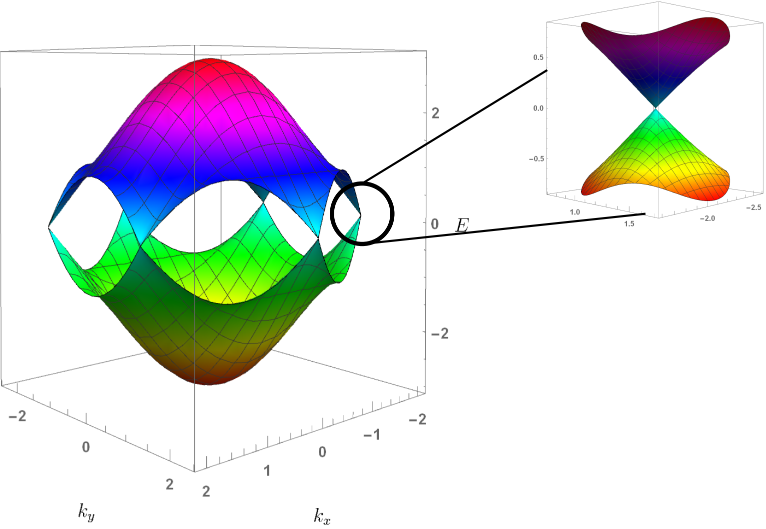
$$H = -\sum_{s=\uparrow\downarrow}\sum_{\langle x,y\rangle} c^\dagger_{x,s} c_{y,s} + \frac{1}{2}U\sum_x q_x^2\,, \quad q_x = c^\dagger_{x,\uparrow}c_{x,\uparrow} + c^\dagger_{x,\downarrow}c_{x,\downarrow} - 1$$





Phase transition: chiral Heisenberg universality class [Janssen & Herbut *PRB* **89** (2014)]

2 bands $\times$ 2 Dirac points $\times$ 2 spins $=$ 2 flavors $\times$ 4 spinor



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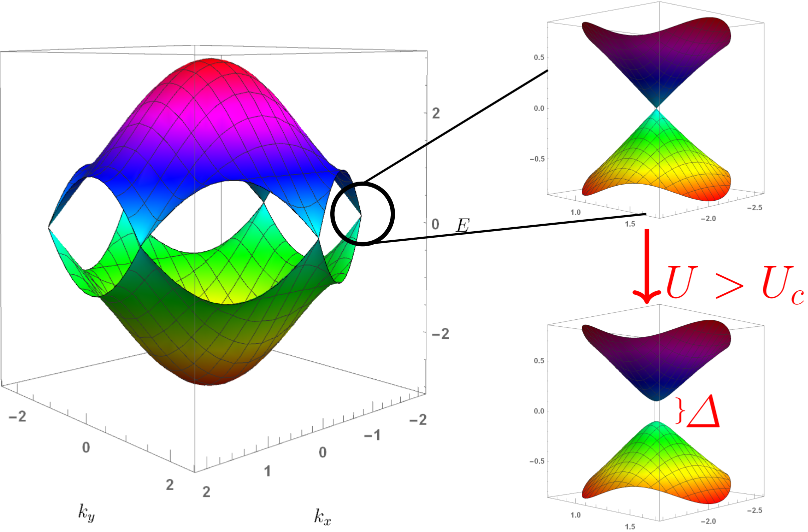

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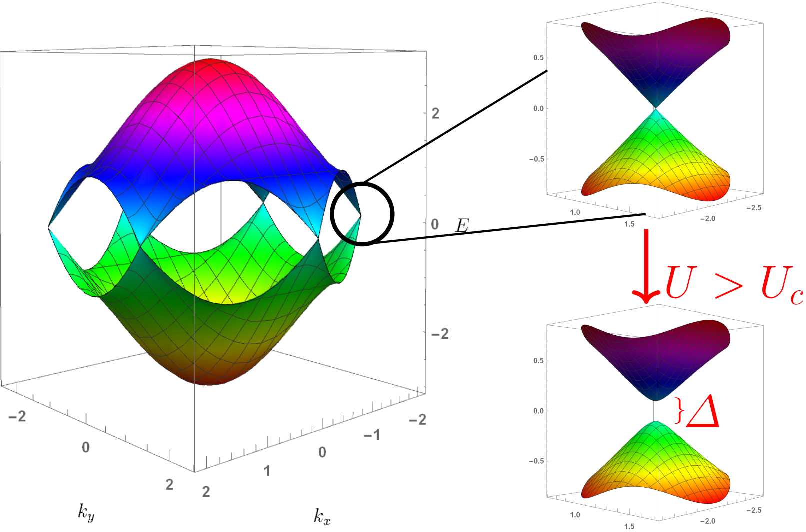

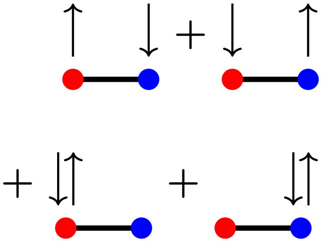

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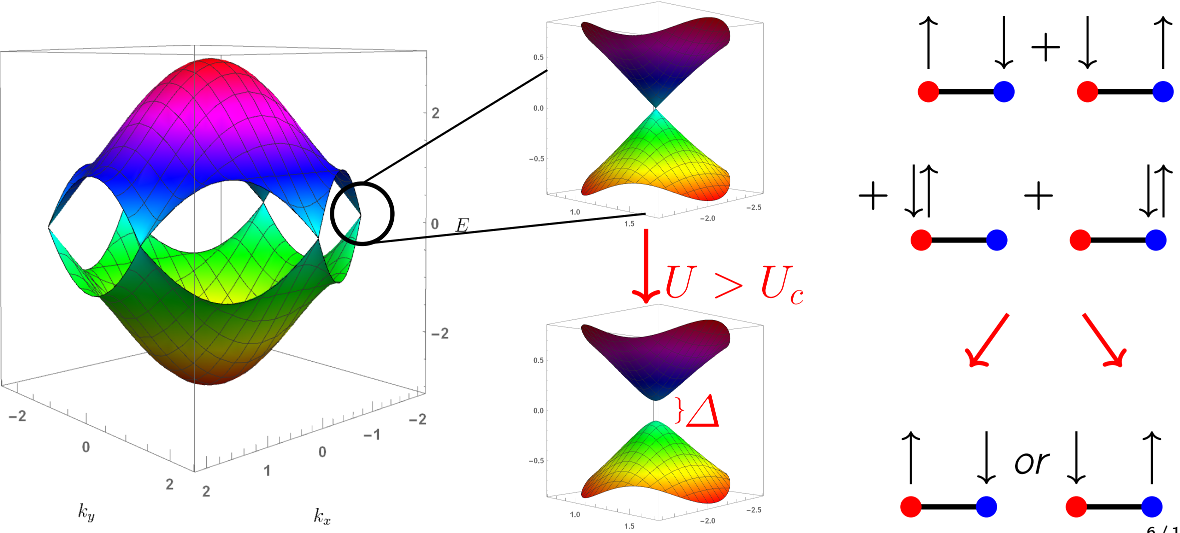

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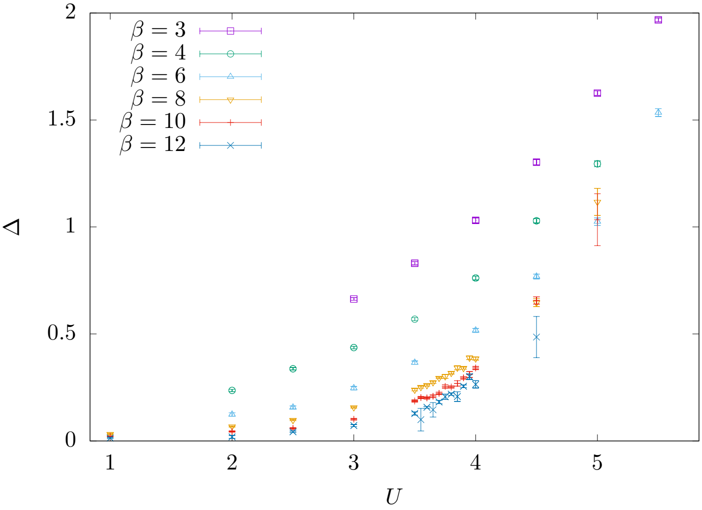

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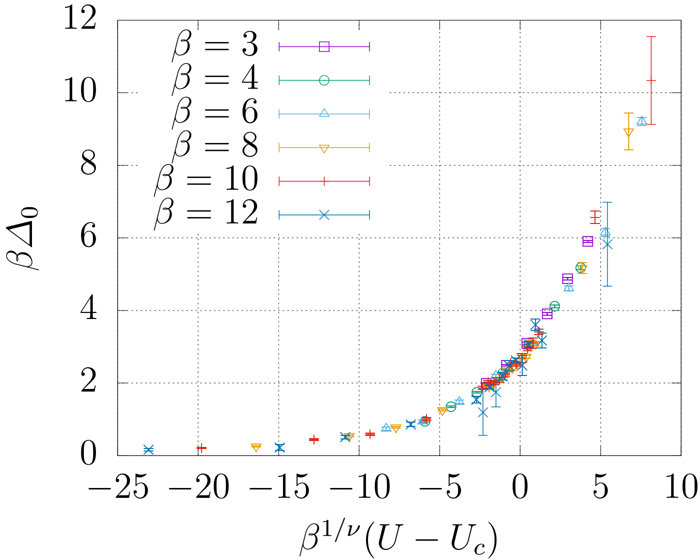

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The simulation [Kaplan *PLB* **288** (1992); Kennedy+ *NuclPhysB* **73** (1999)]

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- Domain Wall Fermions $\Psi(x,s)$ in $(3+1)$D

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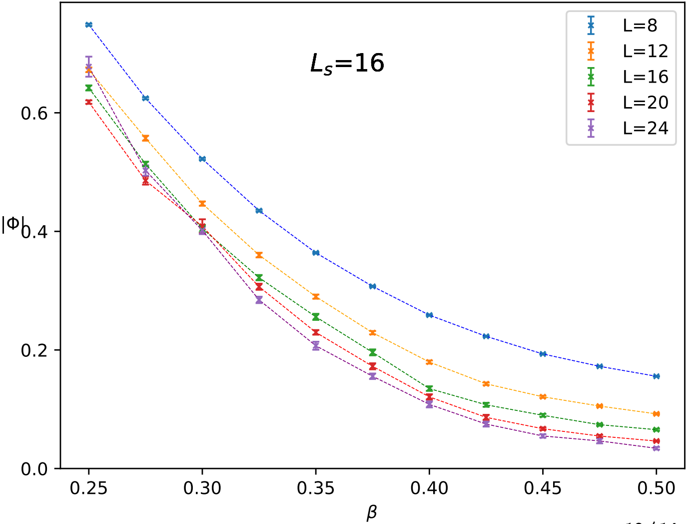
- Rational Hybrid Monte Carlo $\phi \sim \sqrt{\det \mathcal{M}^\dagger_{\mathsf{DW}}\mathcal{M}_{\mathsf{DW}}}$

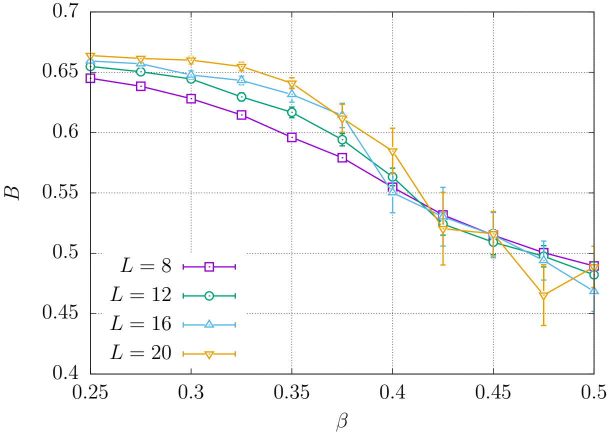
The order parameter

$$|\Phi| = \frac{1}{L^3}\sqrt{\sum_{a=1}^{3}\left(\sum_x \phi_a(x)\right)^2}$$



Binder cumulant

$$B = 1 - \frac{\left\langle |\Phi|^4 \right\rangle}{3 \left\langle |\Phi|^2 \right\rangle^2}$$



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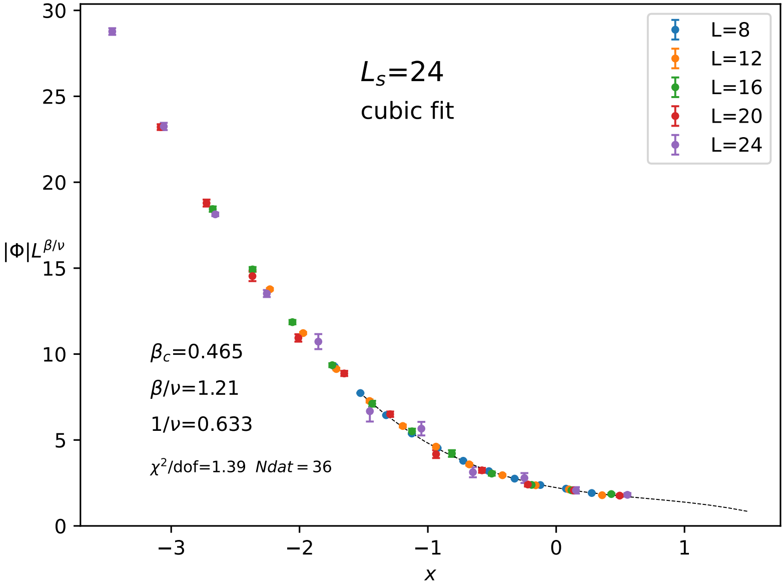
- Universal function:

$$|\Phi| L^{\frac{\beta_\Phi}{\nu}} = f(tL^{\frac{1}{\nu}})\,,$$
$$t \equiv (\beta - \beta_c)/\beta_c$$

Data collapse fit



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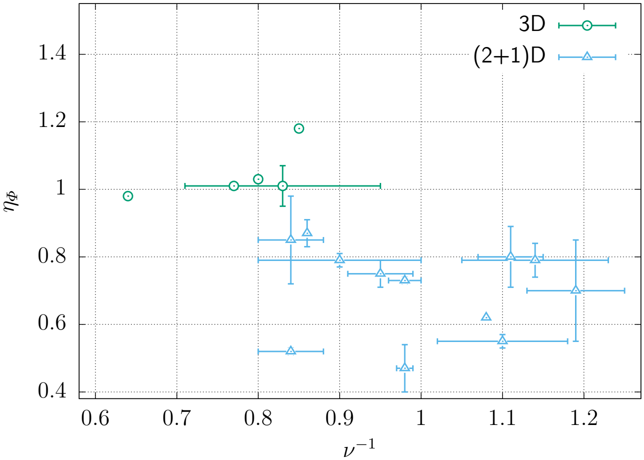

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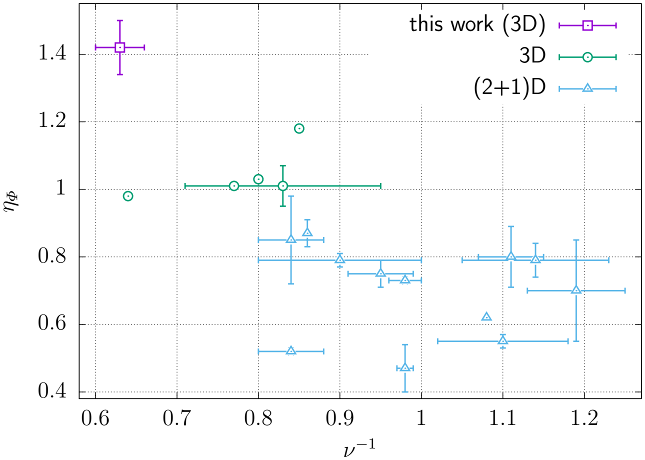

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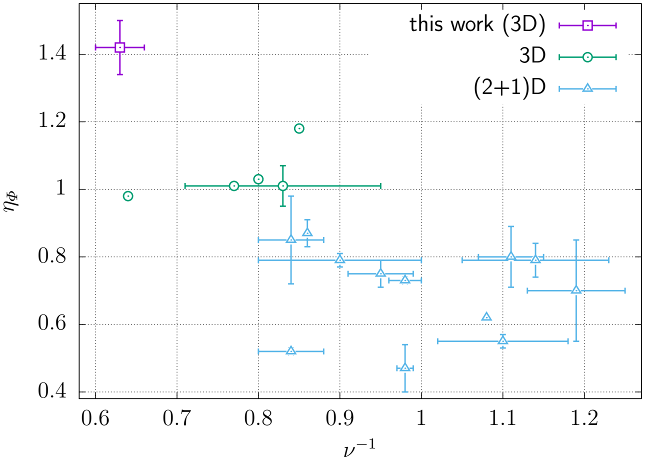
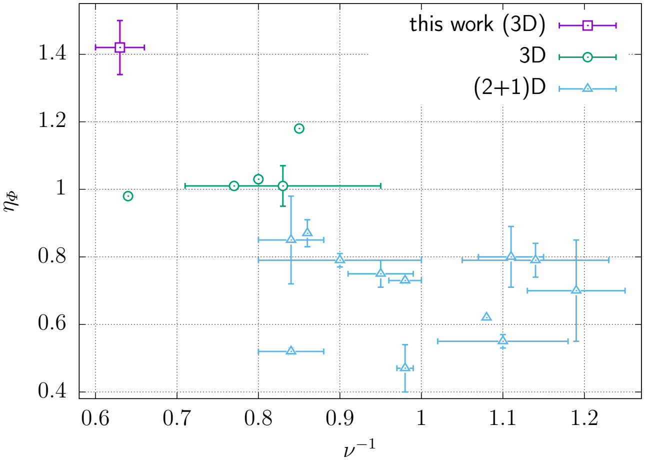


How did we make such a mess?

The SU(2) Gross-Neveu-Yukawa "chiral Heisenberg" *universality* class

[Hands & JO *PRB* **accepted** (2026)]
[Buividovich + *PRB* **98** (2018), *PRB* **99** (2019)]
[Gracey *PRD* **97** (2018)]
[Janssen & Herbut *PRB* **89** (2014)]
[Knorr *PRB* **97** (2018)]
[Ladovrechis + *PRB* **107** (2023)]
[Lang & Läuchli *PRB* **112** (2025), *PRL* **123** (2019)]
[Liu + *PRB* **104** (2021), *NatCommun* **10** (2019)]
[JO, Berkowitz + *PRB* **102** (2020), *PRB* **104** (2021)]
[Otsuka + *PRB* **102** (2020), *PRX* **6** (2016)]
[Parisen Toldin + *PRB* **91** (2015)]
[Wang + *cond-mat.str-el*/**2602**.03656]
[Xu & Grover *PRL* **126** (2021)]
[Zerf + *PRD* **96** (2017)]



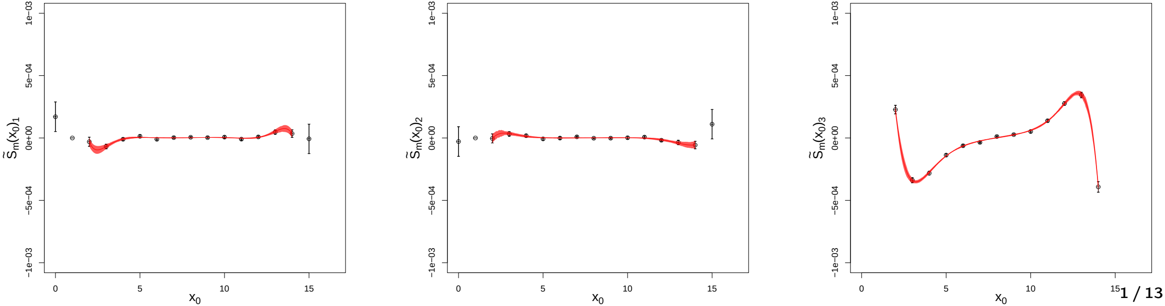
How did we make such a mess?

How can we clean it up?

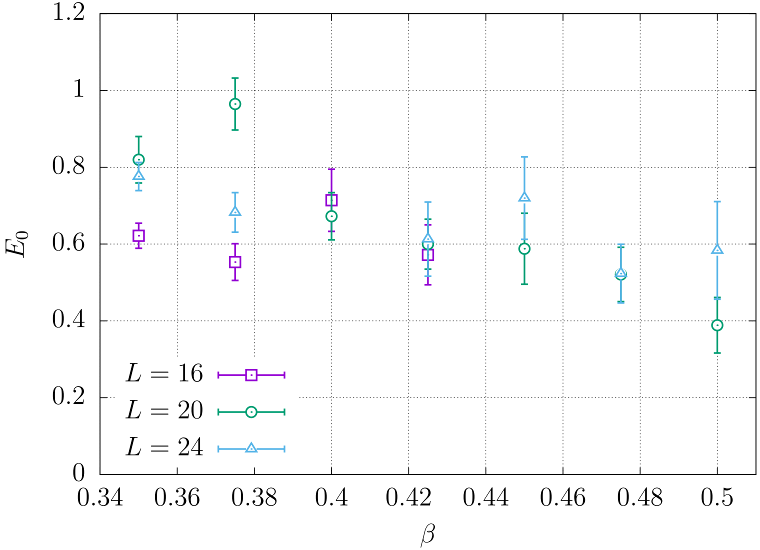
Fermion correlators [JO & Urbach *hep-lat*/**2510**.15500]

$$\langle \psi(0)\bar{\psi}(x_0)\rangle = \begin{pmatrix} z_3 & z_1 - \mathrm{i}\, z_2 \\ z_1 + \mathrm{i}\, z_2 & -z_3 \end{pmatrix} \equiv z_i \sigma_i \,,$$

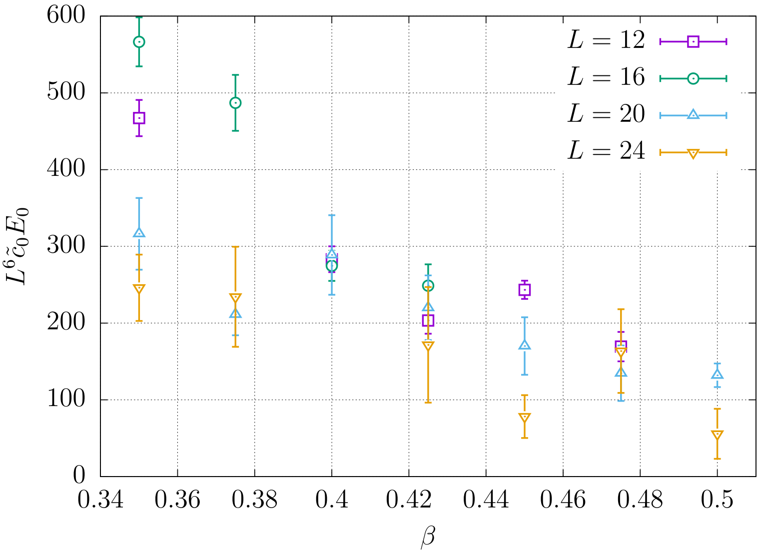
$$z_i = \sum_{l=1}^{k/2} c_{i,l} \sinh\left(E_l(x_0 - L/2)\right)$$



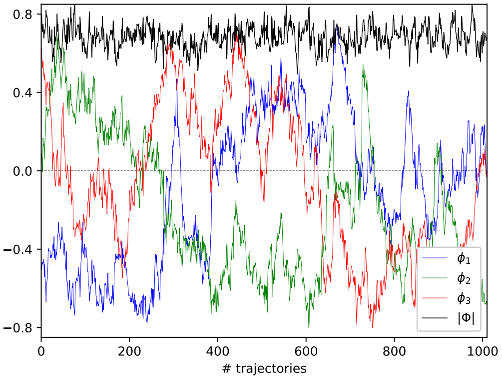
Fermion ground state energy



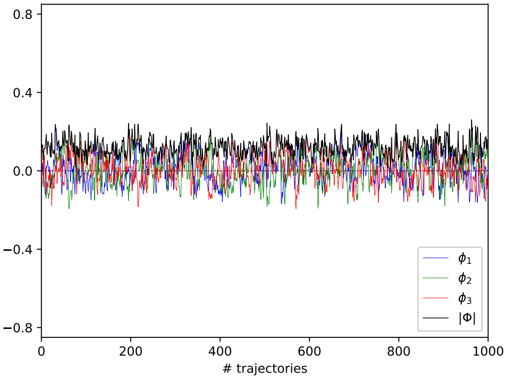
Another data collapse?



Ergodicity verification



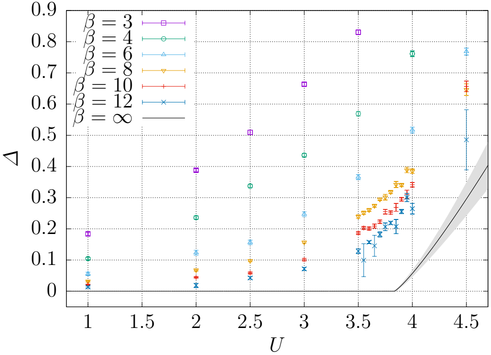
$$\beta = 0.25$$

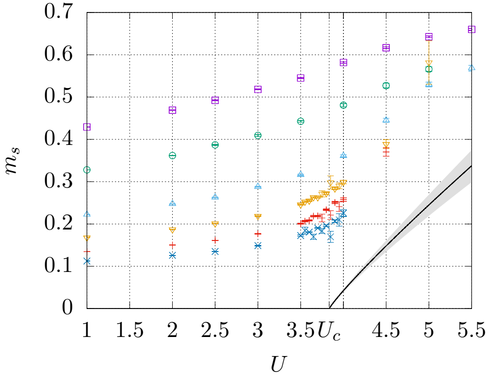


$$\beta = 0.45$$

$T = 0$ quantum phase transition [JO, Berkowitz + *PRB* **102** (2020), *PRB* **104** (2021)]

$$H = -\sum_{s=\uparrow\downarrow} \vec{c}_s^{\,\dagger} \kappa \vec{c}_s + \frac{1}{2} U \sum_x q_x^2$$





Domain wall fermion action

$$S = S_{\text{kin}} + S_{\text{int}} + S_\phi = \bar{\Psi}\mathcal{M}_{\mathsf{DW}}\Psi + S_\phi \,,$$

$$S_{\text{kin}} = \sum_{x,y}\sum_{s,s'} \bar{\Psi}(x,s)[\delta_{s,s'}D_W(x|y) + \delta_{x,y}D_3(s|s')]\Psi(y,s'),$$

$$S_{\text{int}} = \sum_x \vec{\phi}(x)\cdot[\bar{\Psi}(x,L_s)\mathcal{P}_-\vec{\tau}\Psi(x,1) + \bar{\Psi}(x,1)\mathcal{P}_+\vec{\tau}\Psi(x,L_s)],$$

$$S_\phi = \beta\sum_x \vec{\phi}(x)\cdot\vec{\phi}(x)\,,$$

$$\psi(x) = \mathcal{P}_-\Psi(x,1) + \mathcal{P}_+\Psi(x,L_s); \quad \bar{\psi}(x) = \bar{\Psi}(x,L_s)\mathcal{P}_- + \bar{\Psi}(x,1)\mathcal{P}_+,$$

$$D_W(M)_{x,y} = -\frac{1}{2}\sum_\mu [(1-\gamma_\mu)\delta_{x+\hat{\mu},y} + (1+\gamma_\mu)\delta_{x-\hat{\mu},y}] + (3-M)\delta_{x,y}$$

Fit results

L_s	β_c	β_Φ/ν	$1/\nu$	χ^2/dof
8	0.453(9)	1.17(4)	0.634(28)	1.2
16	0.461(5)	1.18(4)	0.629(22)	0.8
24	0.465(11)	1.21(4)	0.633(26)	1.4

$$\eta_\Phi = 2\frac{\beta_\Phi}{\nu} - 1 = 1.42(8)$$

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