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in collaboration with:
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Comparing single-hadron and multi-hadron treatments of a narrow ρ in $B \to \rho \ell \bar{\nu}$

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Institut "Jožef Stefan" Ljubljana, Slovenija

University of Ljubljana
FMF
Faculty of Mathematics and Physics

Briceño and Hansen once wrote

... and I used this formula to estimate the FV effects for a generic R many times ...

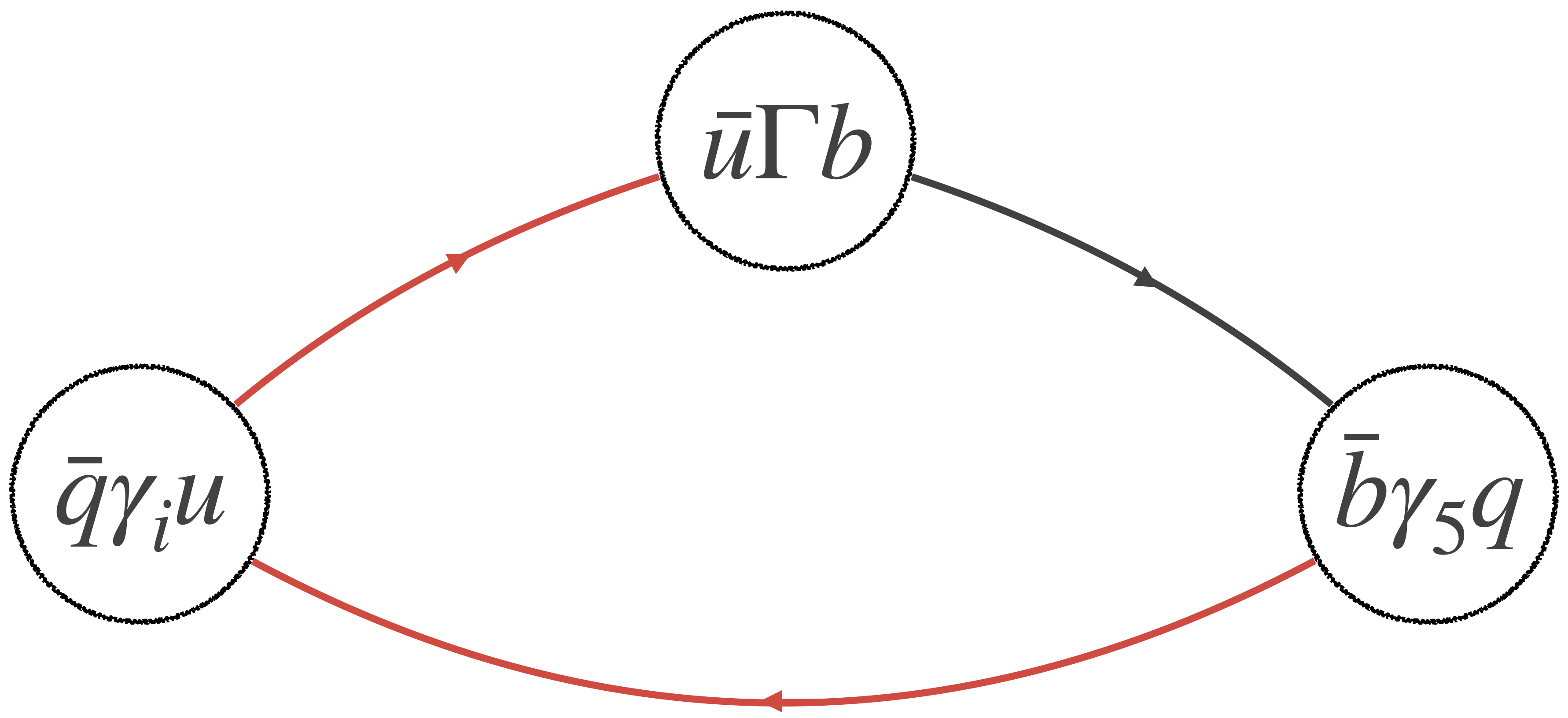
Multichannel $0 \to 2$ and $1 \to 2$ transition amplitudes for arbitrary spin particles in a finite volume

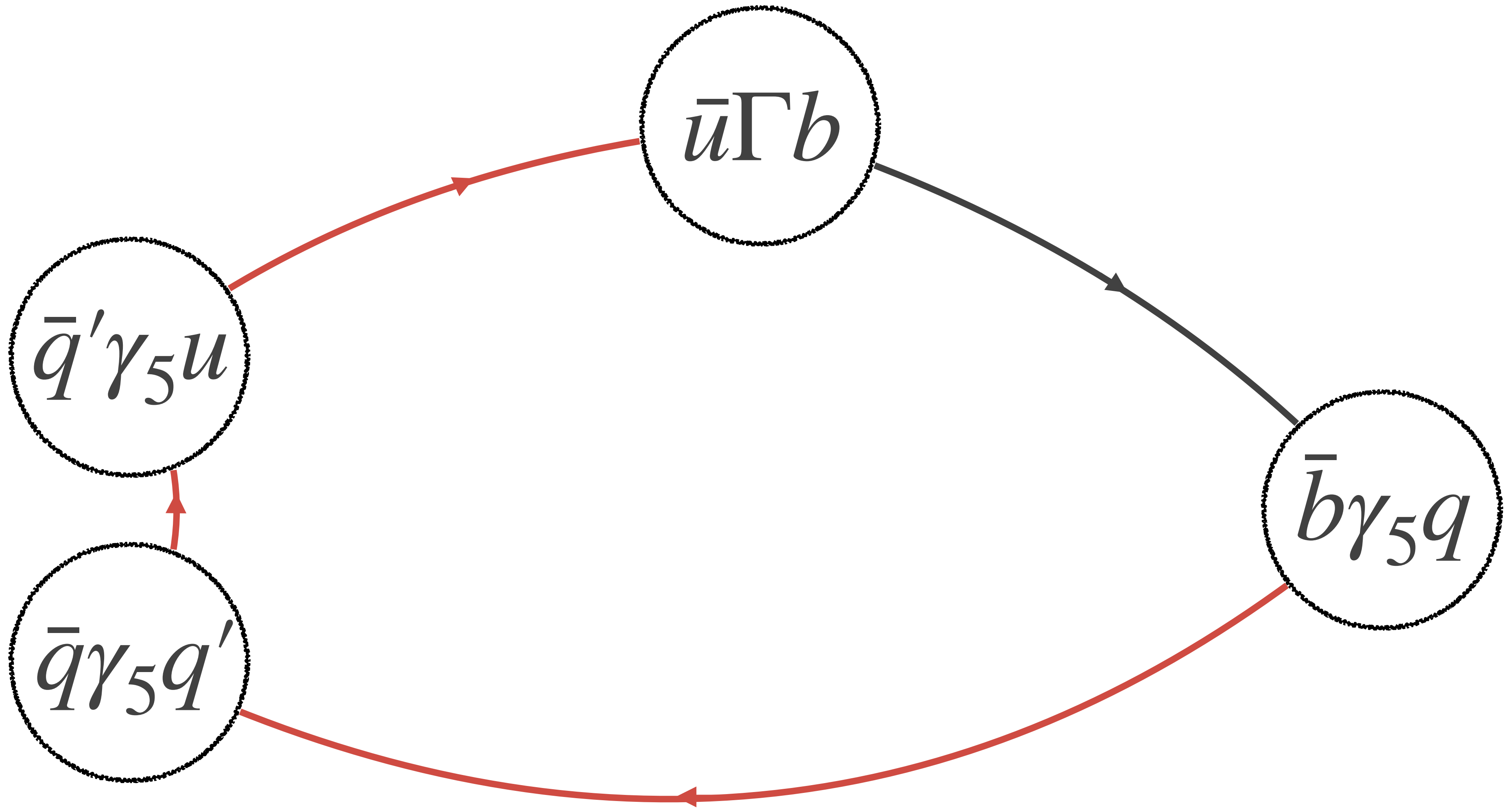
Raúl A. Briceño[1,*] and Maxwell T. Hansen[2,†]

$$|\langle E_R, \vec{p}_R, L \,|\, J \,|\, E, \vec{p} \rangle|^2 = |F_{1\to R}|^2 \left[1 + \mathcal{O}\left(\frac{\Gamma_R}{m_R} \right) \right]$$

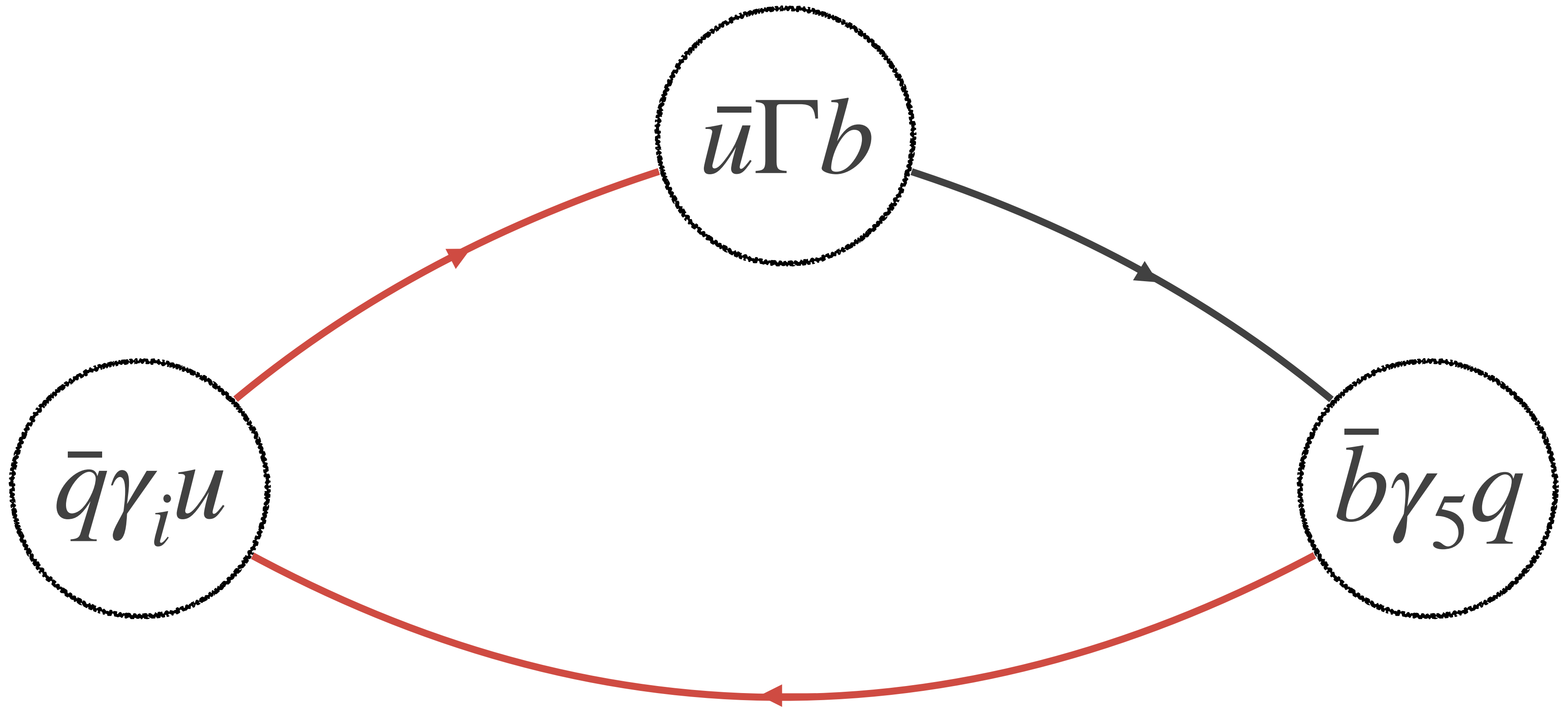
multi-hadron vs. single-hadron

- interpolating fields
- irreps
- variational analysis
- state projection
- normalization (LL-factor)
- matrix elements
- transition amplitude
- continuation to the pole
- form factors



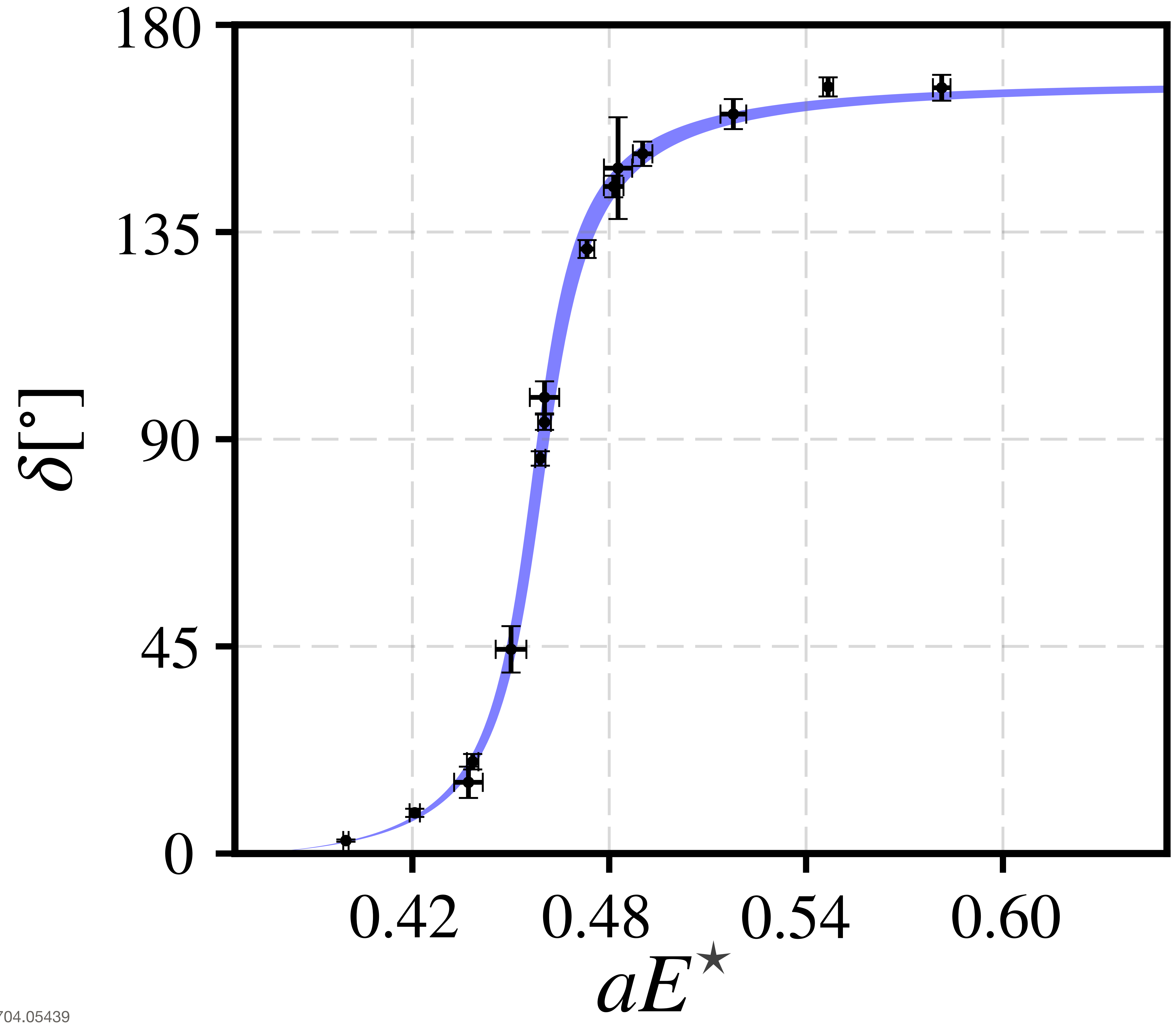


multi-hadron vs. single-hadron



- single bilinear
- matrix elements
- form factors

C13 - a great laboratory



- $m_\pi \approx 317$ MeV
- $m_\rho \approx 799$ MeV
- $\Gamma_\rho \approx 40$ MeV

$$\frac{\Gamma_\rho}{m_\rho} = \frac{40}{799} = 5\,\%$$

narrow!

$$T(E^\star) = \frac{16\pi E^\star}{k} \frac{E^\star \Gamma_\rho(E^\star)}{m_\rho^2 - E^{\star 2} - iE^\star \Gamma_\rho(E^\star)}$$

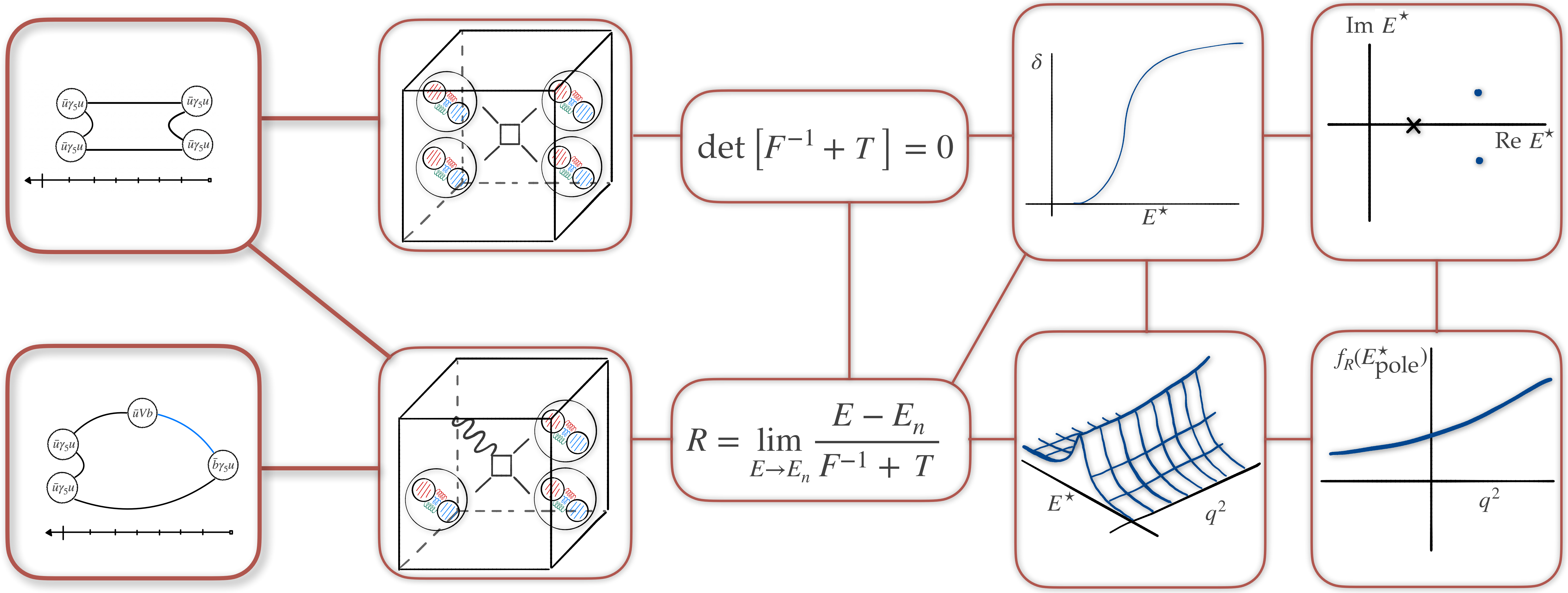
$$\Gamma_\rho(E^\star) = \frac{g^2}{6\pi} \frac{k^3}{E^{\star 2}}$$

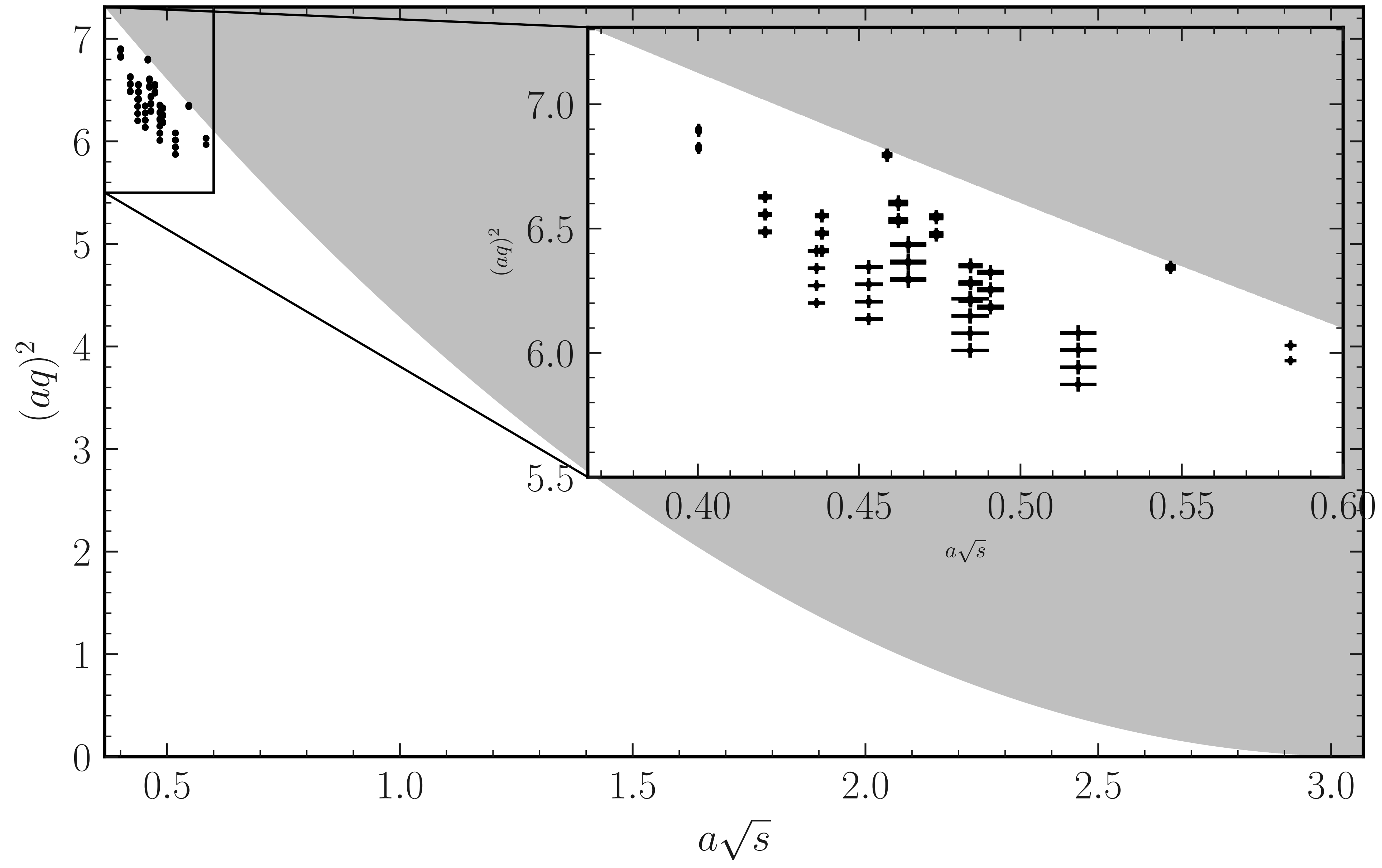
$am_R = 0.4616(19)$

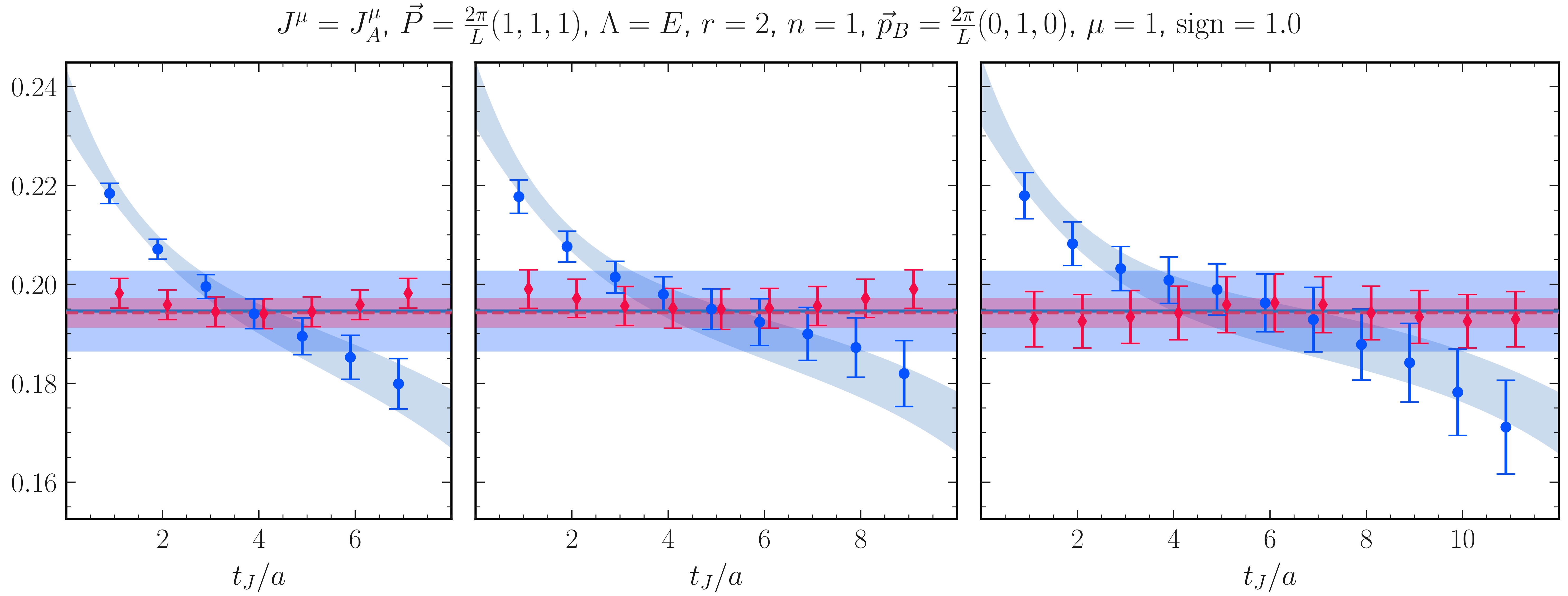
$g_{\rho\pi\pi} = 5.76(16)$

1704.05439

multi-hadron: $\langle n \,|\, \Gamma_J \,|\, B \rangle_L^{(data)}$







Luscher NPB354
Rummukainen, Gottlieb hep-lat/9503028
Kim, Sharpe, Sachrajda hep-lat/0507006
LL, Prelovsek hep-lat/1202.2145
Briceno 1401.3312
Briceno, Dudek, Young 1706.06223
Woss, Wilson, Dudek 2001.08474
[and many more]

Lellouch, Luscher hep-lat/0003023
Lin, Sachrajda, Testa hep-lat/0104006
…
Briceno, Hansen, Walker-Loud 1406.5965
Briceno, Hansen 1502.04314
Briceno, Dudek, LL 2105.02017

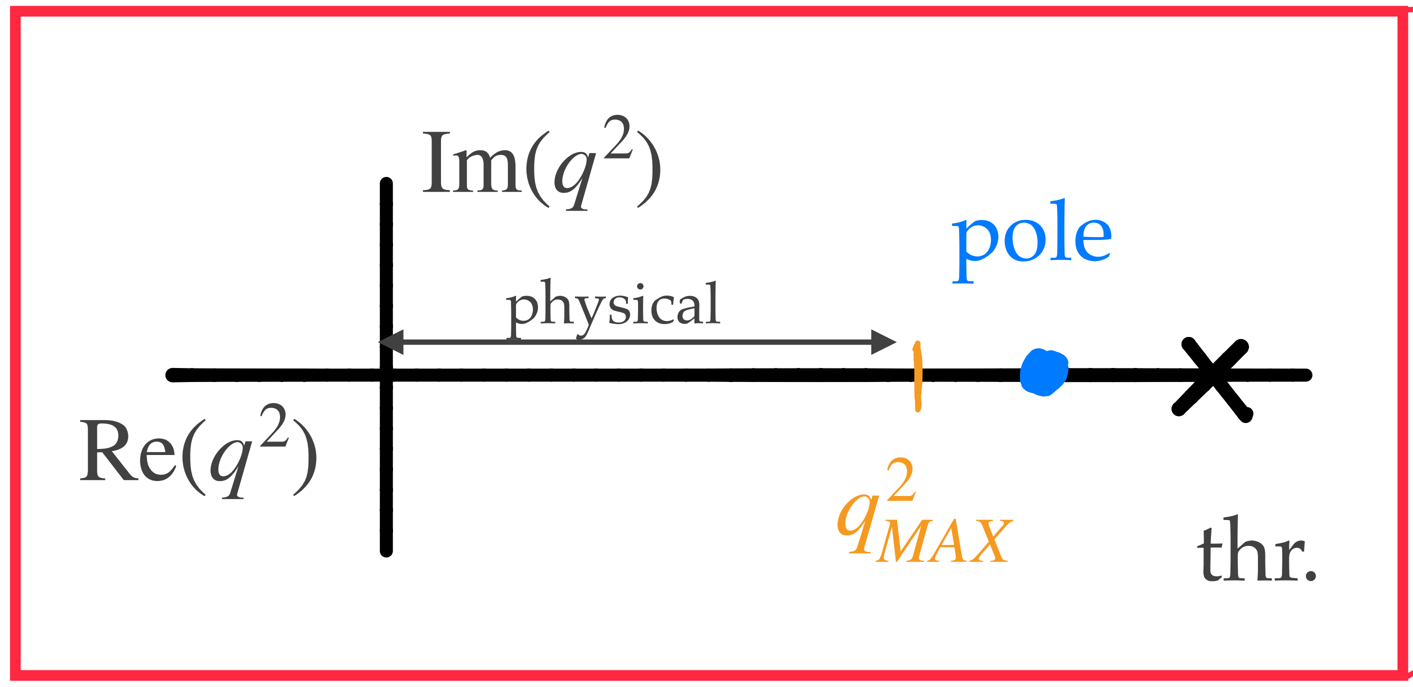
multi-hadron: $\langle n | \Gamma_J | B \rangle_L^{(model)}$

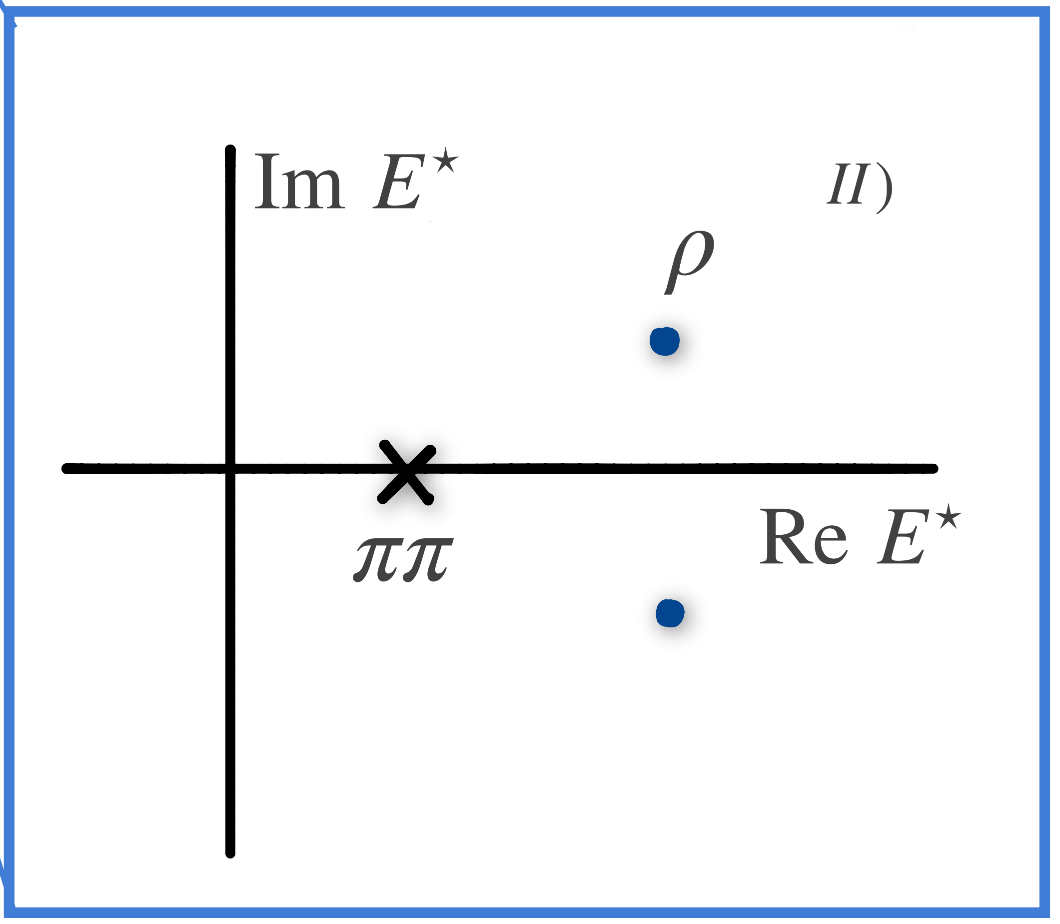
$$\boxed{\langle n | \Gamma_J | B \rangle_L = \sqrt{R_n}\ \langle \pi\pi, E^\star | \Gamma_J | B, p_B \rangle_\infty}$$

"Lellouch-Lüscher" factor

$$\langle \pi\pi, E^\star, \epsilon | \Gamma_J^\mu | B, p_B \rangle_\infty = \sum_{i=V,A_0A_1,A_{12}} K_i^\mu(P, p_B, \epsilon)\, f^{(i)}(q^2, s)$$

$$f^{(i)}(E^\star, q^2) = \boxed{F^{(i)}(E^\star, q^2)}\ \boxed{\frac{T(E^\star)}{k}}$$





Boyd, Grinstein, Lebed hep-ph/9412324
Bourrely, Caprini, Lellouch 0807.2722
Alexandrou, LL, Meinel et al. 1807.08357
Briceño, Dudek, LL 2105.02017

multi-hadron: $\langle n | \Gamma_J | B \rangle_L^{(model)}$

$$f^{(i)}(E^\star, q^2) = \boxed{F^{(i)}(E^\star, q^2)}\, \boxed{\frac{T(E^\star)}{k}}$$

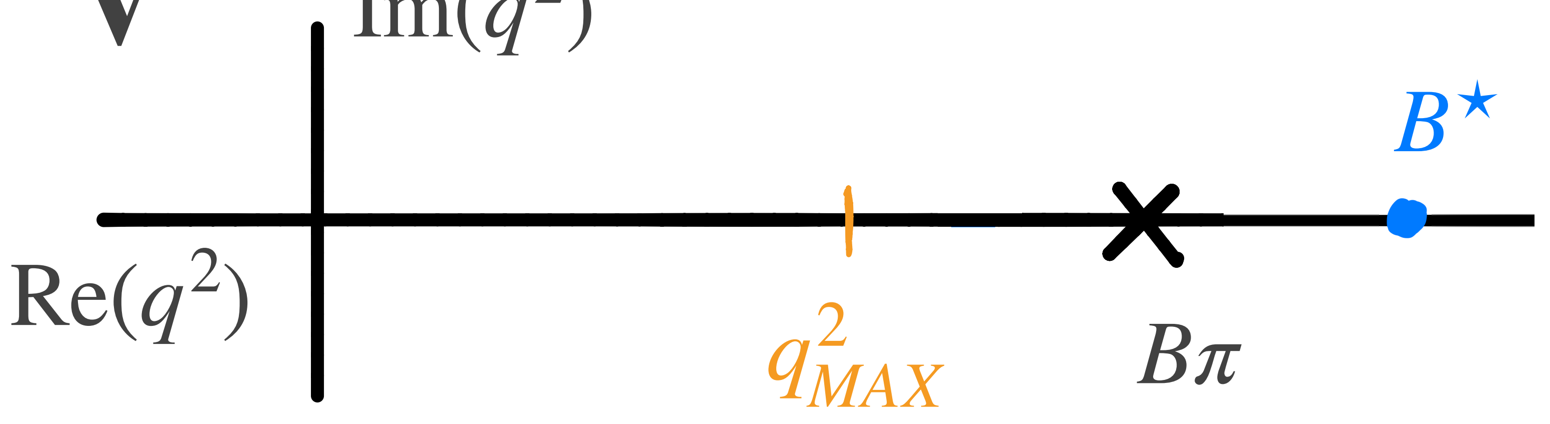
$$T(E^\star) = \frac{16\pi E^\star}{k} \frac{E^\star \Gamma_\rho(E^\star)}{m_\rho^2 - E^{\star 2} - iE^\star \Gamma_\rho(E^\star)}$$

$$\Gamma_\rho(E^\star) = \begin{cases} \dfrac{g^2}{6\pi} \dfrac{k^3}{E^{\star 2}}, & \text{BWI} \\[2ex] \dfrac{g^2}{6\pi} \dfrac{k^3}{E^{\star 2}} \dfrac{1 + (k_\rho r_0)^2}{1 + (k r_0^2)}, & \text{BWII} \end{cases}$$

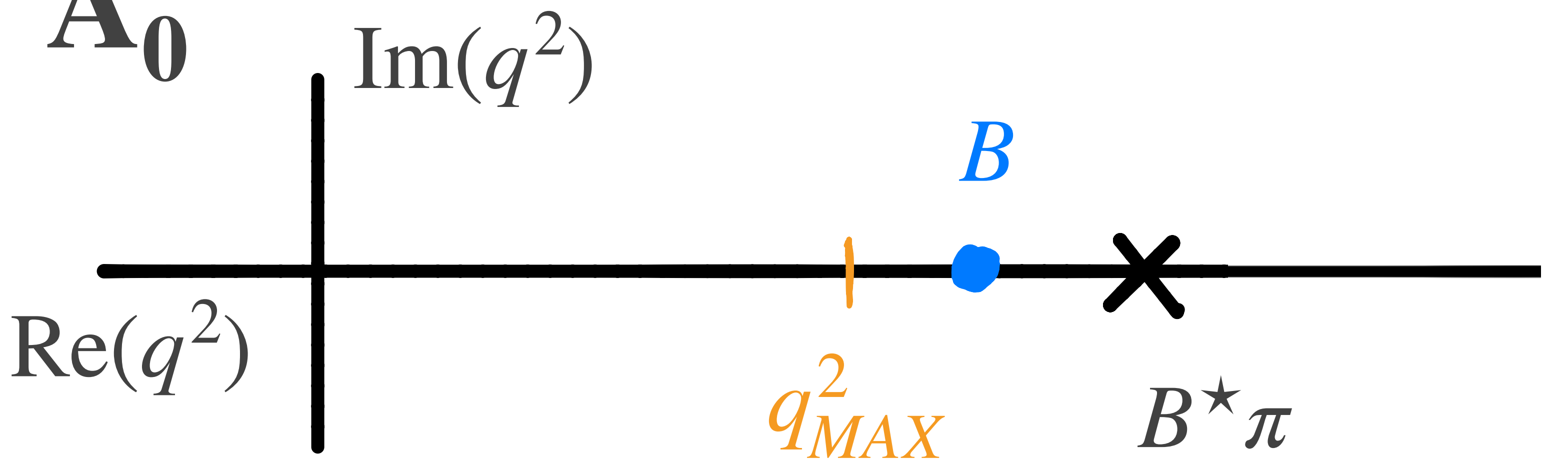
$$F^{(i)}(E^\star, q^2) = \frac{1}{1 - \dfrac{q^2}{\left(m_{\text{pole}}^{(i)}\right)^2}} \sum_{n=0,m=0}^{N,M} a_{nm}^{(i)} \left[z^{(i)}\right]^n \left(\frac{E^{\star 2} - E^{\star 2}_{\text{ref}}}{E^{\star 2}_{\text{ref}}} \right)^m$$

$$z^{(i)}(q^2) = \frac{\sqrt{t_+^{(i)} - q^2} - \sqrt{t_+^{(i)} - t_0^{(i)}}}{\sqrt{t_+^{(i)} - q^2} + \sqrt{t_+^{(i)} - t_0^{(i)}}}$$

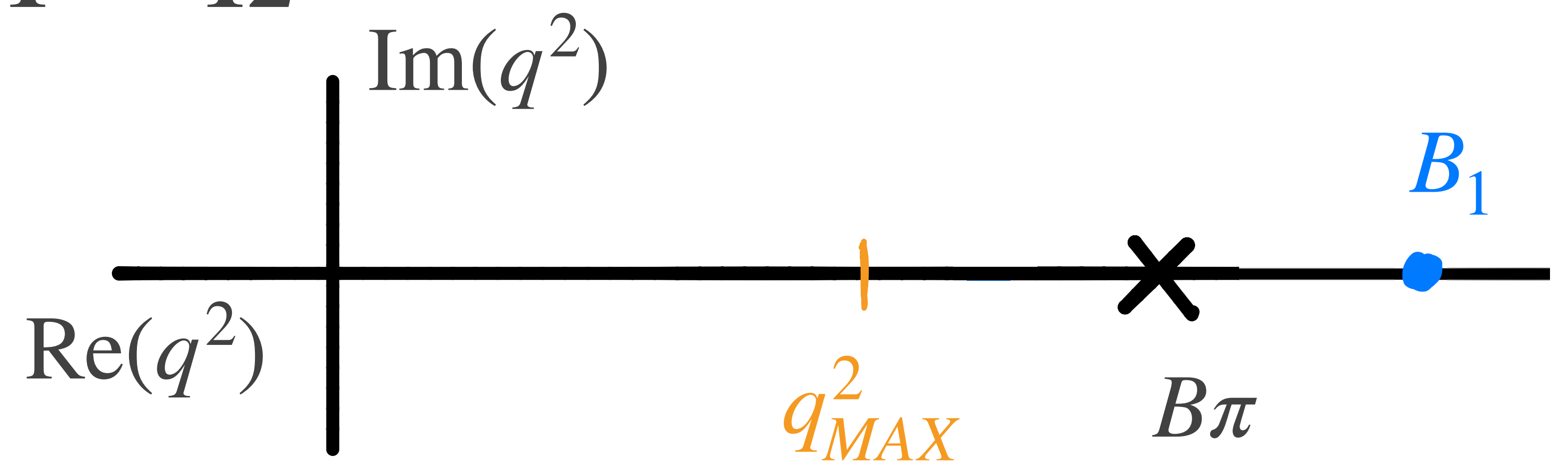
V



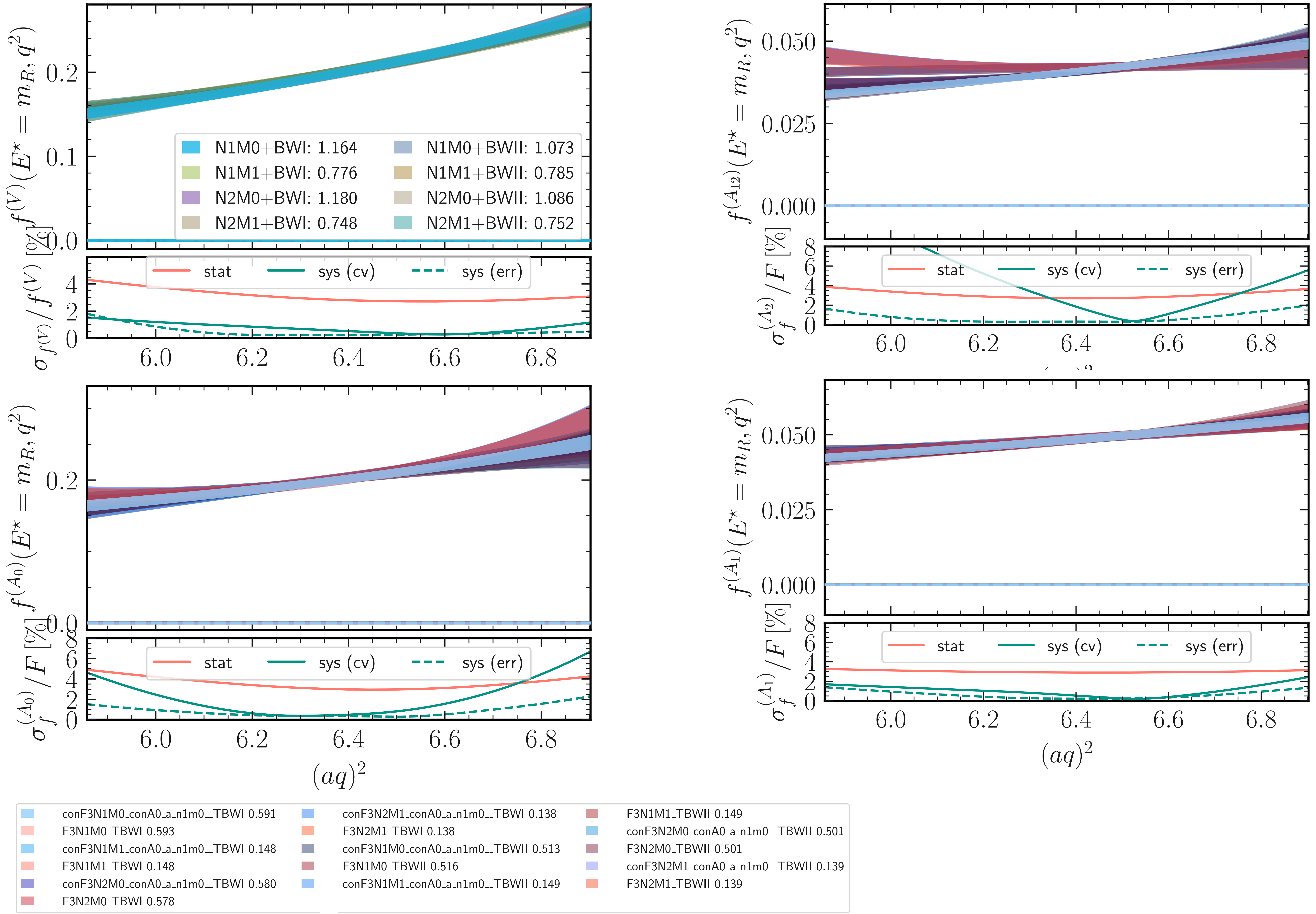
A_0



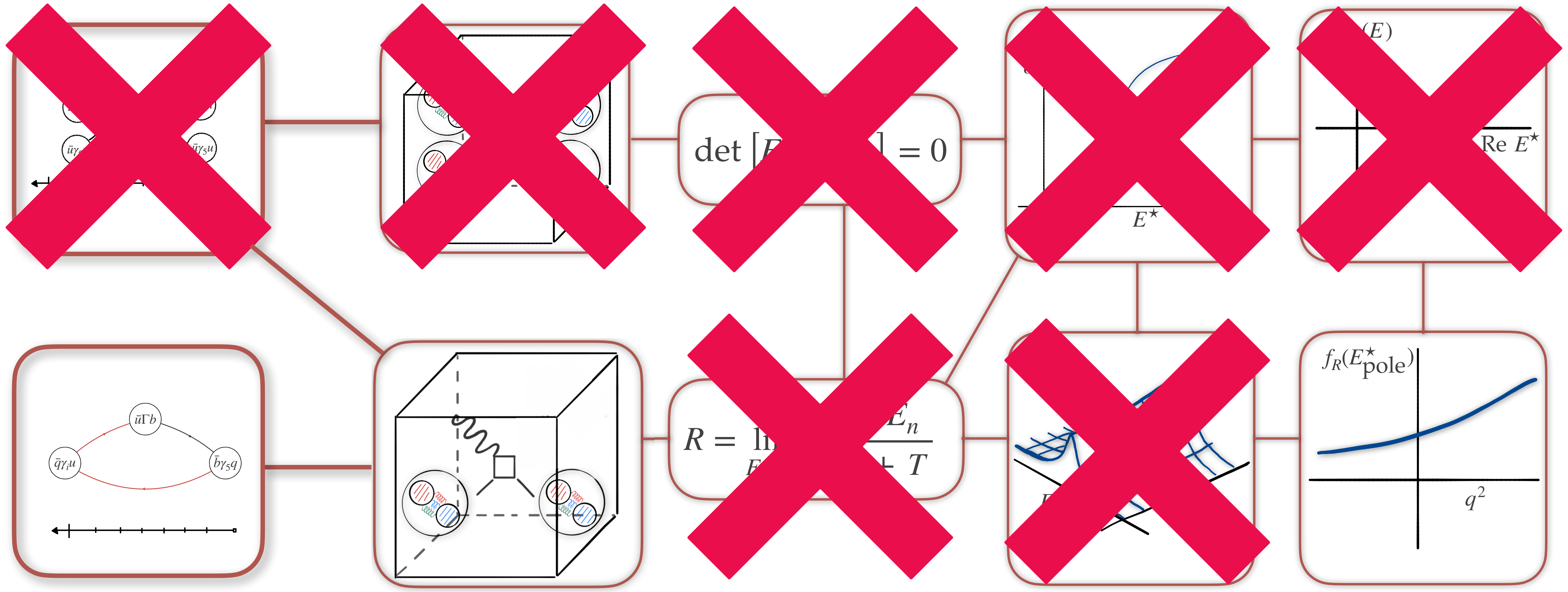
A_1, A_{12}

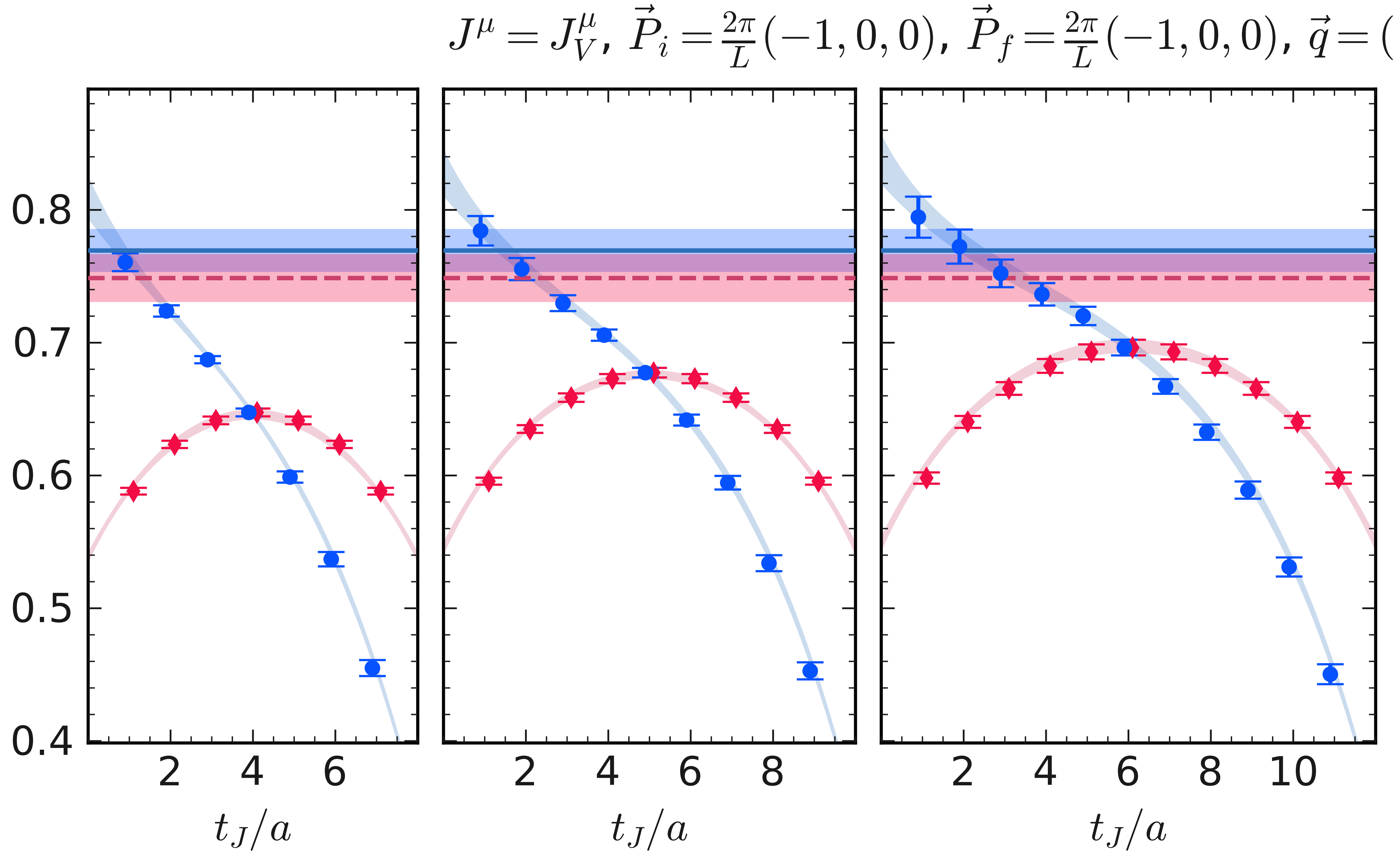


multi-hadron: $f^{(i)}$

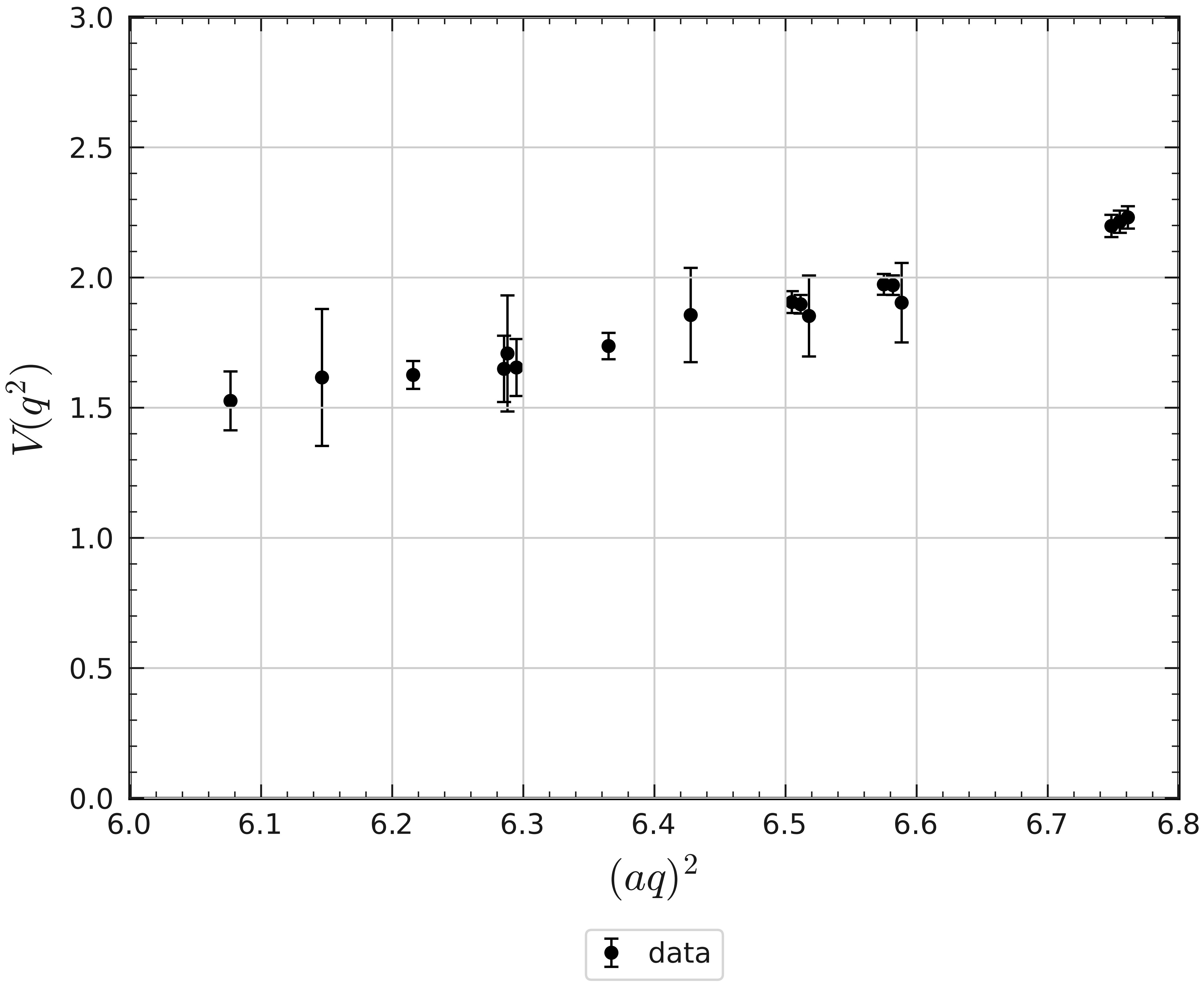


single-hadron: form factors





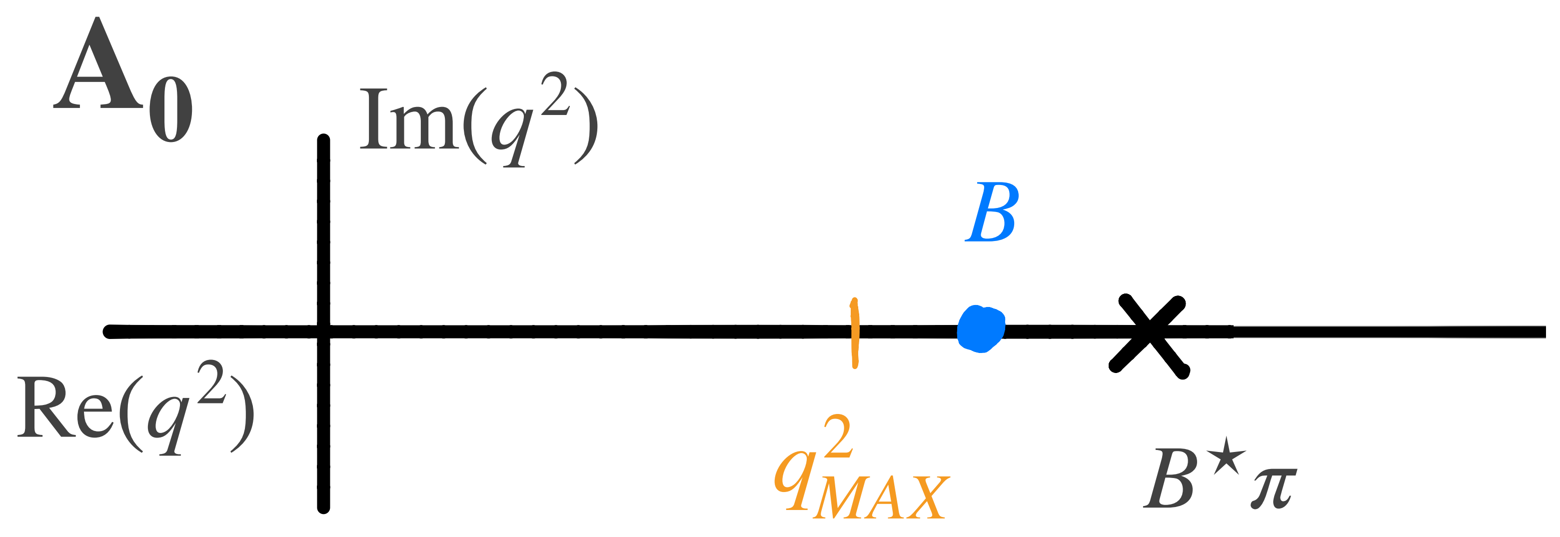
- $V \rightarrow 17$ points
- $A_0 \rightarrow 9$ points
- $A_1 \rightarrow 3$ points
- $A_{12} \rightarrow 3$ points

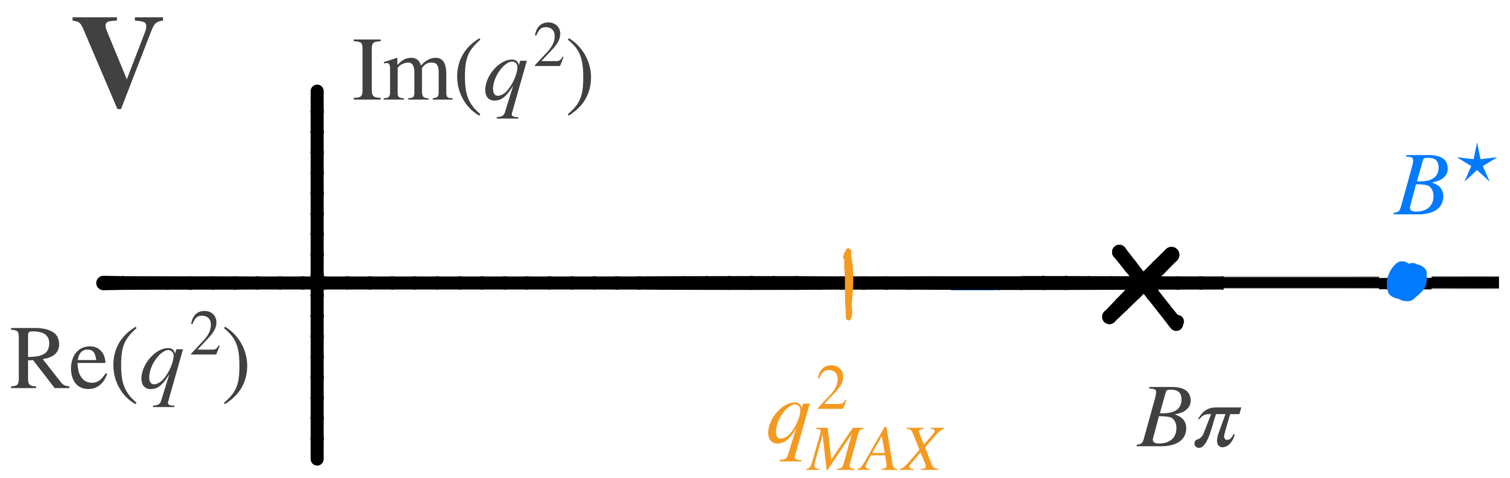


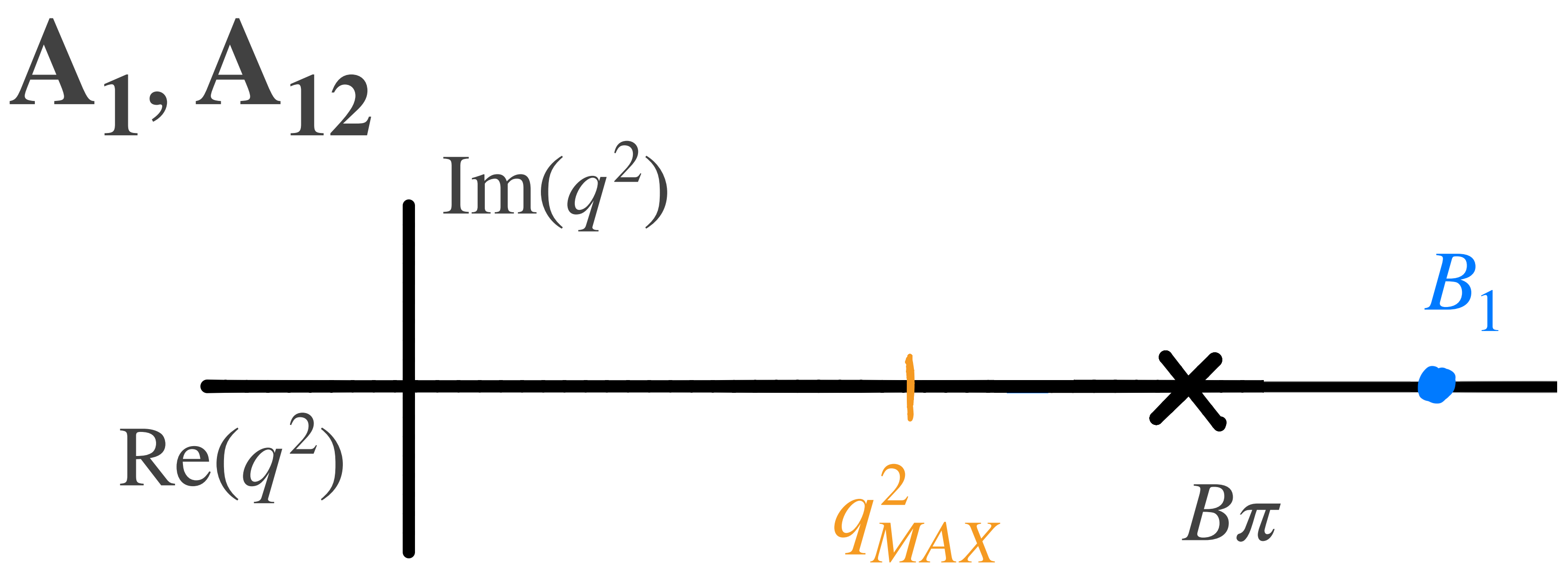
Horgan et al. 1310.3722

single-hadron: form factors

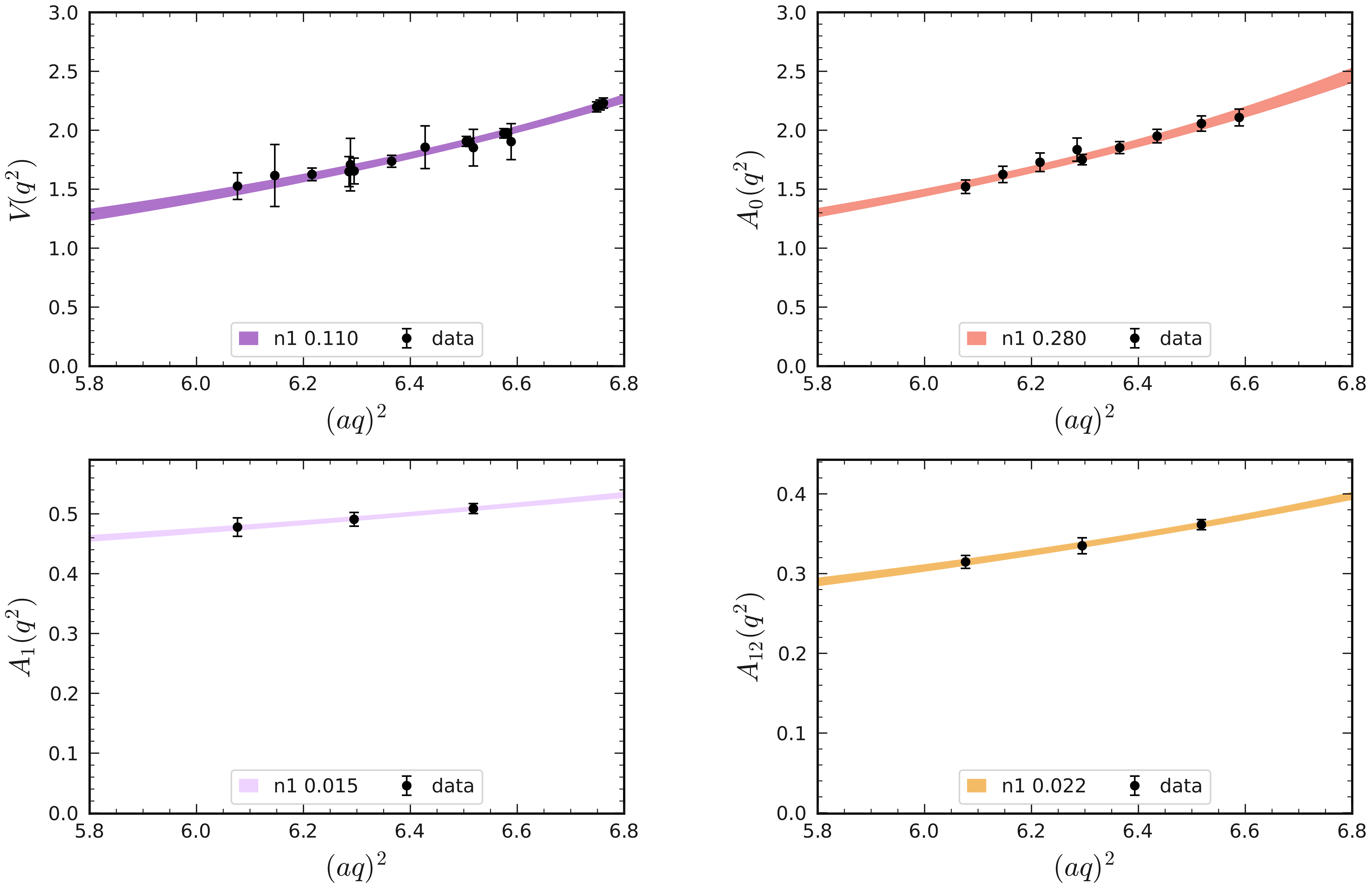
$$F^{(i)}(q^2) = \frac{1}{1 - \frac{q^2}{\left(m_{\text{pole}}^{(i)}\right)^2}} \sum_{n=0}^{1} a_n^{(i)} \left[z^{(i)}\right]^n \qquad z^{(i)}(q^2) = \frac{\sqrt{t_+^{(i)} - q^2} - \sqrt{t_+^{(i)} - t_0^{(i)}}}{\sqrt{t_+^{(i)} - q^2} + \sqrt{t_+^{(i)} - t_0^{(i)}}}$$



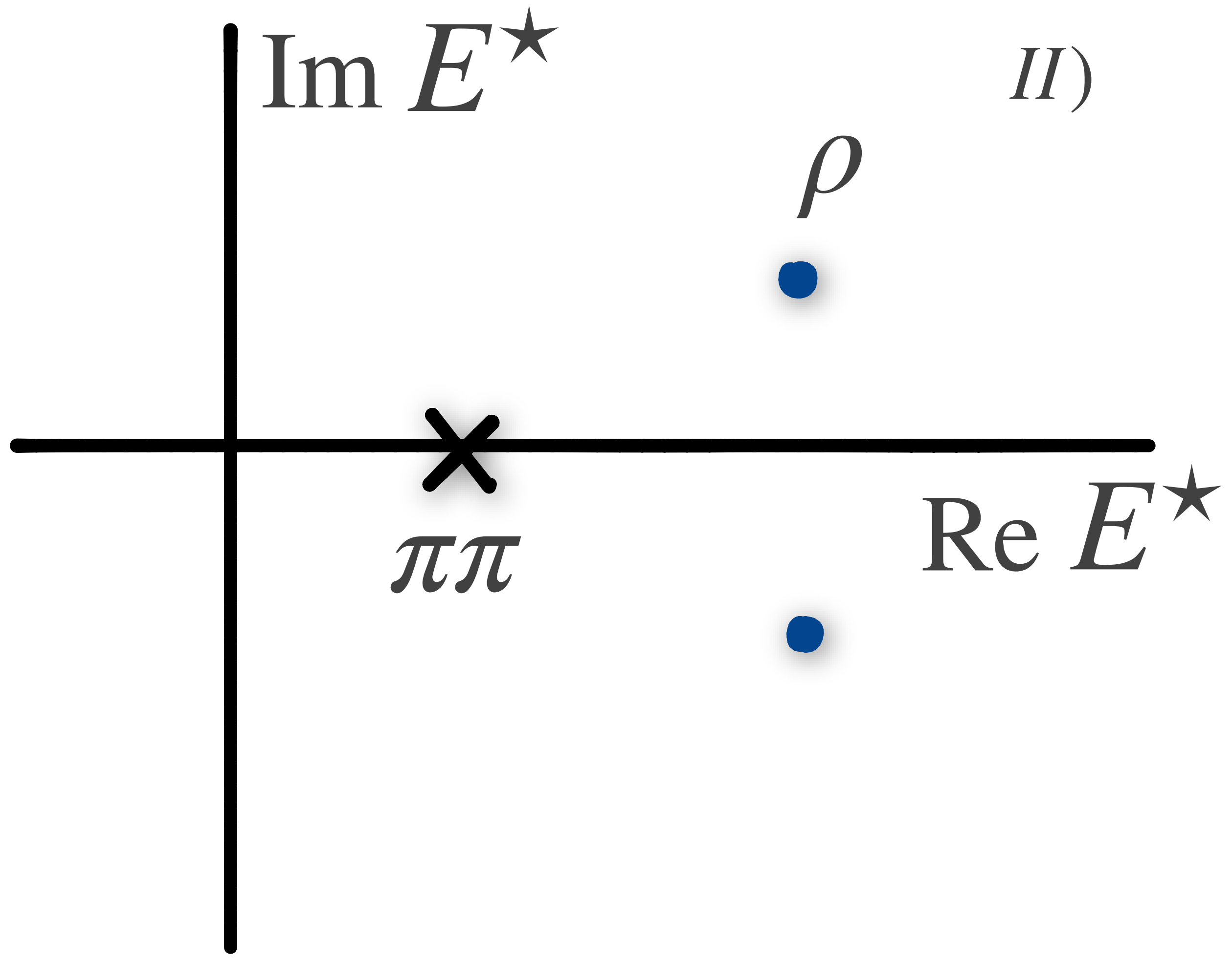


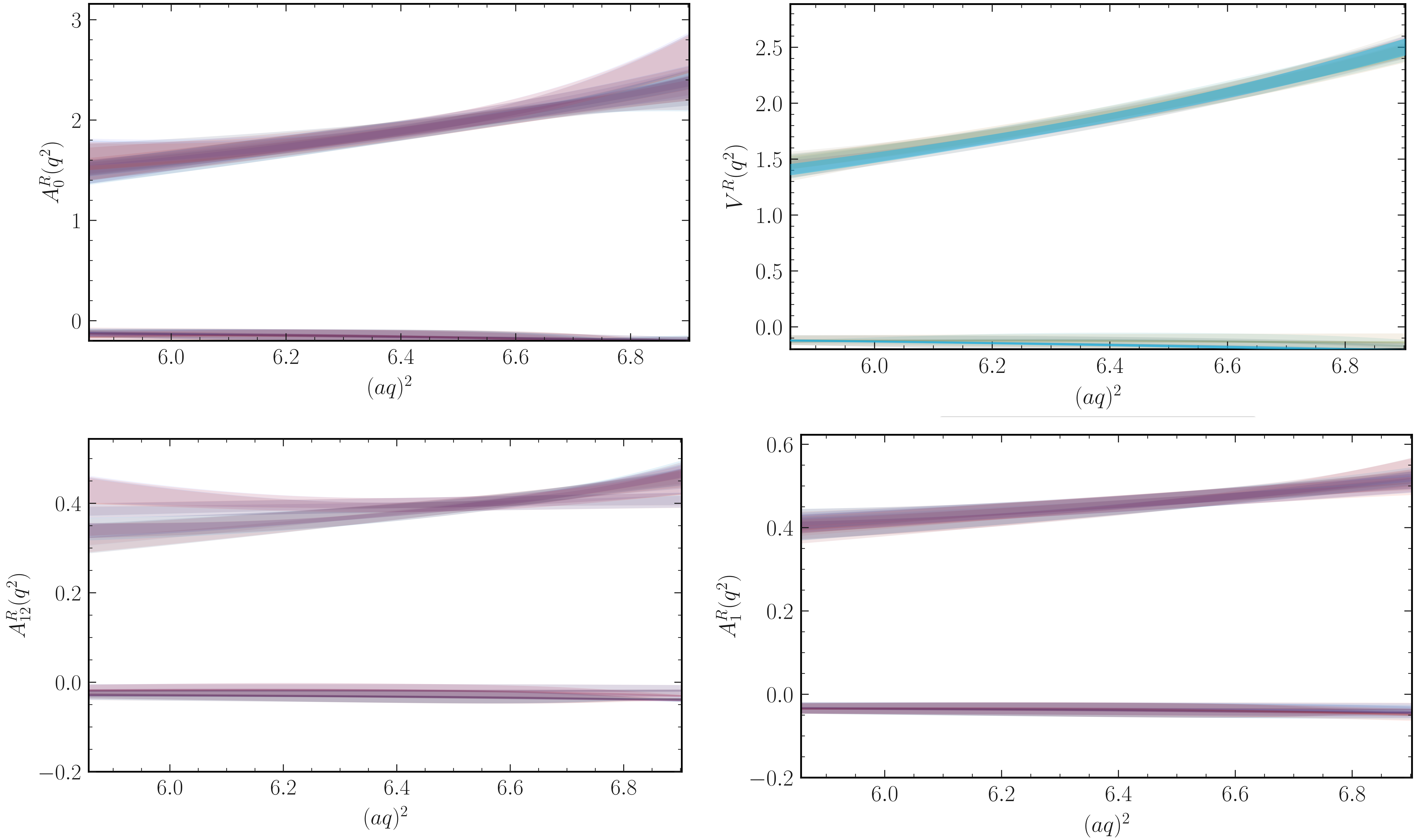


single-hadron: form factors

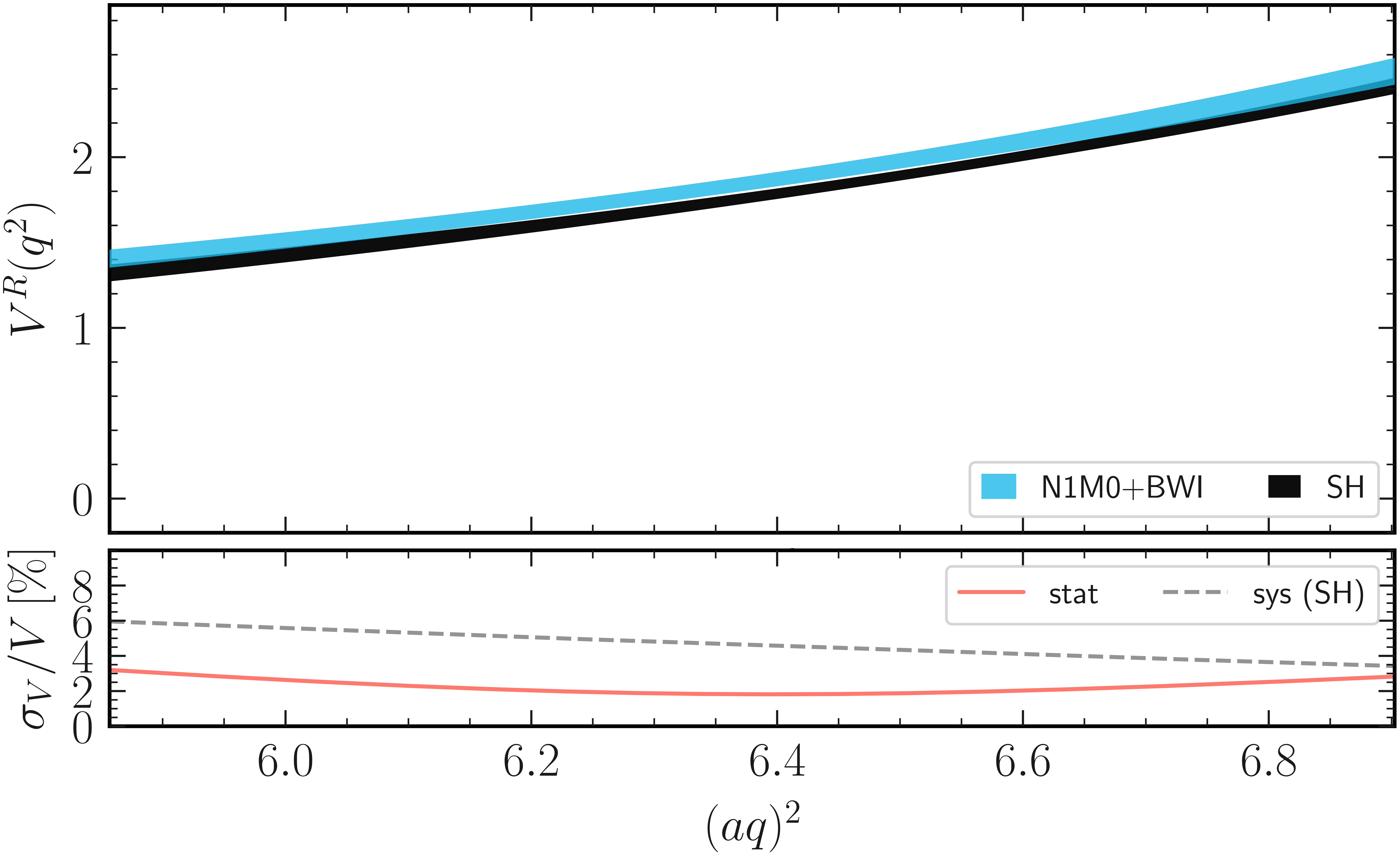


comparing the MH with the SH

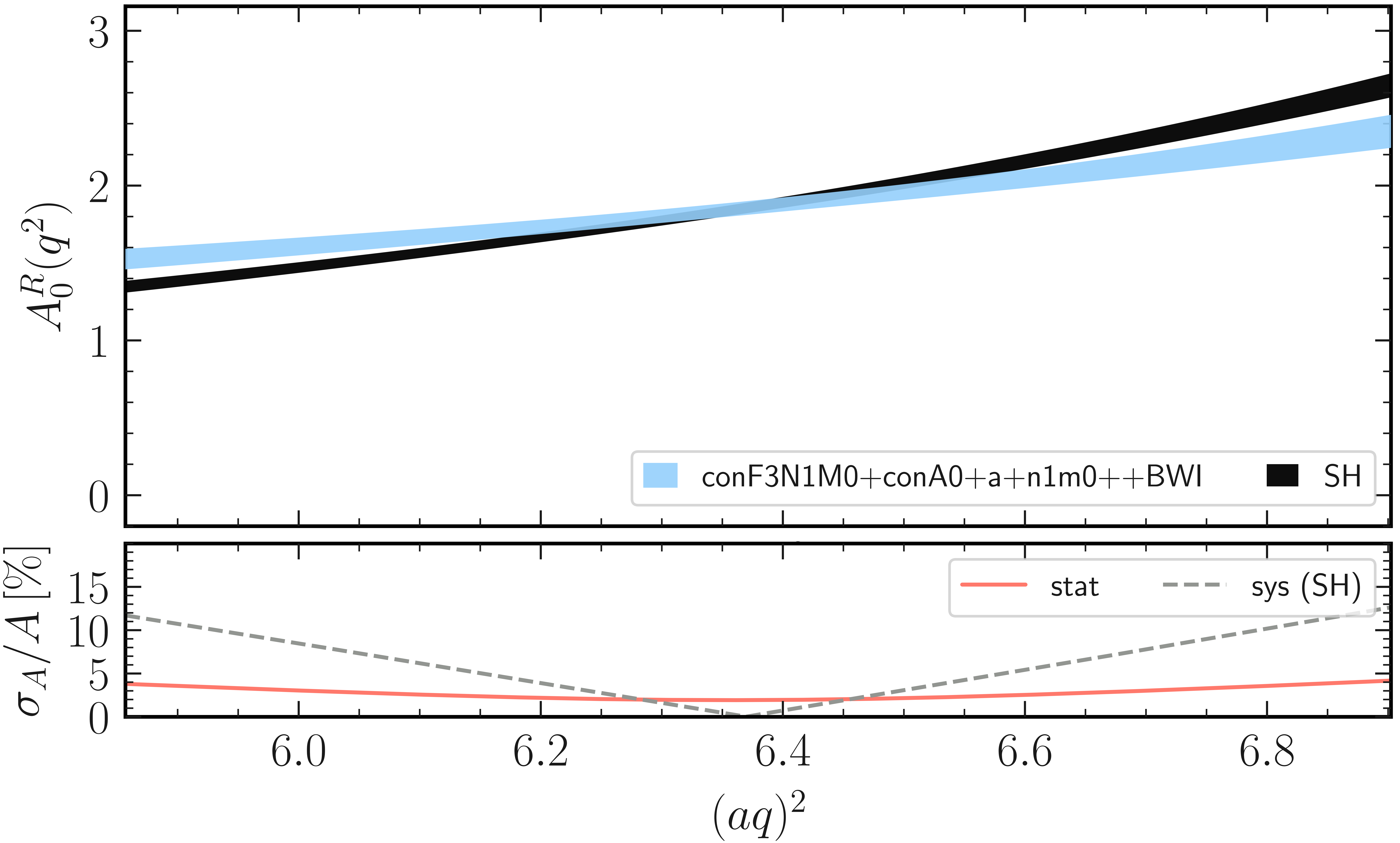




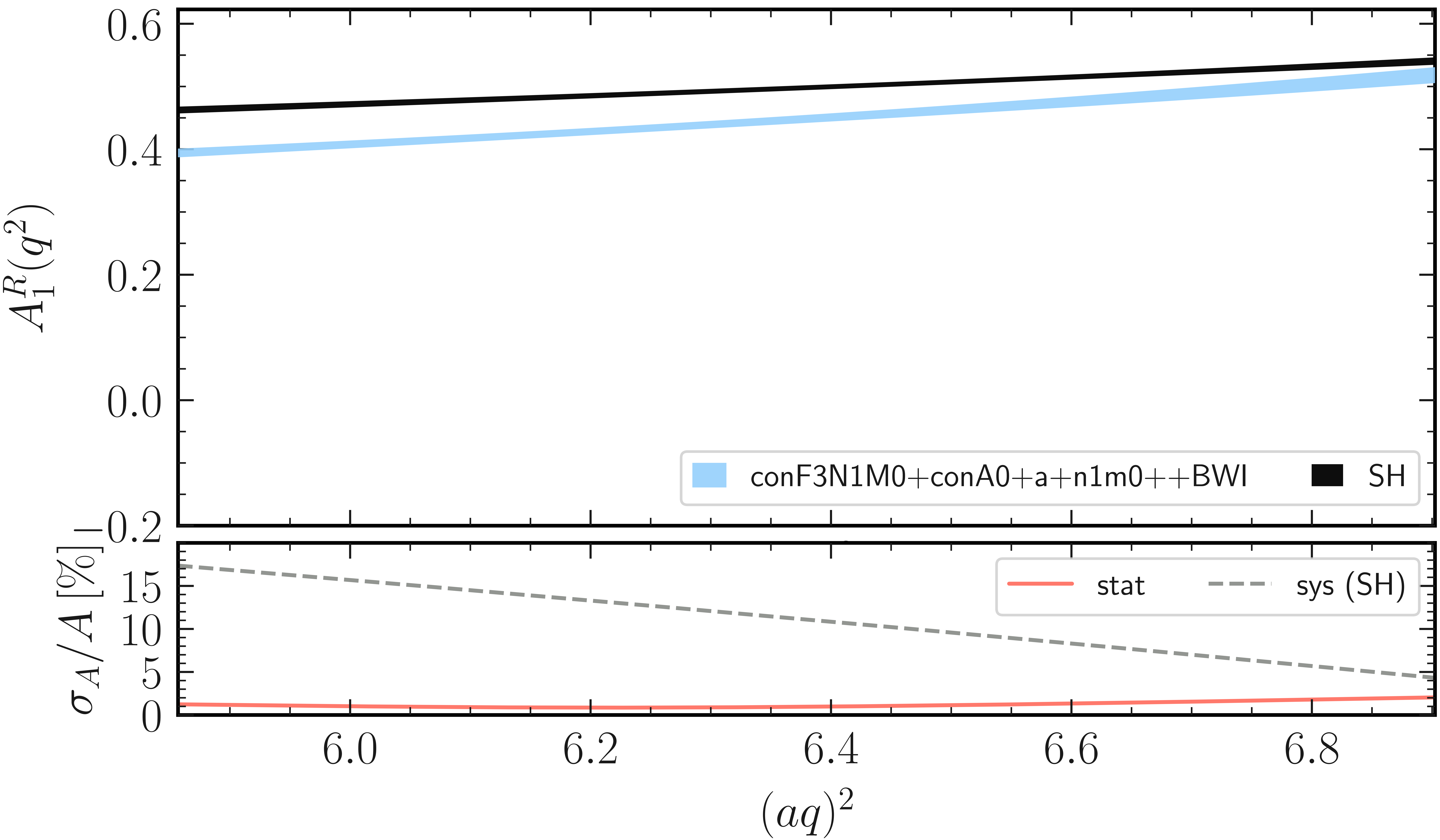
$$(i)^R(q^2) = \frac{c_\rho}{k_\rho} F^{(i)}(E^\star = E_\rho^{pole}, q^2)$$



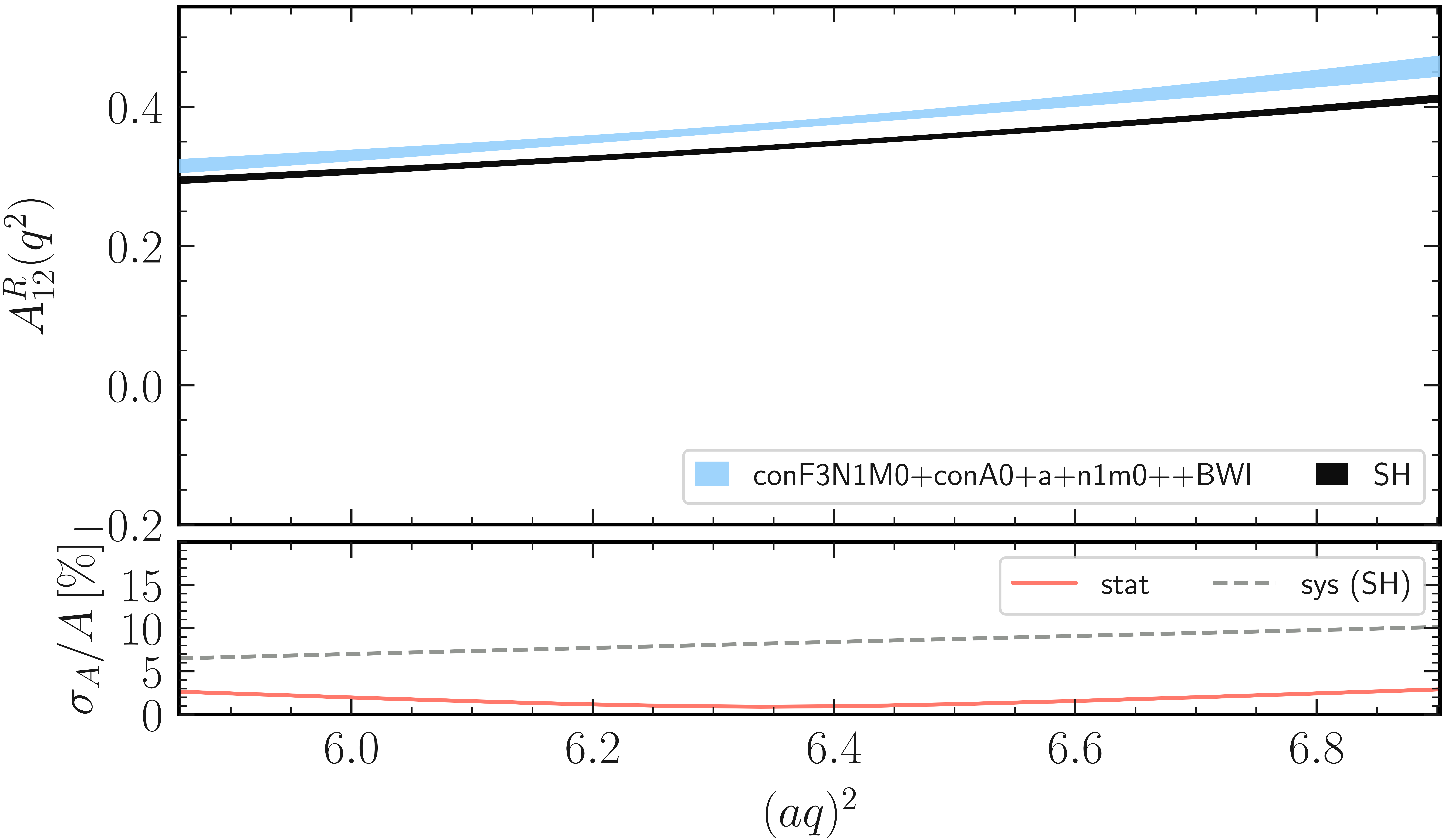
$V^R(q^2)$
0
1
2
N1M0+BWI
SH
σ_V/V [%]
0
2
4
6
8
stat
sys (SH)
6.0
6.2
6.4
6.6
6.8
$(aq)^2$



$A_0^R(q^2)$
conF3N1M0+conA0+a+n1m0++BWI
SH
$\sigma_A/A\,[\%]$
stat
sys (SH)
$(aq)^2$



0.6
0.4
0.2
0.0
−0.2
$A_1^R(q^2)$
conF3N1M0+conA0+a+n1m0++BWI
SH
15
10
5
0
$\sigma_A/A\,[\%]$
stat
sys (SH)
6.0
6.2
6.4
6.6
6.8
$(aq)^2$


$A_{12}^R(q^2)$
0.4
0.2
0.0
−0.2
conF3N1M0+conA0+a+n1m0++BWI
SH
$\sigma_A/A\,[\%]$
15
10
5
0
stat
sys (SH)
6.0
6.2
6.4
6.6
6.8
$(aq)^2$

Summary

Briceño and Hansen are right

Multichannel $0 \to 2$ and $1 \to 2$ transition amplitudes for arbitrary spin particles in a finite volume

Raúl A. Briceño[1,*] and Maxwell T. Hansen[2,†]

$$|\langle E_R, \vec{p}_R, L \,|\, J \,|\, E, \vec{p} \rangle|^2 = |F_{1\to R}|^2 \left[1 + \mathcal{O}\left(\frac{\Gamma_R}{m_R} \right) \right]$$

but it's not simple

the Lellouch-Lüscher factor affects the state

so the caveat:

- straightforward if a single form factor in the current (vector current)
- complicated for multiple form factors (axial current)