Mellin Moments of Proton Unpolarized GPDs at Nonzero Skewness From Lattice QCD With Neural Networks

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GPDs Mellin Moments | **Lattice QCD** | **Artificial Neural Networks**

Goal: internal structure of the nucleon

- Internal structure of hadrons → quantified by partonic functions
- Partons → quarks and gluons viewed in a high-energy scattering
- Independent constituents carrying fractions of the hadron momentum

Generalized Parton Distribution Functions (GPDs)

- GPDs → different initial and final hadron states:

$$\text{GPD: } |P_i\rangle \neq |P_f\rangle$$

- Functions of:

x → fraction of the hadron's momentum carried by a parton

$t \equiv \Delta^2$ → square of the momentum transfer Δ^μ

ξ → skewness, quantifies the fraction of momentum transferred in the longitudinal direction

Unpolarized Proton GPDs definition

$$F^{\gamma^+}_{s's}(x,\xi,t) = \int \frac{dz^-}{4\pi}\, e^{ixP^+z^-}\, \langle P', s' | \bar{q}\left(-\tfrac{z}{2}\right) \gamma^+ W\left(-\tfrac{z}{2}, \tfrac{z}{2}\right) q\left(\tfrac{z}{2}\right) | P, s\rangle \Big|_{z^+=0,\, \mathbf{z}_T=0}$$

- Wilson line $\to W\left(\frac{-z}{2}, \frac{z}{2}\right)$

$|P, s\rangle \to$ In this work, proton state

- $\gamma^+ \to$ unpolarized quark sector

$$\text{Light-cone coordinates} \;\; \to z^{\pm} = \frac{z^0 \pm z^3}{\sqrt{2}}, \qquad \vec{z}_T = (z^1, z^2)$$

$$z^+ = 0,\ \vec{z}_T = \vec{0}_T,\ z^- \neq 0 \implies \boxed{z^2 = 0}$$

Unpolarized Proton GPDs definition

$$F^{\gamma^+}_{s's}(x,\xi,t) = \int \frac{dz^-}{4\pi}\, e^{ixP^+z^-}\, \langle P', s' \big| \bar{q}\left(-\tfrac{z}{2}\right) \gamma^+ W\left(-\tfrac{z}{2}, \tfrac{z}{2}\right) q\left(\tfrac{z}{2}\right) \big| P, s \rangle \Big|_{z^+=0,\, \mathbf{z}_T=0}$$

- OPE + Parameterization on ME above → twist-2 GPDs

$$F^{\gamma^+}_{s's}(x,\xi,t) = \frac{1}{2P^+}\bar{u}(P',s')\left[\gamma^+ H(x,\xi,t) + \frac{i\sigma^{+\alpha}\Delta_\alpha}{2m_N} E(x,\xi,t)\right] u(P,s)$$

Unpolarized proton twist-2 GPDs → H and E

Mellin moments and polynomiality

$$\text{Mellin moments of } H \rightarrow \int_{-1}^{1} dx\, x^n\, H(x,\xi,t) = \sum_{\substack{i=0\\ \text{even}}}^{n} (2\xi)^i\, A_{n+1,i}(t) \; + \; \mathrm{mod}(n,2)\,(2\xi)^{n+1}\, C_{n+1}(t)$$

$$\text{Mellin moments of } E \rightarrow \int_{-1}^{1} dx\, x^n\, E(x,\xi,t) = \sum_{\substack{i=0\\ \text{even}}}^{n} (2\xi)^i\, B_{n+1,i}(t) \; - \; \mathrm{mod}(n,2)\,(2\xi)^{n+1}\, C_{n+1}(t).$$

Generalized form factors (GFFs) $\rightarrow$ $A_{n+1,i}(t), B_{n+1,i}(t), C_{n+1}(t)$

Physical meaning of GFFs

- A_{10} and B_{10} → information on the electromagnetic structure of the nucleon

- A_{20} and B_{20}, Ji's quark total angular momentum sum rule → $J = 1/2[A_{20}(0) + B_{20}(0)]$

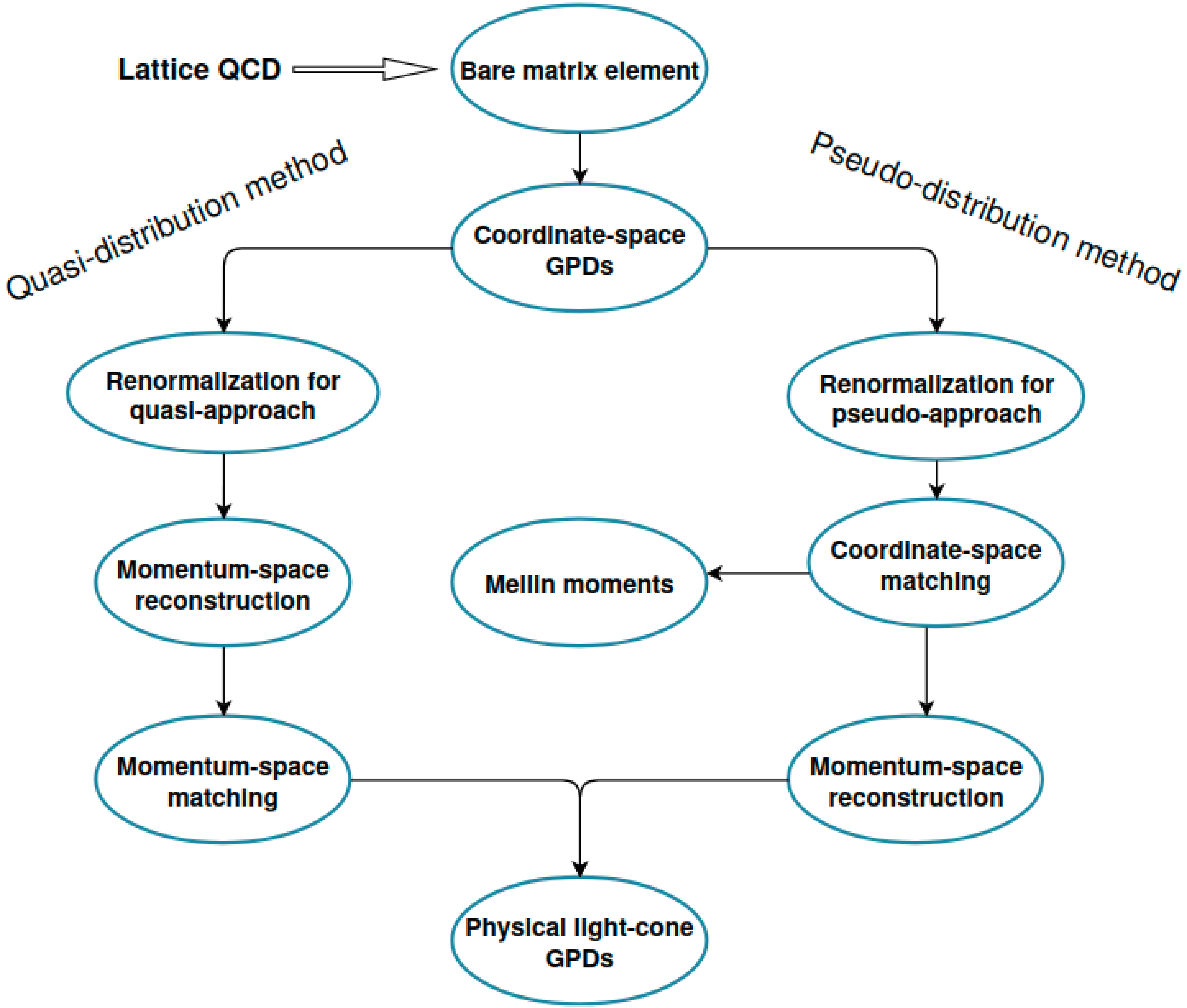
- C_2 → only contributes at $\xi \neq 0$
  → related to the D-term structure of the energy-momentum tensor
  → information on "mechanical" structure inside hadrons

Internal structure of the nucleon

Methodology

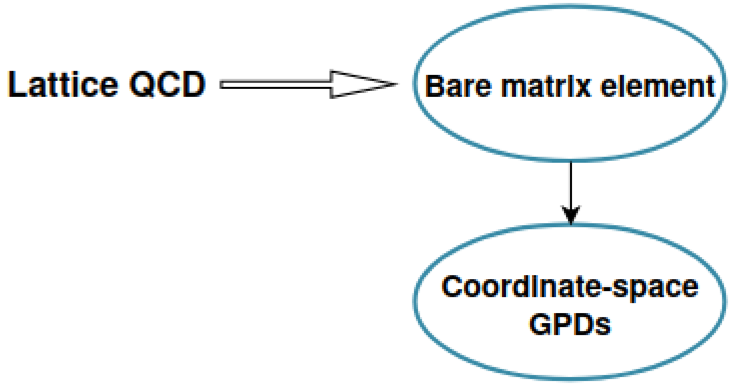
How to use lattice QCD and ANNs compute these GFFs?

Methodology



Coordinate-space GPDs from Lattice Matrix Elements

- Coordinate-space GPDs:
  - Linear combination of lattice matrix elements
  - Discrete and defined in coordinate-space



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Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

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Shohini Bhattacharya[1,*], Krzysztof Cichy[2], Martha Constantinou[3,†], Jack Dodson[3], Xiang Gao[4], Andreas Metz[3], Swagato Mukherjee[1], Aurora Scapellato[3], Fernanda Steffens[5] *et al.*

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Phys. Rev. D **106**, 114512 – **Published 26 December, 2022**

DOI: https://doi.org/10.1103/PhysRevD.106.114512

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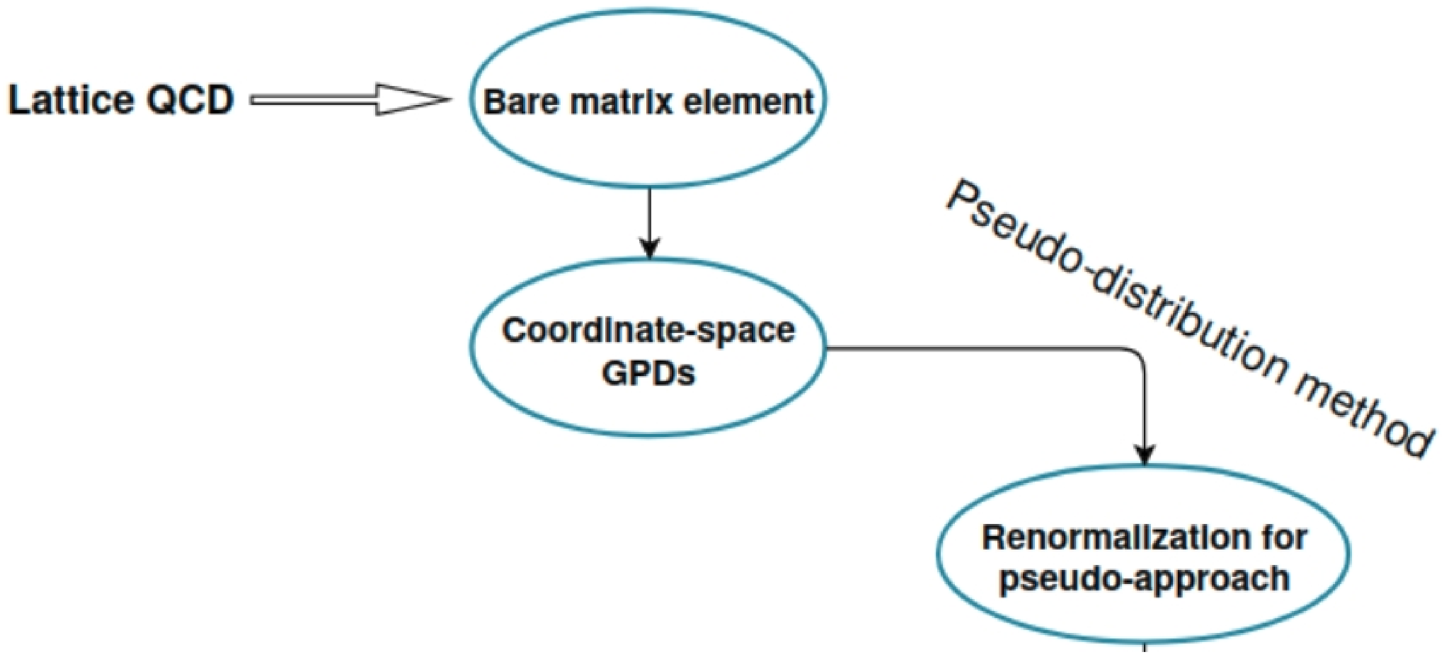
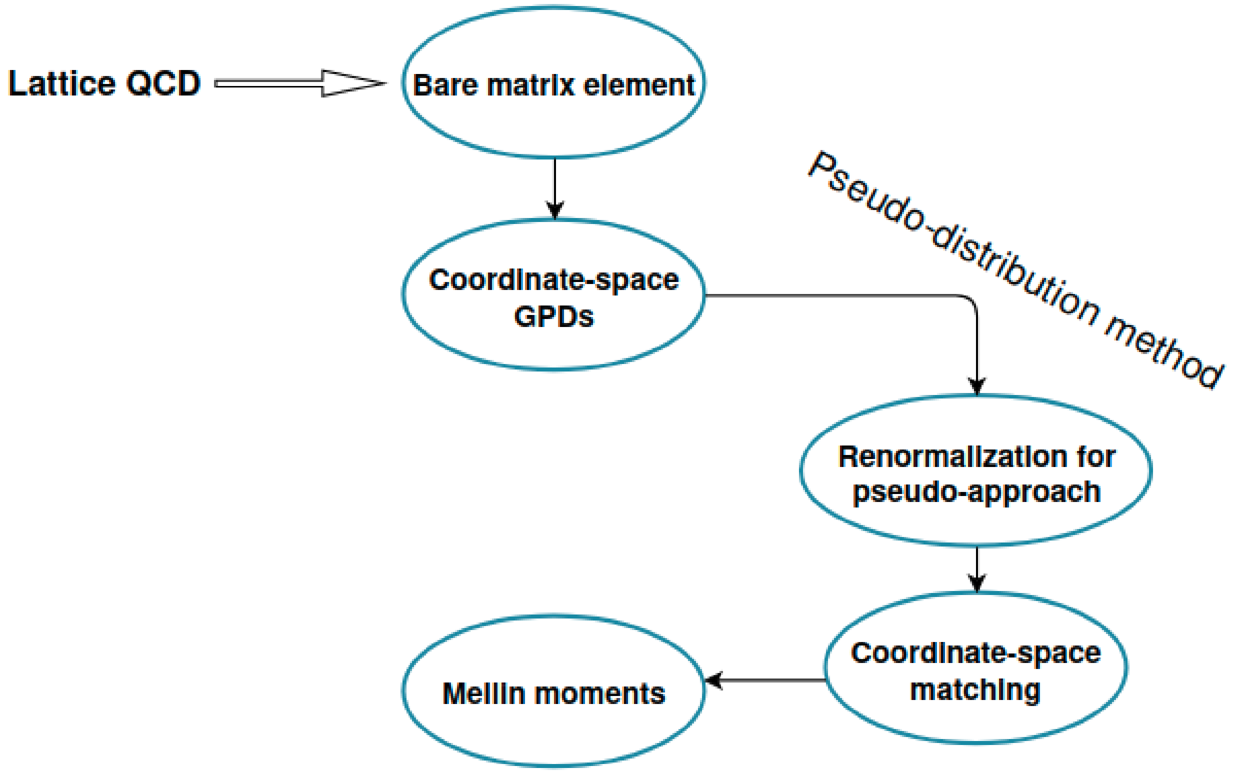
Reduced Ioffe-time Distributions (ITDs)



- Renormalization → ratio-scheme → divide by PDF → cancel divergent factors
- Renormalized H and E → Reduced ITDs → $F(\nu, t, \xi, z^2),\ \ \nu := P \cdot z,\ \ P := (P_i + P_f)/2$

Pseudo-distribution – Short Distance Matching



$$\underbrace{F_n(\xi,t,z^2)}_{\substack{\text{finite-}z^2\\ \text{lattice quantity}}} = \sum_m \underbrace{C_{nm}(z^2\mu^2)}_{\substack{\text{Wilson matching}\\ \text{coefficient}}} \underbrace{\overline{F}_m(\xi,t)}_{\substack{\text{light-cone } z^2=0\\ \text{GPD moment}}} + \underbrace{\mathcal{O}\big(z^2\Lambda^2_{\mathrm{QCD}},\, z^2 m_N^2,\, z^2 t\big)}_{\substack{\text{finite-}z^2 \text{ power corrections}\\ \text{vanish as } z^2\to 0}}$$

From Matching + Polynomiality and Truncation to GFFs

$$\mathrm{Re}\, F^{(H)}(\nu, t, \xi; z) = \left(c_{1,0} - \frac{\nu^2}{2}\xi^2 c_{3,2}\right) A_{1,0}(t) - \frac{\nu^2}{2} c_{3,0}\, A_{3,0}(t) - \frac{\nu^2}{2}\xi^2 c_{3,0}\, A_{3,2}(t)(t) + O(\nu^4)$$

$$\mathrm{Im}\, F^{(H)}(\nu, t, \xi; z) = \left(-\nu c_{2,0} + \frac{\nu^3}{6}\xi^2 c_{4,2}\right) A_{2,0}(t) + \left(-\nu \xi^2 c_{2,0} + \frac{\nu^3}{6}\xi^4 c_{4,2}\right) C_2(t)$$
$$+ \frac{\nu^3}{6} c_{4,0}\, A_{4,0}(t) + \frac{\nu^3}{6}\xi^2 c_{4,0}\, A_{4,2}(t) + \frac{\nu^3}{6}\xi^4 c_{4,0}\, C_4(t) + O(\nu^5)$$

$$\mathrm{Re}\, F^{(E)}(\nu, t, \xi; z) = \left(c_{1,0} - \frac{\nu^2}{2}\xi^2 c_{3,2}\right) B_{1,0}(t) - \frac{\nu^2}{2} c_{3,0}\, B_{3,0}(t) - \frac{\nu^2}{2}\xi^2 c_{3,0}\, B_{3,2}(t) + O(\nu^4)$$

$$\mathrm{Im}\, F^{(E)}(\nu, t, \xi; z) = \left(-\nu c_{2,0} + \frac{\nu^3}{6}\xi^2 c_{4,2}\right) B_{2,0}(t) + \left(\nu \xi^2 c_{2,0} - \frac{\nu^3}{6}\xi^4 c_{4,2}\right) C_2(t)$$
$$+ \frac{\nu^3}{6} c_{4,0}\, B_{4,0}(t) + \frac{\nu^3}{6}\xi^2 c_{4,0}\, B_{4,2}(t) - \frac{\nu^3}{6}\xi^4 c_{4,0}\, C_4(t) + O(\nu^5)$$

Kinematics of the data points – correlation between (ν,t,ξ)

$$\mathrm{Re}\, F^{(H)}(\nu,t,\xi;z) = \left(c_{1,0} - \frac{\nu^2}{2}\xi^2 c_{3,2}\right) A_{1,0}(t) - \frac{\nu^2}{2} c_{3,0}\, A_{3,0}(t) - \frac{\nu^2}{2}\xi^2 c_{3,0}\, A_{3,2}(t)$$

$$\mathrm{Re}\, F^{(E)}(\nu,t,\xi;z) = \left(c_{1,0} - \frac{\nu^2}{2}\xi^2 c_{3,2}\right) B_{1,0}(t) - \frac{\nu^2}{2} c_{3,0}\, B_{3,0}(t) - \frac{\nu^2}{2}\xi^2 c_{3,0}\, B_{3,2}(t)$$

$$\mathrm{Im}\, F^{(H)}(\nu,t,\xi;z) = \left(-\nu c_{2,0} + \frac{\nu^3}{6}\xi^2 c_{4,2}\right) A_{2,0}(t) + \left(-\nu\xi^2 c_{2,0} + \frac{\nu^3}{6}\xi^4 c_{4,2}\right) C_2(t) + \frac{\nu^3}{6} c_{4,0}\, A_{4,0}(t) + \frac{\nu^3}{6}\xi^2 c_{4,0}\, A_{4,2}(t) + \frac{\nu^3}{6}\xi^4 c_{4,0}\, C_4(t)$$

$$\mathrm{Im}\, F^{(E)}(\nu,t,\xi;z) = \left(-\nu c_{2,0} + \frac{\nu^3}{6}\xi^2 c_{4,2}\right) B_{2,0}(t) + \left(\nu\xi^2 c_{2,0} - \frac{\nu^3}{6}\xi^4 c_{4,2}\right) C_2(t) + \frac{\nu^3}{6} c_{4,0}\, B_{4,0}(t) + \frac{\nu^3}{6}\xi^2 c_{4,0}\, B_{4,2}(t) - \frac{\nu^3}{6}\xi^4 c_{4,0}\, C_4(t)$$

- For $z=(0,0,0,z_3)$: $\nu = z_3 P_3, \quad \xi = -\dfrac{\Delta_3}{2P_3}, \quad t = (E_f - E_i)^2 - \boldsymbol{\Delta}^2, \quad E_{f/i} = \sqrt{m_N^2 + \boldsymbol{P}_{f/i}^2}$

In lattice units ($z_3 = 1$) $\rightarrow$

P_3	Δ_3	ν	ξ	t GeV^2
2	1	2	$-1/4$	-1.25
3	1	3	$-1/6$	-1.12
3	2	3	$-1/3$	-1.53

Changing P_3 changes ν and also changes t.

Changing Δ_3 changes ξ and also changes t.

Generally, data do not provide independent samples at fixed t

Artificial Neural Network and GFFs

$$\mathrm{Re}\,F^{(H)}(\nu,t,\xi;z) = \left(c_{1,0} - \frac{\nu^2}{2}\xi^2 c_{3,2}\right) A_{1,0}(t) - \frac{\nu^2}{2} c_{3,0}\, A_{3,0}(t) - \frac{\nu^2}{2}\xi^2 c_{3,0}\, A_{3,2}(t)$$

$$\mathrm{Re}\,F^{(E)}(\nu,t,\xi;z) = \left(c_{1,0} - \frac{\nu^2}{2}\xi^2 c_{3,2}\right) B_{1,0}(t) - \frac{\nu^2}{2} c_{3,0}\, B_{3,0}(t) - \frac{\nu^2}{2}\xi^2 c_{3,0}\, B_{3,2}(t)$$

$$\begin{aligned}\mathrm{Im}\,F^{(H)}(\nu,t,\xi;z) &= \left(-\nu c_{2,0} + \frac{\nu^3}{6}\xi^2 c_{4,2}\right) A_{2,0}(t) + \left(-\nu\xi^2 c_{2,0} + \frac{\nu^3}{6}\xi^4 c_{4,2}\right) C_2(t) \\ &+ \frac{\nu^3}{6} c_{4,0}\, A_{4,0}(t) + \frac{\nu^3}{6}\xi^2 c_{4,0}\, A_{4,2}(t) + \frac{\nu^3}{6}\xi^4 c_{4,0}\, C_4(t)\end{aligned}$$

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$$\text{Parameterize GFFs} \to A_{1,0}, A_{2,0}, A_{3,0}, A_{3,2}, A_{4,0}, A_{4,2}, B_{1,0}, B_{2,0}, B_{3,0}, B_{3,2}, B_{4,0}, B_{4,2}, C_2, C_4,$$

$$G^{\mathrm{ANN}}(t) = \sum_{i=1}^{M} W_i^{(G)}\ \tanh\!\left(w_i^{(G)}\, t + b_i^{(G)}\right) + B^{(G)}$$

ANN is just a flexible global fit ansatz for the t-dependence of the GFFs

Artificial Neural Network and GFFs

$$\mathrm{Re}\,F^{(H)}(\nu,t,\xi;z)=\left(c_{1,0}-\frac{\nu^2}{2}\xi^2c_{3,2}\right)A_{1,0}(t)-\frac{\nu^2}{2}c_{3,0}\,A_{3,0}(t)-\frac{\nu^2}{2}\xi^2c_{3,0}\,A_{3,2}(t)$$

$$\mathrm{Re}\,F^{(E)}(\nu,t,\xi;z)=\left(c_{1,0}-\frac{\nu^2}{2}\xi^2c_{3,2}\right)B_{1,0}(t)-\frac{\nu^2}{2}c_{3,0}\,B_{3,0}(t)-\frac{\nu^2}{2}\xi^2c_{3,0}\,B_{3,2}(t)$$

$$\begin{aligned}\mathrm{Im}\,F^{(H)}(\nu,t,\xi;z)=&\left(-\nu c_{2,0}+\frac{\nu^3}{6}\xi^2c_{4,2}\right)A_{2,0}(t)+\left(-\nu\xi^2c_{2,0}+\frac{\nu^3}{6}\xi^4c_{4,2}\right)C_2(t)\\&+\frac{\nu^3}{6}c_{4,0}\,A_{4,0}(t)+\frac{\nu^3}{6}\xi^2c_{4,0}\,A_{4,2}(t)+\frac{\nu^3}{6}\xi^4c_{4,0}\,C_4(t)\end{aligned}$$

$$\begin{aligned}\mathrm{Im}\,F^{(E)}(\nu,t,\xi;z)=&\left(-\nu c_{2,0}+\frac{\nu^3}{6}\xi^2c_{4,2}\right)B_{2,0}(t)+\left(\nu\xi^2c_{2,0}-\frac{\nu^3}{6}\xi^4c_{4,2}\right)C_2(t)\\&+\frac{\nu^3}{6}c_{4,0}\,B_{4,0}(t)+\frac{\nu^3}{6}\xi^2c_{4,0}\,B_{4,2}(t)-\frac{\nu^3}{6}\xi^4c_{4,0}\,C_4(t)\end{aligned}$$

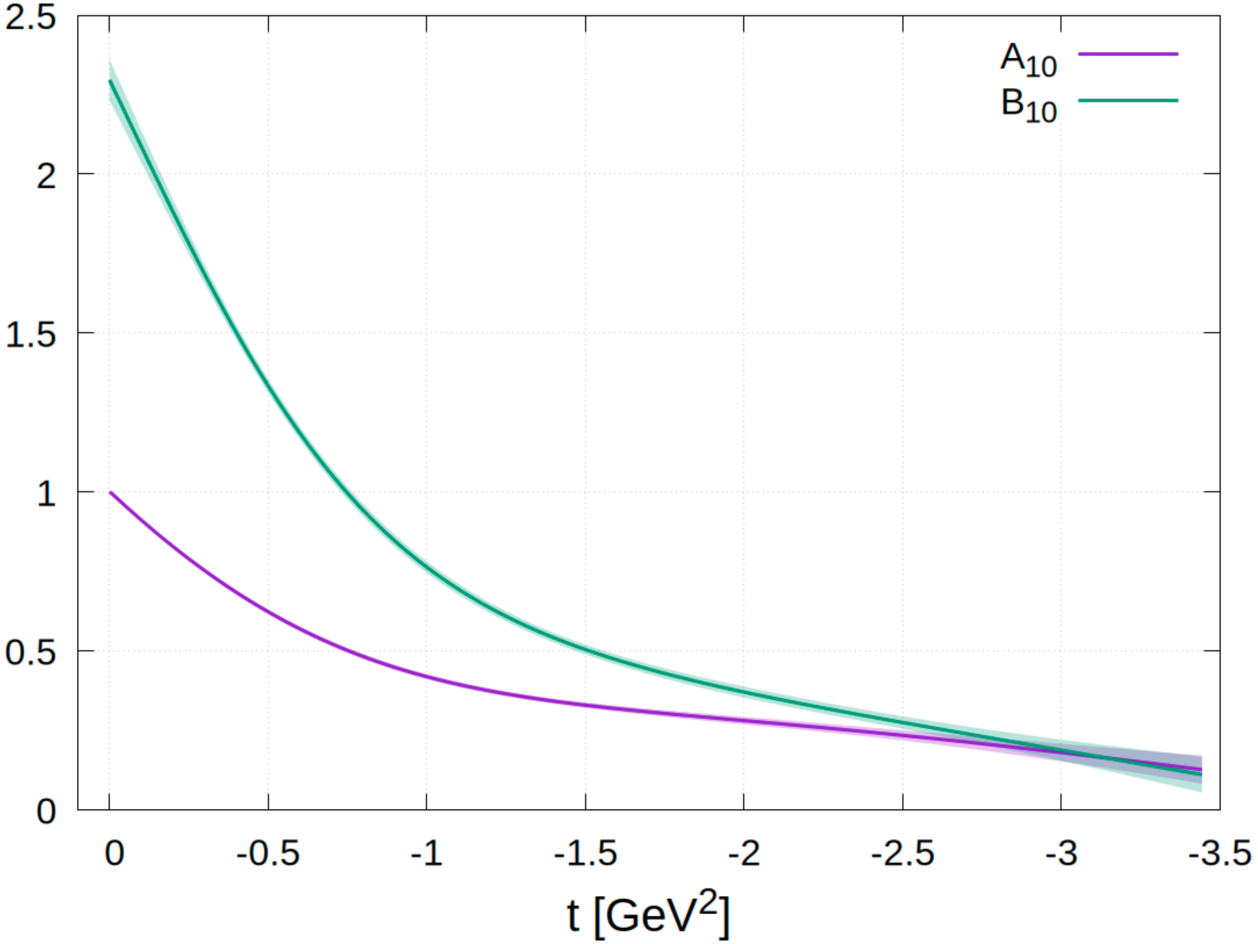
$$G^{\mathrm{ANN}}(t)=\sum_{i=1}^{M}W_i^{(G)}\,\tanh\!\left(w_i^{(G)}\,t+b_i^{(G)}\right)+B^{(G)}$$

$$\text{Train parameters}\to\left\{W_i^{(G)},\,w_i^{(G)},\,b_i^{(G)},\,B^{(G)}\right\}\iff\text{Minimize }\chi^2\sim(\text{data}-\text{theory})^2\leftarrow\text{Global fit}$$

Results: isovector u − d, NLO, $\mu = 2\,\mathrm{GeV}, \overline{\mathrm{MS}}$

- Preliminary proof-of-concept results
    - Controlling lattice systematics
    - Checks of stability of GFFs under small ANN setup variations
    - Some of the data used for training may have large $\underbrace{\mathcal{O}(z^2)}_{\text{power corrections vanish as } z^2 \to 0}$
    - Large logs and α_s in matching $\to$ breakdown of perturbation theory
- Cross-checked for and checked self-consistency
- Soon publication-ready

A_{10} and B_{10}

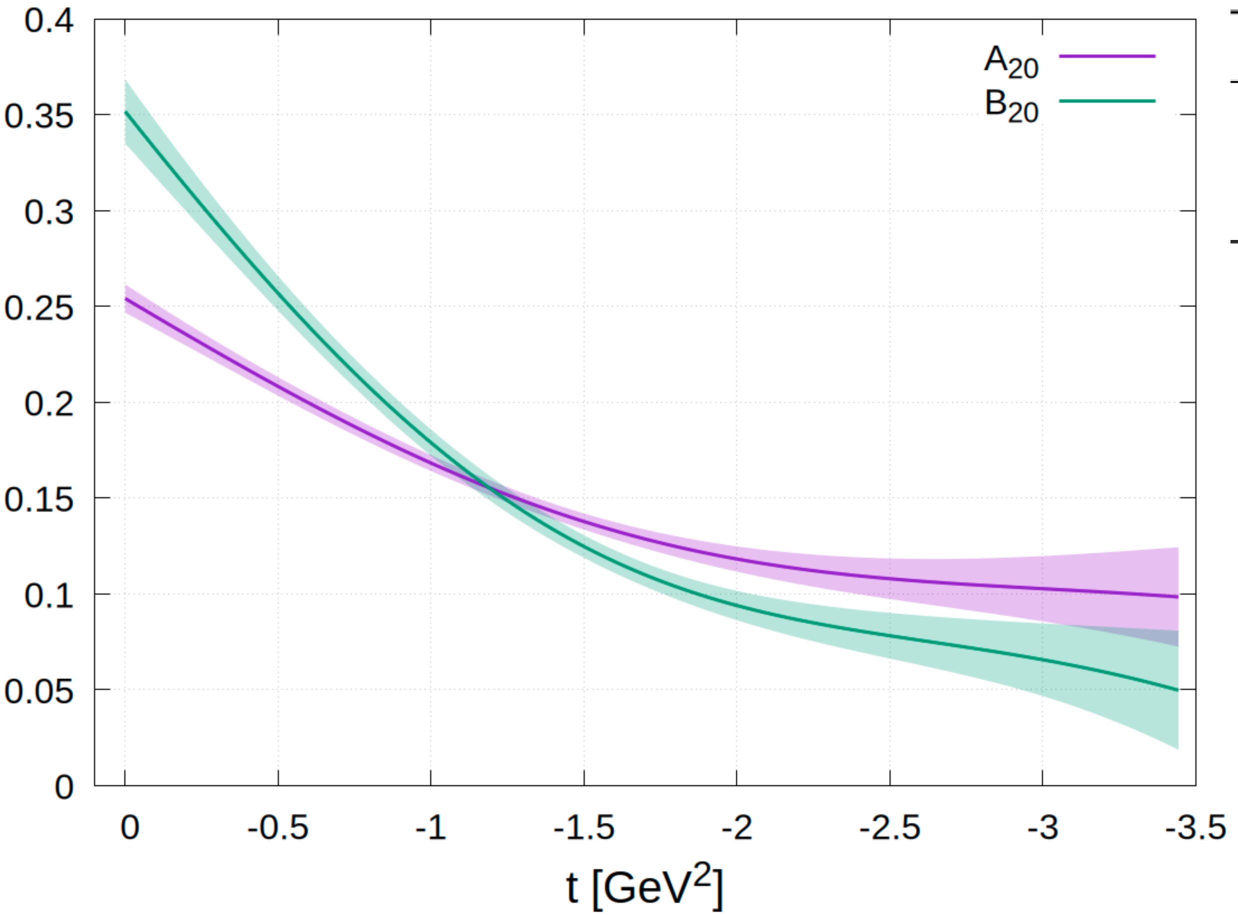


Source	$A_{10}^{u-d}(0)$	$B_{10}^{u-d}(0)$
This work	0.9937(16)	2.30(6)
Bhattacharya et al. 2023 [1]	0.98(5)(3)	2.54(22)(11)
Dutrieux et al. 2026 [2]	0.943(15)	3.17(6)

[1] S. Bhattacharya et al. "Moments of proton GPDs from the OPE of nonlocal quark bilinears up to NNLO". In: *Phys. Rev. D* 108.1 (2023), p. 014507. DOI: 10.1103/PhysRevD.108.014507. arXiv: 2305.11117 [hep-lat].

[2] H. Dutrieux et al. *Reconstructing the full kinematic dependence of GPDs from pseudo-distributions.* 2026. arXiv: 2604.21476 [hep-lat].

A_{20} and B_{20}



Source	$A_{20}^{u-d}(0)$	$B_{20}^{u-d}(0)$	J^{u-d}
This work	0.254(7)	0.352(17)	0.303(9)
Bhattacharya et al. 2023 [1]	0.267(13)(10)	0.30(4)(2)	0.281(21)(11)
Dutrieux et al. 2026 [2]	0.215(10)	0.38(2)	0.298(11)

[1] S. Bhattacharya et al. "Moments of proton GPDs from the OPE of nonlocal quark bilinears up to NNLO". In: *Phys. Rev. D* 108.1 (2023), p. 014507. DOI: 10.1103/PhysRevD.108.014507. arXiv: 2305.11117 [hep-lat].

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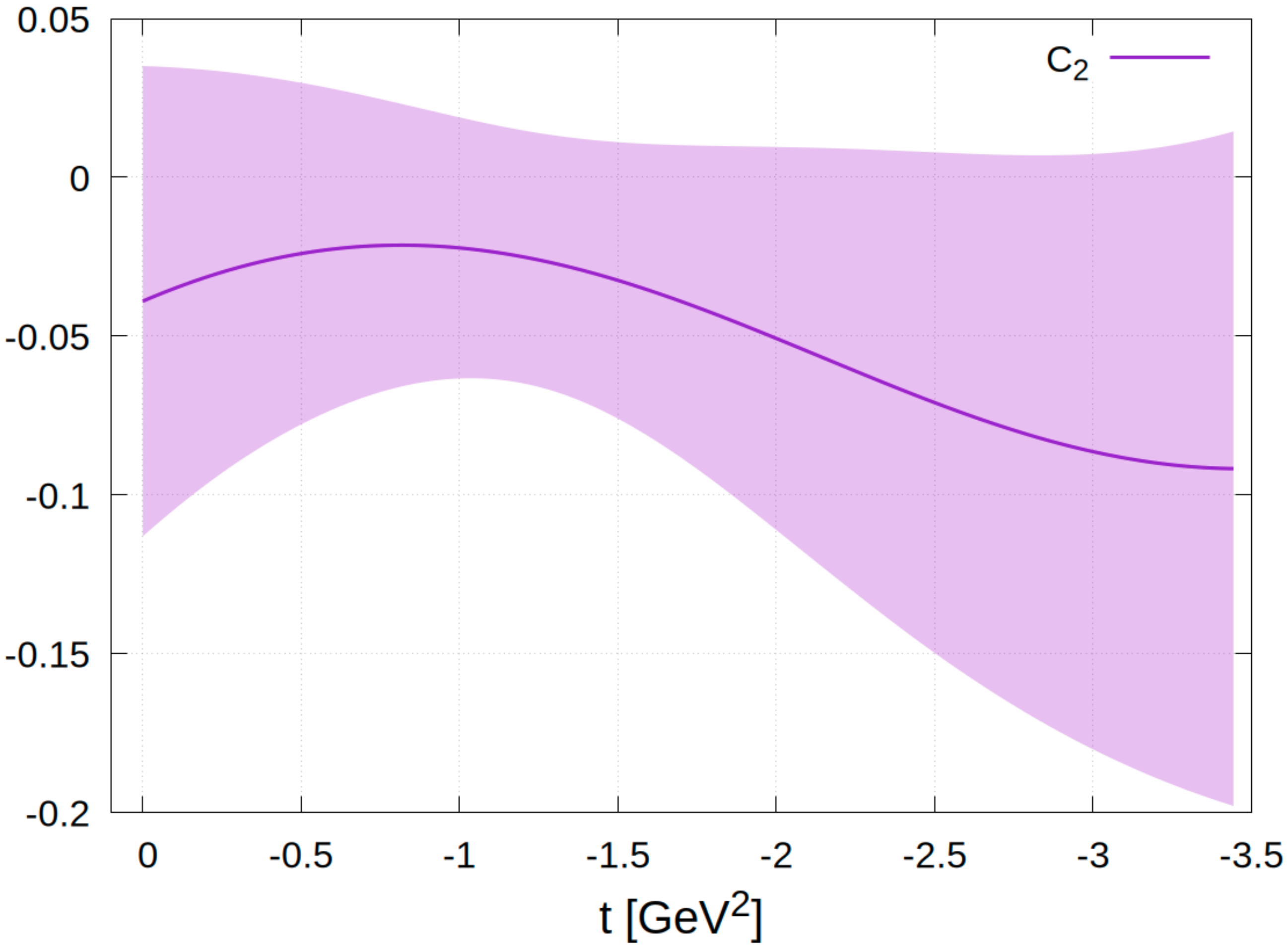
C_2



Source	$C_2^{u-d}(0)$
This work	$-0.04(7)$
Dutrieux et al. 2024 [3]	$0.025(8)$

[3] H. Dutrieux et al. "Towards unpolarized GPDs from pseudo-distributions". In: *JHEP* 08 (2024), p. 162. DOI: 10.1007/JHEP08(2024)162. arXiv: 2405.10304 [hep-lat].

Higher-Type GFFs

- Higher GFFs $\rightarrow$ less directly constrained by the data
- Enter through higher powers of ν and ξ
- More sensitive to fit-systematic effects
- Constraints on the Mellin expansion
- Included to reduce bias on the lower moments due to truncation

Conclusions

- Physical Goal:
    - Extract Mellin moments of proton unpolarized GPDs at nonzero skewness
    - GFFs encode nucleon internal structure

Conclusions

- Methodology – pseudo-distributions:
  - Short-distance matching and polynomiality → relation between GFFs and lattice data

Conclusions

- Methodology – pseudo-distributions:
  - Short-distance matching and polynomiality → relation between GFFs and lattice data
- Methodology –ANNs:
  - ANNs parametrize the GFFs in a flexible way

Main message

- Consistent route to GFFs at nonzero skewness from lattice QCD matrix elements

- Preliminary results show the method works

Backup slides

Lattice setup

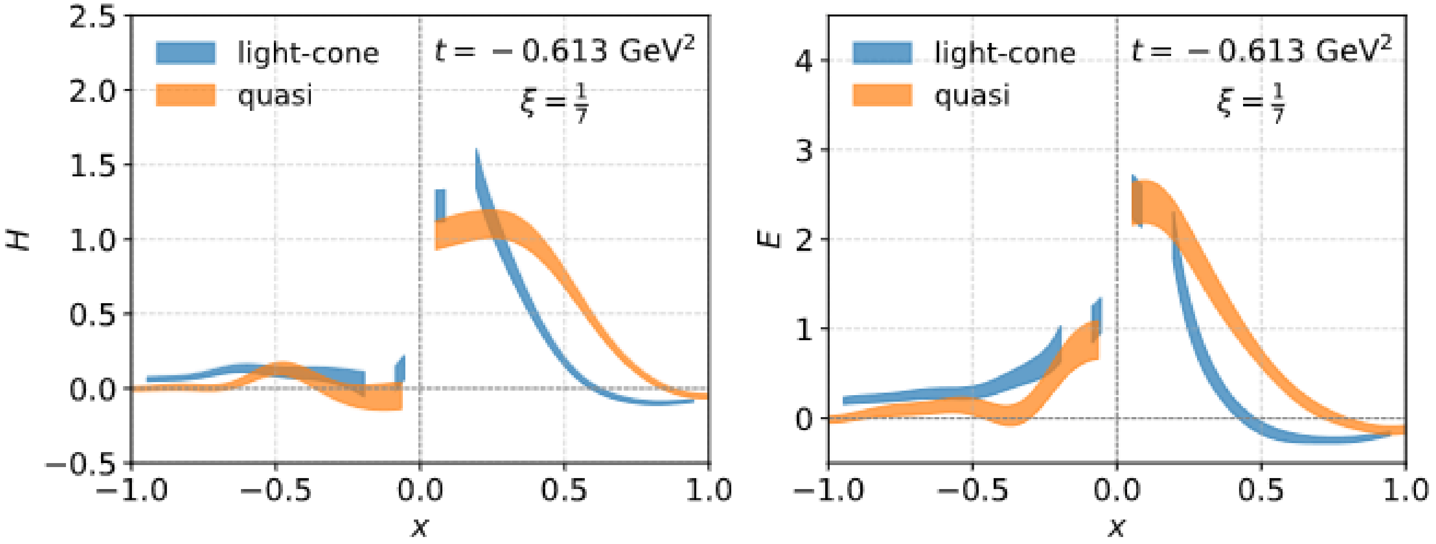
- $N_f = 2 + 1 + 1$ twisted-mass fermions with clover improvement
- Iwasaki-improved gauge action
- $a = 0.093 = \text{fm}, \qquad 32^3 \times 64, \qquad m_\pi = 260\text{MeV}$
- Flavor non-singlet combination: $u - d$
- Source-sink separation:

$$t_s = 10a \simeq 0.93 \text{ fm}$$

Connection to Previous Quasi-GPD Nonzero-Skewness Work

$$\int_{-1}^{1} dx\, x^n\, H(x,\xi,t) = \sum_{\substack{i=0\\ \text{even}}}^{n} (2\xi)^i\, A_{n+1,i}(t) \;+\; \mathrm{mod}(n,2)\,(2\xi)^{n+1}\, C_{n+1}(t)$$

$$\int_{-1}^{1} dx\, x^n\, E(x,\xi,t) = \sum_{\substack{i=0\\ \text{even}}}^{n} (2\xi)^i\, B_{n+1,i}(t) \;-\; \mathrm{mod}(n,2)\,(2\xi)^{n+1}\, C_{n+1}(t).$$

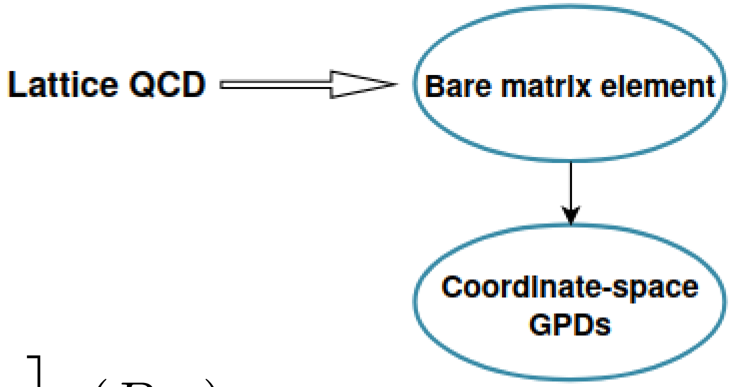


M.-H. Chu et al. "Generalized Parton Distributions from Lattice QCD with Asymmetric Momentum Transfer: Unpolarized Quarks at Nonzero Skewness". In: *Phys. Rev. D* 112 (2025), p. 094510. DOI: 10.1103/ts5s-hvb1. arXiv: 2508.17998 [hep-lat].

Mellin moments → constrain GPDs → help reconstruct these missing regions

Coordinate-space GPDs from Lattice Matrix Elements

- Coordinate-space GPDs:



$$
\begin{aligned}
F^{\mu}(z,P,\Delta) &= \langle P',s'|\bar{q}(-\tfrac{z}{2})\gamma^{\mu}\,W(-\tfrac{z}{2},\tfrac{z}{2})q(\tfrac{z}{2})|P,s\rangle \\
&= \bar{u}(P',s')\Big[\frac{P^{\mu}}{m}A_1 + mz^{\mu}A_2 + \frac{\Delta^{\mu}}{m}A_3 + im\sigma^{\mu z}A_4 \\
&\quad + \frac{i\sigma^{\mu\Delta}}{m}A_5 + \frac{P^{\mu}i\sigma^{z\Delta}}{m}A_6 + mz^{\mu}i\sigma^{z\Delta}A_7 + \frac{\Delta^{\mu}i\sigma^{z\Delta}}{m}A_8\Big]u(P,s)
\end{aligned}
$$

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Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

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Phys. Rev. D **106**, 114512 – **Published 26 December, 2022**

DOI: https://doi.org/10.1103/PhysRevD.106.114512

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Coordinate-space GPDs from Lattice Matrix Elements

$$H = A_1 - 2\xi A_3$$

$$E = -A_1 + 2\xi A_3 + 2A_5 + 2P_3 z A_6 - 4\xi P_3 z A_8$$

$\rightarrow$ Discrete and defined in coordinate-space

- Amplitudes are from found:

$$\text{Matrix element} \rightarrow \Pi_\mu(\Gamma_\nu) = K\,\mathrm{Tr}\left[\Gamma_\nu \left(\frac{-i\not{P}_f + m}{2m}\right) \tilde{F}^\mu \left(\frac{-i\not{P}_i + m}{2m}\right)\right] \rightarrow$$

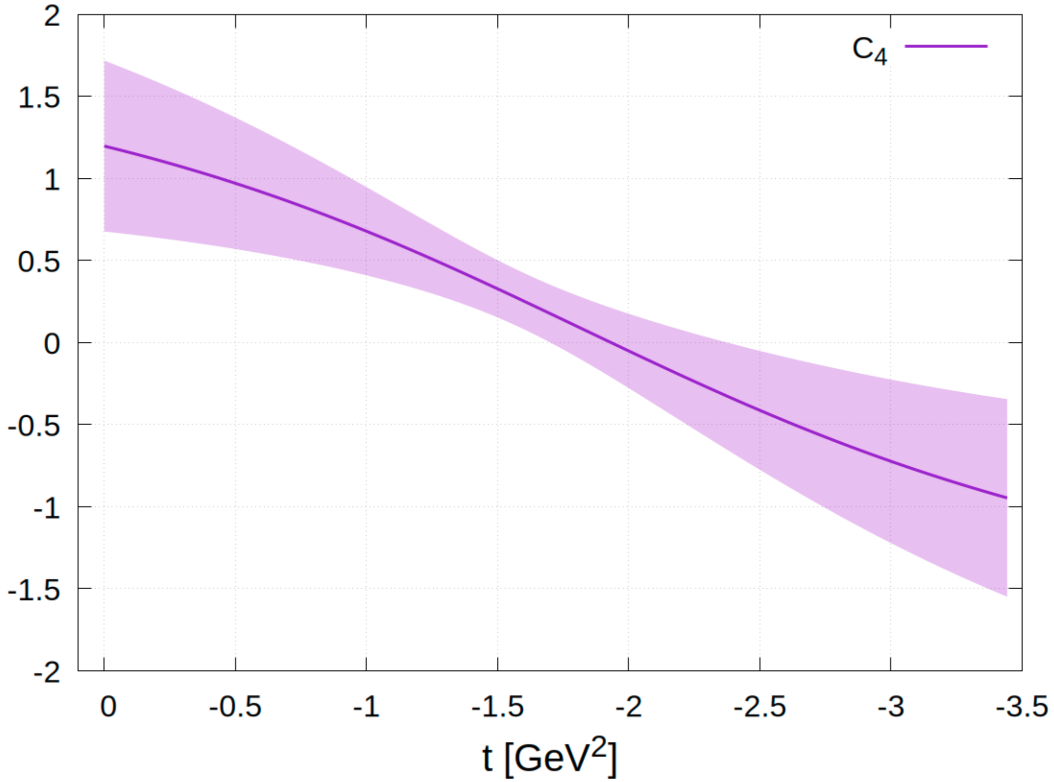
Linear combination A_i
System of linear equations

$$\Gamma_0 = \frac{1}{4}(1+\gamma_0)\,, \quad \Gamma_k = \frac{1}{4}(1+\gamma_0)\, i\gamma_5\gamma_k\,, \quad k = 1,2,3 \qquad K = \frac{2m^2}{\sqrt{E_f E_i (E_f + m)(E_i + m)}}$$

$$\tilde{F}^\mu(z,P,\Delta) = \left[\frac{P^\mu}{m}A_1 + mz^\mu A_2 + \frac{\Delta^\mu}{m}A_3 + im\sigma^{\mu z}A_4 + \frac{i\sigma^{\mu\Delta}}{m}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{m}A_6 + mz^\mu i\sigma^{z\Delta}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{m}A_8\right]$$

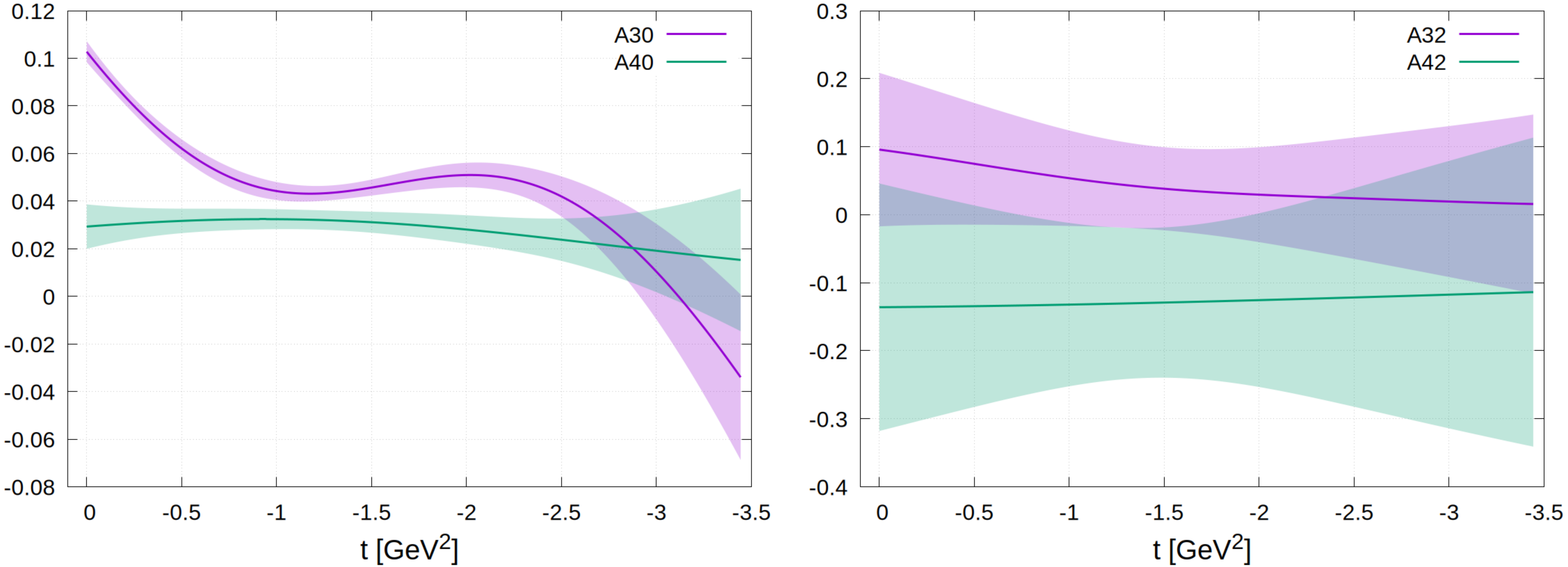
Higher-Type GFFs - C_4

- Disclaimer on lack of control on C_4
    - Highest-skewness coefficient (ξ^4)
    - Appears in both H and E sector
    - Included mainly as a truncation-control term



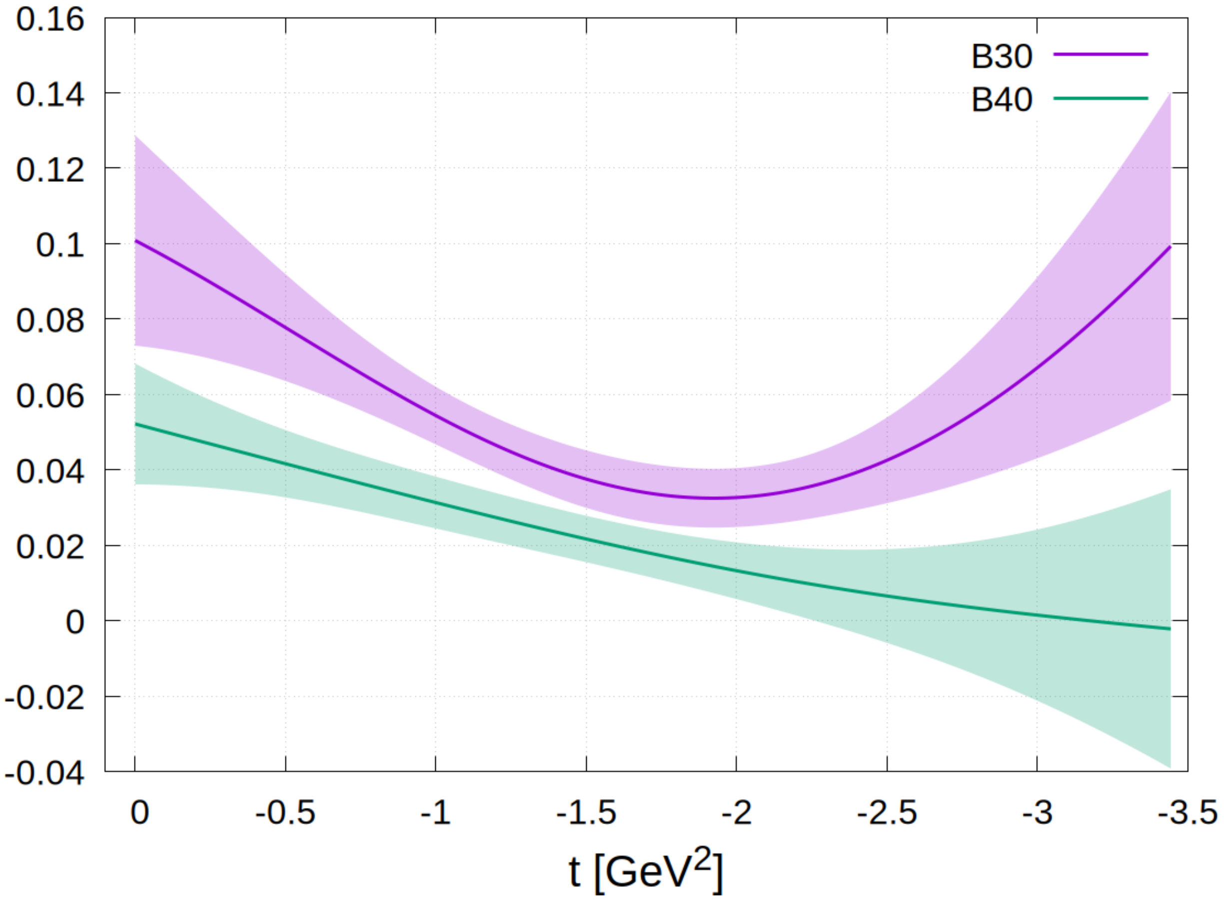
Higher A-type GFFs

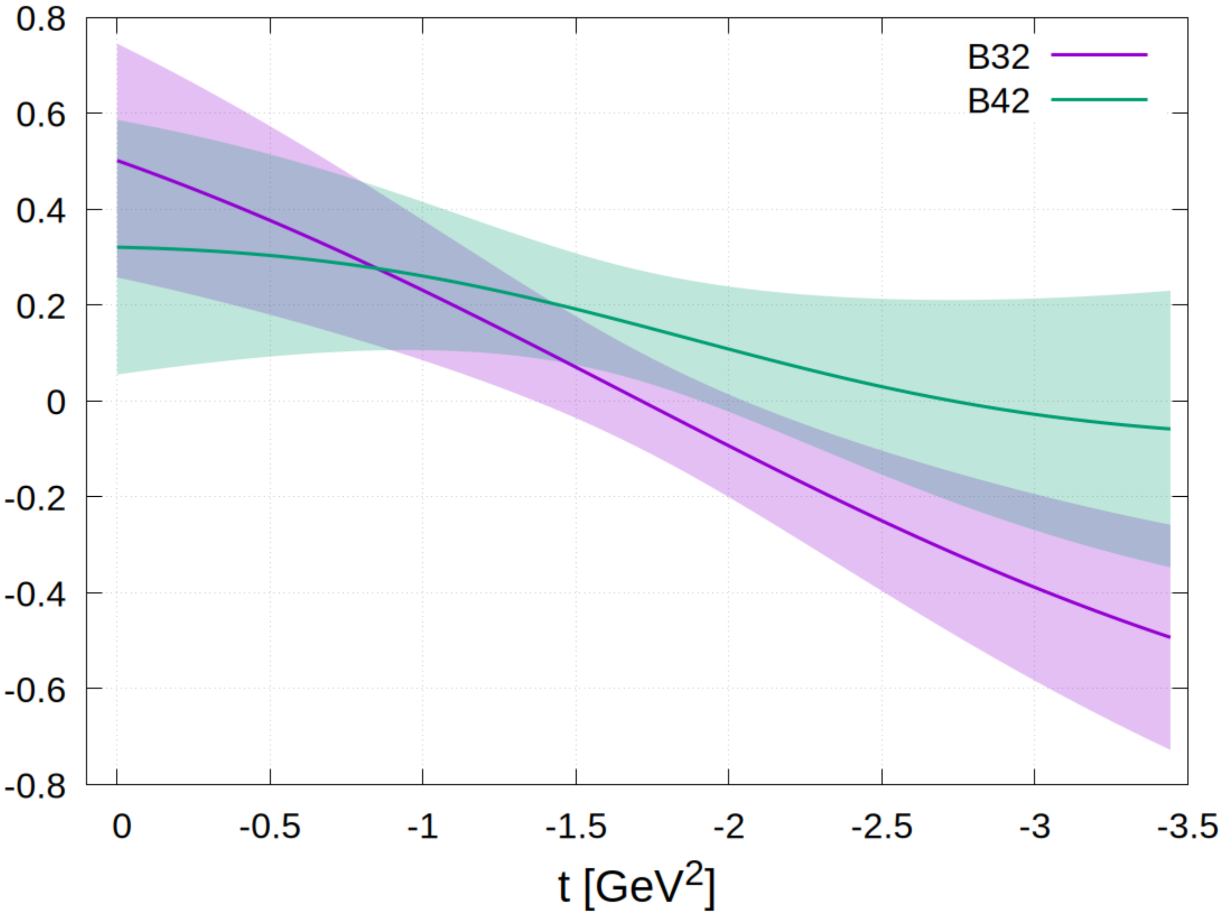
- Skewness-dependent GFFs have larger relative uncertainties



Higher B-type GFFs

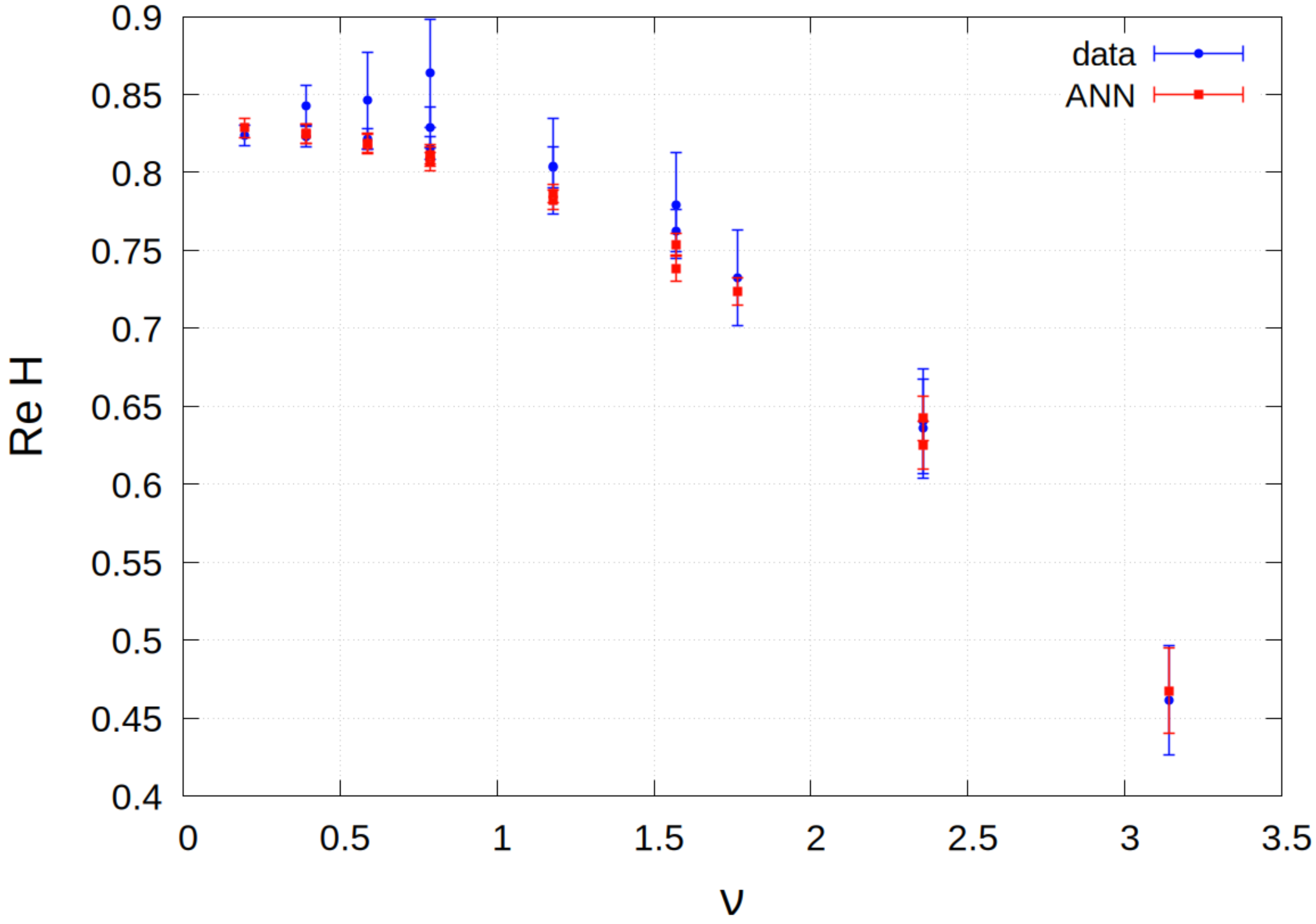
- Skewness-dependent GFFs have larger relative uncertainties





ANN Predictions Δ=(1,0,0)

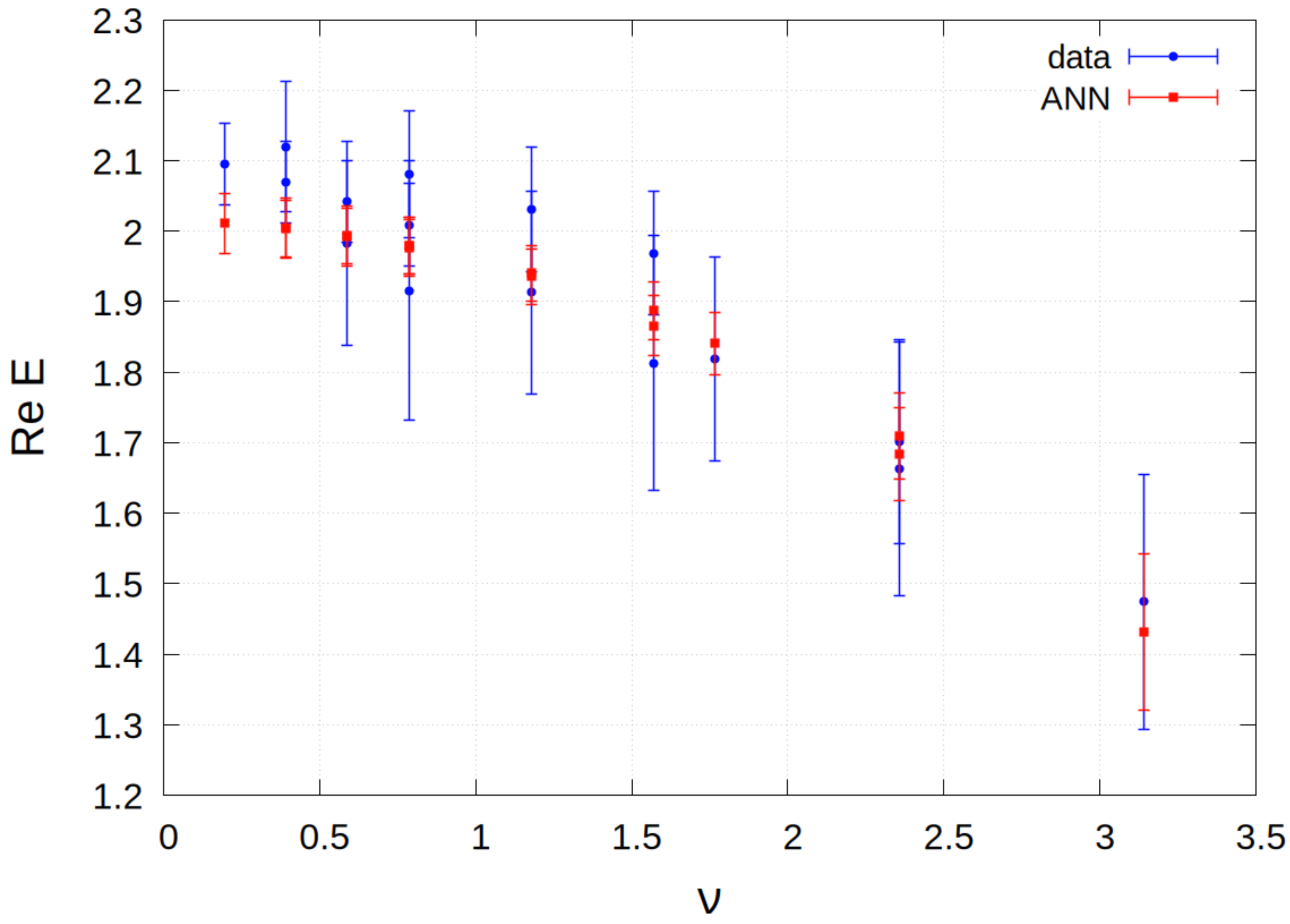
$$\mathrm{Re}\, F^{(H)}(\nu, t, \xi; z) = \left(c_{1,0} - \frac{\nu^2}{2}\xi^2 c_{3,2}\right) A_{1,0}(t) - \frac{\nu^2}{2} c_{3,0}\, A_{3,0}(t) - \frac{\nu^2}{2}\xi^2 c_{3,0}\, A_{3,2}(t)$$



The error bars of the ANN points are NOT a prediction of the error bar of the data points!

ANN Predictions Δ=(1,0,0)

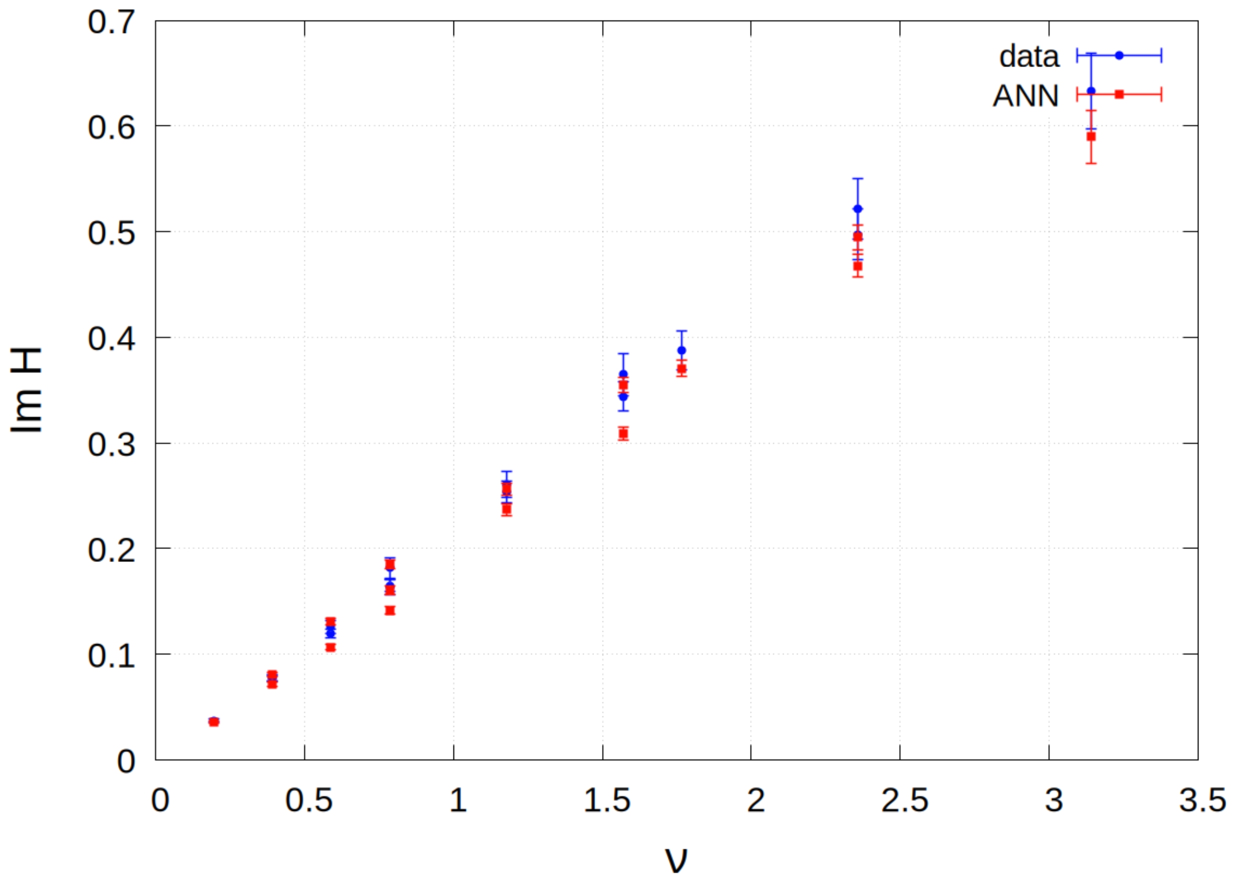
$$\mathrm{Re}\, F^{(E)}(\nu,t,\xi;z) = \left(c_{1,0} - \frac{\nu^2}{2}\xi^2 c_{3,2}\right) B_{1,0}(t) - \frac{\nu^2}{2} c_{3,0}\, B_{3,0}(t) - \frac{\nu^2}{2}\xi^2 c_{3,0}\, B_{3,2}(t)$$



The error bars of the ANN points are NOT a prediction of the error bar of the data points!

ANN Predictions Δ=(1,0,0)

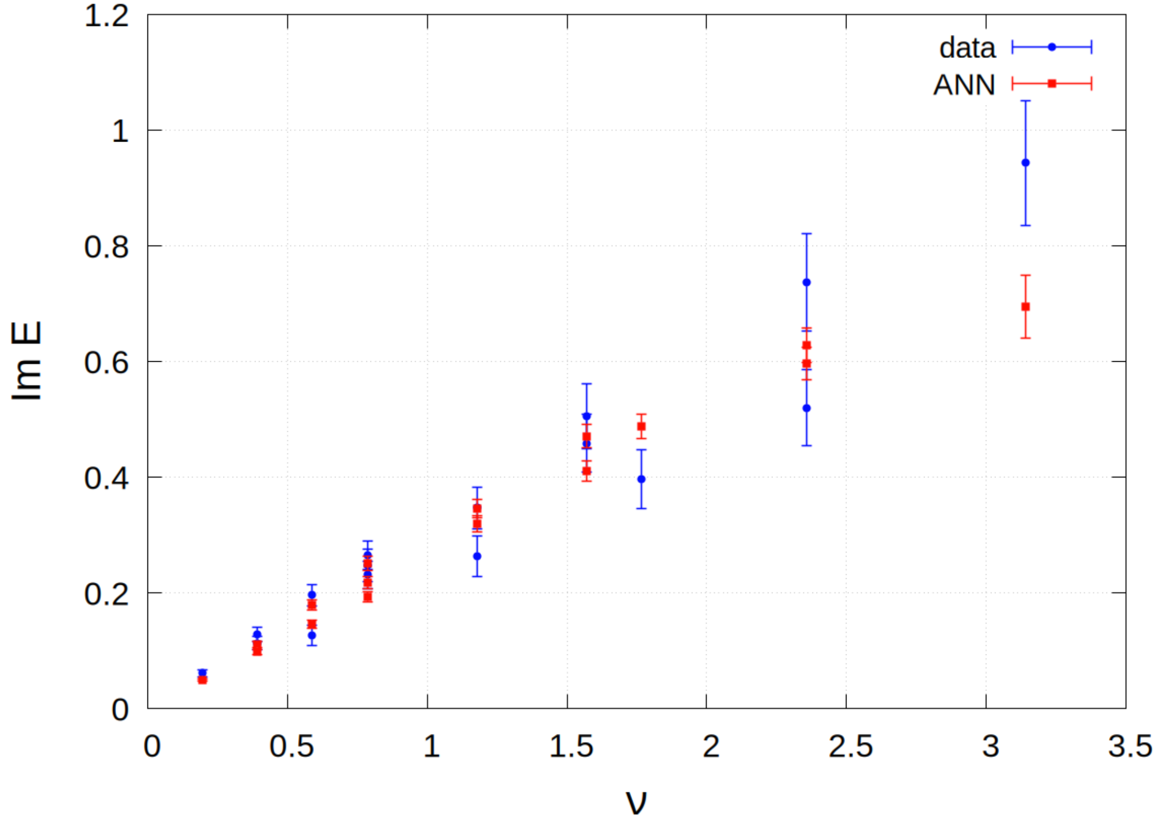
$$\mathrm{Im}\,F^{(H)}(\nu,t,\xi;z) = \left(-\nu c_{2,0} + \frac{\nu^3}{6}\xi^2 c_{4,2}\right) A_{2,0}(t) + \left(-\nu\xi^2 c_{2,0} + \frac{\nu^3}{6}\xi^4 c_{4,2}\right) C_2(t)$$
$$+ \frac{\nu^3}{6} c_{4,0}\, A_{4,0}(t) + \frac{\nu^3}{6}\xi^2 c_{4,0}\, A_{4,2}(t) + \frac{\nu^3}{6}\xi^4 c_{4,0}\, C_4(t)$$



The error bars of the ANN points are NOT a prediction of the error bar of the data points!

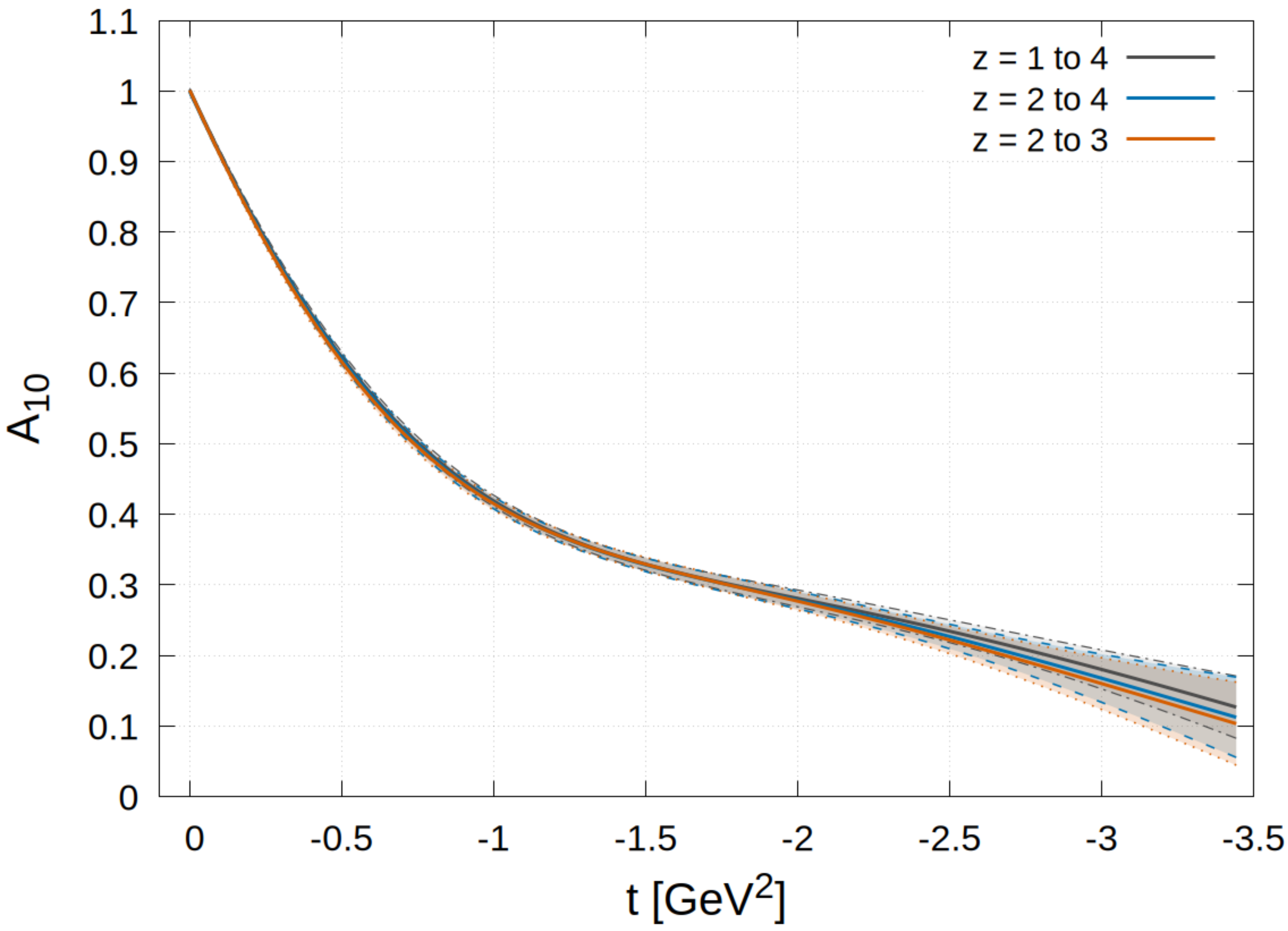
ANN Predictions Δ=(1,0,0)

$$\mathrm{Im}\, F^{(E)}(\nu,t,\xi;z) = \left(-\nu c_{2,0} + \frac{\nu^3}{6}\xi^2 c_{4,2}\right) B_{2,0}(t) + \left(\nu\xi^2 c_{2,0} - \frac{\nu^3}{6}\xi^4 c_{4,2}\right) C_2(t)$$
$$+ \frac{\nu^3}{6} c_{4,0}\, B_{4,0}(t) + \frac{\nu^3}{6}\xi^2 c_{4,0}\, B_{4,2}(t) - \frac{\nu^3}{6}\xi^4 c_{4,0}\, C_4(t)$$

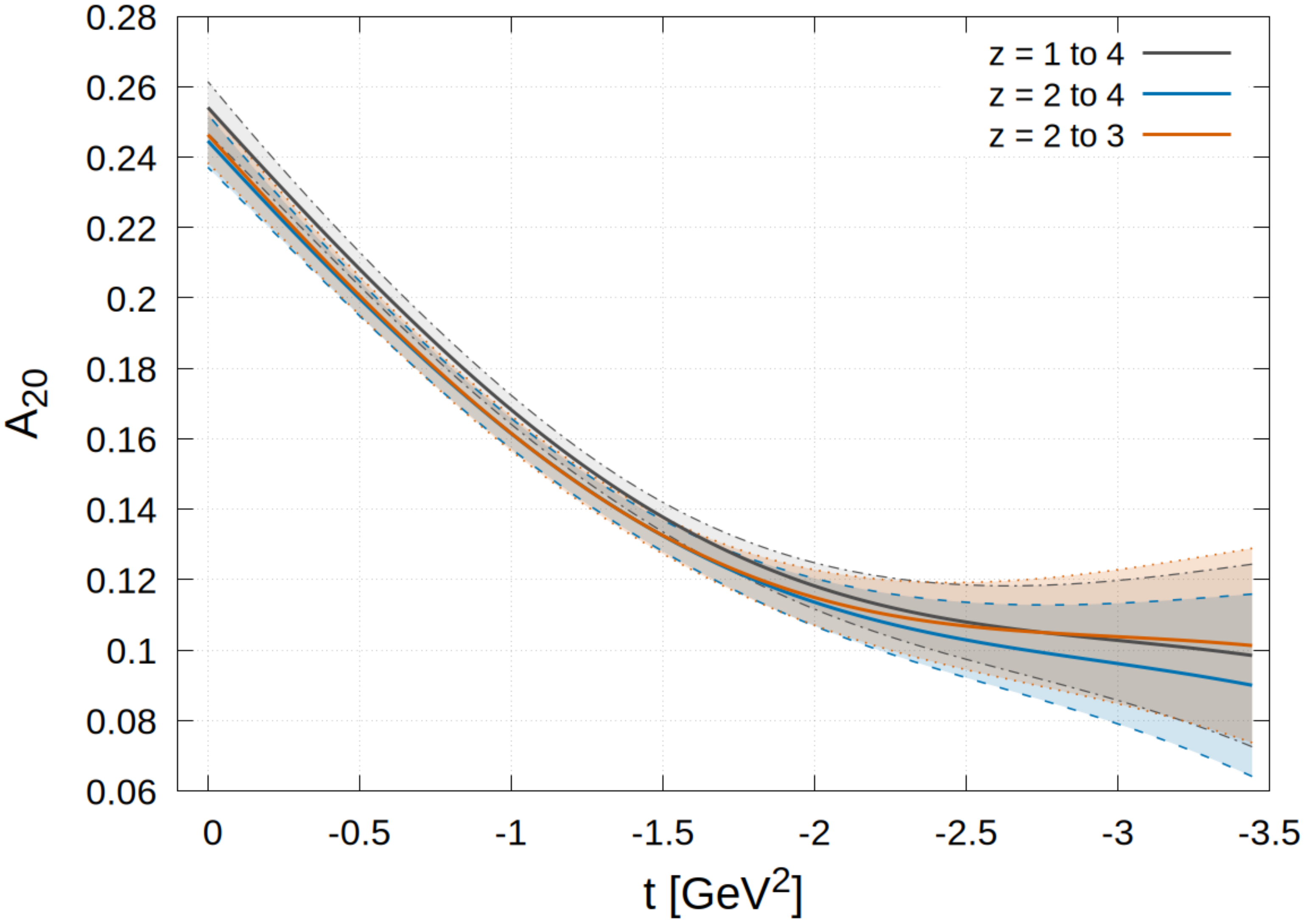


The error bars of the ANN points are NOT a prediction of the error bar of the data points!

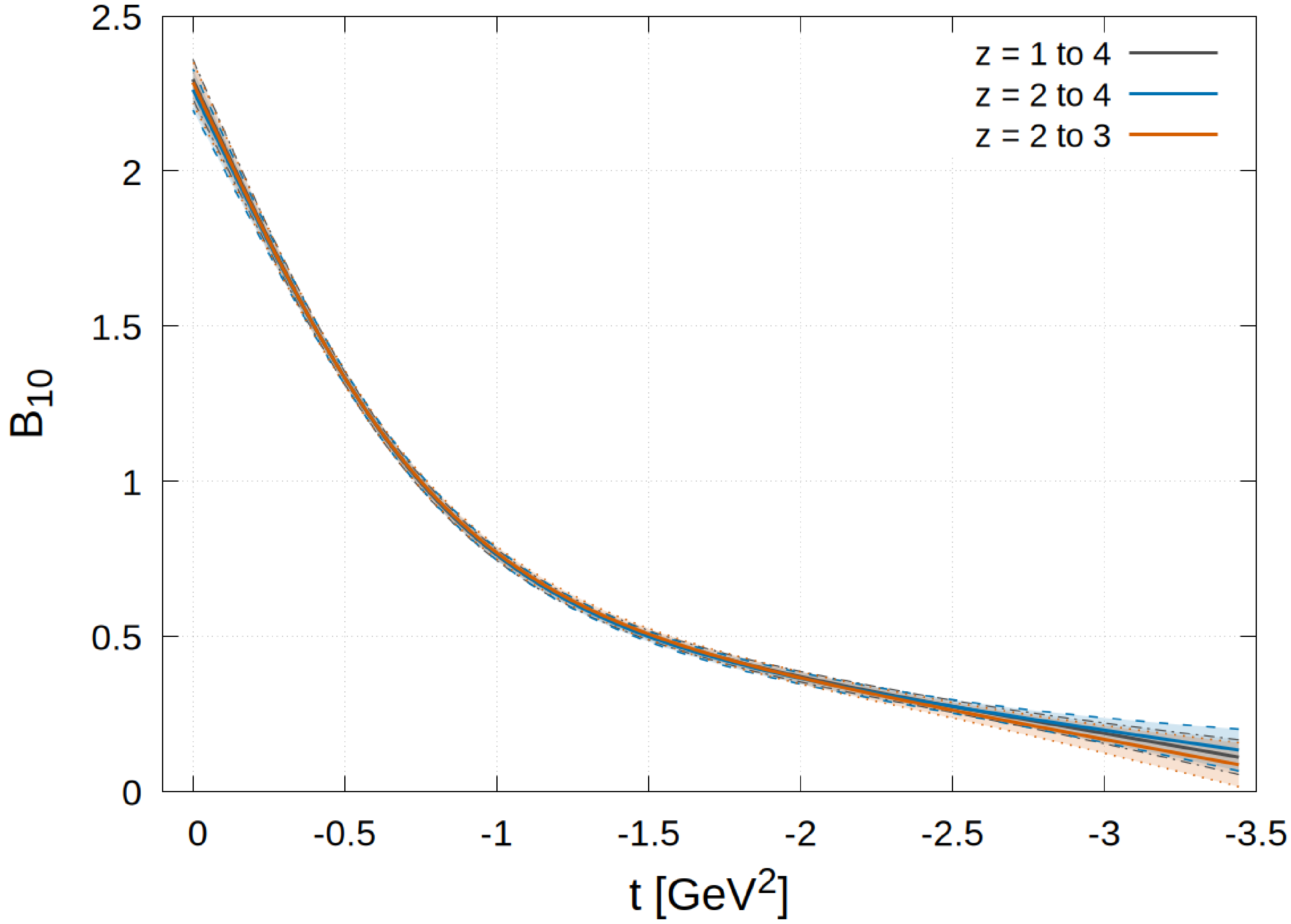
Using different z-data to train the ANN



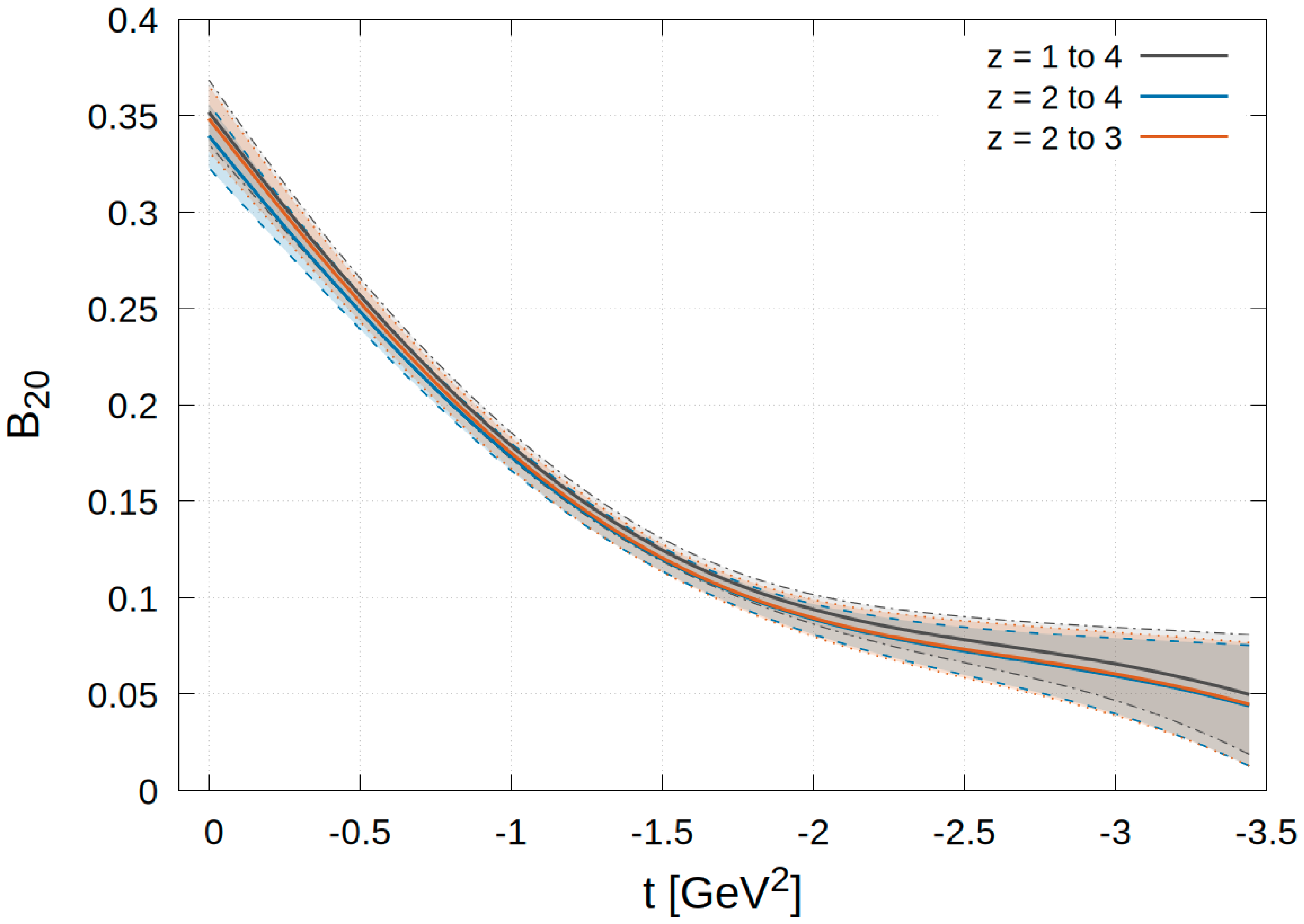
Using different z-data to train the ANN



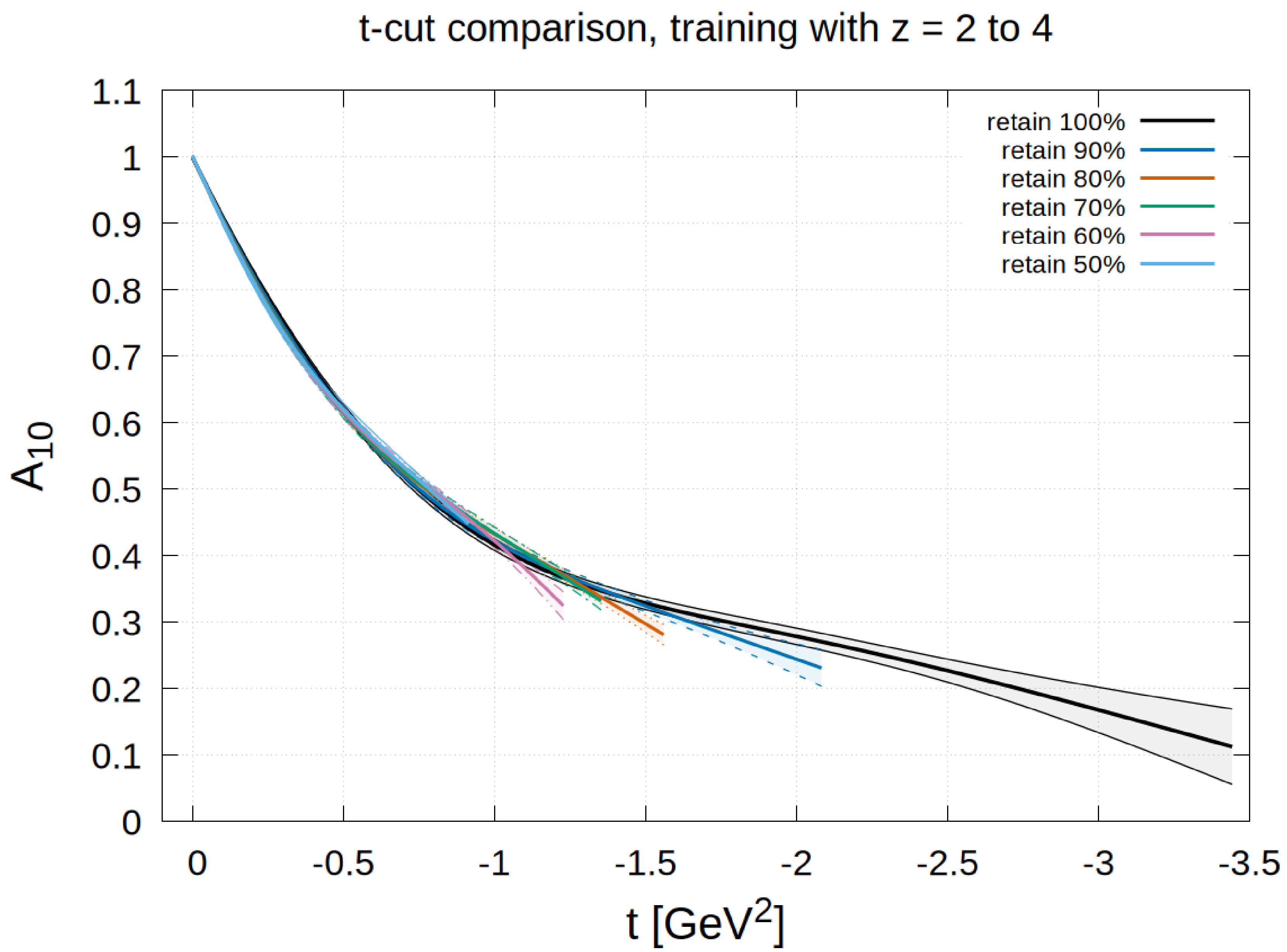
Using different z-data to train the ANN



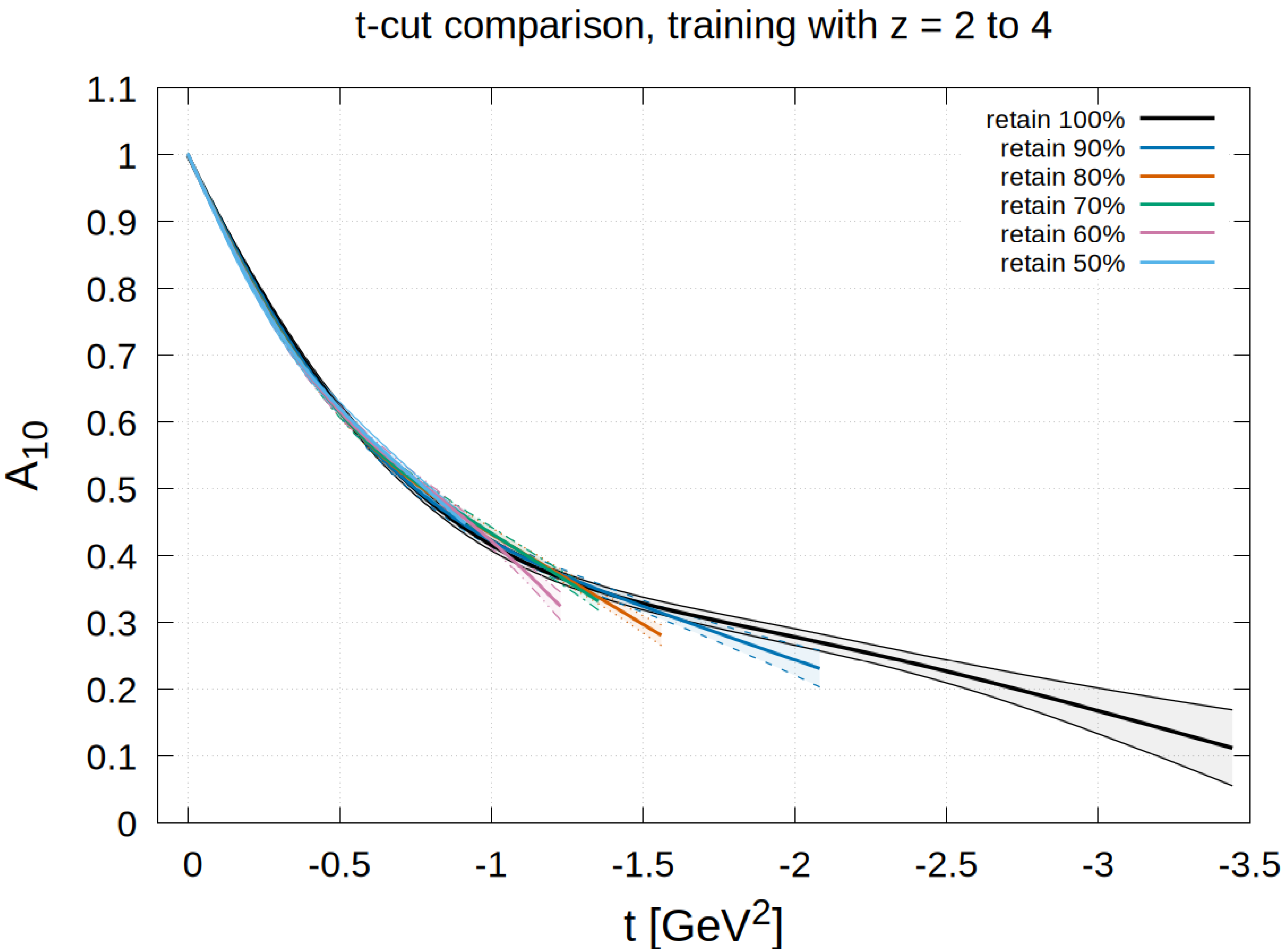
Using different z-data to train the ANN



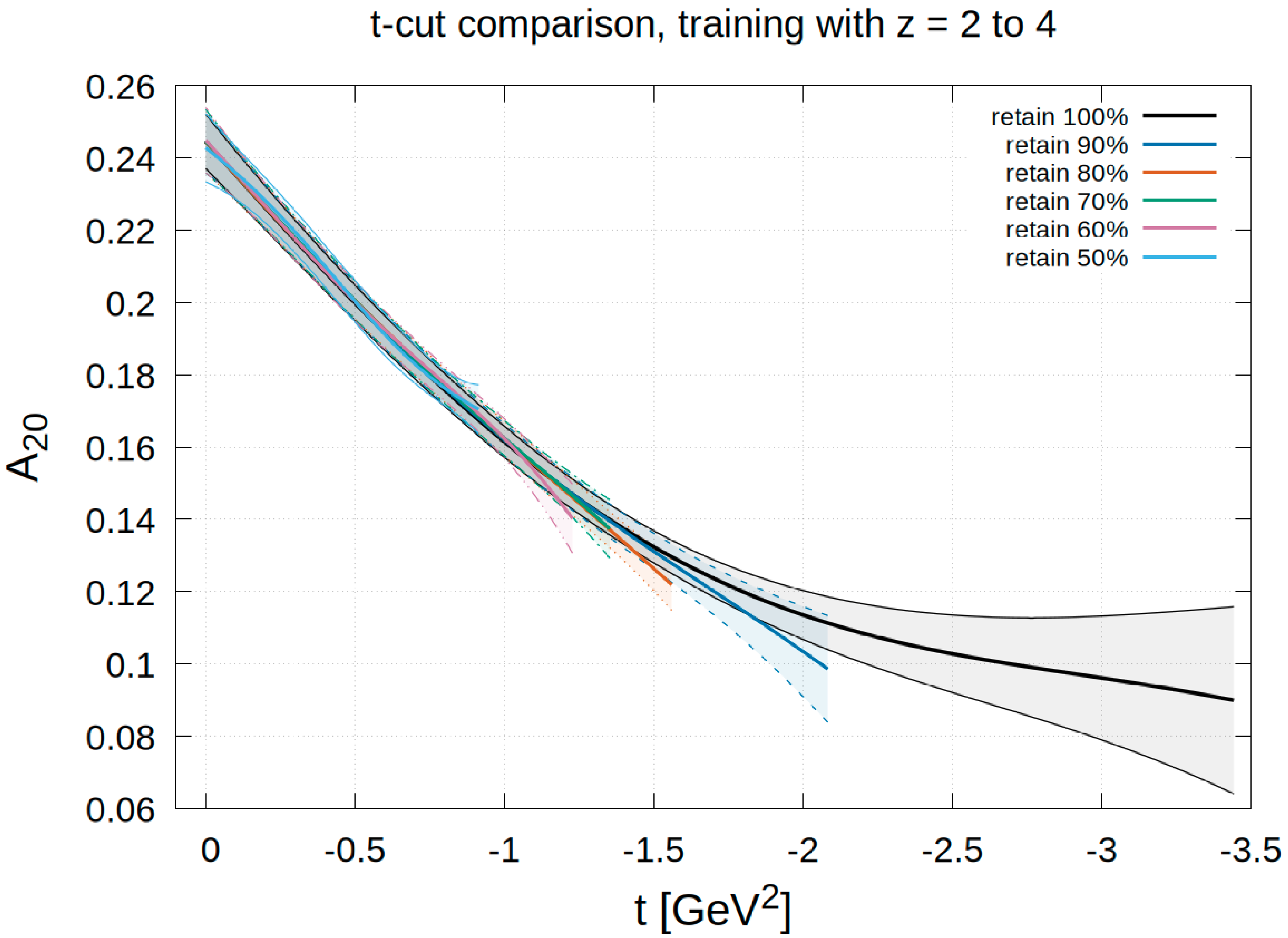
Using different t-data to train the ANN



Using different t-data to train the ANN



Using different t-data to train the ANN



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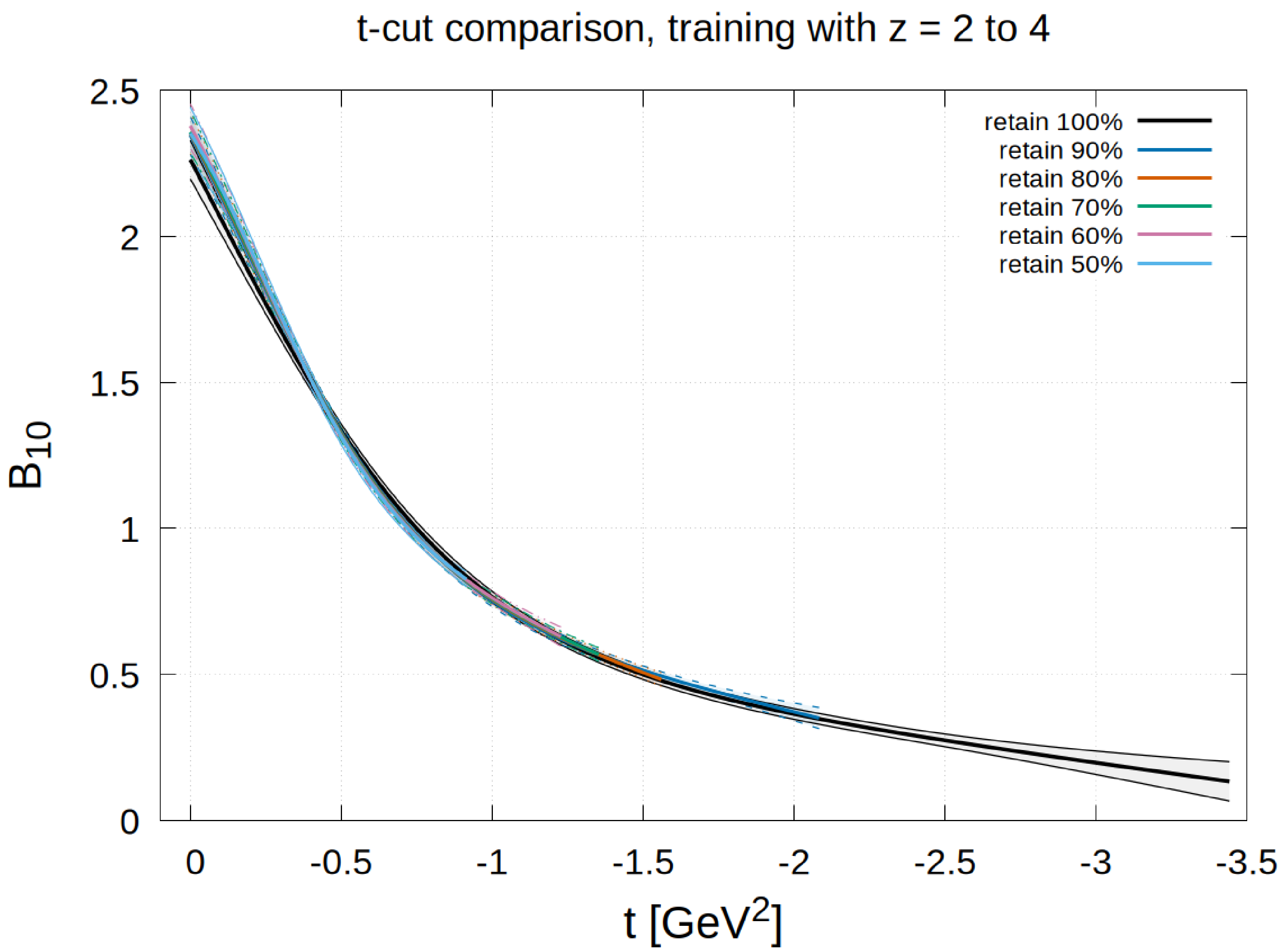
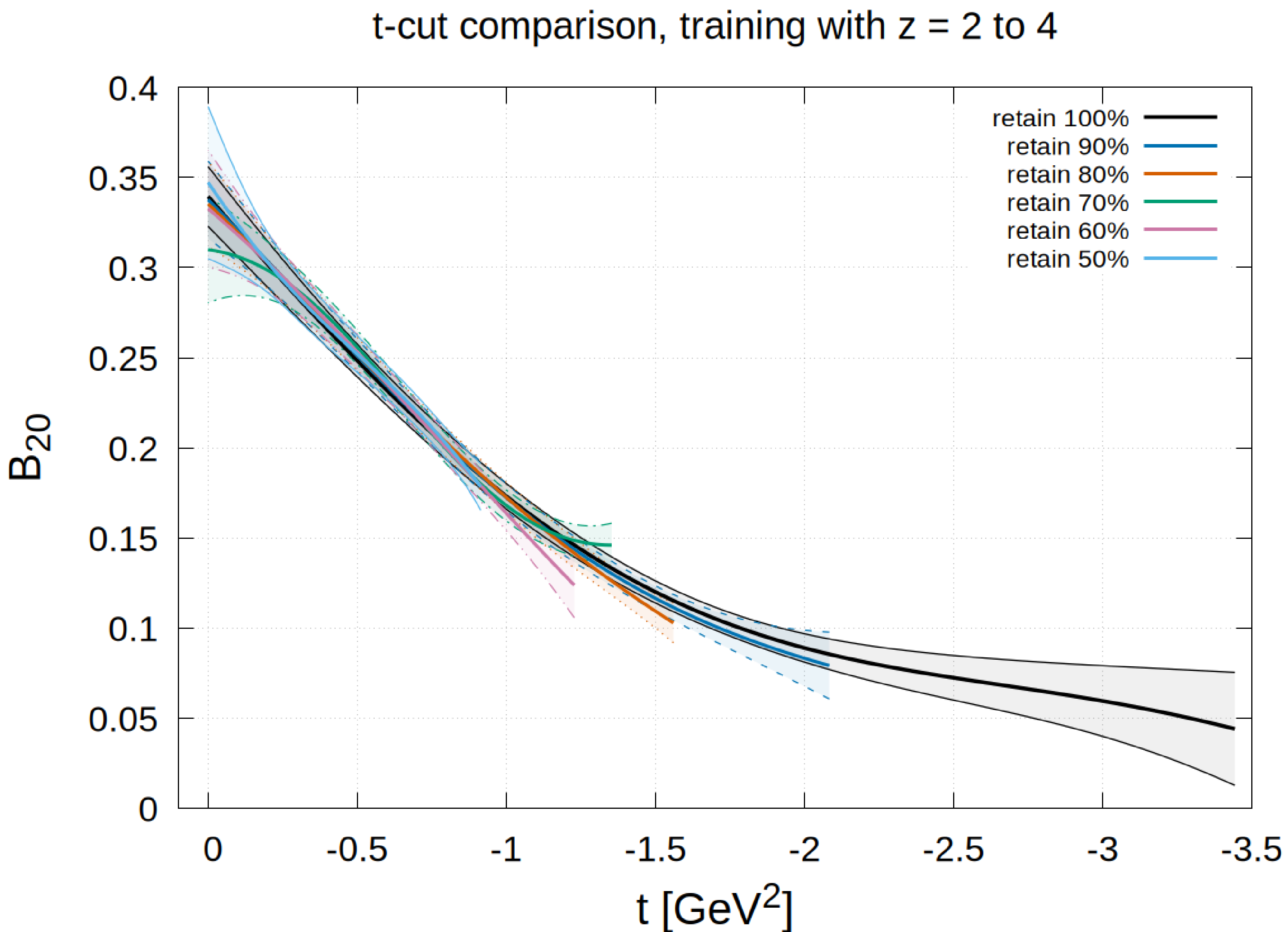
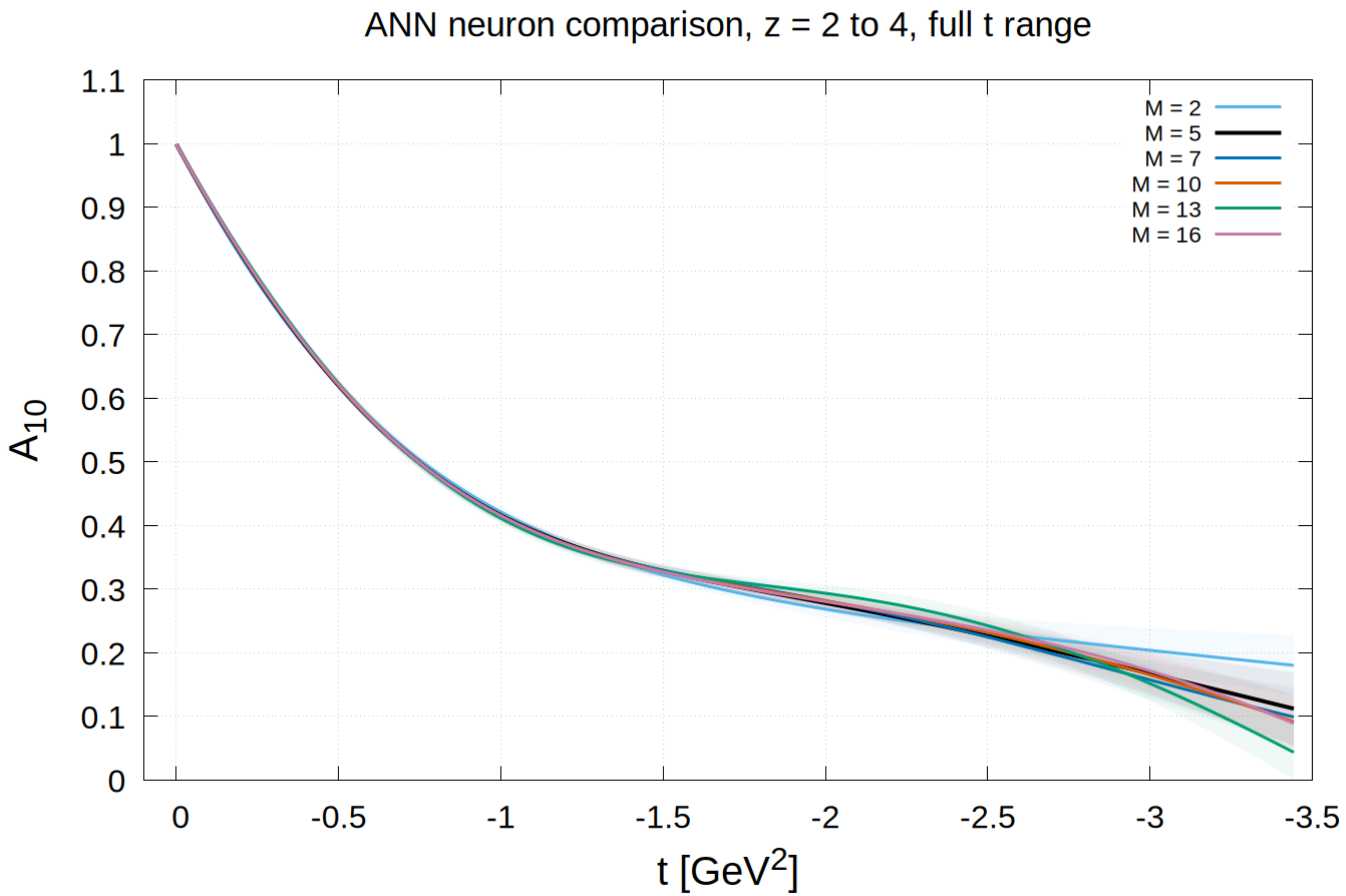


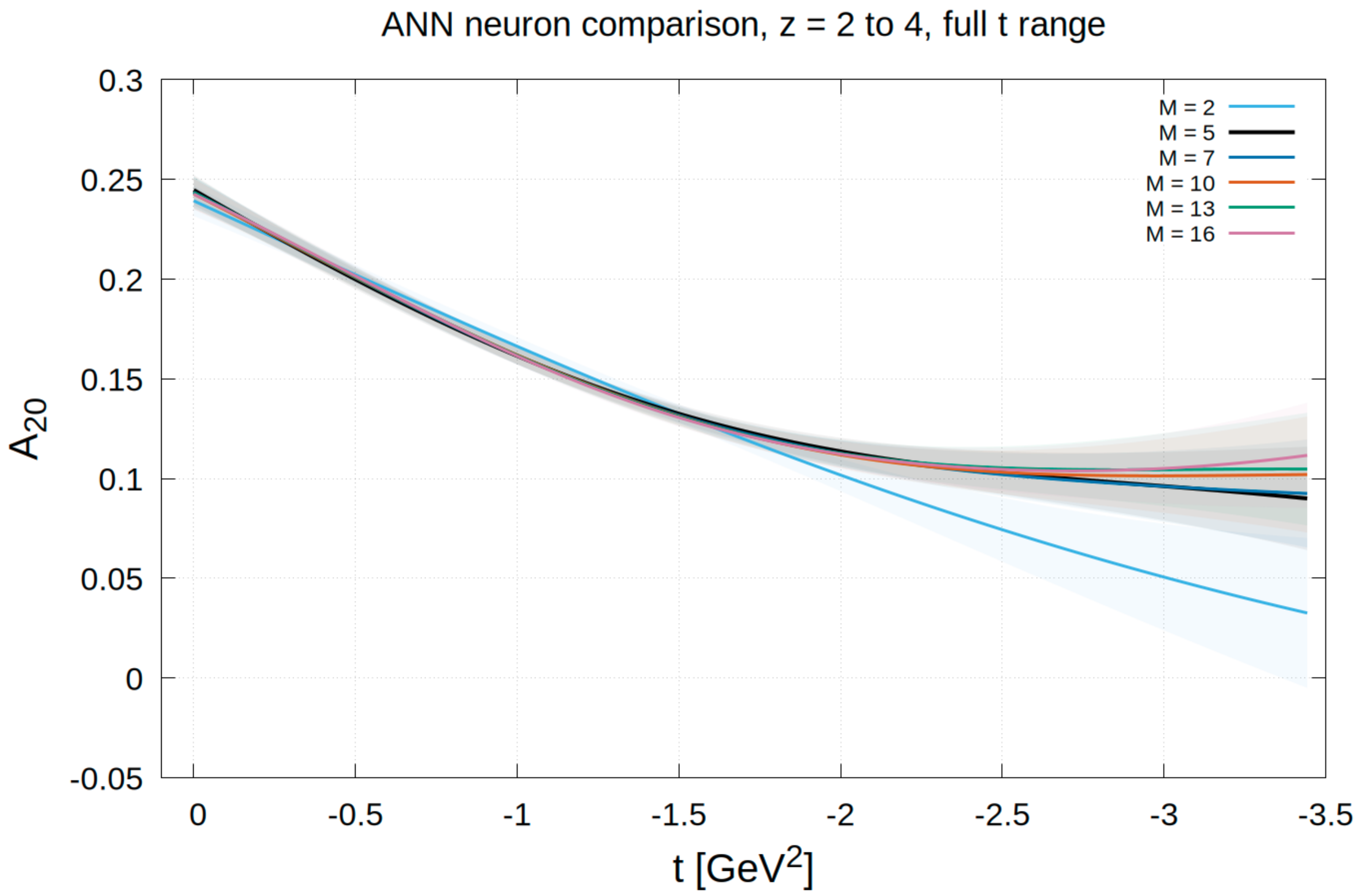
Using different t-data to train the ANN



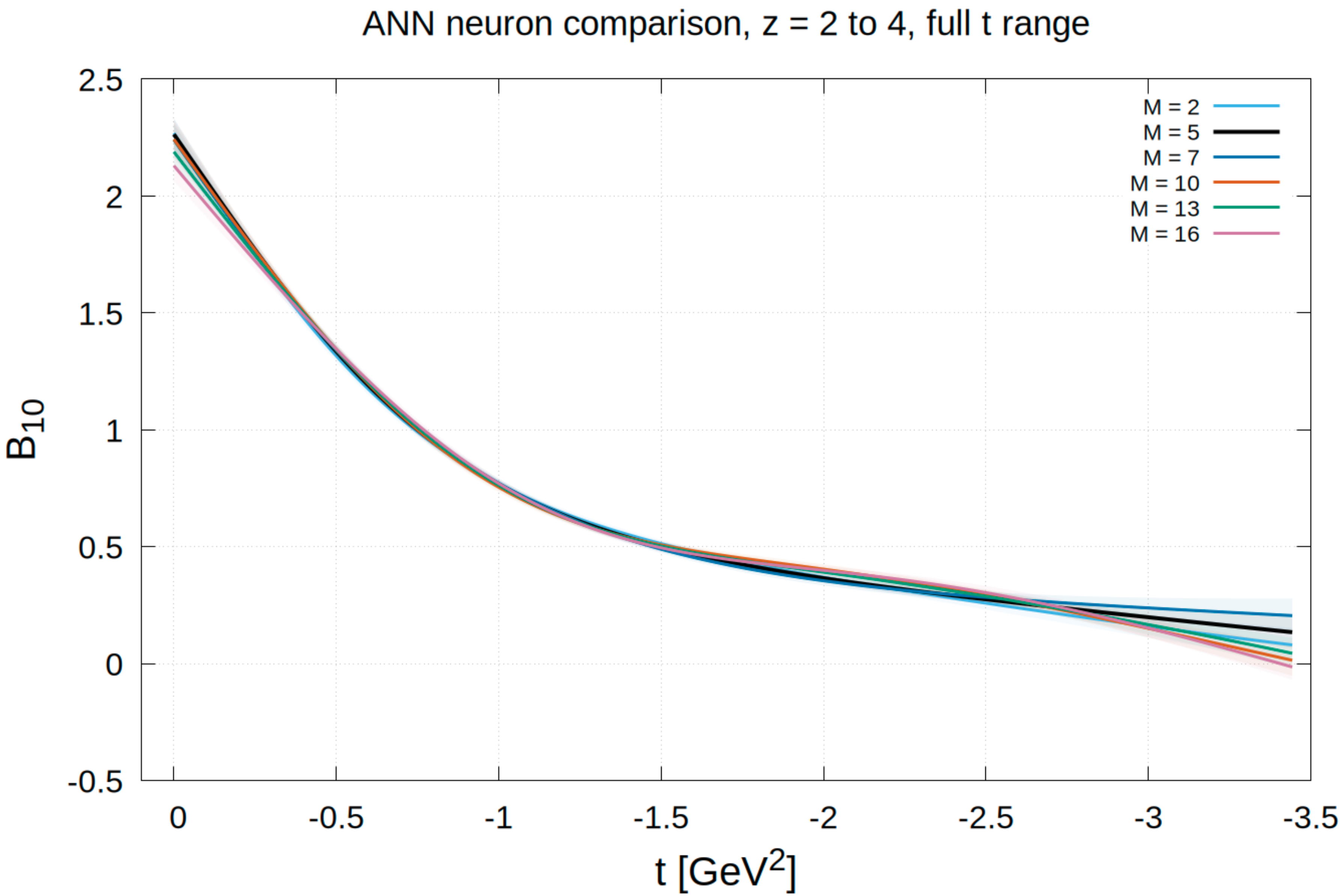
Using different neurons



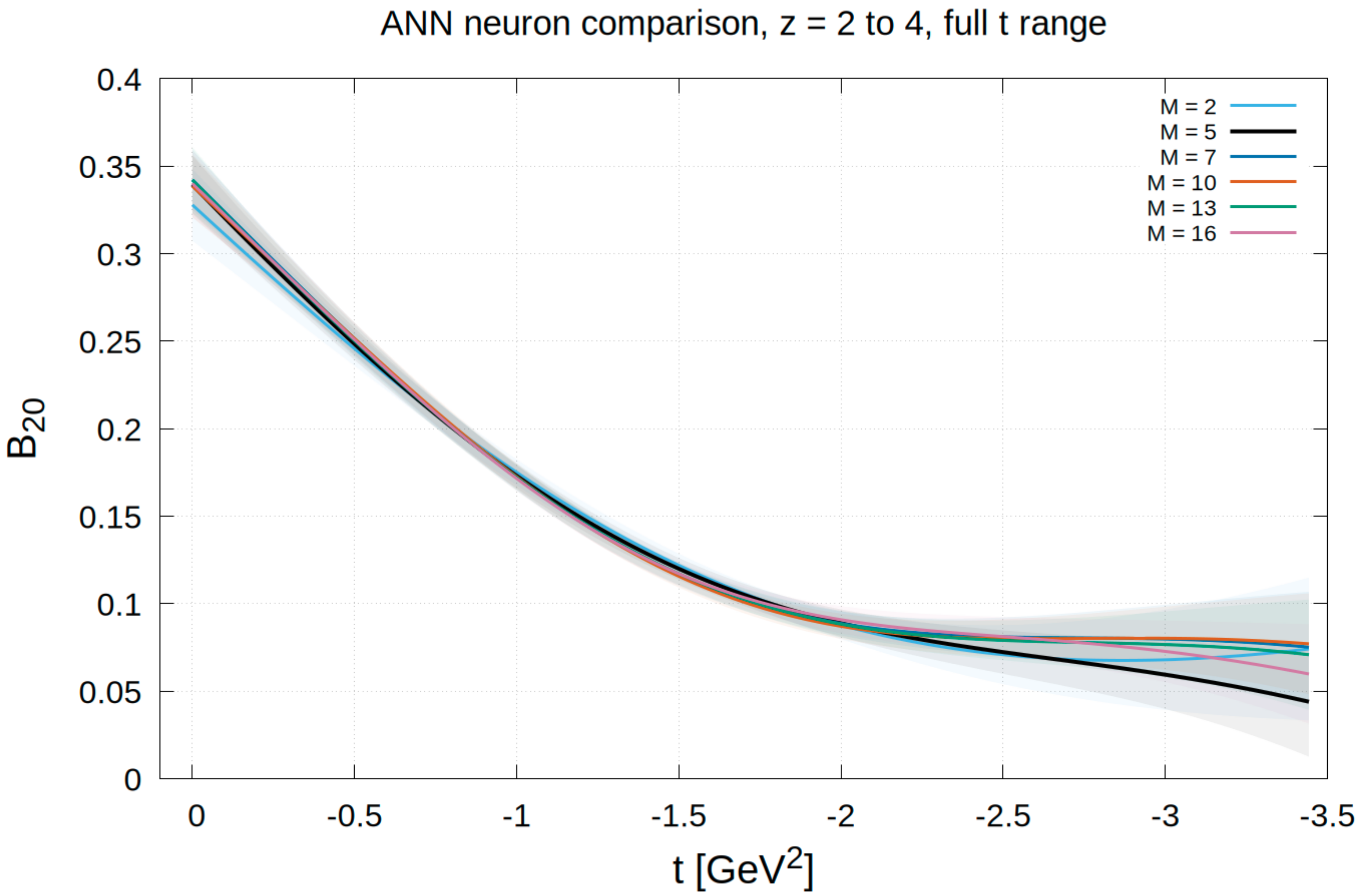
Using different neurons



Using different neurons



Using different neurons



Using different neurons