



UNIVERSITÀ DI PADOVA
Dipartimento
di Fisica e Astronomia
"Galileo Galilei" - DFA



QUANTUM
Information and Matter



Preparation and detection of quasiparticles for quantum simulations of scattering

M. MORGAVI, P. MAJCEN, M. RIGOBELLO, S. MONTANGERO, P. SILVI

The 43rd International Symposium on Lattice Field Theory - University of Maryland, College Park

Wednesday 29, 2026



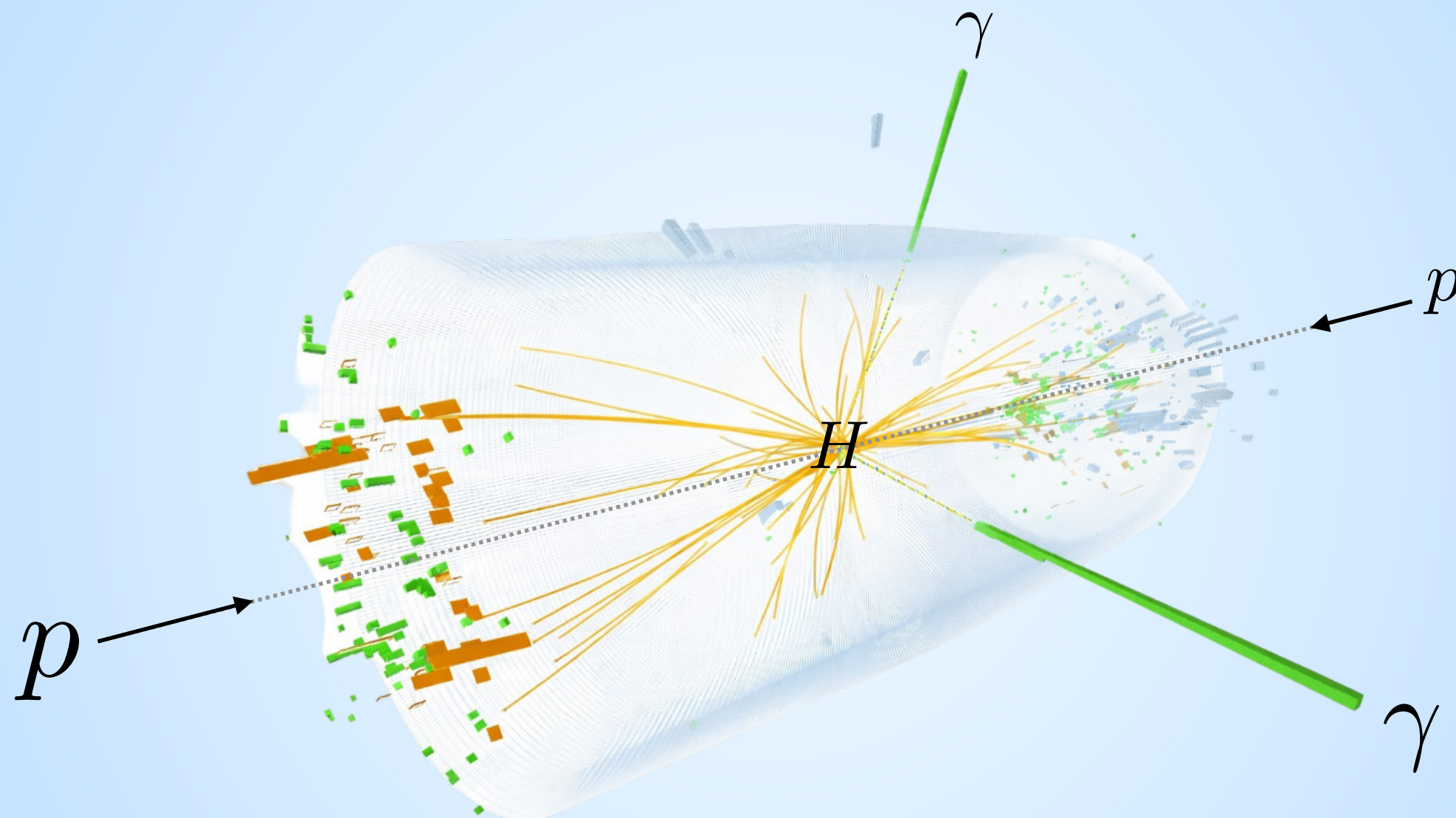
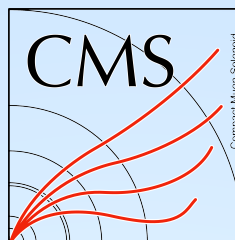
QUANTUM
COMPUTING
AND
SIMULATION
CENTER

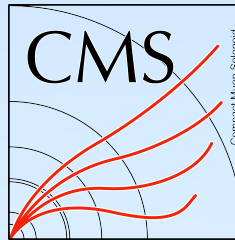


Centro Nazionale di Ricerca in HPC,
Big Data and Quantum Computing

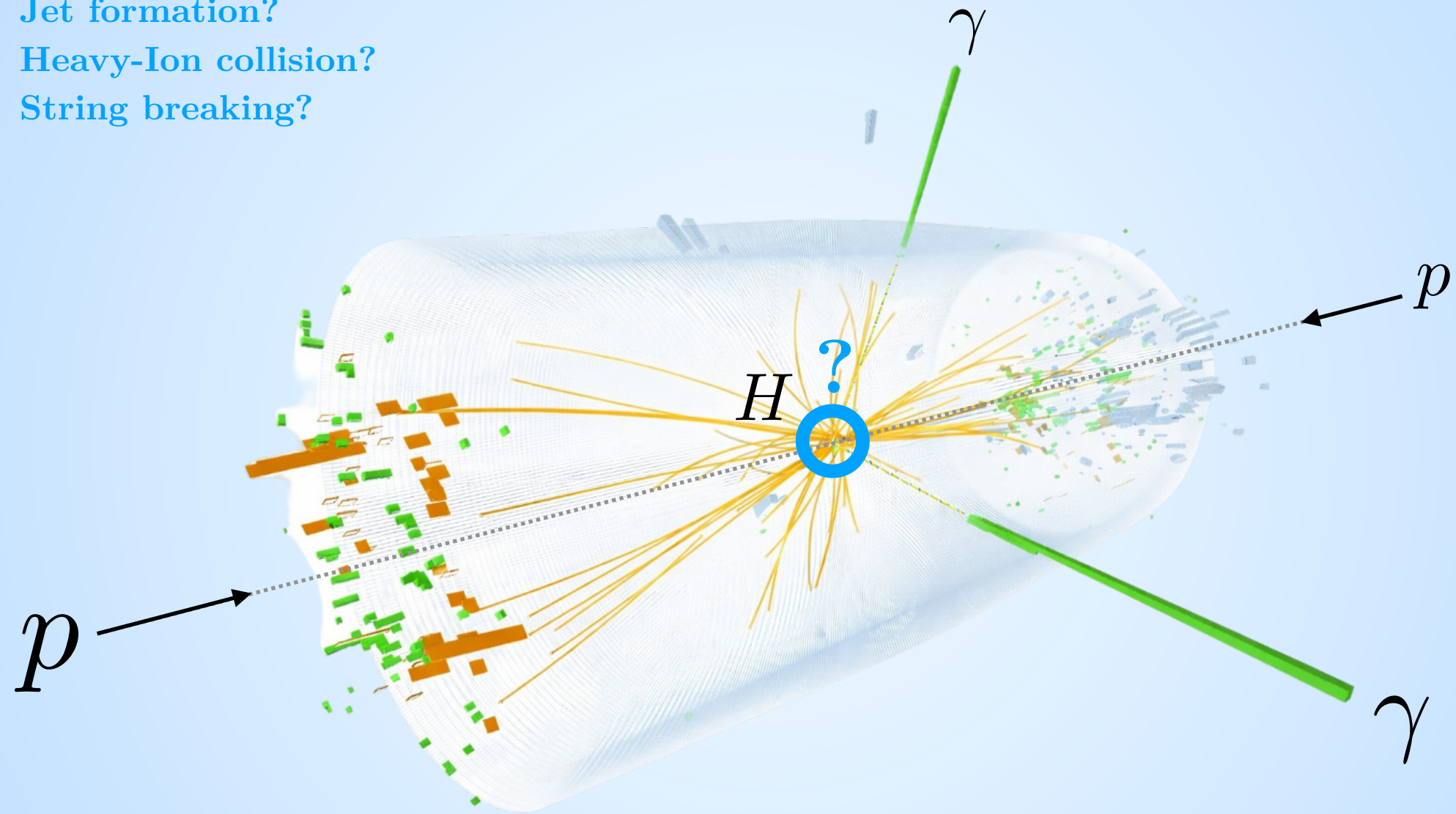


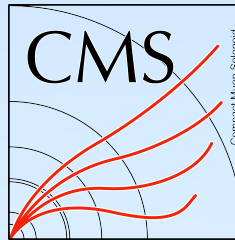
I. MOTIVATION



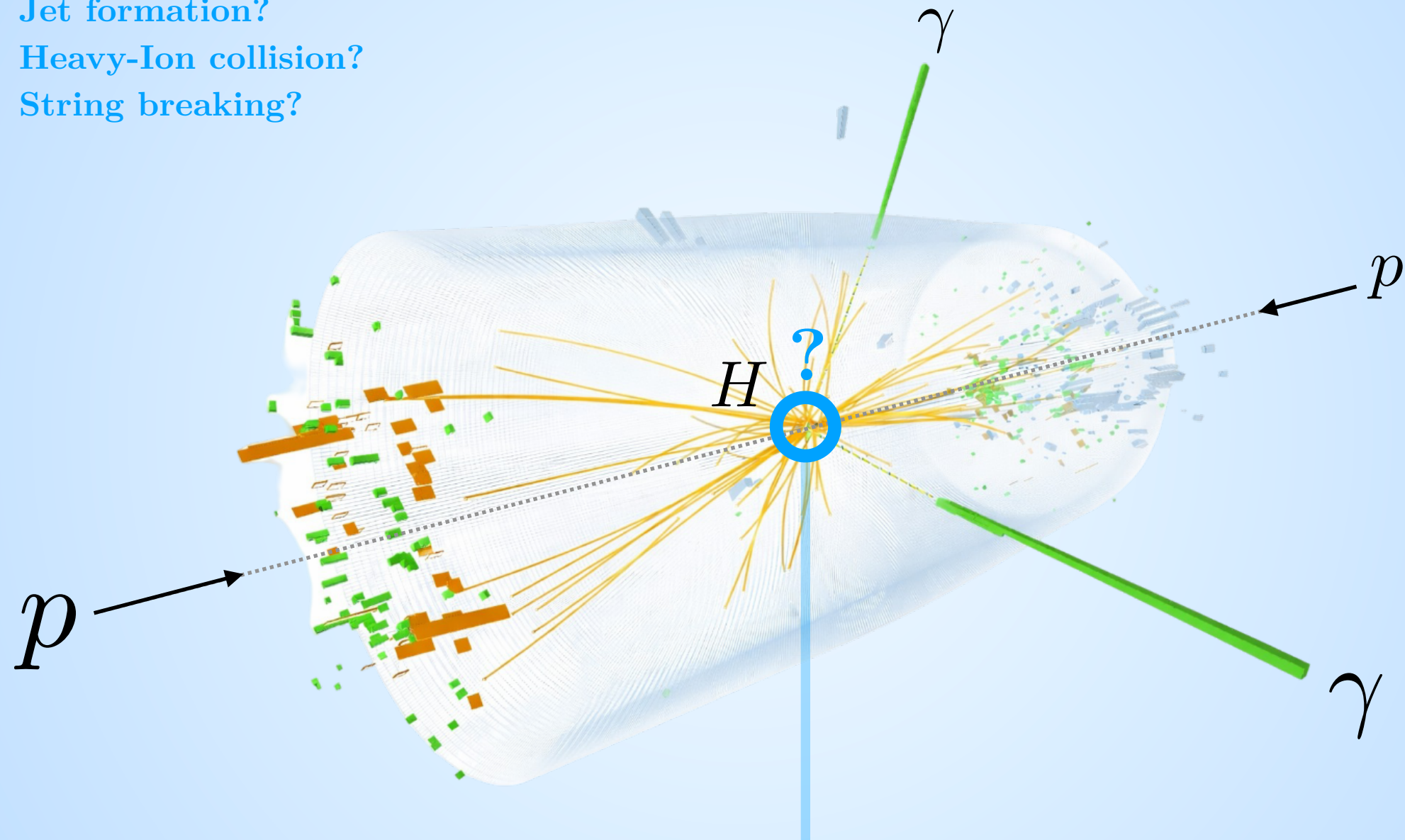


Hadronization?
Jet formation?
Heavy-Ion collision?
String breaking?

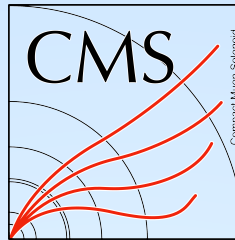




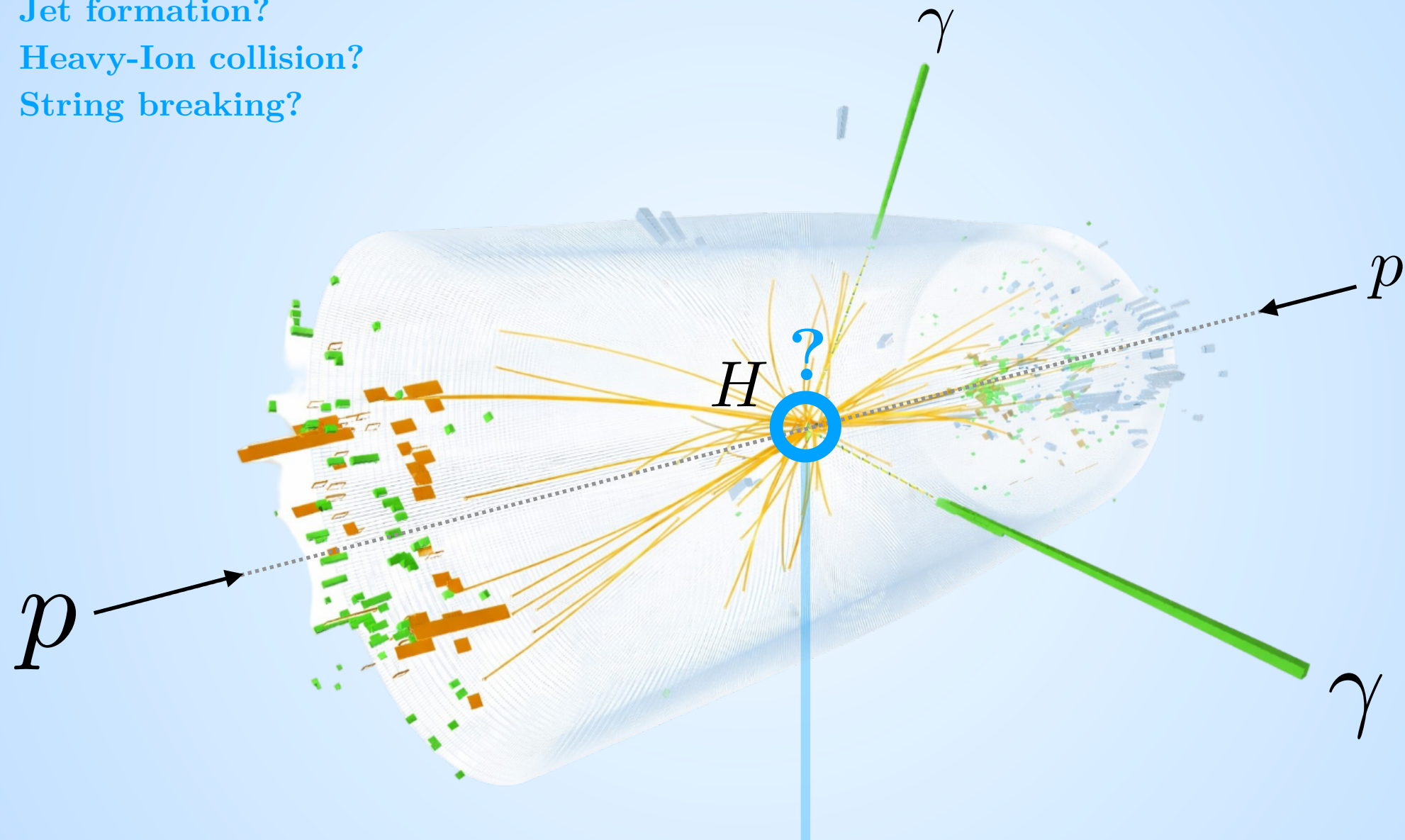
Hadronization?
Jet formation?
Heavy-Ion collision?
String breaking?



Problem 1: framework for **real-time simulation**



Hadronization?
Jet formation?
Heavy-Ion collision?
String breaking?



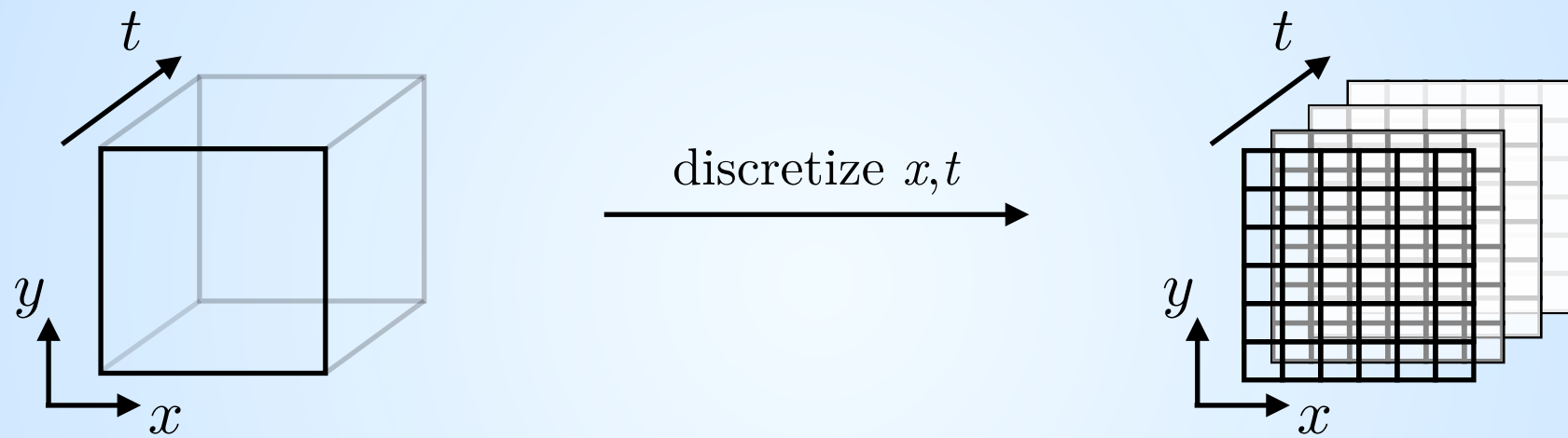
Problem **1**: framework for **real-time simulation**

Problem **2**: prepare **initial scattering states**

Problem **1**: framework for **real-time simulation**

Problem **1**: framework for **real-time simulation**

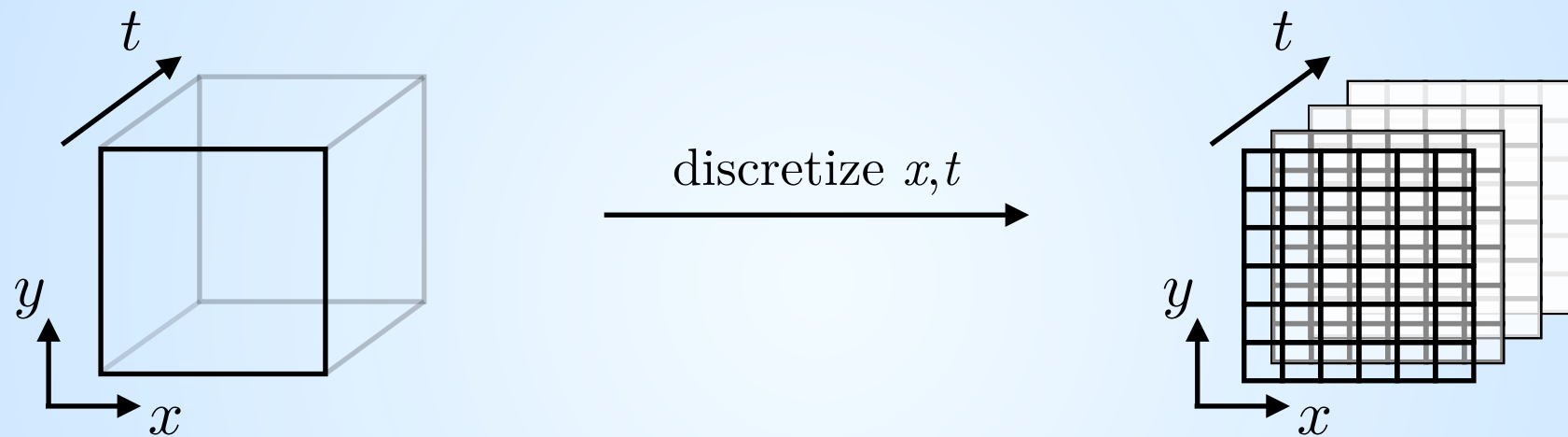
Monte Carlo Methods



$$\langle \hat{\mathcal{O}} \rangle = \frac{1}{Z} \int \mathcal{D}\phi \, \hat{\mathcal{O}}[\phi] e^{-S_E[\phi]/\hbar}$$

Problem 1: framework for **real-time simulation**

~~Monte Carlo Methods~~



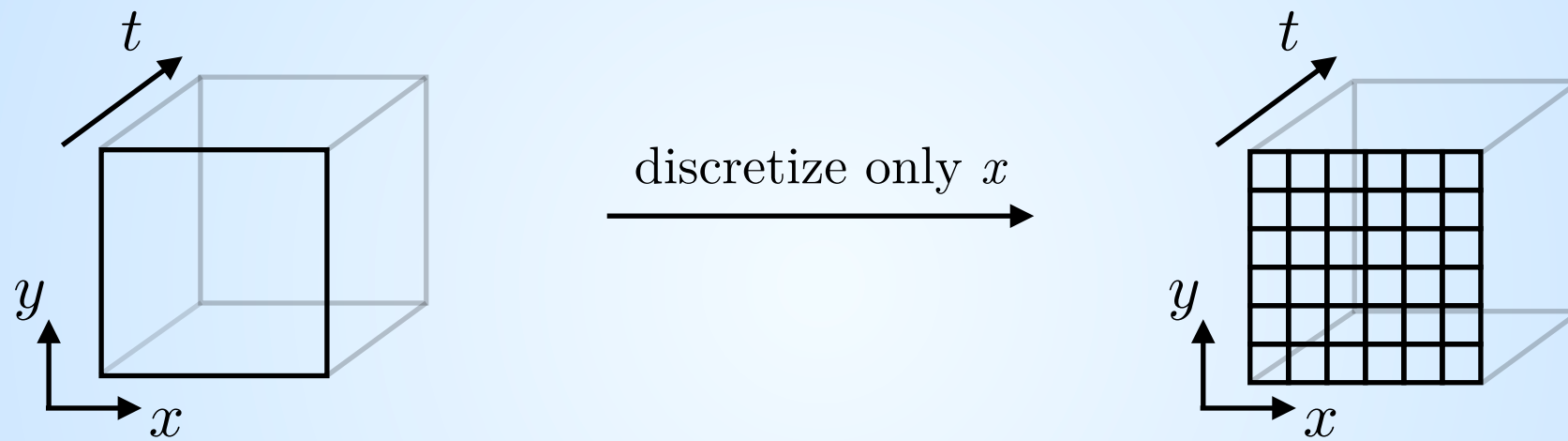
$$\langle \hat{\mathcal{O}} \rangle = \frac{1}{Z} \int \mathcal{D}\phi \, \hat{\mathcal{O}}[\phi] e^{-S_E[\phi]/\hbar}$$



sign problem!

Problem **1**: framework for **real-time simulation**

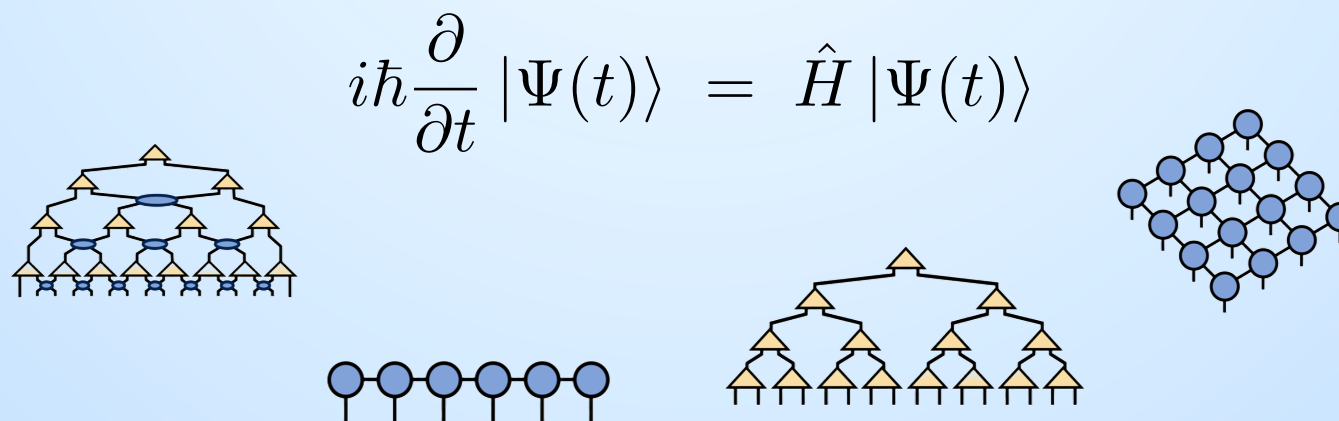
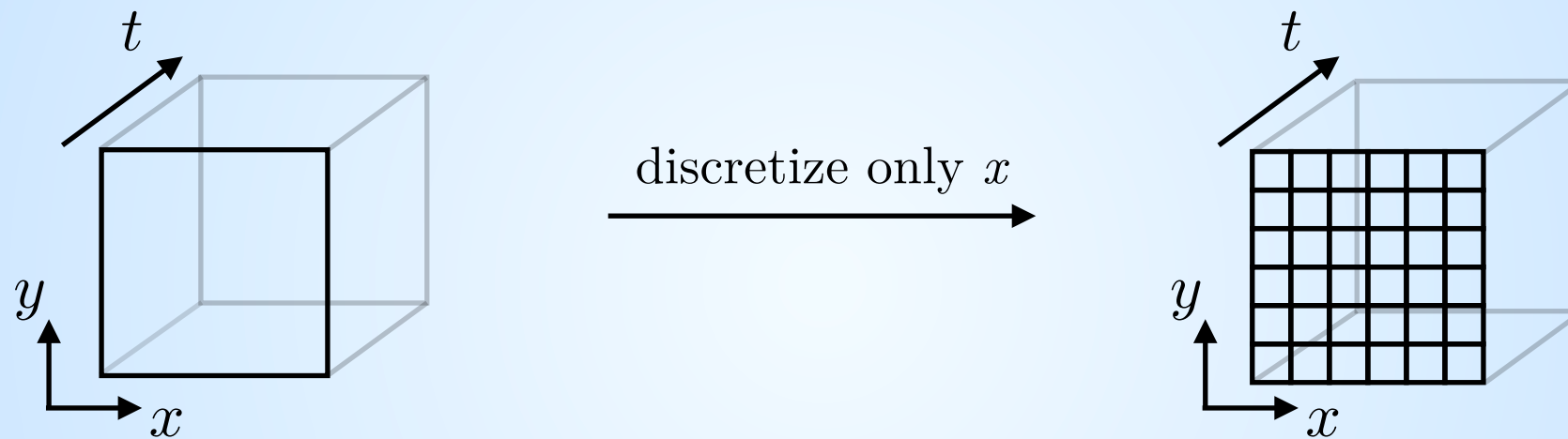
Hamiltonian Lattice Gauge Theory



$$i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle = \hat{H} |\Psi(t)\rangle$$

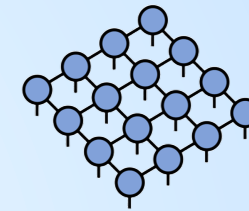
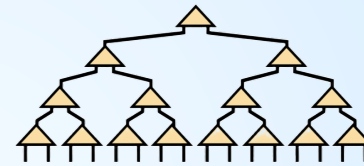
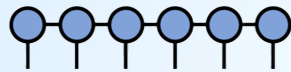
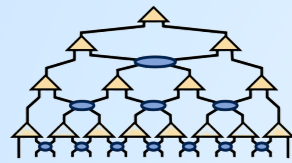
Problem 1: framework for **real-time simulation**

Hamiltonian Lattice Gauge Theory



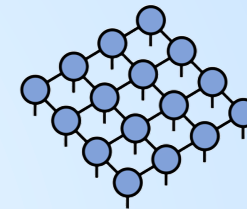
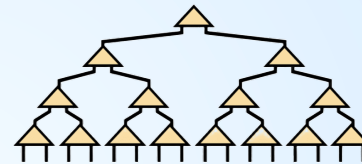
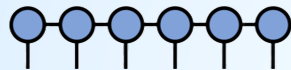
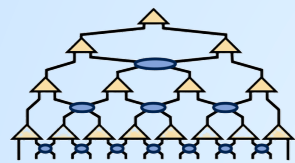
Problem 1: framework for **real-time simulation**

Tensor Networks

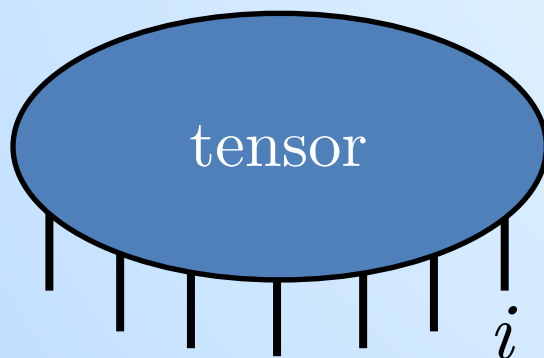


Problem 1: framework for **real-time simulation**

Tensor Networks



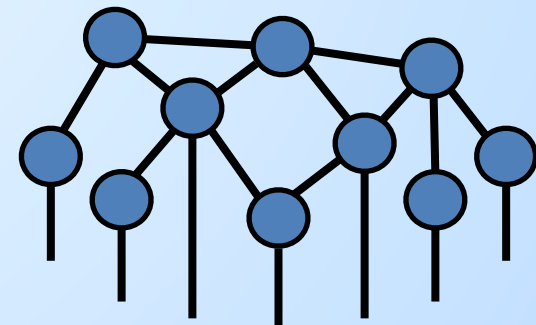
$|\psi\rangle$



ansatz



$|\psi\rangle_{\text{TN}}$

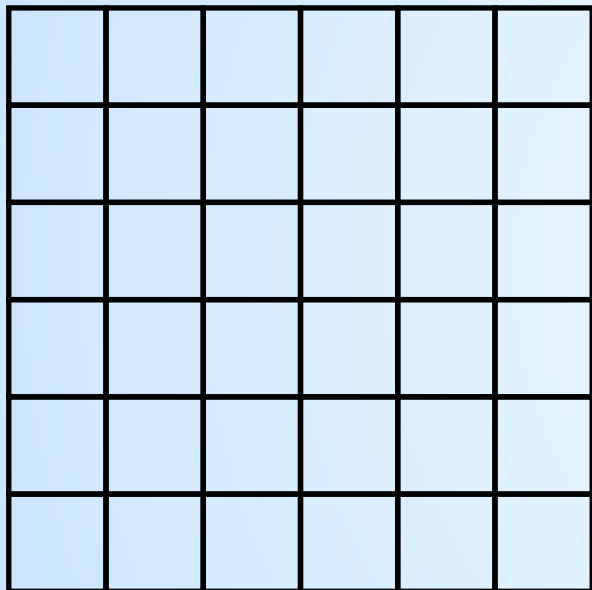


Problem **1**: framework for **real-time simulation** ✓

Problem **2**: prepare **initial scattering states** ?

Problem **1**: framework for **real-time simulation** ✓

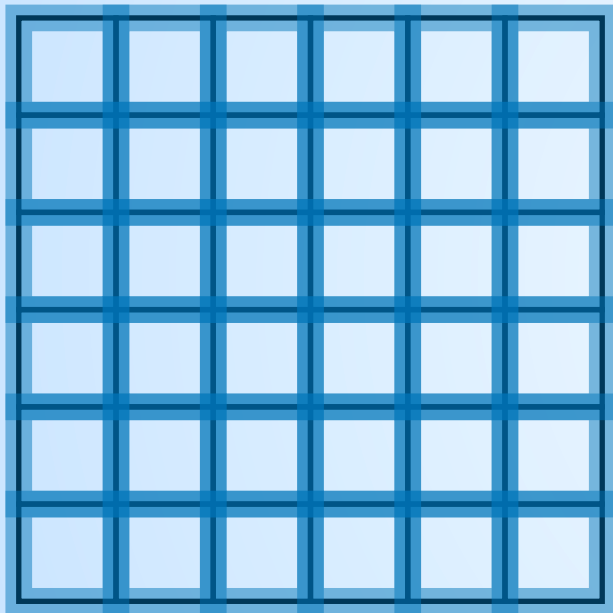
Problem **2**: prepare **initial scattering states** ?



$$\hat{H}$$

Problem **1**: framework for **real-time simulation** ✓

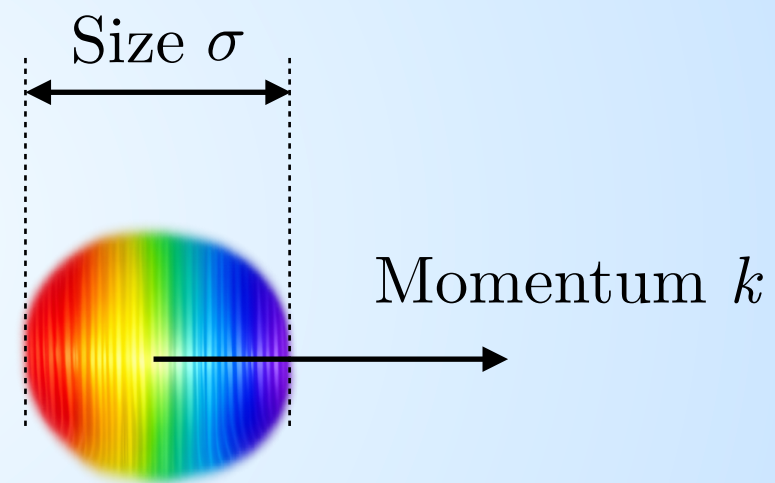
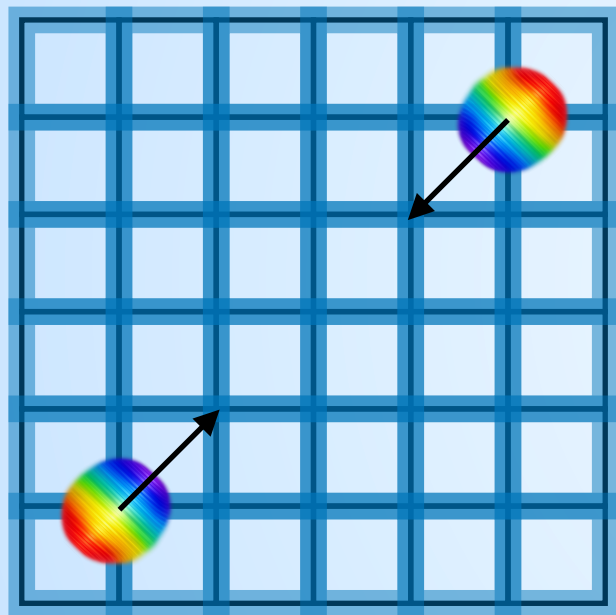
Problem **2**: prepare **initial scattering states** ?



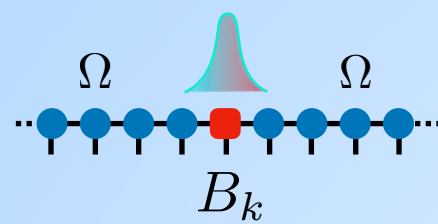
$$\hat{H} \longrightarrow |\Omega\rangle$$

Problem 1: framework for **real-time simulation** ✓

Problem 2: prepare **initial scattering states** ?



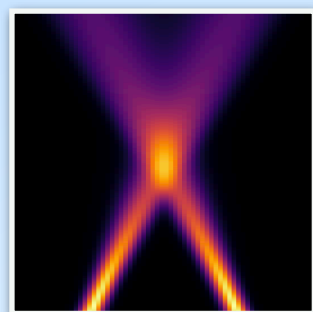
$\hat{H} \longrightarrow |\Omega\rangle \longrightarrow \text{Wave-packets state}$



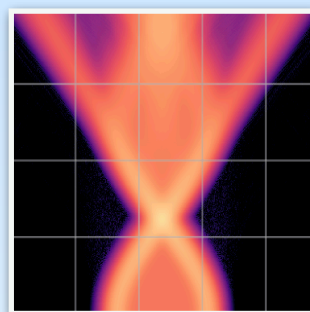
Tangent space



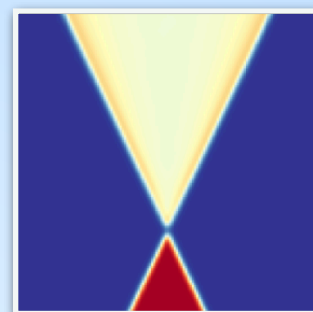
Purely TN strategy



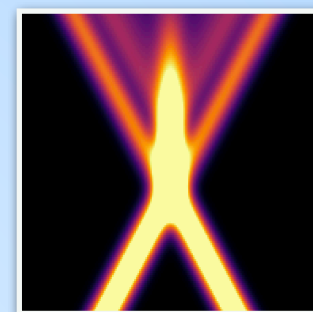
PRR 3, 013078 (2021)



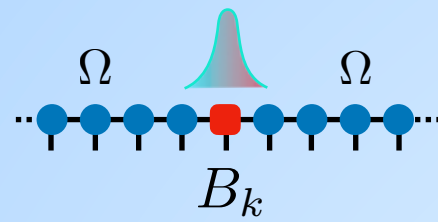
PRX-Q 3, 020316 (2022)



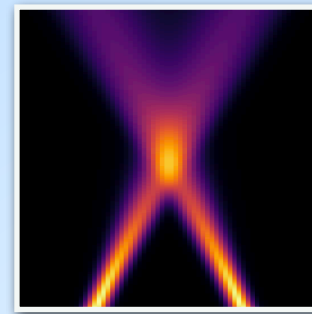
PRL 132, 091903 (2024)



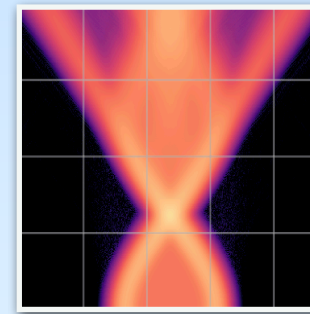
PRR 7, 023266 (2025)



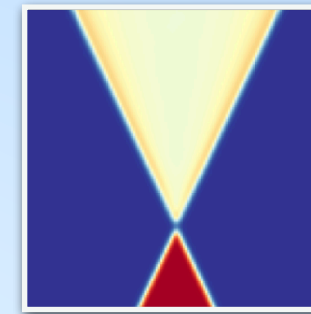
Tangent space



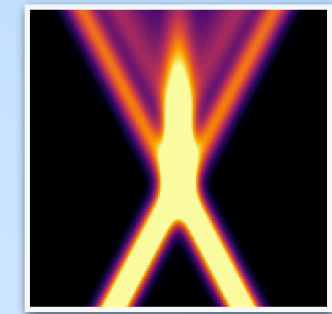
PRR 3, 013078 (2021)



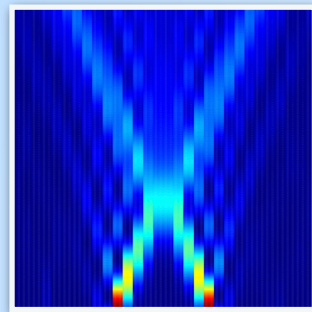
PRX-Q 3, 020316 (2022)



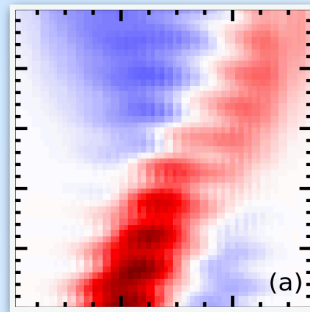
PRL 132, 091903 (2024)



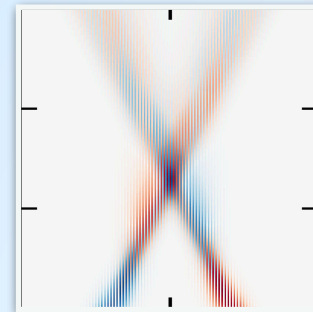
PRR 7, 023266 (2025)



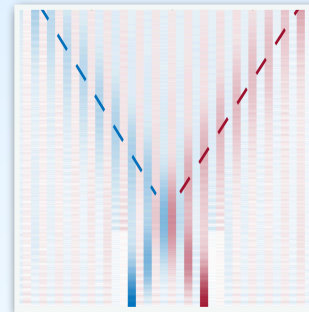
PRX 6, 011023 (2016)



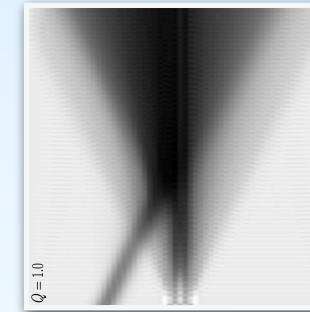
New J. Phys. 23 062001 (2021)



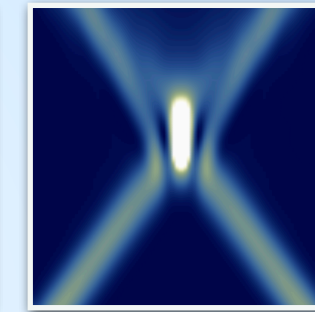
PRD 104, 114501 (2021)



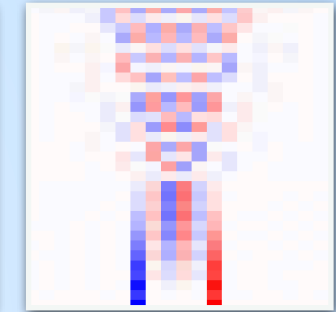
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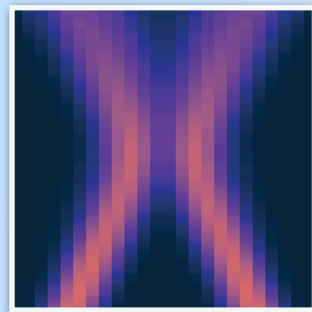
arXiv:2502.17558



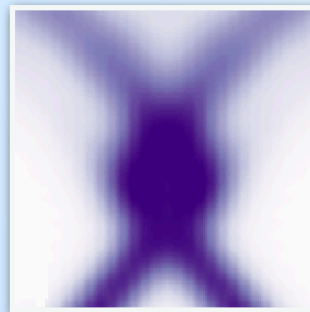
arXiv:2505.03878



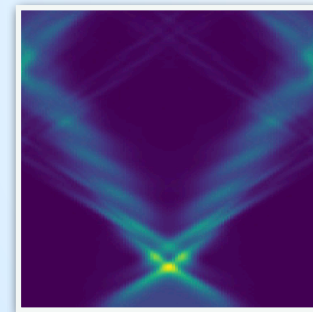
arXiv:2505.20387



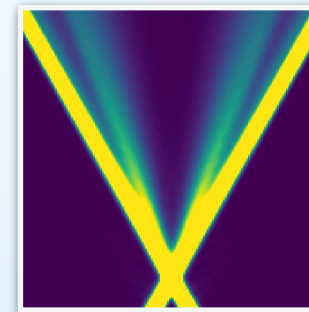
Quantum 9, 1773 (2025)



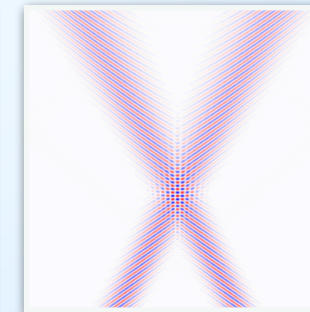
arXiv:2505.21240



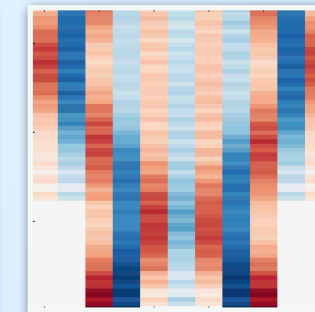
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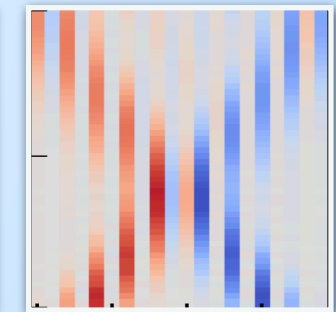
arXiv:2505.03111



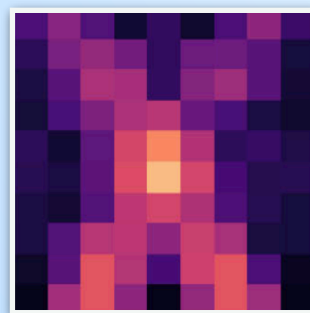
PRA 112, 012614 (2025)



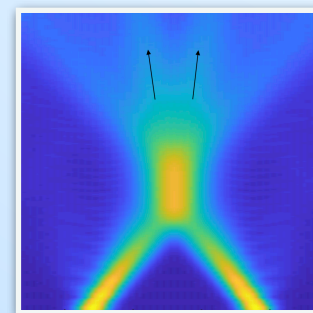
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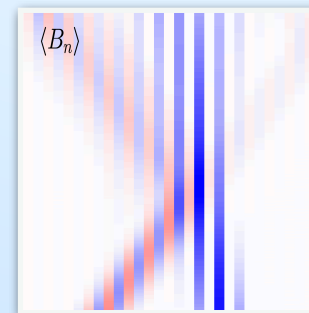
Quantum 9, 1638 (2025)



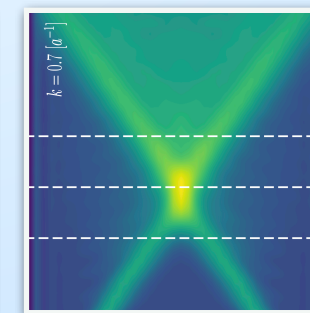
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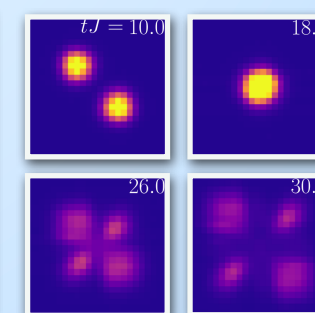
PRD 111, 014504 (2025)



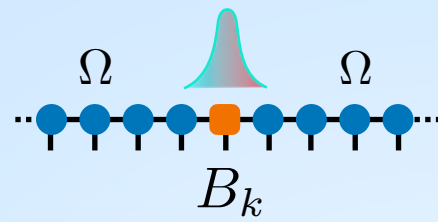
arXiv:2511.00154



arXiv:2510.16101

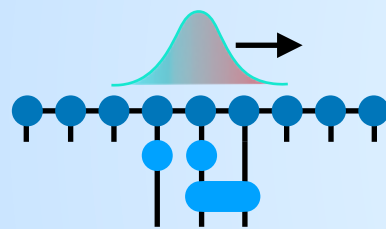


arXiv:2509.02702

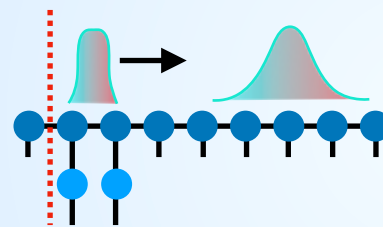


Tangent space

TN

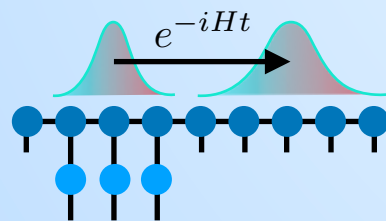


Ad-hoc Ansatzes

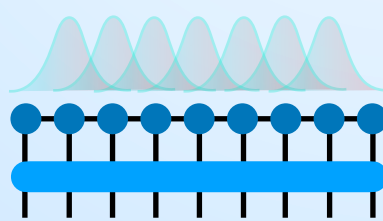


Potential wall

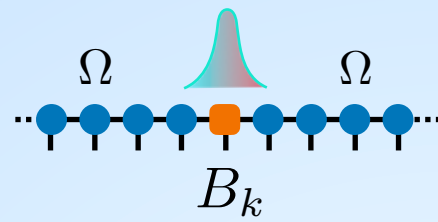
QS



Adiabatic Theorem

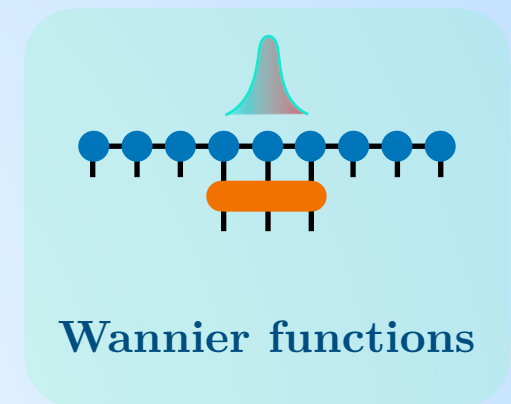


W-states



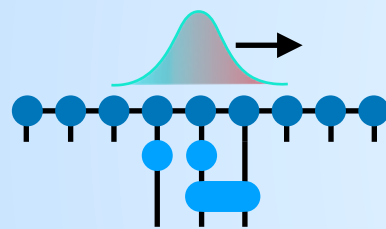
Tangent space

TN

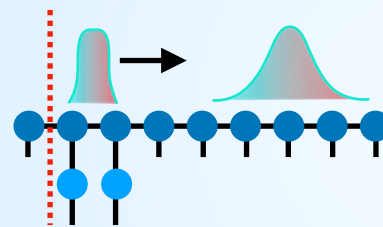


Wannier functions

- ✓ Model independent
- ✓ Easy detection

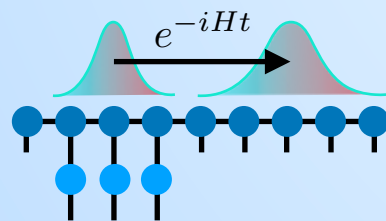


Ad-hoc Ansatzes

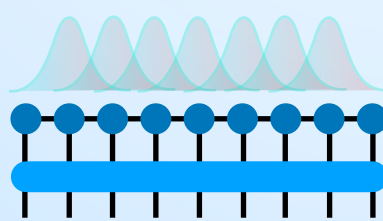


Potential wall

QS

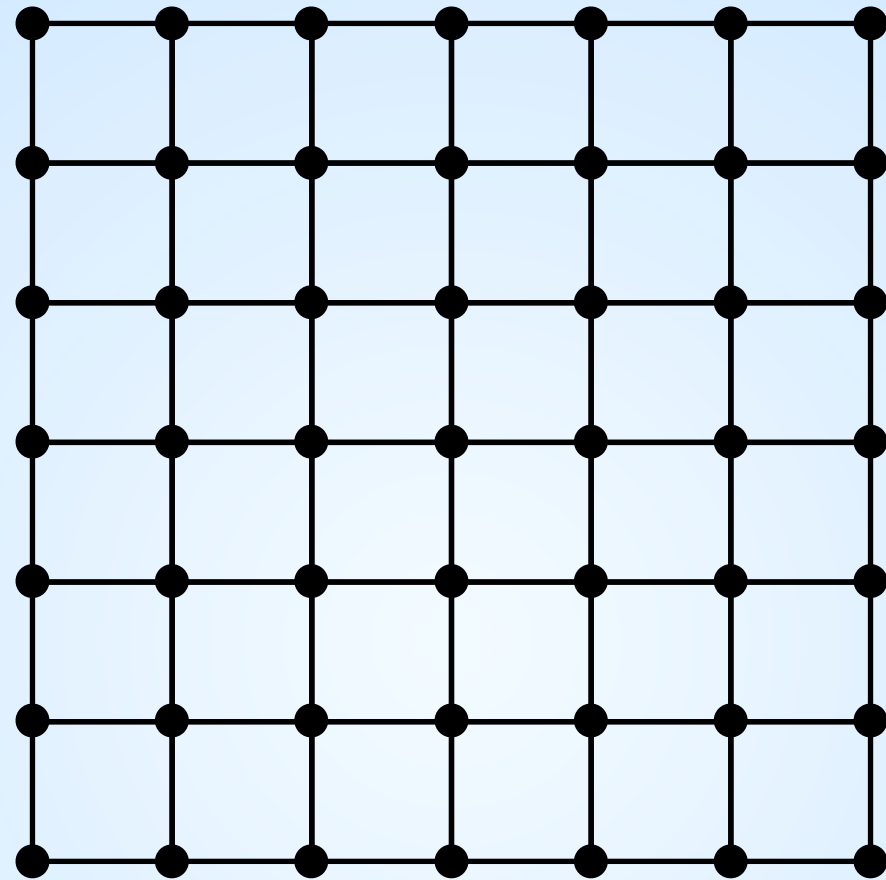


Adiabatic Theorem

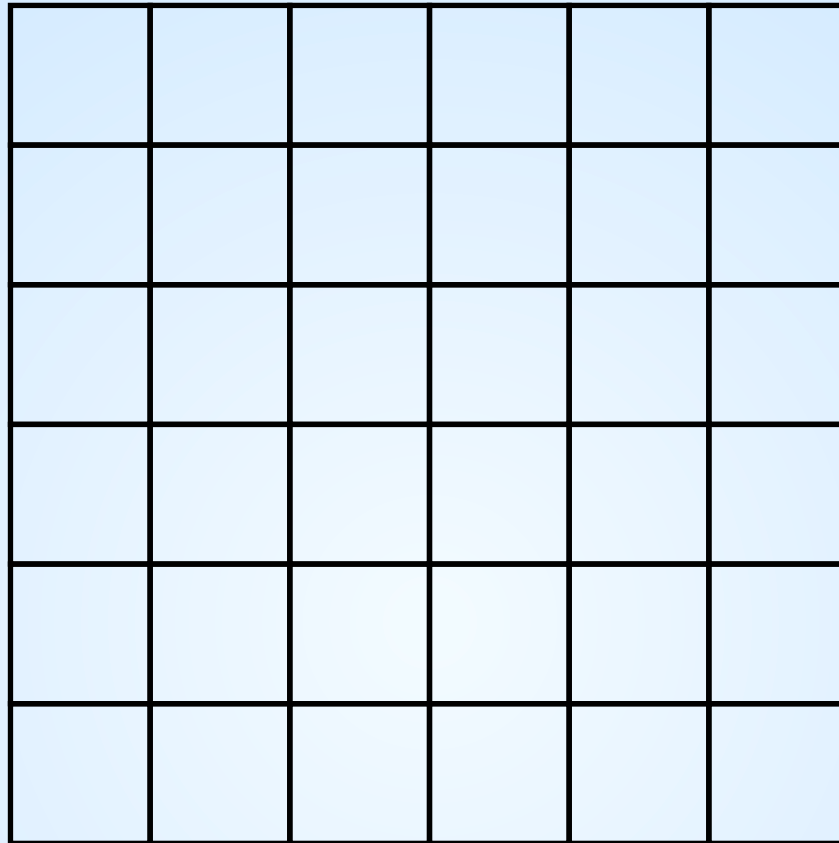


W-states

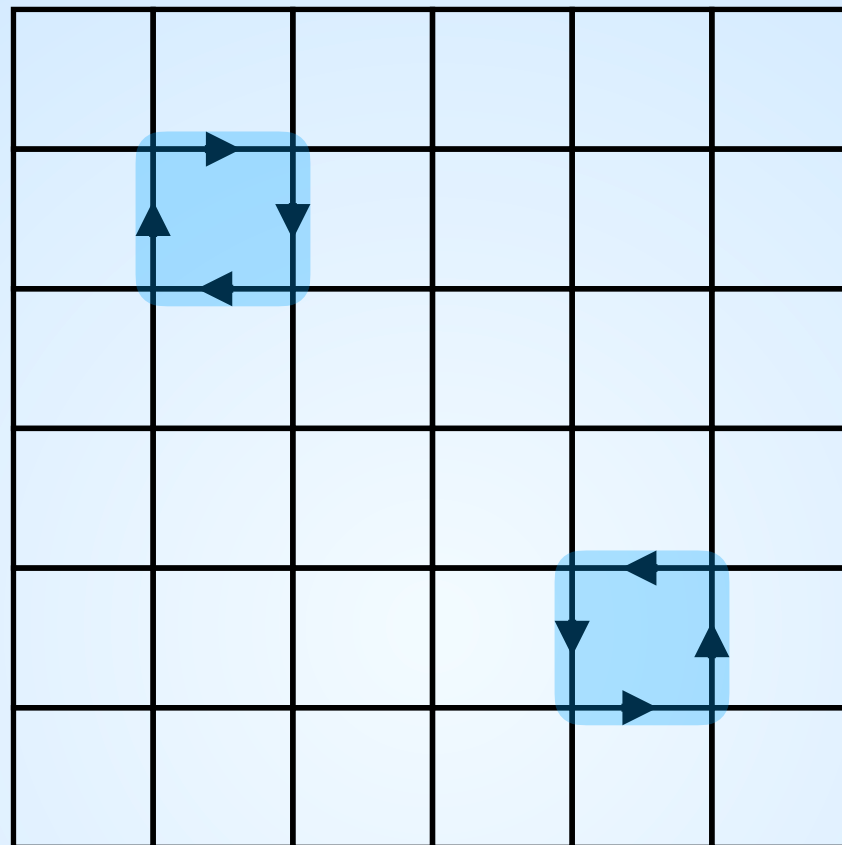
II. MODEL



Hamiltonian Lattice Gauge Theory

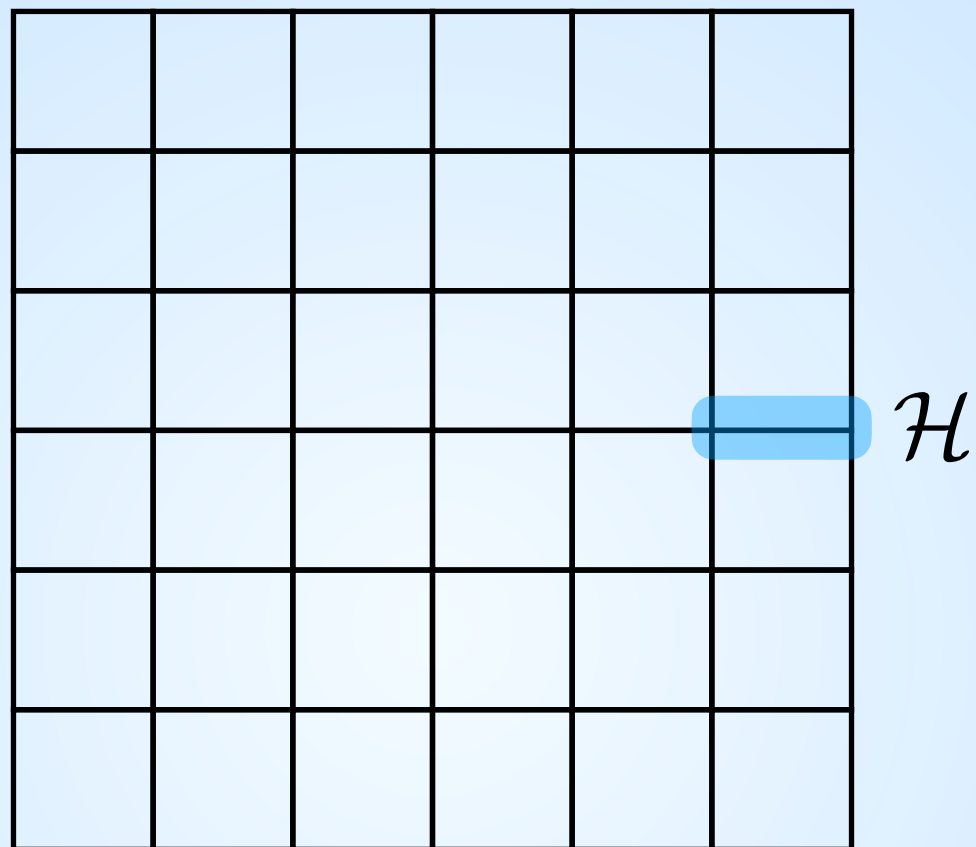


$$\hat{H} = \frac{g^2}{2a} \sum_{\ell} \hat{E}_{\ell}^2 - \frac{1}{2g^2 a} \sum_{\square} \text{Tr}(\hat{U}_{\square} + \hat{U}_{\square}^{\dagger})$$



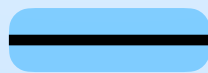
“glueballs”

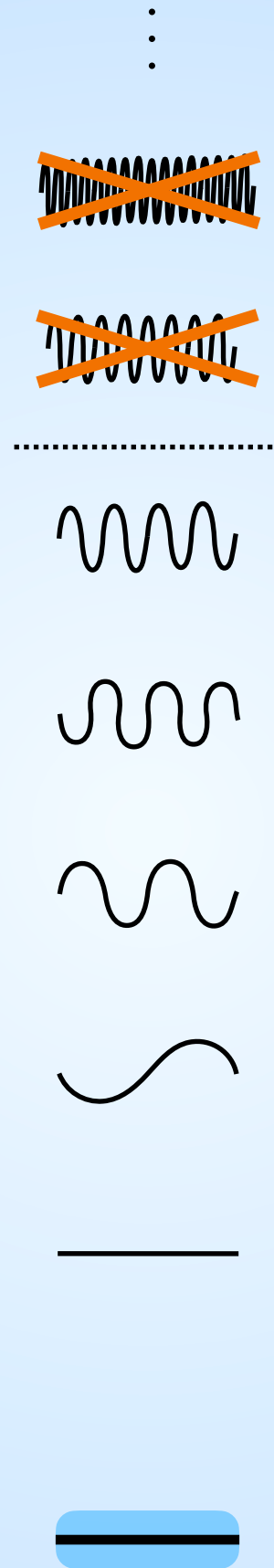
$$\hat{H} = \frac{g^2}{2a} \sum_{\ell} \hat{E}_{\ell}^2 - \frac{1}{2g^2 a} \sum_{\square} \text{Tr}(\hat{U}_{\square} + \hat{U}_{\square}^{\dagger})$$

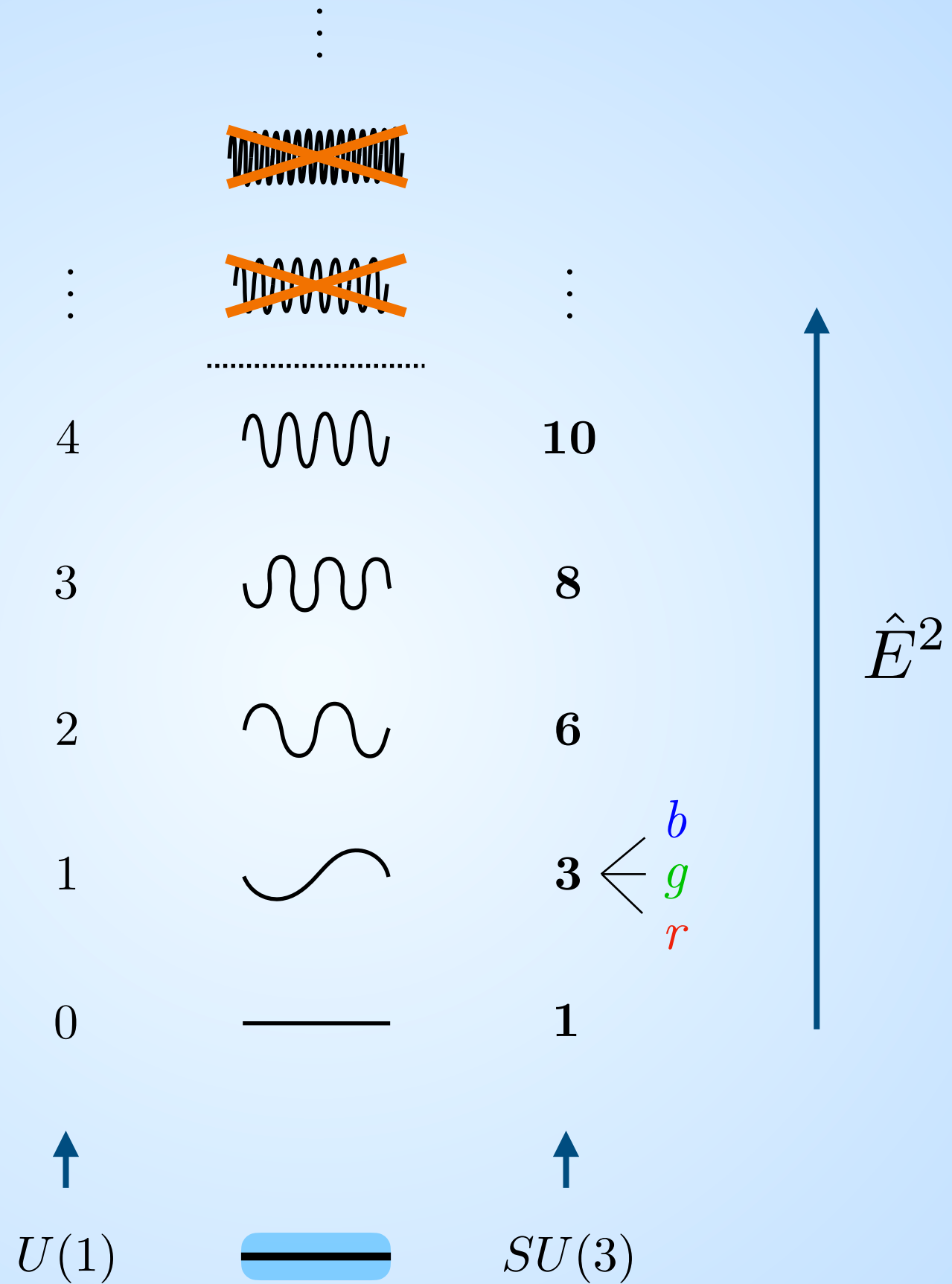


$$\hat{H} = \frac{g^2}{2a} \sum_{\ell} \hat{E}_{\ell}^2 - \frac{1}{2g^2 a} \sum_{\square} \text{Tr}(\hat{U}_{\square} + \hat{U}_{\square}^{\dagger})$$

⋮







(Clebsch-Gordan)

$\hat{U}$



$U(1)$



0

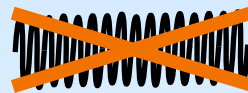
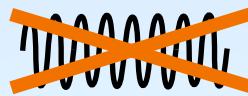
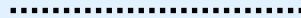
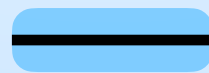
1

2

3

4

$\vdots$



$\vdots$

$SU(3)$



1

3

6

8

10

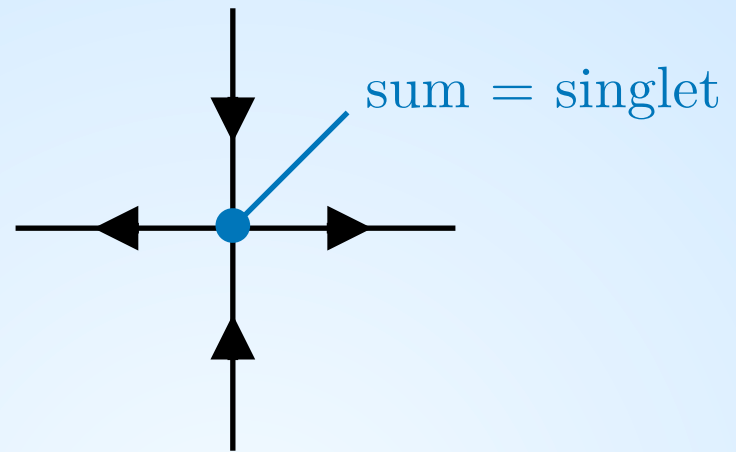
$\vdots$

$\begin{matrix} b \\ g \\ r \end{matrix} \leftarrow$

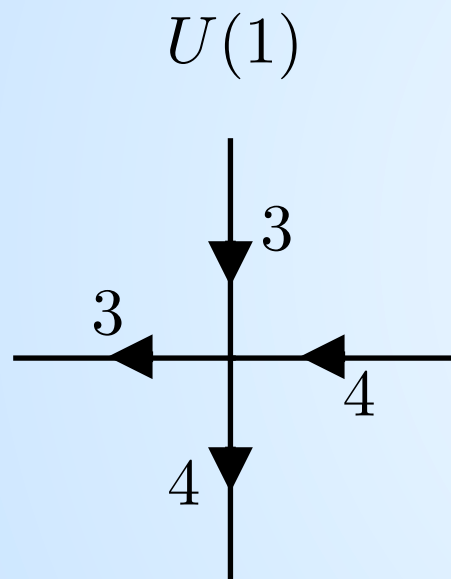
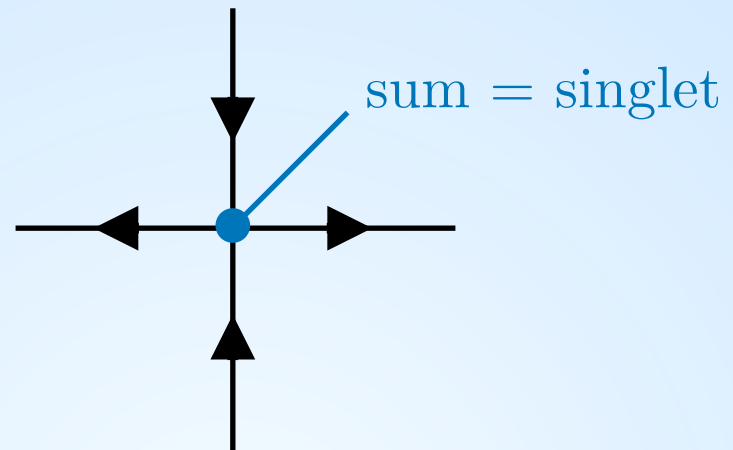
$\hat{E}^2$



Gauge invariance $\iff$ Gauss' Law



Gauge invariance $\iff$ Gauss' Law

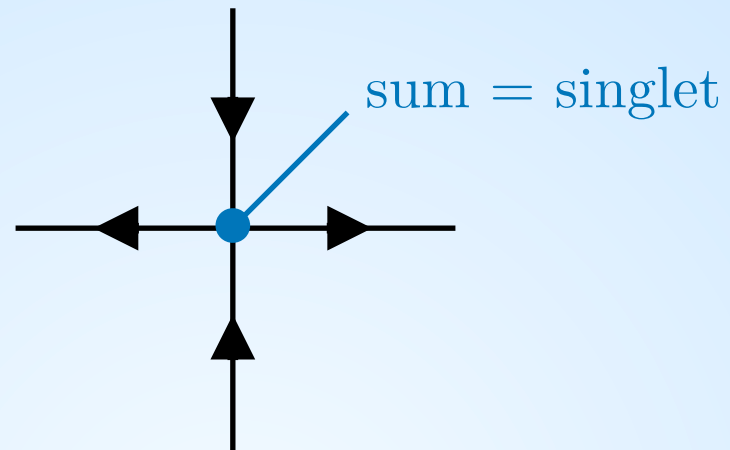


Abelian fusion (trivial)

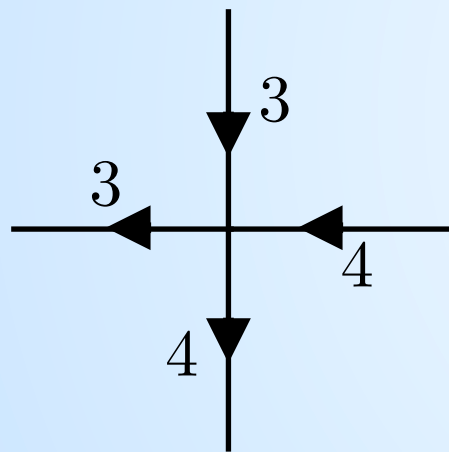
$$3 - 3 + 4 - 4 = 0$$

singlet

Gauge invariance $\iff$ Gauss' Law



$U(1)$

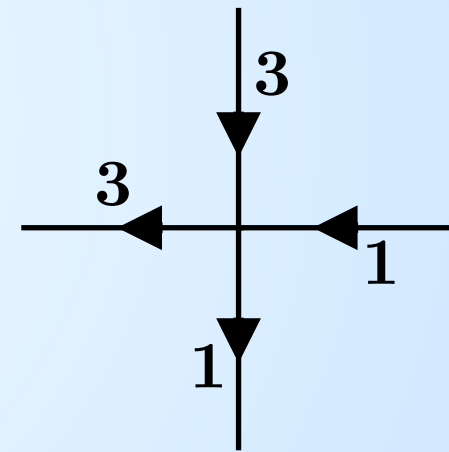


Abelian fusion (trivial)

$$3 - 3 + 4 - 4 = \boxed{0}$$

singlet

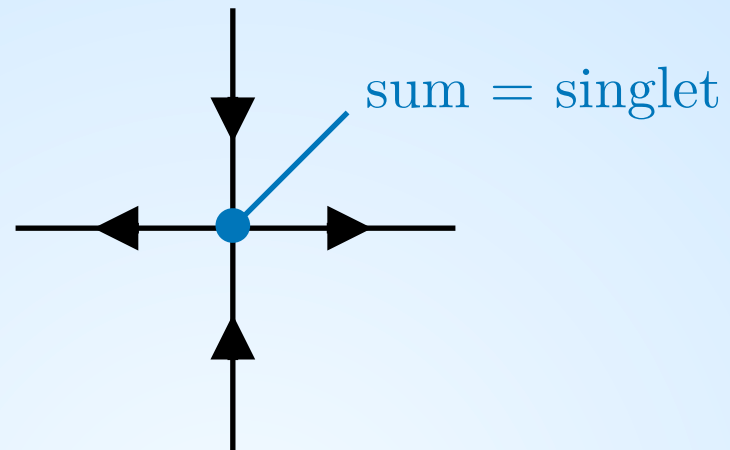
$SU(3)$



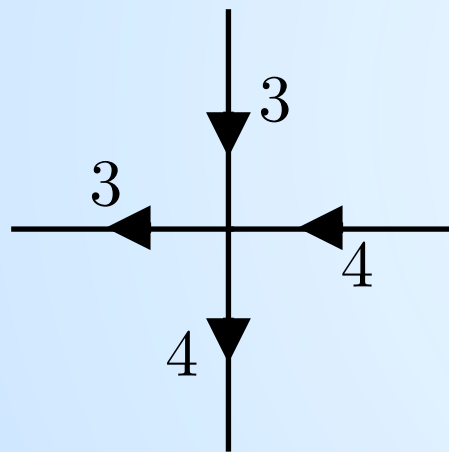
Non-abelian fusion

$$\mathbf{1} \oplus \mathbf{8}$$

Gauge invariance $\iff$ Gauss' Law



$U(1)$

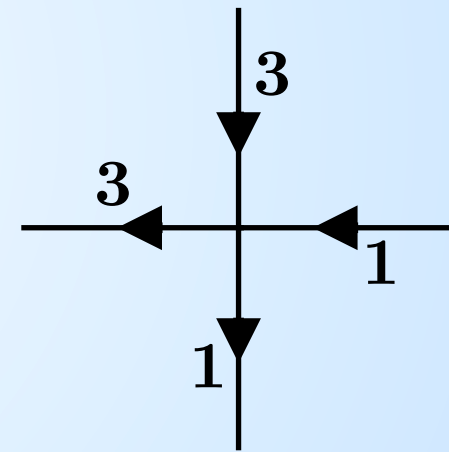


Abelian fusion (trivial)

$$3 - 3 + 4 - 4 = \boxed{0}$$

singlet

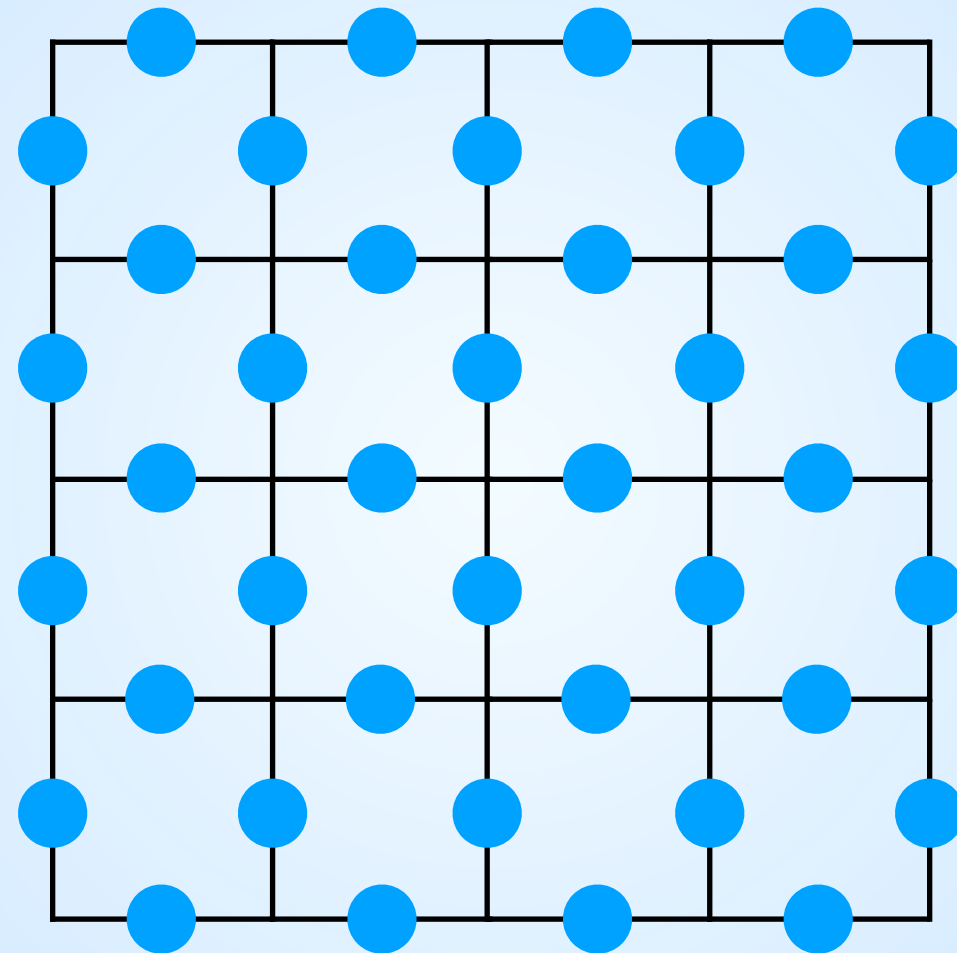
$SU(3)$

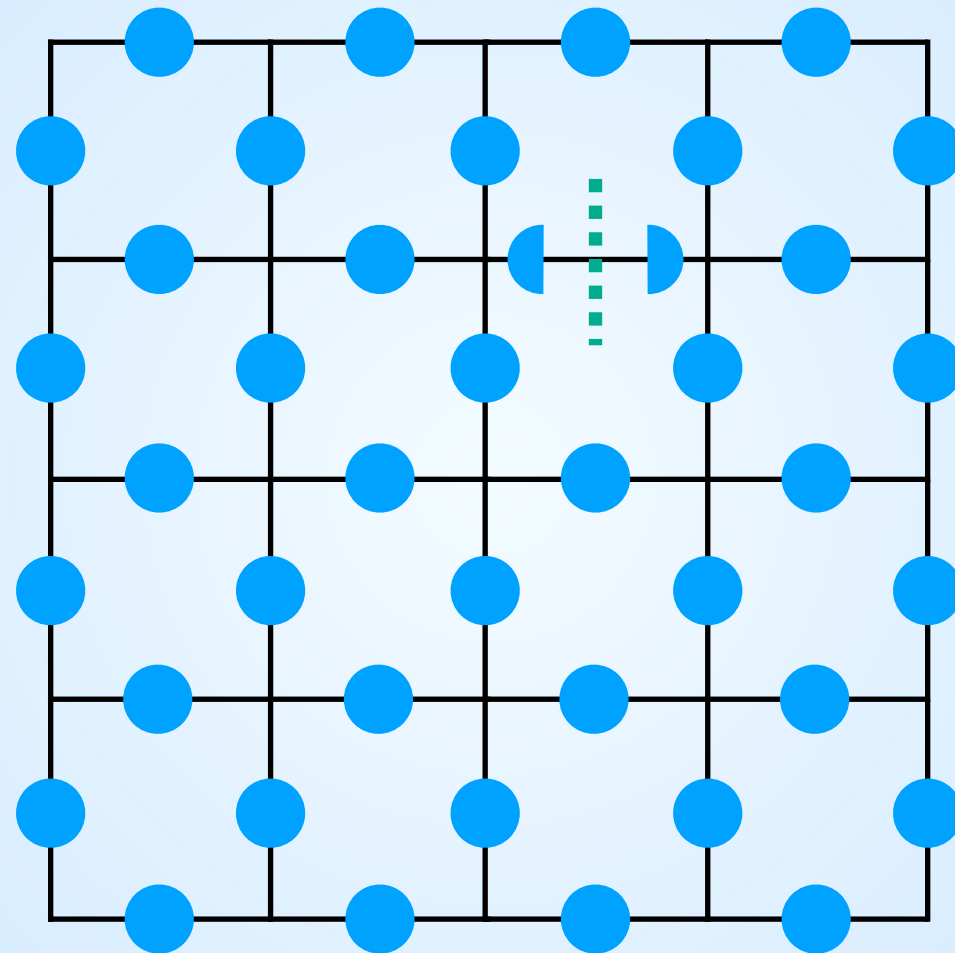


Non-abelian fusion

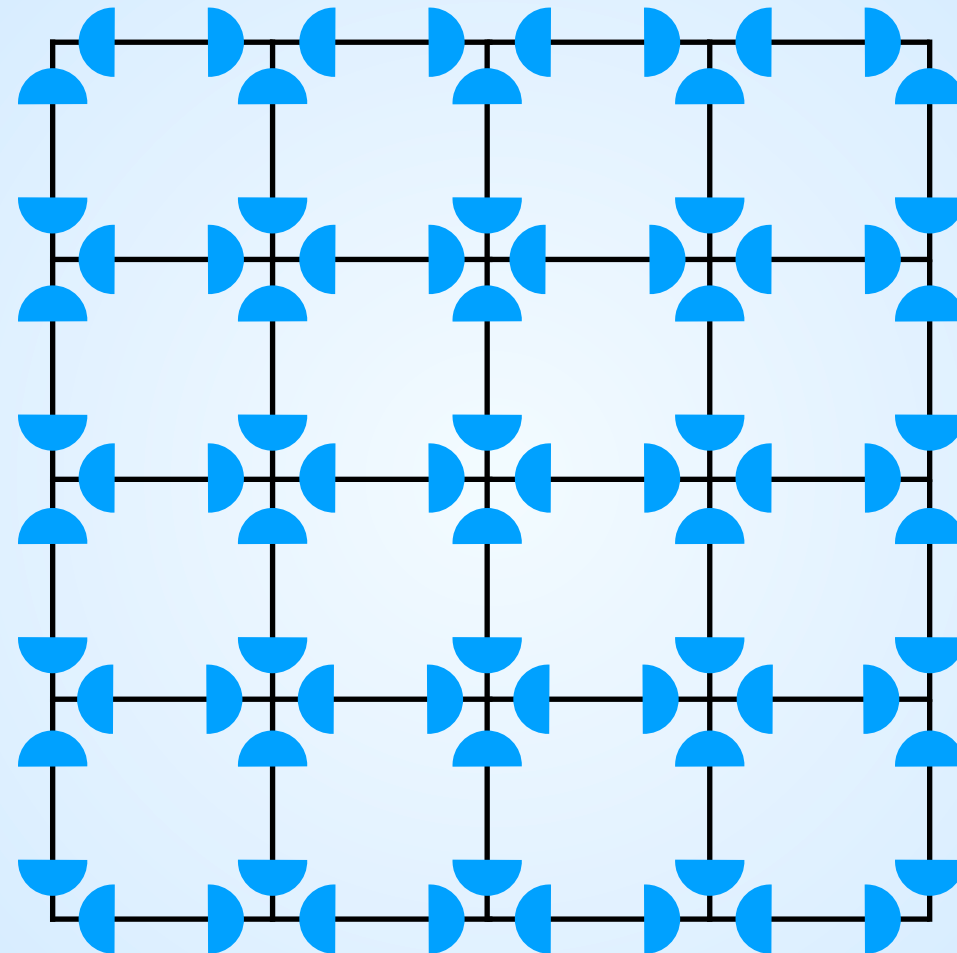
$$\boxed{1} \oplus \cancel{8}$$

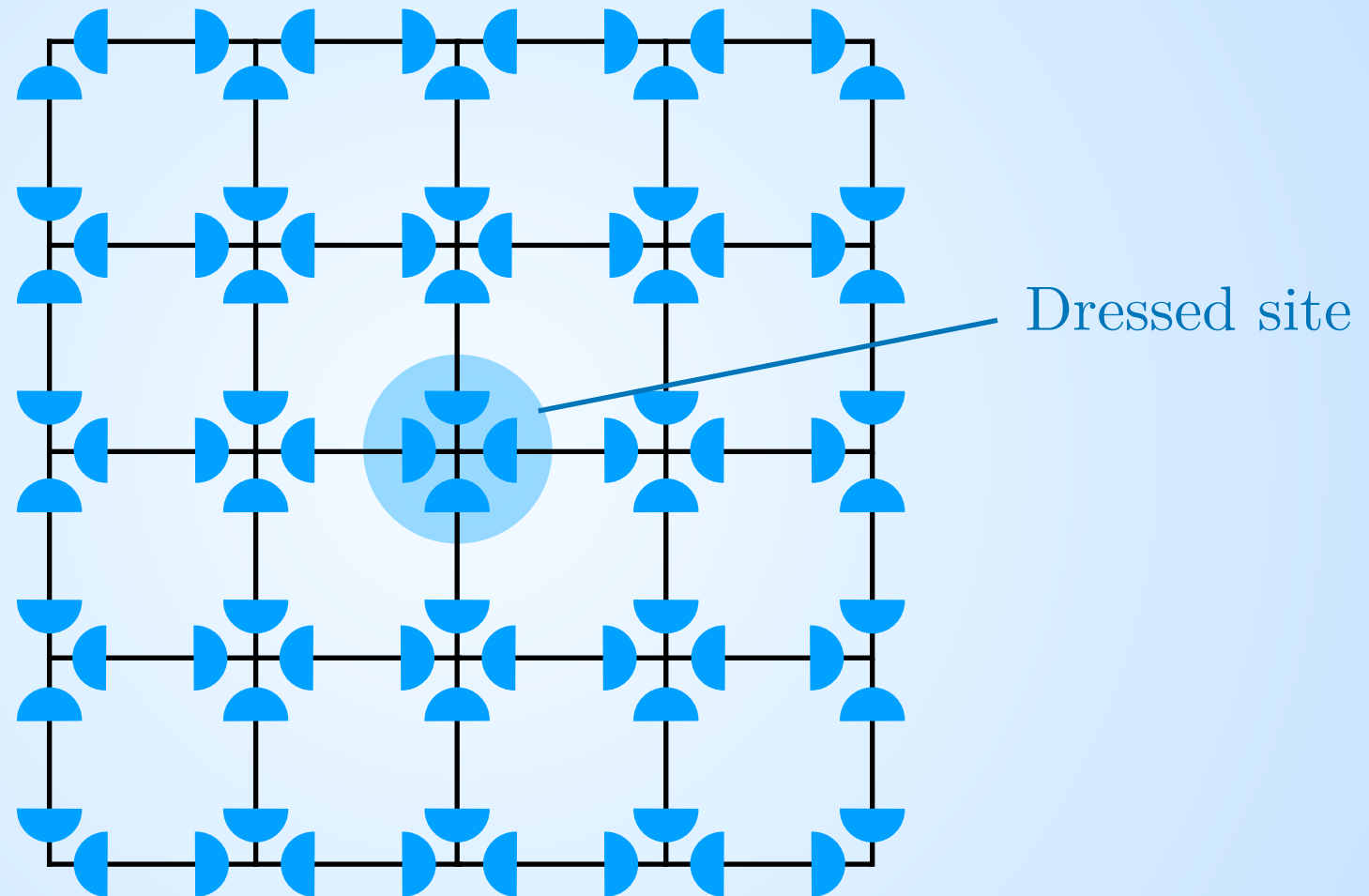
singlet



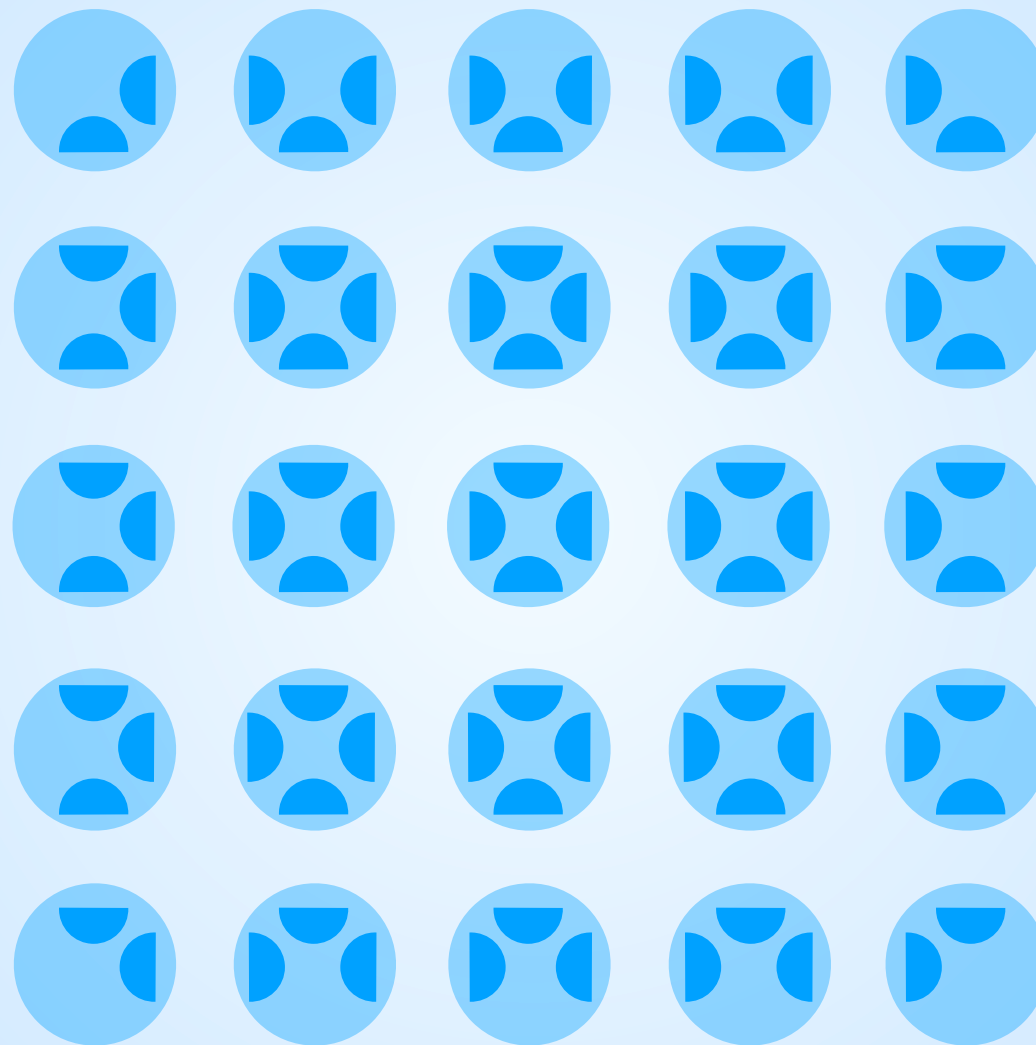


We “split” the links (semi-links)

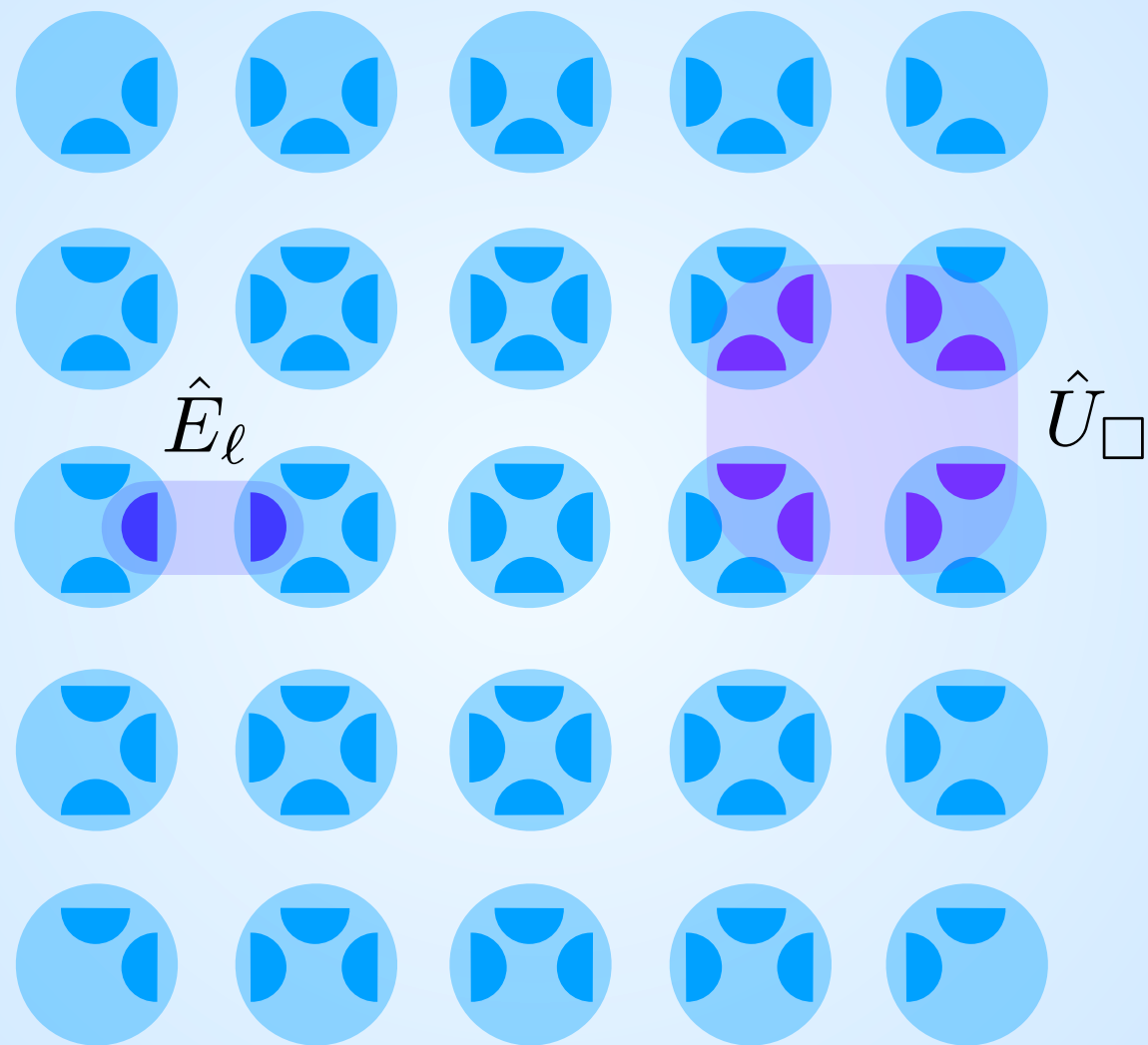


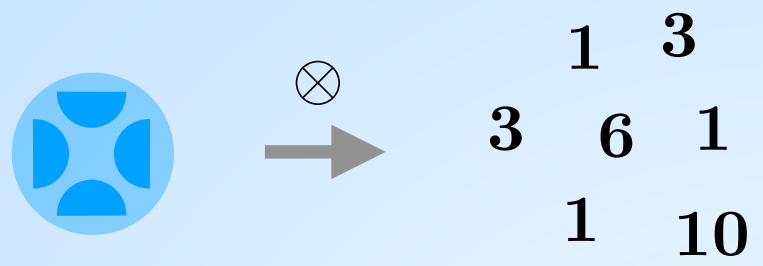


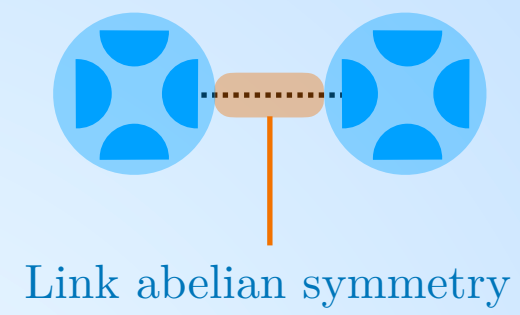
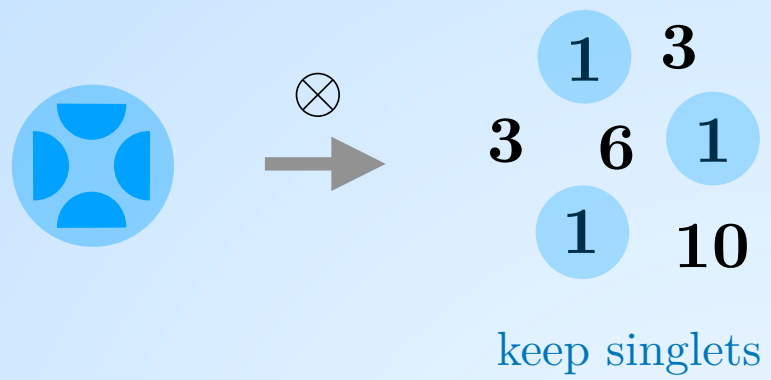
Dressed site basis

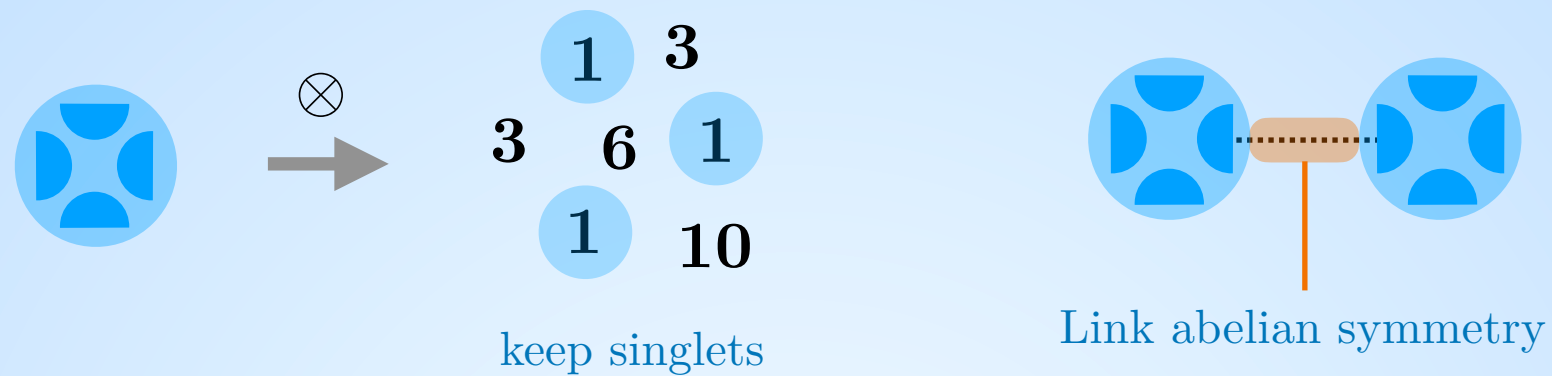


Dressed site basis









∇ group, dimension and truncation?
 You need a [library](#)!



Dresau6

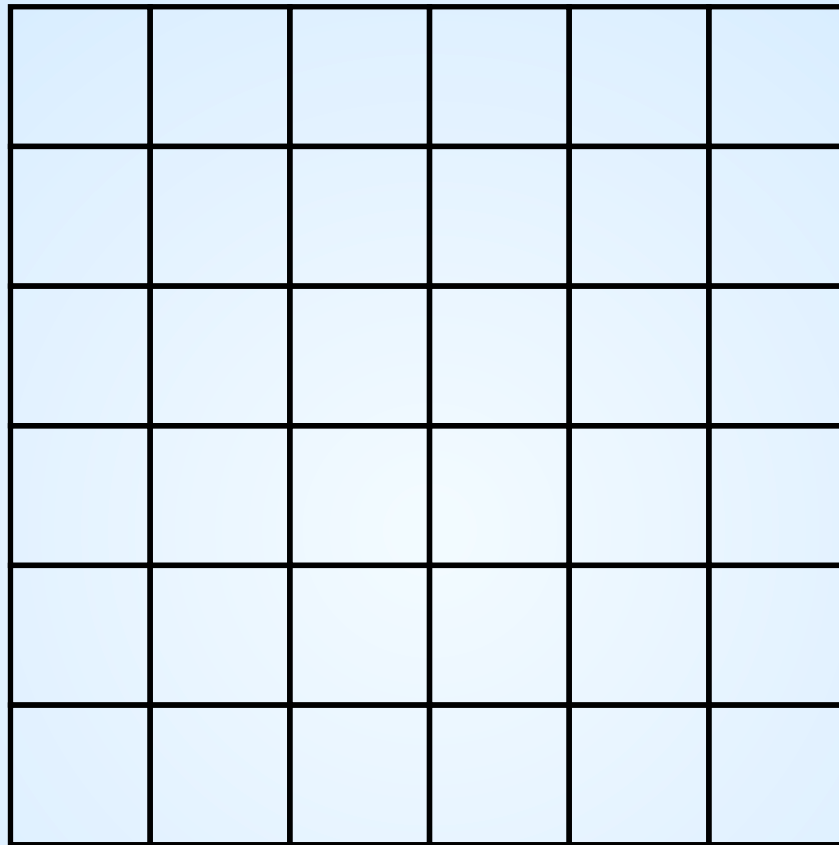


M. Rigobello



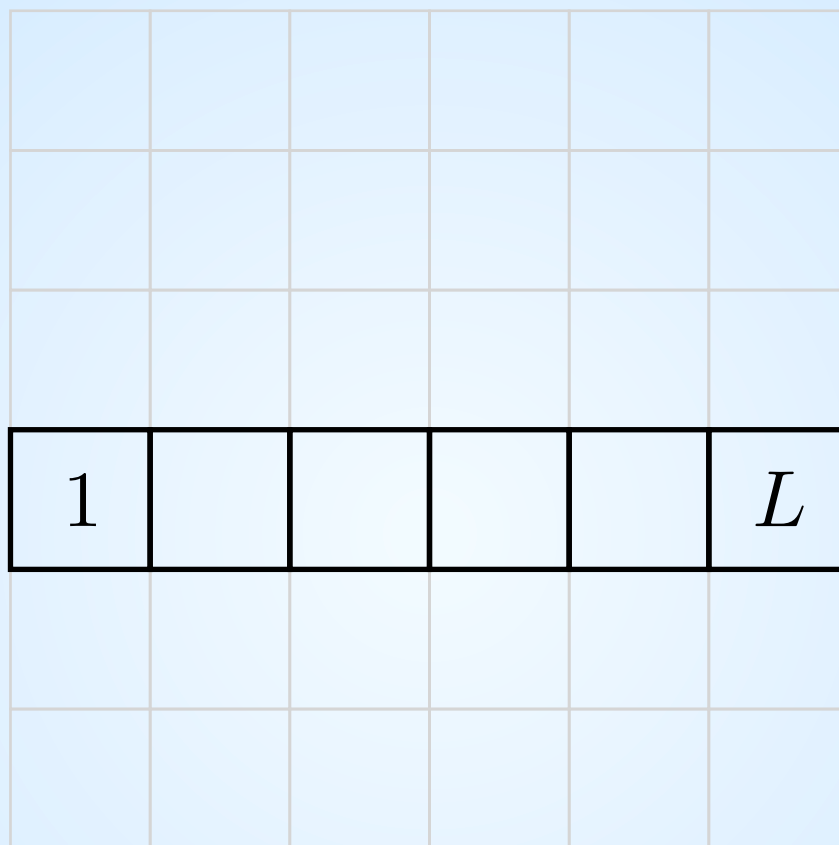
P. Majcen

Back to our model...



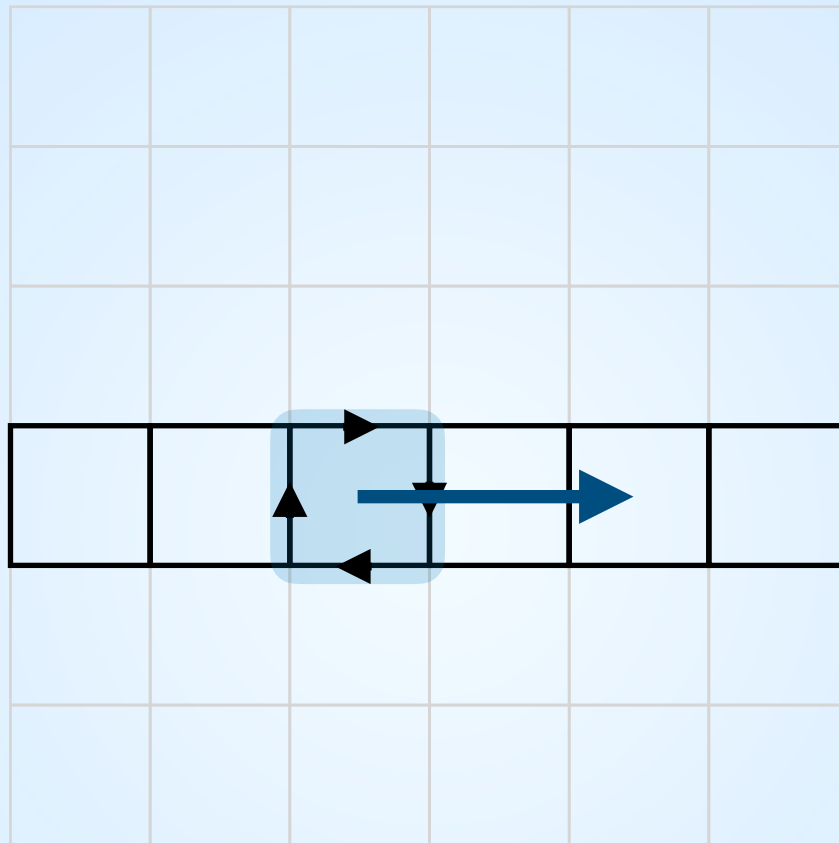
$$\hat{H} = \frac{g^2}{2a} \sum_{\ell} \hat{E}_{\ell}^2 - \frac{1}{2g^2 a} \sum_{\square} \text{Tr}(\hat{U}_{\square} + \hat{U}_{\square}^{\dagger})$$

“Ladder geometry”



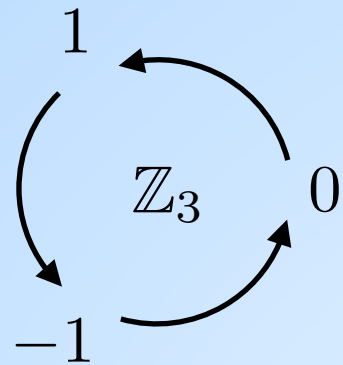
$$\hat{H} = \frac{g^2}{2a} \sum_{\ell} \hat{E}_{\ell}^2 - \frac{1}{2g^2 a} \sum_{\square} \text{Tr}(\hat{U}_{\square} + \hat{U}_{\square}^{\dagger})$$

“Ladder geometry”

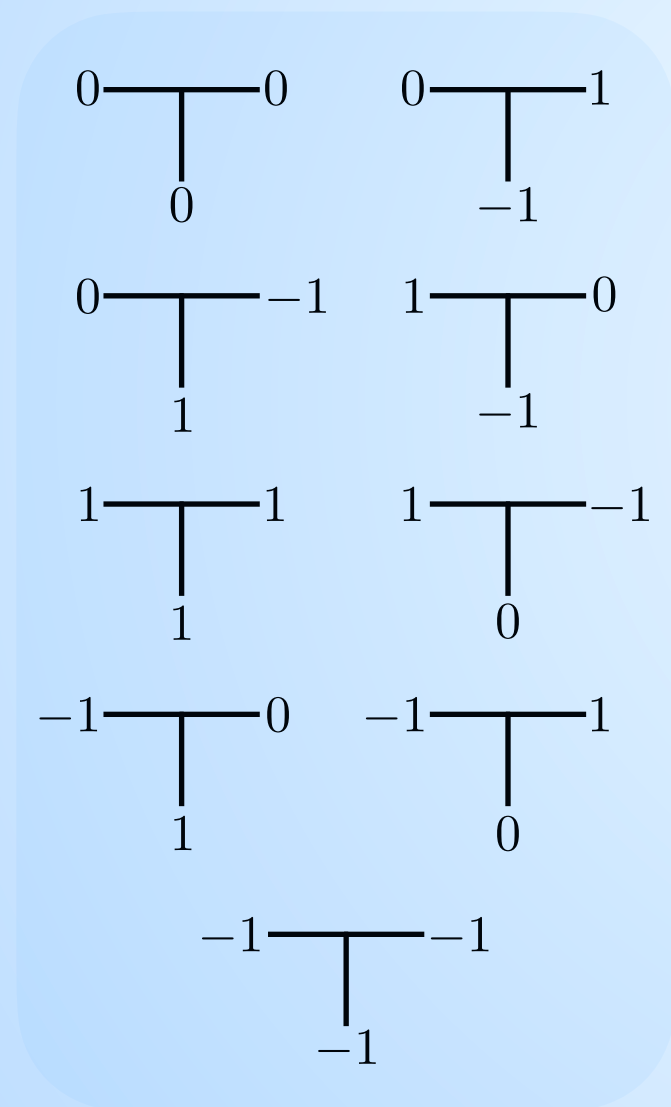


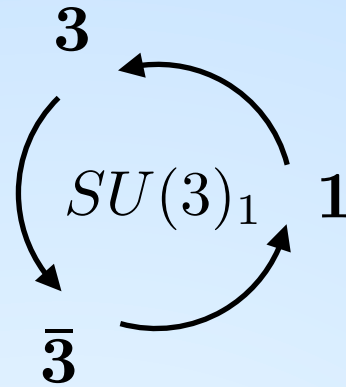
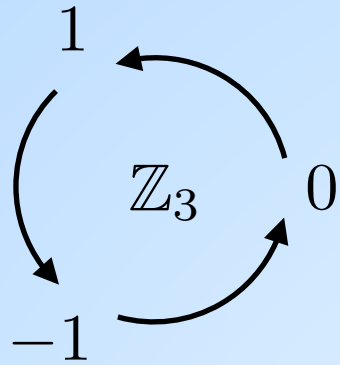
$$\hat{H} = \frac{g^2}{2a} \sum_{\ell} \hat{E}_{\ell}^2 - \frac{1}{2g^2 a} \sum_{\square} \text{Tr}(\hat{U}_{\square} + \hat{U}_{\square}^{\dagger})$$

(2-nd order hopping)

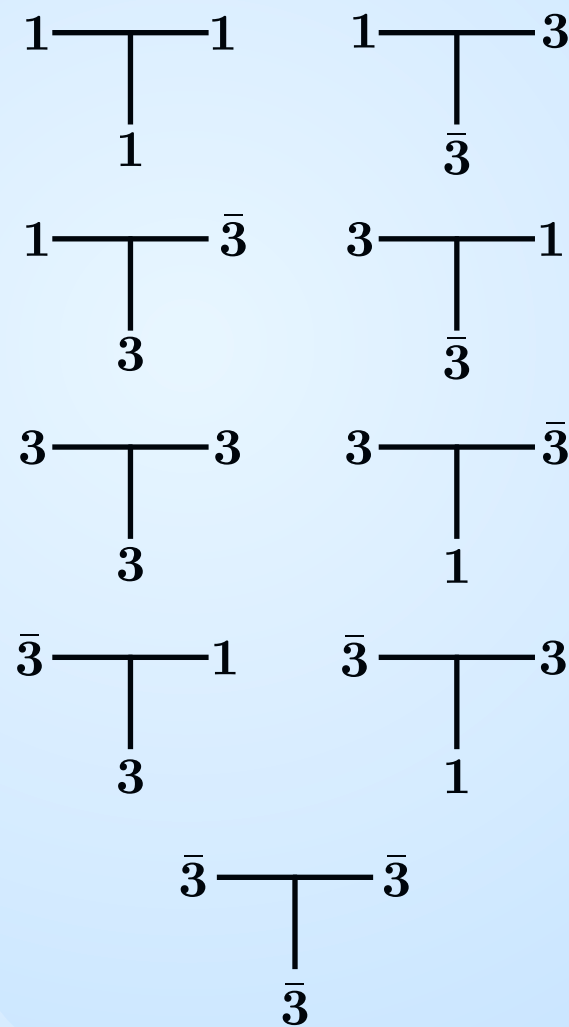
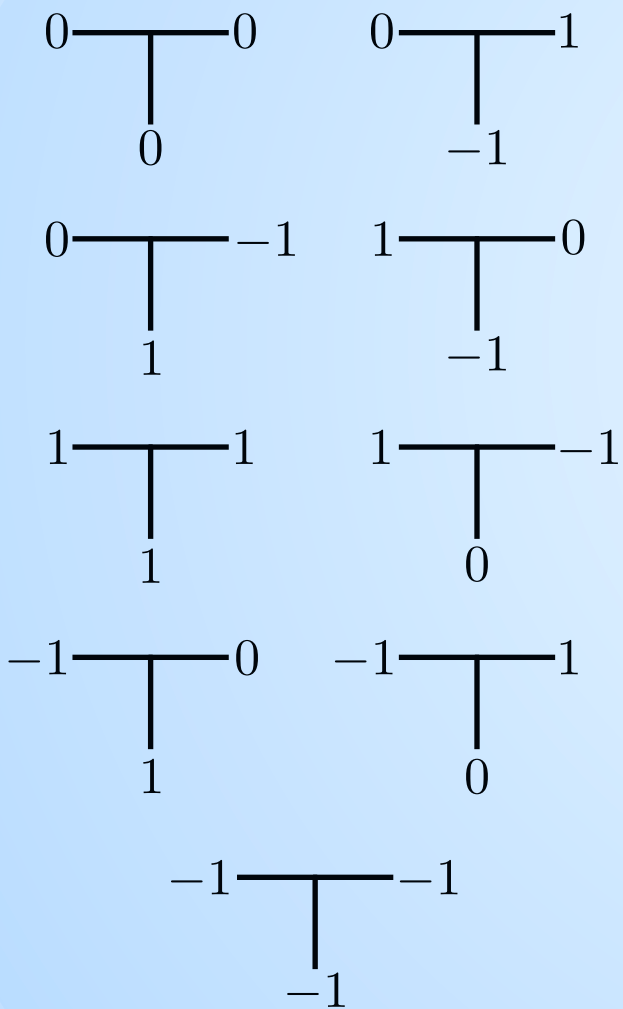


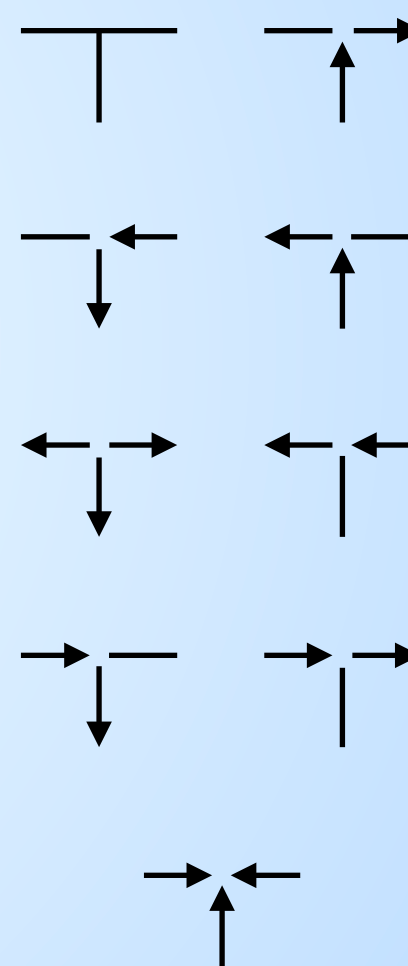
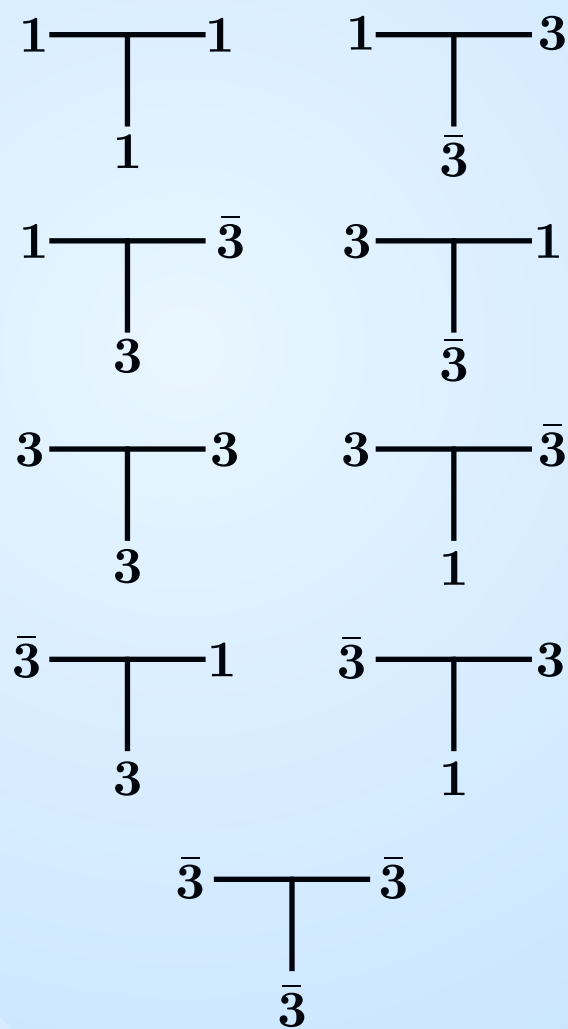
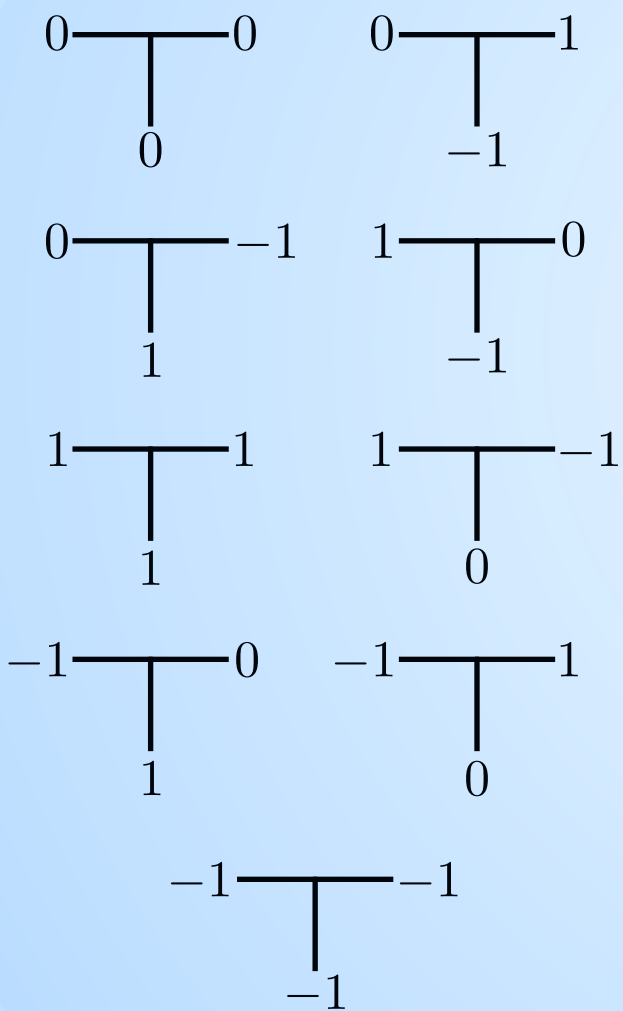
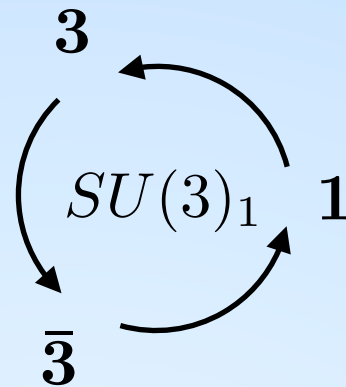
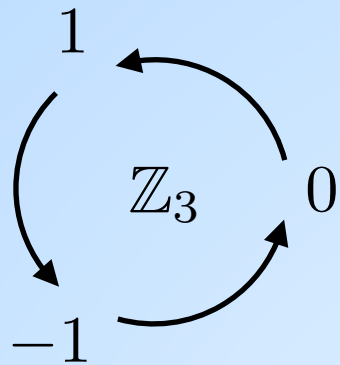
Subgroup approximation





“Hardcore gluons”



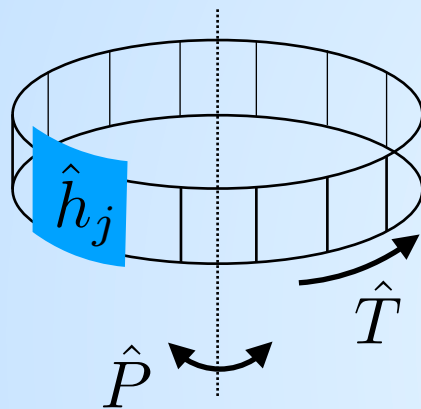


III. METHOD AND RESULTS

Summary of the algorithm

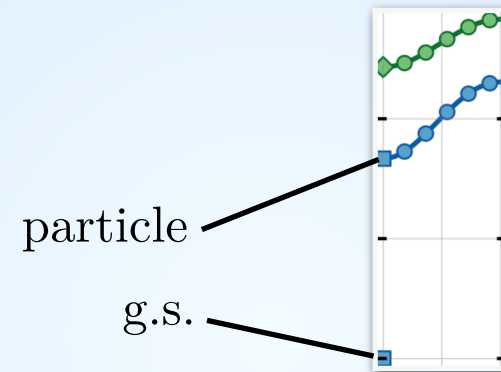
STEP 0.

Model definition



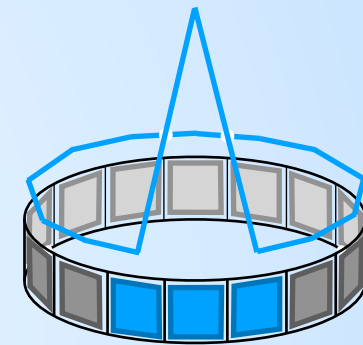
STEP 1.

Dispersion relation



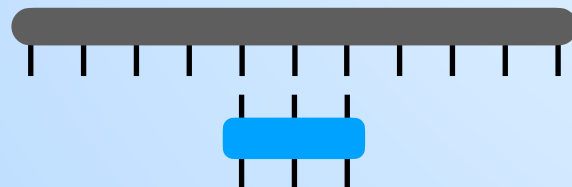
STEP 2.

WF localization



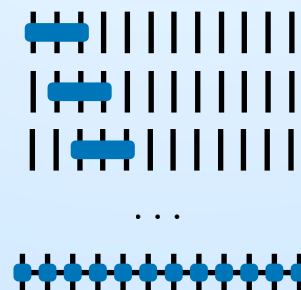
STEP 3.

Creation operator



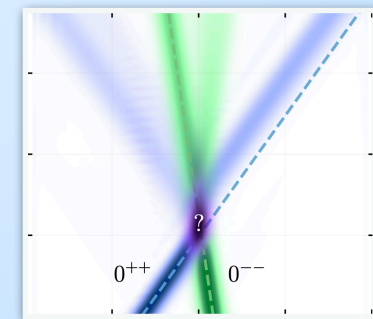
STEP 4.

Wavepacket



STEP 5.

Evolve & detect



Assumptions (Model independent!)

A. $\hat{H} = \sum_j \hat{h}_j$

B. $[\hat{H}, \hat{T}] = 0$

C. $\exists! |\Omega\rangle$

Optional but useful:

D. $[\hat{H}, \hat{P}] = 0$

Assumptions (Model independent!)

A. $\hat{H} = \sum_j \hat{h}_j$

B. $[\hat{H}, \hat{T}] = 0$

C. $\exists! |\Omega\rangle$

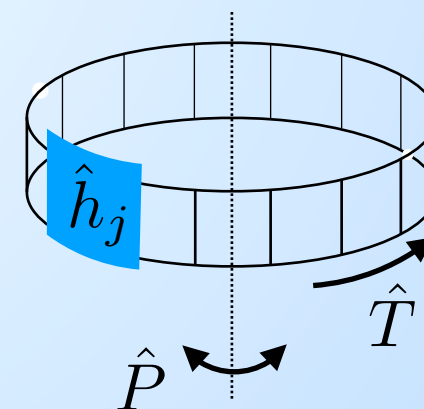
Optional but useful:

D. $[\hat{H}, \hat{P}] = 0$

STEP 1. Dispersion relation

1. Periodic boundary conditions
2. Intermediate system size

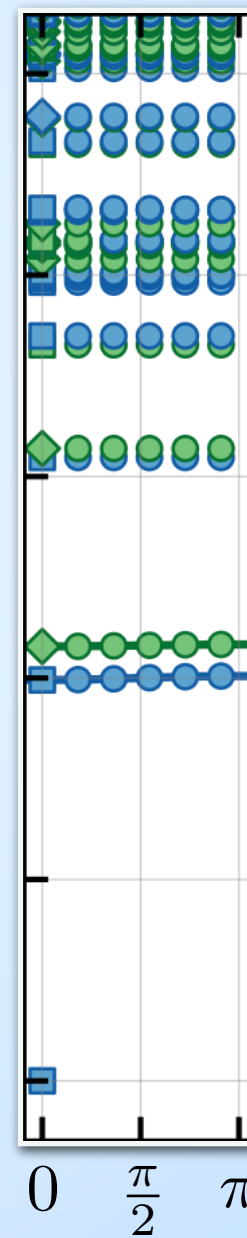
$$L = 11$$



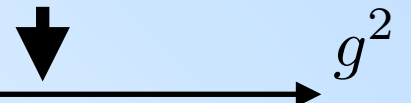
$$U(1) \rightarrow \mathbb{Z}_3$$


 g^2

$$g^2 = 3$$



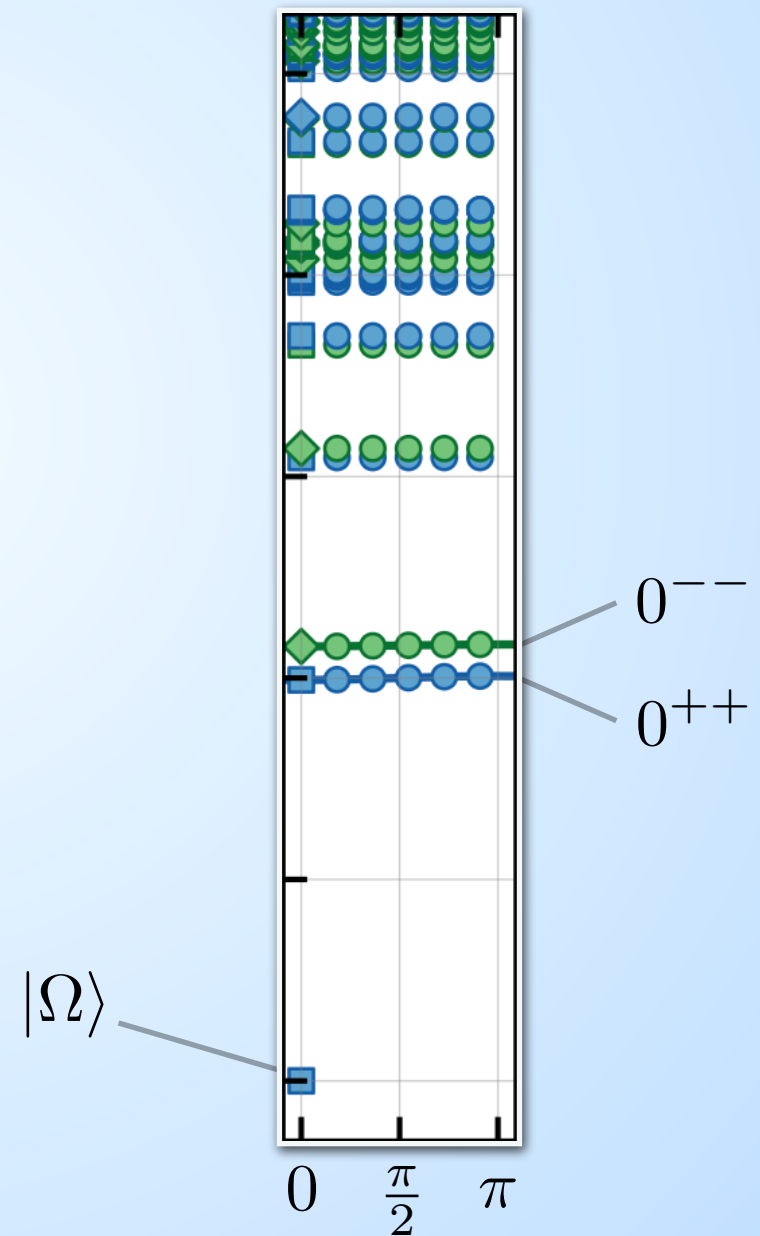
$$U(1) \rightarrow \mathbb{Z}_3$$



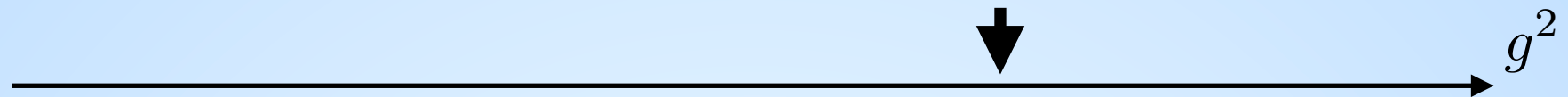
$$g^2 = 3$$

$$|0^{++}\rangle = \frac{|\text{clockwise}\rangle + |\text{counterclockwise}\rangle}{\sqrt{2}}$$

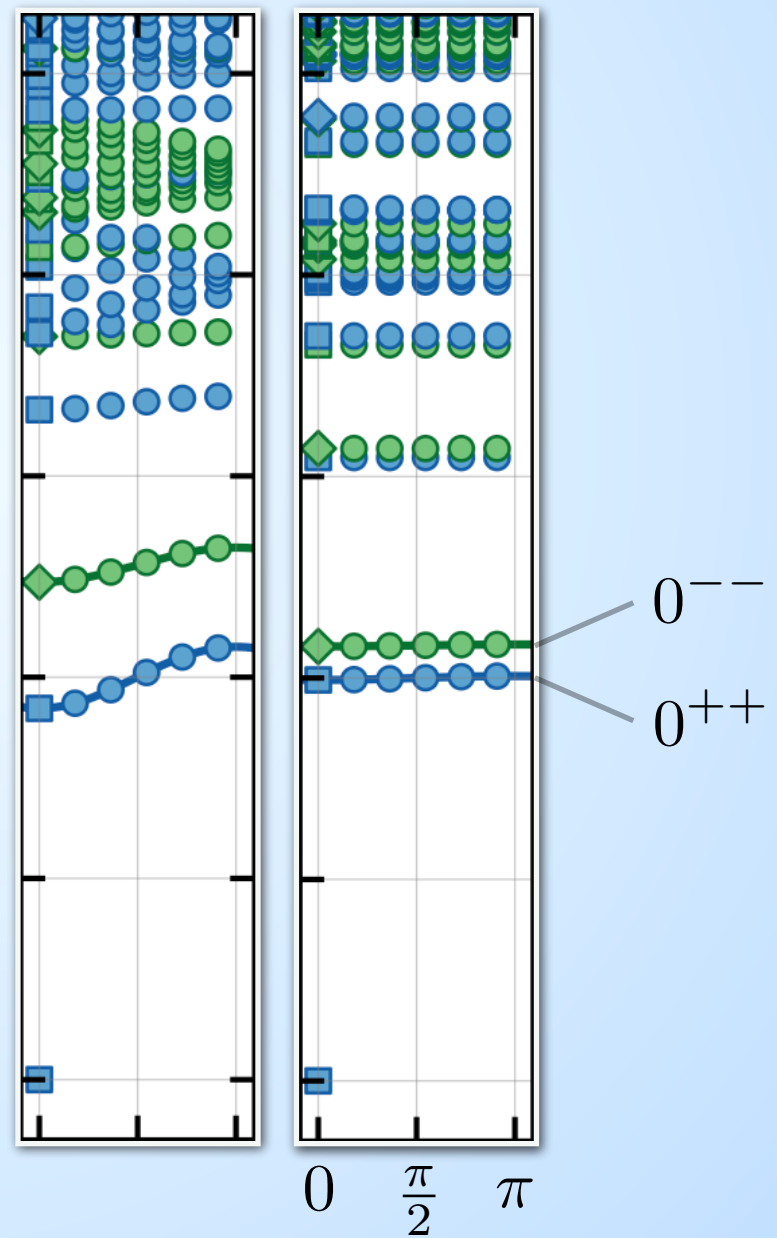
$$|0^{--}\rangle = \frac{|\text{clockwise}\rangle - |\text{counterclockwise}\rangle}{\sqrt{2}}$$



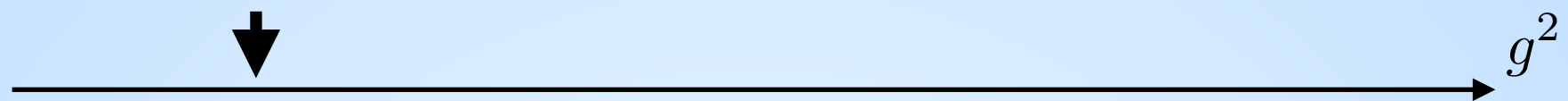
$$U(1) \rightarrow \mathbb{Z}_3$$



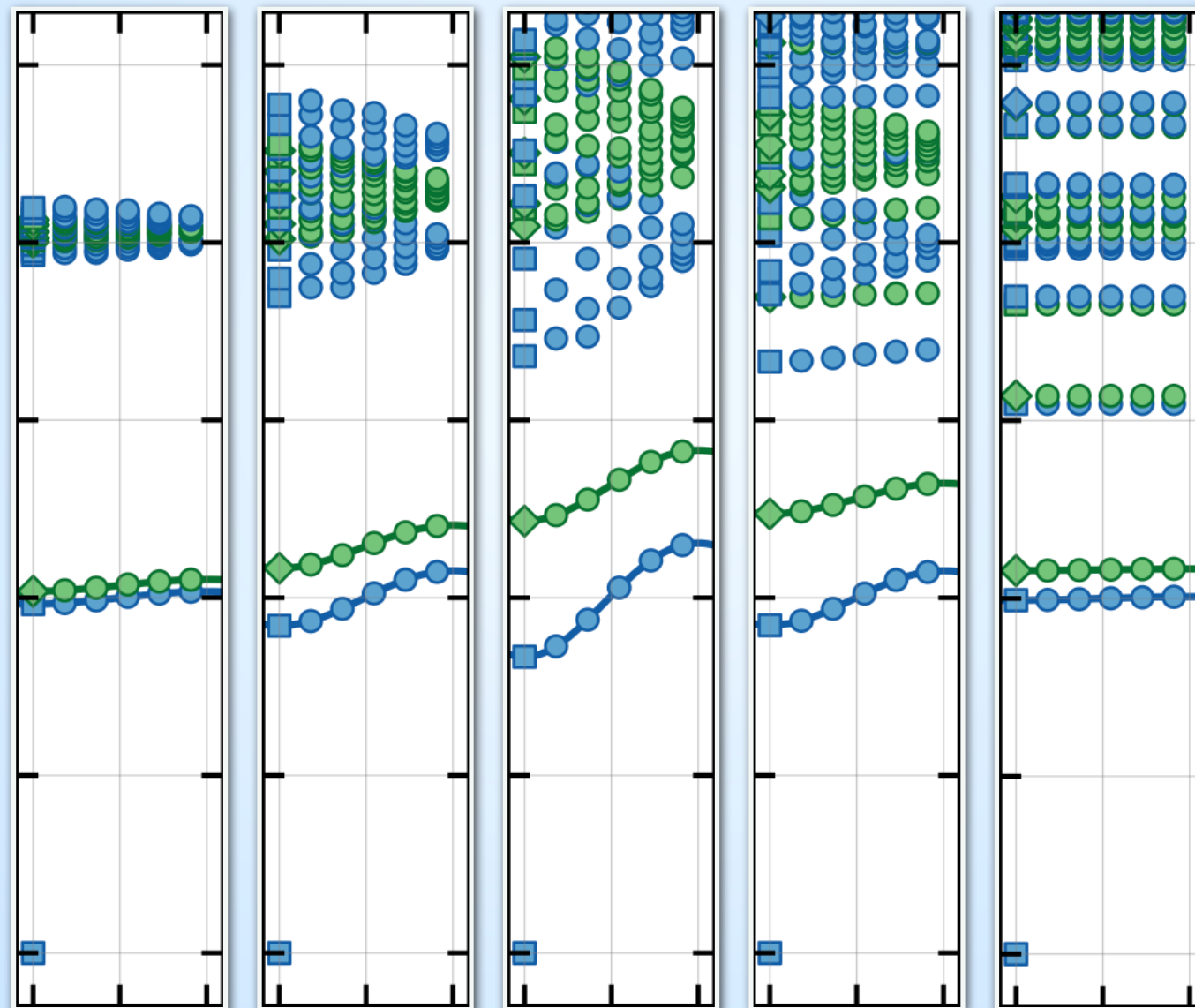
$$g^2 = 1.5 \quad g^2 = 3$$



$$U(1) \rightarrow \mathbb{Z}_3$$



$g^2 = 0.3$ $g^2 = 0.7$ $g^2 = 1$ $g^2 = 1.5$ $g^2 = 3$



$U(1)$ trivial
for
 $g^2 \rightarrow 0$

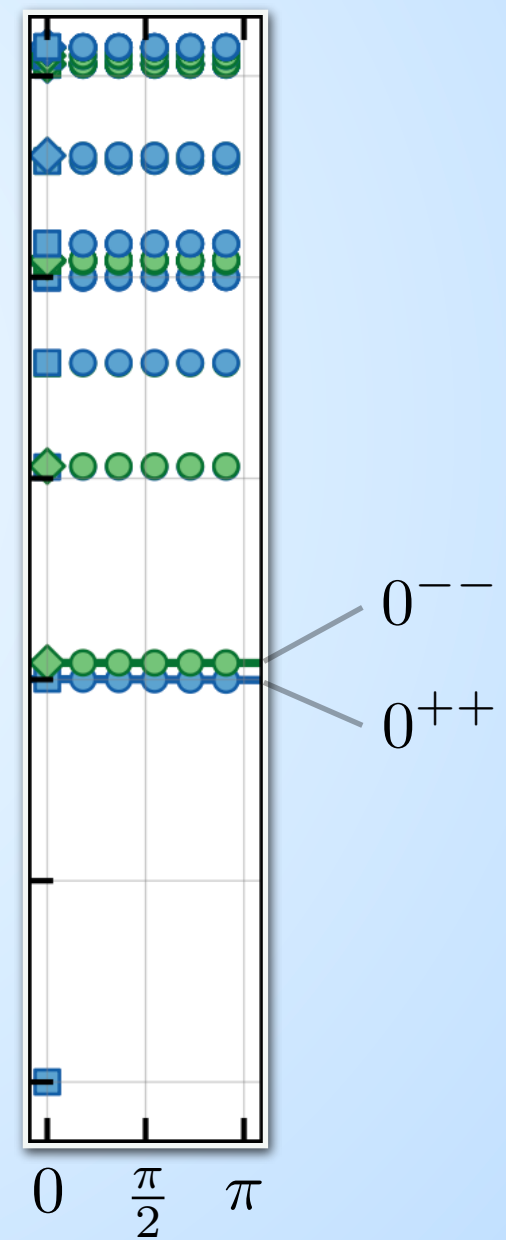
0^{--}
 0^{++}

0 $\frac{\pi}{2}$ π

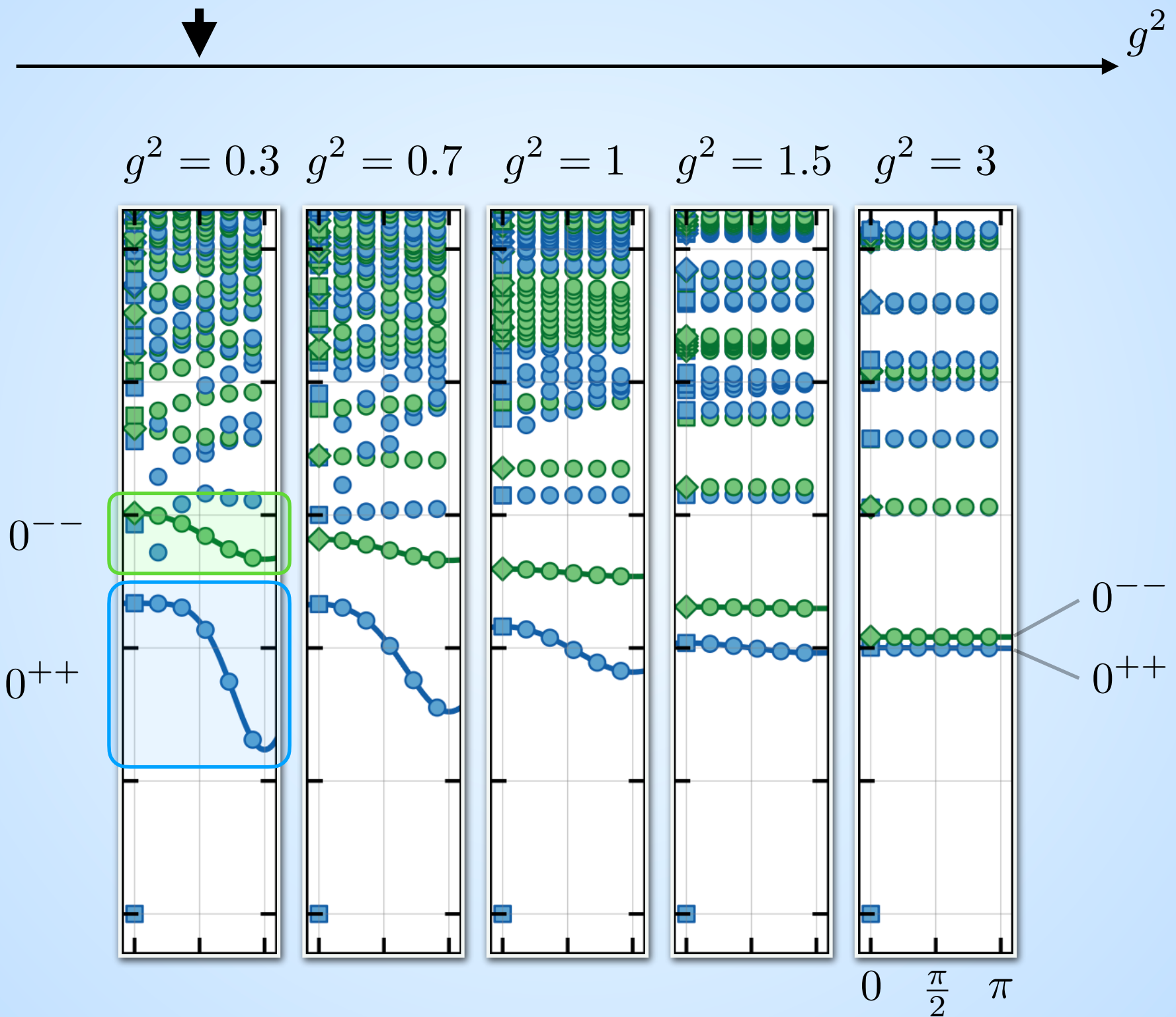
$SU(3)_1$

$$g^2$$

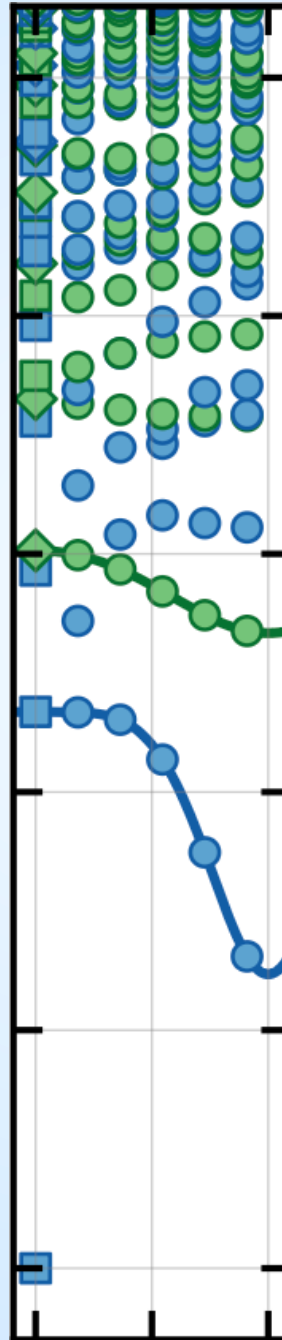
$$g^2 = 3$$



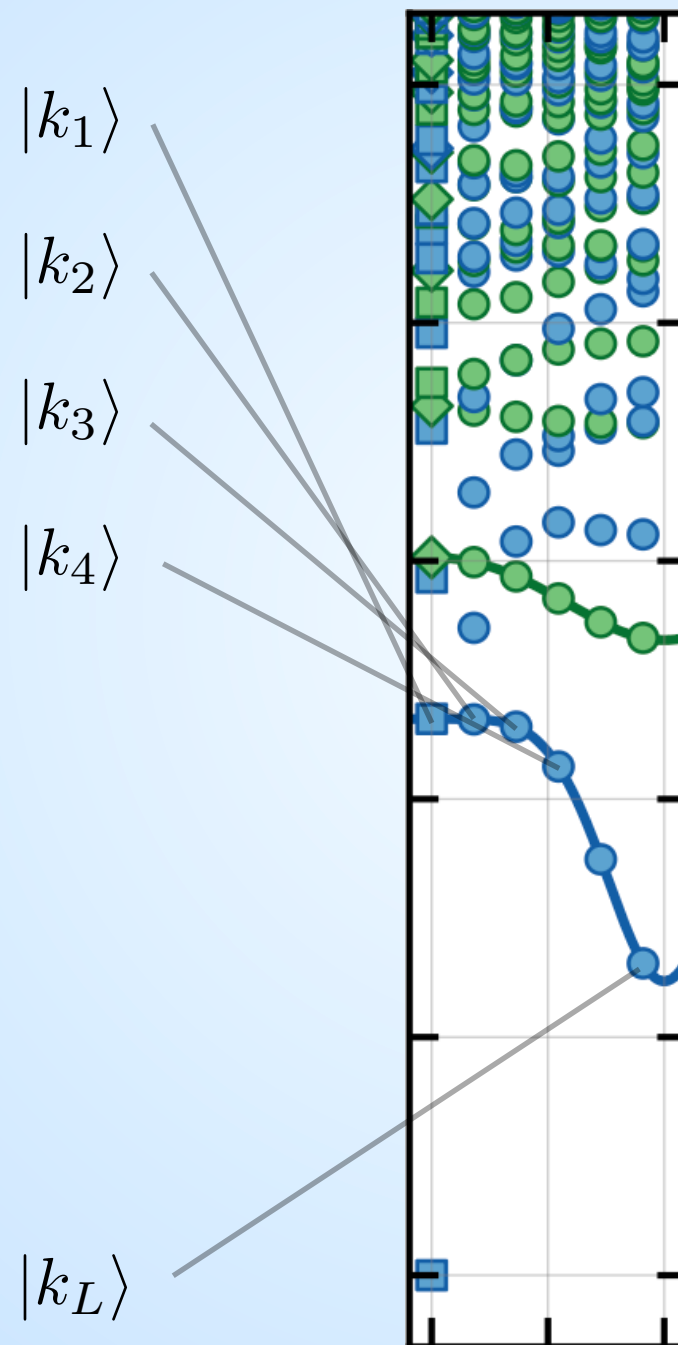
$$SU(3)_1$$



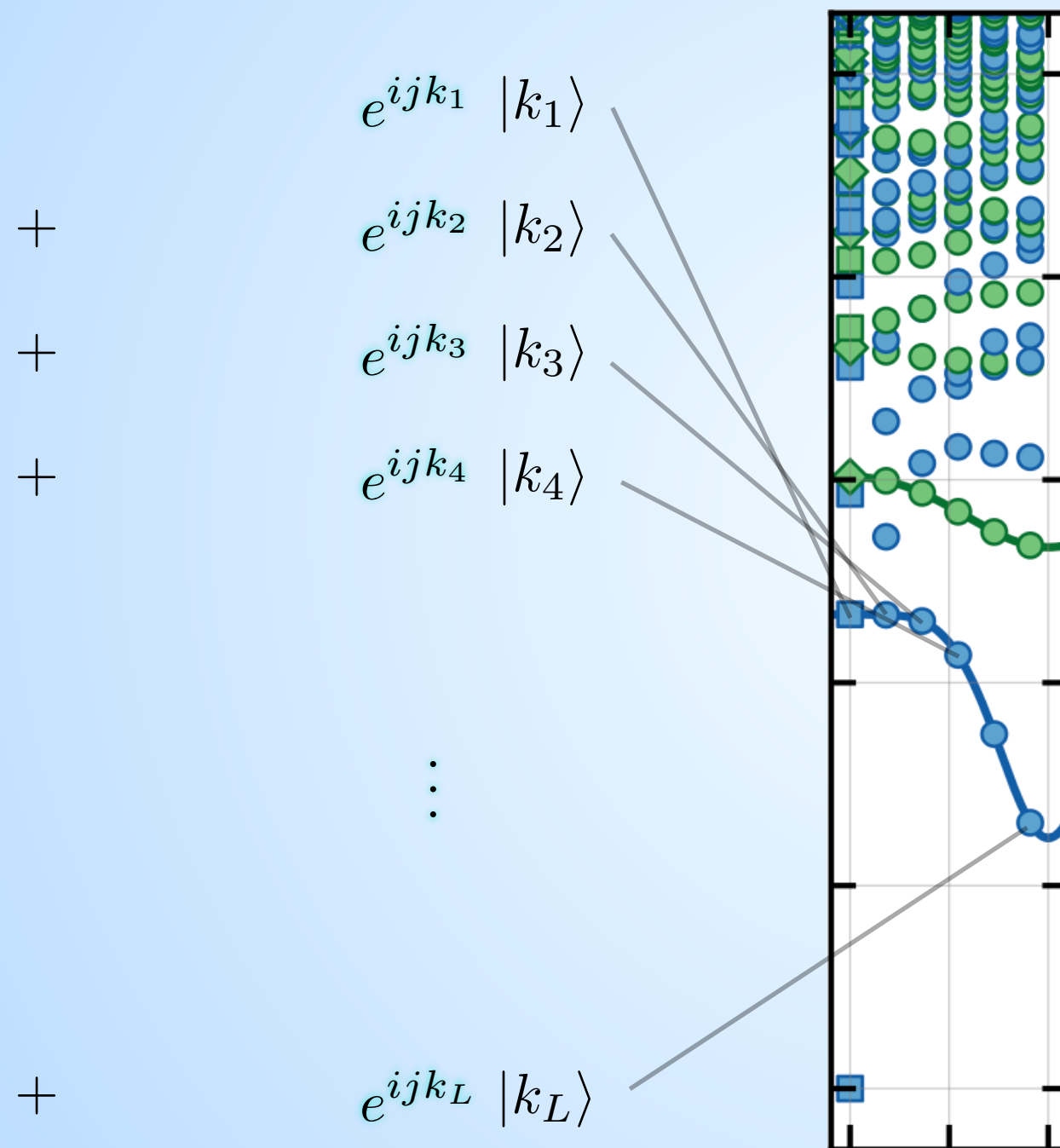
STEP 2.



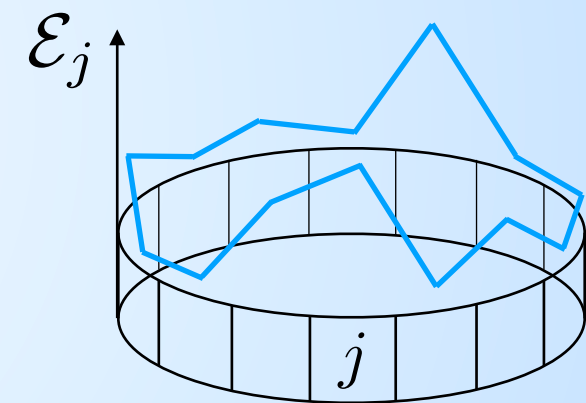
STEP 2. Band localization



STEP 2.



Fourier transform



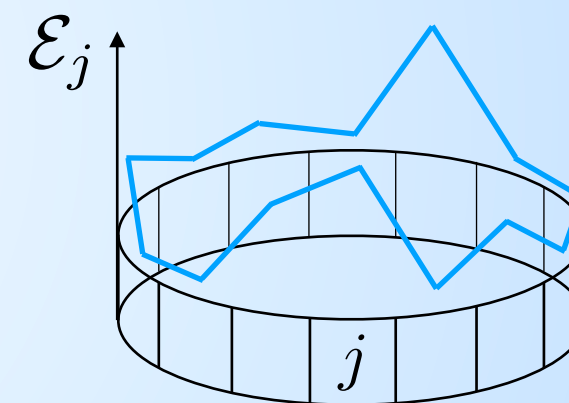
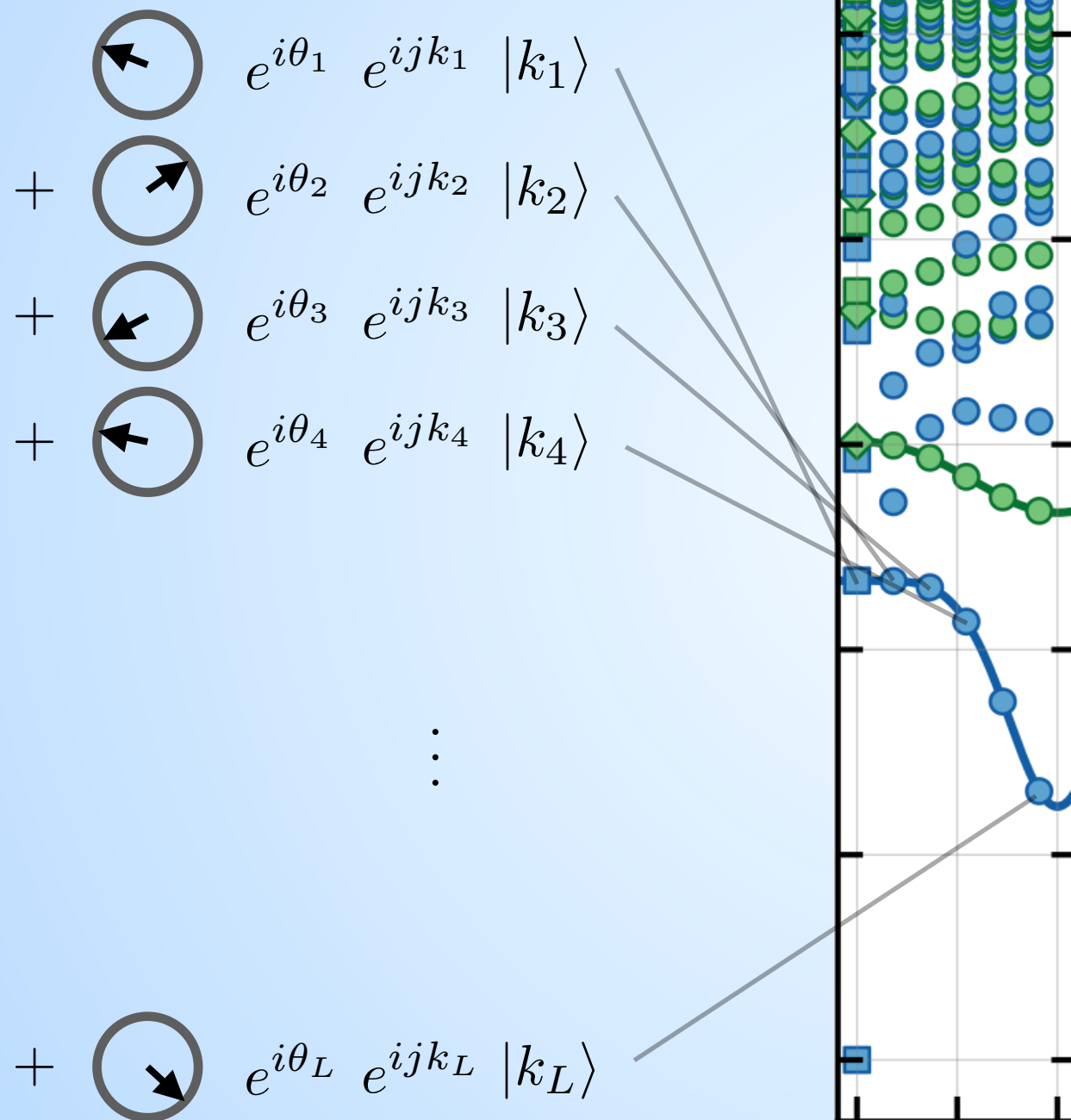
$$\varepsilon_j = |\langle \psi | \hat{h}_j | \psi \rangle - \langle \Omega | \hat{h}_j | \Omega \rangle|$$

Energy excess

$$|W[\theta]\rangle$$

$$\{\theta_k\}$$

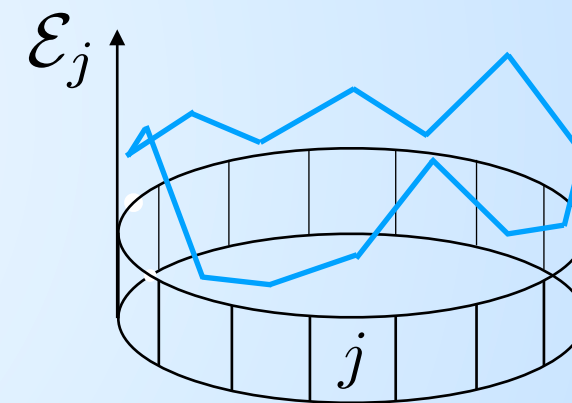
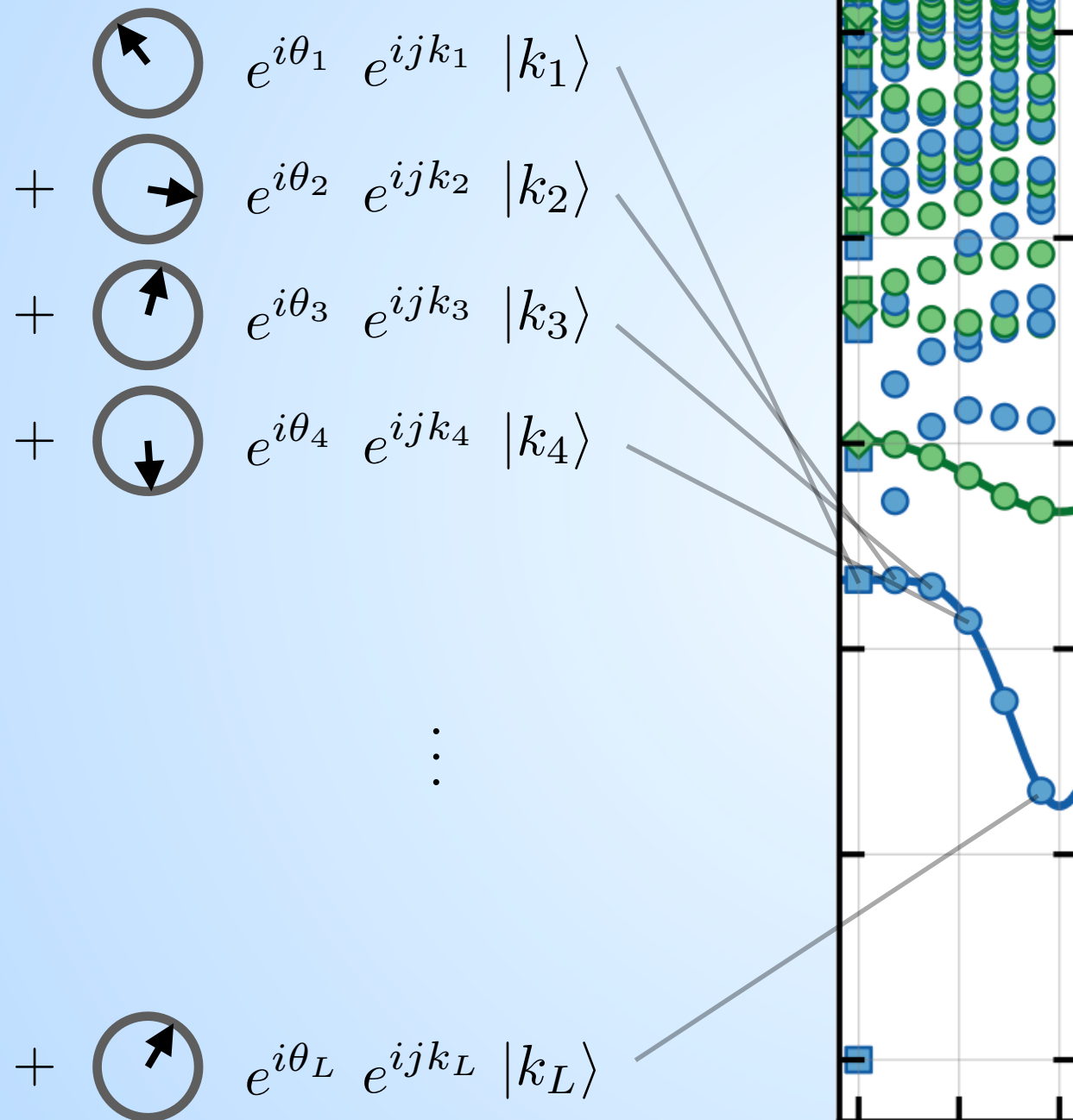
STEP 2.



$$|W[\theta]\rangle$$

$$\{\theta_k\}$$

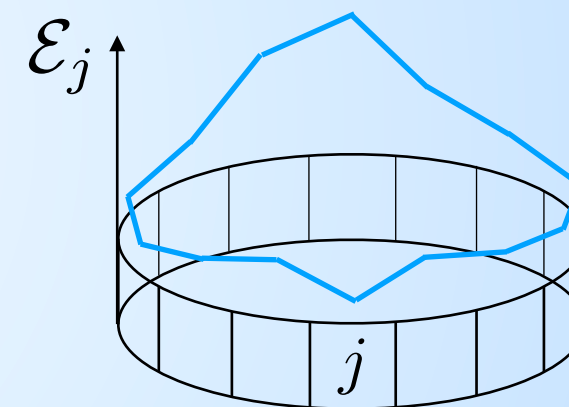
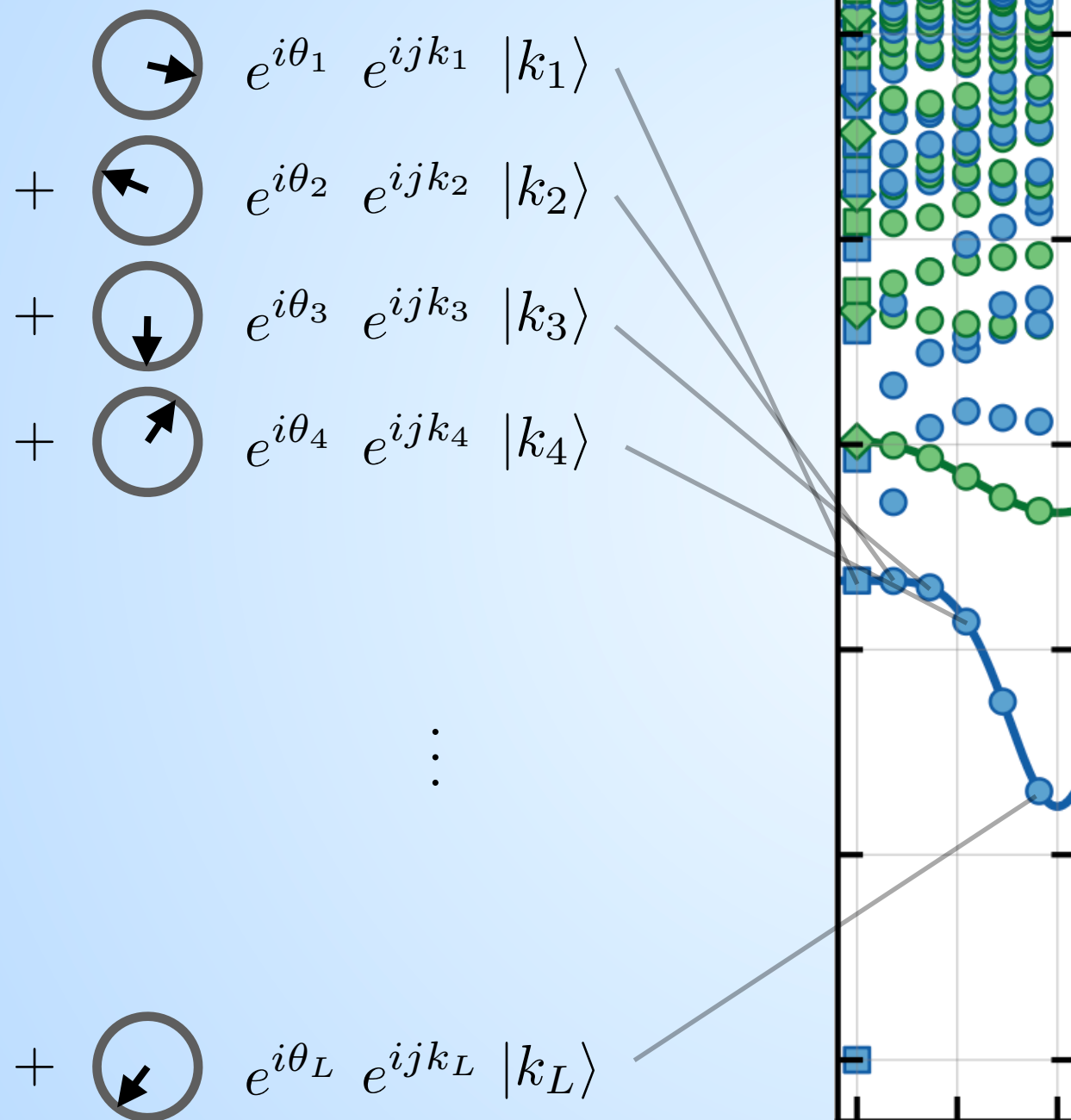
STEP 2.



$$|W[\theta]\rangle$$

$$\{\theta_k\}$$

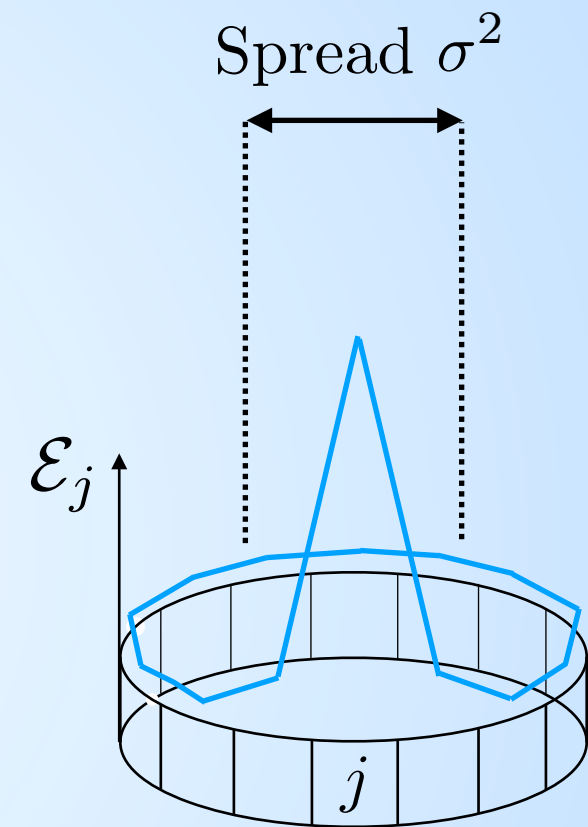
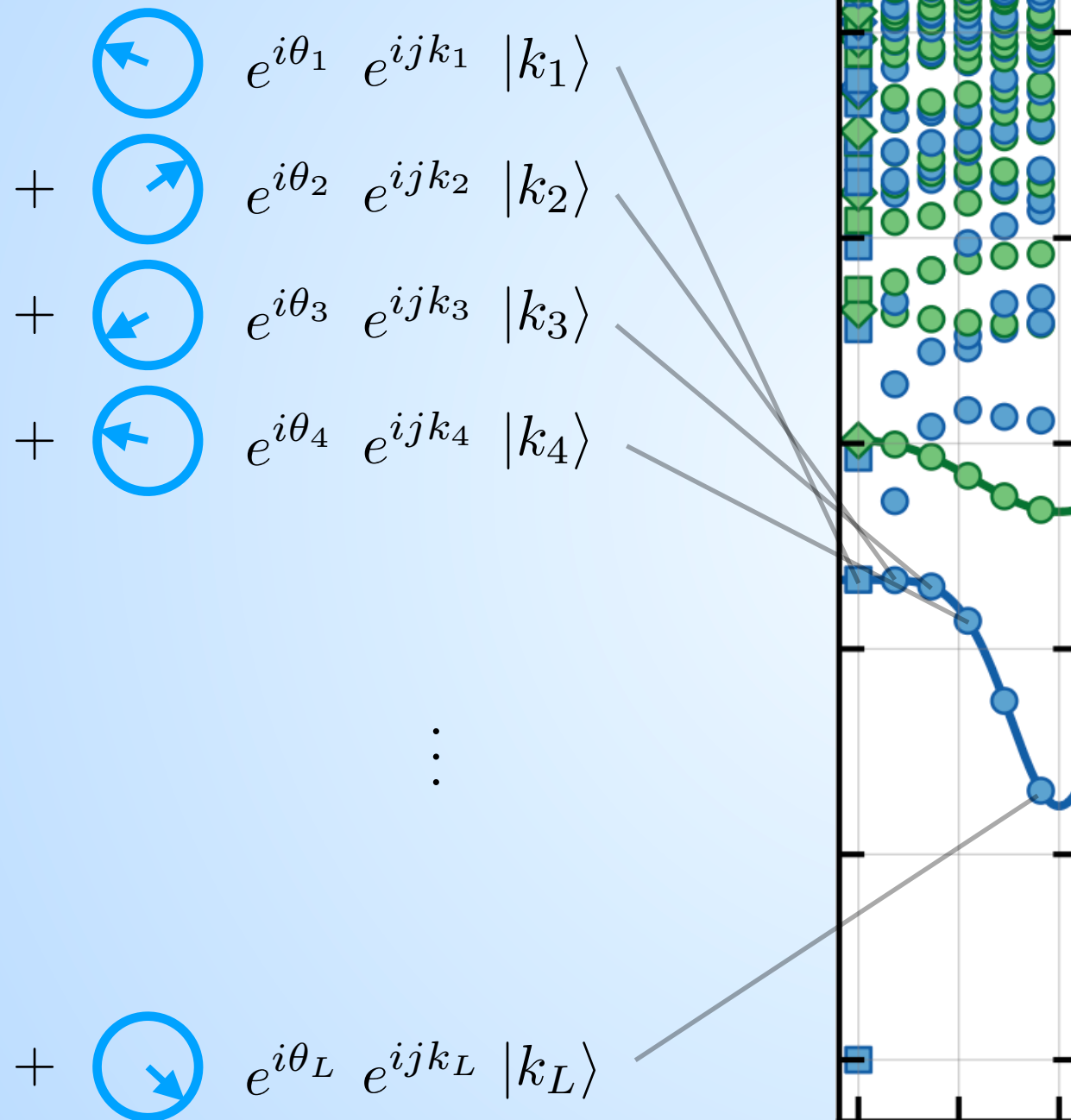
STEP 2.



$$|W[\theta]\rangle$$

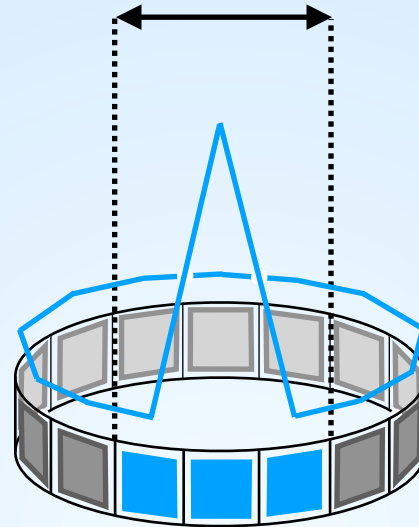
$$\{\theta_k\}$$

STEP 2.

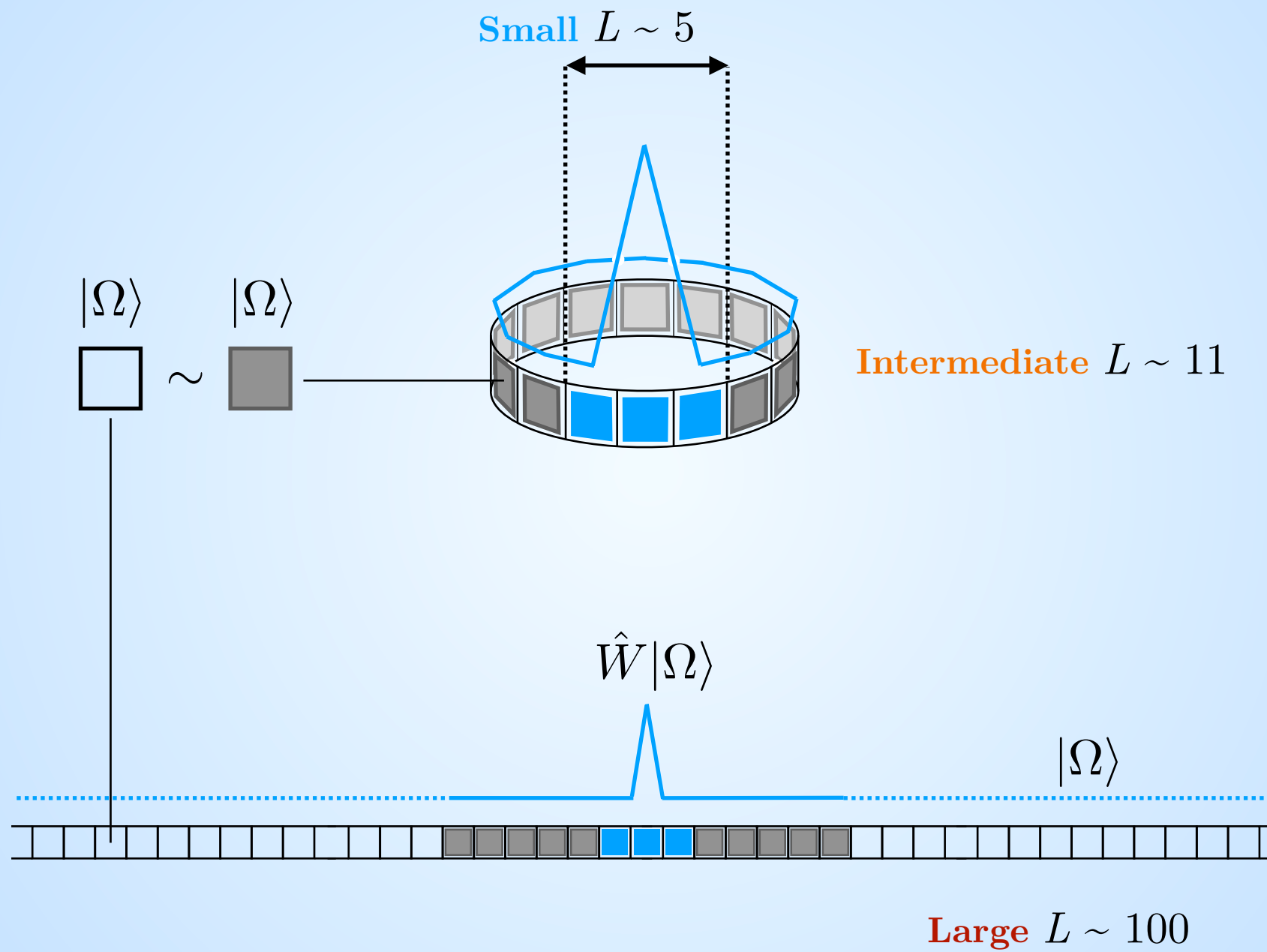


Maximally localized
Wannier function

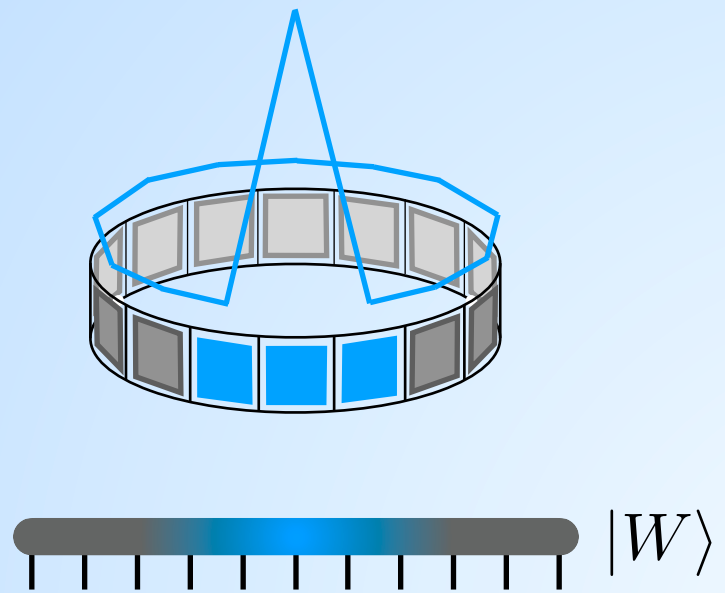
Small $L \sim 5$



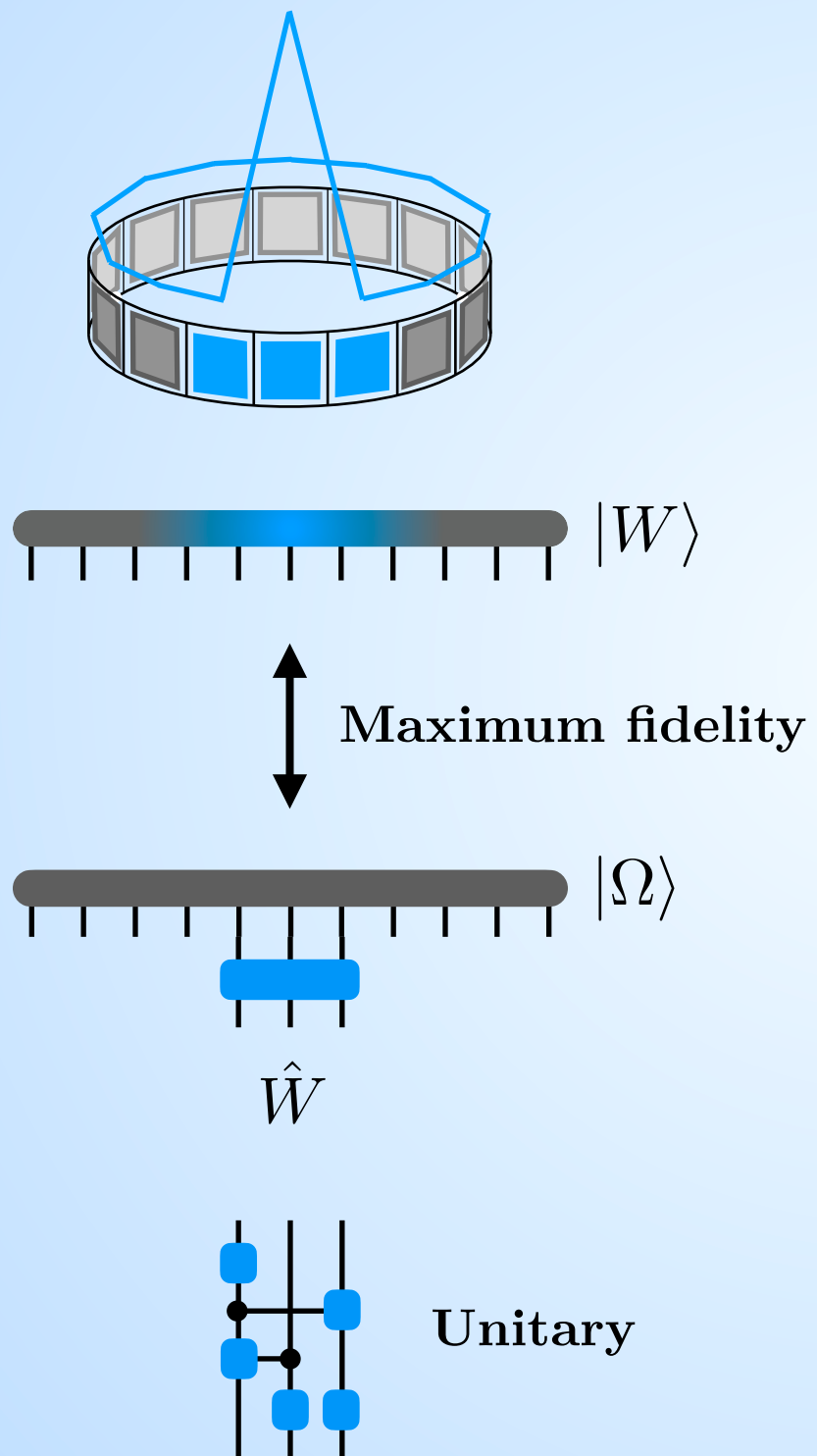
Intermediate $L \sim 11$



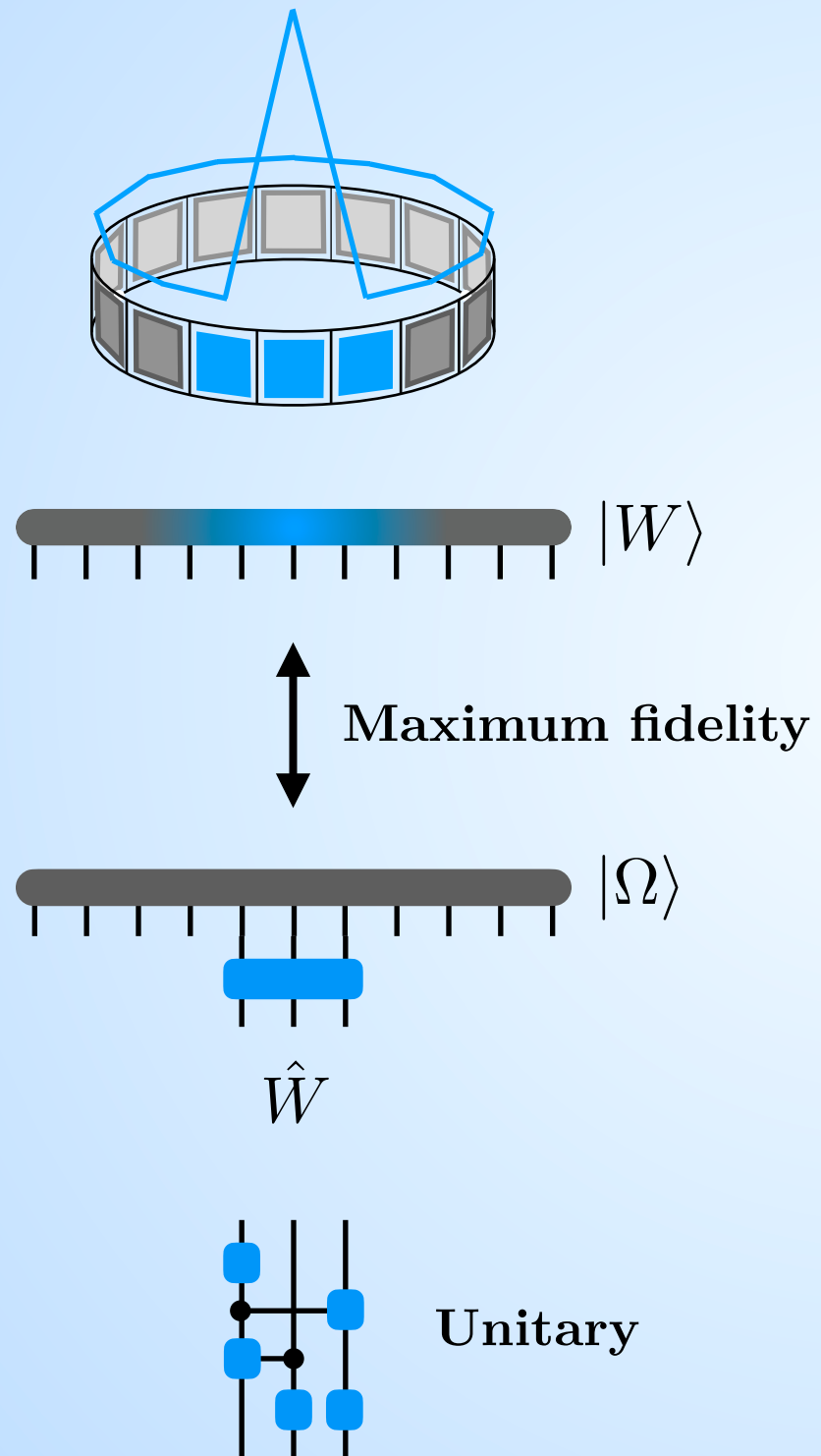
STEP 3.



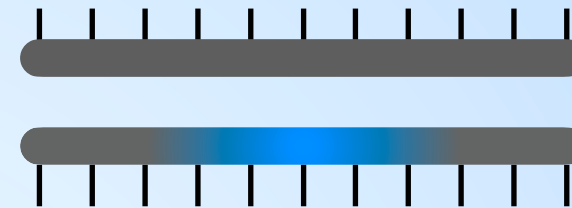
STEP 3.



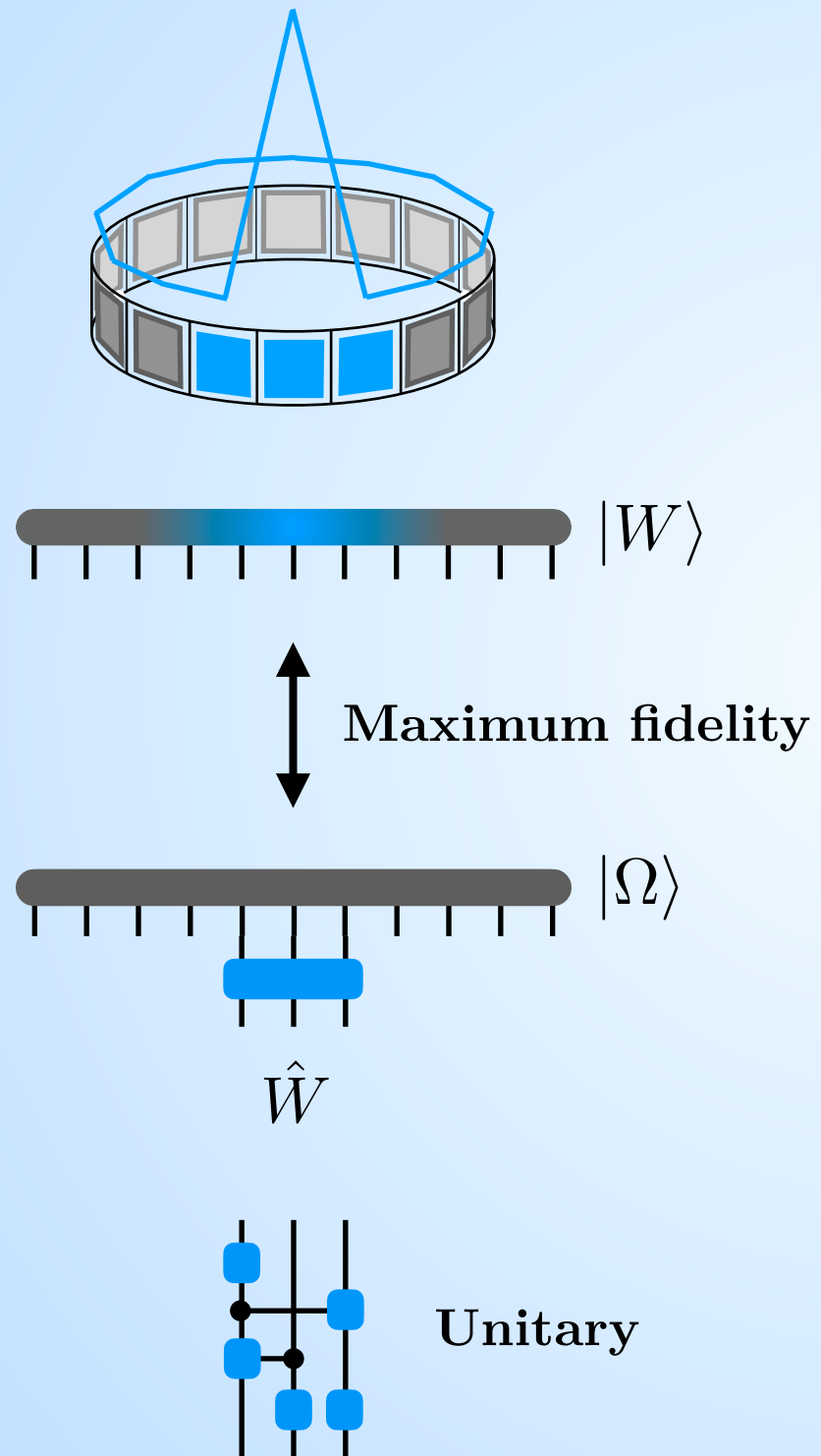
STEP 3.



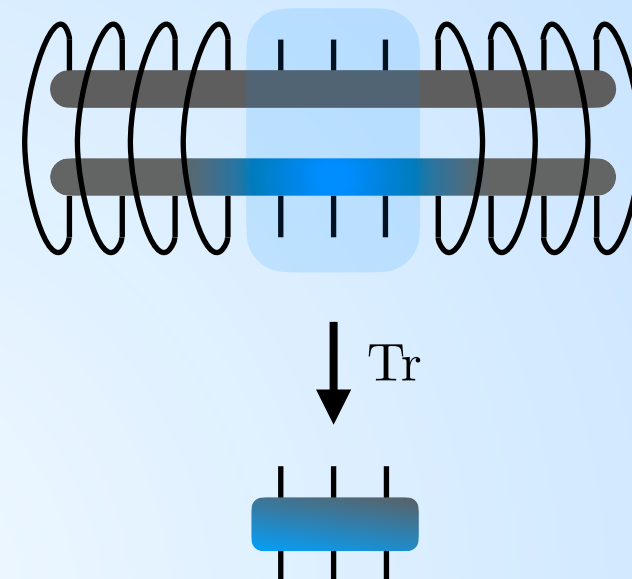
Orthogonal Procrustes Problem



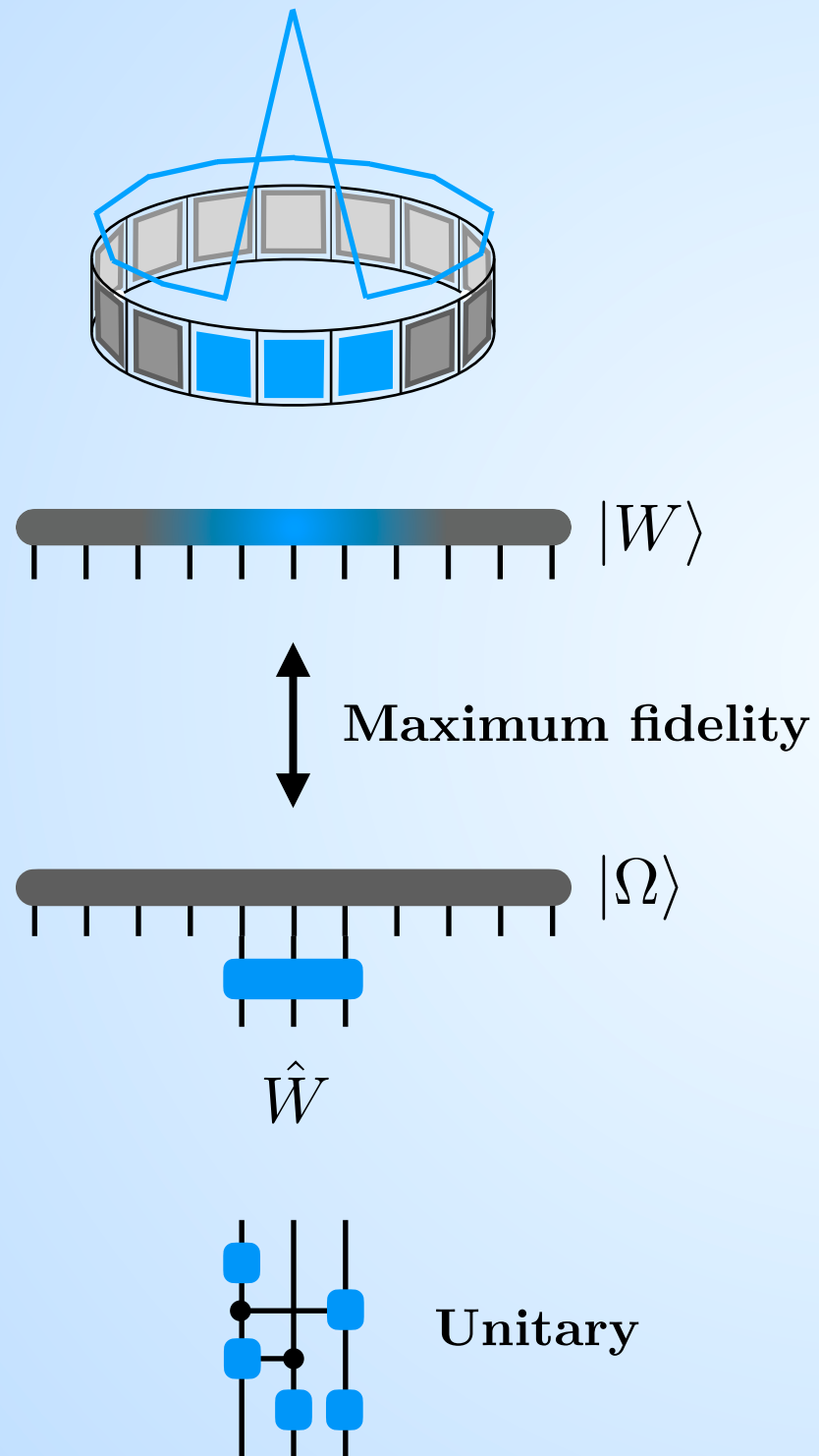
STEP 3.



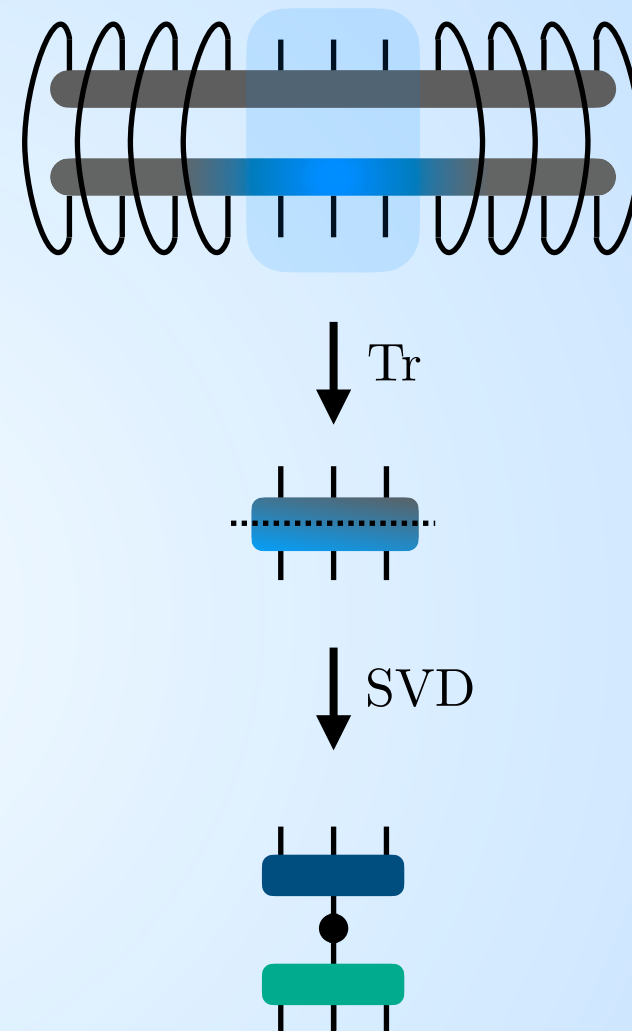
Orthogonal Procrustes Problem



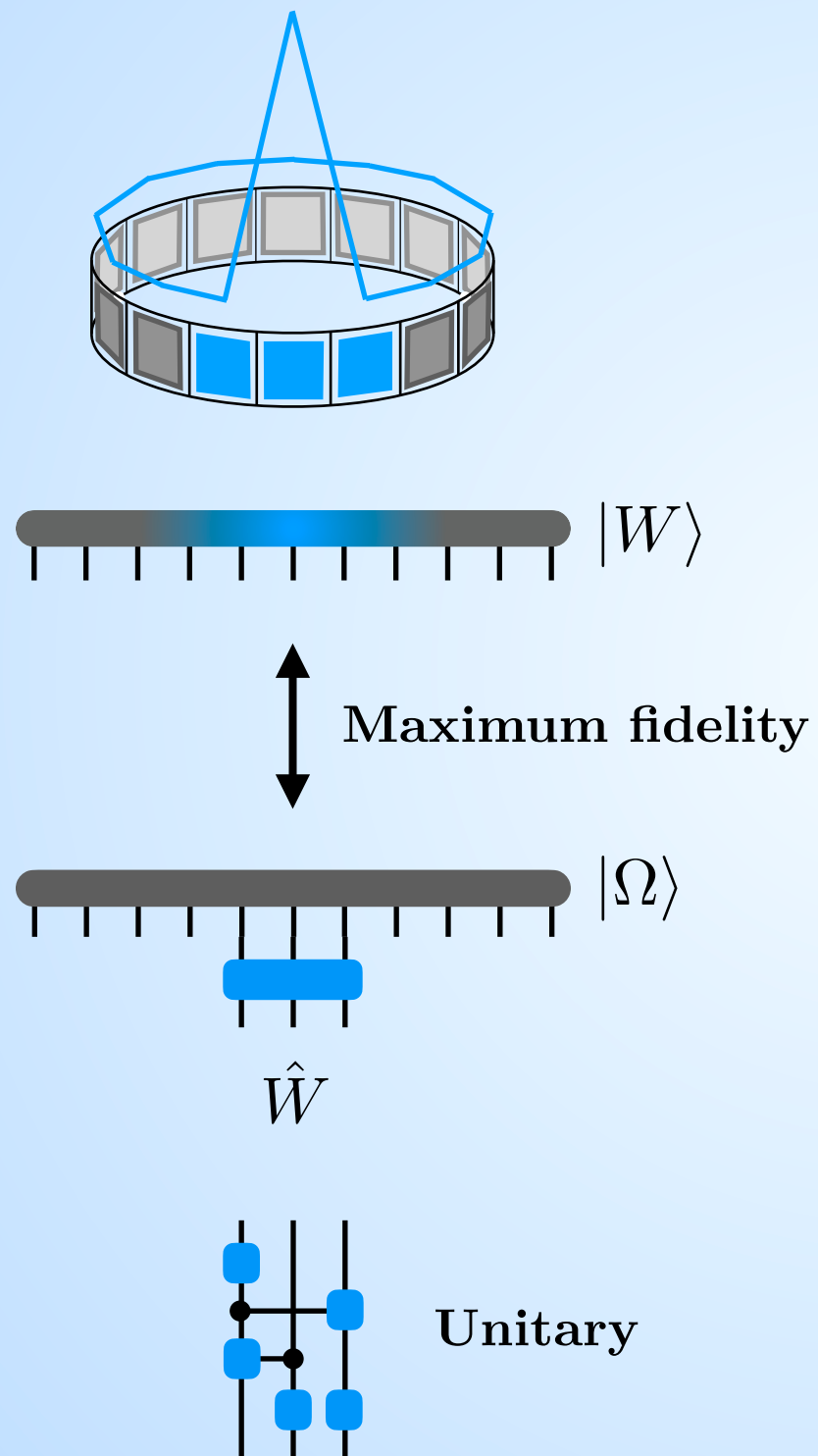
STEP 3.



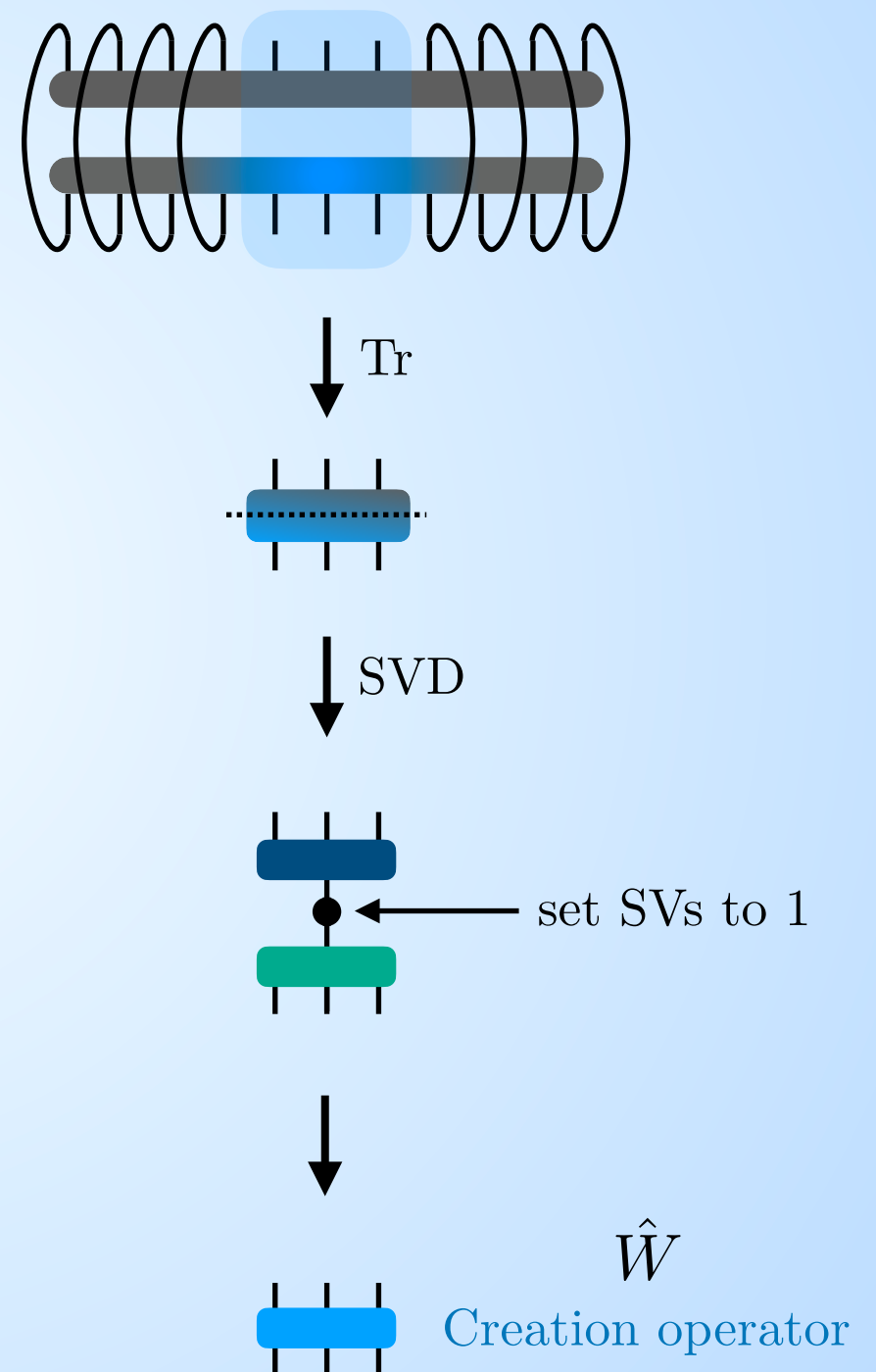
Orthogonal Procrustes Problem



STEP 3.

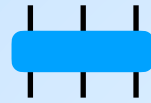


Orthogonal Procrustes Problem

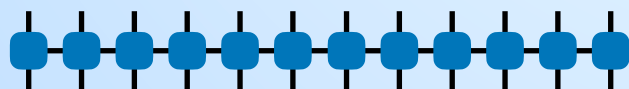
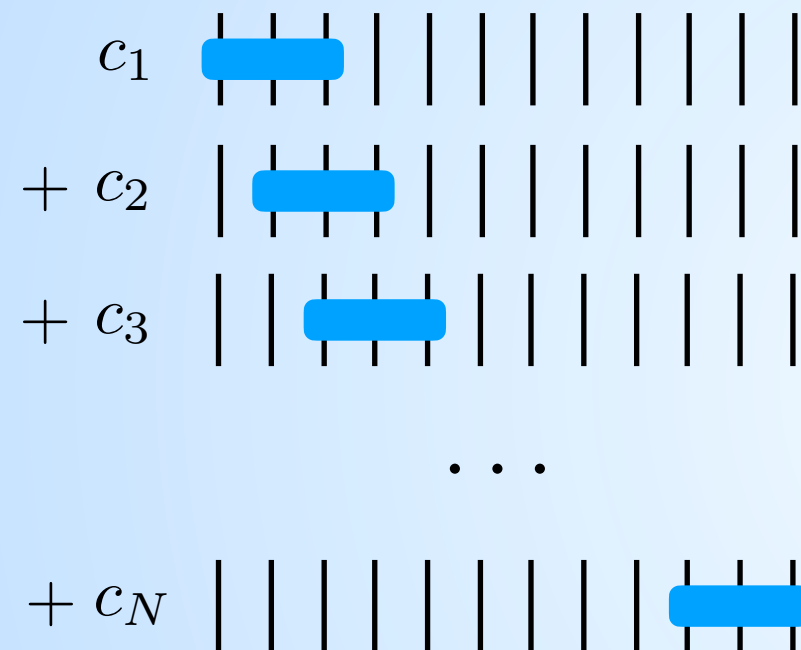
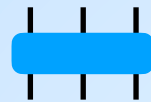


$$\hat{W}$$

Creation operator

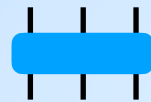


$\hat{W}$
Creation operator

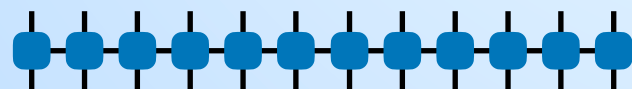
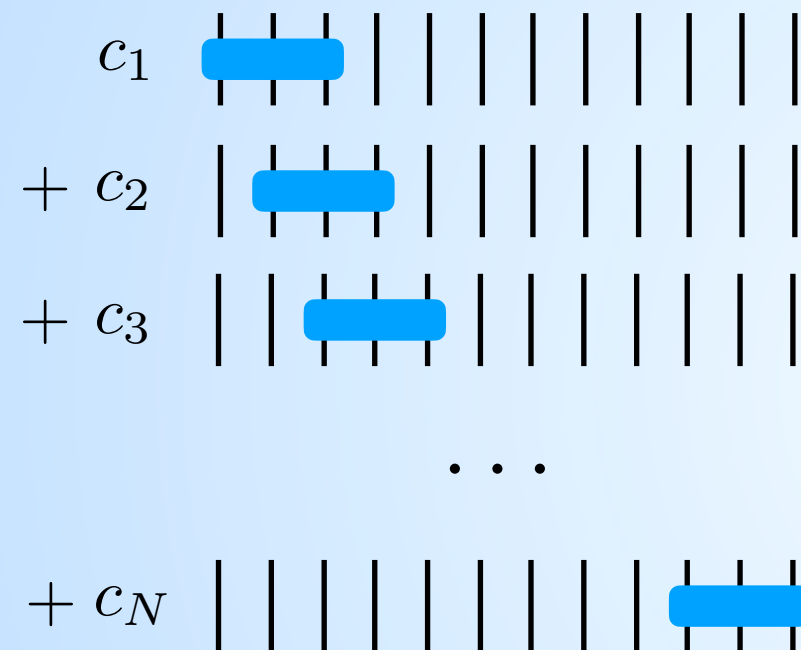
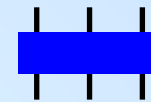


STEP 4.

$\hat{W}$
Creation operator

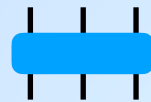


$\hat{\rho}$
Local detector

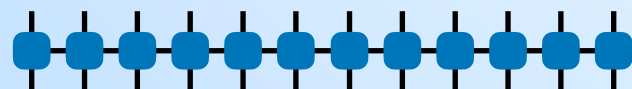
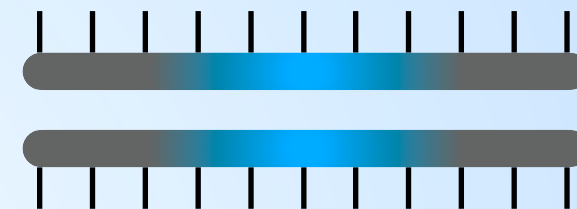
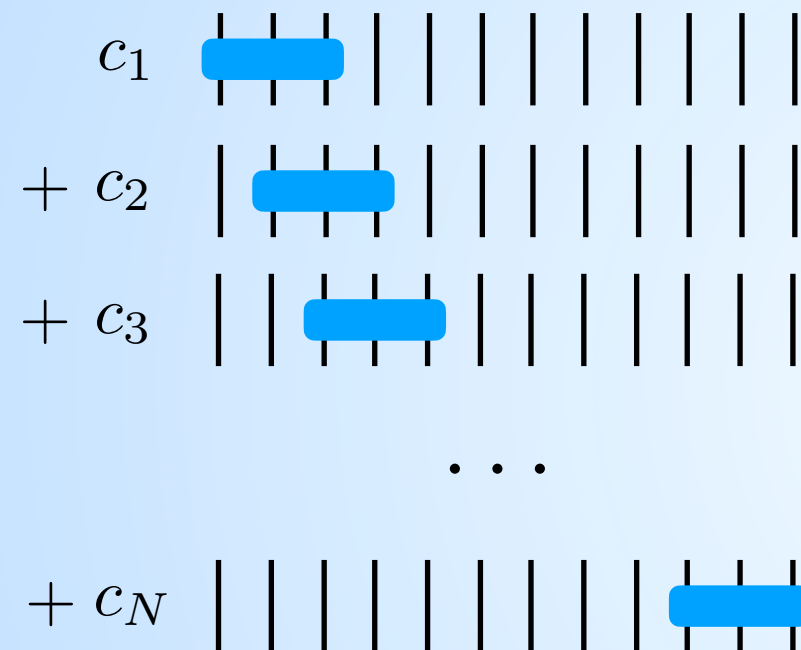
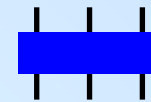


STEP 4.

$\hat{W}$
Creation operator

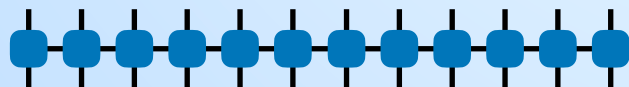
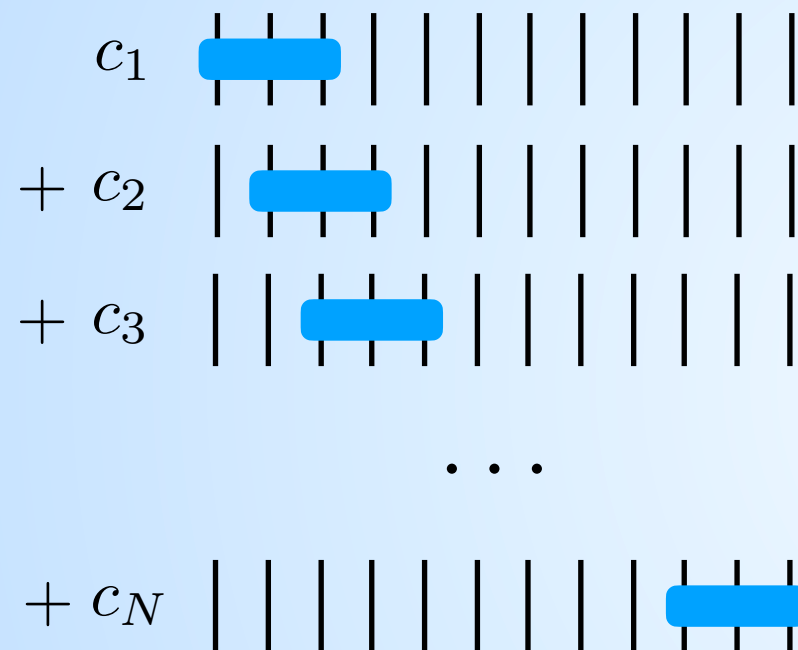
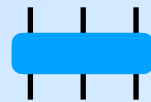


$\hat{\rho}$
Local detector



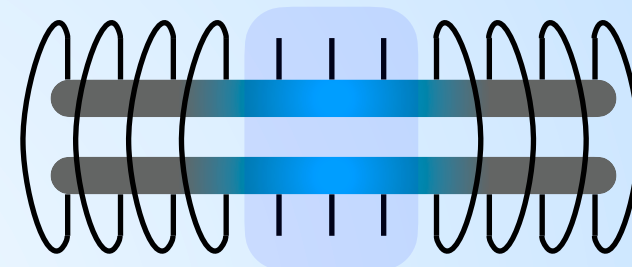
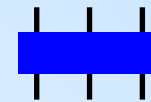
STEP 4.

$\hat{W}$
Creation operator

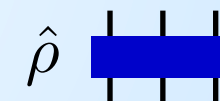


STEP 4.

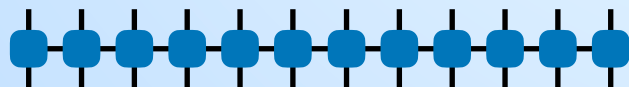
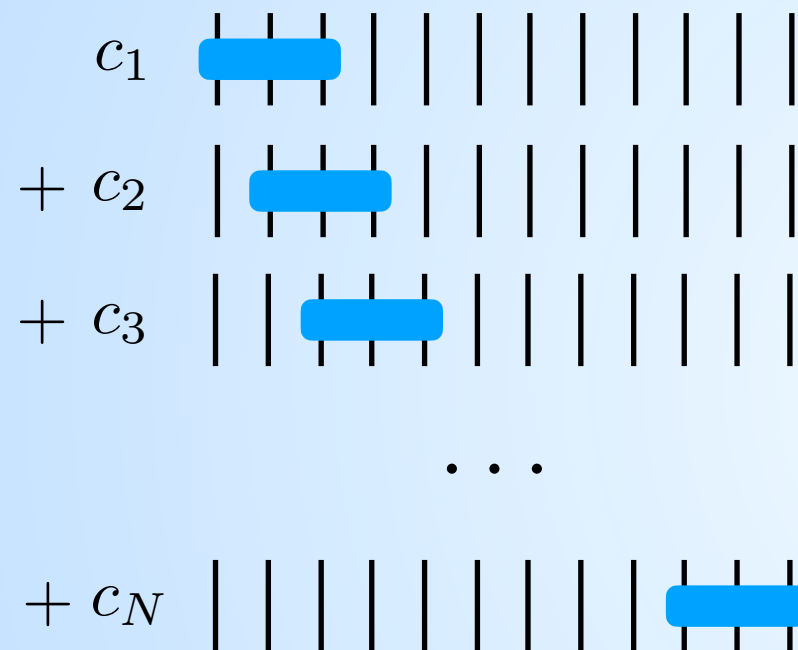
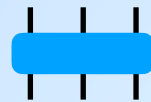
$\hat{\rho}$
Local detector



↓ RDM

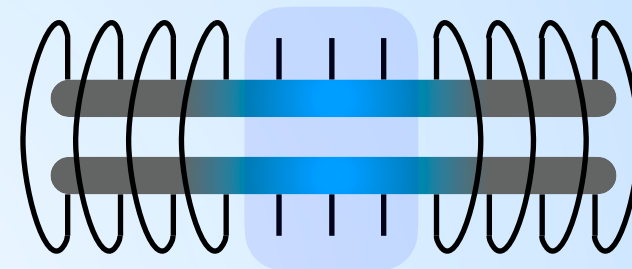
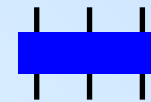


$\hat{W}$
Creation operator

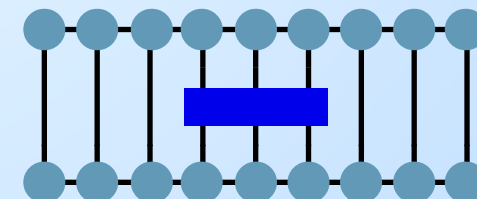
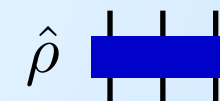


STEP 4.

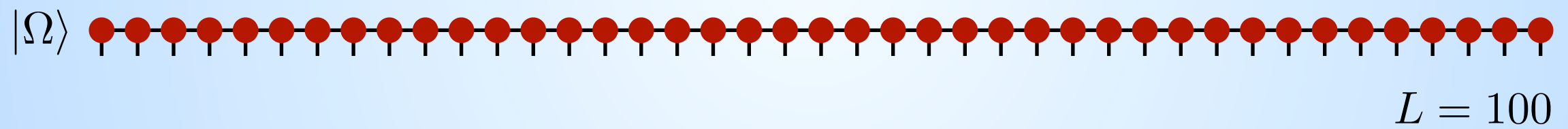
$\hat{\rho}$
Local detector



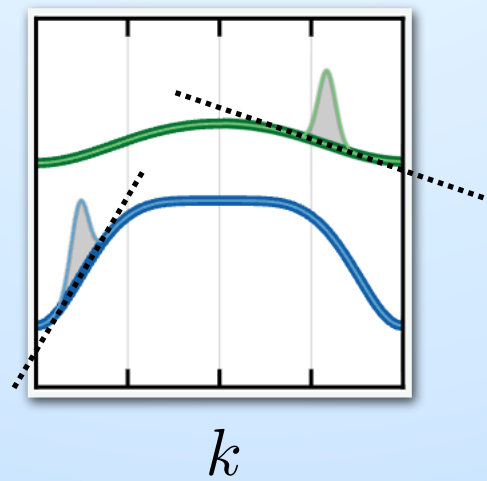
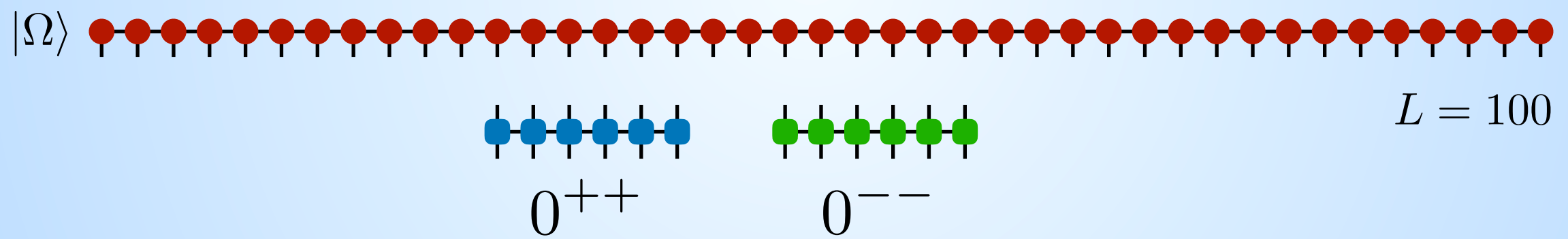
↓ RDM



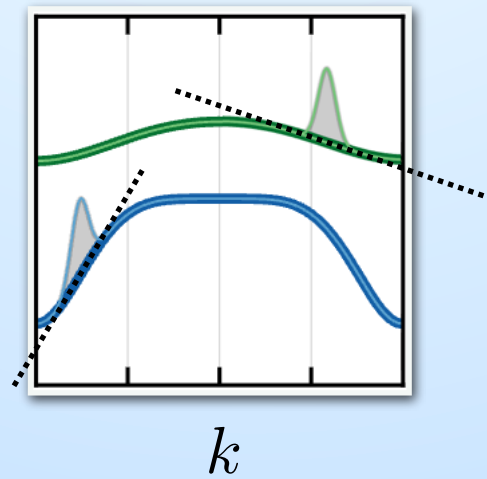
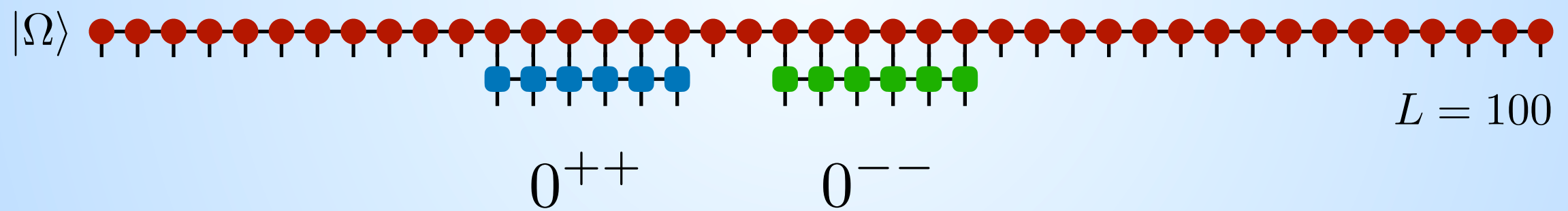
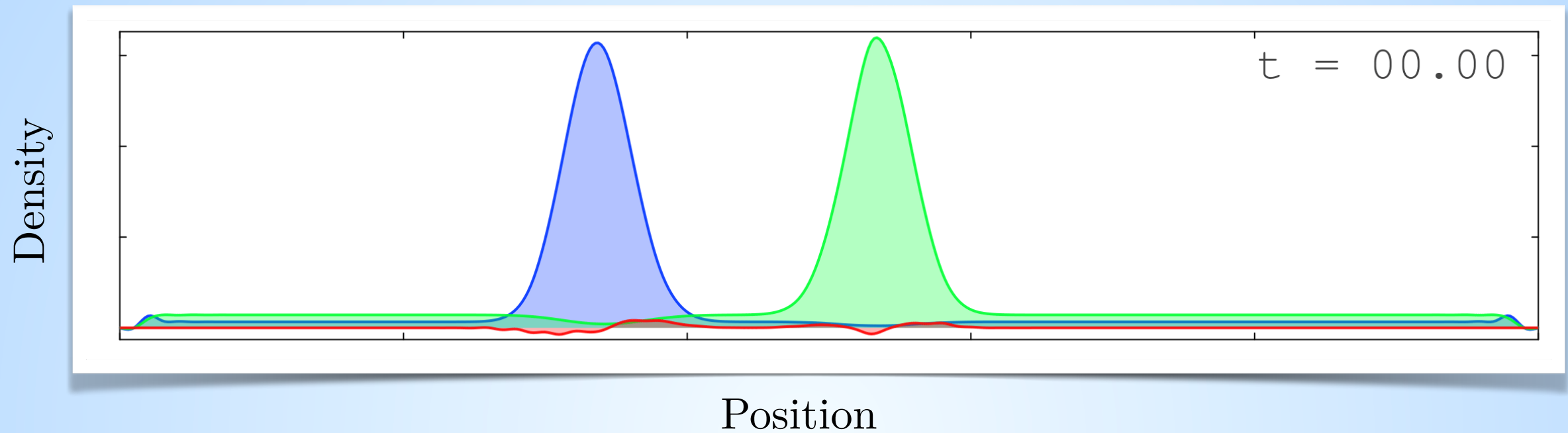
$$SU(3)_1, \ g^2 = 0.3$$

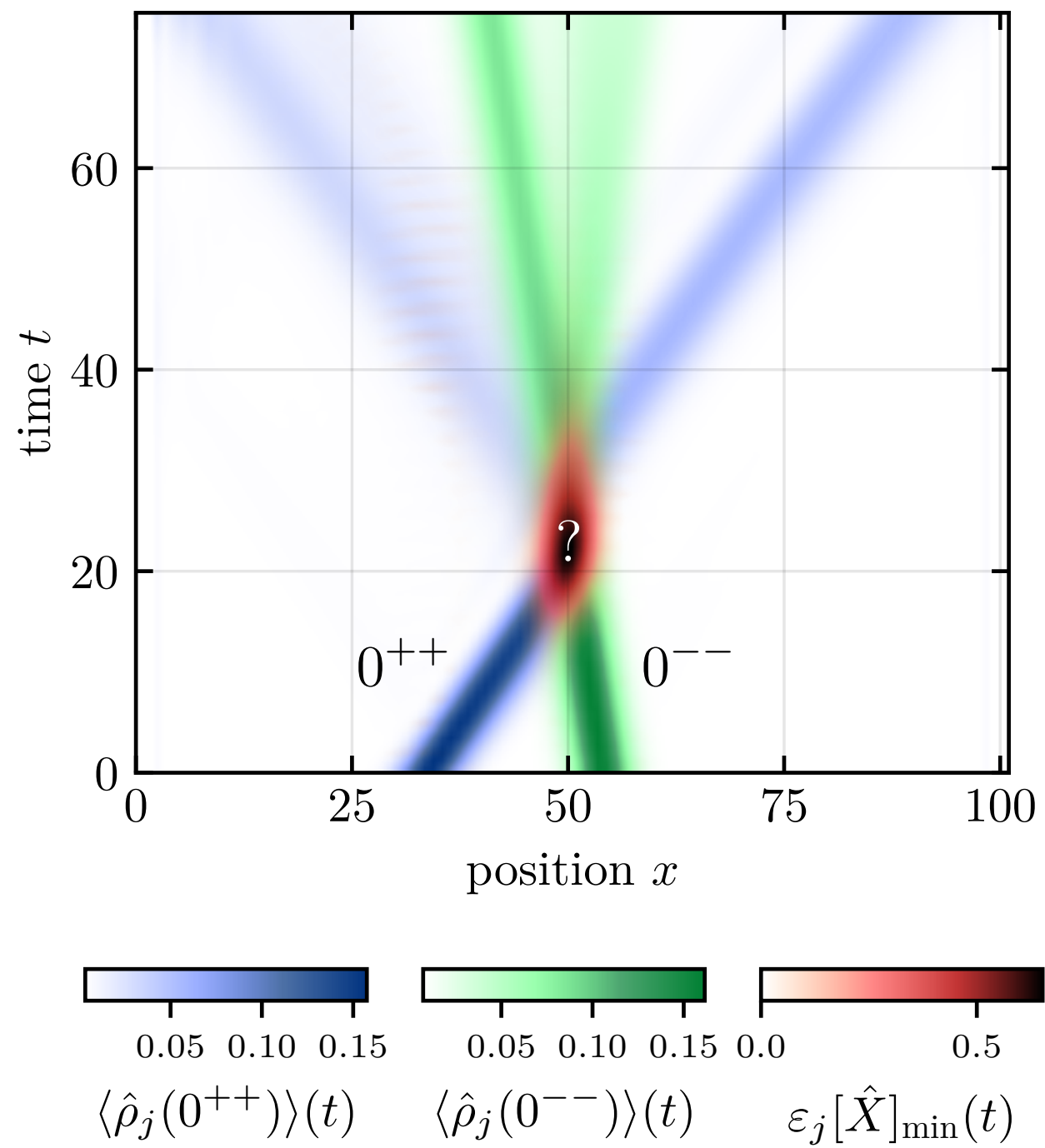


$$SU(3)_1, \ g^2 = 0.3$$

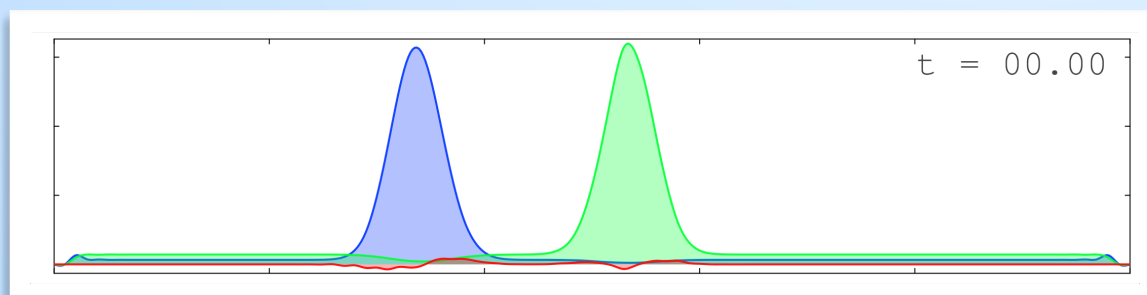
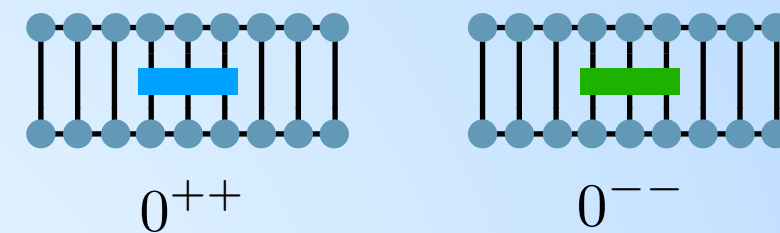


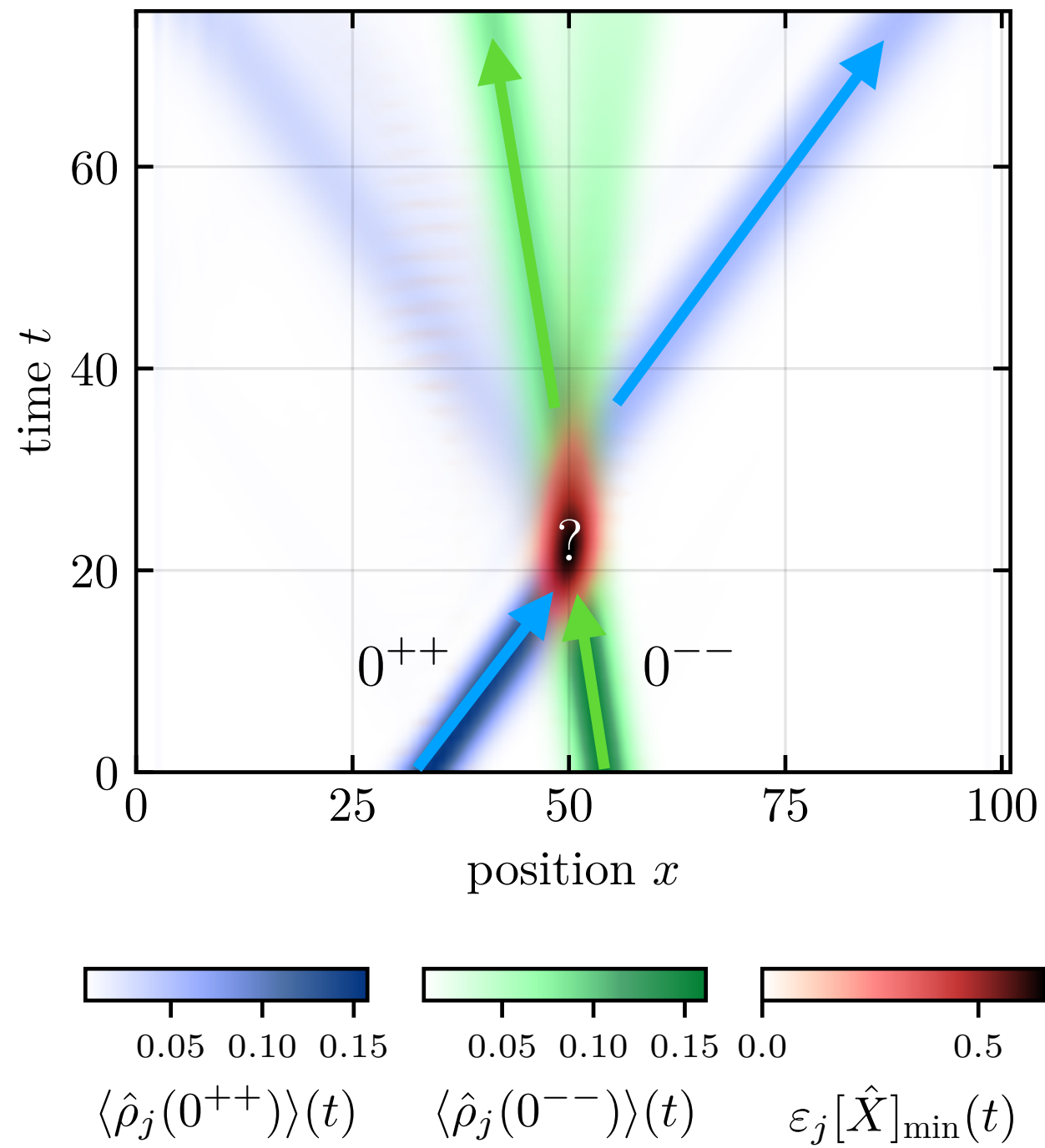
$$SU(3)_1, g^2 = 0.3$$



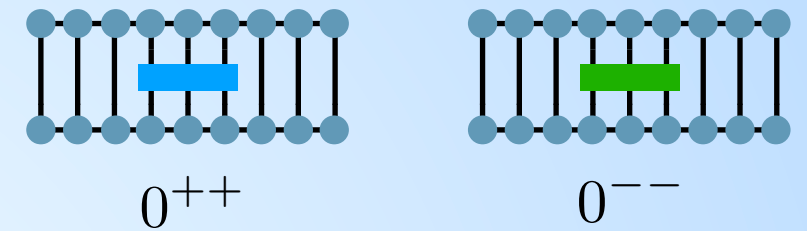


Detection

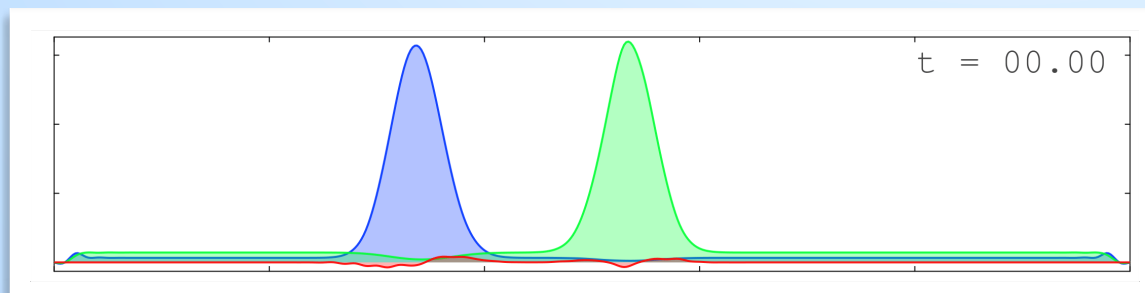
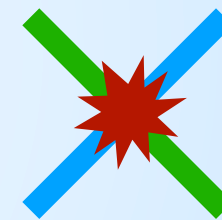


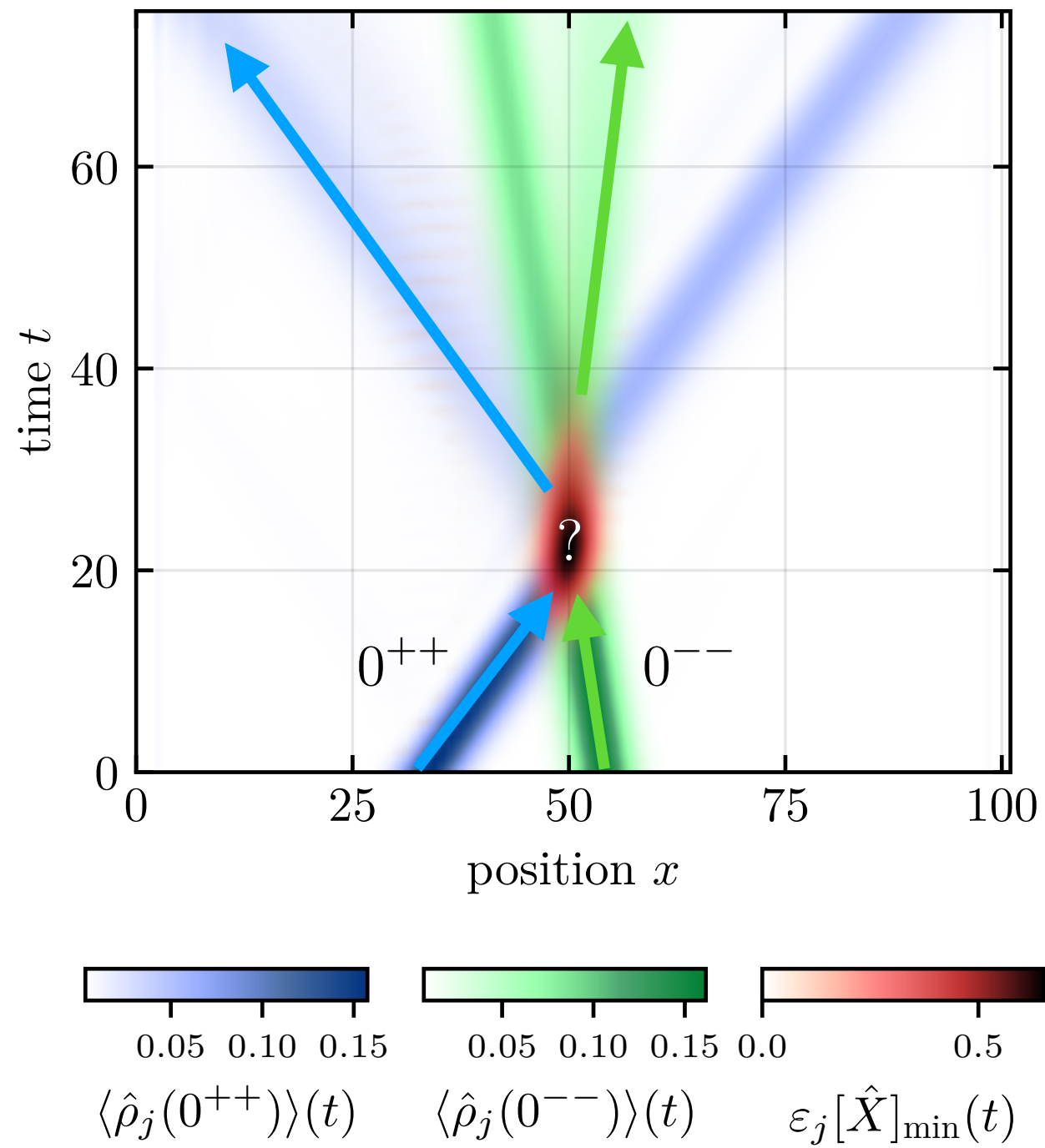


Detection

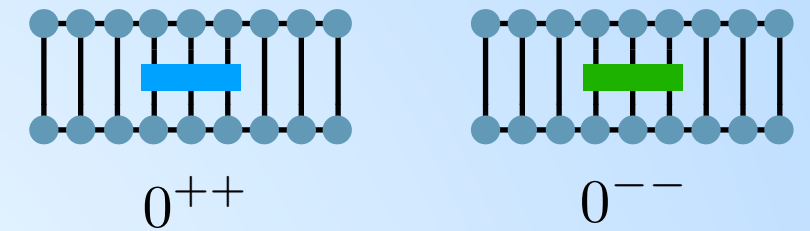


Two main channels

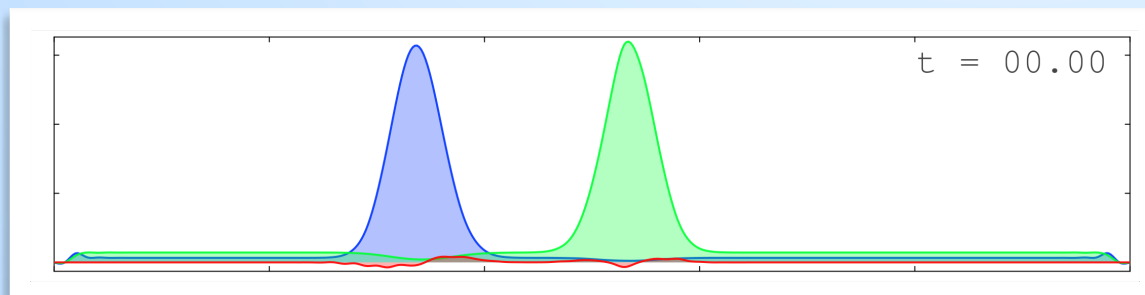


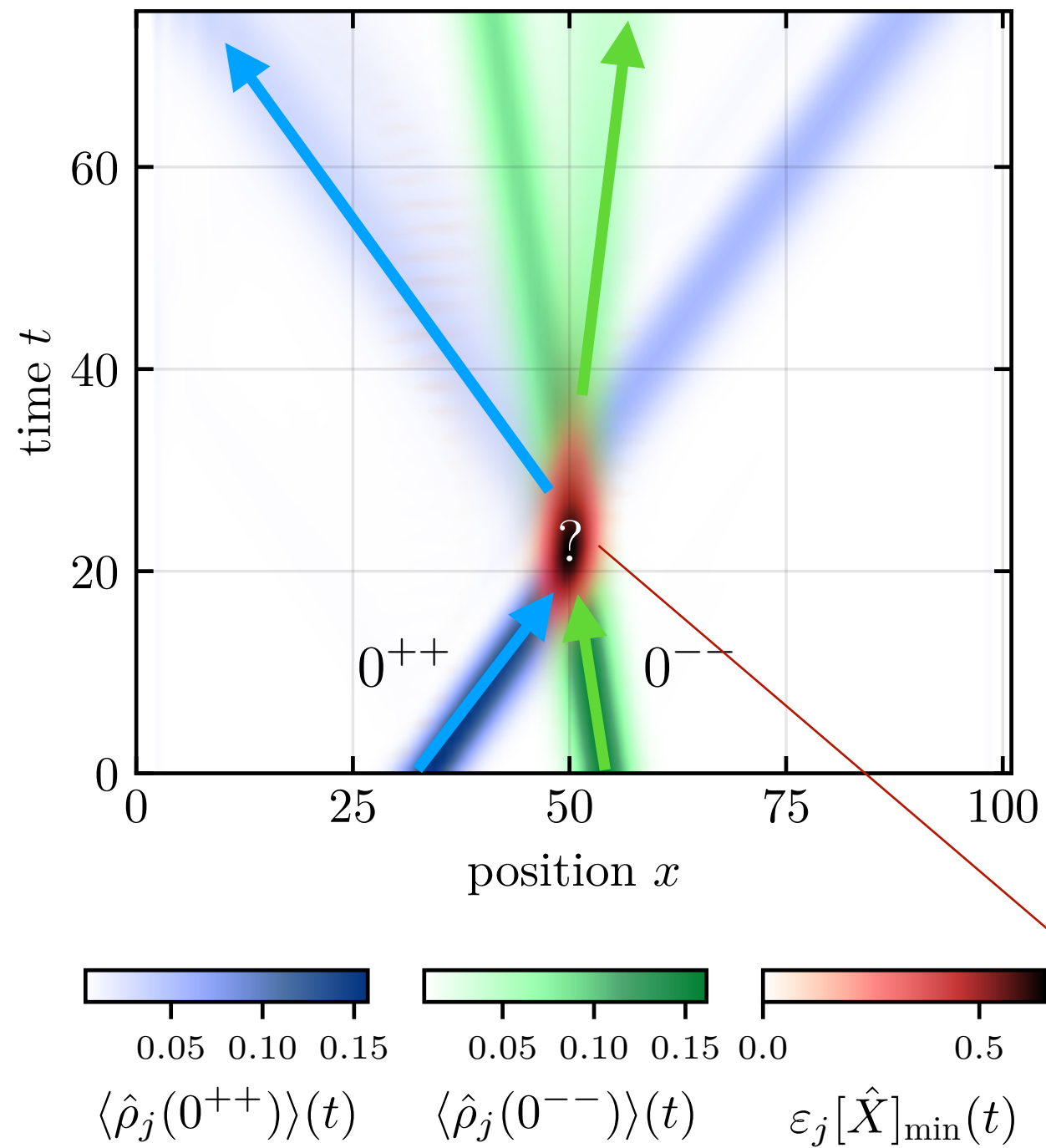


Detection

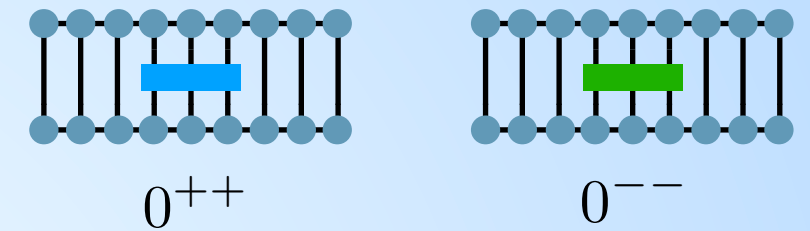


Two main channels





Detection

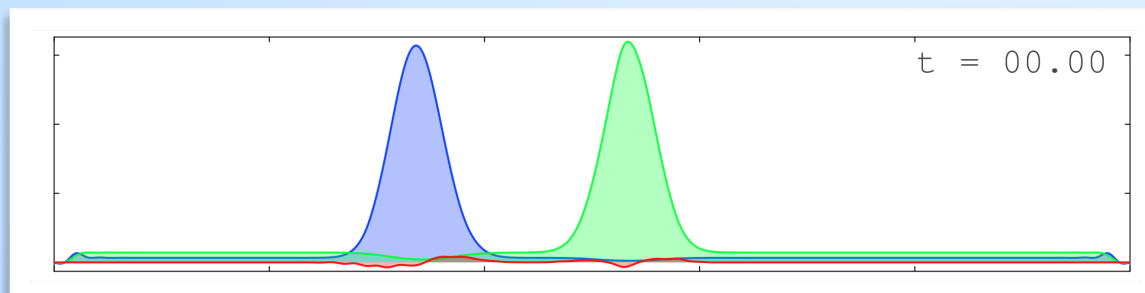


Two main channels



Resonance

$$\mathcal{E}_{X,j} = \mathcal{E}_{\text{tot}} - \mathcal{E}_{0^{++}} - \mathcal{E}_{0^{--}}$$



$$0^{++}0^{++} \rightarrow 0^{++}0^{++}$$

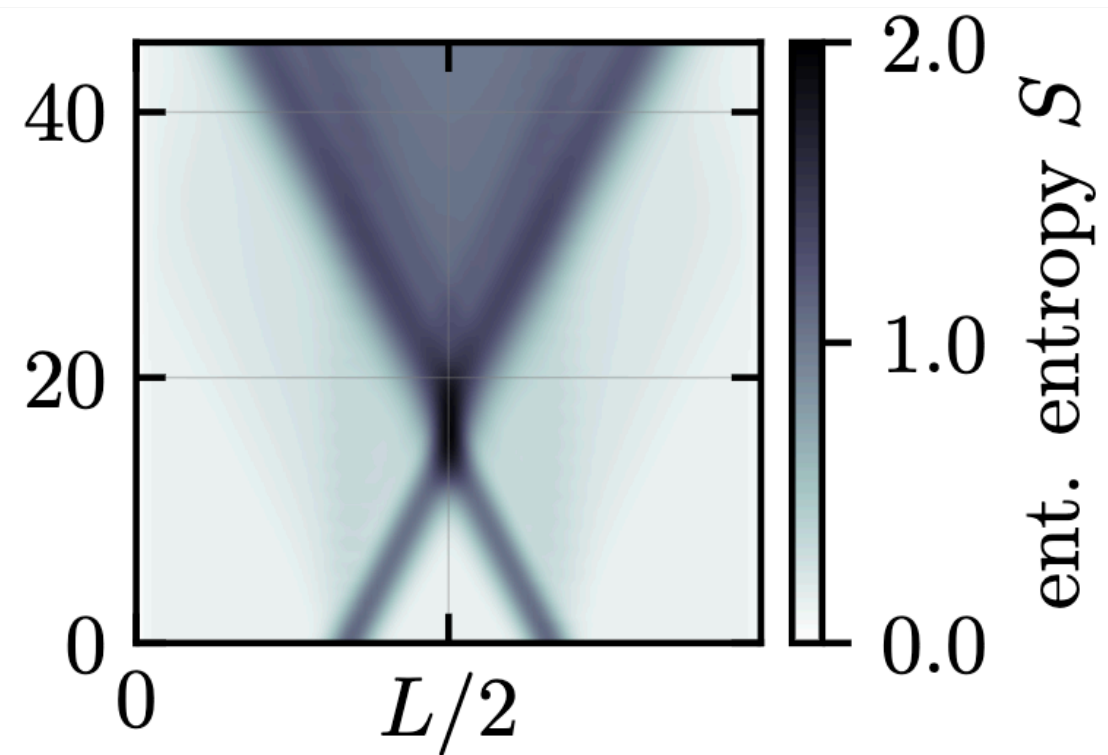
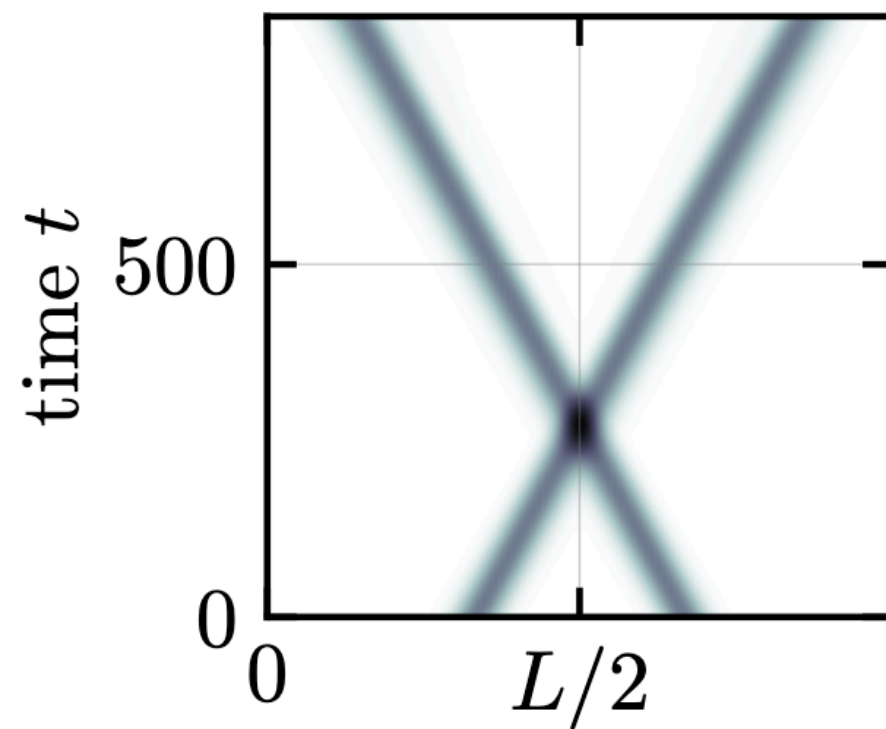
Entanglement entropy in the weak coupling regime

$$0^{++}0^{++} \rightarrow 0^{++}0^{++}$$

Entanglement entropy in the weak coupling regime

Abelian

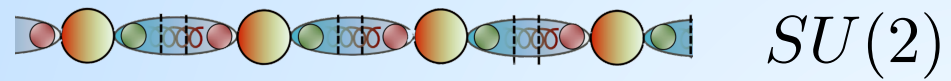
Non-Abelian



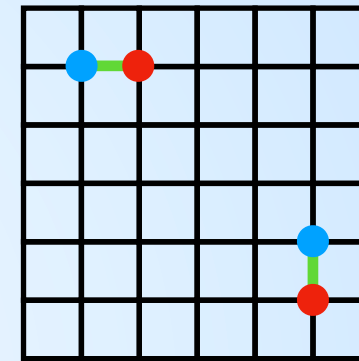
IV. OUTLOOK

Future goals

1D non-abelian models with matter

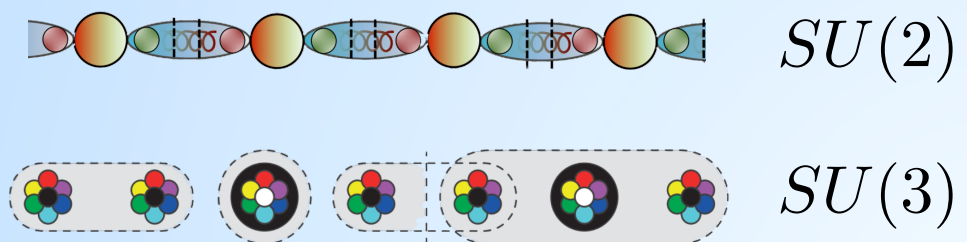


2D QED in high- g with adiabatic

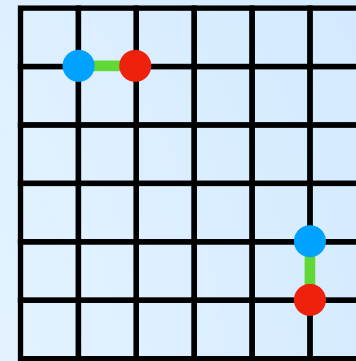


Future goals

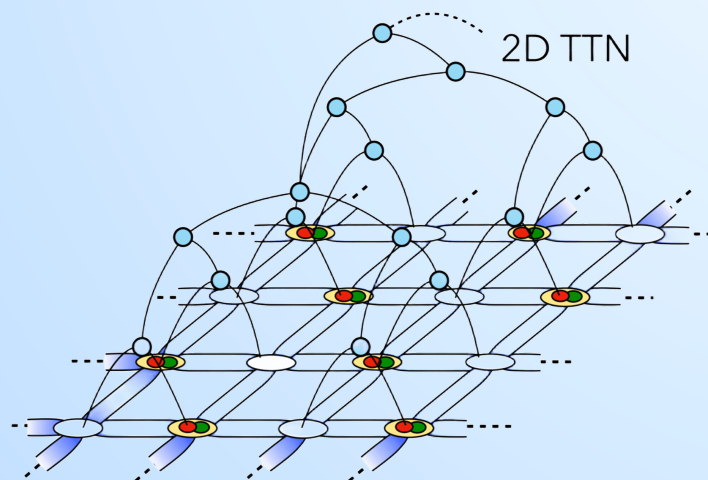
1D non-abelian models with matter



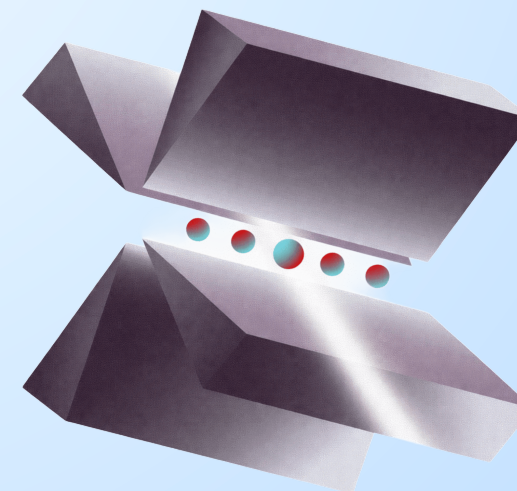
2D QED in high- g with adiabatic



Wannier functions in 2D ?



Quantum simulation ?





UNIVERSITÀ DI PADOVA
Dipartimento
di Fisica e Astronomia
“Galileo Galilei” - DFA



QUANTUM
Information and Matter



Thank you for your attention!



P. Silvi



P. Majcen



S. Montangero



M. Rigobello

Eprint



QUANTUM
FLAGSHIP



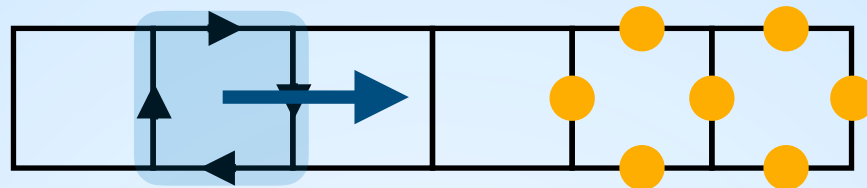
QUANTUM
COMPUTING
AND
SIMULATION
CENTER



Centro Nazionale di Ricerca in HPC,
Big Data and Quantum Computing

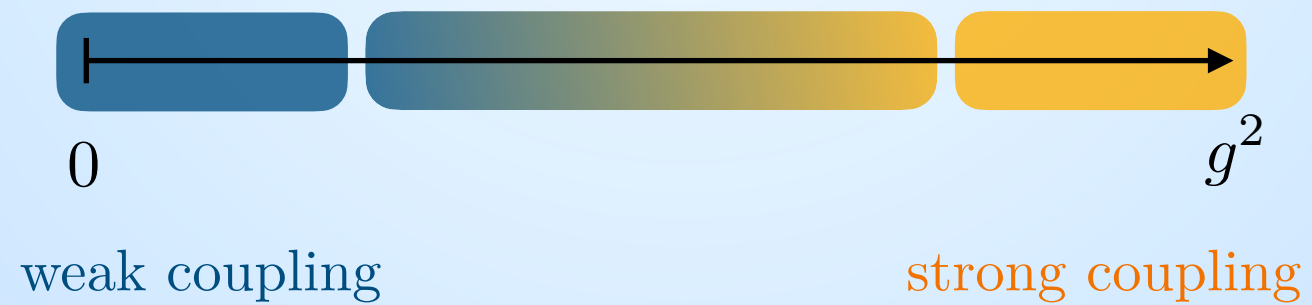
SUPPLEMENTAL MATERIAL

separable
(flat bands)



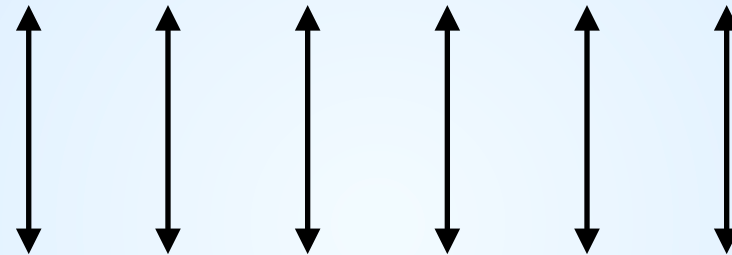
$$\hat{H} = \frac{g^2}{2a} \sum_{\ell} \hat{E}_{\ell}^2 - \frac{1}{2g^2 a} \sum_{\square} \text{Tr}(\hat{U}_{\square} + \hat{U}_{\square}^{\dagger})$$

(2-nd order hopping)

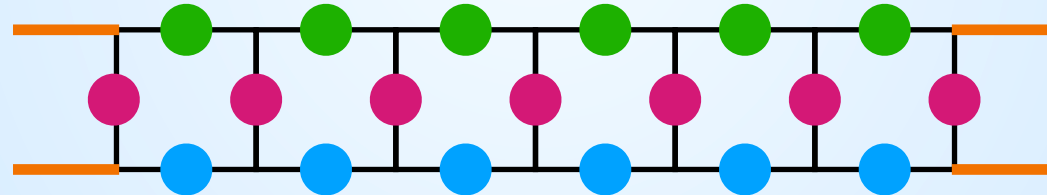


Known duality for $\mathbb{Z}_3$

3-Potts model



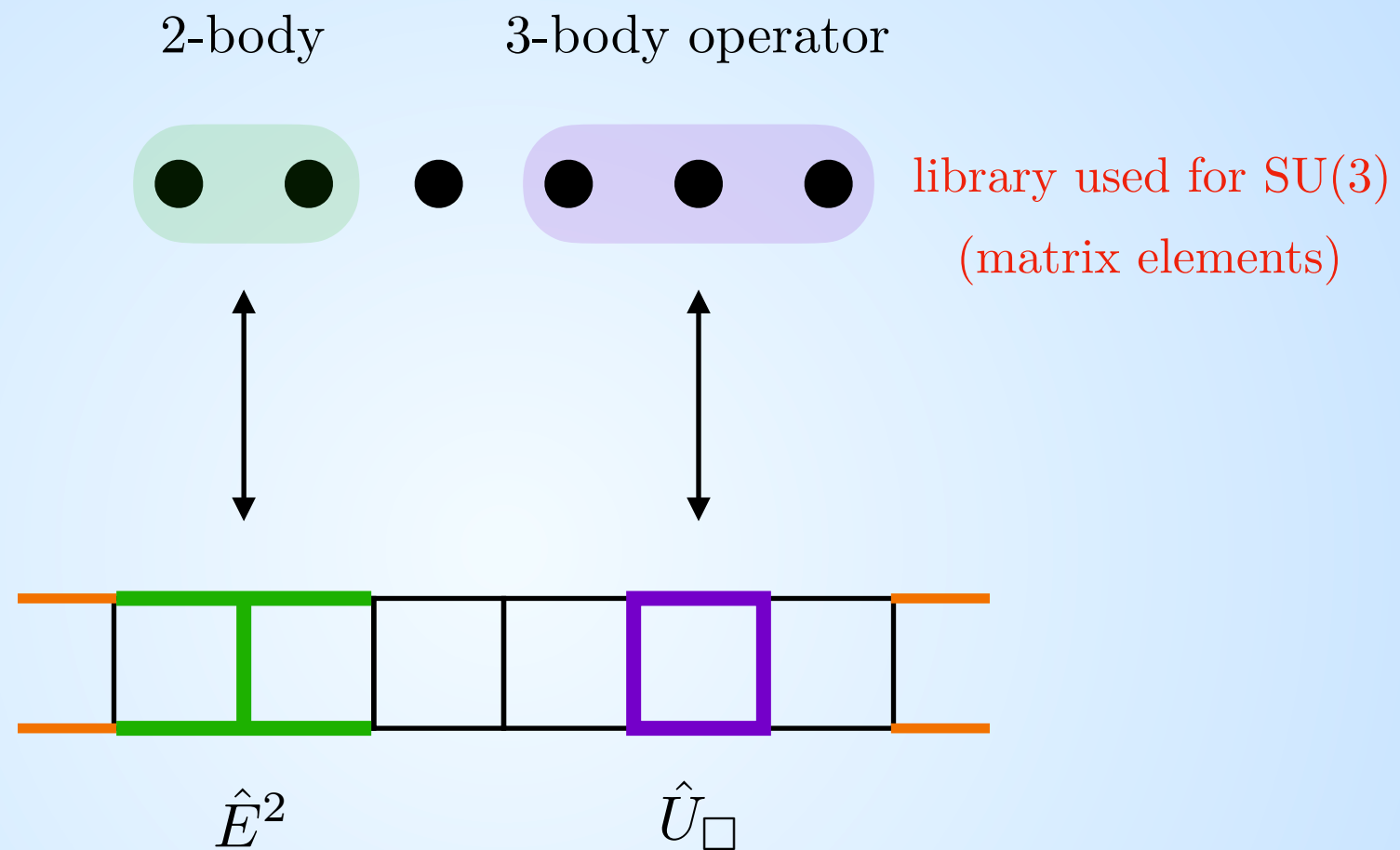
Ladder



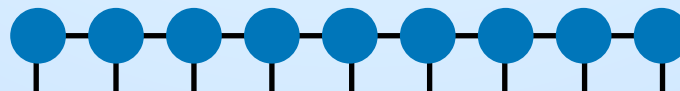
fixing OBCs

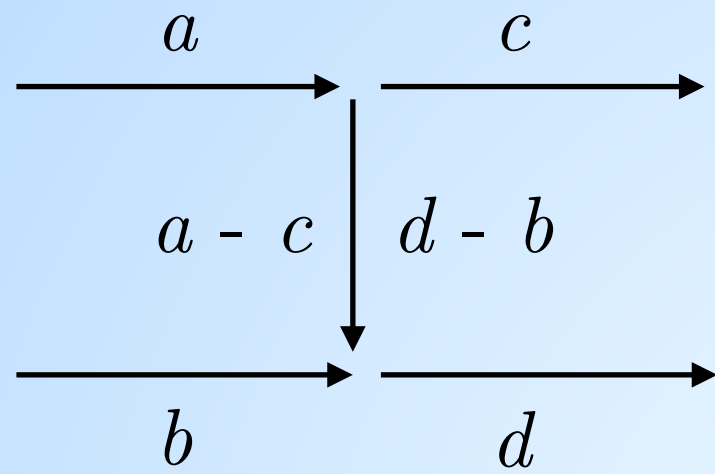
If the states of the top legs are known
from longitudinal polarization
and from Gauss' Law
(all links are known)

$d = 3$ Quantum simulator?



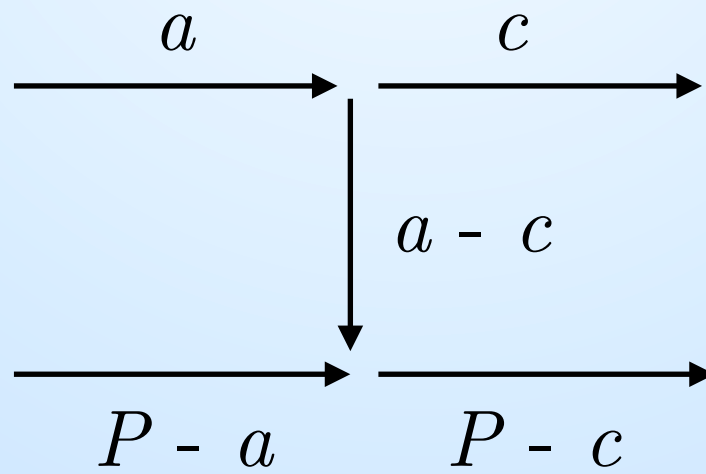
This is the encoding used with MPS





$$a - c = d - b \longrightarrow a + b = c + d$$

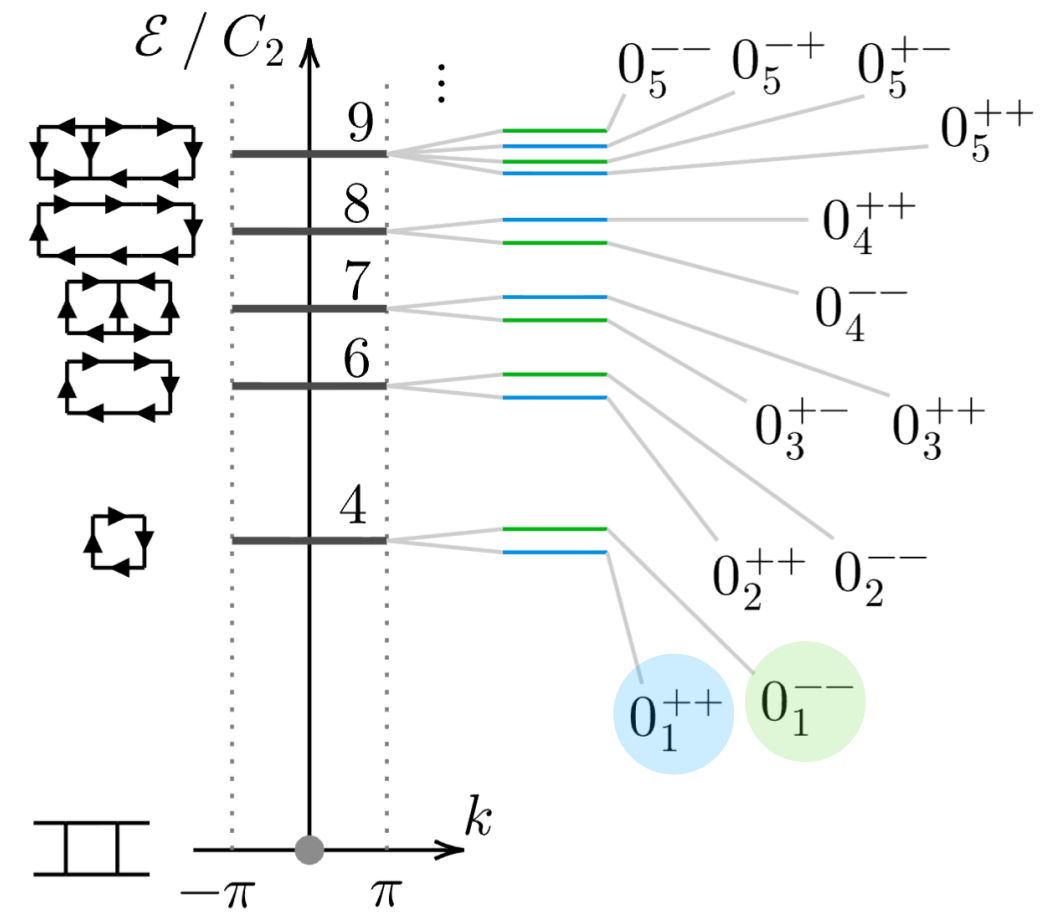
$$P = a + b = c + d = \dots$$



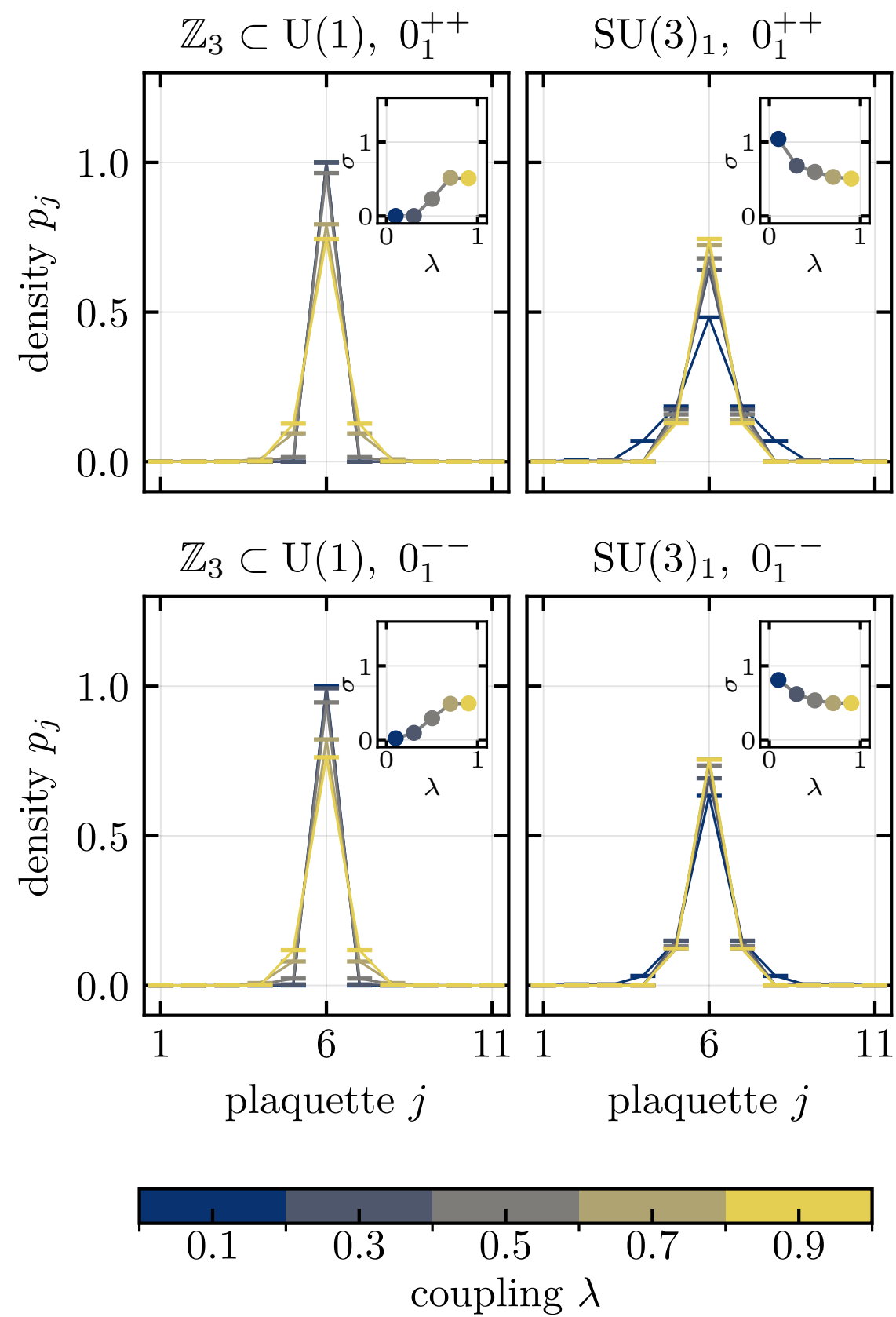
Excitation basis	$\mathcal{E}$	P	C	J_n^{PC}
$ \begin{array}{c} \text{---} \end{array}\rangle \xleftrightarrow{P, C} \begin{array}{c} \text{---} \end{array}\rangle$	$4C_2$	$+$	$+$	0_1^{++} 0_1^{--}
$ \begin{array}{c} \text{---} \end{array}\rangle \xleftrightarrow{P, C} \begin{array}{c} \text{---} \end{array}\rangle$	$6C_2$	$+$	$+$	0_2^{++} 0_2^{--}
$P \begin{array}{c} \text{---} \end{array} \xleftrightarrow{C} \begin{array}{c} \text{---} \end{array} \xrightarrow{P}$	$7C_2$	$+$	$+$	0_3^{++} 0_3^{+-}
$ \begin{array}{c} \text{---} \end{array}\rangle \xleftrightarrow{P, C} \begin{array}{c} \text{---} \end{array}\rangle$	$8C_2$	$+$	$+$	0_4^{++} 0_4^{--}
$ \begin{array}{c} \text{---} \end{array}\rangle \xleftrightarrow{C} \begin{array}{c} \text{---} \end{array}\rangle$ $\updownarrow P$	$9C_2$	$+$	$+$	0_5^{++}
$ \begin{array}{c} \text{---} \end{array}\rangle \xleftrightarrow{C} \begin{array}{c} \text{---} \end{array}\rangle$		$+$	$-$	0_5^{+-}
$ \begin{array}{c} \text{---} \end{array}\rangle \xleftrightarrow{C} \begin{array}{c} \text{---} \end{array}\rangle$		$-$	$+$	0_5^{-+}
$ \begin{array}{c} \text{---} \end{array}\rangle \xleftrightarrow{C} \begin{array}{c} \text{---} \end{array}\rangle$		$-$	$-$	0_5^{--}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

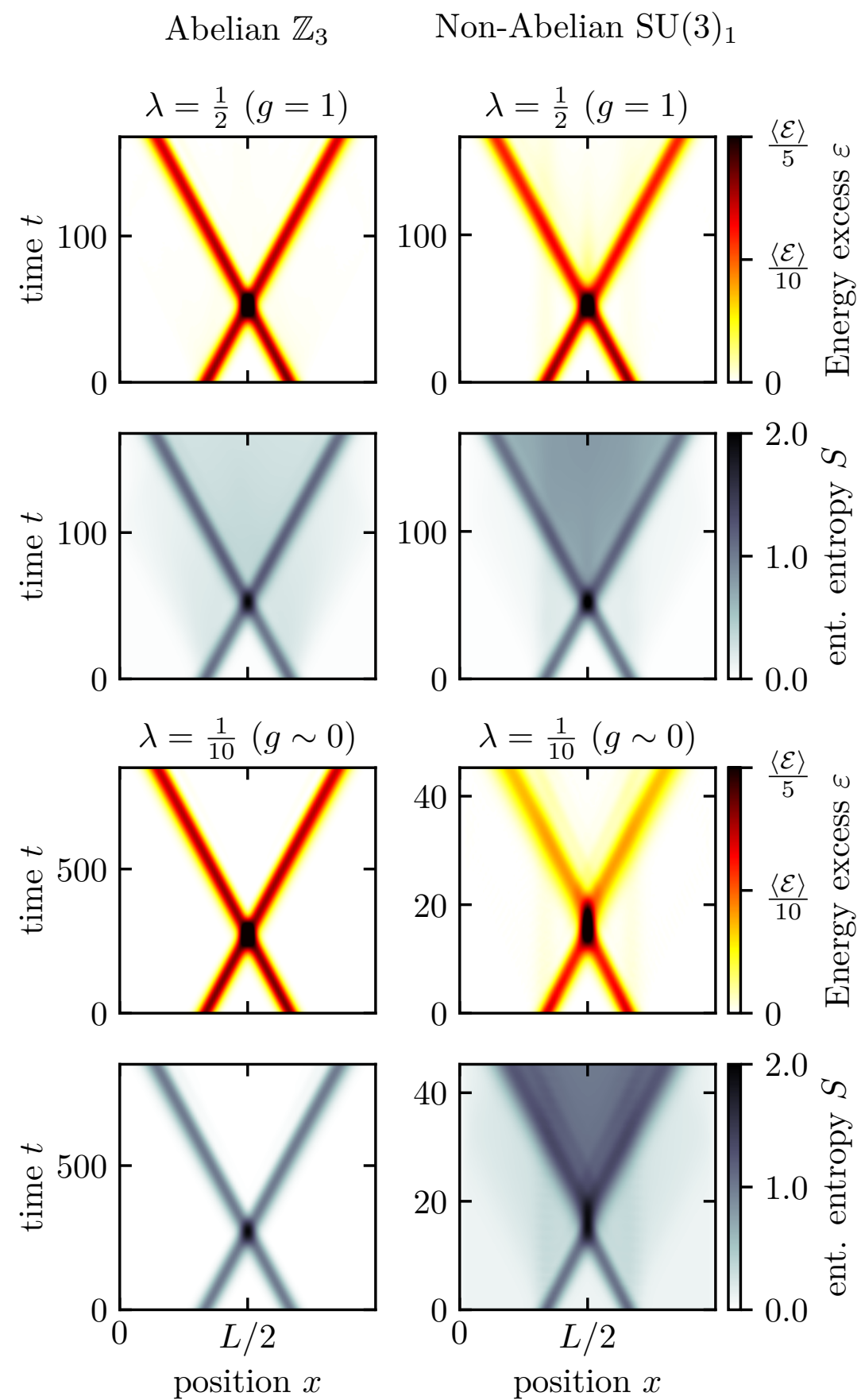
$$|0_1^{++}\rangle = \frac{|\begin{array}{c} \text{---} \end{array}\rangle + |\begin{array}{c} \text{---} \end{array}\rangle}{\sqrt{2}}$$

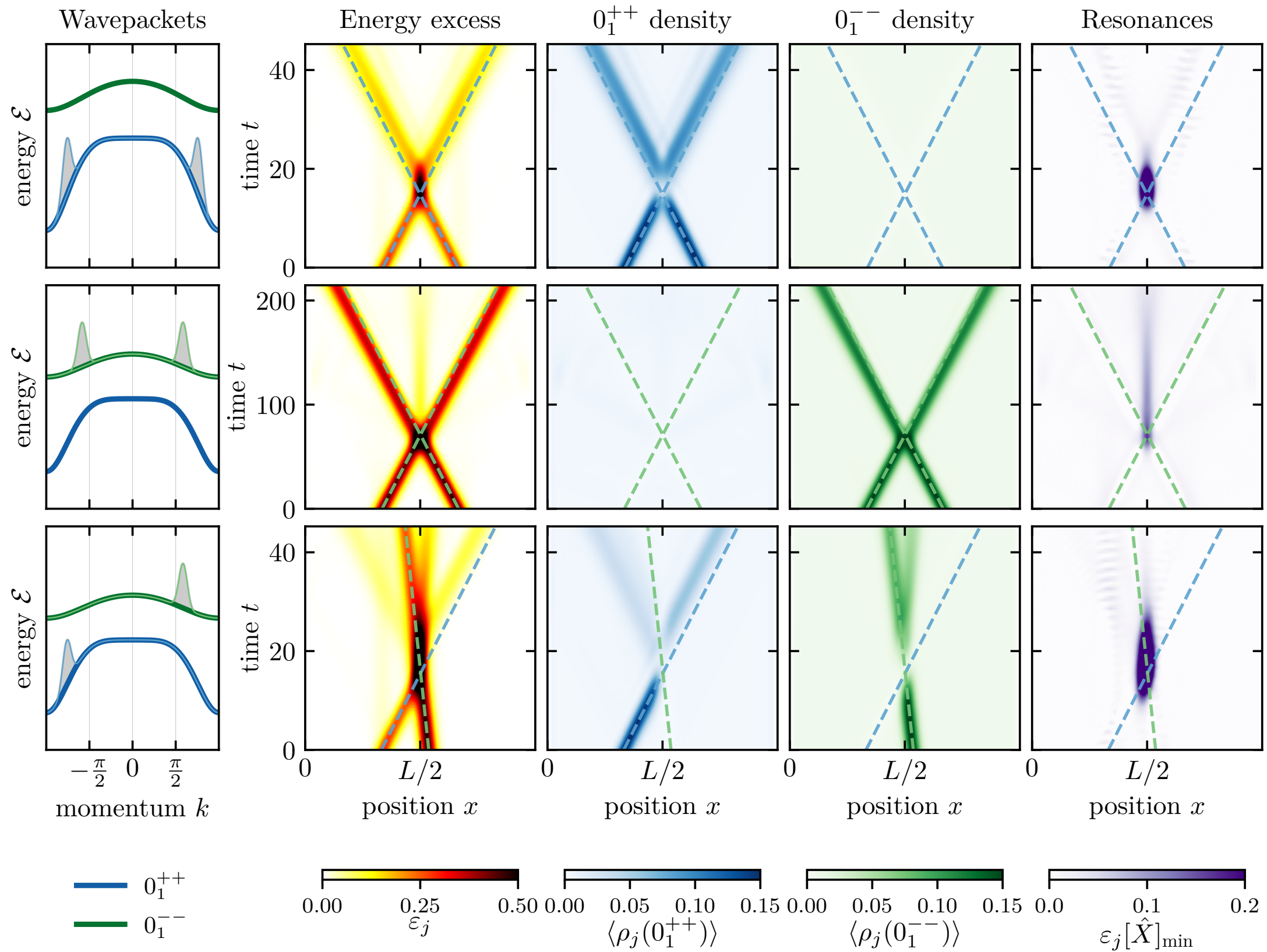
$$|0_1^{--}\rangle = \frac{|\begin{array}{c} \text{---} \end{array}\rangle - |\begin{array}{c} \text{---} \end{array}\rangle}{\sqrt{2}}$$



$I_x = P$	$I_y = C$	J^{PC}	Sector name
$+$	$+$	0^{++}	Scalar
$-$	$+$	0^{-+}	Pseudoscalar
$+$	$-$	0^{+-}	Exotic scalar
$-$	$-$	0^{--}	Exotic pseudoscalar

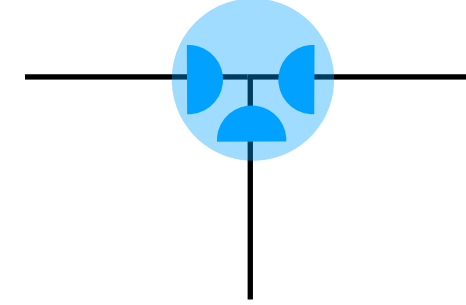




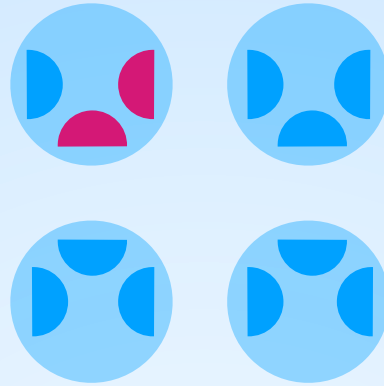


$SU(3)$ singlets on a T-junction

$$\begin{aligned}
 \left| \begin{array}{c} 1 \\ \hline 1 \end{array} \right\rangle &= \left| \begin{array}{c} w \\ \hline w \end{array} \right\rangle \\
 \left| \begin{array}{c} 1 \\ \hline \bar{3} \end{array} \right\rangle &= \frac{1}{\sqrt{3}} \left(\left| \begin{array}{c} w \\ \hline c \end{array} \right\rangle + \left| \begin{array}{c} w \\ \hline m \end{array} \right\rangle + \left| \begin{array}{c} w \\ \hline y \end{array} \right\rangle \right) \\
 \left| \begin{array}{c} 1 \\ \hline 3 \end{array} \right\rangle &= \frac{1}{\sqrt{3}} \left(\left| \begin{array}{c} w \\ \hline r \end{array} \right\rangle + \left| \begin{array}{c} w \\ \hline g \end{array} \right\rangle + \left| \begin{array}{c} w \\ \hline b \end{array} \right\rangle \right) \\
 \left| \begin{array}{c} 3 \\ \hline \bar{3} \end{array} \right\rangle &= \frac{1}{\sqrt{3}} \left(\left| \begin{array}{c} r \\ \hline c \end{array} \right\rangle + \left| \begin{array}{c} g \\ \hline m \end{array} \right\rangle + \left| \begin{array}{c} b \\ \hline y \end{array} \right\rangle \right) \\
 \left| \begin{array}{c} \bar{3} \\ \hline 1 \end{array} \right\rangle &= \frac{1}{\sqrt{3}} \left(\left| \begin{array}{c} c \\ \hline r \end{array} \right\rangle + \left| \begin{array}{c} m \\ \hline g \end{array} \right\rangle + \left| \begin{array}{c} y \\ \hline b \end{array} \right\rangle \right) \\
 \left| \begin{array}{c} 3 \\ \hline 1 \end{array} \right\rangle &= \frac{1}{\sqrt{3}} \left(\left| \begin{array}{c} r \\ \hline w \end{array} \right\rangle + \left| \begin{array}{c} g \\ \hline w \end{array} \right\rangle + \left| \begin{array}{c} b \\ \hline w \end{array} \right\rangle \right) \\
 \left| \begin{array}{c} \bar{3} \\ \hline 3 \end{array} \right\rangle &= \frac{1}{\sqrt{3}} \left(\left| \begin{array}{c} c \\ \hline w \end{array} \right\rangle + \left| \begin{array}{c} m \\ \hline w \end{array} \right\rangle + \left| \begin{array}{c} y \\ \hline w \end{array} \right\rangle \right) \\
 \left| \begin{array}{c} 3 \\ \hline 3 \end{array} \right\rangle &= \frac{1}{\sqrt{6}} \left(\left| \begin{array}{c} r \\ \hline b \end{array} \right\rangle + \left| \begin{array}{c} b \\ \hline g \end{array} \right\rangle + \left| \begin{array}{c} g \\ \hline r \end{array} \right\rangle - \left| \begin{array}{c} r \\ \hline g \end{array} \right\rangle - \left| \begin{array}{c} b \\ \hline r \end{array} \right\rangle - \left| \begin{array}{c} g \\ \hline b \end{array} \right\rangle \right) \\
 \left| \begin{array}{c} \bar{3} \\ \hline \bar{3} \end{array} \right\rangle &= \frac{1}{\sqrt{6}} \left(\left| \begin{array}{c} c \\ \hline y \end{array} \right\rangle + \left| \begin{array}{c} y \\ \hline m \end{array} \right\rangle + \left| \begin{array}{c} m \\ \hline c \end{array} \right\rangle - \left| \begin{array}{c} c \\ \hline m \end{array} \right\rangle - \left| \begin{array}{c} y \\ \hline c \end{array} \right\rangle - \left| \begin{array}{c} m \\ \hline y \end{array} \right\rangle \right)
 \end{aligned}$$

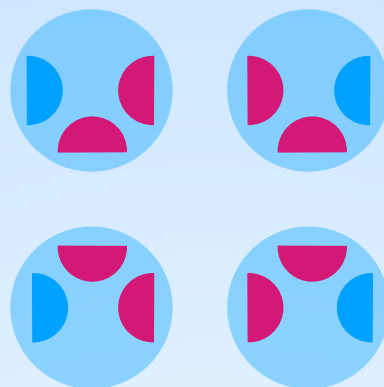
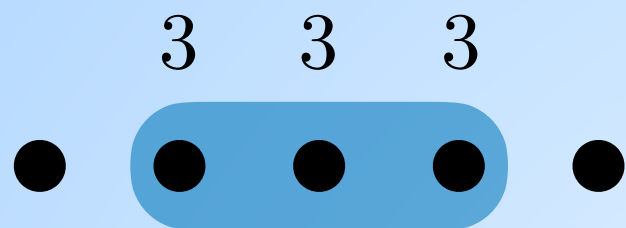


Corner operator



$$\begin{aligned}
 \left\langle \begin{array}{c} 1 \\ \mathbf{T} \\ 3 \end{array} \begin{array}{c} \mathbf{\bar{3}} \end{array} \middle| \hat{C} \middle| \begin{array}{c} 1 \\ \mathbf{T} \\ 1 \end{array} \begin{array}{c} 1 \end{array} \right\rangle &= 1, & \left\langle \begin{array}{c} \mathbf{\bar{3}} \\ \mathbf{T} \\ 3 \end{array} \begin{array}{c} 1 \end{array} \middle| \hat{C} \middle| \begin{array}{c} \mathbf{\bar{3}} \\ \mathbf{T} \\ 1 \end{array} \begin{array}{c} 3 \end{array} \right\rangle &= \frac{1}{\sqrt{3}}, & \left\langle \begin{array}{c} 3 \\ \mathbf{T} \\ 3 \end{array} \begin{array}{c} 3 \end{array} \middle| \hat{C} \middle| \begin{array}{c} 3 \\ \mathbf{T} \\ 1 \end{array} \begin{array}{c} \mathbf{\bar{3}} \end{array} \right\rangle &= \frac{1}{\sqrt[4]{3}}, \\
 \left\langle \begin{array}{c} \mathbf{\bar{3}} \\ \mathbf{T} \\ 3 \end{array} \begin{array}{c} \mathbf{\bar{3}} \end{array} \middle| \hat{C} \middle| \begin{array}{c} \mathbf{\bar{3}} \\ \mathbf{T} \\ 3 \end{array} \begin{array}{c} 1 \end{array} \right\rangle &= \frac{1}{\sqrt[4]{3}}, & \left\langle \begin{array}{c} 3 \\ \mathbf{T} \\ 3 \end{array} \begin{array}{c} 1 \end{array} \middle| \hat{C} \middle| \begin{array}{c} 3 \\ \mathbf{T} \\ 3 \end{array} \begin{array}{c} 3 \end{array} \right\rangle &= \frac{1}{\sqrt{3}}, & \left\langle \begin{array}{c} 1 \\ \mathbf{T} \\ 3 \end{array} \begin{array}{c} 3 \end{array} \middle| \hat{C} \middle| \begin{array}{c} 1 \\ \mathbf{T} \\ 3 \end{array} \begin{array}{c} \mathbf{\bar{3}} \end{array} \right\rangle &= \frac{1}{\sqrt[4]{3}}, \\
 \left\langle \begin{array}{c} 3 \\ \mathbf{T} \\ 1 \end{array} \begin{array}{c} \mathbf{\bar{3}} \end{array} \middle| \hat{C} \middle| \begin{array}{c} 3 \\ \mathbf{T} \\ 3 \end{array} \begin{array}{c} 1 \end{array} \right\rangle &= \frac{1}{\sqrt{3}}, & \left\langle \begin{array}{c} 1 \\ \mathbf{T} \\ 1 \end{array} \begin{array}{c} 1 \end{array} \middle| \hat{C} \middle| \begin{array}{c} 1 \\ \mathbf{T} \\ 3 \end{array} \begin{array}{c} 3 \end{array} \right\rangle &= 1, & \left\langle \begin{array}{c} \mathbf{\bar{3}} \\ \mathbf{T} \\ 1 \end{array} \begin{array}{c} 3 \end{array} \middle| \hat{C} \middle| \begin{array}{c} \mathbf{\bar{3}} \\ \mathbf{T} \\ 3 \end{array} \begin{array}{c} \mathbf{\bar{3}} \end{array} \right\rangle &= \frac{1}{\sqrt[4]{3}}.
 \end{aligned}$$

In $U(1)$ case, all these coefficients are 1



Plaquette operator

$$\hat{U}_{\square}$$

27×27 matrix with 27 non-vanishing matrix elements

$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = 1,$	$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{\sqrt{3}},$	$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{3},$
$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = 1,$	$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{3},$	$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{\sqrt{3}},$
$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = 1,$	$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{\sqrt{3}},$	$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{\sqrt{3}},$
$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{\sqrt{3}},$	$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{3},$	$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{3\sqrt{3}},$
$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{3},$	$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{9},$	$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{3\sqrt{3}},$
$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{\sqrt{3}},$	$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{3},$	$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{3},$
$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{3},$	$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{3\sqrt{3}},$	$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{9},$
$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{\sqrt{3}},$	$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{3\sqrt{3}},$	$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{3},$
$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{\sqrt{3}},$	$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{3},$	$\langle \text{diagram} \rangle$	$\hat{U}_{\square}$	$\langle \text{diagram} \rangle = \frac{1}{3}.$

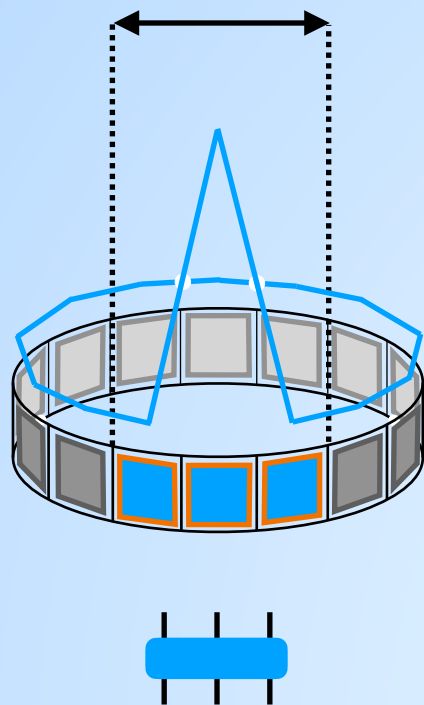
In $U(1)$ case, all these coefficients are 1

$$H = \frac{g^2}{2a} H_E + \frac{1}{2ag^2} H_B$$

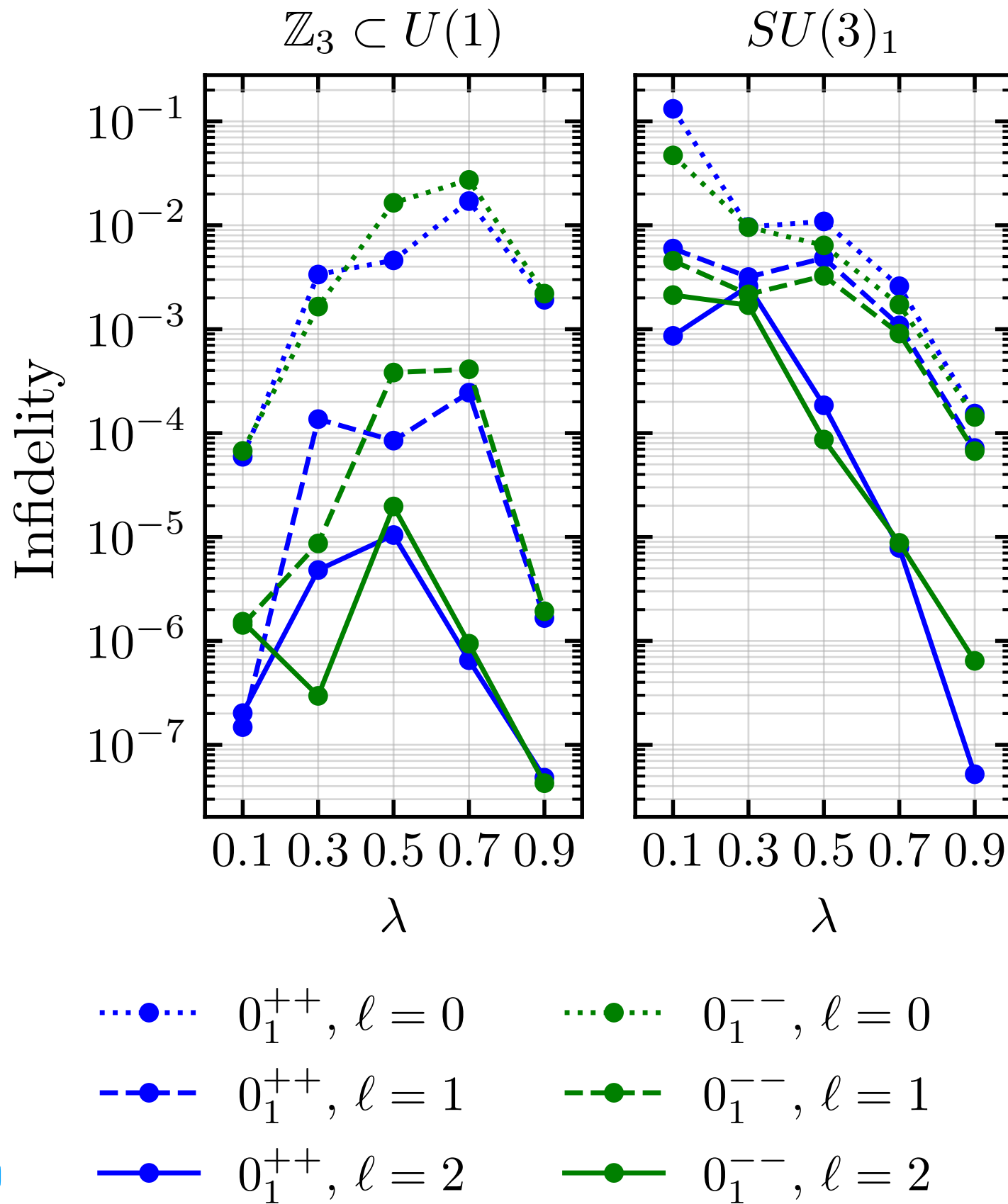
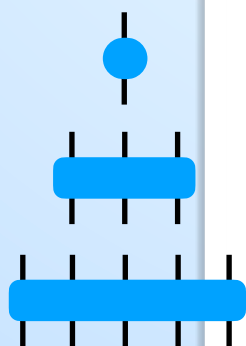
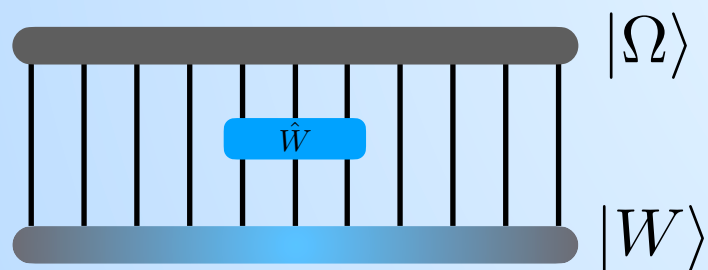
$$H_E = \sum_j \left[2\mathbb{I} - 2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_j \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{j+1} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}_j \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}_{j+1} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}_j \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{j+1} \right]$$

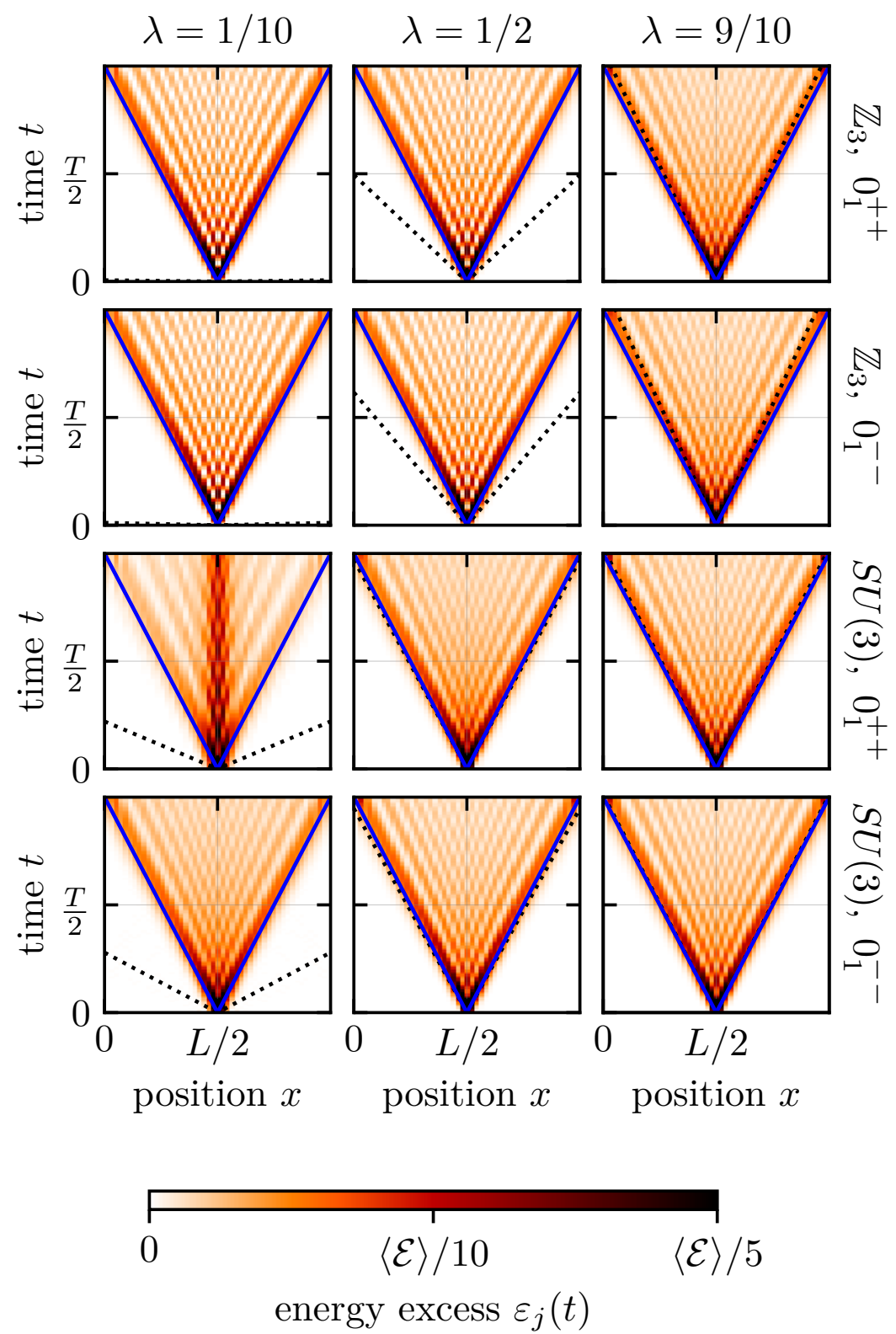
$$\begin{aligned} H_B = & \sum_j \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{\sqrt{3}} \end{pmatrix}_j \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}_{j+1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{\sqrt{3}} \end{pmatrix}_{j+2} \\ & + \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{3}} \end{pmatrix}_j \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}_{j+1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{3}} \end{pmatrix}_{j+2} \\ & + \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}_j \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}_{j+1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}_{j+2} \end{aligned}$$

$$w = 2\ell + 1$$

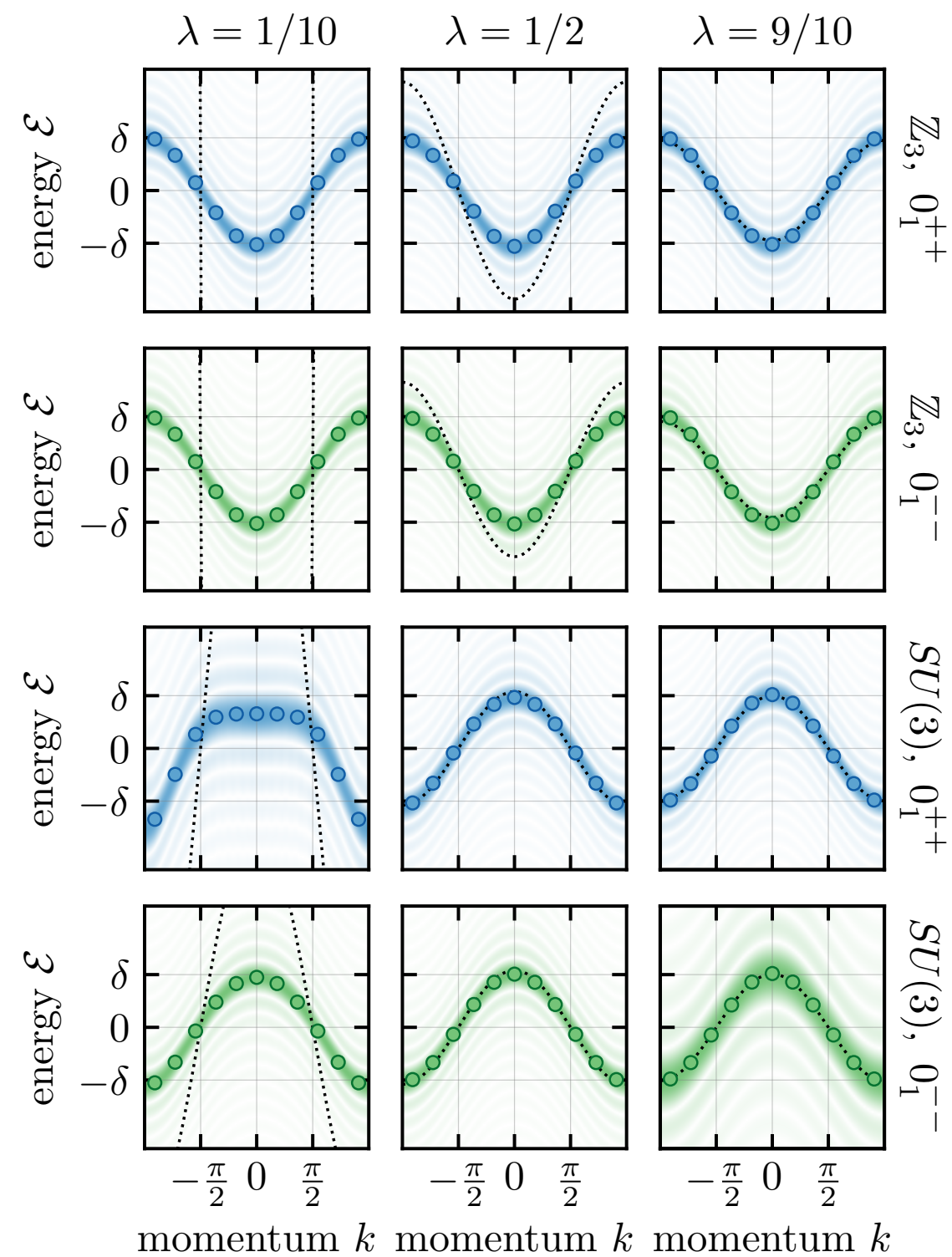
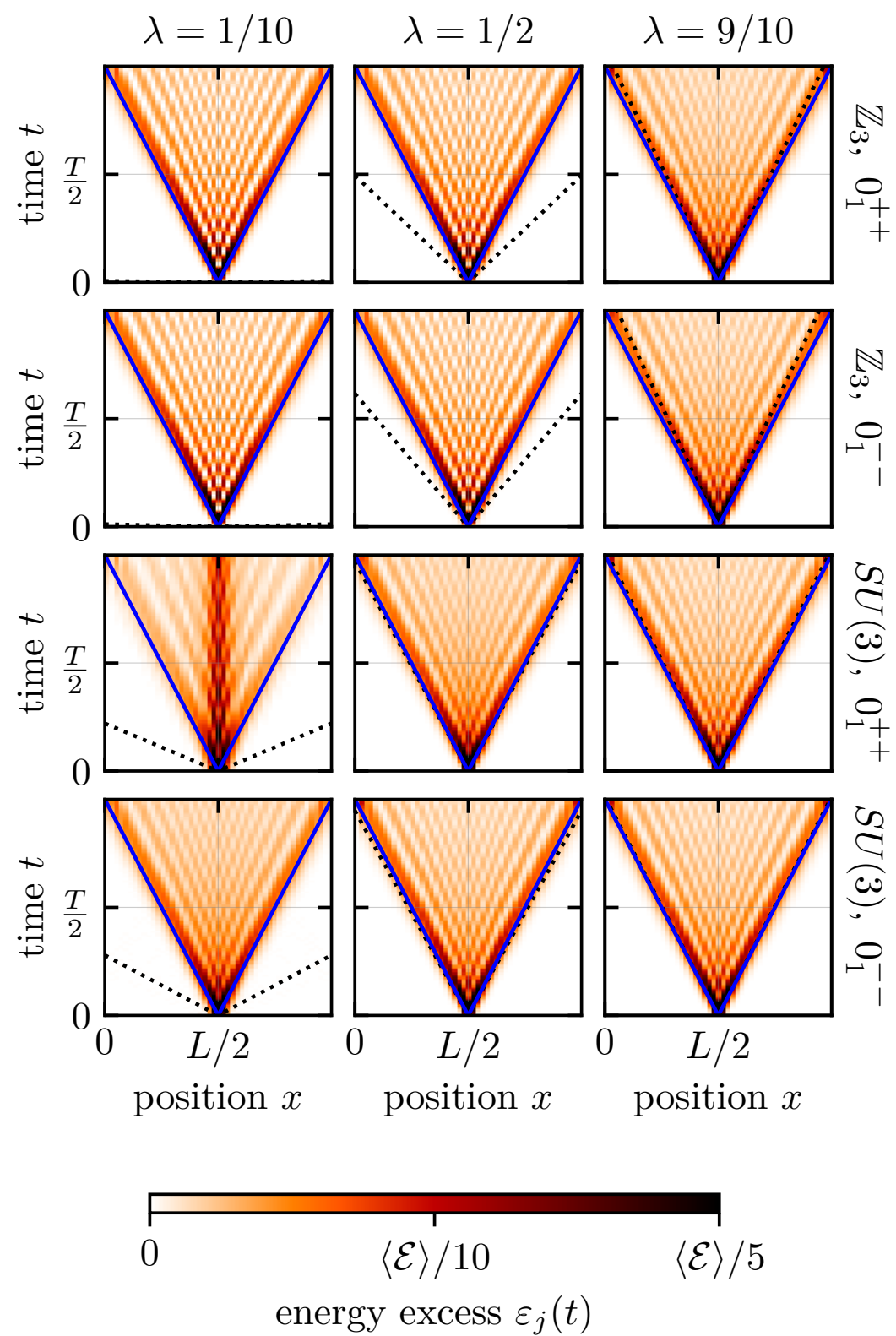


Maximize fidelity:





$$A(k, \omega)$$



Resonance detector

$$c_1 \hat{\rho}_1 + c_2 \hat{\rho}_2 + \hat{X} = \mathbb{I} \quad c_1, c_2 > 0$$

Non-linear energy functional

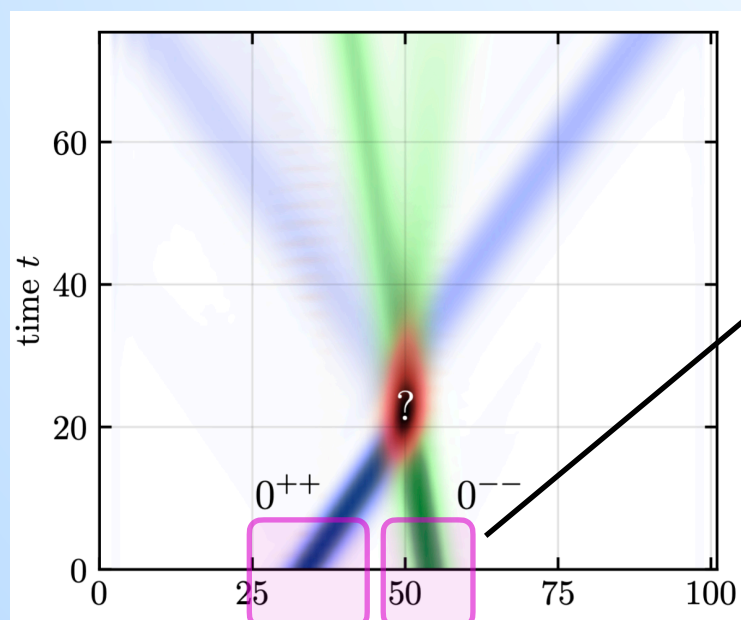
$$\varepsilon_j[\hat{O}] = |\langle \psi(t) | \hat{O} \hat{h}_j | \psi(t) \rangle - \langle \Omega | \hat{O} \hat{h}_j | \Omega \rangle|$$

triangular inequality $\varepsilon_j(t) \leq \varepsilon_j[\hat{X}](t) + c_1 \varepsilon_j[\hat{\rho}_1](t) + c_2 \varepsilon_j[\hat{\rho}_2](t)$

$$\varepsilon_j[\hat{X}](t) \geq \boxed{\varepsilon_j(t)} - \boxed{c_1 \varepsilon_j[\hat{\rho}_1](t) + c_2 \varepsilon_j[\hat{\rho}_2](t)} \equiv \varepsilon_j[\hat{X}]_{\min}(t)$$

total energy excess

contributions from known particles



c_1, c_2 are tuned in a way that this X contribution at the beginning of time evolution is vanishing