Multi-ensemble analysis of ρ resonance

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INTRODUCTION

- ρ meson calculation with 4 ensembles
- Extrapolation to physical ρ:
  - Continuum extrapolation
  - Quark mass extrapolation

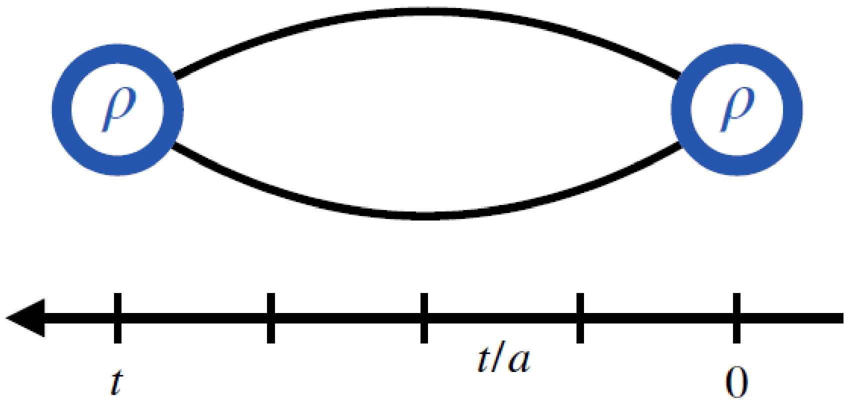
Ensembles

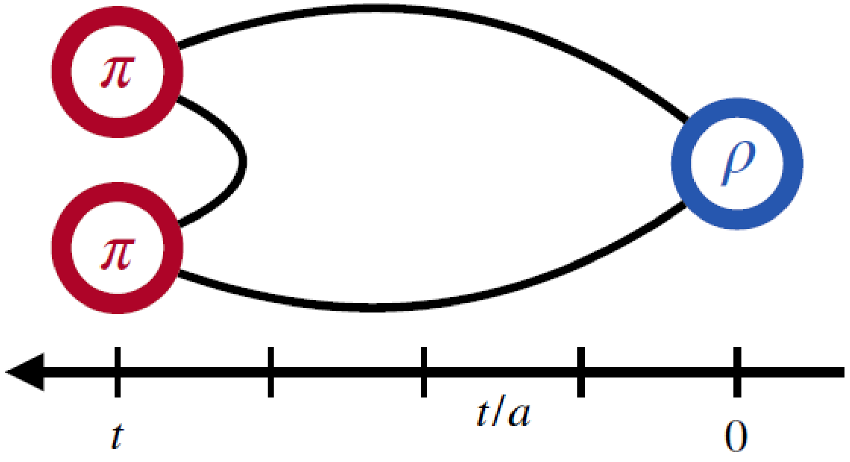
- $N_f = 2 + 1$ dynamical quark flavors
- Clover improved Wilson action

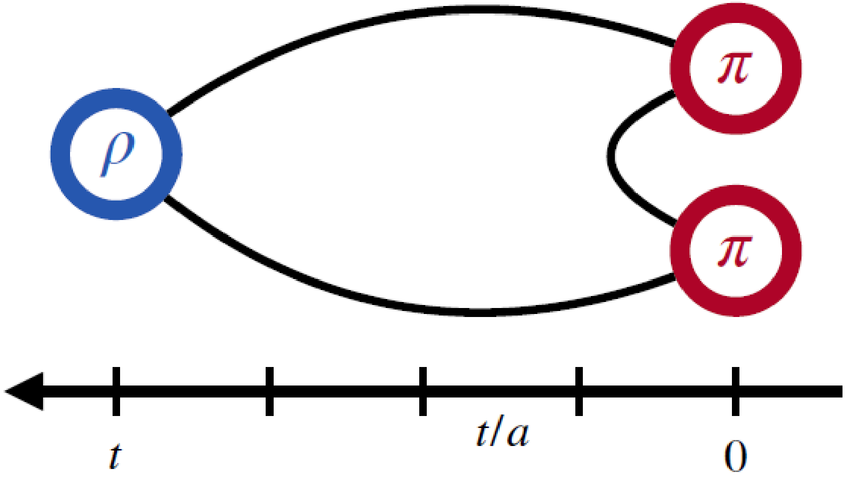
	C13	D5	D6	E5
$N_s^3 \times N_t$	$32^3 \times 96$	$32^3 \times 64$	$48^3 \times 96$	$48^3 \times 128$
a [fm]	0.11403(77)	0.08990(82)	0.08766(79)	0.06827(55)
m_π [MeV]	316.6(2.2)	282.8(2.8)	174.7(1.8)	289.9(2.4)
m_{η_s} [MeV]	671.8(5.6)	702.8(6.4)	702.8(6.3)	821.5(6.7)
$N_{configs} \times N_{sources}$	1039 x 8	467 x 16	621 x 16	297 x 16

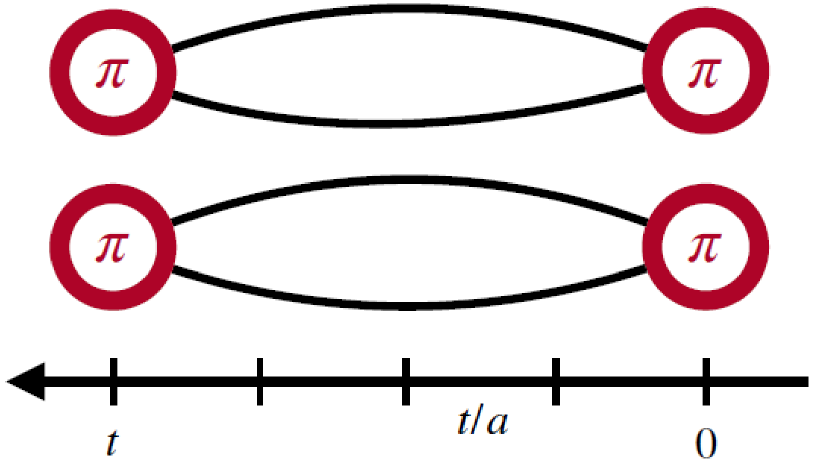
2-point correlation functions

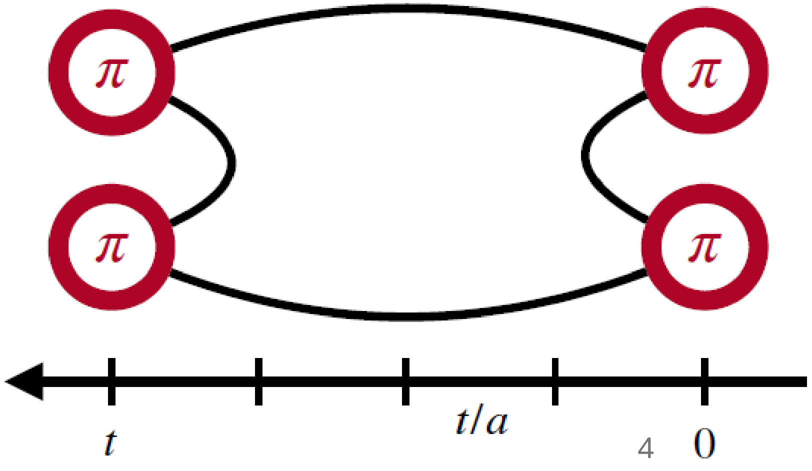
$$C_{ij}(t) = \langle O_i^\dagger(t) O_j(0) \rangle = \sum_n \frac{Z_i^{n*} Z_j^n}{2E_n} e^{-E_n t}$$











Interpolators

Interpolators in momentum space:

- $O_\rho(\boldsymbol{P}) = \bar{q}\gamma_i q(\boldsymbol{P})$
- $O_{\pi\pi}(\boldsymbol{p_1}, \boldsymbol{p_2}) = [\bar{q}\gamma_5 q](\boldsymbol{p_1})[\bar{q}\gamma_5 q](\boldsymbol{p_2})$

$\boldsymbol{p_1} + \boldsymbol{p_2} = \boldsymbol{P}$

For all:

- $P = |\boldsymbol{P}|^2 \leq 3$;
- $|\boldsymbol{p_1}|^2, |\boldsymbol{p_2}|^2 \leq 3$

Project to $\Lambda, r, \boldsymbol{P}$

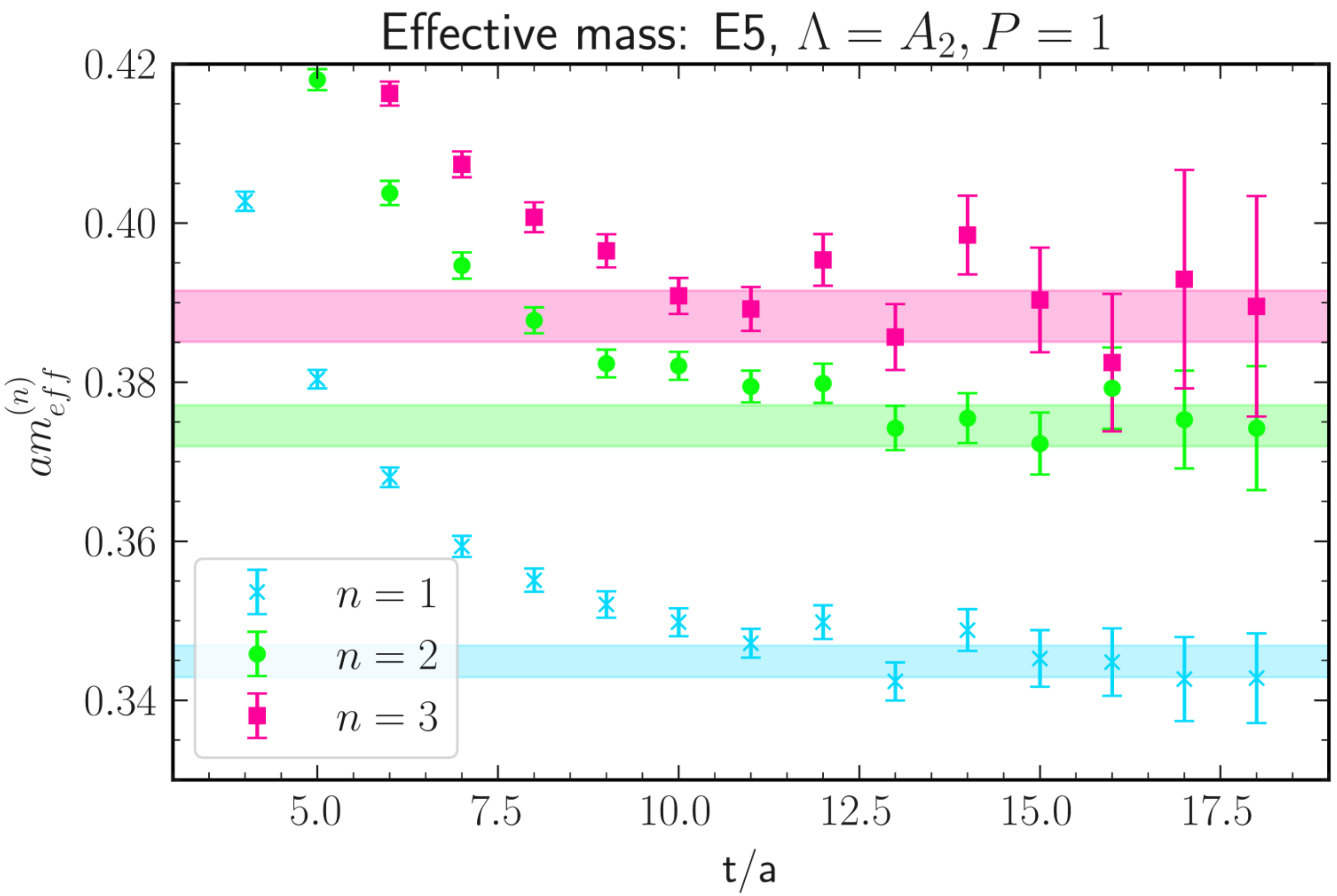
Example: $O_{\pi\pi}^{\Lambda,r,\boldsymbol{P}}(E)$

- $O_{\pi\pi}^{E,1,(001)}(E_1)$,
- $O_{\pi\pi}^{E,1,(010)}(E_1)$,
- $O_{\pi\pi}^{E,2,(001)}(E_1)$
- …

$\xrightarrow{\text{Average } C^{\Lambda,\mathrm{r},\mathbf{P}} \text{ over } r, \boldsymbol{P}}$ $O_{\pi\pi}^{\Lambda=E,P=1}(E_1)$

GEVP:

$$C_{ij}^{\Lambda,P}(t)v_j^{\Lambda,P,n}(t) = \lambda^{\Lambda,P,n}(t,t_0)C_{ij}^{\Lambda,P}(t_0)v_j^{\Lambda,P,n}(t)$$

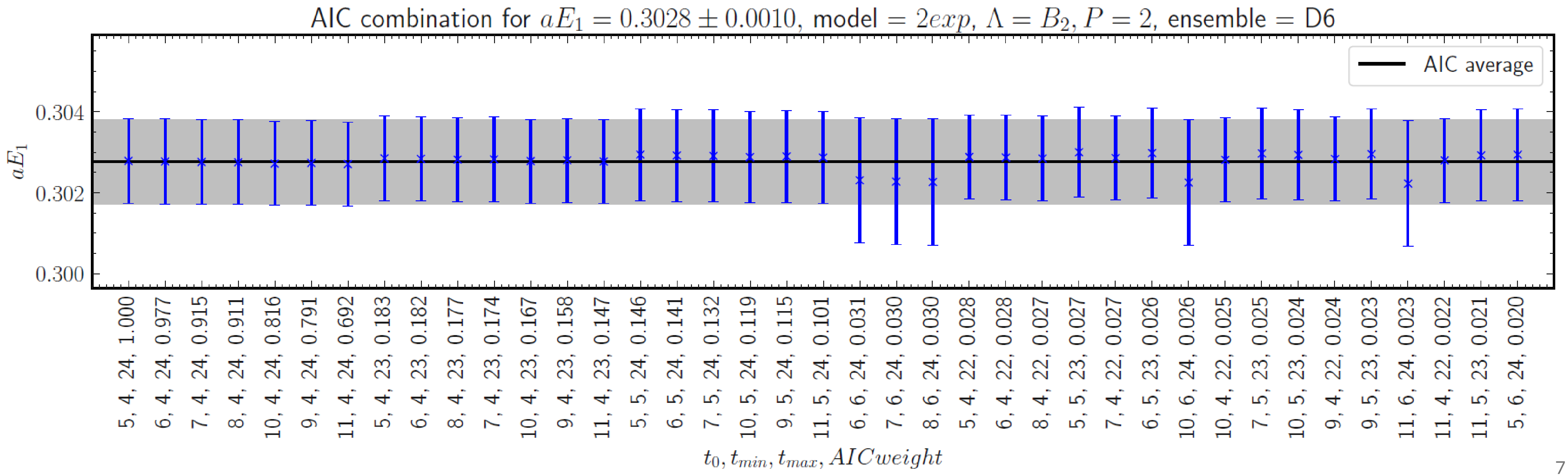


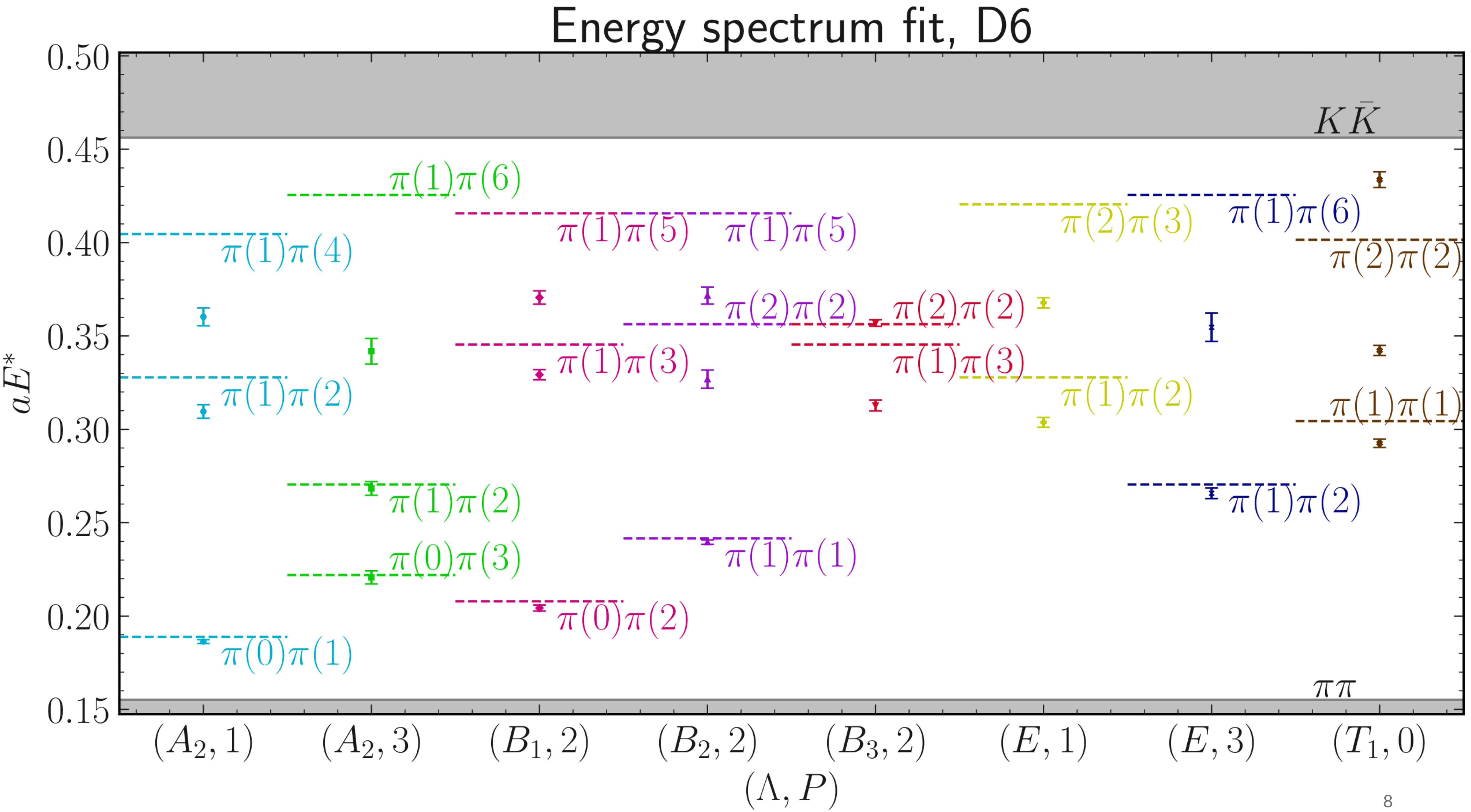
AIC: Jay – Neil, arXiv:2008.01069

Energy fit:

$$\lambda_n^{1\exp}(t, t_0) = A \exp[-E_n(t - t_0)]$$

$$\lambda_n^{2\exp}(t, t_0) = (1 - A) \exp[-E_n(t - t_0)] + A \exp[-E_m(t - t_0)]$$





Energy spectrum fit, D6
$K\bar{K}$
$\pi\pi$
aE^*
(Λ, P)
$(A_2, 1)$
$(A_2, 3)$
$(B_1, 2)$
$(B_2, 2)$
$(B_3, 2)$
$(E, 1)$
$(E, 3)$
$(T_1, 0)$
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
$\pi(0)\pi(1)$
$\pi(1)\pi(2)$
$\pi(1)\pi(4)$
$\pi(0)\pi(3)$
$\pi(1)\pi(6)$
$\pi(0)\pi(2)$
$\pi(1)\pi(3)$
$\pi(1)\pi(5)$
$\pi(1)\pi(1)$
$\pi(2)\pi(2)$
$\pi(2)\pi(3)$

Global fit

$$\chi^2 = (E_{data} - E_{model})^T C^{-1} (E_{data} - E_{model})$$

$$E_{data} = [\,\{E_{C13}\}, \{E_{D5}\}, \{E_{D6}\}, \{E_{E5}\}]$$

$$N_{data} = 57$$

$$C = \begin{bmatrix} C_{C13} & & & \\ & C_{D5} & & \\ & & C_{D6} & \\ & & & C_{E5} \end{bmatrix}$$

Global fit – model energies

$$E^*_{model} = E^*_T - E^*_d$$

QC in continuum

Discretization effects

QC: Briceno-Dudek-Young: arXiv:1706.06223

Quantization condition in continuum

$$\det\left[\,T^{-1}(E_T^*) + \rho\, F(E_T^*)\right] = 0$$

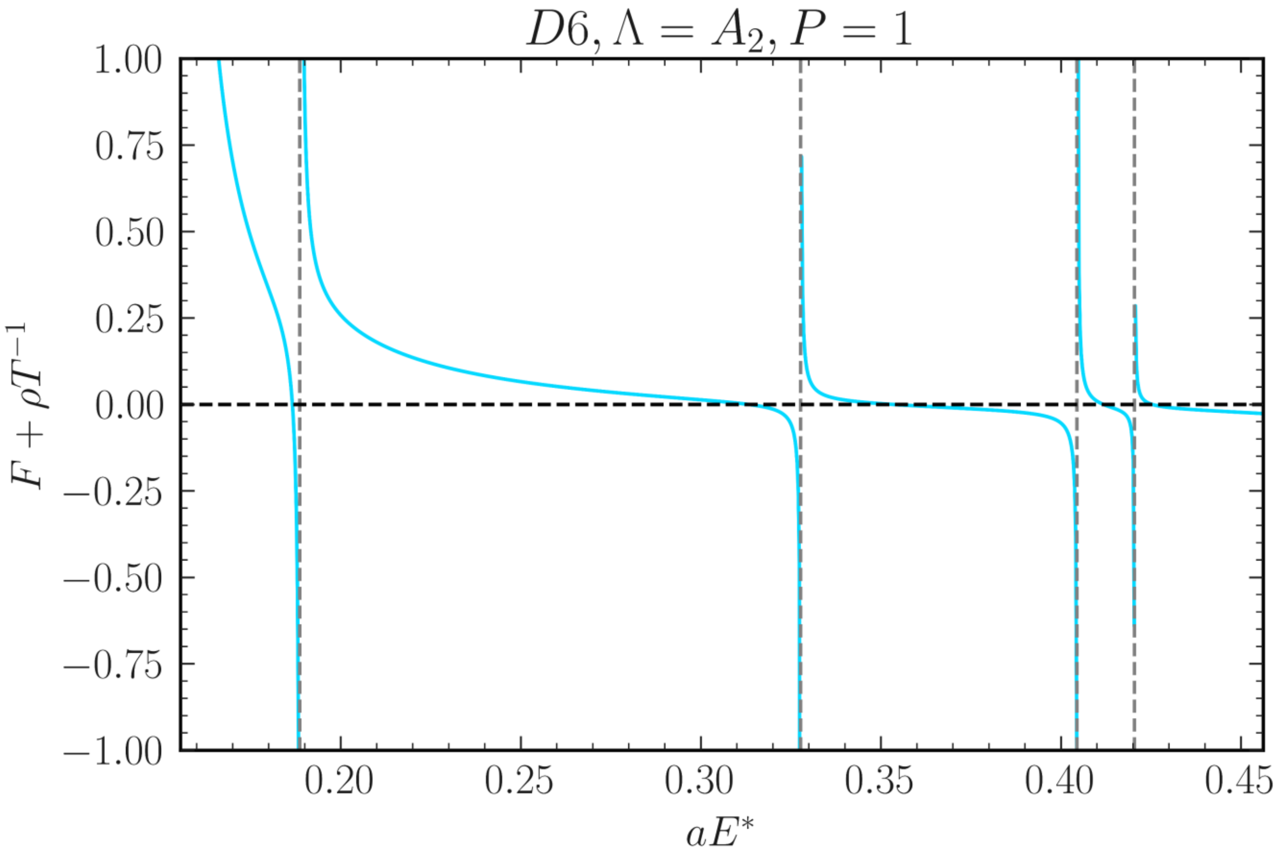
$$T^{-1} = K^{-1} - i\,\rho$$

$$K = \widetilde{K}\,k^{2l}$$

$$\rho = \frac{k}{16\pi\sqrt{s}}$$

Functions of parameters

$$\widetilde{K} = \frac{g^2(m_\pi, m_{\eta_s})}{m_\rho^2(m_\pi, m_{\eta_s}) - s}$$



Mass dependence

Motivation: Bruns-Meisner, arXiv:hep-ph/0411223

$$m_\rho = m_{\rho,phys} + c_2\, M_\pi^2 + c_{2log}\, M_\pi^{2log} + c_3\, M_\pi^3 + c_{4log}\, M_\pi^{4log} + c_{\eta_s}\, M_{\eta_s}^2$$

$$g_\rho^2 = g_{phys}^2 + d_2\, M_\pi^2 + d_{2log}\, M_\pi^{2log} + d_3\, M_\pi^3 + d_{4log}\, M_\pi^{4log} + d_{\eta_s}\, M_{\eta_s}^2$$

$$M_i^2 = \frac{a^2 m_i^2 - a^2 m_{i,phys}^2}{a^2 m_{i,phys}^2} \qquad M_i^{2\log} = \frac{a^2 m_i^2}{a^2 m_{i,phys}^2} 2\ln\left(\frac{m_i}{\mu}\right) - 2\ln\left(\frac{m_{i,phys}}{\mu}\right)$$

$$M_i^3 = \frac{a^3 m_i^3 - a^3 m_{i,phys}^3}{a^3 m_{i,phys}^3} \qquad M_i^{4\log} = \frac{a^4 m_i^4}{a^4 m_{i,phys}^4} 2\ln\left(\frac{m_i}{\mu}\right) - 2\ln\left(\frac{m_{i,phys}}{\mu}\right)$$

Motivation: Hansen-Peterken, arXiv:2408.07062

Discretization

Additive models:

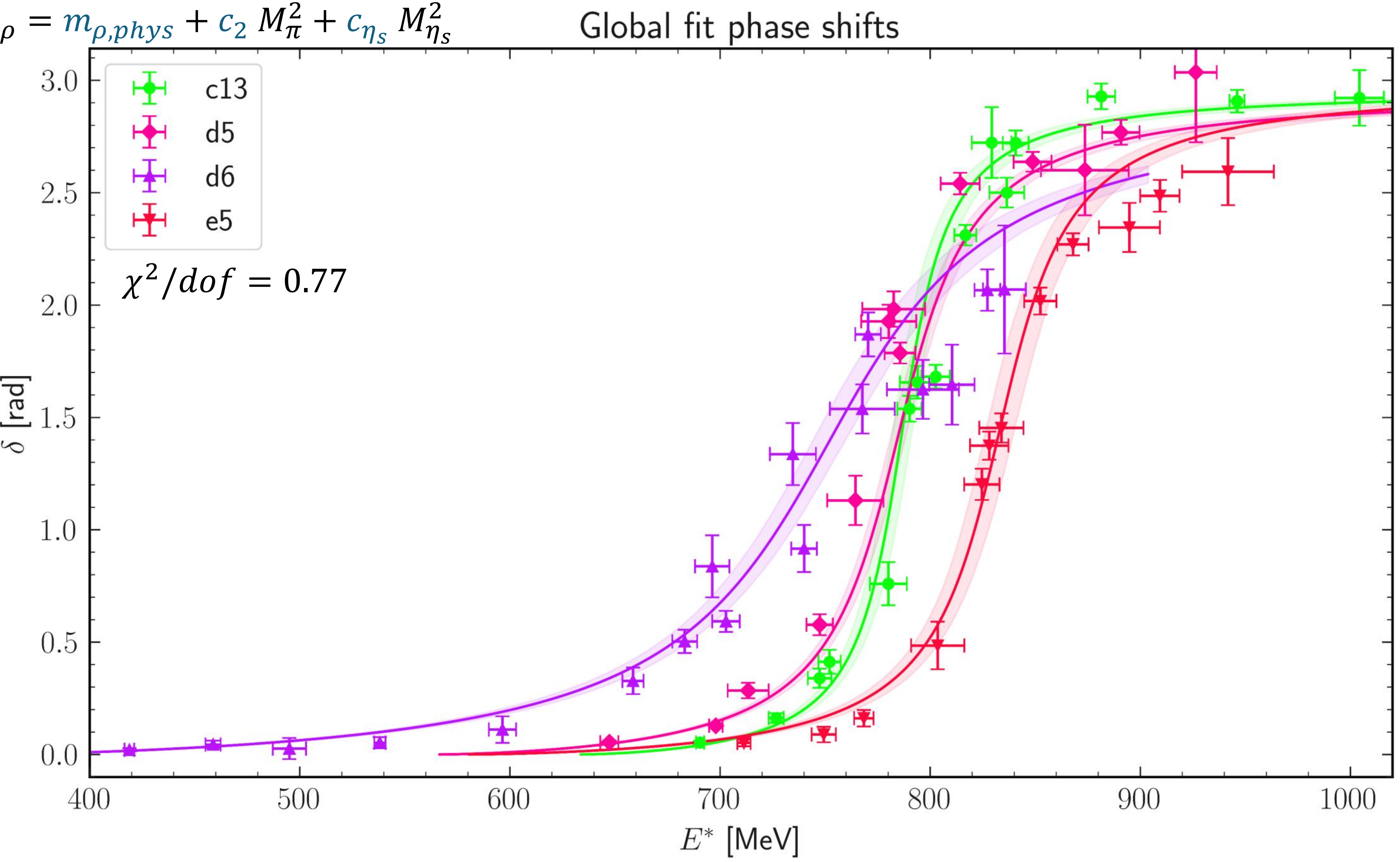
$$aE^*_{d,a} = c_P a(aP)^2 + c_k a(ak)^2 + c_C a^3 \Lambda^2_{QCD}$$

Multiplicative models:

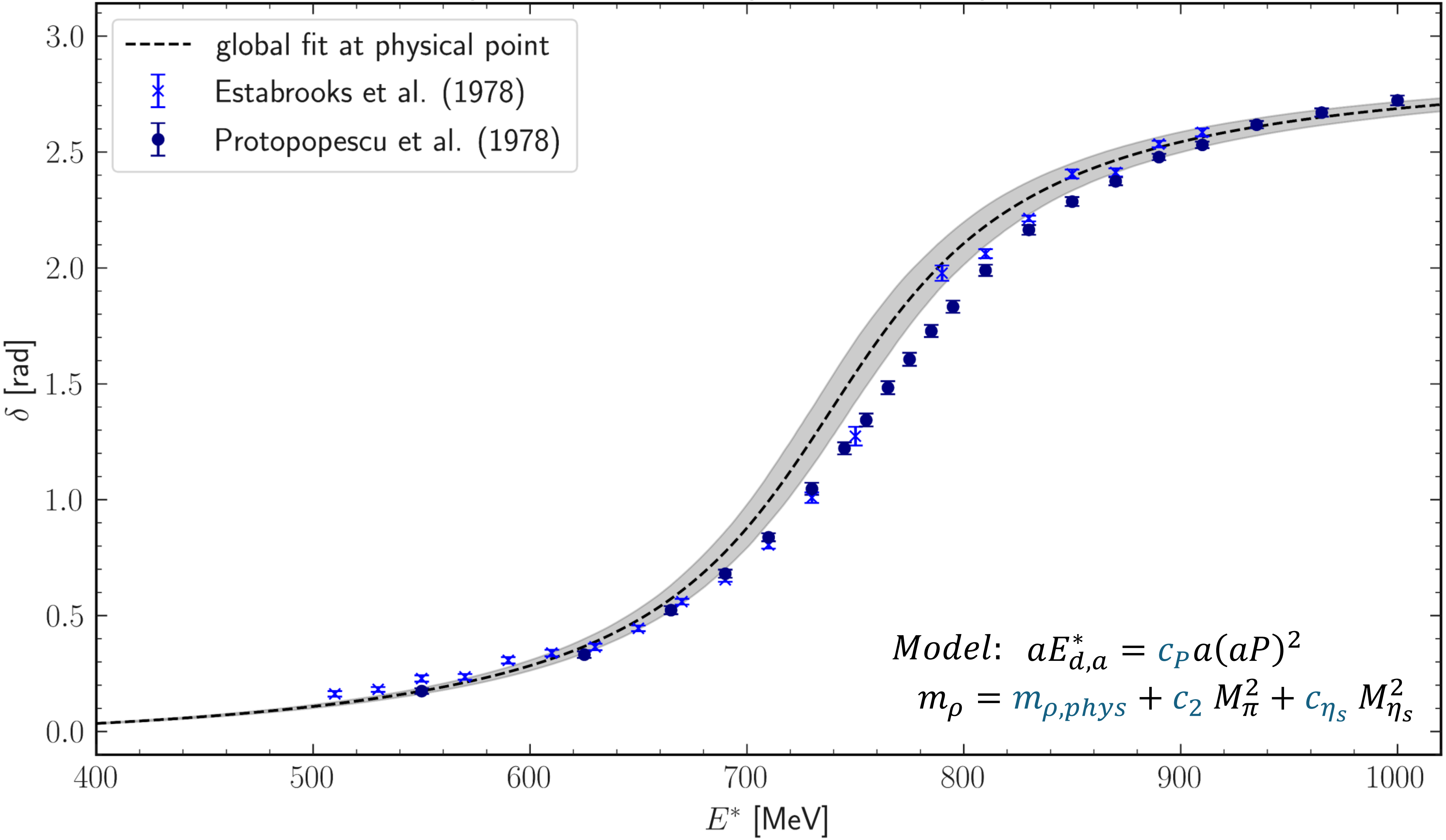
$$aE^*_{d,m} = E^*_T\left[c_P (aP)^2 + c_k (ak)^2 + c_C a^2 \Lambda^2_{QCD}\right]$$

$Model$: $aE^*_{d,a} = c_P a(aP)^2$

$m_\rho = m_{\rho,phys} + c_2\, M_\pi^2 + c_{\eta_s}\, M_{\eta_s}^2$



Global fit phase shift in comparison with experimental data

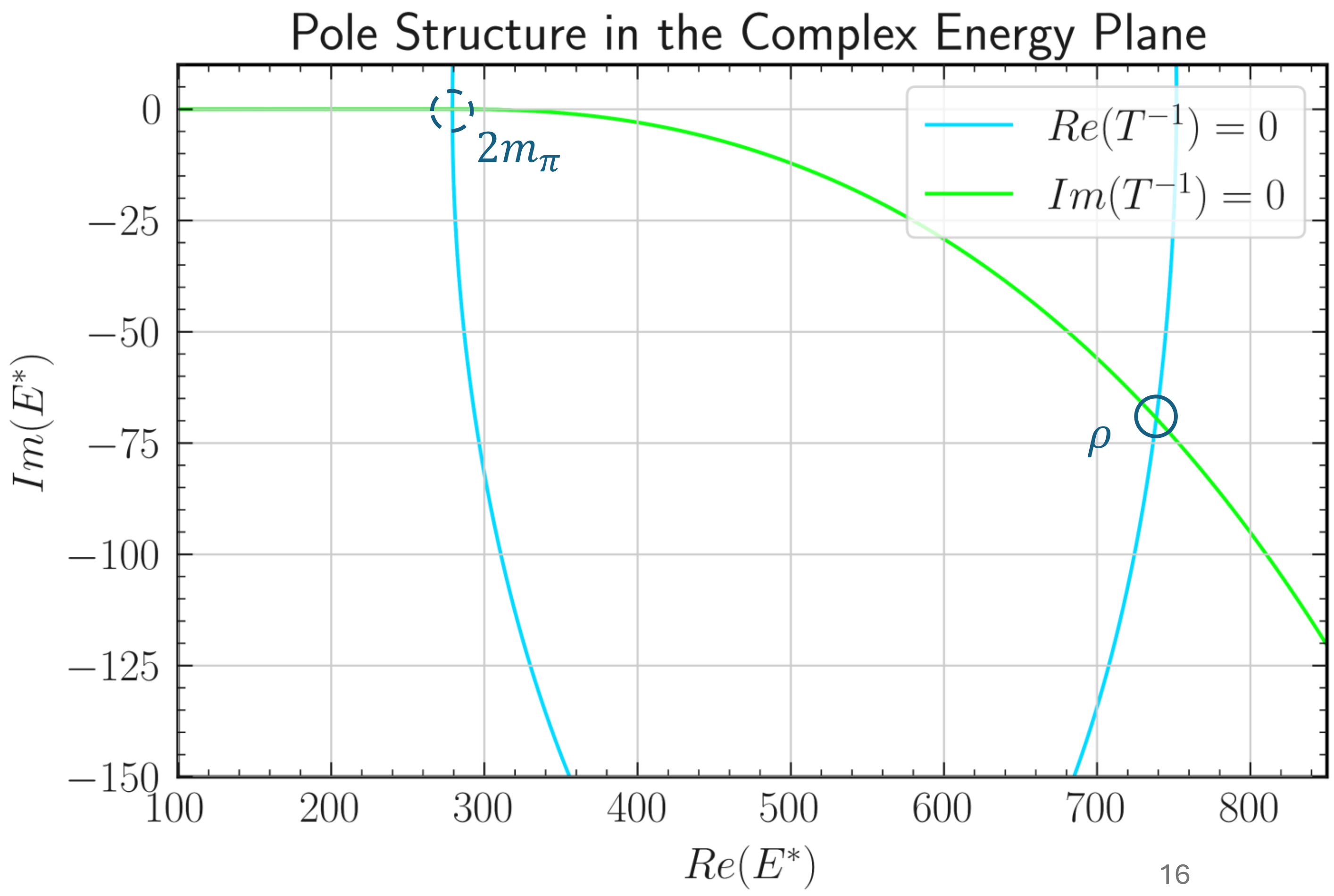


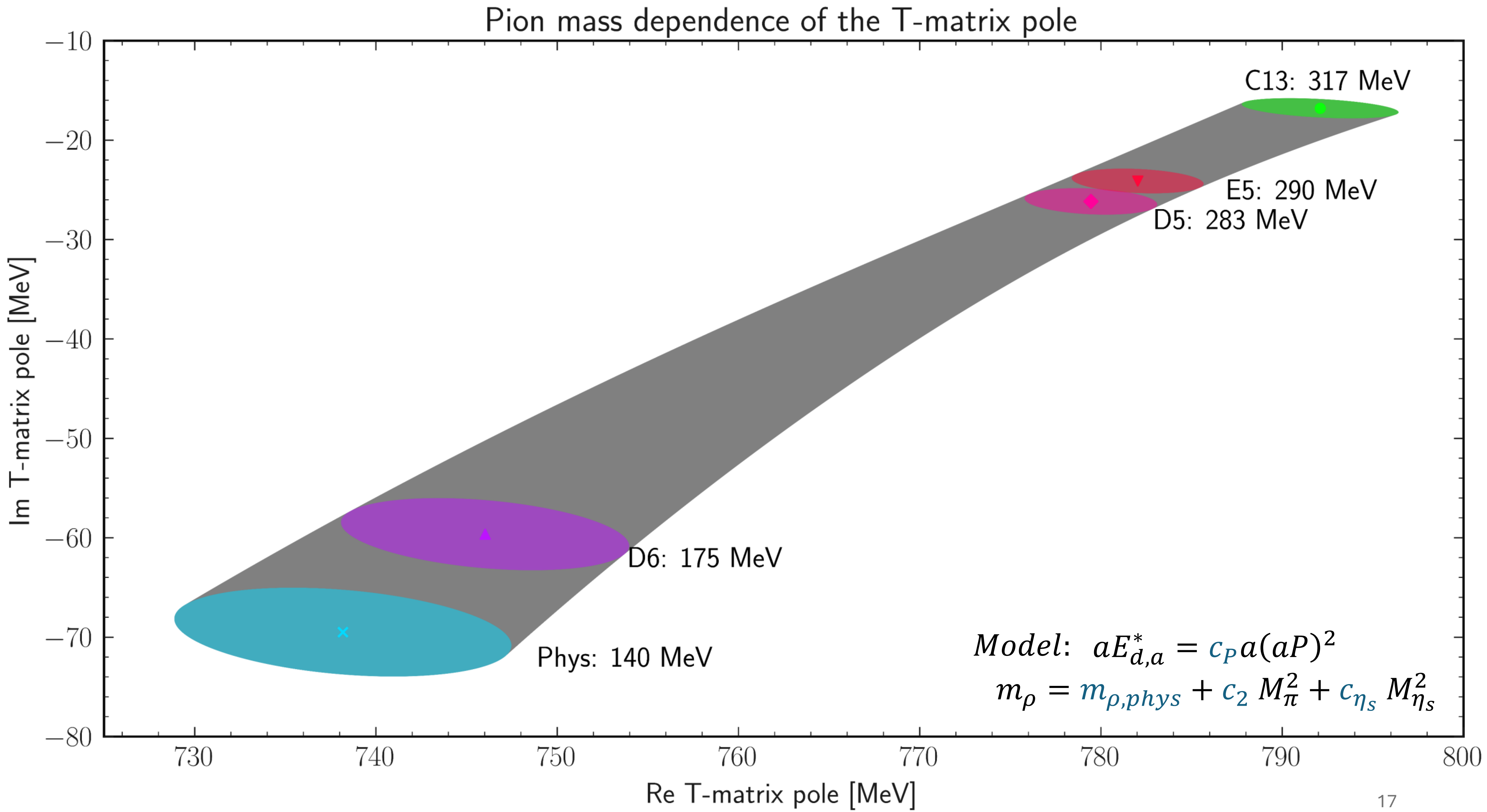
$$Model:\;\; aE^*_{d,a} = c_P\, a(aP)^2$$

$$m_\rho = m_{\rho,phys} + c_2\, M_\pi^2 + c_{\eta_s}\, M_{\eta_s}^2$$

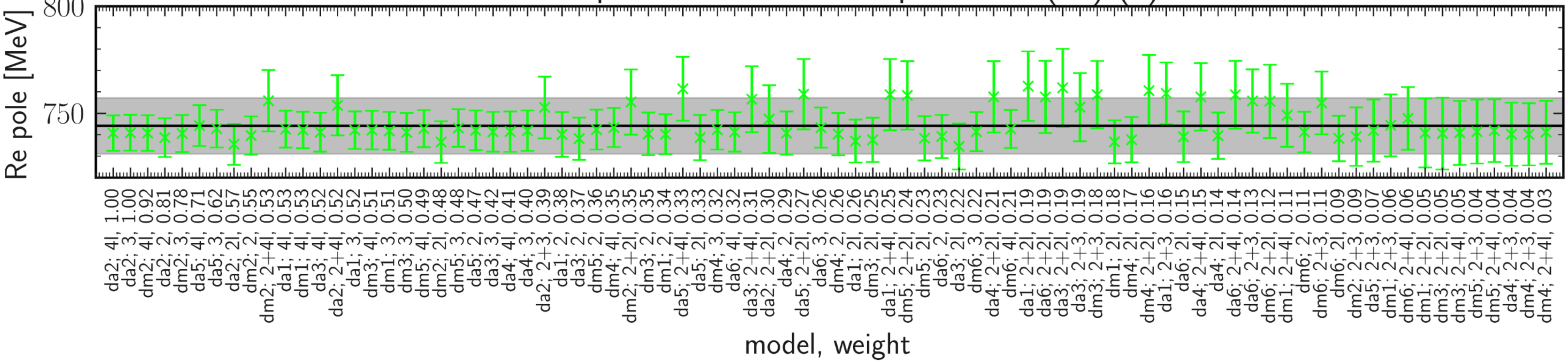
T-matrix pole

$$T^{-1}(E^*) = K^{-1}(E^*) - i\,\rho(E^*) = 0$$

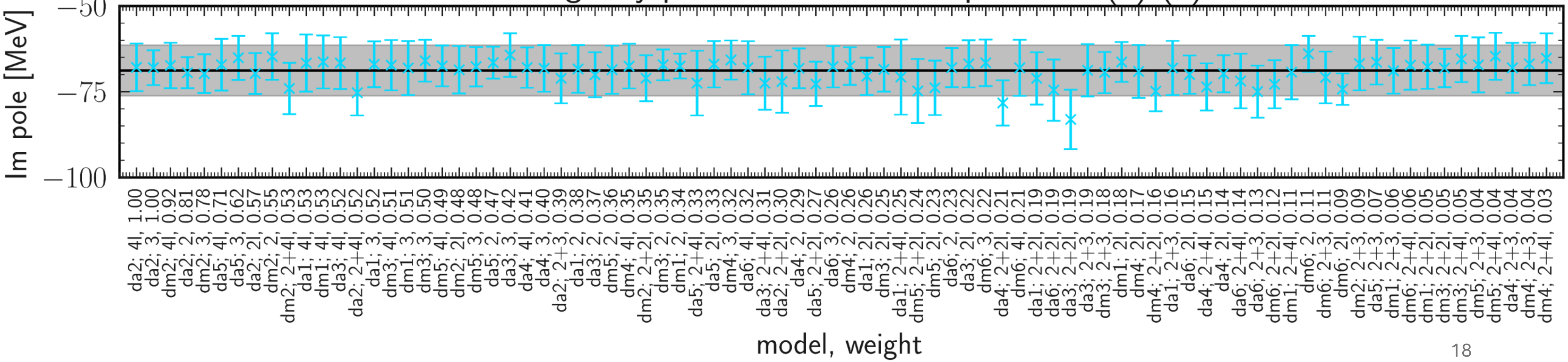


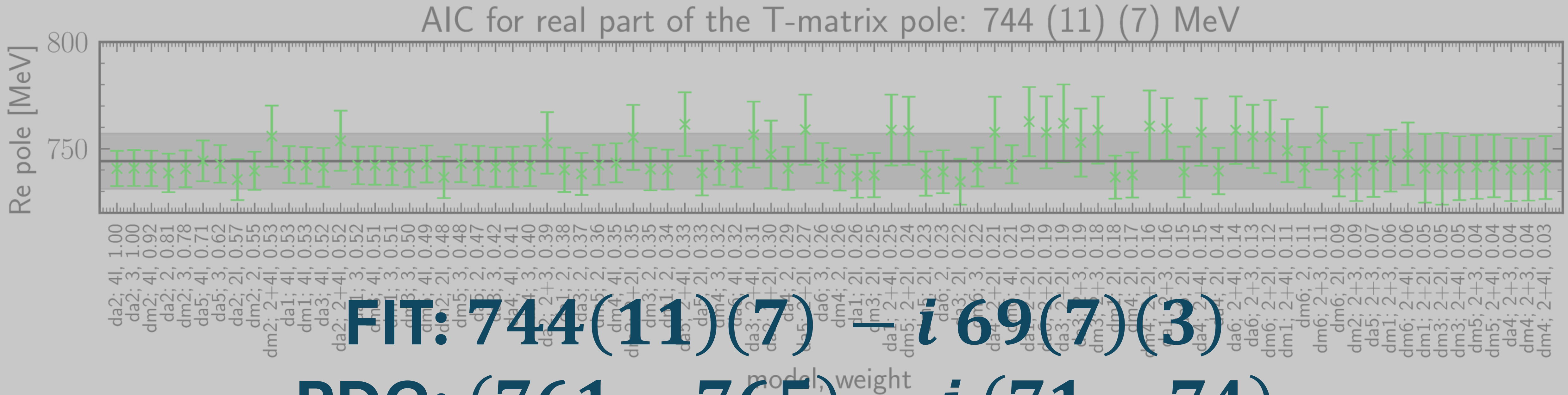

Pion mass dependence of the T-matrix pole
C13: 317 MeV
E5: 290 MeV
D5: 283 MeV
D6: 175 MeV
Phys: 140 MeV
Im T-matrix pole [MeV]
Re T-matrix pole [MeV]
−10
−20
−30
−40
−50
−60
−70
−80
730
740
750
760
770
780
790
800
$Model$: $aE^*_{d,a} = c_P a(aP)^2$
$m_\rho = m_{\rho,phys} + c_2\, M_\pi^2 + c_{\eta_s}\, M_{\eta_s}^2$

AIC for real part of the T-matrix pole: 744 (11) (7) MeV



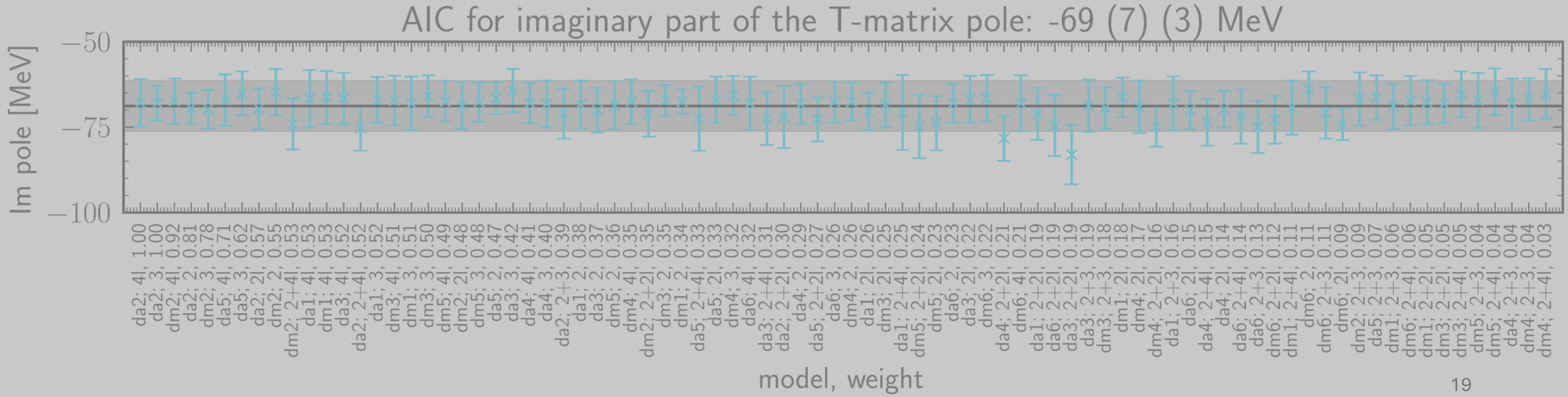
AIC for imaginary part of the T-matrix pole: -69 (7) (3) MeV





FIT: $744(11)(7) - i\,69(7)(3)$

PDG: $(761 - 765) - i\,(71 - 74)$



OUTLOOK

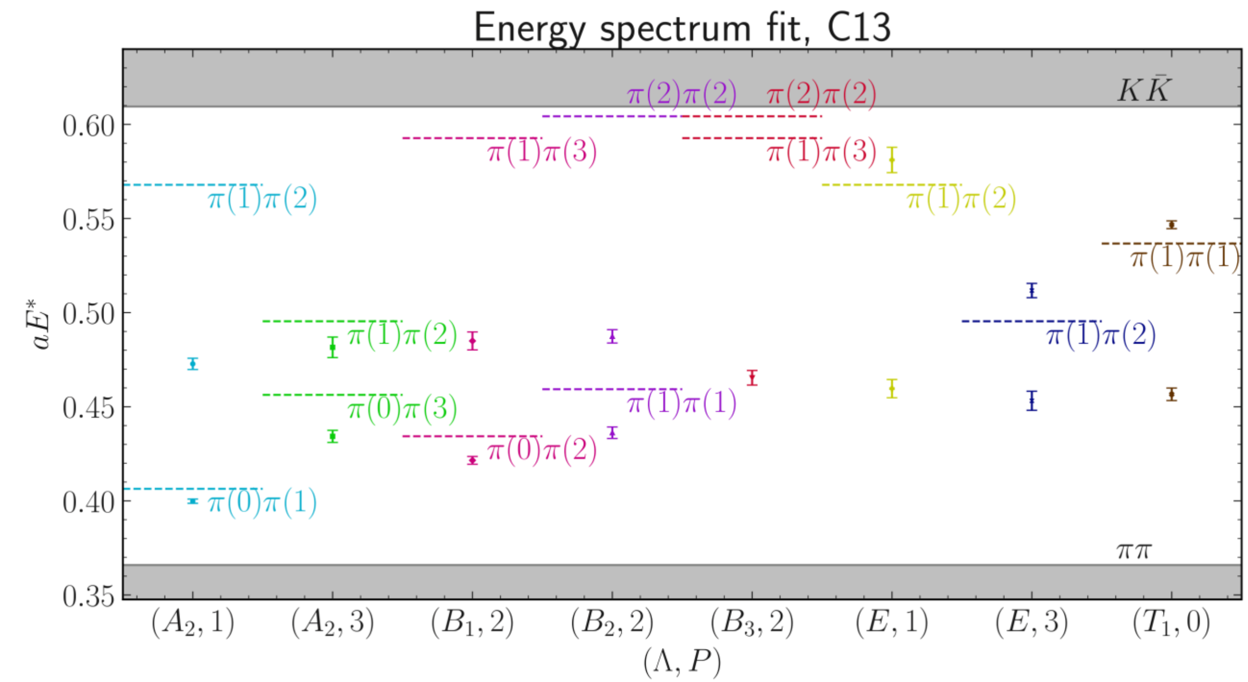
Same approach to:

- Other resonances
- Transitions

	C13	D5	D6	E5
$N_s^3 \times N_t$	$32^3 \times 96$	$32^3 \times 64$	$48^3 \times 96$	$48^3 \times 128$
a [fm]	0.11403(77)	0.08990(82)	0.08766(79)	0.06827(55)
L [fm]	3.649(25)	2.877(26)	4.208(38)	3.277(26)
m_π [MeV]	316.6(2.2)	282.8(2.8)	174.7(1.8)	289.9(2.4)
m_K [MeV]	527.4(3.6)	536.3(4.9)	513.4(4.6)	526.3(4.3)
m_{η_s} [MeV]	671.8(5.6)	702.8(6.4)	702.8(6.3)	821.5(6.7)
N_{config}	1039	467	621	297
$N_{sources}$	8	16	16	16

$$m_{phys,\eta_s} = 688.5 \text{ MeV}$$

η_s: Dowdall et al., arXiv:1110.6887


Energy spectrum fit, C13
aE^*
0.35
0.40
0.45
0.50
0.55
0.60
$K\bar{K}$
$\pi\pi$
$\pi(\bar{1})\pi(2)$
$\pi(\bar{0})\pi(1)$
$\pi(\bar{1})\pi(2)$
$\pi(\bar{0})\pi(3)$
$\pi(\bar{1})\pi(3)$
$\pi(\bar{0})\pi(2)$
$\pi(2)\pi(2)$
$\pi(\bar{1})\pi(1)$
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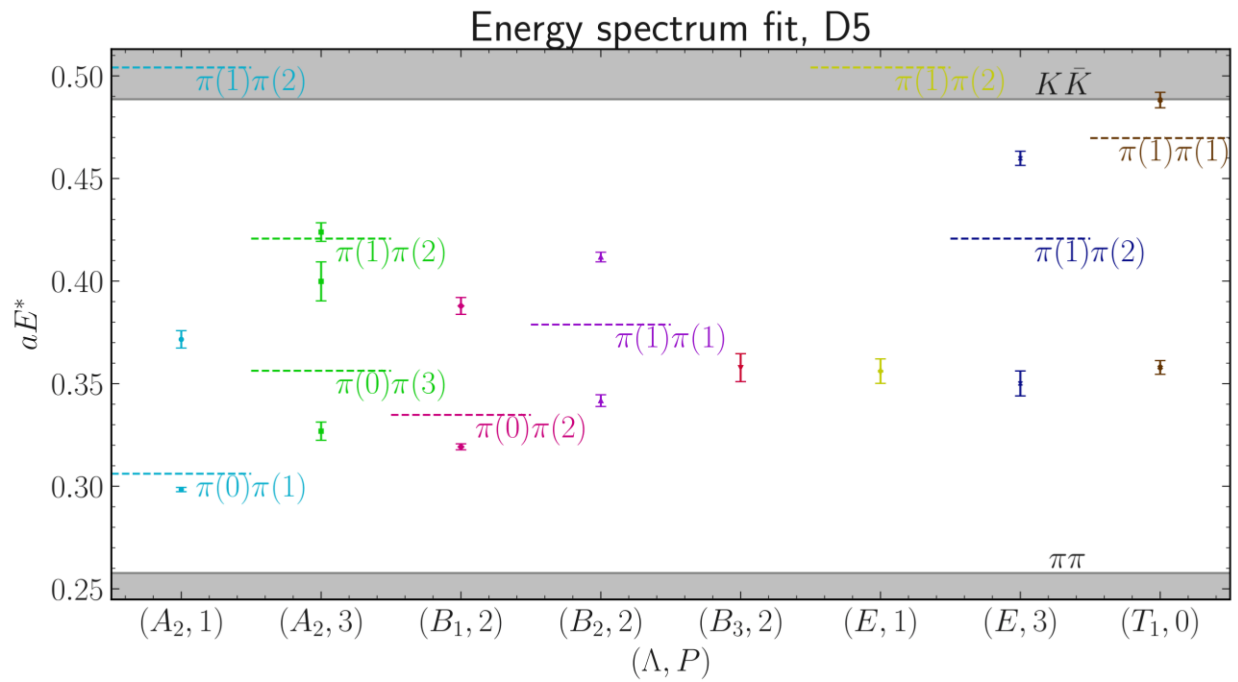
$(A_2,1)$ $(A_2,3)$ $(B_1,2)$ $(B_2,2)$ $(B_3,2)$ $(E,1)$ $(E,3)$ $(T_1,0)$
(Λ,P)


Energy spectrum fit, D5
aE^*
0.25
0.30
0.35
0.40
0.45
0.50
$K\bar{K}$
$\pi\pi$
$\pi(\bar{1})\pi(2)$
$\pi(\bar{0})\pi(1)$
$\pi(\bar{1})\pi(2)$
$\pi(\bar{0})\pi(3)$
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$\pi(\bar{1})\pi(1)$
$\pi(\bar{1})\pi(2)$
$\pi(\bar{1})\pi(2)$
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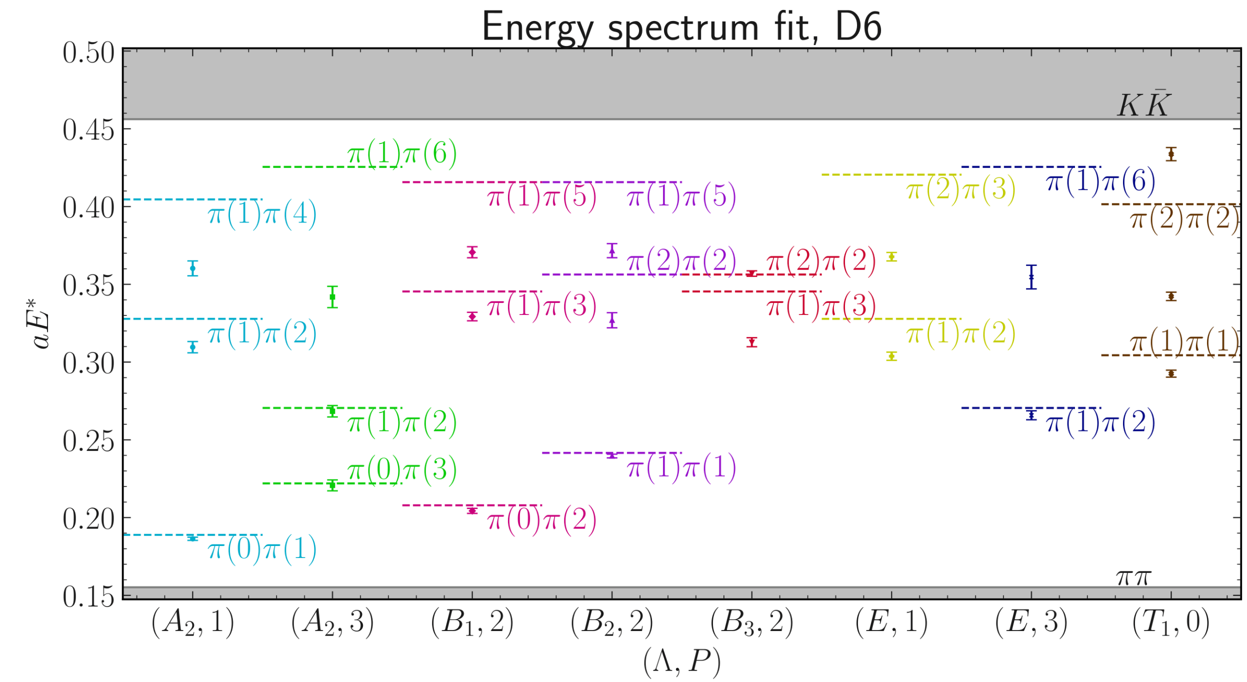
$(A_2,1)$ $(A_2,3)$ $(B_1,2)$ $(B_2,2)$ $(B_3,2)$ $(E,1)$ $(E,3)$ $(T_1,0)$
(Λ,P)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$\pi(\bar{1})\pi(2)$
$\pi(\bar{0})\pi(3)$
$\pi(\bar{1})\pi(5)$
$\pi(\bar{1})\pi(3)$
$\pi(\bar{0})\pi(2)$
$\pi(\bar{1})\pi(5)$
$\pi(\bar{2})\pi(2)$
$\pi(\bar{1})\pi(1)$
$\pi(\bar{2})\pi(2)$
$\pi(\bar{1})\pi(3)$
$\pi(\bar{2})\pi(3)$
$\pi(\bar{1})\pi(2)$
$\pi(\bar{1})\pi(6)$
$\pi(\bar{1})\pi(2)$
$\pi(2)\pi(2)$
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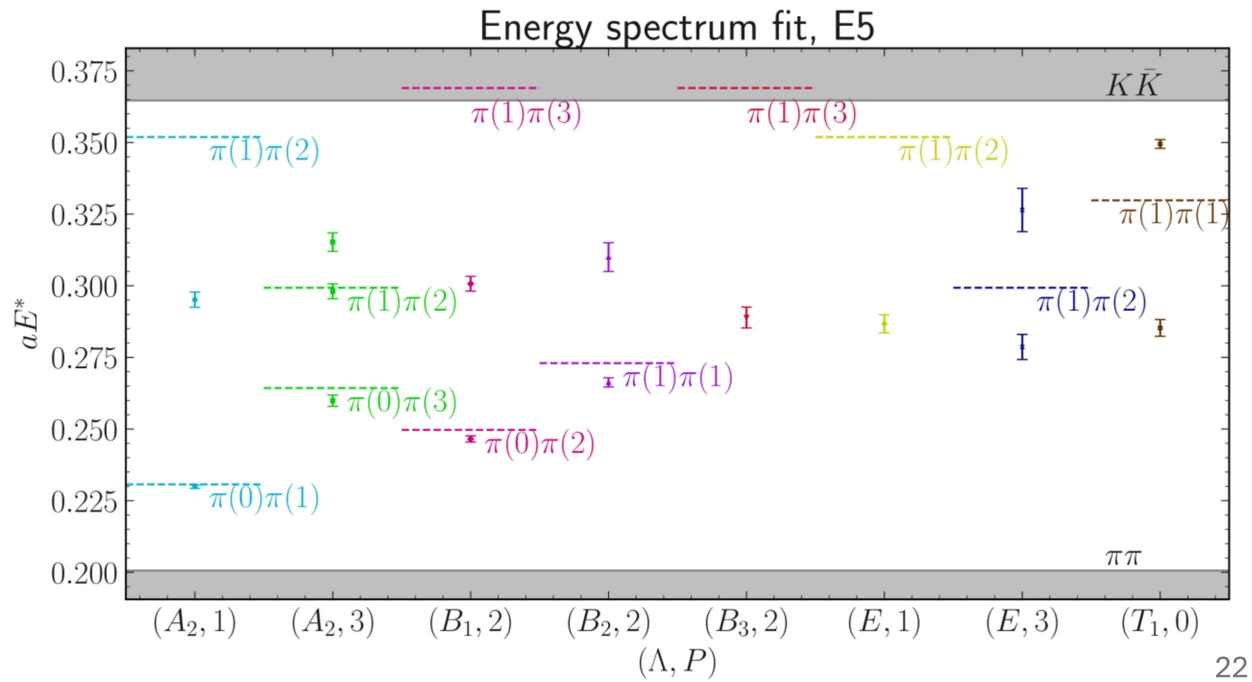
$(A_2,1)$ $(A_2,3)$ $(B_1,2)$ $(B_2,2)$ $(B_3,2)$ $(E,1)$ $(E,3)$ $(T_1,0)$
(Λ,P)


Energy spectrum fit, E5
aE^*
0.200
0.225
0.250
0.275
0.300
0.325
0.350
0.375
$K\bar{K}$
$\pi\pi$
$\pi(\bar{1})\pi(2)$
$\pi(\bar{0})\pi(1)$
$\pi(\bar{1})\pi(2)$
$\pi(\bar{0})\pi(3)$
$\pi(1)\pi(3)$
$\pi(\bar{0})\pi(2)$
$\pi(\bar{1})\pi(1)$
$\pi(1)\pi(3)$
$\pi(\bar{1})\pi(2)$
$\pi(\bar{1})\pi(2)$
$\pi(\bar{1})\pi(1)$
$(A_2,1)$ $(A_2,3)$ $(B_1,2)$ $(B_2,2)$ $(B_3,2)$ $(E,1)$ $(E,3)$ $(T_1,0)$
(Λ,P)