The isospin violating part of the hadronic vacuum polarization

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29th July 2026

Lattice conference 2026

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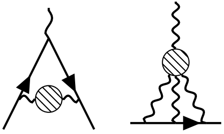
Motivation

Current experimental average for $(g-2)_\mu$ [Phys.Rev.Lett. 135 (2025) 10, 101802]:

$$a_\mu^{\mathrm{exp}} = 116\ 592\ 071.5(14.5) \times 10^{-11}$$

Current Standard Model prediction from WP 2025 [Phys.Rept. 1143 (2025) 1-158]:

Contribution	
QED	116 584 718.8(2)
EW	154.4(4)
HVP (LO + NLO + NNLO)	7045(61)
HLbL (phenomenology + lattice + NLO)	115.5(9.9)
Total SM Value	116 592 033(62)



a) HVP b) HLbL

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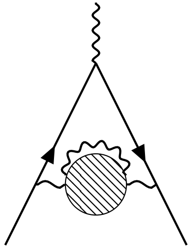

a) EM correction to HVP

QED on the lattice

- Approach 1: QCD + QED ensembles
- Approach 2: Perturbative method (pioneered in [1303.4896])
  - Expand correlation functions around isosymmetric point in powers of α
  - Specifics of the implementation can vary drastically

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  - Expand correlation functions around isosymmetric point in powers of α
  - Specifics of the implementation can vary drastically
  - Our calculation uses infinite-volume photon propagator with a double Pauli-Villars regularization (QED_∞)

Doubly Pauli-Villars-regulated photon propagator [JHEP 03 (2023) 194]

$$[\mathcal{G}(x)]_\Lambda = \frac{1}{4\pi^2|x|^2} - 2G^{(1)}_{\frac{\Lambda}{\sqrt{2}}}(x) + G^{(1)}_{\Lambda}(x)$$
$$G^{(1)}_m(x) = \frac{mK_1\left(m|x|\right)}{4\pi^2|x|}$$

- Recently validated by calculating electromagnetic pion mass splitting [2605.29902]
- Introduces additional scale to calculation
  $\rightarrow$ Limit $\Lambda \rightarrow \infty$ necessary
- PV-mass should be smaller than inverse lattice spacing

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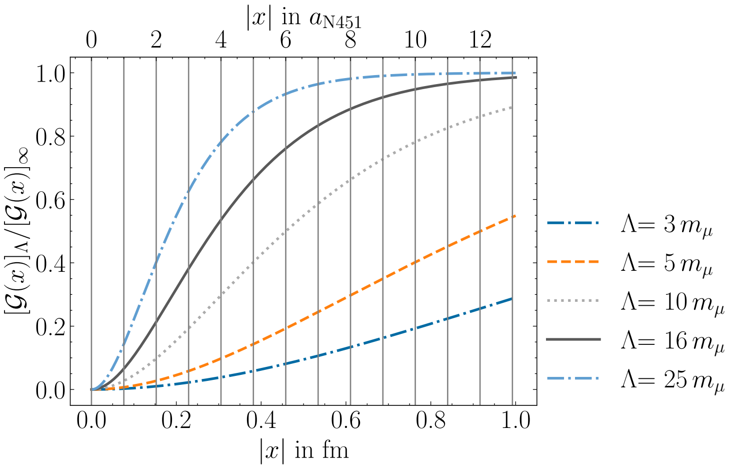

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$$
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- Allows for crosschecks of 'gluonless' ensembles with continuum calculations

a_μ^{HVP} from CCS [1706.01139]

$$a_\mu^{\mathrm{HVP}} = \int_z H_{\lambda\sigma}(z) G_{\lambda\sigma}(z), \qquad H_{\lambda\sigma}(z) = -\delta_{\lambda\sigma}\mathcal{H}_1(|z|) + \frac{z_\lambda z_\sigma}{|z|^2}\mathcal{H}_2(|z|) + \partial_\lambda\left(z_\sigma f(|z|)\right)$$

$H_{\lambda\sigma}(z)$: Covariant coordinate-space (CCS) kernel (not unique)

$G_{\lambda\sigma}(z) = \langle j_\lambda^{em}(z) j_\sigma^{em}(0)\rangle_{\mathrm{QCD+QED}}$: vector-vector correlator of the e.m. vector current

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Use decomposition of em current, $j_\mu^{em}(x) = j_\mu^3(x) + j_\mu^8(x)$, and focus on 38 part

LO is exactly zero, only NLO survives $\rightarrow$ "The total isospin-violating part of a_μ^{HVP}"

The isospin-violating part of a_μ^{HVP}

$$a_\mu^{\text{HVP},38} = -\frac{e^2}{2}\int_{x,y,z} H_{\lambda\sigma}(z)\delta_{\mu\nu}[\mathcal{G}(x-y)]_\Lambda \langle j_\lambda^3(z) j_\mu^{em}(x) j_\nu^{em}(y) j_\sigma^8(0)\rangle + C_T(\Lambda)$$

'Traceless' version of CCS Kernel:

$$H_{\lambda\sigma}(z) = \left(-\delta_{\lambda\sigma} + 4\frac{z_\lambda z_\sigma}{z^2}\right)\mathcal{H}_2(|z|)$$

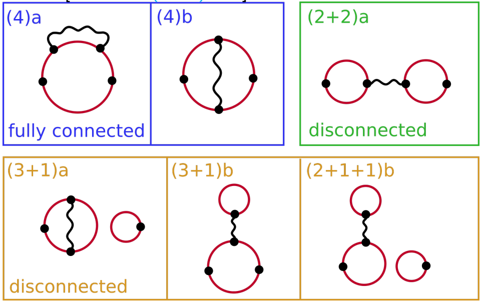
PV-regulated photon propagator :

$$[\mathcal{G}(x)]_\Lambda = \frac{1}{4\pi^2|x|^2} - 2G^{(1)}_{\frac{\Lambda}{\sqrt{2}}}(x) + G^{(1)}_\Lambda(x)$$

$$G^{(1)}_m(x) = \frac{mK_1(m|x|)}{4\pi^2|x|}$$

[JHEP 10 (2025) 157]
[JHEP 07 (2026) 068]

[JHEP 07 (2025) 201]



Conventions

$$a_\mu^{\mathrm{HVP},38} = -\frac{e^2}{2}\int_{x,y,z} H_{\lambda\sigma}(z)\delta_{\mu\nu}\left[\mathcal{G}(x-y)\right]_\Lambda \left\langle j_\lambda^3(z) j_\mu^{em}(x) j_\nu^{em}(y) j_\sigma^8(0)\right\rangle + C_T(\Lambda)$$

$$j_\lambda^Q(x) = \bar{\psi}(x)\gamma_\lambda \mathcal{Q}^{(Q)}\psi(x), \qquad \mathcal{Q}^{(em)} = \mathrm{diag}(2/3, -1/3, -1/3),$$
$$\mathcal{Q}^{(3)} = \mathrm{diag}(1/2, -1/2, 0), \qquad \mathcal{Q}^{(8)} = \mathrm{diag}(1/6, 1/6, -1/3)$$

Notation: XdYdZd, d$\in$\{l, c\} , l: local, c: conserved

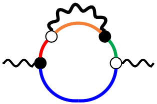
The two versions of the photon propagator

Problem: We need to avoid Volume^2 sum.

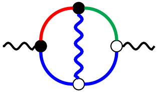
Solution 1:

2-point source method (2PS)

$$a_{\mu,em}^{\mathrm{HVP},38}(\Lambda) = \pi^2 e^2 \int_0^\infty d|y|\,|y|^3\, a^8 \sum_{x,z} H_{\lambda\sigma}(z)\,\delta_{\nu\rho}\,[\mathcal{G}(x-y)]_\Lambda \langle j^3_\lambda(z) j^{em}_\nu(y) j^{em}_\rho(x) j^8_\sigma(0)\rangle$$



a) (4)a



b) (4)b

Black dots $\hat{=}$ Point-source

White dots $\hat{=}$ Sum over lattice

The two versions of the photon propagator

Solution 2:

5D propagator method [JHEP 07 (2026) 068]

$$G_m^{(1)}(x-y) = 4\int d^4w\; G_m^{(\frac{3}{2})}(w-y)\, G_m^{(\frac{3}{2})}(x-w), \qquad G_m^{(\frac{3}{2})}(w-x) = e^{-m|w-x|}\frac{m|w-x|+1}{8\pi^2|w-x|^3}$$

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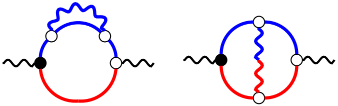
$$a_{\mu,em}^{\mathrm{HVP},38}(\Lambda) = -\frac{e^2}{2}\, 4\int d^4w \; a^{12}\sum_{x,y,z} H_{\lambda\sigma}(z)\delta_{\nu\rho}\; G_m^{(\frac{3}{2})}(w-y)\, G_m^{(\frac{3}{2})}(x-w)\, \langle j_\lambda^3(z) j_\nu^{em}(y) j_\rho^{em}(x) j_\sigma^8(0)\rangle$$



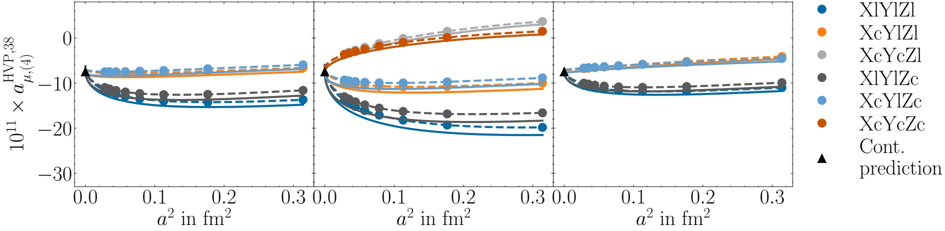
a) (4)a   b) (4)b

Black dots $\hat{=}$ Point-source

White dots $\hat{=}$ Sum over lattice

Crosscheck in QED with gluonless ensembles ((4)a + (4)b)

Fit function: $f_{fit}(a) = c_0 + c_1\, a + c_2\, a^2 + c_3\, a^3$, PV-mass: $\Lambda = 3\, m_\mu$



a) 2PS b) 5D Propagator c) Mixed

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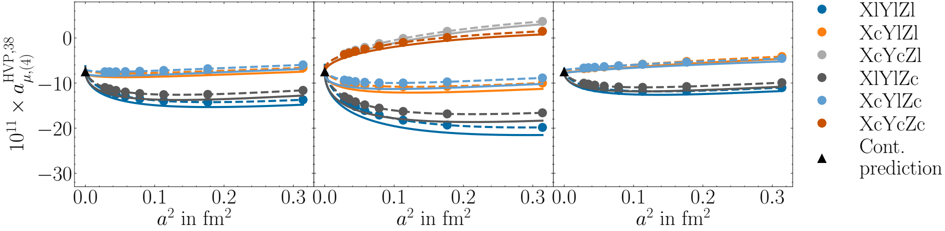


a) 2PS b) 5D Propagator c) Mixed

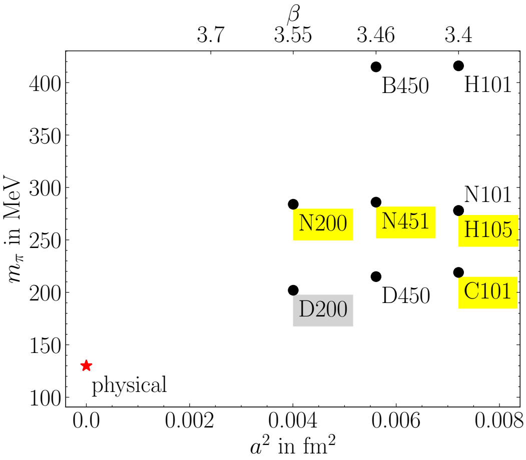
Table: The values are given in units of 10^{-11}. The expected value is -7.50×10^{-11}.

	XlYlZl	XcYlZl	XcYcZl	XlYlZc	XcYlZc	XcYcZc
2PS	-7.11	-7.68	-	-7.40	-7.74	-
5D propagator	-7.40	-7.65	-7.54	-7.67	-7.82	-7.58
Mixed	-7.29	-7.70	-	-7.57	-7.80	-

CLS Ensembles

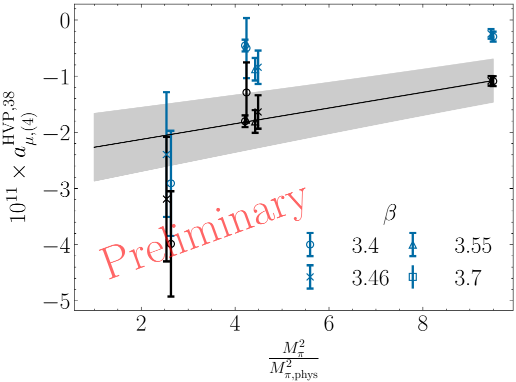
- $N_f = 2 + 1$ non-perturbatively $\mathcal{O}(a)$-improved Wilson Fermions

- White: Only connected diagrams
- Yellow: Connected and disconnected diagrams
- Gray: in the works

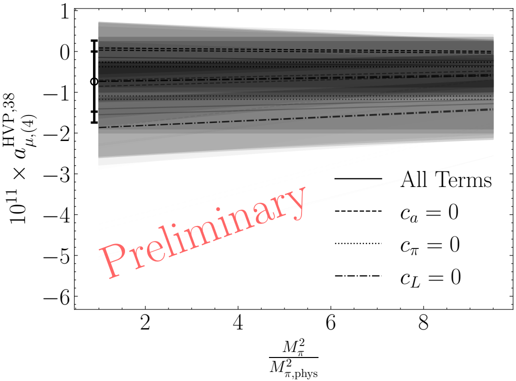


QCD extrapolation to the physical point ((4)a + (4)b)

Fit function: $f_{fit}(a, M_\pi, M_\pi L) = c_0 + c_a a^2 + c_\pi M_\pi^2 + c_L e^{-\frac{M_\pi L}{2}}$, PV-mass: $\Lambda = 16\, m_\mu$



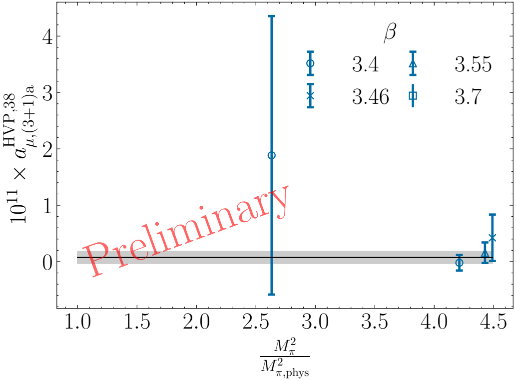
a) Example model: All terms, XcYlZl, TL, $\chi^2/DOF = 1.62$.



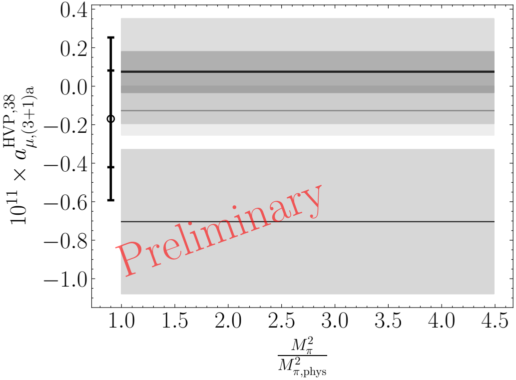
b) Model average using AIC. The resulting value is $a_{\mu,(4)}^{\mathrm{HVP},38}(\Lambda = 16\, m_\mu) = -0.74(74)_{stat}(69)_{syst} \times 10^{-11}$.

QCD extrapolation to the physical point ((3+1)a with 2PS)

Fit function: $f_{fit}(a, M_\pi, M_\pi L) = c_0$, PV-mass: $\Lambda = 16\, m_\mu$

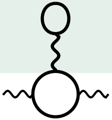


a) Example model: Constant, XcYlZl, TL, $\chi^2/DOF = 0.68$.

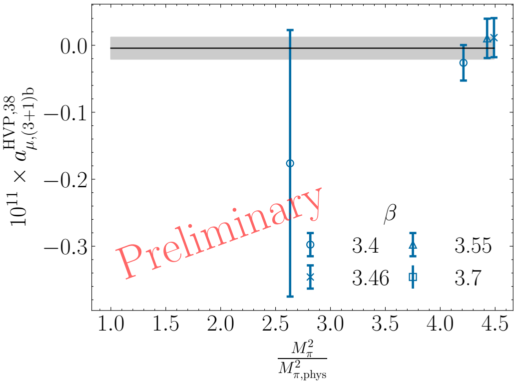


b) Model average using AIC. The resulting value is $a_{\mu,(3+1)\mathrm{a}}^{\mathrm{HVP},38}(\Lambda = 16\, m_\mu) = -0.17(26)_{stat}(34)_{syst} \times 10^{-11}$.

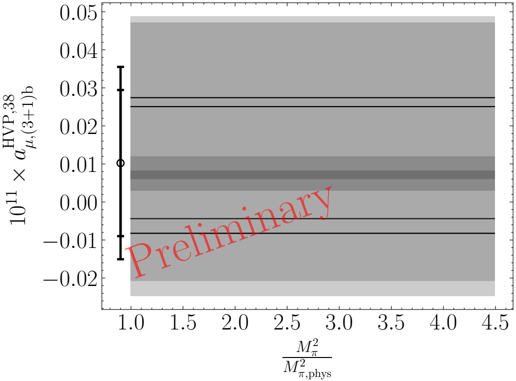
QCD extrapolation to the physical point ((3+1)b with 2PS)



Fit function: $f_{fit}(a, M_\pi, M_\pi L) = c_0$, PV-mass: $\Lambda = \infty$



a) Example model: Constant, XcYlZl, TL, $\chi^2/DOF = 0.54$.



b) Model average using AIC. The resulting value is $a^{\mathrm{HVP},38}_{\mu,(3+1)\mathrm{b}}(\Lambda = \infty) = -0.01(2)_{stat}(2)_{syst} \times 10^{-11}$.

Counterterm

Master formula:

$$a_\mu^{\text{HVP},38} = a_{\mu,em}^{\text{HVP},38}(\Lambda) + \overbrace{\frac{\Delta M_K^{phys} - \Delta M_K^{em}(\Lambda)}{\frac{\partial\, \Delta M_K}{\partial(m_u - m_d)}} \times \frac{\partial a_\mu^{\text{HVP},38}}{\partial(m_u - m_d)}}^{C_T(\Lambda)}$$

$$= a_{\mu,em}^{\text{HVP},38}(\Lambda) + (\Delta M_K^{phys} - \Delta M_K^{em}(\Lambda)) \times R_{38K}$$

- Work on counterterm still in progress

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&= a_{\mu,em}^{\mathrm{HVP},38}(\Lambda) + \left(\Delta M_K^{phys} - \Delta M_K^{em}(\Lambda)\right) \times R_{38K}
\end{aligned}
$$

- Work on counterterm still in progress
- $\Delta M_K^{phys} = -3.934(20)$ MeV
- $\Delta M_K^{em}(\Lambda)$: Talk by Harvey Meyer, Tue. 4:50 pm
- $\frac{\partial\, \Delta M_K}{\partial(m_u - m_d)}$ and $\frac{\partial a_\mu^{\mathrm{HVP},38}}{\partial(m_u - m_d)}$: Talk by Sebastian Lahrtz (right after this talk)

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- $\frac{\partial\, \Delta M_K}{\partial(m_u - m_d)}$ and $\frac{\partial a_\mu^{\mathrm{HVP},38}}{\partial(m_u - m_d)}$: Talk by Sebastian Lahrtz (right after this talk)
- We will use pheno estimate for R_{38K}: $-2.50(63) \times 10^{-11}$ MeV^{-1} [JHEP 10 (2025) 157]

Summary of Results (preliminary)

UV-finite quantities:

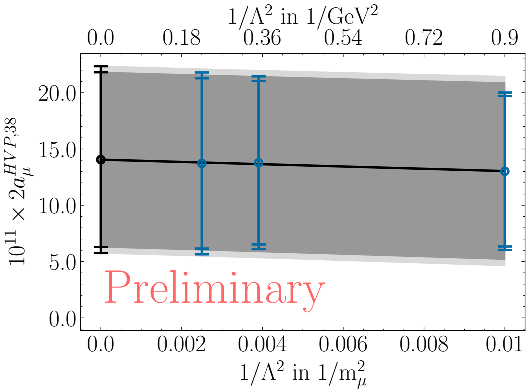
- $R_{38K} = -2.50(63) \times 10^{-11}$ MeV^{-1} (Phenomenological estimate, [JHEP 10 (2025) 157])
- $a^{\mathrm{HVP},38}_{\mu,(3+1)\mathrm{b}} = -0.01(2)_{stat}(2)_{syst} \times 10^{-11}$
- $a^{\mathrm{HVP},38}_{\mu,(2+2)\mathrm{a}} = -5.78(80)_{stat}(90)_{syst} \times 10^{-11}$ [JHEP 07 (2025) 201]

	$\Lambda = 10\ m_\mu$	$\Lambda = 16\ m_\mu$	$\Lambda = 20\ m_\mu$
$10^{11} \times a^{\mathrm{HVP},38}_{\mu,(4)\mathrm{a}}$	$-2.07(39)_{stat}(47)_{syst}$	$-3.28(71)_{stat}(95)_{syst}$	$-4.15(96)_{stat}(1.36)_{syst}$
$10^{11} \times a^{\mathrm{HVP},38}_{\mu,(4)\mathrm{b}}$	$1.73(32)_{stat}(22)_{syst}$	$2.16(43)_{stat}(28)_{syst}$	$2.33(47)_{stat}(32)_{syst}$
$10^{11} \times a^{\mathrm{HVP},38}_{\mu,(4)}$	$-0.33(48)_{stat}(43)_{syst}$	$-0.74(74)_{stat}(69)_{syst}$	$-1.08(90)_{stat}(87)_{syst}$
$10^{11} \times a^{\mathrm{HVP},38}_{\mu,(3+1)\mathrm{a}}$	$-0.03(11)_{stat}(13)_{syst}$	$-0.17(26)_{stat}(34)_{syst}$	$-0.31(38)_{stat}(52)_{syst}$
ΔM_K^{em} in MeV	$1.13(12)_{stat}(10)_{syst}$	$1.50(15)_{stat}(14)_{syst}$	$1.68(17)_{stat}(16)_{syst}$
$10^{11} \times a^{\mathrm{HVP},38}_{\mu}$	$6.51(3.34)_{stat}(1.04)_{syst}$	$6.89(3.63)_{stat}(1.24)_{syst}$	$6.86(3.78)_{stat}(1.42)_{syst}$

Charge factors in backup slides

$\Lambda \to \infty$ (Preliminary)

Linear extrapolation with 100% correlation:



Current estimate based on phenomenological estimate of R_{38K}:
$2a_\mu^{\mathrm{HVP},38} = 14.1(7.8)_{stat}(3.0)_{syst} \times 10^{-11}$

On par with experimental uncertainty:
14.5×10^{-11}

Missing for total isospin-breaking effects:
$a_\mu^{\mathrm{HVP},33}$ and $a_\mu^{\mathrm{HVP},88}$

Outlook

- Fine tune analysis of $a_{\mu,em}^{\mathrm{HVP},38}(\Lambda)$
- Calculate R_{38K} from lattice
- Perform combined extrapolation to physical point and infinite Λ

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Other talks of Mainz group

Connected to this talk:

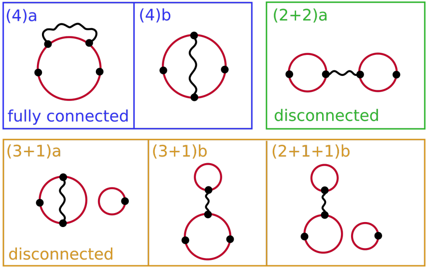
- Sebastian Lahrtz, Wed. 11:30 am
- David Albandea, Wed. 11:50 pm
- Harvey Meyer, Tue. 4:50 pm

Further HVP talks:

- Alessandro Conigli, Wed. 09:00 am
- Arnau Beltran, Wed. 09:20 am
- Alessandro De Santis, Fri. 05:10 pm

Additional Slides

Charge factors



For detailed breakdown of 38 charge factors see JHEP 07 (2026) 068.

1-loops consist of light minus strange. All other loops consist only of light quarks.

	$(4)a$	$(4)b$	$(2+2)a$	$(3+1)a$	$(3+1)b$	$(2+1+1)b$
38	$1/36$	$1/36$	$1/36$	$1/18$	$1/36$	$1/18$
33	$5/36$	$5/36$	$1/4$	0	$1/36$	0
88	$5/324$	$5/324$	$1/324$	$5/162$	$1/324$	$1/162$
Full	$17/81$	$17/81$	$25/81$	$7/81$	$7/81$	$5/81$