

The isospin violating part of the hadronic vacuum polarization

Dominik Erb¹, Harvey Meyer^{1,2,3}

¹PRISMA⁺⁺ Cluster of Excellence &
Institut für Kernphysik, Johannes Gutenberg-Universität Mainz, D-55099 Mainz, Germany

²Helmholtz Institut Mainz, Staudingerweg 18, D-55128 Mainz, Germany

³Theoretical Physics Department, CERN, 1211 Geneva 23, Switzerland

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Lattice conference 2026

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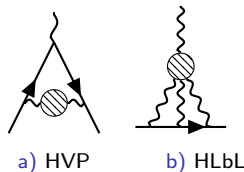
Motivation

Current experimental average for $(g - 2)_\mu$ [[Phys.Rev.Lett. 135 \(2025\) 10, 101802](#)]:

$$a_\mu^{\text{exp}} = 116\,592\,071.5(14.5) \times 10^{-11}$$

Current Standard Model prediction from WP 2025 [[Phys.Rept. 1143 \(2025\) 1-158](#)]:

Contribution	
QED	116 584 718.8(2)
EW	154.4(4)
HVP (LO + NLO + NNLO)	7045(61)
HLbL (phenomenology + lattice + NLO)	115.5(9.9)
Total SM Value	116 592 033(62)



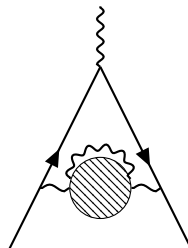
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a) EM correction to HVP

QED on the lattice

- ▶ Approach 1: QCD + QED ensembles
- ▶ Approach 2: Perturbative method (pioneered in [[1303.4896](#)])
 - ▶ Expand correlation functions around isosymmetric point in powers of α
 - ▶ Specifics of the implementation can vary drastically

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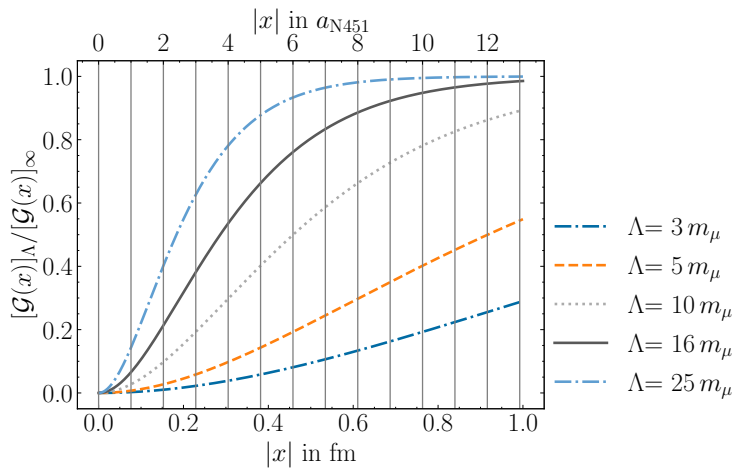
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 - ▶ Specifics of the implementation can vary drastically
 - ▶ Our calculation uses infinite-volume photon propagator with a double Pauli-Villars regularization (QED_∞)

Doubly Pauli-Villars-regulated photon propagator [[JHEP 03 \(2023\) 194](#)]

$$[\mathcal{G}(x)]_\Lambda = \frac{1}{4\pi^2|x|^2} - 2G_{\frac{\Lambda}{\sqrt{2}}}^{(1)}(x) + G_\Lambda^{(1)}(x)$$
$$G_m^{(1)}(x) = \frac{mK_1(m|x|)}{4\pi^2|x|}$$

- ▶ Recently validated by calculating electromagnetic pion mass splitting [[2605.29902](#)]
- ▶ Introduces additional scale to calculation
→ Limit $\Lambda \rightarrow \infty$ necessary
- ▶ PV-mass should be smaller than inverse lattice spacing

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 - Easy comparisons between collaborations possible
- ▶ Allows for crosschecks of 'gluonless' ensembles with continuum calculations

a_μ^{HVP} from CCS [1706.01139]

$$a_\mu^{\text{HVP}} = \int_z H_{\lambda\sigma}(z) G_{\lambda\sigma}(z), \quad H_{\lambda\sigma}(z) = -\delta_{\lambda\sigma} \mathcal{H}_1(|z|) + \frac{z_\lambda z_\sigma}{|z|^2} \mathcal{H}_2(|z|) + \partial_\lambda (z_\sigma f(|z|))$$

$H_{\lambda\sigma}(z)$: Covariant coordinate-space (CCS) kernel (not unique)

$G_{\lambda\sigma}(z) = \langle j_\lambda^{em}(z) j_\sigma^{em}(0) \rangle_{\text{QCD+QED}}$: vector-vector correlator of the e.m. vector current

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Use decomposition of em current, $j_\mu^{em}(x) = j_\mu^3(x) + j_\mu^8(x)$, and focus on 38 part
LO is exactly zero, only NLO survives $\rightarrow$ "The total isospin-violating part of a_μ^{HVP} "

The isospin-violating part of a_μ^{HVP}

$$a_\mu^{\text{HVP},38} = -\frac{e^2}{2} \int_{x,y,z} H_{\lambda\sigma}(z) \delta_{\mu\nu} [\mathcal{G}(x-y)]_\Lambda \langle j_\lambda^3(z) j_\mu^{em}(x) j_\nu^{em}(y) j_\sigma^8(0) \rangle + C_T(\Lambda)$$

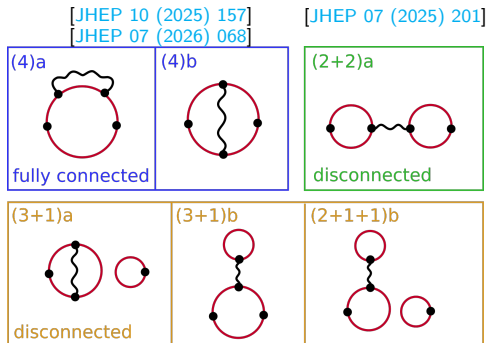
'Traceless' version of CCS Kernel:

$$H_{\lambda\sigma}(z) = \left(-\delta_{\lambda\sigma} + 4 \frac{z_\lambda z_\sigma}{z^2} \right) \mathcal{H}_2(|z|)$$

PV-regulated photon propagator :

$$[\mathcal{G}(x)]_\Lambda = \frac{1}{4\pi^2|x|^2} - 2G_{\frac{\Lambda}{\sqrt{2}}}^{(1)}(x) + G_\Lambda^{(1)}(x)$$

$$G_m^{(1)}(x) = \frac{mK_1(m|x|)}{4\pi^2|x|}$$



Conventions

$$a_{\mu}^{\text{HVP},38} = -\frac{e^2}{2} \int_{x,y,z} H_{\lambda\sigma}(z) \delta_{\mu\nu} [\mathcal{G}(x-y)]_{\Lambda} \langle j_{\lambda}^3(z) j_{\mu}^{em}(x) j_{\nu}^{em}(y) j_{\sigma}^8(0) \rangle + C_T(\Lambda)$$

$$j_{\lambda}^Q(x) = \bar{\psi}(x) \gamma_{\lambda} Q^{(Q)} \psi(x), \quad Q^{(em)} = \text{diag}(2/3, -1/3, -1/3), \\ Q^{(3)} = \text{diag}(1/2, -1/2, 0), \quad Q^{(8)} = \text{diag}(1/6, 1/6, -1/3)$$

Notation: $X_d Y_d Z_d$, $d \in \{l, c\}$, l : local, c : conserved

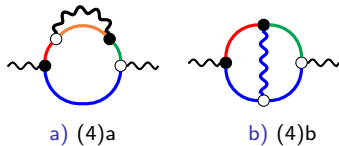
The two versions of the photon propagator

Problem: We need to avoid Volume^2 sum.

Solution 1:

2-point source method (2PS)

$$a_{\mu,em}^{\text{HVP},38}(\Lambda) = \pi^2 e^2 \int_0^\infty d|y| |y|^3 a^8 \sum_{x,z} H_{\lambda\sigma}(z) \delta_{\nu\rho} [\mathcal{G}(x-y)]_\Lambda \langle j_\lambda^3(z) j_\nu^{em}(y) j_\rho^{em}(x) j_\sigma^8(0) \rangle$$



Black dots $\hat{=}$ Point-source

White dots $\hat{=}$ Sum over lattice

The two versions of the photon propagator

Solution 2:

5D propagator method

$$G_m^{(1)}(x-y) = 4 \int d^4 w \, G_m^{(\frac{3}{2})}(w-y) G_m^{(\frac{3}{2})}(x-w), \quad G_m^{(\frac{3}{2})}(w-x) = e^{-m|w-x|} \frac{m|w-x| + 1}{8\pi^2 |w-x|^3}$$

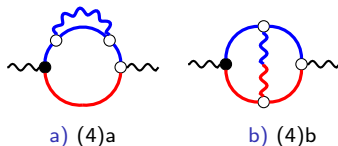
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$$a_{\mu,em}^{\text{HVP},38}(\Lambda) = -\frac{e^2}{2} 4 \int d^4 w a^{12} \sum_{x,y,z} H_{\lambda\sigma}(z) \delta_{\nu\rho} G_m^{(\frac{3}{2})}(w-y) G_m^{(\frac{3}{2})}(x-w) \langle j_\lambda^3(z) j_\nu^{em}(y) j_\rho^{em}(x) j_\sigma^8(0) \rangle$$

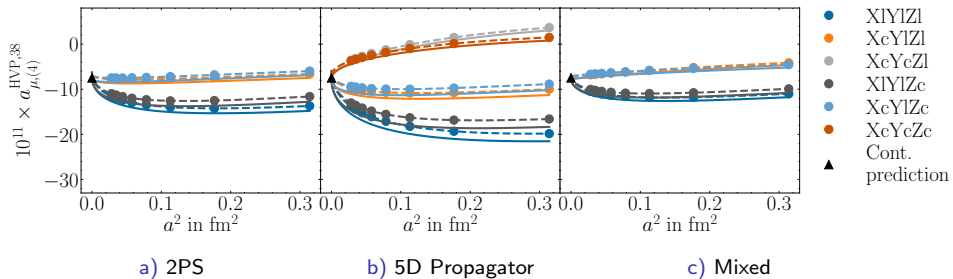


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Crosscheck in QED with gluonless ensembles ((4)a + (4)b)

Fit function: $f_{fit}(a) = c_0 + c_1 a + c_2 a^2 + c_3 a^3$, PV-mass: $\Lambda = 3 m_\mu$



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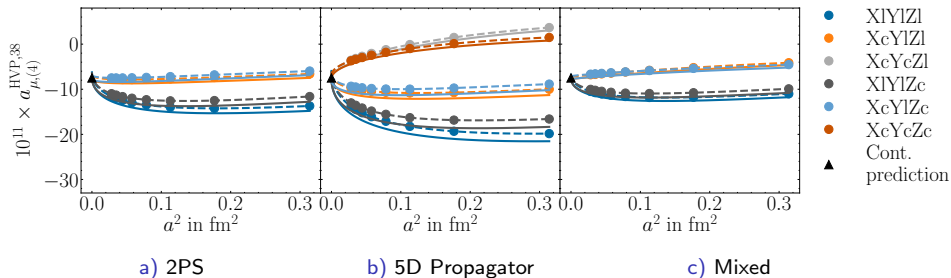
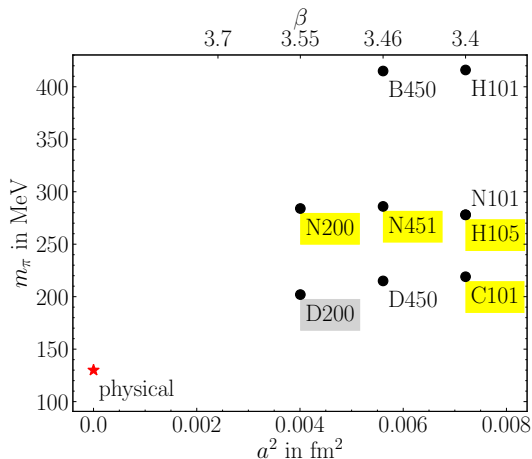


Table: The values are given in units of 10^{-11} . The expected value is -7.50×10^{-11} .

	XIYZI	XcYIZI	XcYcZI	XIYZc	XcYIZc	XcYcZc
2PS	-7.11	-7.68	-	-7.40	-7.74	-
5D propagator	-7.40	-7.65	-7.54	-7.67	-7.82	-7.58
Mixed	-7.29	-7.70	-	-7.57	-7.80	-

CLS Ensembles

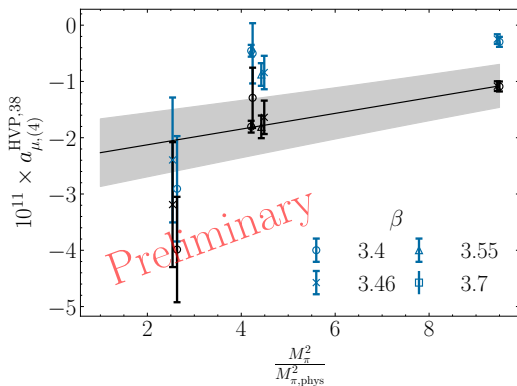
- ▶ $N_f = 2 + 1$ non-perturbatively $\mathcal{O}(a)$ -improved Wilson Fermions
- ▶ White: Only connected diagrams
- ▶ Yellow: Connected and disconnected diagrams
- ▶ Gray: in the works



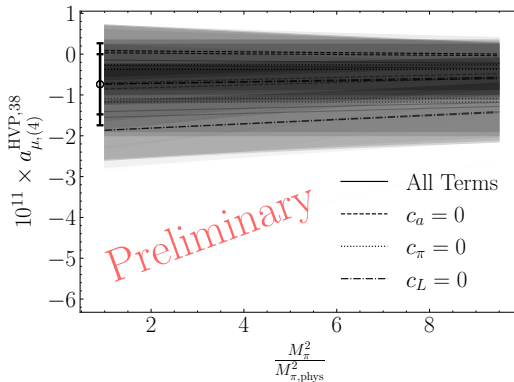
QCD extrapolation to the physical point ((4)a + (4)b)



Fit function: $f_{\text{fit}}(a, M_\pi, M_\pi L) = c_0 + c_a a^2 + c_\pi M_\pi^2 + c_L e^{-\frac{M_\pi L}{2}}$, PV-mass: $\Lambda = 16 m_\mu$

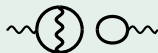


a) Example model: All terms, XcYIZI, TL,
 $\chi^2/\text{DOF} = 1.62$.

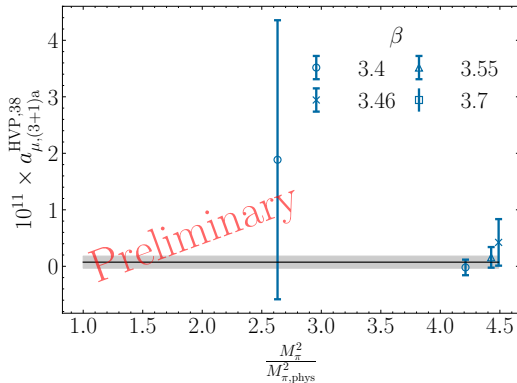


b) Model average using AIC. The resulting value is
 $a_{\mu,(4)}^{\text{HVP},38}(\Lambda = 16 m_\mu) = -0.74(74)_{\text{stat}}(69)_{\text{syst}} \times 10^{-11}$.

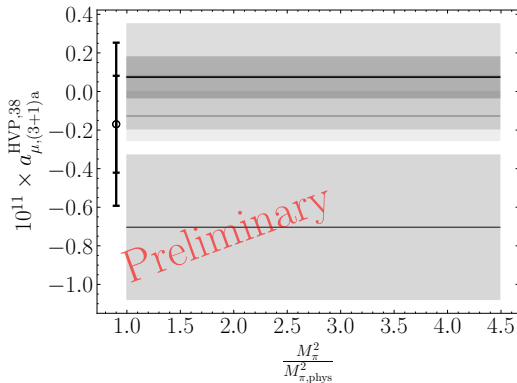
QCD extrapolation to the physical point ((3+1)a with 2PS)



Fit function: $f_{\text{fit}}(a, M_\pi, M_\pi L) = c_0$, PV-mass: $\Lambda = 16 m_\mu$

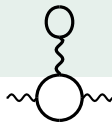


a) Example model: Constant, XcYIZI, TL,
 $\chi^2/\text{DOF} = 0.68$.

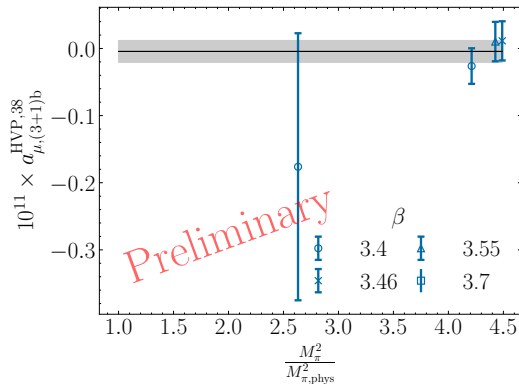


b) Model average using AIC. The resulting value is
 $a_{\mu,(3+1)a}^{\text{HVP},38}(\Lambda = 16 m_\mu) = -0.17(26)_{\text{stat}}(34)_{\text{syst}} \times 10^{-11}$.

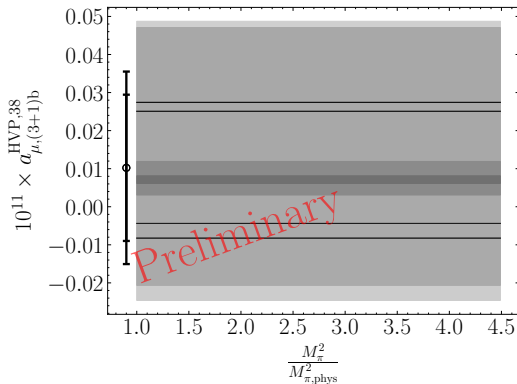
QCD extrapolation to the physical point ((3+1)b with 2PS)



Fit function: $f_{fit}(a, M_\pi, M_\pi L) = c_0$, PV-mass: $\Lambda = \infty$



a) Example model: Constant, XcYIZI, TL,
 $\chi^2/DOF = 0.54$.



b) Model average using AIC. The resulting value is
 $a_{\mu,(3+1)b}^{HVP,38}(\Lambda = \infty) = -0.01(2)_{stat}(2)_{syst} \times 10^{-11}$.

Counterterm

Master formula:

$$\begin{aligned} a_{\mu}^{\text{HVP},38} &= a_{\mu,em}^{\text{HVP},38}(\Lambda) + \overbrace{\frac{\Delta M_K^{\text{phys}} - \Delta M_K^{\text{em}}(\Lambda)}{\frac{\partial \Delta M_K}{\partial(m_u - m_d)}}}^{C_T(\Lambda)} \times \frac{\partial a_{\mu}^{\text{HVP},38}}{\partial(m_u - m_d)} \\ &= a_{\mu,em}^{\text{HVP},38}(\Lambda) + (\Delta M_K^{\text{phys}} - \Delta M_K^{\text{em}}(\Lambda)) \times R_{38K} \end{aligned}$$

- Work on counterterm still in progress

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- ▶ Work on counterterm still in progress
- ▶ $\Delta M_K^{\text{phys}} = -3.934(20) \text{ MeV}$
- ▶ $\Delta M_K^{\text{em}}(\Lambda)$: Talk by Harvey Meyer, [Tue. 4:50 pm](#)
- ▶ $\frac{\partial \Delta M_K}{\partial(m_u - m_d)}$ and $\frac{\partial a_{\mu}^{\text{HVP},38}}{\partial(m_u - m_d)}$: Talk by Sebastian Lahrtz (right after this talk)

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- ▶ We will use pheno estimate for R_{38K} : $-2.50(63) \times 10^{-11} \text{ MeV}^{-1}$ [[JHEP 10 \(2025\) 157](#)]

Summary of Results (preliminary)

UV-finite quantities:

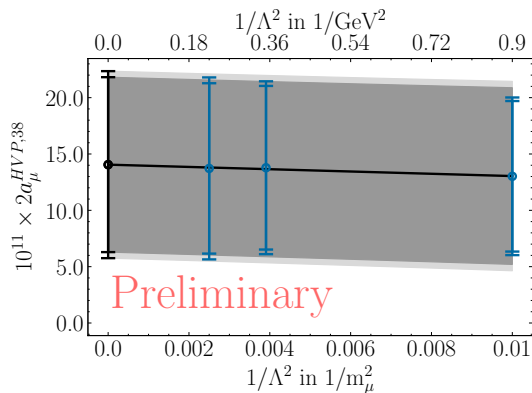
- ▶ $R_{38K} = -2.50(63) \times 10^{-11} \text{ MeV}^{-1}$ (Phenomenological estimate, [[JHEP 10 \(2025\) 157](#)])
- ▶ $a_{\mu,(3+1)b}^{\text{HVP},38} = -0.01(2)_{\text{stat}}(2)_{\text{syst}} \times 10^{-11}$
- ▶ $a_{\mu,(2+2)a}^{\text{HVP},38} = -5.78(80)_{\text{stat}}(90)_{\text{syst}} \times 10^{-11}$ [[JHEP 07 \(2025\) 201](#)]

	$\Lambda = 10 m_\mu$	$\Lambda = 16 m_\mu$	$\Lambda = 20 m_\mu$
$10^{11} \times a_{\mu,(4)a}^{\text{HVP},38}$	$-2.07(39)_{\text{stat}}(47)_{\text{syst}}$	$-3.28(71)_{\text{stat}}(95)_{\text{syst}}$	$-4.15(96)_{\text{stat}}(1.36)_{\text{syst}}$
$10^{11} \times a_{\mu,(4)b}^{\text{HVP},38}$	$1.73(32)_{\text{stat}}(22)_{\text{syst}}$	$2.16(43)_{\text{stat}}(28)_{\text{syst}}$	$2.33(47)_{\text{stat}}(32)_{\text{syst}}$
$10^{11} \times a_{\mu,(4)}^{\text{HVP},38}$	$-0.33(48)_{\text{stat}}(43)_{\text{syst}}$	$-0.74(74)_{\text{stat}}(69)_{\text{syst}}$	$-1.08(90)_{\text{stat}}(87)_{\text{syst}}$
$10^{11} \times a_{\mu,(3+1)a}^{\text{HVP},38}$	$-0.03(11)_{\text{stat}}(13)_{\text{syst}}$	$-0.17(26)_{\text{stat}}(34)_{\text{syst}}$	$-0.31(38)_{\text{stat}}(52)_{\text{syst}}$
ΔM_K^{em} in MeV	$1.13(12)_{\text{stat}}(10)_{\text{syst}}$	$1.50(15)_{\text{stat}}(14)_{\text{syst}}$	$1.68(17)_{\text{stat}}(16)_{\text{syst}}$
$10^{11} \times a_\mu^{\text{HVP},38}$	$6.51(3.34)_{\text{stat}}(1.04)_{\text{syst}}$	$6.89(3.63)_{\text{stat}}(1.24)_{\text{syst}}$	$6.86(3.78)_{\text{stat}}(1.42)_{\text{syst}}$

Charge factors in [backup slides](#)

$\Lambda \rightarrow \infty$ (Preliminary)

Linear extrapolation with 100% correlation:



Current estimate based on

phenomenological estimate of R_{38K} :

$$2a_{\mu}^{\text{HVP},38} = 14.1(7.8)_{\text{stat}}(3.0)_{\text{syst}} \times 10^{-11}$$

On par with experimental uncertainty:

$$14.5 \times 10^{-11}$$

Missing for total isospin-breaking effects:

$$a_{\mu}^{\text{HVP},33} \text{ and } a_{\mu}^{\text{HVP},88}$$

Outlook

- ▶ Fine tune analysis of $a_{\mu,em}^{\text{HVP},38}(\Lambda)$
- ▶ Calculate R_{38K} from lattice
- ▶ Perform combined extrapolation to physical point and infinite Λ

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Other talks of Mainz group

Connected to this talk:

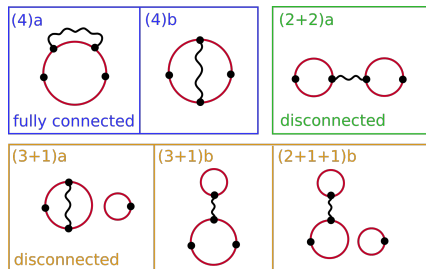
- ▶ Sebastian Lahrtz, [Wed. 11:30 am](#)
- ▶ David Albandea, [Wed. 11:50 pm](#)
- ▶ Harvey Meyer, [Tue. 4:50 pm](#)

Further HVP talks:

- ▶ Alessandro Conigli, [Wed. 09:00 am](#)
- ▶ Arnau Beltran, [Wed. 09:20 am](#)
- ▶ Alessandro De Santis, [Fri. 05:10 pm](#)

Additional Slides

Charge factors



For detailed breakdown of 38 charge factors see [JHEP 07 \(2026\) 068](#).

1-loops consist of light minus strange. All other loops consist only of light quarks.

	$(4)a$	$(4)b$	$(2+2)a$	$(3+1)a$	$(3+1)b$	$(2+1+1)b$
38	$1/36$	$1/36$	$1/36$	$1/18$	$1/36$	$1/18$
33	$5/36$	$5/36$	$1/4$	0	$1/36$	0
88	$5/324$	$5/324$	$1/324$	$5/162$	$1/324$	$1/162$
Full	$17/81$	$17/81$	$25/81$	$7/81$	$7/81$	$5/81$