Qubit-Regularized Gauge Theories on Plaquette Chains

Asymptotic Safety, Continuum Physics, and Glueballs from Finite-Dimensional Hilbert Spaces

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Introduction

This talk is based on our recent papers:

- Continuum limit of a qubit-regularized SU(3) lattice gauge theory with glueballs, arXiv:2603.01215
- Asymptotic freedom and massive glueballs in a qubit-regularized SU(2) gauge theory, PRD **113**, 114522 (2026), 10.1103/2cy4-cp7f.
- Monomer-dimer tensor-network basis for qubit-regularized lattice gauge theories, PRD **111**, 114502 (2025).

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Traditional LFT

- In a traditional Hamiltonian lattice field theory, the total lattice Hilbert space is a tensor product over local ones: $\mathcal{H}_{tot} = \left(\bigotimes_{\ell \in \text{links}} \mathcal{H}_\ell\right) \otimes \left(\bigotimes_{x \in \text{sites}} \mathcal{H}_x\right)$
- Traditionally, if the theory has bosonic degrees of freedom (e.g. scalar fields and gauge fields), the bosonic local Hilbert space is ∞-dimensional.

LFTs with Finite Local Hilbert Spaces

- The quantum simulation community thus attempts to **truncate** the local Hilbert space and studies LFT with finite-dim local Hilbert spaces.
- The idea of studying QFT with finite-dimensional quantum systems is much older than quantum simulation.
- E.g., the $\mathrm{SU}(2)_1$ WZW model naturally emerges as the EFT of the spin-half Heisenberg antiferromagnetic chain [Affleck and Haldane (1987)].
- Recently, new examples have been discovered that also recover massive QFTs with asymptotic freedom [Bhattacharya et al. (2021), Caspar and Singh (2022), Maiti et al. (2024)]

Qubit Regularization Program

- We refer to the idea of obtaining continuum QFTs using spin models with fixed finite-dimensional local Hilbert spaces as **Qubit Regularization (QReg)**, or, more adequately, **qudit** regularization.
- Can the same philosophy be extended from spin models to lattice gauge theories? Yes! E.g., Hersh's talk on chiral U(1) gauge theory.

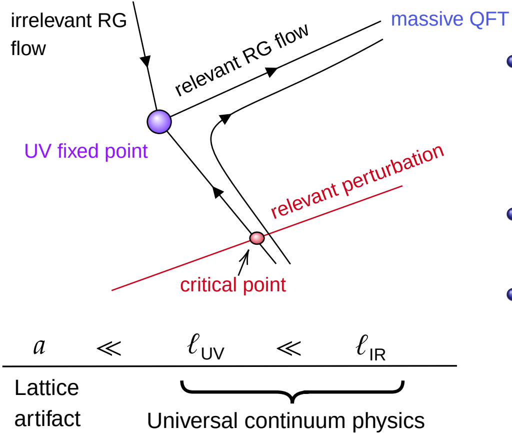
Guiding Question:

Can interesting massive continuum QFTs emerge as the continuum limits of *lattice gauge theories* with finite-dimensional local Hilbert spaces?

- The construction of such LGTs has been previously explored in various contexts: gauge magnets [Orland and Rohrlich (1990)], D-theory and quantum link models [Chandrasekharan and Wiese (1997)], quantum group [Zache et al. (2023)], etc.
- We take a specific kind of route to the continuum.

Massive QFTs from RG of LFT

- Many massive QFTs (e.g. pure SU(N) YM) have UV fixed points in their RG flows.



- The UV fixed point of the massive QFT is the IR fixed point of the quantum critical lattice theory.
- A small relevant perturbation then leads to massive IR QFT.
- The coupling determines $\ell_{\rm UV}$ and $\ell_{\rm IR}$. The box size L helps us to probe the crossover between different scales.

Today's plan

I will show that some SU(2) and SU(3) pure LGTs with finite-dimensional link Hilbert spaces on a quasi-1D spatial lattice (two-legged ladder, plaquette chain) give rise to asymptotically safe, relativistic 1+1D continuum QFTs with massive gauge-invariant excitations in the IR.

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Summary of Recipe

There are 3 main ingredients/steps in our construction.

- **Step 1**: Construct an orthonormal basis for the physical subspace of LGTs.
- **Step 2**: Choose an convenient finite-dimensional truncation.
- **Step 3**: Construct a Hamiltonian with a quantum critical point and relevant perturbation, and study the emergent continuum QFT.

Step 1: Gauge-invariant Basis

- This talk: only pure gauge SU(N), without matter.
- Traditional link Hilbert space:

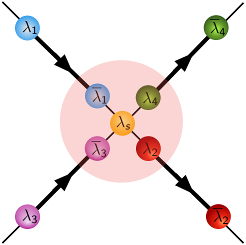
$$\mathcal{H}_\ell^{\text{trad}} \simeq \bigoplus_{\lambda_\ell \text{ irrep}} V_{\lambda_\ell} \otimes V_{\bar{\lambda}_\ell} \tag{1}$$

- The factorization corresponds to the independent actions of the left and right momentum operators, L_ℓ^a, R_ℓ^a $(a = 1, \cdots, \dim(G))$ generating left/right translation on the gauge group G.
- The gauge generators are of the form

$$\mathcal{G}_s^a = \sum_{\ell,\ell'} (L_\ell^a + R_{\ell'}^a) \tag{2}$$

for $a = 1, \cdots, \dim(G)$, where ℓ, ℓ' are the links whose left and right ends, respectively, are at the site s.

- At each site s, consider the dressed site Hilbert space $\mathcal{H}_s^g$ constructed from the irreps corresponding to the ends of the links meeting at s, and label an orthonormal basis of gauge singlets by α_s.
- The resulting orthonormal basis $|\{\lambda_\ell\}, \{\lambda_s\}, \{\alpha_s\}\rangle$ for the physical subspace $\mathcal{H}_{\text{phys}}^{\text{trad}}$ of the traditional LGT is what we call the Monomer-Dimer Tensor-Network (MDTN) basis [Chandrasekharan, **RXS**, Bhattacharya (2025)]



$$\mathcal{H}_s^g \;=\; V_{\bar{\lambda}_1} \otimes V_{\lambda_2} \otimes V_{\bar{\lambda}_3} \otimes V_{\lambda_4} \otimes V_{\lambda_s}$$

Step 2: Choice of truncation

- For each subset Q of irreps of G, the qubit regularized link Hilbert space with QReg scheme Q is

$$\mathcal{H}_\ell^Q = \bigoplus_{\lambda \in Q} V_\lambda \otimes V_{\bar{\lambda}} \tag{3}$$

- In this talk, for convenience, we only include antisym irreps (1-column Young diagrams) in Q. For SU(2), $Q = \{\mathbf{1}, \mathbf{2}\}$; for SU(3), $Q = \{\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}}\}$.
- At this stage, once given a lattice, it is possible to enumerate all physical (gauge-invariant) states of the qubit regularized Hilbert space $\mathcal{H}^Q$ using the MDTN basis.

Step 3: Construction of Hamiltonian

- A simple family of Hamiltonians is

$$H \;=\; H_E + H_B \;=\; \sum_{\ell} \kappa_\ell \, \hat{\mathcal{E}}_\ell - \sum_P g_P \left(\hat{\mathcal{U}}_P + \hat{\mathcal{U}}_P^\dagger \right) \tag{4}$$

- The electric density operator $\hat{\mathcal{E}}_\ell$ is diagonal in the MDTN basis, and supported on a link ℓ.
- Magnetic-density/plaquette operator $\hat{\mathcal{U}}_P$ is supported on all $\ell \in P$, is purely off-diagonal, and need not be the standard plaquette op.
- It can be chosen so that the model is sign-problem free in any dim, amenable to quantum MCMC.
- Is there a quantum critical point?

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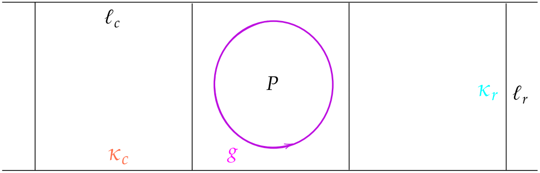
- Periodic plaquette chain consists of L vertical rungs and two horizontal chains:



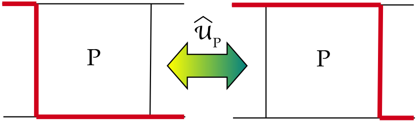
- Recently, we proposed and studied the qubit-regularized pure SU(N) LGTs ($N = 2, 3$) [**RXS**, Chandrasekharan, Bhattacharya (2026a, 2026b)]

$$H = \kappa_c \sum_{\ell_c} \hat{\mathcal{E}}_{\ell_c} + \kappa_r \sum_{\ell_r} \hat{\mathcal{E}}_{\ell_r} - g \sum_P \frac{1}{1+\delta_{2,N}} \left(\hat{\mathcal{U}}_P + \hat{\mathcal{U}}_P^\dagger\right), \quad (5)$$

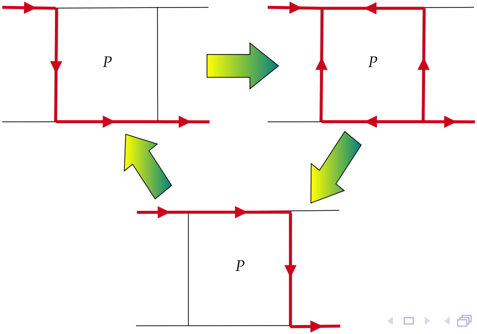
where $\hat{\mathcal{E}}_\ell |\lambda_\ell\rangle = (1 - \delta_{\lambda_\ell,\mathbf{1}})|\lambda_\ell\rangle$, and $\hat{\mathcal{U}}_P$ cyclically permutes the irreps of each link in the plaquette with unit amplitude, without CG coefficients.

Plaquette operator

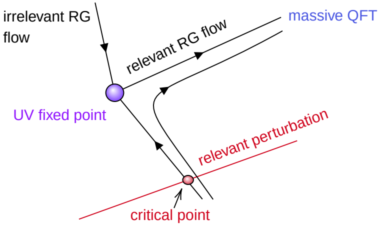
For SU(2), $\hat{\mathcal{U}}_P$ sends $\mathbf{1} \leftrightarrow \mathbf{2}$ on each link in the plaquette.



For SU(3), $\hat{\mathcal{U}}_P$ sends $\mathbf{1} \to \mathbf{3} \to \bar{\mathbf{3}} \to \mathbf{1}$ on each link in the plaquette.



Continuum QFTs from QReg Plaquette Chain LGTS



- Can be tuned to a quantum critical point at $\boxed{\kappa_c = 0, \kappa_r = gN}$
- Relativistic CFTs from quantum critical points acts as UV fixed points:
  - SU(2): 2D Ising CFT
  - SU(3): 2D 3-state Potts CFT
- $\kappa_c \neq 0$ acts as a relevant perturbation, flow from the UV fixed point CFTs to massive QFTs in the IR:
  - $SU(2)$: integrable E_8 QFT [Zamolodchikov (1989)]
  - $SU(3)$: magnetically-deformed Potts field theory [Lepori, Toth, and Delfino (2009)]

$$a \ll \ell_{\mathrm{UV}} \ll \ell_{\mathrm{IR}}$$

Lattice artifact

$$\underbrace{\ell_{\mathrm{UV}} \ll \ell_{\mathrm{IR}}}_{\text{Universal continuum physics}}$$

- The dimensionless scaling variable

$$\mu = (2^{N-2}\kappa_c)^{\nu} L \propto \frac{L}{\xi(L, \kappa_c)} \tag{6}$$

  helps distinguish UV vs IR physics, where 2^{N-2} is just a convention, and $\nu = \frac{1}{2-\Delta} > 0$ is related to the scaling dimension Δ of the relevant deformation.

- The universal continuum physics at a particular scale is obtained in the limit of $\kappa_c \to 0^+$, $L \to \infty$ at fixed μ. E.g.,

$$Obs^{\mathrm{cont}}(\mu) := \lim_{L\to\infty} Obs^{\mathrm{lat}}(L, \kappa_c)\big|_{\kappa_c = 2^{2-N}(\mu/L)^{1/\nu}} \tag{7}$$

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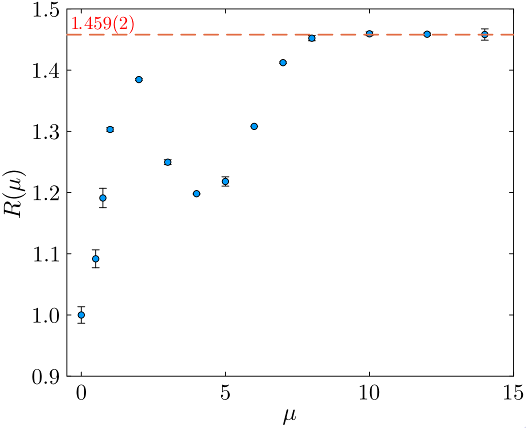
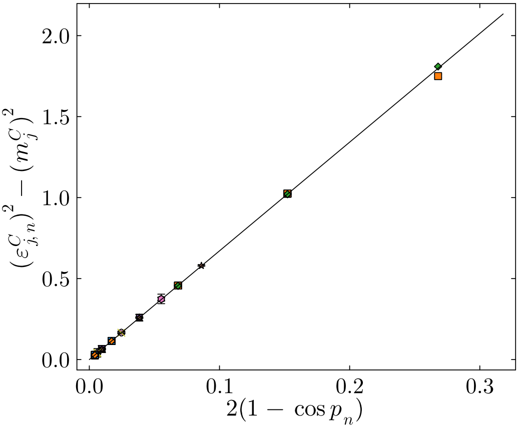
Massive "Glueballs" from SU(3) Plaquette Chain

At every μ, we used DMRG to compute the mass ratio $R(\mu) = m^- / m^+$ of the lightest two "glueball" masses of opposite charge conjugation parity as a function of the scaling variable μ from the UV to the IR. We estimate $\boxed{R(\infty) = 1.459(2)}$, consistent with the Potts results from Lepori et al.

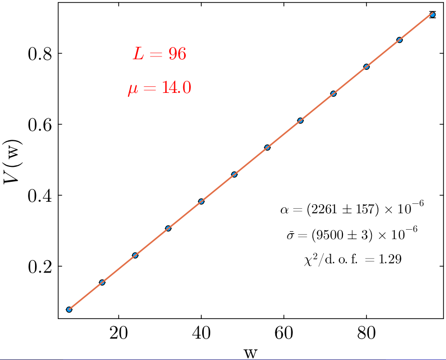


For sufficiently large μ, we found relativistic dispersion relations in the low-lying excited states:



String Tension in the SU(3) Plaquette Chain

- QReg allows us to interpret these continuum QFTs that emerged earlier from lattice spin models as gauge theories with string tensions.
- Using DMRG, we computed the static quark-antiquark potential $V(\mathrm{w}) = \alpha + \bar{\sigma}\mathrm{w}$ as a function of the separation w of the $q\bar{q}$ pair.
- Properly rescaled string tension σ yields the dimensionless universal ratio, which is a new result: $\boxed{\sqrt{\sigma}/m^+ = 0.2648(2)}$



Comparison with pure Yang-Mills

- The SU(2) LGT has analytic results derivable from the integrable Ising field theory: Delfino (2004)
- The pure YM result from MC: Athenodorou and Teper (2017)
- SU(2) table: for YM, $R(\infty)$ is defined using the lowest two scalar glueballs.

Observables	SU(2)	
	Our 1+1D LGT	2+1D YM
$R(\infty)$	1.61803...	1.449(3)
$\sqrt{\sigma}/m^+$	0.24845239...	0.2111(3)

- SU(3) table: for YM, $R = m^-/m^+$, the m^- is the mass of 0^{--}.

Observables	SU(3)	
	Our 1+1D LGT	2+1D YM
$R(\infty)$	1.459(2)	1.463(4)
$\sqrt{\sigma}/m^+$	0.2648(2)	0.2289(4)

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Summary

- We have shown that QReg SU(2) and SU(3) LGTs on plaquette chains show interesting relativistic continuum physics:
  - asymptotically-safe in the UV
  - massive particles like glueballs in the IR
  - linear confinement of static quarks
- QReg allows us to interpret these continuum QFTs that emerged from lattice spin models as gauge theories.
- We computed universal dimensionless quantities that characterize these gauge theories, and found that they lie in the Yang–Mills ballpark.

Outlook and open problems

- We could try including matter fields, moving to higher dimensions, changing the QReg scheme.
- There are also a few fundamental questions:
    - Can we reach different continuum QFTs from LGTs with the same gauge group and matter content?
    - What is the landscape of QFTs that can be reached by these new LGTs?
    - Which continuum QFTs lie beyond the reach of QReg, and how do we characterize the obstruction(s)?
    - Does confinement impose a degree of universality on infrared observables?
- I DO NOT know the answer to these questions (yet), please tell me if you know something!

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Thank you!

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Appendix A: Finite Size Scaling 1

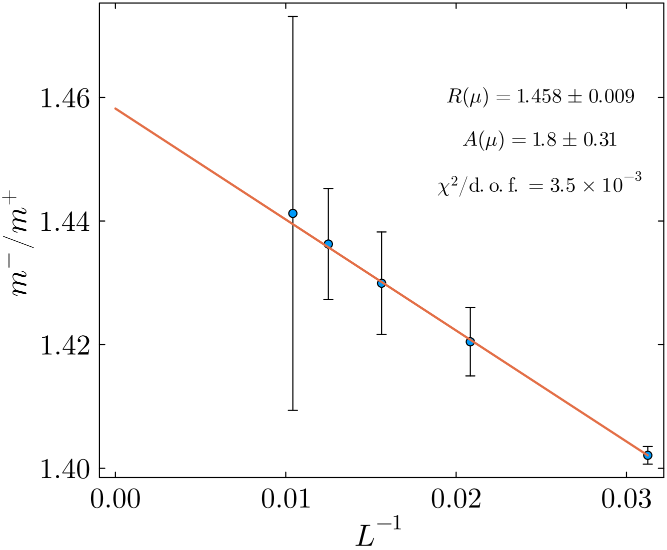


Figure: At $\mu = 14.0$, a linear fit is used to extrapolate the mass ratio's value at

Appendix B: Finite Size Scaling 2

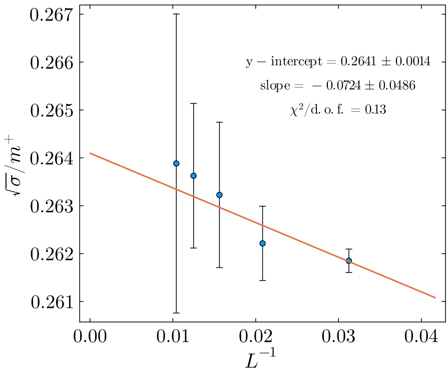
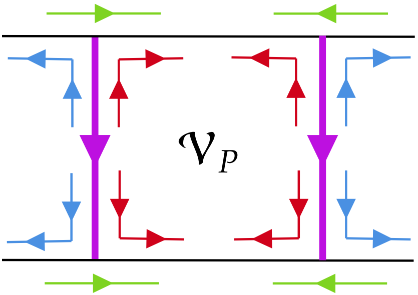
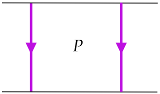
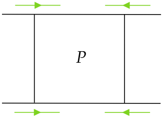
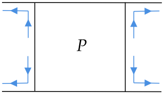
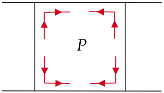


Figure: At $\mu = 14.0$, we use DMRG to compute the rescaled string tension and the corresponding dimensionless ratio. Linear fit is used to extrapolate the ratio's value at $L \to \infty$.

Plaq. dressing:



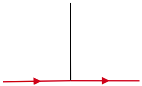
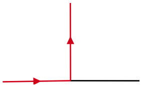
$\mathcal{V}_P = \Bigg(1 \; +$  $+ \; 3$ 

$+$  $- \; 2$  $\Bigg)$ (8)

Decorated plaq. op.:

$$\hat{\mathcal{U}}_P = \mathcal{V}_P \mathcal{U}_P \ , \ \mathcal{U}_P = \mathcal{U}_{\ell_1,ij}\mathcal{U}_{\ell_2,jk}\mathcal{U}^\dagger_{\ell_3,lk}\mathcal{U}^\dagger_{\ell_4,il} \tag{9}$$

Electric Hamiltonian:

$$H_E = J_1 \sum_{\ell} \delta_{\lambda_\ell,\mathbf{2}} + \frac{1}{2}\sum_{v}\Bigg[\ J_2 \tag{10}$$

$$+\ J_3$$

$$\Bigg]$$

Plaq/Magnetic Hamiltonian:

$$H_B = -g\sum_P [\hat{\mathcal{U}}_P + h.c.] \tag{11}$$

Total Hamiltonian:

$$H_{SU(2)} = H_E + H_B \tag{12}$$

On the physical Hilbert space, the qubit regularized SU(2) plaquette chain Hamiltonian reduces to the ferromagnetic transverse field Ising chain with a longitudinal term:

$$H_{SU(2)} \;=\; -\sum_{P} \left(\frac{\kappa_r}{2}\, \sigma^z_P \sigma^z_{P+1} + \kappa_c\, \theta\, \sigma^z_P + g\, \sigma^x_P \right) + constant \tag{13}$$

where

$$\kappa_r = J_1 + \frac{1}{2} J_2 - J_3\ ,\ \ \kappa_c = J_1 - \frac{1}{2} J_2\ ,\ \ \theta = \begin{cases} 1 & \text{(even sector)} \\ 0 & \text{(odd sector)} \end{cases} \tag{14}$$

Self-dual point: $g = \kappa_r/2$, $\kappa_c = 0$.

Appendix D: Explicit Form of SU(3) Hamiltonian

On the physical Hilbert space, the SU(3) Hamiltonians reduces to the 1D 3-state quantum clock chain:

$$H_{SU(3)} = -\sum_{P} \left\{ \frac{\kappa_r}{3} \hat{Z}_P \hat{Z}^\dagger_{P+1} + \frac{2\kappa_c}{3} \theta \, \hat{Z}_P + g \, \hat{X}_P + \text{h.c.} \right\} + constant \tag{15}$$

$$\hat{X} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \qquad \hat{Z} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i2\pi/3} & 0 \\ 0 & 0 & e^{-i2\pi/3} \end{pmatrix} \tag{16}$$

Self-dual point: $g = \kappa_r/3$, $\kappa_c = 0$.