Perturbative study of higher-moment operators in the Wilson-flow framework on the lattice

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Introduction - Motivation

- **Mellin moments of PDFs**
  • Study of key observables: parton momentum fractions, etc.
  • Connect PDFs to local operator matrix elements calculable in LQCD [X. Ji., J. Phys. G 24 (1998) 1181]
  • Help in interpreting current and future experimental data (EIC) [R. A. Khalek et al., Nucl. Phys. A1026 (2022) 122447]
- **Gradient/Wilson flow** [M. Lüscher, JHEP 1008 (2010) 071]
  • Powerful technique that suppresses ultraviolet divergences exponentially $\Longrightarrow$ simplify the renormalization procedure
  • Combined with the short-flow time expansion (SFTX): Allows the direct calculation of moments of any order in lattice QCD beyond the current limitations [A. Shindler, Phys. Rev. D110 (2024) L051503]
- **Perturbative calculation of flowed moment operators on the lattice:**
  • Check for possible residual finite operator mixing effects allowed by hypercubic symmetry

Study of PDF moments with conventional methods

- **Matrix elements of twist-2 operators with multiple covariant derivatives**
  For example, unpolarized PDFs:

$$\left\langle h(p) \left| \mathcal{O}^{\{\mu_1 \cdots \mu_n\}} \right| h(p) \right\rangle = 2p^{\mu_1} p^{\mu_2} \cdots p^{\mu_n} \langle x^{n-1} \rangle,$$

$$\mathcal{O}^{\{\mu_1 \cdots \mu_n\}} = i^{n-1} \bar{\psi} \gamma^{\{\mu_1} \overleftrightarrow{D}^{\mu_2} \cdots \overleftrightarrow{D}^{\mu_n\}} \psi$$

  where $\{\cdots\}$ denotes symmetrization over the Lorentz indices and subtraction of the traces.

- **Renormalization of twist-2 operators on the lattice:**
  • Larger mixing pattern than in the continuum due to the reduction of $O(4)$ into $H(4)$, e.g., $\mathcal{O}^{\{44\}}$ renormalizes differently from $\mathcal{O}^{\{4i\}}$

  • Mixing with operators of the same or lower dimension. The mixing with lower-dimensional operators is power-divergent $(1/a^d,\ d > 0)$

  • Operators protected from mixing (nonsinglet, zero momentum insertion):

$$\mathcal{O}^{\{\mu\mu\}}, \qquad \mathcal{O}^{\{\mu_1 \neq \mu_2 \neq \cdots \neq \mu_n\}}$$

  Restricted up to operators with three derivatives, which give $\langle x \rangle$, $\langle x^2 \rangle$, $\langle x^3 \rangle$.

Study of PDF moments with conventional methods

- **Operators protected from mixing (nonsinglet, zero momentum insertion):**

  • $\mathcal{O}^{\{\mu_1 \neq \mu_2 \neq \cdots \neq \mu_n\}}$: Requires a **boosted frame**: More noisy, Large cutoff effects [G. Beccarini at al., Nucl. Phys. B 456, (1995) 271]

  ∗ $\mathcal{O}^{\{4i\}}$: one non-zero component of the boost momentum

  ∗ $\mathcal{O}^{\{4ij\}}$: two non-zero components of the boost momentum

  ∗ $\mathcal{O}^{\{4ijk\}}$: three non-zero components of the boost momentum

  • **Recent calculations by ETMC at multiple lattice-spacing ensembles:**

  ∗ Pion and kaon moments: $\langle x \rangle$, $\langle x^2 \rangle$, $\langle x^3 \rangle$ [ETMC, arXiv:2605.29998]

  ∗ Nucleon moments: $\langle x \rangle$, $\langle x^2 \rangle$, $\langle x^3 \rangle$ [ETMC, arXiv:2607.03458, arXiv:2607.20337, arXiv:2607.21230]

  ↪ See talks by **C. Alexandrou (Thursday at 14:00)** and **C. Kummer (Monday at 15:20)**

- **Operators** $\mathcal{O}^{\{44\cdots4\}}$ can be evaluated in the lab frame (less noisy) but suffer from power-divergent mixing ⇒ Not typically selected in conventional methods

Gradient flow

- **Gradient flow equations** [M. Lüscher, JHEP 08 (2010) 071, JHEP 04 (2013) 123]

$$\partial_t B_\mu(t) = D_\nu G_{\nu\mu}(t), \qquad \partial_t \chi(t) = \Delta\chi(t), \qquad \partial_t \bar{\chi}(t) = \bar{\chi}(t)\overleftarrow{\Delta},$$
$$B_\mu(0) = A_\mu, \qquad \chi(0) = \psi, \qquad \bar{\chi}(0) = \bar{\psi},$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + ig[B_\mu, B_\nu], \qquad D_\mu = \partial_\mu + ig[B_\mu, \cdot],$$
$$\Delta = (\partial_\mu + igB_\mu)(\partial_\mu + igB_\mu), \qquad \overleftarrow{\Delta} = (\overleftarrow{\partial}_\mu - igB_\mu)(\overleftarrow{\partial}_\mu - igB_\mu).$$

- The flow time acts as a UV regulator and smooths out the fields over region $\sim \sqrt{8t}$
- **Proof of renormalizability:** [M. Lüscher, P. Weisz, JHEP 02 (2011) 051; M. Lüscher, JHEP 04 (2013) 123; K. Hieda, H. Makino, H. Suzuki, Nucl.Phys.B 918 (2017) 23]
  "Any correlation functions and composite operators of flowed fields are made finite by the conventional renormalization of gauge theory and wave function renormalization of the flowed fermion fields."

$$\mathcal{O}^R(\underbrace{\bar{\chi},\ldots,\bar{\chi}}_{n},\underbrace{\chi,\ldots,\chi}_{m},B_{\mu_1},\ldots,B_{\mu_l}) = Z_\chi^{(n+m)/2}\mathcal{O}(\underbrace{\bar{\chi},\ldots,\bar{\chi}}_{n},\underbrace{\chi,\ldots,\chi}_{m},B_{\mu_1},\ldots,B_{\mu_l}).$$

Gradient flow applied to twist-2 operators

[A. Shindler, Phys. Rev. D 110 (2024) L051503]

- Twist-2 flowed operators $\mathcal{O}^{\{\mu_1\cdots\mu_n\}}(t)$ are multiplicatively renormalizable with the same renormalization factor, regardless of their Dirac structure Γ, the choice of Lorentz indices, or the number of covariant derivatives.
  We can choose to study $\mathcal{O}^{\{4\cdots4\}}(t)$
- Renormalization by using "ringed" fermion fields [H. Makino, H. Suzuki, PTEP 2014 (2014) 063B02]:

$$\left\langle \mathring{\bar{\chi}}_r(x,t)\overleftrightarrow{\not{D}}\mathring{\chi}_r(x,t) \right\rangle = -\frac{N_c}{(4\pi)^2 t^2}\,.$$

- Alternatively: Take ratios of the matrix elements of two different twist-2 operators to cancel the renormalization factor [A. Francis et al., Phys.Rev.Lett. 136 (2026) 171903, Phys. Rev. D 113 (2026) 074520]:

$$\frac{\langle h(p)|\mathcal{O}^{\{4\cdots4\}}(t)|h(p)\rangle}{\langle h(p)|\mathcal{O}^{\{44\}}(t)|h(p)\rangle} = \frac{\langle h(p)|\mathring{\mathcal{O}}^{\{4\cdots4\}}(t)|h(p)\rangle}{\langle h(p)|\mathring{\mathcal{O}}^{\{44\}}(t)|h(p)\rangle}$$

- Then the continuum limit ($a \to 0$) can be taken at fixed t.

Gradient flow applied to twist-2 operators

- **SFTX** [M. Lüscher, PoS LATTICE2013, (2024) 016]
  Perturbative matching $\zeta_n(t,\mu)$ [Harlander et al., Eur.Phys.J.C 86 (2026) 439] to $\overline{\mathrm{MS}}$ at zero flow time through OPE-like expansion for small t:

$$\lim_{a\to 0} \frac{\langle h(p)|\mathcal{O}^{\{4\cdots 4\}}(t)|h(p)\rangle}{\langle h(p)|\mathcal{O}^{\{44\}}(t)|h(p)\rangle} = \frac{\zeta_n(t,\mu)}{\zeta_2(t,\mu)} \times \frac{\langle h(p)|\mathcal{O}^{\{4\cdots 4\}}|h(p)\rangle^{\overline{\mathrm{MS}}}(\mu)}{\langle h(p)|\mathcal{O}^{\{44\}}|h(p)\rangle^{\overline{\mathrm{MS}}}(\mu)} + \mathcal{O}(t)$$

- Extrapolate to $t \to 0$ to remove mixing with higher-dimensional continuum operators multiplied by positive powers of t.

- **Theoretical concern:**
  The renormalizability proof is formulated in dimensional regularization. What about residual **finite operator-mixing effects** on the lattice allowed by hypercubic symmetry? This is relevant when matching to continuum schemes, where such effects are absent.

  ∗ Analogous investigations of (unflowed) nonlocal lattice operators: [M. Constantinou, H. Panagopoulos, Phys. Rev. D96, (2017) 054506; G. Spanoudes et al., Phys. Rev. D109 (2024) 114501]
  Finite mixing has been revealed when matching to continuum schemes, even in the continuum limit

Calculation

- Examine possible finite mixing effects in lattice perturbation theory
- One-loop calculation using Wilson/clover fermions and Symanzik-improved gluons
- Study of non-singlet twist-2 flowed operators with 0, 1, ..., 5 covariant derivatives and for a general Γ matrix:

$$\tilde{\mathcal{O}}^{\mu_1\cdots\mu_n}_{\Gamma;f_1 f_2}(t,x) = \bar{\chi}_{f_1}(t,x)\,\Gamma \overleftrightarrow{D}^{\mu_1}\cdots\overleftrightarrow{D}^{\mu_n}\chi_{f_2}(t,x), \qquad n \in [0,5]$$

- Consider off-shell amputated Green's functions of the flowed operators with external (unflowed) quark fields:

$$\Lambda^{\mu_1\cdots\mu_n}_{\Gamma}(t,p) \equiv \langle \psi_{f_1}(p)|\sum_x \tilde{\mathcal{O}}^{\mu_1\cdots\mu_n}_{\Gamma;f_1 f_2}(t,x)|\bar{\psi}_{f_2}(p)\rangle_{\rm amp}$$

- **Lattice Wilson flow equations:** $(a=1)$

$$\partial_t U_\mu(t,x) = Q_\mu(t,x)U_\mu(t,x), \qquad \partial_t\chi(t) = \Delta\chi(t), \qquad \partial_t\bar{\chi}(t) = \bar{\chi}(t)\overleftarrow{\Delta},$$
$$U_\mu(0,x) = U_\mu(x), \qquad \chi(0,x) = \psi(x), \qquad \bar{\chi}(0,x) = \bar{\psi}(x),$$

$$Q_\mu(t,x) = \mathcal{P}\{V_\mu(t,x)U^\dagger_\mu(t,x)\},\ \mathcal{P}\{M\} = \frac{1}{2}(M - M^\dagger) - \frac{1}{2N_c}\mathrm{tr}(M - M^\dagger),$$

$$V_\mu(t,x) = U^\dagger_\nu(t,x-\hat{\nu})U_\mu(t,x-\hat{\nu})U_\nu(x-\hat{\nu}+\hat{\mu}) + U_\nu(t,x)U_\mu(t,x+\hat{\nu})U^\dagger_\nu(x+\hat{\mu}),$$

$$\Delta\chi(t,x) = [U_\mu(t,x)\chi(t,x+\hat{\mu}) + U^\dagger_\mu(t,x-\hat{\mu})\chi(t,x-\hat{\mu}) - 2\chi(t,x)],$$

$$\bar{\chi}(t,x)\overleftarrow{\Delta} = [\bar{\chi}(t,x+\hat{\mu})U^\dagger_\mu(t,x) + \chi(t,x-\hat{\mu})U_\mu(t,x-\hat{\mu}) - 2\bar{\chi}(t,x)]$$

Calculation

- **Expand the flowed fields in terms of the unflowed fields:**

$$U_\mu(t,x) = \exp(ig_0 T^a B^a_\mu(t,x)) = \mathbf{1} + ig_0 T^a B^a_\mu(t,x) - g_0^2 T^a T^b B^a_\mu(t,x) B^b_\mu(t,x) + \mathcal{O}(g_0^3),$$

$$B^a_\mu(t,x) = B^{a,(0)}_\mu(t,x) + g_0 B^{a,(1)}_\mu + \mathcal{O}(g_0^2),$$

$$\chi(t,x) = \chi^{(0)}(t,x) + g_0 \chi^{(1)}(t,x) + g_0^2 \chi^{(2)}(t,x) + \mathcal{O}(g_0^3),$$

$$\bar\chi(t,x) = \bar\chi^{(0)}(t,x) + g_0 \bar\chi^{(1)}(t,x) + g_0^2 \bar\chi^{(2)}(t,x) + \mathcal{O}(g_0^3)$$

- **Solve the lattice Wilson-flow equations order by order**
  - **Gauge flowed field** [M. Ammer, S. Dürr, Phys.Rev.D 110 (2024) 054504]:

$$B^{a,(0)}_\mu(t,x) = \int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4} e^{ik\cdot(x+\hat\mu/2)} \tilde B_{\mu\rho}(t,k) A^a_\rho(k),$$

$$B^{a,(1)}_\mu(t,x) = \int_{-\pi}^{\pi} \frac{d^4k_1}{(2\pi)^4} \int_{-\pi}^{\pi} \frac{d^4k_2}{(2\pi)^4} e^{i(k_1+k_2)\cdot(x+\hat\mu/2)} \tilde B^{abc}_{\mu\rho_1\rho_2}(t,k_1,k_2) A^b_{\rho_1}(k_1) A^c_{\rho_2}(k_2),$$

where $\tilde B_{\mu\nu}(t,p) = e^{-t\hat p^2}[\delta_{\mu\nu} - (\hat p_\mu \hat p_\nu)/\hat p^2] + (\hat p_\mu \hat p_\nu)/\hat p^2,$

$$\tilde B^{abc}_{\mu\nu\rho}(t,p_1,p_2) = (-i) f^{abc} \tilde g_{\sigma_1\sigma_2\sigma_3}(p_1,p_2)\times \int_0^t dt' \tilde B_{\mu\sigma_1}(t-t',p_1+p_2) \tilde B_{\sigma_2\nu}(t',p_1) \tilde B_{\sigma_3\rho}(t',p_2),$$

$$\tilde g_{\mu\nu\rho}(p_1,p_2) = -\delta_{\mu\nu}\sin(p_1+p_2/2)_\rho \cos(p_2/2)_\mu + \delta_{\mu\rho}\sin(p_1/2+p_2)_\nu \cos(p_1/2)_\mu + \delta_{\nu\rho}\sin((p_1-p_2)/2)_\mu \cos((p_1+p_2)/2)_\nu$$

Calculation

- **Fermion flowed field:**

$$\chi^{(0)}(t,x) = \int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4} e^{ik\cdot x} \tilde{R}(t,k)\psi(k),$$

$$\chi^{(1)}(t,x) = \int_{-\pi}^{\pi} \frac{d^4k_1}{(2\pi)^4} \int_{-\pi}^{\pi} \frac{d^4k_2}{(2\pi)^4} e^{i(k_1+k_2)\cdot x} \tilde{R}_\rho(t,k_1,k_2)(-iT^a)A^a_\rho(k_1)\psi(k_2),$$

$$\chi^{(2)}(t,x) = \int_{-\pi}^{\pi} \frac{d^4k_1}{(2\pi)^4} \int_{-\pi}^{\pi} \frac{d^4k_2}{(2\pi)^4} \int_{-\pi}^{\pi} \frac{d^4k_3}{(2\pi)^4} e^{i(k_1+k_2+k_3)\cdot x} \int_0^t dt' R(t-t',k_1+k_2+k_3) \times$$
$$\{s_\sigma(k_1,k_2)\tilde{B}^{abc}_{\sigma\rho_1\rho_2}(t',k_1,k_2)(T^c/2)R(t',k_3)$$
$$-\frac{1}{4}[\frac{1}{N_c}\delta^{ab} + (d^{abc} + if^{abc})T^c]s_\sigma(k_1,k_2+k_3)\tilde{B}_{\sigma\rho_1}(t',k_1)\tilde{R}_{\rho_2}(t',k_2,k_3)$$
$$-\frac{1}{4}[\frac{1}{N_c}\delta^{ab} + (d^{abc} - if^{abc})T^c]s_\sigma(k_2,k_1+k_3)\tilde{B}_{\sigma\rho_2}(t',k_2)\tilde{R}_{\rho_1}(t',k_1,k_3)$$
$$-\frac{1}{4}[\frac{1}{N_c}\delta^{ab} + d^{abc}T^c]c_\sigma(k_1+k_2,k_3)\tilde{B}_{\sigma\rho_1}(t',k_1)\tilde{B}_{\sigma\rho_2}(t',k_2)R(t',k_3)\}$$
$$A^a_{\rho_1}(k_1)A^b_{\rho_2}(k_2)\psi(k_3),$$

where $\tilde{R}(t,p) = e^{-t\hat{p}^2}$,

$\tilde{R}_\mu(t,p_1,p_2) = s_\rho(p_1,p_2)\int_0^t dt' R(t-t',p_1+p_2)\,\tilde{B}_{\rho\mu}(t',p_1)\,R(t',p_2)$

$s_\mu(p_1,p_2) = 2i\sin(p_1/2+p_2)_\mu,\ c_\mu(p_1,p_2) = 2\cos(p_1/2+p_2)_\mu$

Calculation

- **Feynman rules for twist-2 operators:**

$$\tilde{\mathcal{O}}^{\mu_1\cdots\mu_n}_{\Gamma;f_1 f_2}(t,x) = \bar{\chi}_{f_1}(t,x)\,\Gamma\overleftrightarrow{D}^{\mu_1}(t,x)\cdots\overleftrightarrow{D}^{\mu_n}(t,x)\chi_{f_2}(t,x), \qquad n\in[0,5]$$

$$\overleftrightarrow{D}^{\mu}(t,x) = \frac{1}{2}(\overrightarrow{D}^{\mu}(t,x) - \overleftarrow{D}^{\mu}(t,x))$$

$$\overrightarrow{D}_{\mu}(t,x)\chi(t,x) = \frac{1}{2}[U_{\mu}(t,x)\chi(t,x+\hat{\mu}) - U^{\dagger}_{\mu}(t,x-\hat{\mu})\chi(t,x-\hat{\mu})],$$

$$\bar{\chi}(t,x)\overleftarrow{D}_{\mu}(t,x) = \frac{1}{2}[\bar{\chi}(t,x+\hat{\mu})U^{\dagger}_{\mu}(t,x) - \bar{\chi}(t,x-\hat{\mu})U_{\mu}(t,x-\hat{\mu})],$$

• Expand the flowed quark fields and covariant derivatives in terms of the unflowed gauge and fermion fields.
• Example gluon-quark-antiquark vertex of one-derivative flowed operator:

$$\sum_x \tilde{\mathcal{O}}^{\mu_1}_{\Gamma;f_1 f_2}(t,x) = \int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\int\frac{d^4k_3}{(2\pi)^4}\,\bar{\psi}_{f_1}(k_2)V^{\mu_1,\nu;c}_{\Gamma}(t,k_1,k_2,k_3)A^c_{\mu}(k_1)\psi(k_3),$$

$$V^{\mu_1,\nu;c}_{\Gamma}(t,k_1,k_2,k_3) = ig_0\,\delta^{(4)}(k_1-k_2+k_3)\,\Gamma T^c\times$$

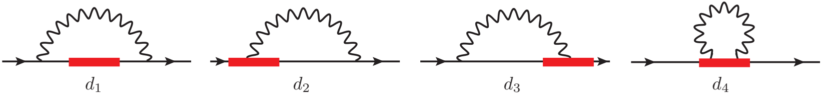
$$\{\,B_{\mu_1\nu}(t,k_1)\,\cos(-k_1/2+k_2)_{\mu_1}\;e^{-t\left(\hat{k}_2^2+\widehat{k_3}^{\,2}\right)}$$

$$-\,2\,B_{\rho\nu}(t',k_1)\int_0^t dt'\,\sin(-k_1/2+k_2)_{\rho}$$

$$\times\,[e^{-2t\,\hat{k}_2^2}\,e^{t'\left(\hat{k}_2^2-\widehat{k_3}^{\,2}\right)}\sin(k_2)_{\mu_1} + e^{-2t\,\widehat{k_3}^{\,2}}\,e^{-t'\left(\hat{k}_2^2-\widehat{k_3}^{\,2}\right)}\sin(k_3)_{\mu_1}]\}.$$

Calculation

- **Feynman diagrams:**



Wavy (straight) lines represent gluons (quarks). The red square represents the flowed operators.

- **Calculation of Feynman integrals:**

• Extend method of Kawai et al. [Kawai et al., Nucl. Phys. B189, 40]

Lattice integrals at flow time t/a^2 and external momentum aq are decomposed into (a) continuum integrals at finite t and q, and (b) scaleless lattice integrals.

∗ For example, from diagrams d_2, d_3:

$$I(t/a^2, aq) = \int_{-\pi}^{\pi} \frac{d^4p}{(2\pi)^4} \int_0^{t/a^2} dt' \, \frac{e^{-t'(\widehat{p}^2 + \widehat{p+aq}^2 - \widehat{aq}^2)} \sin(p_{\rho_1}) \sin(p_{\rho_2})}{\widehat{p}^2 \widehat{p+aq}^2}$$

Calculation

$$I(t/a^2, aq) = \int_{-\pi}^{\pi} \frac{d^4p}{(2\pi)^4} \int_0^{t/a^2} dt' \, \frac{e^{-t'(\hat{p}^2 + \widehat{p+aq}^2 - \widehat{aq}^2)} \sin(p_{\rho_1}) \sin(p_{\rho_2})}{\hat{p}^2 \widehat{p+aq}^2}$$

$$= \int_{-\pi}^{\pi} \frac{d^4p}{(2\pi)^4} \frac{\sin(p_{\rho_1}) \sin(p_{\rho_2})}{(\hat{p}^2 + \widehat{p+aq}^2 - \widehat{aq}^2)\hat{p}^2 \widehat{p+aq}^2} - \int_{-\pi}^{\pi} \frac{d^4p}{(2\pi)^4} \frac{e^{-t/a^2(\hat{p}^2 + \widehat{p+aq}^2 - \widehat{aq}^2)} \sin(p_{\rho_1}) \sin(p_{\rho_2})}{(\hat{p}^2 + \widehat{p+aq}^2 - \widehat{aq}^2)\hat{p}^2 \widehat{p+aq}^2}$$

- The second integral is UV finite due to the Gaussian damping factor. Take the continuum limit and transform into continuum integral.
- The first integral is decomposed according to Kawai et al.:

$$\int_{-\pi}^{\pi} \frac{d^4p}{(2\pi)^4} \frac{\sin(p_{\rho_1}) \sin(p_{\rho_2})}{(\hat{p}^2 + \widehat{p+aq}^2 - \widehat{aq}^2)\hat{p}^2 \widehat{p+aq}^2} =$$

$$\lim_{d\to 4} \Big\{ \int_{-\pi}^{\pi} \frac{d^dp}{(2\pi)^d} \Big[\frac{\sin(p_{\rho_1}) \sin(p_{\rho_2})}{(\hat{p}^2 + \widehat{p+aq}^2 - \widehat{aq}^2)\hat{p}^2 \widehat{p+aq}^2} - \frac{\sin(p_{\rho_1}) \sin(p_{\rho_2})}{2(\hat{p}^2)^3} \Big]$$

$$+ \int_{-\pi}^{\pi} \frac{d^dp}{(2\pi)^d} \frac{\sin(p_{\rho_1}) \sin(p_{\rho_2})}{2(\hat{p}^2)^3} \Big\}$$

- The first integral is UV finite. Take the continuum limit and transform into continuum integral. The second integral is scaleless lattice integral. Spurious IR divergences are canceled once we add the two integrals.

Calculation

- **Continuum integrals:**

$$I_{t,s}(n,m;u_1,u_2,u_3,v_1,v_2,v_3) \equiv \int \frac{d^dp}{(2\pi)^d} \int_0^1 ds \, \frac{e^{-t((u_1+sv_1)p^2+(u_2+sv_2)(p+q)^2+(u_3+sv_3)q^2)}}{(p^2)^n((p+q)^2)^m},$$

$$J_t(n,m;u_1,u_2,u_3) \equiv I_{t,s}(n,m;u_1,u_2,u_3,0,0,0)$$

- **Recursive formula 1:**

$$\begin{aligned} -v_3q^2 I_{t,s}(n,m;u_1,u_2,u_3,v_1,v_2,v_3) &= v_1 I_{t,s}(n-1,m;u_1,u_2,u_3,v_1,v_2,v_3) \\ &+ v_2 I_{t,s}(n,m-1;u_1,u_2,u_3,v_1,v_2,v_3) \\ &+ 1/t J_t(n,m;u_1+v_1,u_2+v_2,u_3+v_3) \\ &- 1/t J_t(n,m;u_1,u_2,u_3) \end{aligned}$$

- **Recursive formula 2:**

$$\begin{aligned} J_t(n,m;u_1,u_2,u_3) &= e^{-tu_3q^2} J_0(n,m;u_1,u_2,u_3) \\ &- tu_1 I_{t,s}(n-1,m;0,0,u_3,u_1,u_2,0) \\ &- tu_2 I_{t,s}(n,m-1;0,0,u_3,u_1,u_2,0) \end{aligned}$$

- **Master integral:** $(n > 0)$

$$J_t(n,0;u_1,u_2,u_3) = \frac{[t(u_1+u_2)]^{n-d/2}}{2^d\pi^{d/2}} e^{-tu_3q^2} \sum_{k=1}^{n-1} \frac{(-1)^{n-1-k}}{k!(n-1-k)!} \frac{{}_1F_1(\frac{d}{2}-1-k, \frac{d}{2}-k, \frac{tu_2^2q^2}{(u_1+u_2)})}{\frac{d}{2}-1-k}.$$

Results

- **Compare lattice vs $\overline{\text{MS}}$-renormalized Green's functions at finite t:** (Landau gauge: $\alpha = 0$)

$$
\begin{aligned}
\left[\Lambda_\Gamma^{\mu_1\cdots\mu_n}(t,q)\right]^L =& \left[\Lambda_\Gamma^{\mu_1\cdots\mu_n}(t,q,\bar\mu)\right]^{\overline{\text{MS}}} \\
&+ \frac{g_{\overline{\text{MS}}}^2 C_F}{16\pi^2}\left[\Lambda_\Gamma^{\mu_1\cdots\mu_n}(t,q)\right]^{\text{tree}} \times \left(c_0 + 3\log(a^2\mu^2)\right) + \mathcal{O}(g_{\overline{\text{MS}}}^4)
\end{aligned}
$$

where $\log(a^2\mu^2) = \log(a^2\bar\mu^2) + \gamma_E - \log(4\pi)$

Gluon action	$\boldsymbol{c_0}$
Wilson	-5.514603
Tree-level Symanzik	-2.644348
Iwasaki	1.580444

Table: The errors from numerical integration are smaller than the last digit presented.

- The difference is proportional to the tree-level structure. $\Rightarrow$ **There are no finite mixing effects at one loop for any twist-2 operator ($\forall\Gamma, \forall n, \forall$ choices of Lorentz indices).**

Results

- **Renormalization factor of twist-2 flowed operators on the lattice:** (Landau gauge: $\alpha = 0$)

$$(Z_\psi^{L,\overline{\rm MS}})^{-1} Z_\chi^{L,\overline{\rm MS}} [\Lambda_\Gamma^{\mu_1\ldots\mu_n}(t,q)]^L = [\Lambda_\Gamma^{\mu_1\ldots\mu_n}(t,q,\bar\mu)]^{\overline{\rm MS}},$$

$$Z_\chi^{L,\overline{\rm MS}} = 1 - \frac{g_{\overline{\rm MS}}^2 C_F}{16\pi^2}\Big[16\pi^2 P_2 + \boldsymbol{e_1^\psi} + \boldsymbol{e_2^\psi} c_{SW} + \boldsymbol{e_3^\psi} c_{SW}^2 + \boldsymbol{c_0} + 3\log\left(a^2\mu^2\right)\Big] + \mathcal{O}(g_{\overline{\rm MS}}^4)$$

where $P_2 = 0.024013$

Gluon action	$\boldsymbol{e_1^\psi}$	$\boldsymbol{e_2^\psi}$	$\boldsymbol{e_3^\psi}$
Wilson	11.852404	-2.248869	-1.397367
Tree-level Symanzik	8.231263	-2.015425	-1.242203
Iwasaki	3.324557	-1.601011	-0.973207

- The renormalization factor is the same $\forall \Gamma$, $\forall n$, $\forall$ choices of Lorentz indices.
- According to the renormalization theorems, this is also the renormalization of the fermion flow field.

Summary and outlook

- We apply the Wilson-flow framework in lattice perturbation theory
- We solve the lattice gauge and fermion flow equations perturbatively
- We develop the methodology for calculating Feynman integrals on the lattice at finite flow time
- We perform a one-loop calculation of Green's functions of twist-2 operators up to 5 covariant derivatives
- We verify to one loop that there are no residual finite operator-mixing effects on the lattice
- The renormalization factor of the fermion flowed operators on the lattice is independent of the Γ structure, the number of covariant derivatives and the choices of Lorentz indices, in accordance with the continuum.

THANK YOU FOR YOUR ATTENTION!

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