Chemical potential dependence of hadron masses in two-color QCD

Etsuko Itou (YITP/ RIKEN iTHEMS)

Recent Review (Phase diagram and EoS):
EI "Lattice results for the equation of state in dense QCD-like theories"
arXiv: 2508.03090

K. Iida, EI, K. Murakami, D. Suenaga, arXiv2606.13974

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

iTHEMS Interdisciplinary Theoretical & Mathematical Sciences

科研費 KAKENHI

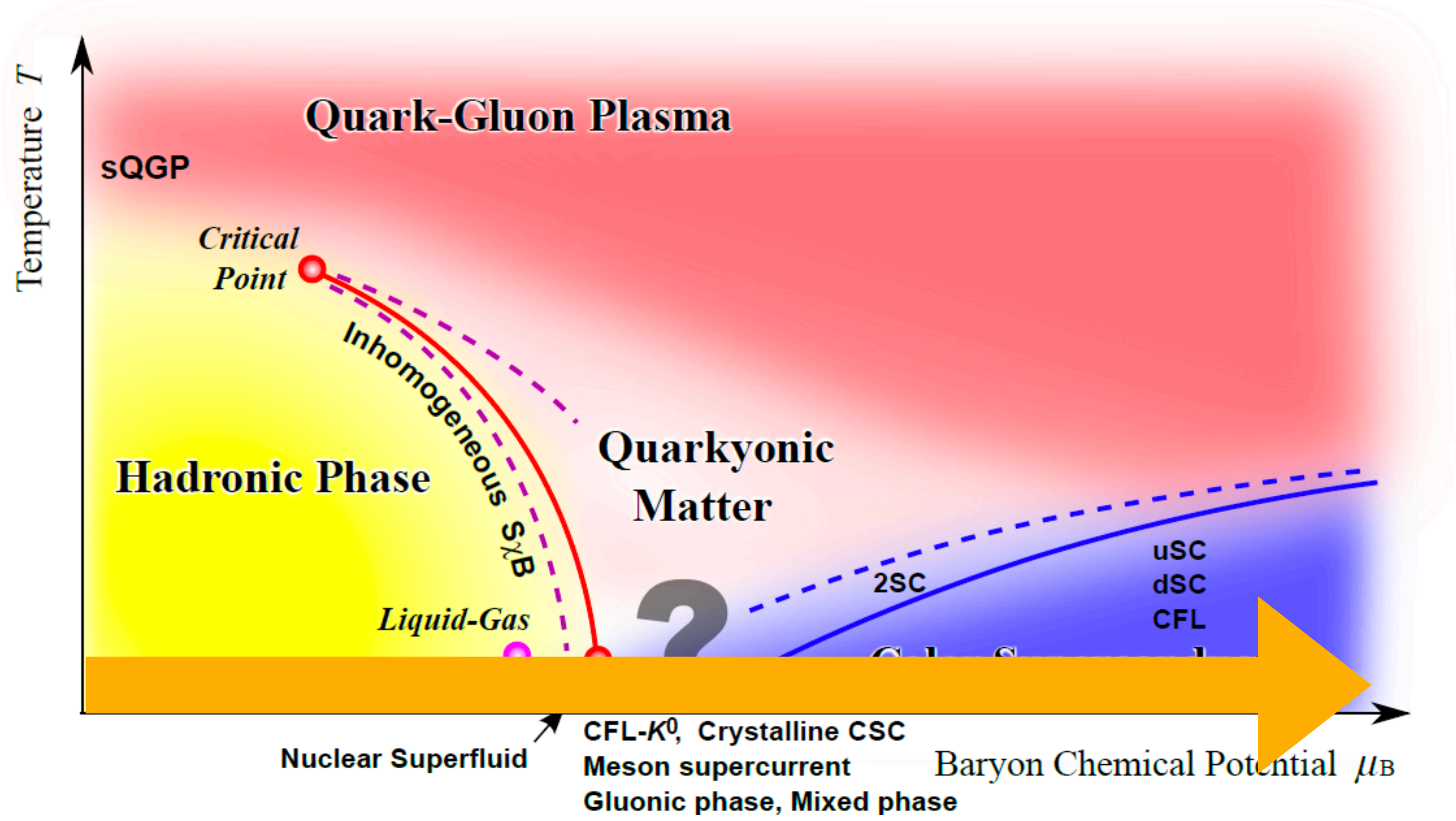
CREST

The 43rd International Symposium on Lattice Field Theory(Lattice 2026), University of Maryland, 2026/07/30

Phase diagram in T-μ plane

3 color QCD
expected phase diagram

Fukushima-Hatsuda (2010)



- Mass spectrum
  in low-T and high-density

- In the superfluid (color-super-conductor) phase, the chiral condensate decreases.
  Some hadron masses should change.

Brown-Rho (1991)

At $\mu \neq 0$, what is happened? (3color QCD)

- At $\mu = 0$

  (1) γ_5-hermiticity: $\gamma_5 D \gamma_5 = D^\dagger$

  (2) No disconnected diagram

From the QCD inequality for $^\forall\Gamma$ **mesons ($\bar{q}\Gamma q$)**,

PS meson (pion, $\Gamma = \gamma_5$) is the lightest!

- $\mu \neq 0$: γ_5-hermiticity is broken : $\gamma_5 D(\mu) \gamma_5 = D^\dagger(-\mu) \neq D^\dagger(\mu)$

  In dense-QCD, neither positivity nor inequality holds

  (cf:in high-density, color-flavor-locking phase, $m_\pi > m_{\eta'}$?) Son-Stepanov (1999)

- According to QCD sum rules, Hatsuda and Lee, PRC 46(92)R34, PRC 52(95)3364 arXiv:nucl-th/9608037

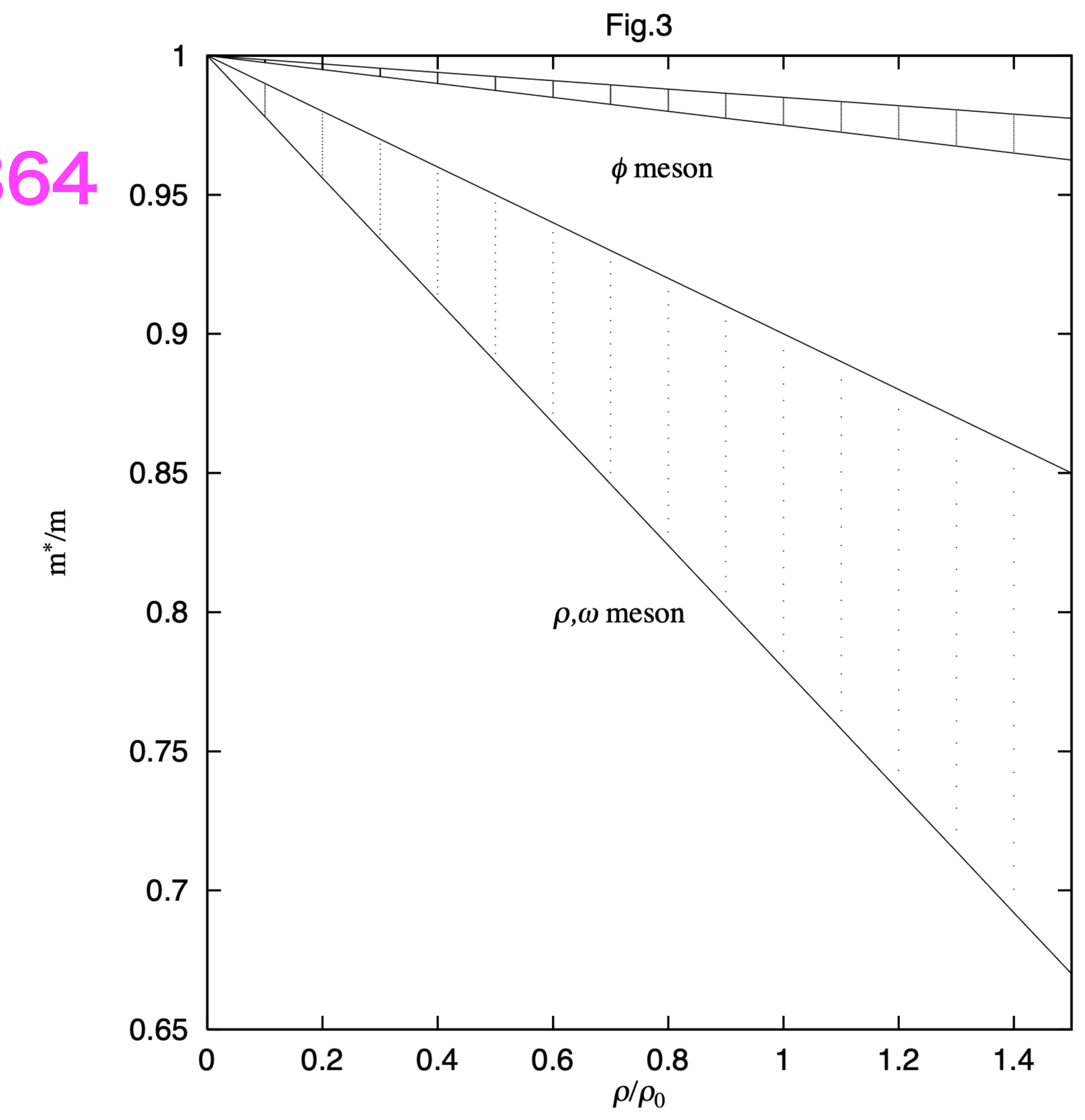
  vector mesons become lighter?

  However, subsequent studies suggest

  ρ and ω masses do not necessarily become lighter.

  (ϕ can be said to do so). S. Zschocke, O.P. Pavlenko, B. Kämpfer, Eur. Phys. J. A 15 (2002) 529–534

- E16 experiment at J-PARC

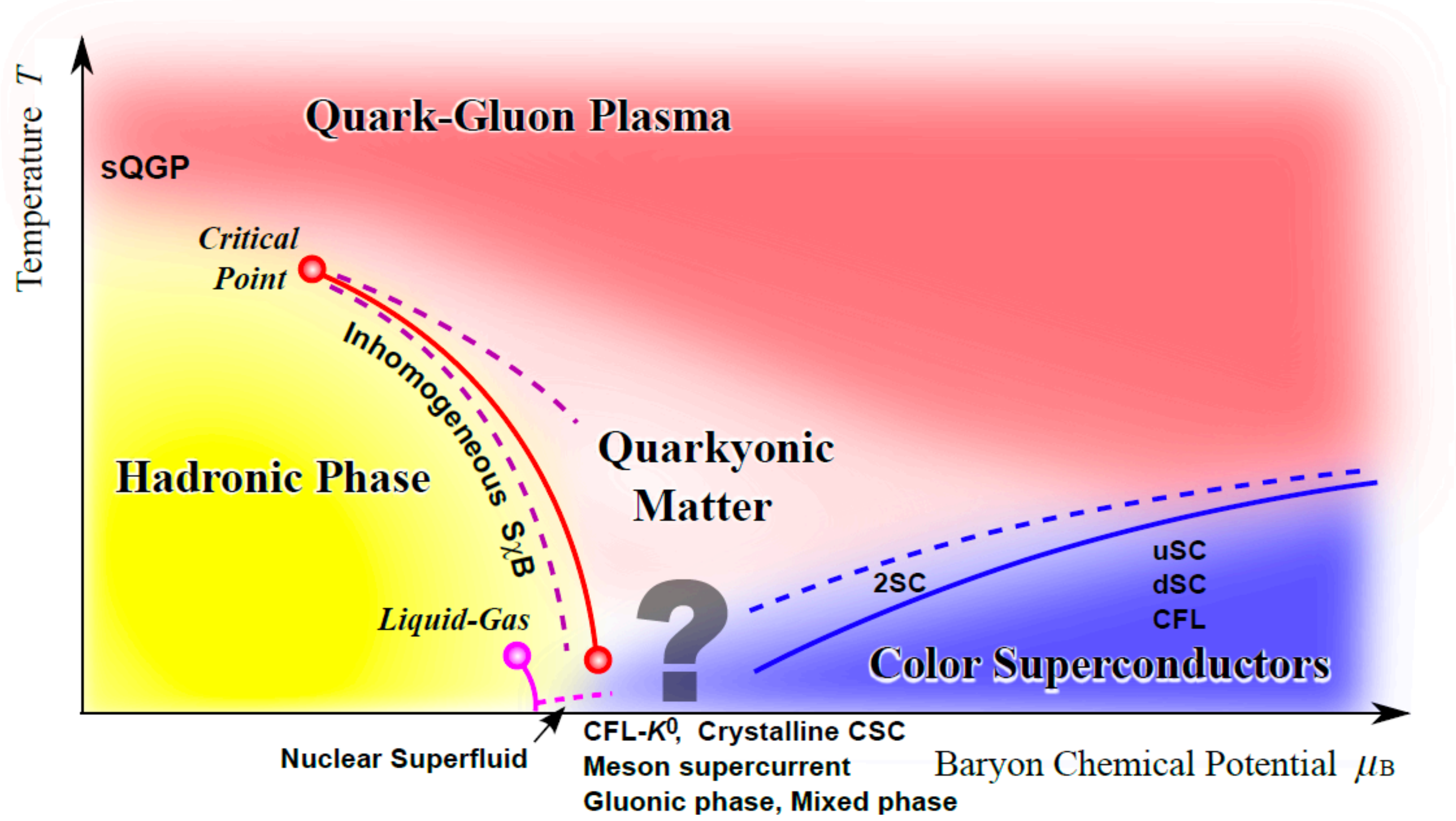


2color QCD

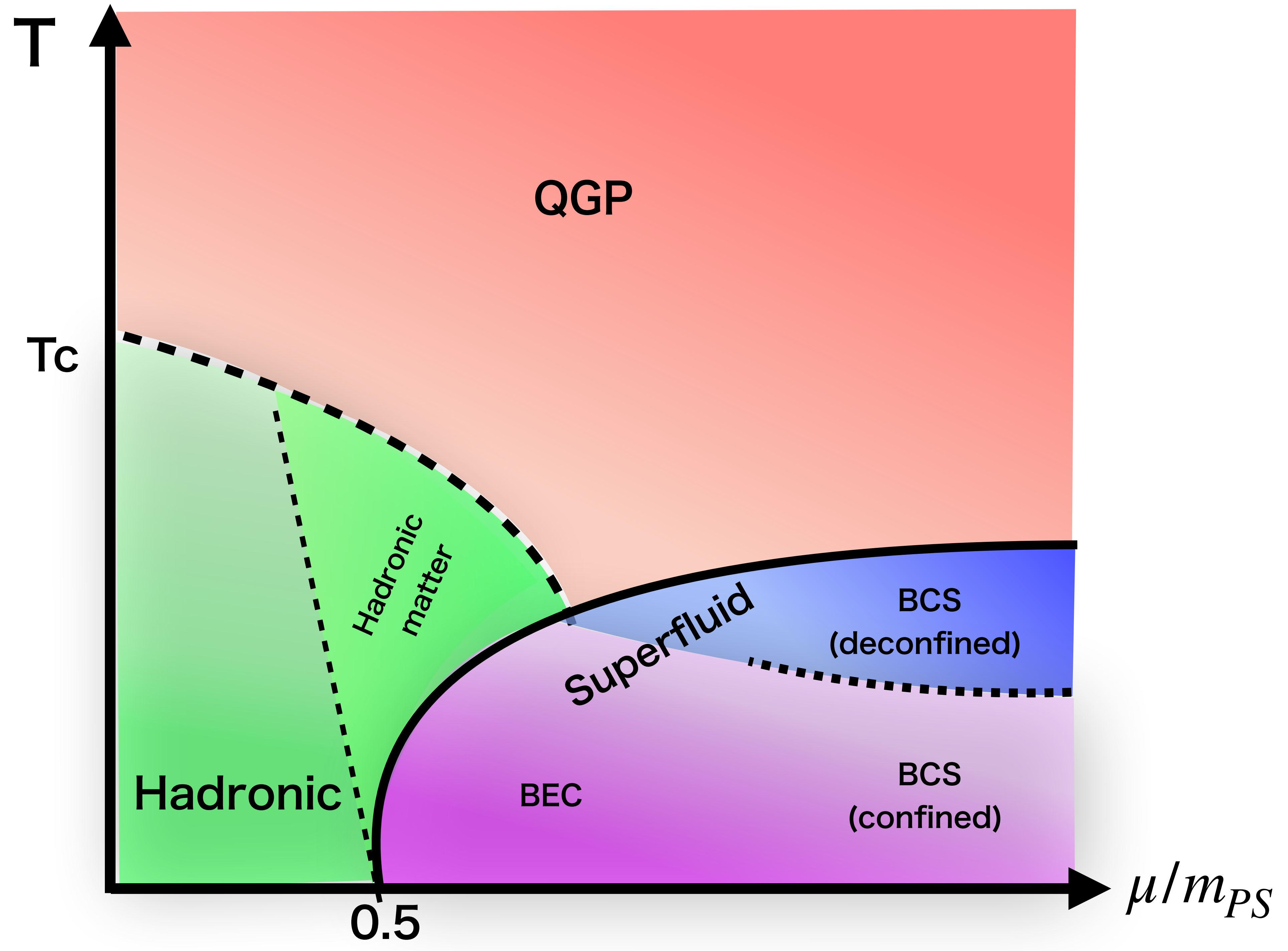
Phase diagram in T-μ plane

3 color QCD
expected phase diagram

Fukushima-Hatsuda (2010)

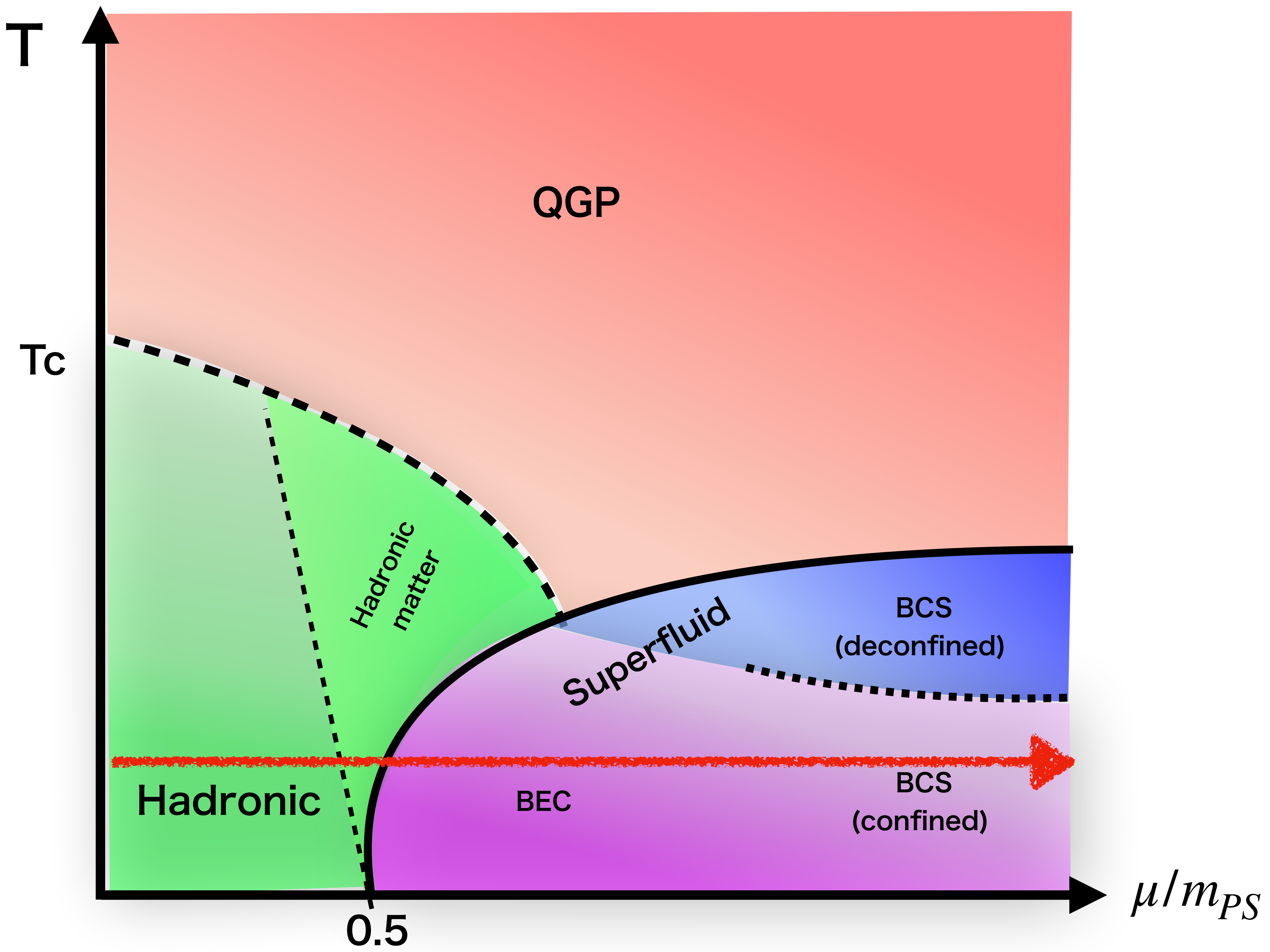


2 color QCD
numerically emerging phase diagram



Our model: 2color 2flavor dense-QCD

2color QCD Phase diagram



40MeV (32^4 lattices)

K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
K.Iida, EI, K.Murakami, D.Suenage, JHEP 10 (2024) 022
Recent Review (Phase diagram and EoS):
EI "Lattice results for the equation of state in dense QCD-like theories" arXiv: 2508.03090

- In Superfluid phase, diquark cond.$\langle qq \rangle \neq 0$
  (color-singlet in 2color QCD)

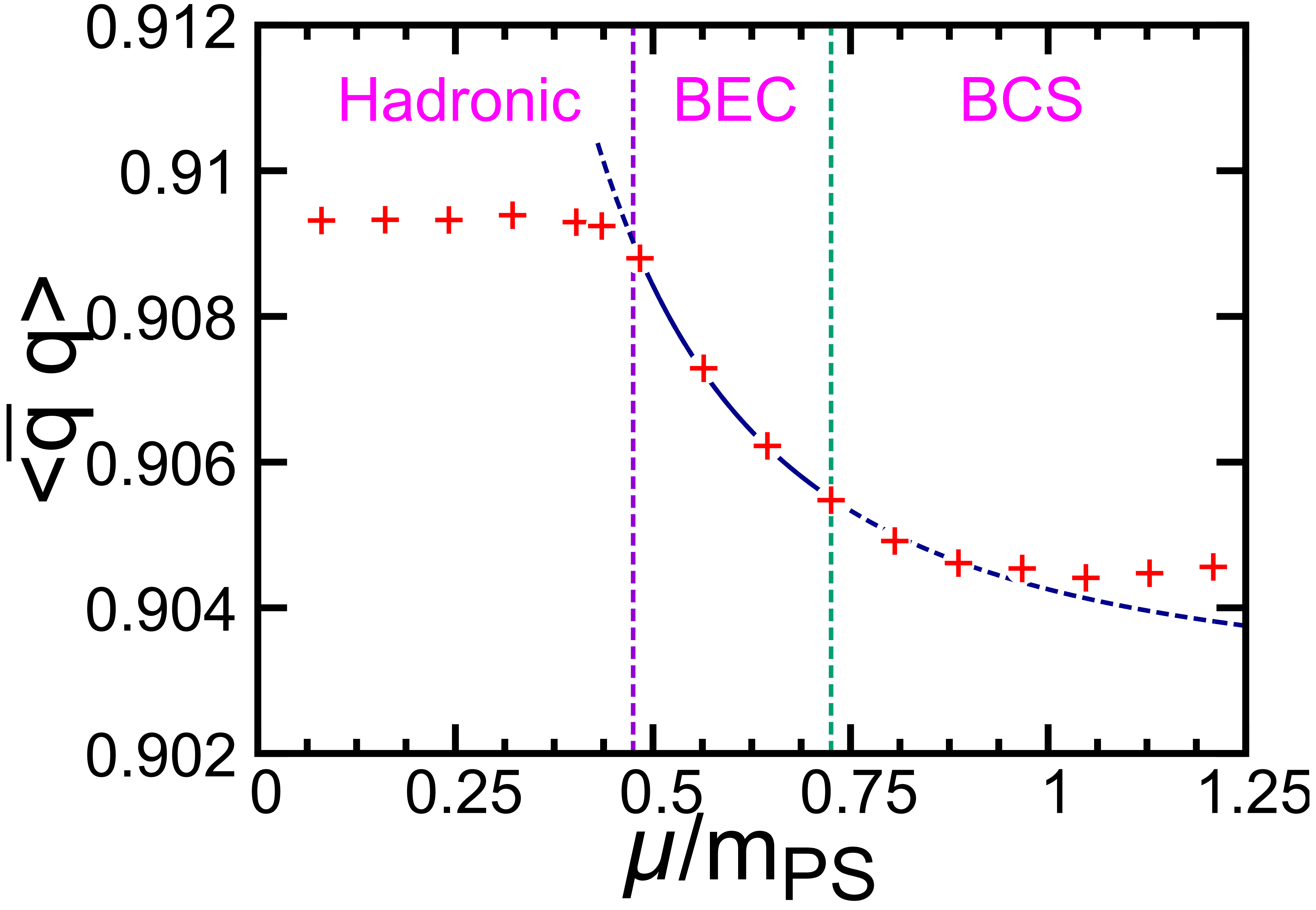
- Bose-Einstein-Condensate (BEC)
  strong coupling regime
  Well explained by ChPT
  $\langle qq \rangle \propto (\mu - \mu_c)^{1/2}$, EoS
  Kogut et al., NPB 582 (2000) 477
  Son and Stephanov (2001)

- BCS phase
  weak coupling regime
  $\langle qq \rangle \propto \mu^2$: weak coupling analysis
  $\langle n_q \rangle \approx n_q^{free}$
  T. Schaefer, NPB 575 (2000) 269
  confinement remains
  nontrivial instanton contributions
  Quarkyonic-matter (free-quark matter with confining gluons)
  Sound velocity exceeds the conformal bound
  ($c_s^2/c^2 > 1/3$)

Chiral symmetry and hadron spectra

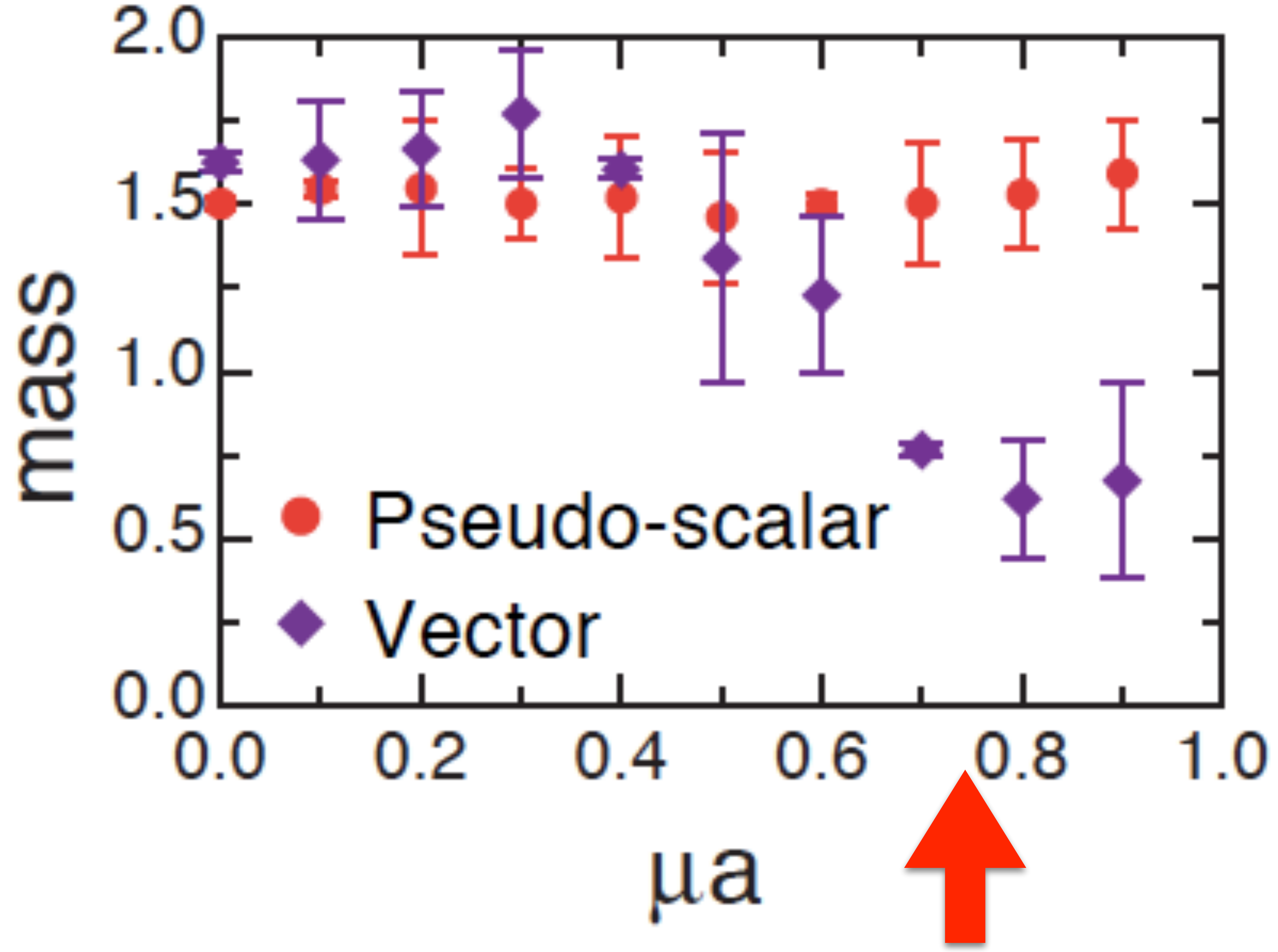
- In the superfluid phase,
  the chiral condensate decreases.

K.Iida et al., JHEP 10 (2024) 022



Muroya et al. Phys.Lett. B551 (2003) 305

2color QCD, Lattice size 4^3 x 8



$\mu/m_{PS} = 0.5$

Special properties in 2color QCD

- $\mu \neq 0$: γ_5-hermiticity is broken : $\gamma_5 D(\mu)\gamma_5 = D^\dagger(-\mu) \neq D^\dagger(\mu)$

- In dense 2color QCD, quarks take a pseudo-real reps.$\gamma_5 C\tau_2 D(\mu)\gamma_5 C\tau_2 = D^*(\mu)$, here C is charge conjugation
  From the QCD inequality, the iso-singlet scalar diquark (baryon) $M_{qq} = \psi^T C\tau_2\gamma_5\psi$ must be the lightest hadron

- In hadronic phase, violation of time-reversal symmetry ($\tau \leftrightarrow (N_\tau - \tau)$) of C(tau)
  $E(\vec{p},\mu) = \sqrt{\vec{p}^2 + m^2} - \mu n_O$, it can be regarded as a mass shift: $m(\mu) = m(\mu=0) - n_O\mu$

- In the SuperFluid (SF) phase, meson-baryon mixing
  $U(1)_B$ is broken, no difference between mesons and baryons
  Technically, consider all possible contractions of $\bar{q}$-q and q-q.

$|0\rangle$: vacuum state at $\mu = 0$ ($\hat{N}|0\rangle = 0$)
(This is valid only for Hadronic phase)
On lattice, $\langle\Omega_i|\hat{N}|\Omega_i\rangle = 0$ for each conf. in $\mu < m_\pi/2$
K. Nagata, PPNP(2022), A.Alexandru et al., JHEP(2016)

Flavor symmetry and its breaking in Nc=Nf=2

$\boxed{SU(4) \text{ at } m = \mu = 0}$

$\downarrow$ $m > 0$

$Sp(4) \simeq SO(5)$

5 pseudo-Goldstone bosons
$3\pi, qq, \bar{q}\bar{q}$

$\downarrow$ $0 < \mu < \mu_c$

$SU(2)_V \times U(1)_B$
meson-baryon sym.

diquark condensation $\downarrow$ $\mu_c < \mu$

$Sp(1)_V \simeq SU(2)_V$

- Extended multiplet: $\Psi \equiv \begin{pmatrix} \psi \\ K\bar{\psi}^T \end{pmatrix}$

  Lagrangian is invariant under $U \in SU(4)$;

  $\Psi \to U\Psi$

- pion $P^a(x)$ and I=0 S diquark $qq(x)$

  $\delta P^a(x) = i\epsilon[X, P^a(x)] = i\epsilon c^a(qq)$

  for broken generator $X^\alpha \in SU(4)/Sp(4)$,

  $\alpha = 1, \cdots 5$ => 5-dim. multiplet $\{P^a(x), qq(x), \bar{q}\bar{q}(x)\}$

- Rho $V_i^a(x)$ and I=1 AV diquark $D_{AV}^{a,i}(x)$

  same multiplet (3dim.) of unbroken $Sp(4)$

2pt fn. formula using quark propagators

Mesons: $\bar{\psi}\Gamma\psi$

- iso-single (I=0) or iso-triplet (I=1)
- $\Gamma = \{1, \gamma^5, \gamma^1, i\gamma^5\gamma^1\}$={S, P, V, AV}

$$\begin{aligned}
\langle M^0(x)M^{0\dagger}(0)\rangle_F &= -2\mathrm{tr}\langle t,\vec{x};c,\beta|S_N\Gamma|t,\vec{x};c,\beta\rangle \cdot \mathrm{tr}\langle 0,\vec{0};c,\beta|S_N\bar{\Gamma}|0,\vec{0};c,\beta\rangle \\
&\quad +\mathrm{tr}\langle t,\vec{x};a,\rho|S_N\bar{\Gamma}|0,\vec{0},c;\beta\rangle \left(\langle t,\vec{x};a,\rho|\Gamma^\dagger S_N^\dagger|0,\vec{0};c,\beta\rangle\right)^* \\
&\quad +\mathrm{tr}\langle t,\vec{x};a,\rho|S_A\bar{\Gamma}^T|0,\vec{0},c;\beta\rangle \left(\langle t,\vec{x};a,\rho|\Gamma^\dagger \bar{S}_A^\dagger|0,\vec{0};c,\beta\rangle\right)^* \\
\langle M^1(x)M^{1\dagger}(0)\rangle_F &= \mathrm{tr}\langle t,\vec{x};a,\rho|S_N\bar{\Gamma}|0,\vec{0},c;\beta\rangle \left(\langle t,\vec{x};a,\rho|\Gamma^\dagger S_N^\dagger|0,\vec{0};c,\beta\rangle\right)^* \\
&\quad -\mathrm{tr}\langle t,\vec{x};a,\rho|S_A\bar{\Gamma}^T|0,\vec{0},c;\beta\rangle \left(\langle t,\vec{x};a,\rho|\Gamma^\dagger \bar{S}_A^\dagger|0,\vec{0};c,\beta\rangle\right)^*
\end{aligned}$$

Diquarks: $\psi_1^T K\Gamma\psi_2$, $K \equiv C\gamma_5\tau_2$

- iso-single (I=0) or iso-triplet (I=1)
- $\Gamma = \{1, \gamma^5, \gamma^1, i\gamma^5\gamma^1\}$={S, P, V, AV}

$$\begin{aligned}
\langle D^0(x)D^{0\dagger}(0)\rangle_F &= -2\mathrm{tr}\langle t,\vec{x};c,\beta|\bar{S}_A\Gamma K|t,\vec{x};c,\beta\rangle \cdot \mathrm{tr}\langle 0,\vec{0};c,\beta|S_A K\bar{\Gamma}|0,\vec{0};c,\beta\rangle \\
&\quad +\mathrm{tr}\langle t,\vec{x};a,\rho|S_N\Gamma K|0,\vec{0},c;\beta\rangle \left(\langle t,\vec{x};a,\rho|\bar{\Gamma}^\dagger K \bar{S}_N^\dagger|0,\vec{0};c,\beta\rangle\right)^* \\
&\quad +\mathrm{tr}\langle t,\vec{x};a,\rho|S_N K\Gamma^T|0,\vec{0};c,\beta\rangle \left(\langle t,\vec{x};a,\rho|\bar{\Gamma}^\dagger K \bar{S}_N^\dagger|0,\vec{0};c,\beta\rangle\right)^* \\
\langle D^1(x)D^{1\dagger}(0)\rangle_F &= -\mathrm{tr}\langle t,\vec{x};a,\rho|S_N\Gamma K|0,\vec{0},c;\beta\rangle \left(\langle t,\vec{x};a,\rho|\bar{\Gamma}^\dagger K \bar{S}_N^\dagger|0,\vec{0};c,\beta\rangle\right)^* \\
&\quad +\mathrm{tr}\langle t,\vec{x};a,\rho|S_N K\Gamma^T|0,\vec{0};c,\beta\rangle \left(\langle t,\vec{x};a,\rho|\bar{\Gamma}^\dagger K \bar{S}_N^\dagger|0,\vec{0};c,\beta\rangle\right)^*
\end{aligned}$$

Updated points from Hands et al. (PLB 2007) and Murakami et al. (2023, Proceeding of Lattice 2023)

Disconnected contribution is included

$\mathrm{Tr}_{\vec{x},\mathrm{color},\mathrm{spinor}}[O\langle\tau,\vec{x}\,|\,\tau,\vec{x}\rangle] - \langle \sum_{\tau} \mathrm{Tr}_{\vec{x},\mathrm{color},\mathrm{spinor}} O(\tau,\vec{x})/N_\tau\rangle$ and its at $(\tau = 0, \vec{x} = \vec{0})$

Mesons has 1pt fn. of $\mathrm{Tr}[S_N\Gamma]$, which are relatively large.

Diquark has the one of $\mathrm{Tr}[S_A\Gamma]$, which are proportional to $J^2 \ll 1$ and it is negrigible

All hadrons and its qualities of signals

Note that to satisfy Fermi statistics, diquarks satisfy $\psi_1^T(C\gamma_5\Gamma)(\tau^a/2)\psi_2 = -\psi_2^T(C\gamma_5\Gamma)(\tau^a/2)\psi_1$.

I=1 channel, spin structure $C\gamma_5\Gamma$ should be symmetric => AV is allowed

I=0 channel, spin structure should be anti-symmetric => S, P, V are allowed

In the words in meson sector, $a_1, \sigma, \eta, \omega$ mesons have a baryon-mixing in the SF phase.

(To disentangle the mixing, we have to solve a generalized eigenvalue problem (GEVP))

	Meson (Hadronic)	Diquark (Hadronic)	Higgs/NG (Hadronic)	Meson(SF)	Diquark(SF)	Higgs/NG (SF)	Disconn.
I=0, 0+ (sigma)	Noisy	OK	OK	OK	OK	Noisy	Yes
I=1, O+ (a0)	Noisy	-	OK	Heavy?	-	OK (massless?)	-
I=0, 0- (eta)	OK	OK	-	Noisy	OK	-	YES
I=1, 0- (pion)	OK	-	-	OK	-	-	-
I=0, 1- (omega)	OK	OK	-	Heavy?	OK ?	-	YES
I=1, 1- (rho)	OK	-	-	OK	-	-	-
I=0, 1+ (f1)	Noisy	-	-	Noisy	-	-	YES
I=1, 1+ (a1)	Noisy	OK	-	OK	OK	-	-

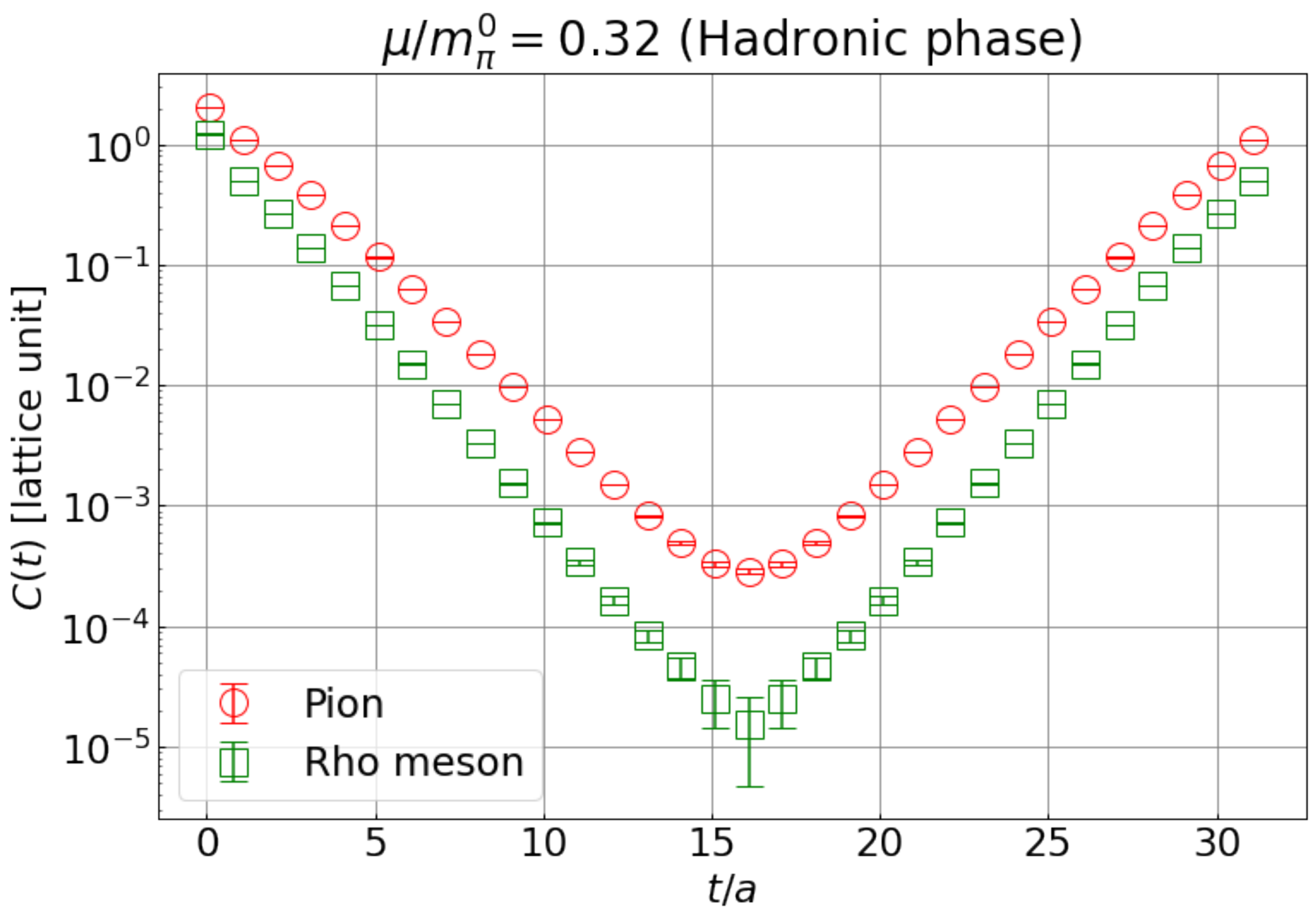
Results

K. Iida, EI, K. Murakami, D. Suenaga, arXiv2606.13974

Correlation fn.: pion and rho meson

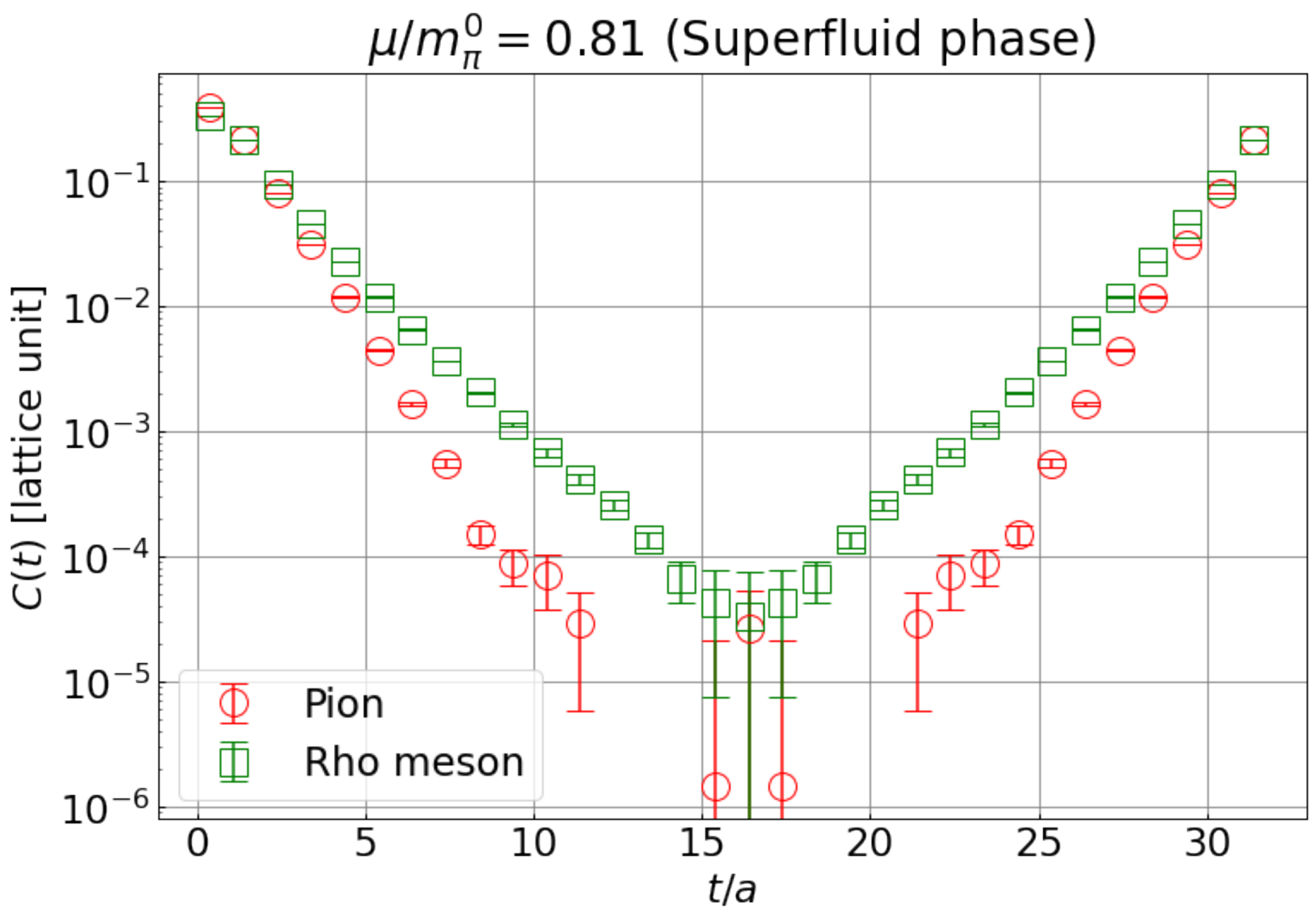
K. Murakami et al., PoS LATTICE2022 (2023) 154

Hadronic phase (low-density)



$C_\pi(\tau) > C_\rho(\tau) \iff m_\pi < m_\rho$

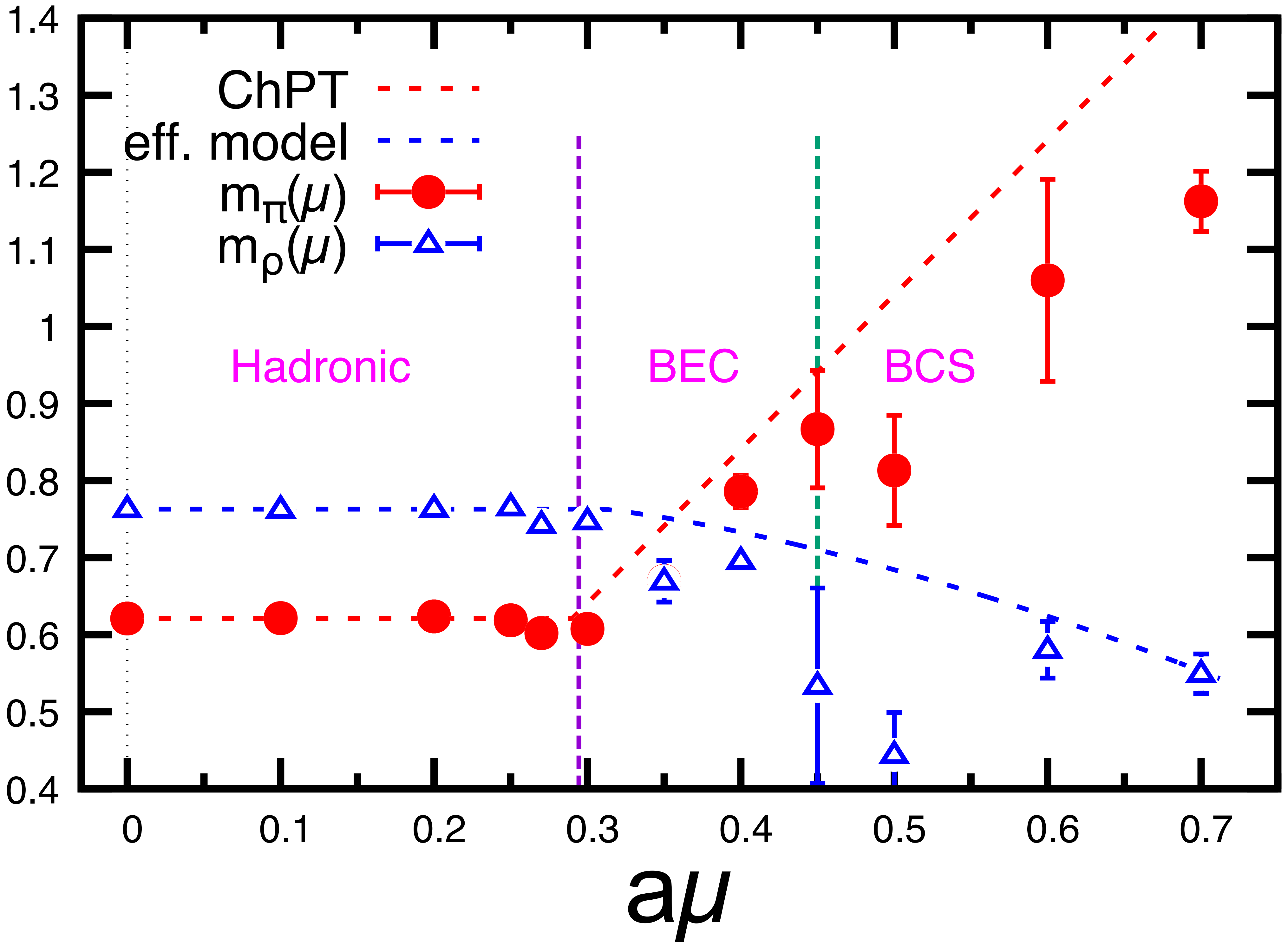
Superfluid phase (high-density)



$C_\pi(\tau) < C_\rho(\tau) \iff m_\pi > m_\rho$

Furthermore, pion may decay to something?

μ-deps of mass: pion and rho meson



- In hadronic phase,
  the pion is lighter than the rho
  (as usual)
  In SF phase, that is opposite.

- ChPT prediction for pion

  $$m_\pi^{\mathrm{ChPT}}(\mu) = m_\pi(\mu = 0) + 2(\mu - \mu_c)\theta(\mu - \mu_c)$$

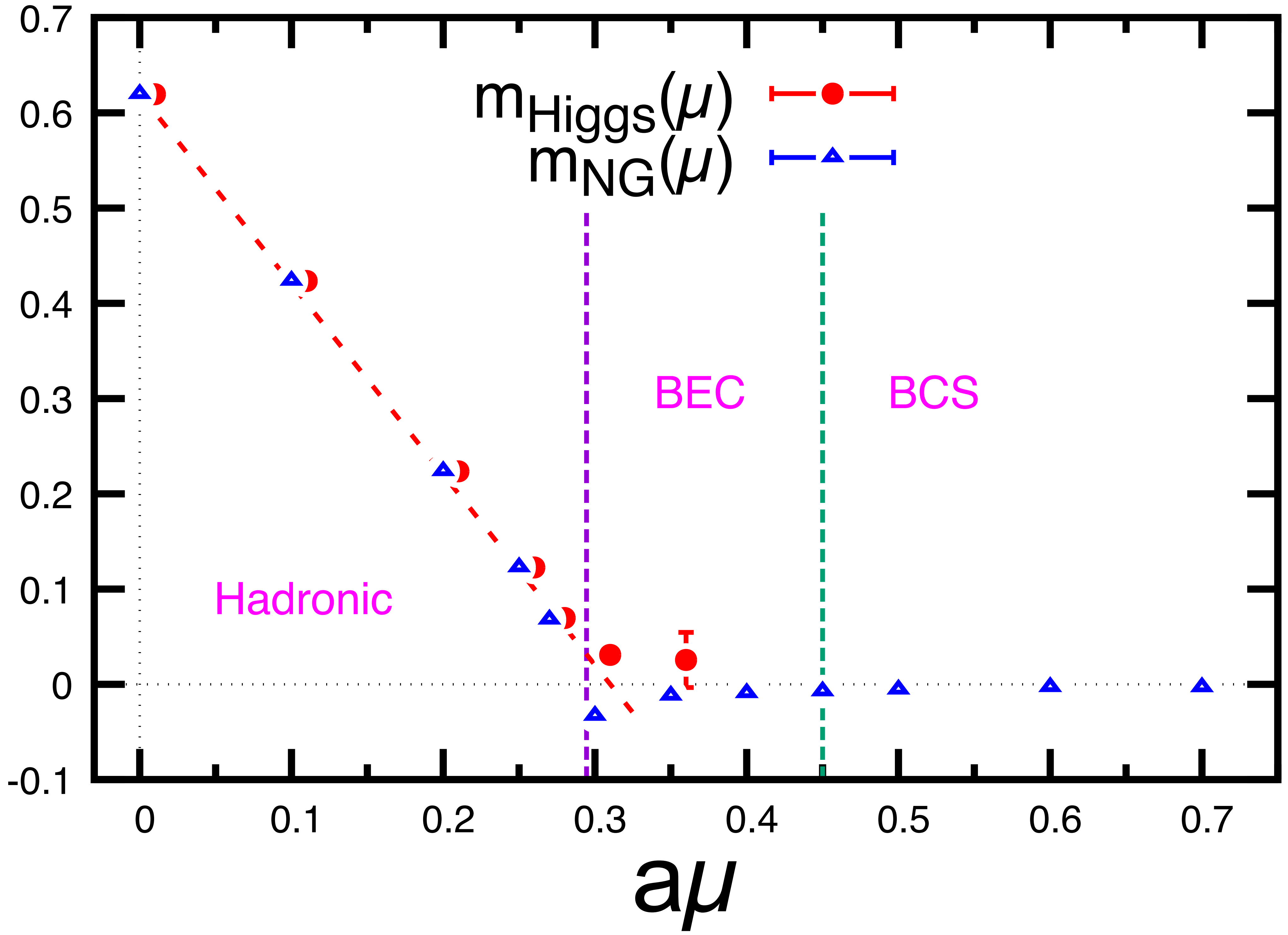
  Kogut et al., NPB(2000)

  Effective model prediction for rho

  Suenaga et al., PRD (2024)

- In SF phase, both signals are noisy.
  It indicates an existence of further lighter hadron?

Mass spectra: Higgs and NG mode of $U(1)_B$ breaking



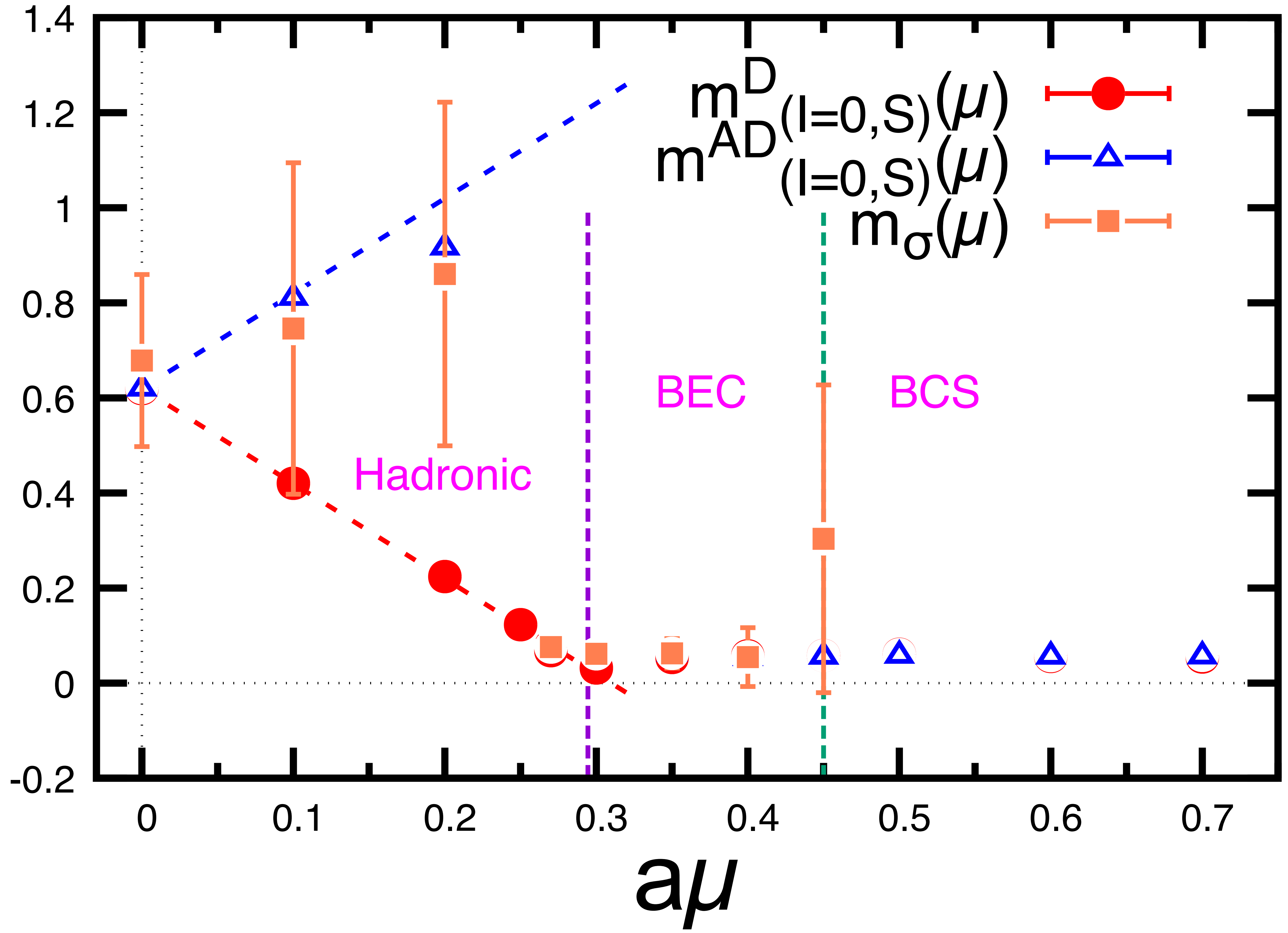
- In hadronic phase, both modes are degenerated.
  They follow the prediction of mass shift

  $m(\mu) = m(\mu = 0) - 2\mu$

- In SF phase, NG mode is almost massless.

- As for the Higgs mode, the signal becomes very noisy, so that we cannot calculate its mass.

Mass spectra: I=0 scalar diquark and meson



- In hadronic phase, the prediction of mass shift

  $m(\mu) = m(\mu = 0) - n_O \mu$

  is reproduced.

  $m(\mu) = m(\mu = 0) - 2\mu$:diquark

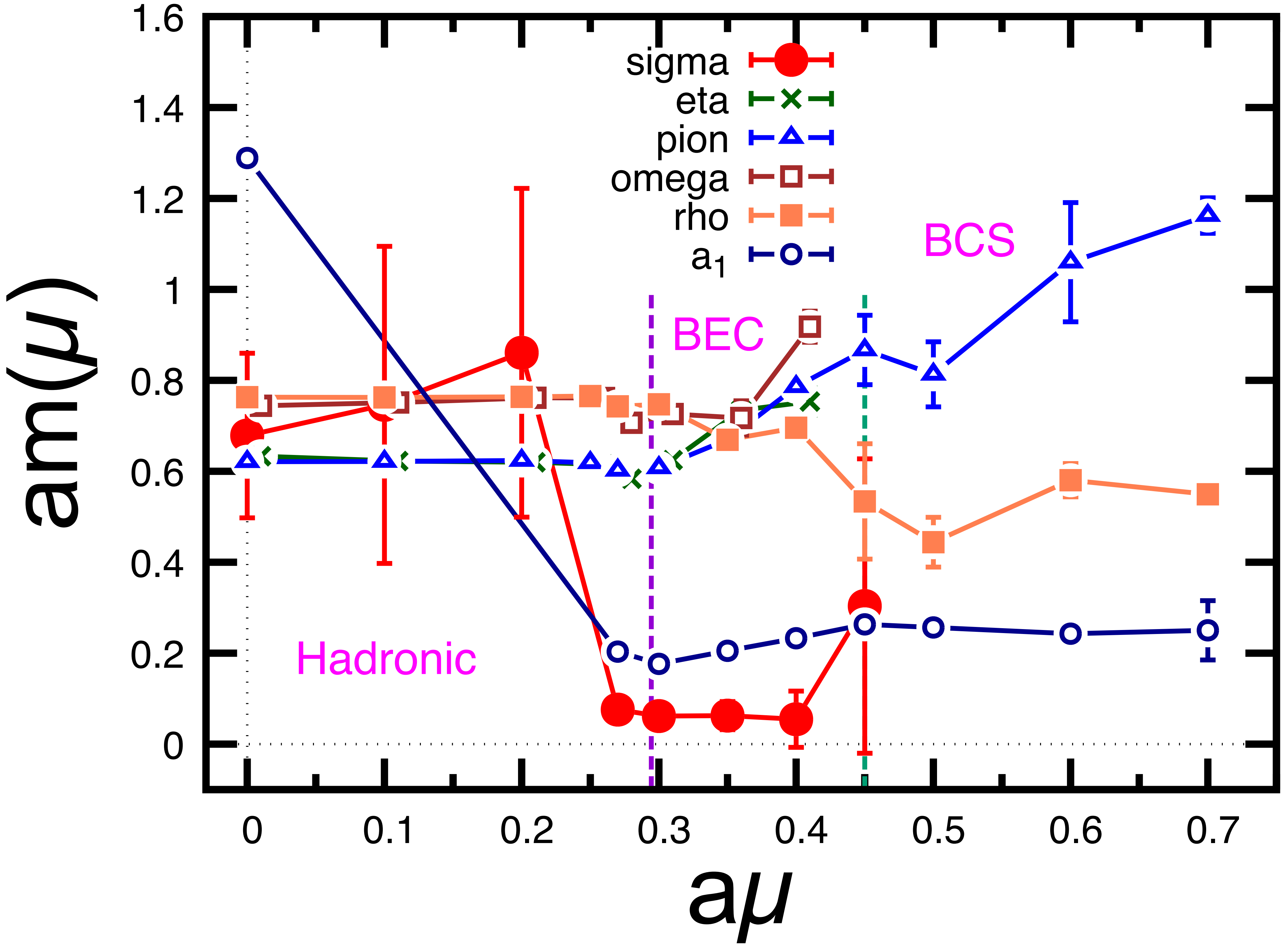
  $m(\mu) = m(\mu = 0) + 2\mu$: anti-diquark

  $m(\mu) = m(\mu = 0)$: sigma meson

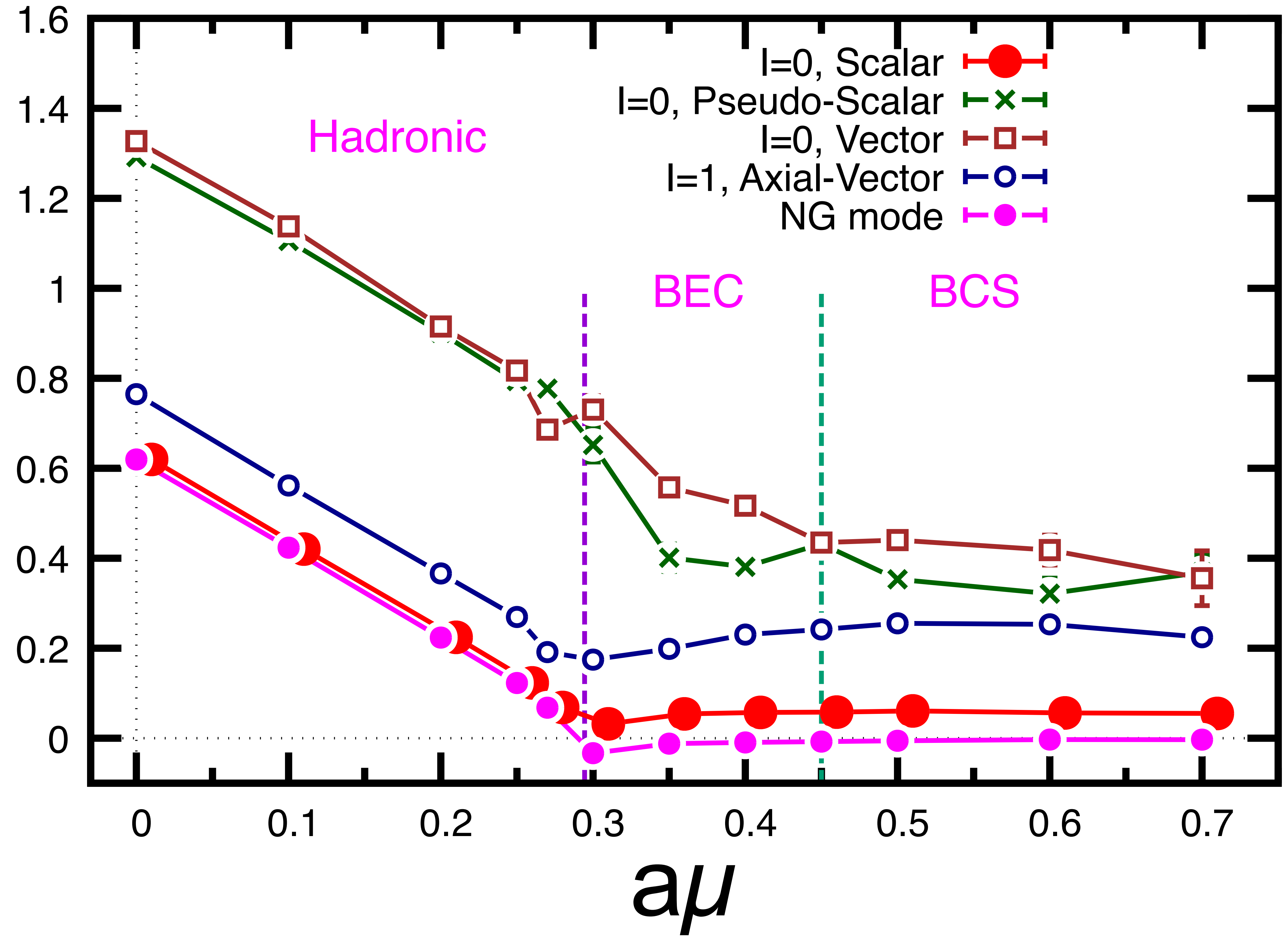
- In SF phase, all three modes are degenerated.
  To resolve the meson-baryon mixing, we will need the variational method (generalized eigenvalue problem (GEVP) method).

Summary of mass spectra

Mesons



Diquarks



In Hadronic phase, $m_\pi \lesssim m_\eta < m_\sigma(\text{noisy}) < m_\rho \sim m_\omega \ll m_{a_1}$

(same with 3color QCD vacua)

In SF phase, $m_\sigma(\text{noisy}) < m_{a_1} < m_\rho < m_\pi \sim m_\eta(\text{noisy}) \ll m_\omega(\text{noisy})$

In both phases, $m_{NG} \lesssim m_{I=0,S} < m_{I=1,AV} < m_{I=0,PS} \lesssim m_{I=0,V}$

(In our notation($\psi_1^T K \Gamma \psi_2$, $K \equiv C\gamma_5\tau_2$), at $\mu = 0$

$\pi \leftrightarrow D_{I=0,S}$, $\rho \leftrightarrow D_{I=1,AV}$ are same multiplets)

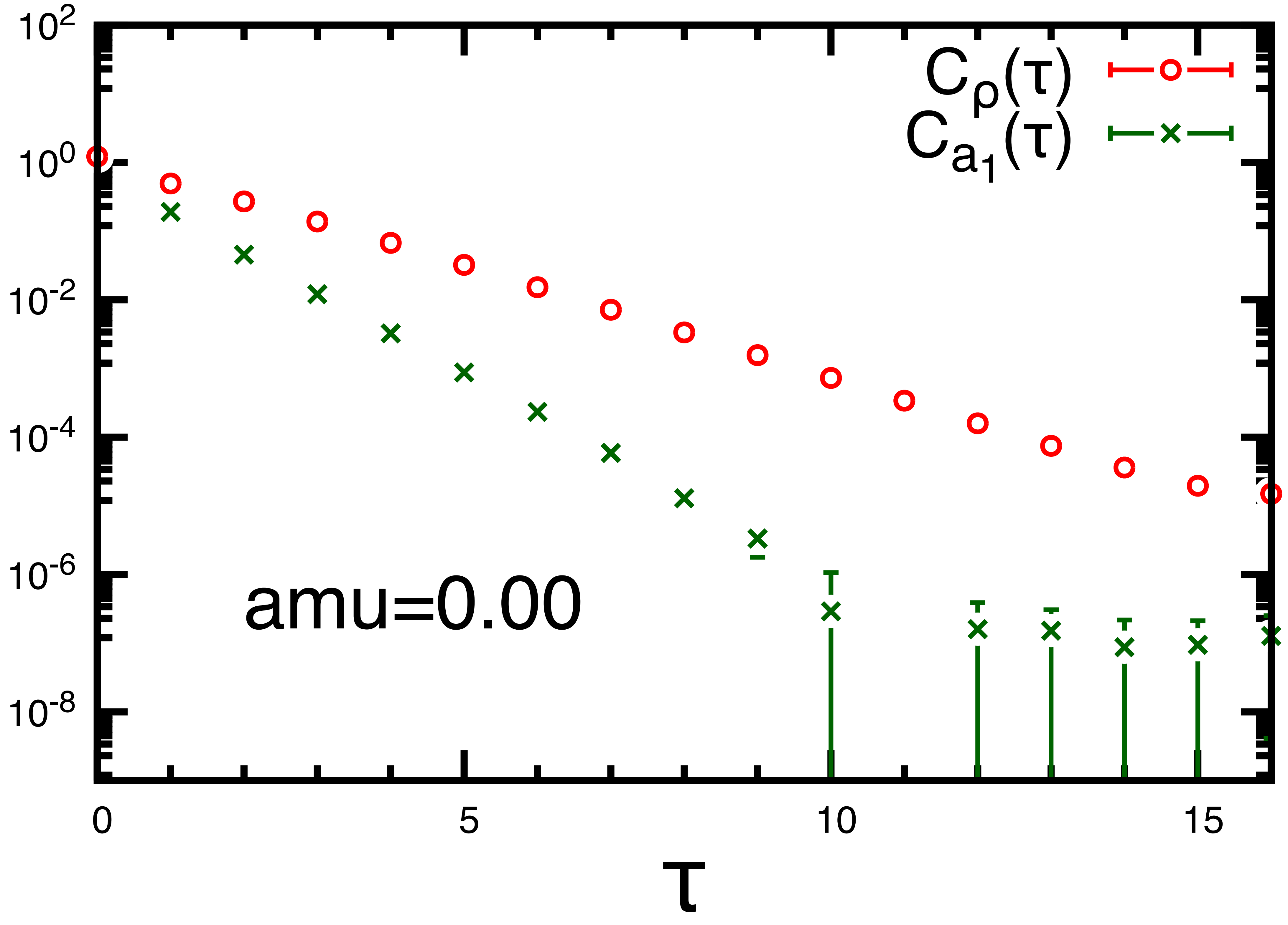
Chiral symmetry and 2pt fn.

K. Iida, EI, K. Murakami, D. Suenaga, arXiv2606.13974

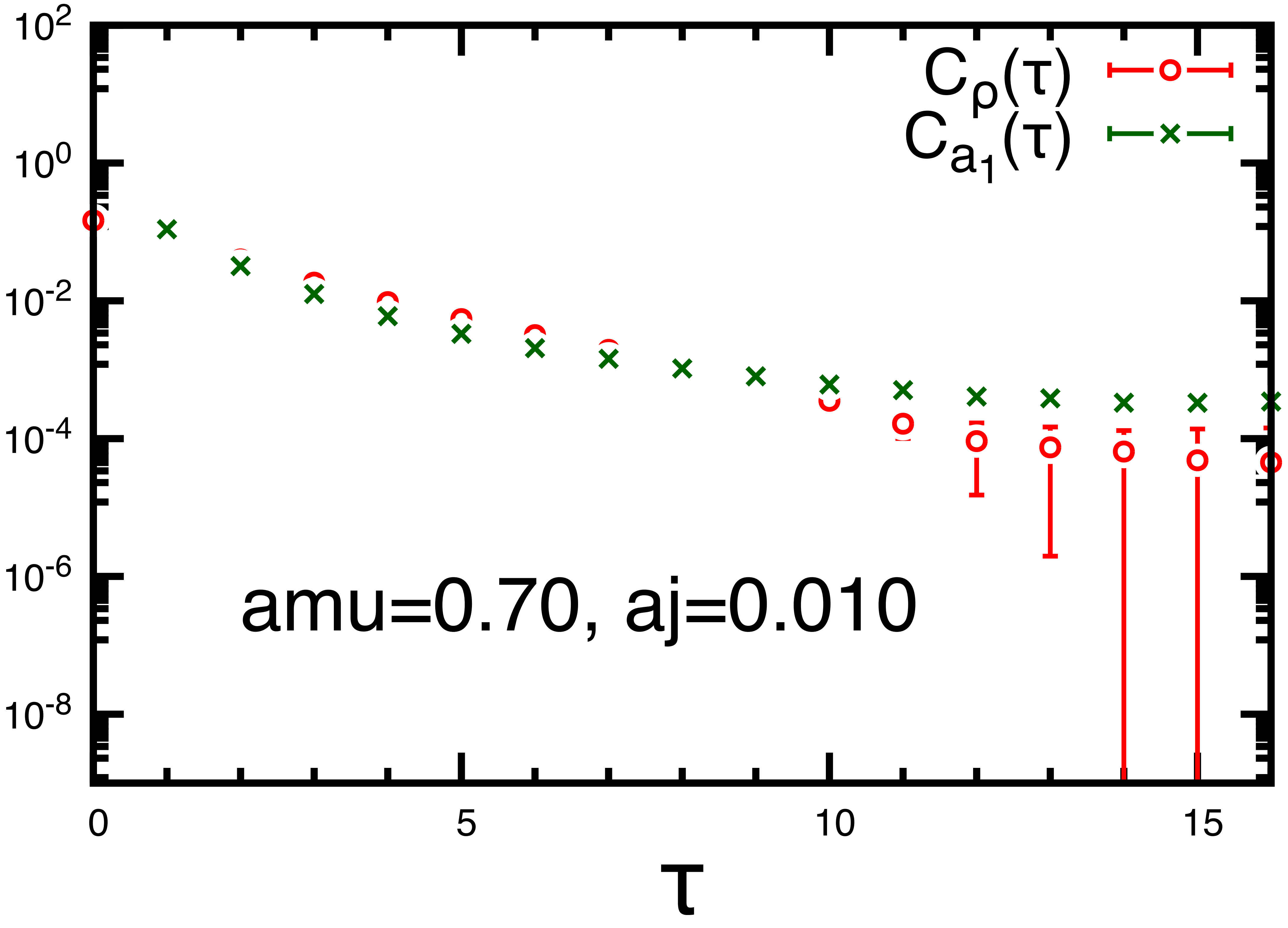
2pt fn. of rho and a1 mesons

We take $\mu = 1$ for V_μ^a and A_μ^a current op. to extract J=1 component.

Hadronic phase (low-density)



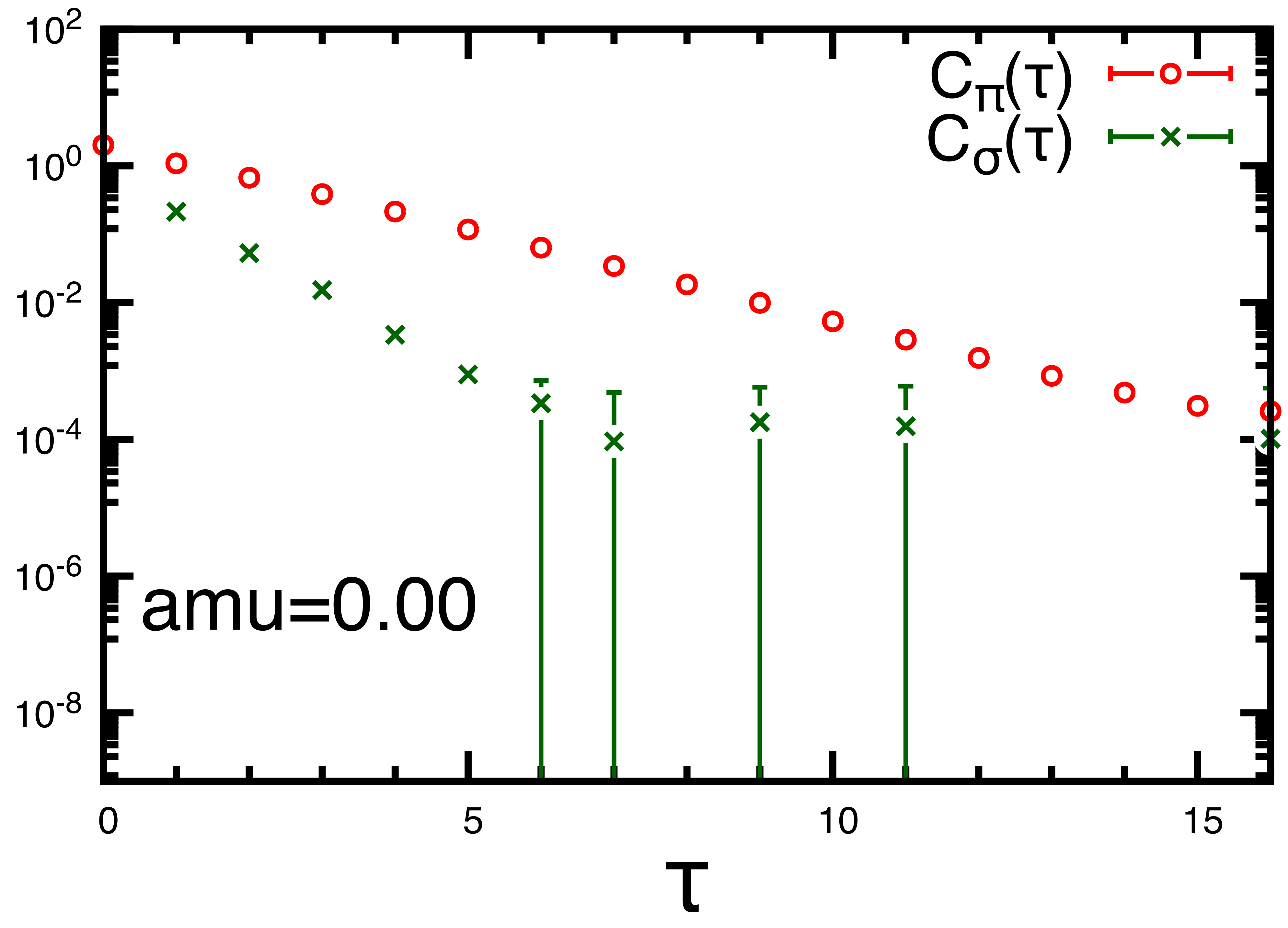
Superfluid phase (high-density)



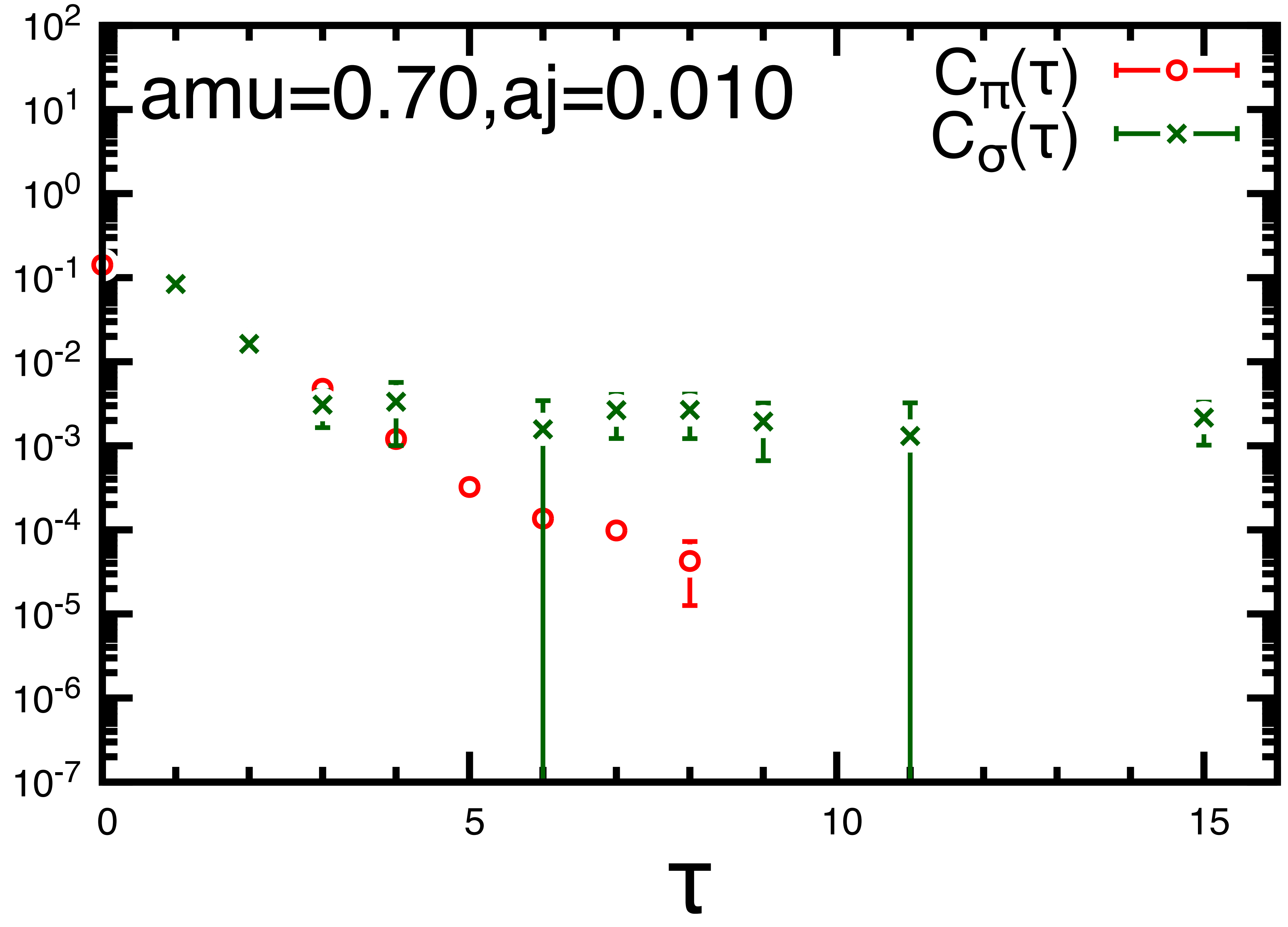
At high density, two 2pt-fns. appear to degenerate.

2pt fn. of pion and sigma meson

Hadronic phase (low-density)



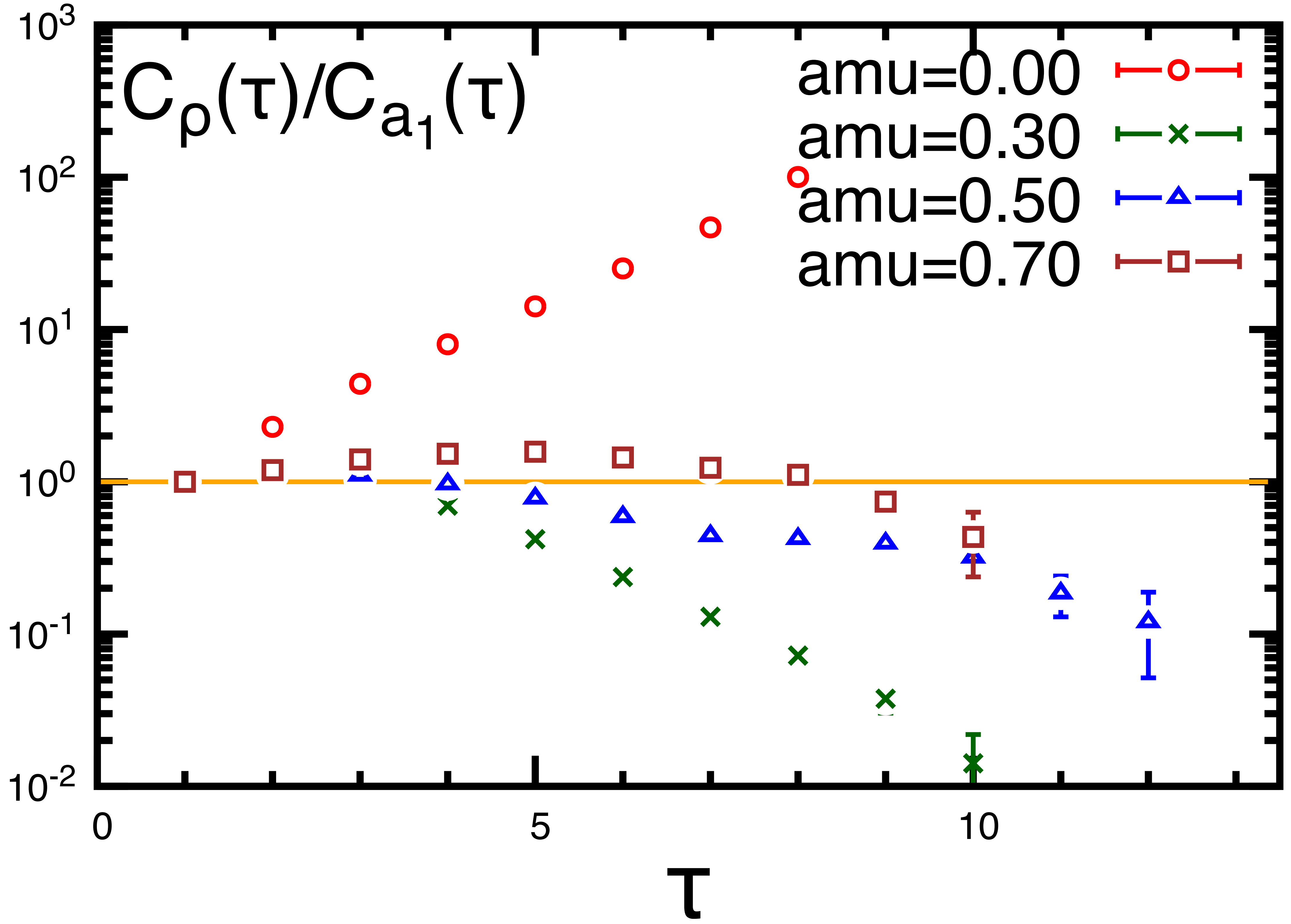
Superfluid phase (high-density)



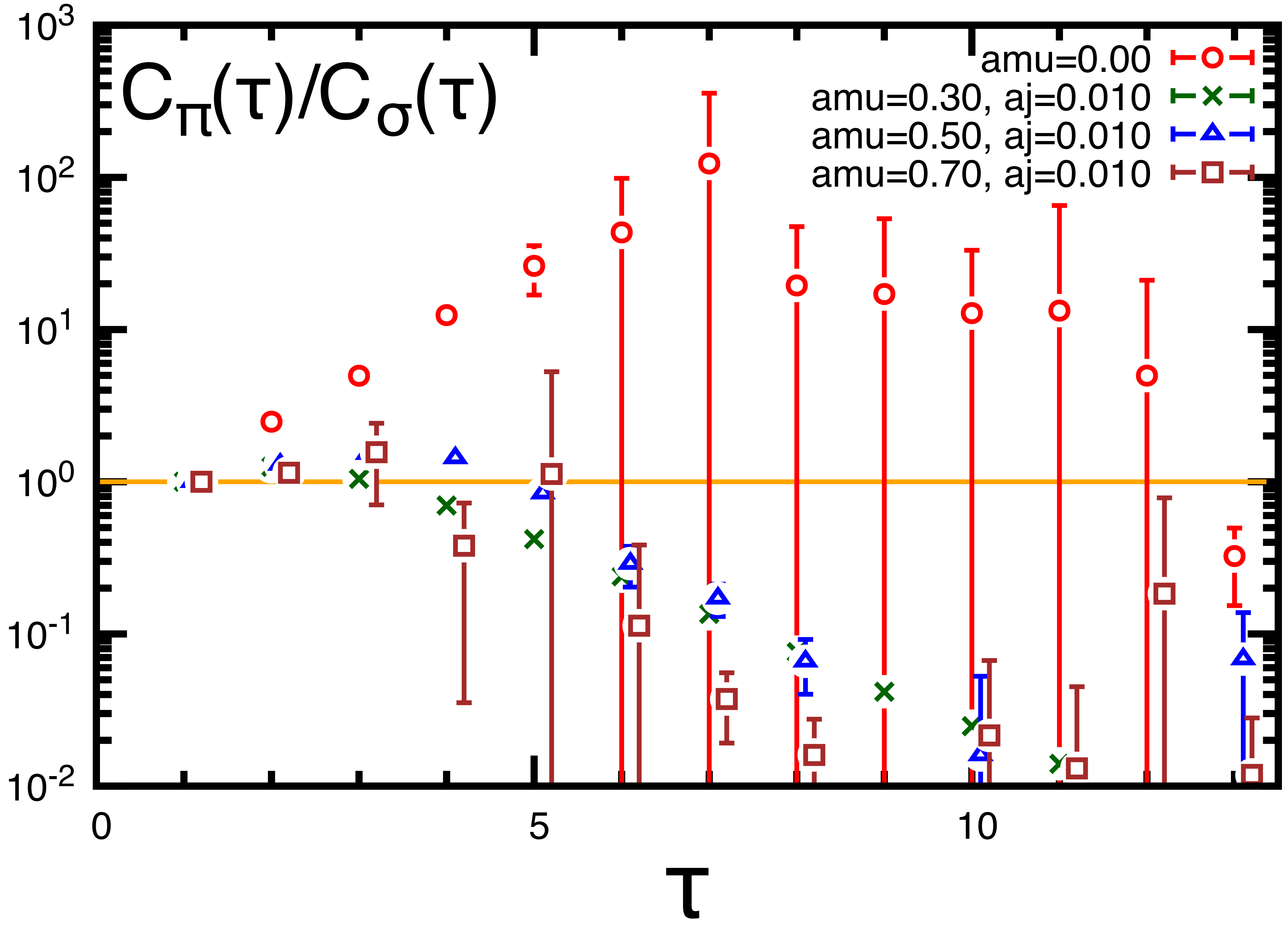
At high density, two 2pt-fns. appear to degenerate.
(but noisy…)

Ratio of 2pt fn. bw chiral partners

Rho and a1



Pion and sigma



Here, the ratio is normalized to 1 with $\tau = 1$.

A region close to 1 extends in higher density.

It suggests the recovery of chiral symmetry.

Summary

Summary and future direction

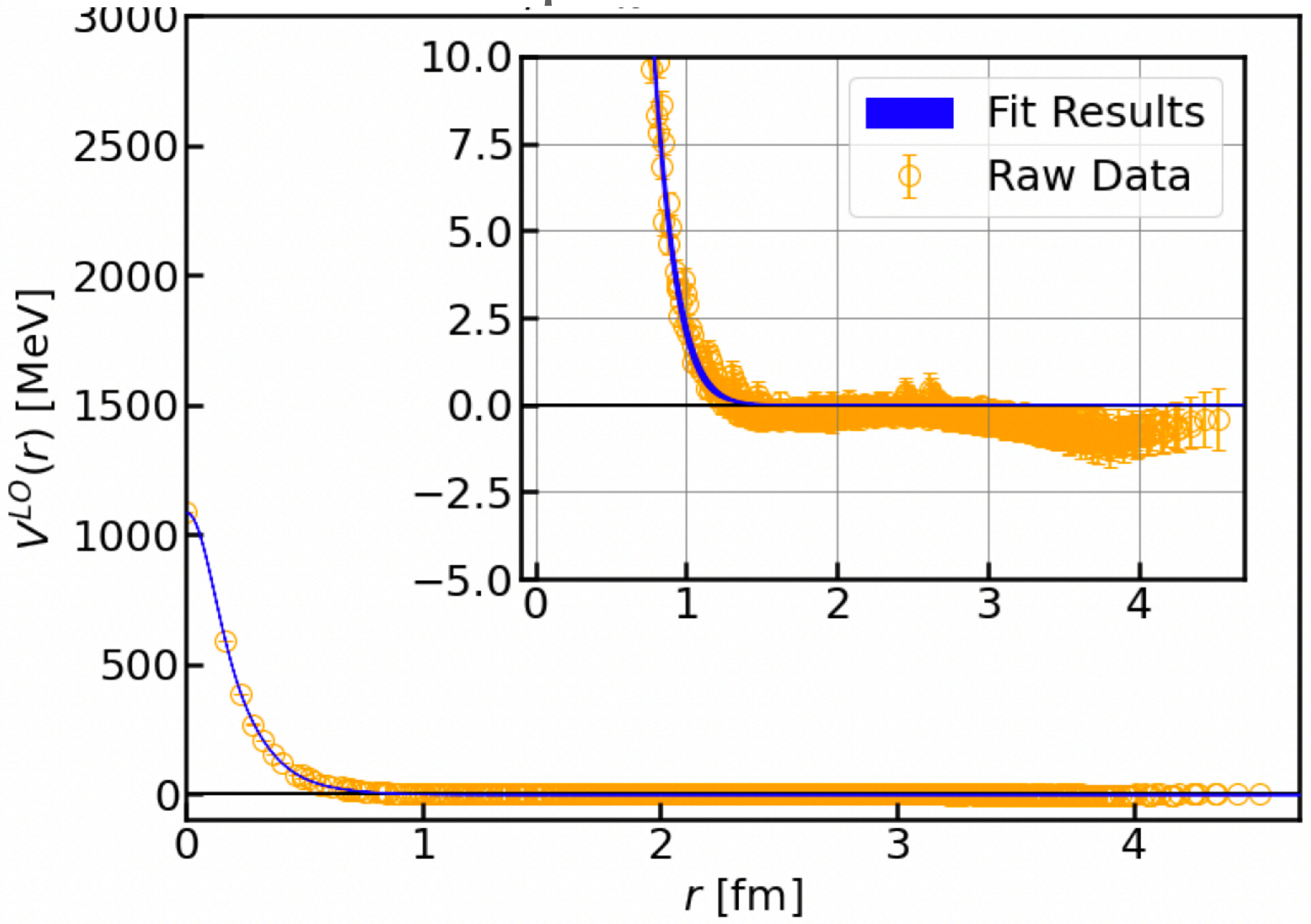
- In two-color QCD, the mass spectra even in the superfluid phase can be calculated using first-principles calculations
- Meson channels may be similar to 3-color QCD.
  In particular, mesons without diquark-mixing, as a0(noisy), pion, rho, f1(noisy)
- The 2pt fn. of diquark/anti-diquark has time-reversal asymmetry in the hadronic phase.
- NG mode for $U(1)_B$ breaking becomes massless in the SF phase.
  It is a special property for 2color QCD.
- A tendency towards the restoration of chiral symmetry is observed from the degeneracy of the 2-pt fn. of chiral partners
- Technically, more detailed analysis to solve the meson-baryon mixing in superfluid phase by a generalized eigenvalue problem (GEVP) is important future work.
- Future directions:
  Extend the HAL QCD method to the SF phase! For the hadronic phase: K. Murakami et al., JHEP 02 (2024) 152
  In finite-T and finite-μ, a talk will be given by K. Tohme (3:20pm, 31st, Fri)

Hadron potential by HAL QCD method

- In hadronic phase, pion and diquark potential are equivalent because of extended flavor symmetry.

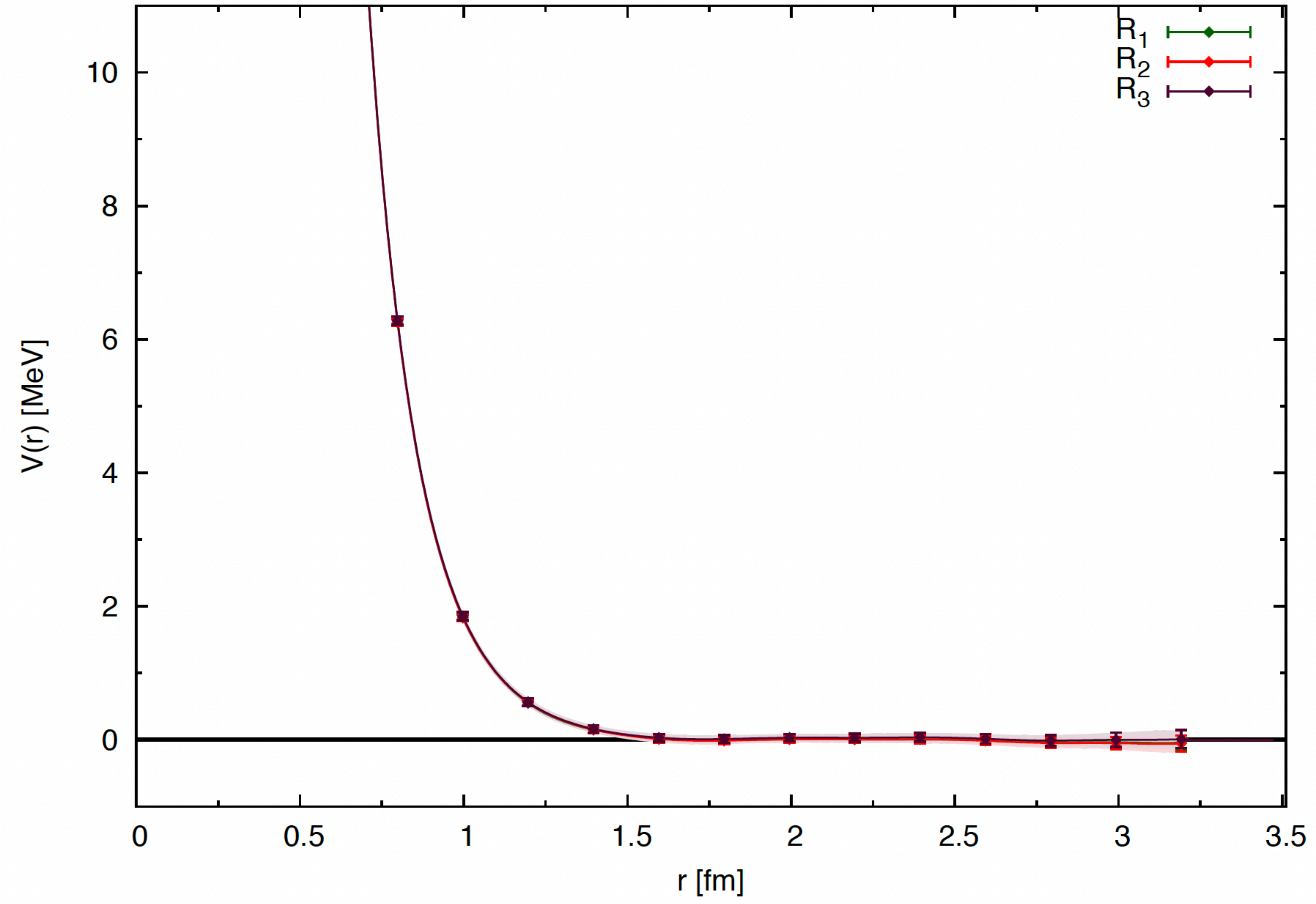
- Pion potential for 2color and 3color QCD are qualitatively same

Diquark-diquark potential
in hadronic phase of 2color QCD



K.Murakami, K.Iida, EI, JHEP 11 (2023) 231

I=2 $\pi\pi$ potential of 3color QCD

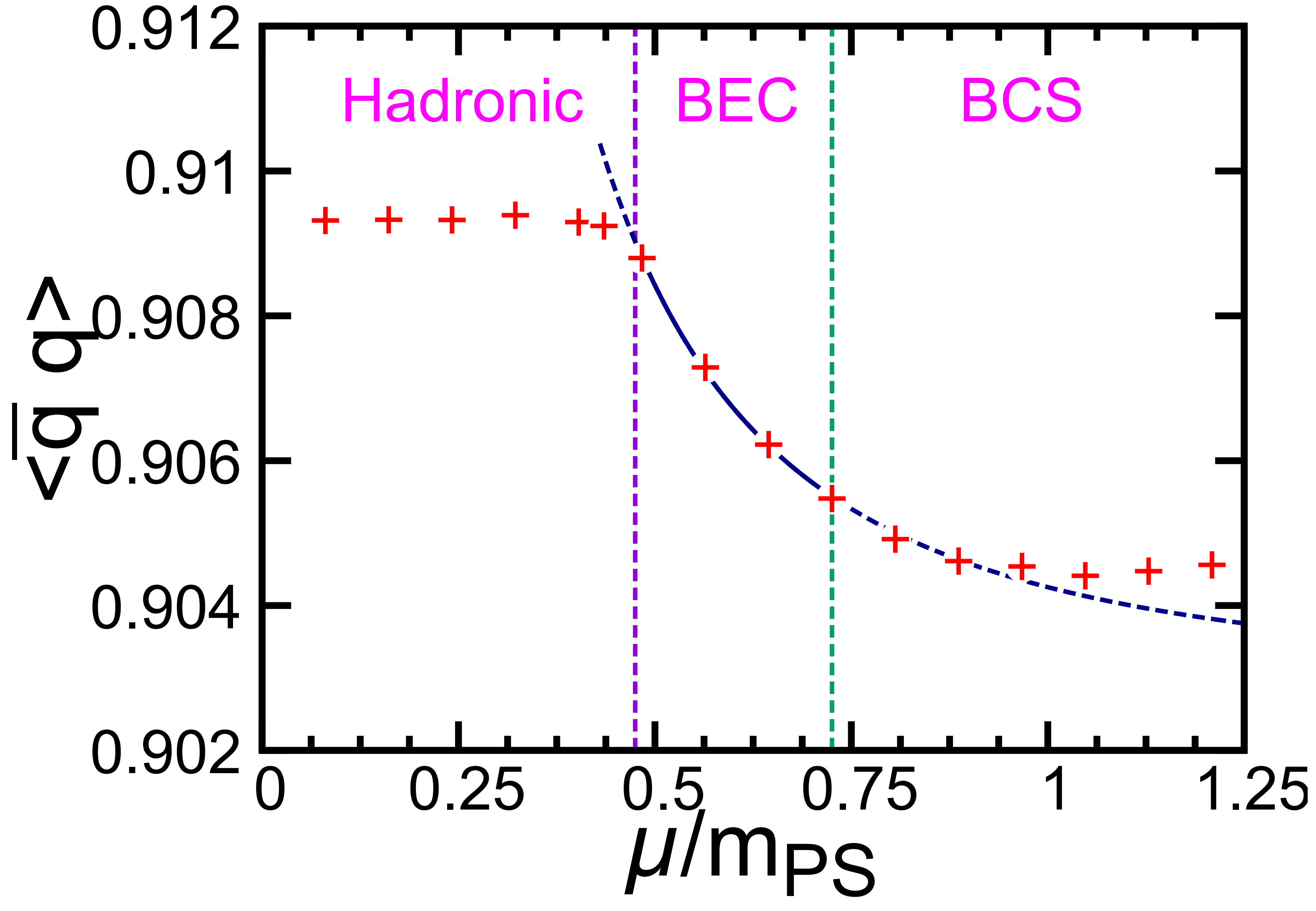


T.Kurth et al.(HAL QCD coll.), JHEP12(2013)015

Backup

Chiral symmetry and hadron spectra

K.Iida et al., JHEP 10 (2024) 022

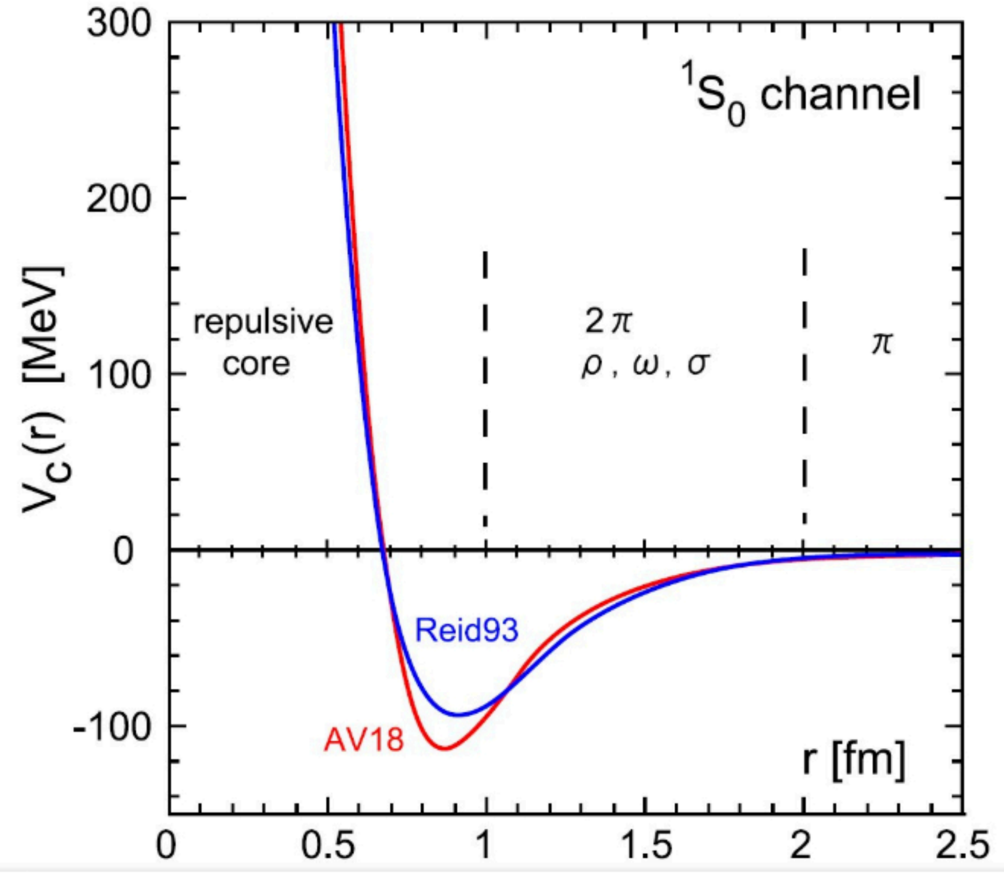


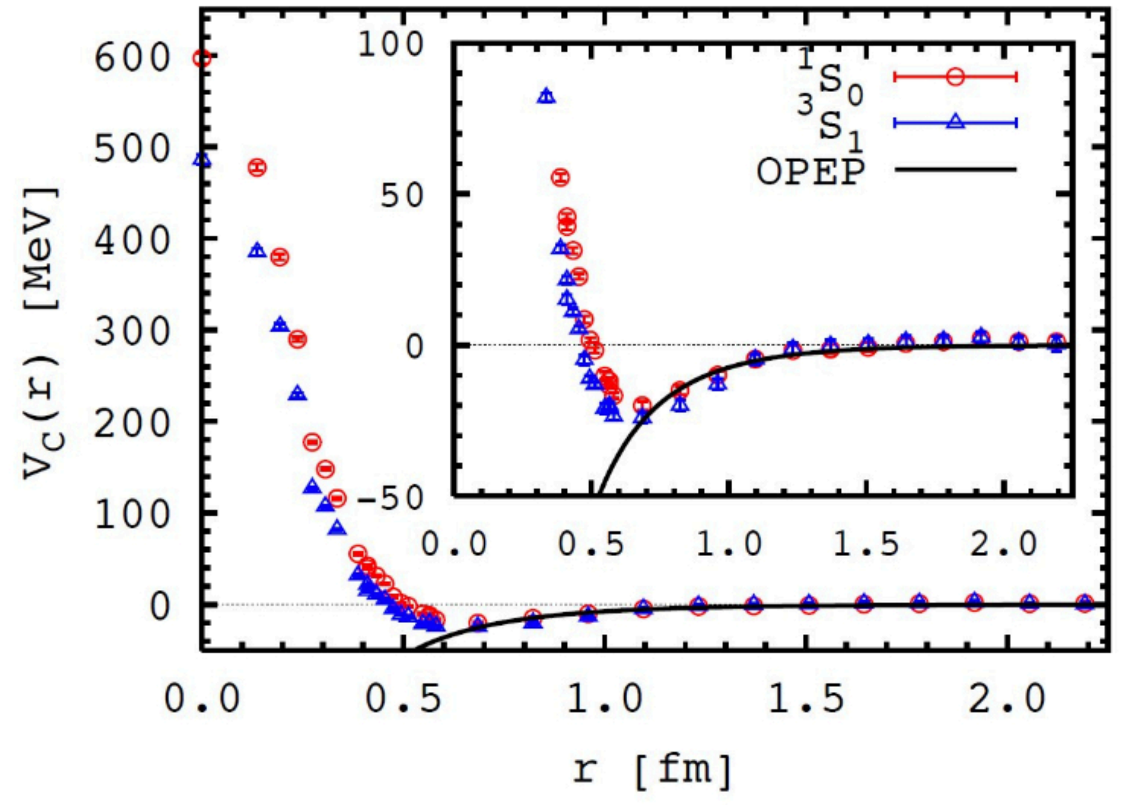
- In the superfluid phase, the chiral condensate decreases.
  Some hadron masses should change.

  Brown-Rho (1991)

- The NN potential may also be modified at finite density if meson masses change.

- At $\mu = 0$, the method for calculating mass spectra and the NN potential is well established.





(left) Examples of phenomenological nuclear potential, (right) The lattice QCD result of nuclear potential

Aoki, Ishii, Hatsuda
HAL QCD coll.(2007 -)

Degeneracy of 2pt fn. bw chiral partners

- Theoretically, the degeneracy of 2pt fn. bw chiral partner directly relate with the chiral restoration.

  Iso-triplet S and PS (a0 and pion) :$\langle S^a(x)S^a(y) - P^a(x)P^a(y)\rangle_0 \neq 0 \to Q_5^a|0\rangle \neq 0$

  Iso-triplet V and AV (rho and a1) :$\langle V^a(x)V^a(y) - A^a(x)A^a(y)\rangle_0 \neq 0 \to Q_5^a|0\rangle \neq 0$

  Here, $Q_5^a(t) = \int \frac{1}{2}(\bar{q}\gamma_0\gamma_5\tau^2 q)(t,x)d^3x$: iso-triplet axial-vector current

  is the generator of chiral transformation.

- If the chiral sym. is restored, then difference bw them must be zero at each point!

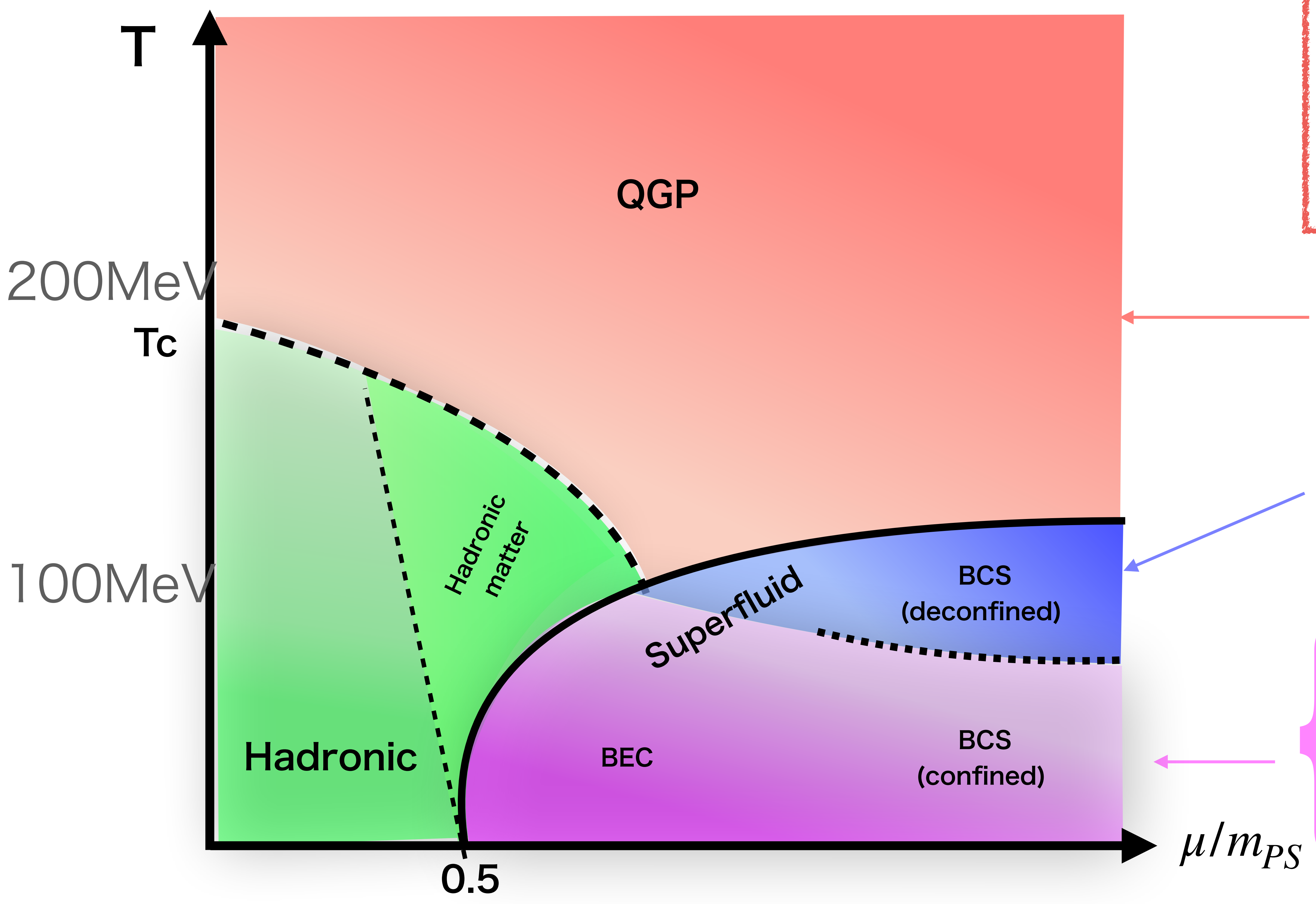
2color QCD phase diagram

(1) K.Iida, EI, K.Murakami, D.Suenage arXiv: 2405.20566 [hep-lat]
(2) K.Iida, K.Ishiguro , EI, arXiv: 2111.13067
(3) K.Iida, EI, T.-G. Lee: PTEP2021(2021) 1, 013B0
(4) K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
(5) T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

Our 2color QCD projects

- K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
  Phase diagram by Lattice simulation (T=80MeV)
- T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253
  Phase diagram by 't Hooft anomaly matching
- K.Iida, EI, T.-G. Lee: PTEP2021(2021) 1, 013B0
  Scale setting of Lattice simulation
- K.Ishiguro, K.Iida, EI, PoS, Lattice 2021
  Flux tube and quark confinement by Lattice simulation
- K.Iida, EI, PTEP 2022 (2022) 11, 111B01
  Velocity of sound by Lattice simulation (T=80MeV, 16^4 lattices)
- D. Suenaga, K.Murakami, EI, K.Iida, PRD 107, 054001 (2023) and
  Mass spectrum using effective model
- K.Murakami, D.Suenaga, K.Iida, EI, PoS, Lattice 2022
  Mass spectrum by Lattice simulation
- K.Murakami, K.Iida, EI, JHEP 02 (2024) 152
  Hadron potential w/ finite μ by Lattice simulation
- K.Iida, EI, K.Murakami, D. Suenaga, JHEP 10 (2024) 022
  Phase diagram and EoS by Lattice simulation (T=40MeV, 32^4 lattices)

Current status on 2color QCD phase diagram



At least 4 independent groups are studying the phase diagram

(1) S. Hands group : Wilson-Plaquette gauge + Wilson fermion
(2) Russian group : tree level improved Symanzik gauge + rooted staggered fermion
(3) Our group : Iwasaki gauge + Wilson fermion, Tc=200 MeV to fix the scale
(4) von Smekal group: Wilson/Improved gauge + rooted staggered fermion

T=158 MeV (**deconfined**, hadron -> QGP phase transition occurs)
T=130 MeV (**deconfined? QGP phase?** , 2019)

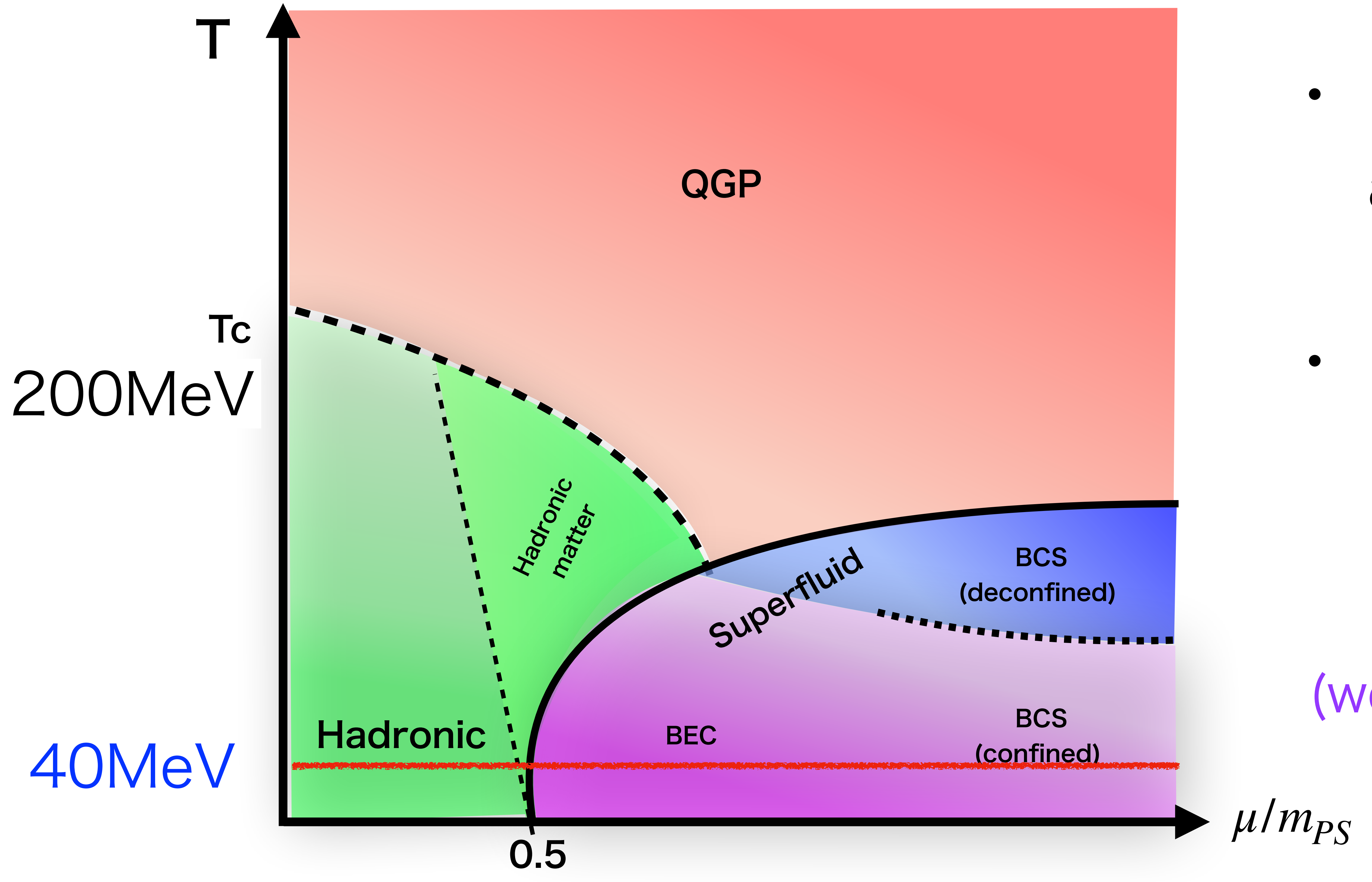
T=140 MeV (**deconfined** in high mu, <qq> is not zero, 2017, 2018, 2020)
T= 93 MeV (**deconfined** in high mu ?, also <qq> is not zero?, 2017)

T=87 MeV (**confined** in 2019)
T=79 MeV (**confined** even in high mu.2019)
T=55 MeV (**confined** in high mu, 2016)
T=47 MeV (**deconfined** coarse lattice in 2012, but **confined** in 2019)
T=45 MeV (**confined** in 2019)
T=40 MeV (**confined, 2024**)

- Even $T \approx 100$MeV and $\mu/m_{PS} = 0.5$, superfluid phase emerges
- T_d (confine/deconfine)$\leq T_{SF}$ (superfluid/QGP) : constraint from 't Hooft anomaly matching

T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

Three phases in low temperature regions

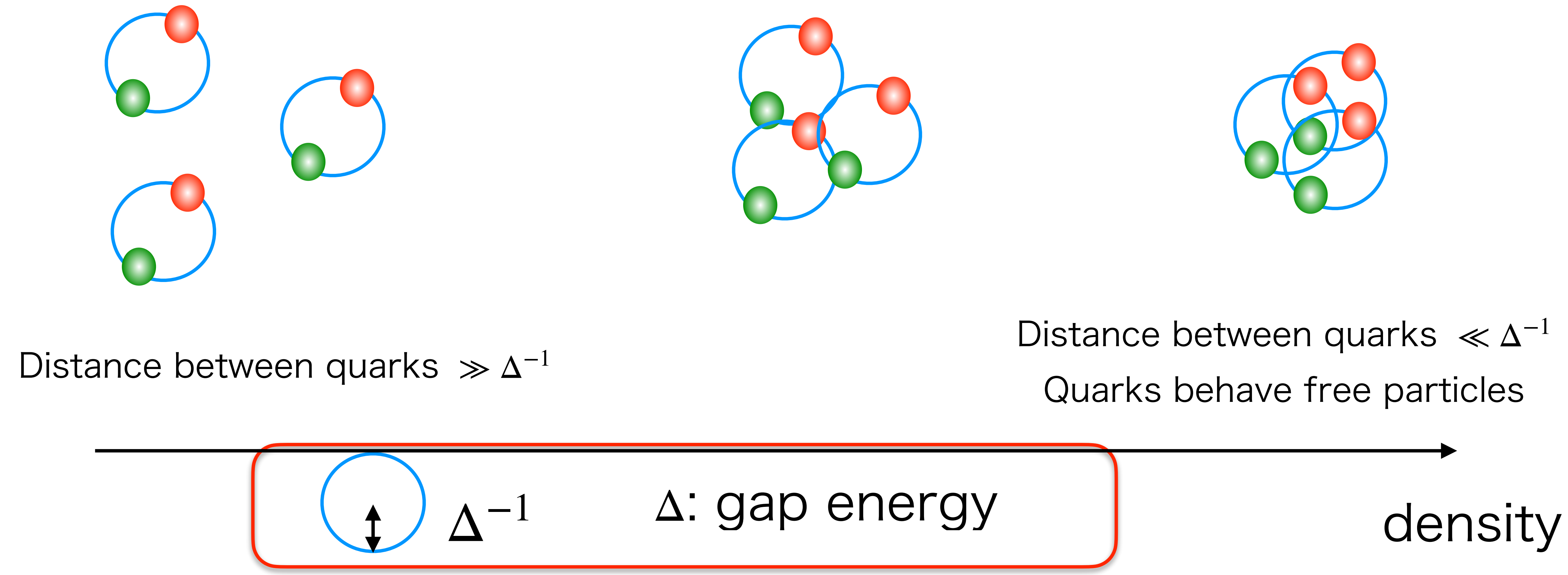


K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
K.Iida, EI, K.Murakami, D.Suenage, JHEP 10 (2024) 022

- We mainly investigated T=40MeV
- Hadronic / Superfluidity phase transition around $\mu \approx m_{\rm PS}/2$
- BEC/BCS crossover in SF phase

BEC phase
strong coupled
(well-described by ChPT)

BCS phase
weakly coupled

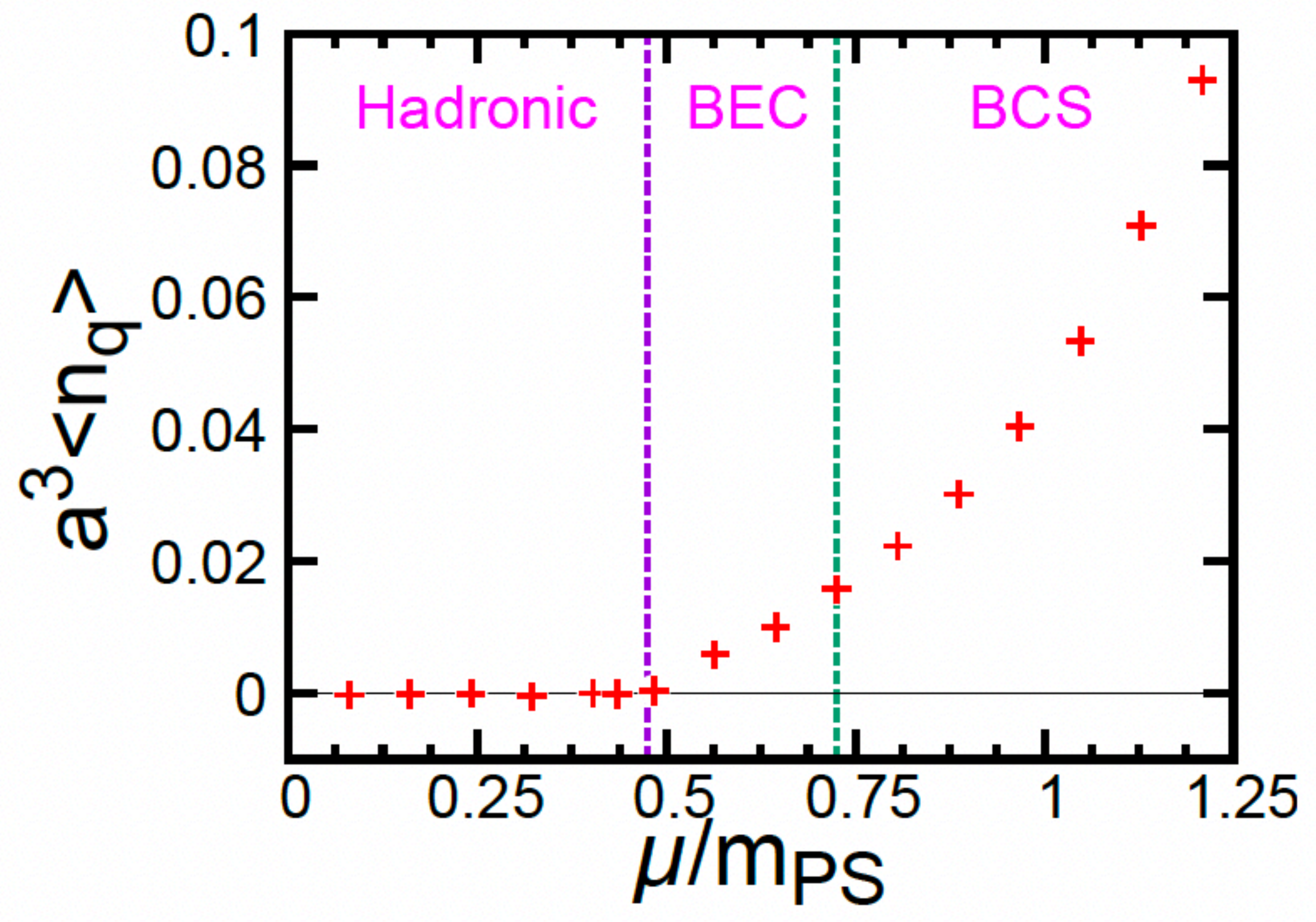


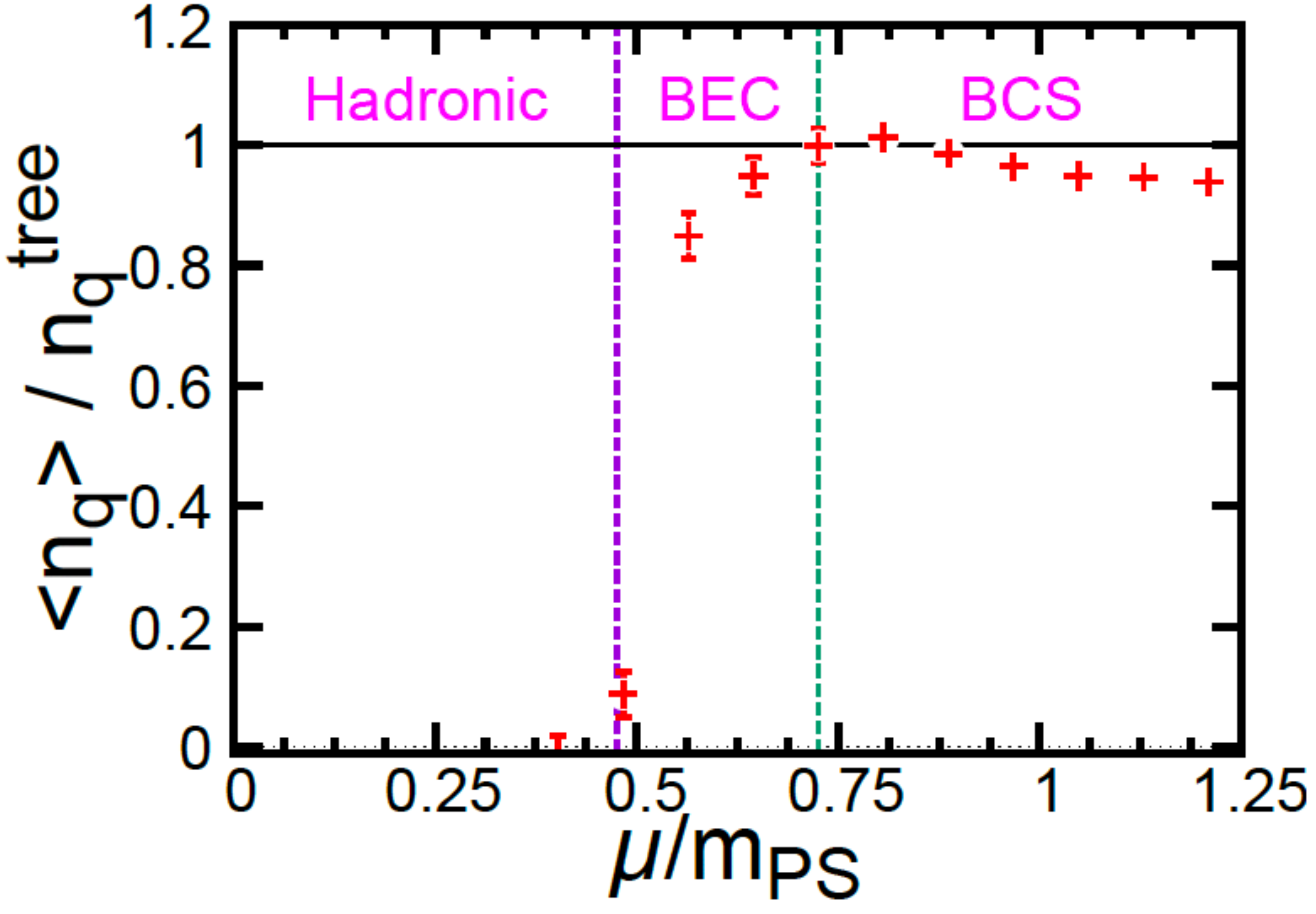
“definition” of BEC/BCS crossover

S. Hands, S. Kim and J.-I. Skullerud(2006)

- If $\langle n_q \rangle \approx n_q^{\text{tree}}$, then the region is identified as BCS phase

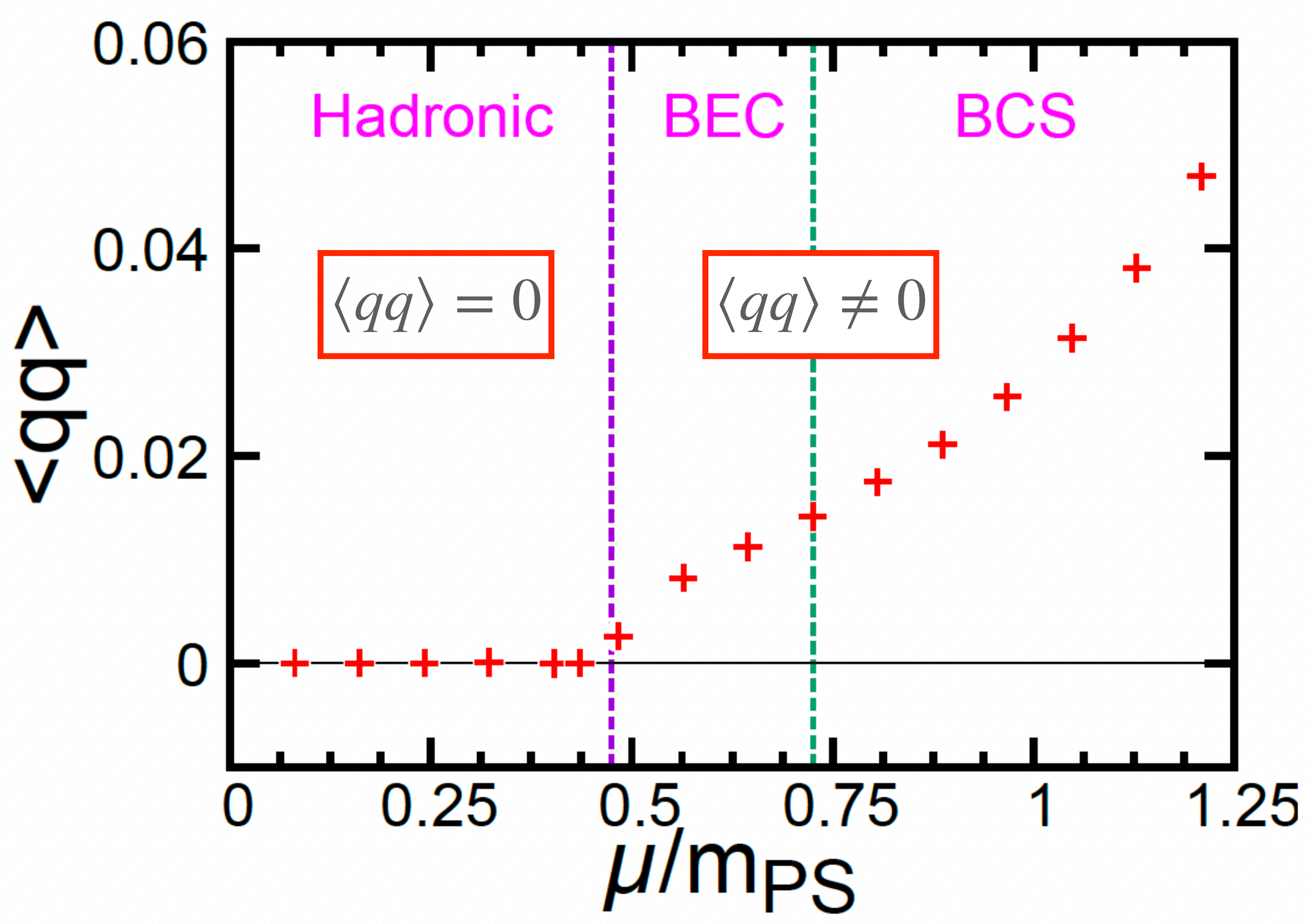
  $\langle n_q \rangle$ denotes a net quark number density



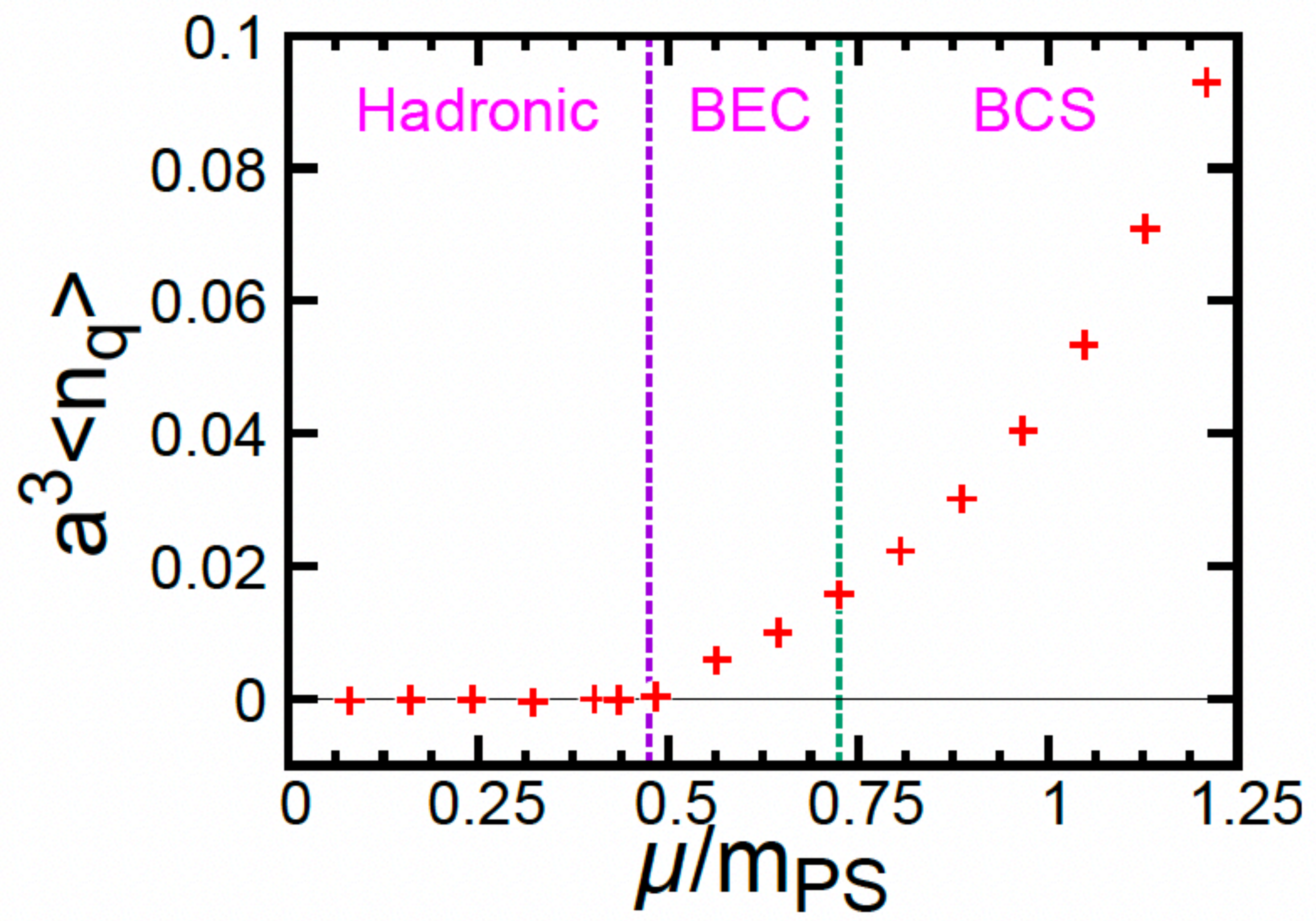


Evidensce of Superfluidity : $\langle qq \rangle$ and $\langle n_q \rangle$

Diquark condensate

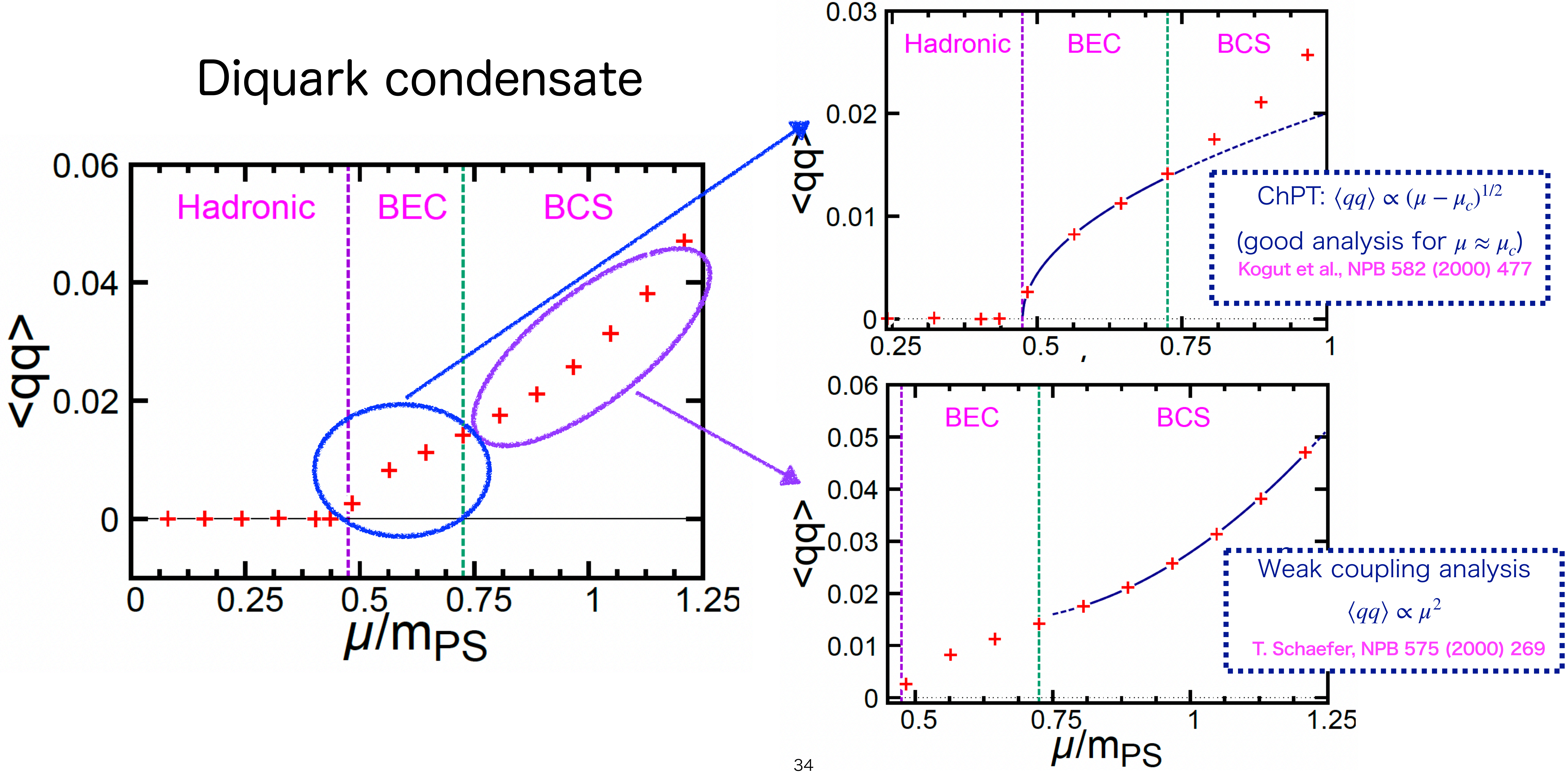


(net) quark number density



Order parameter of Superfluidity : $\langle qq \rangle$

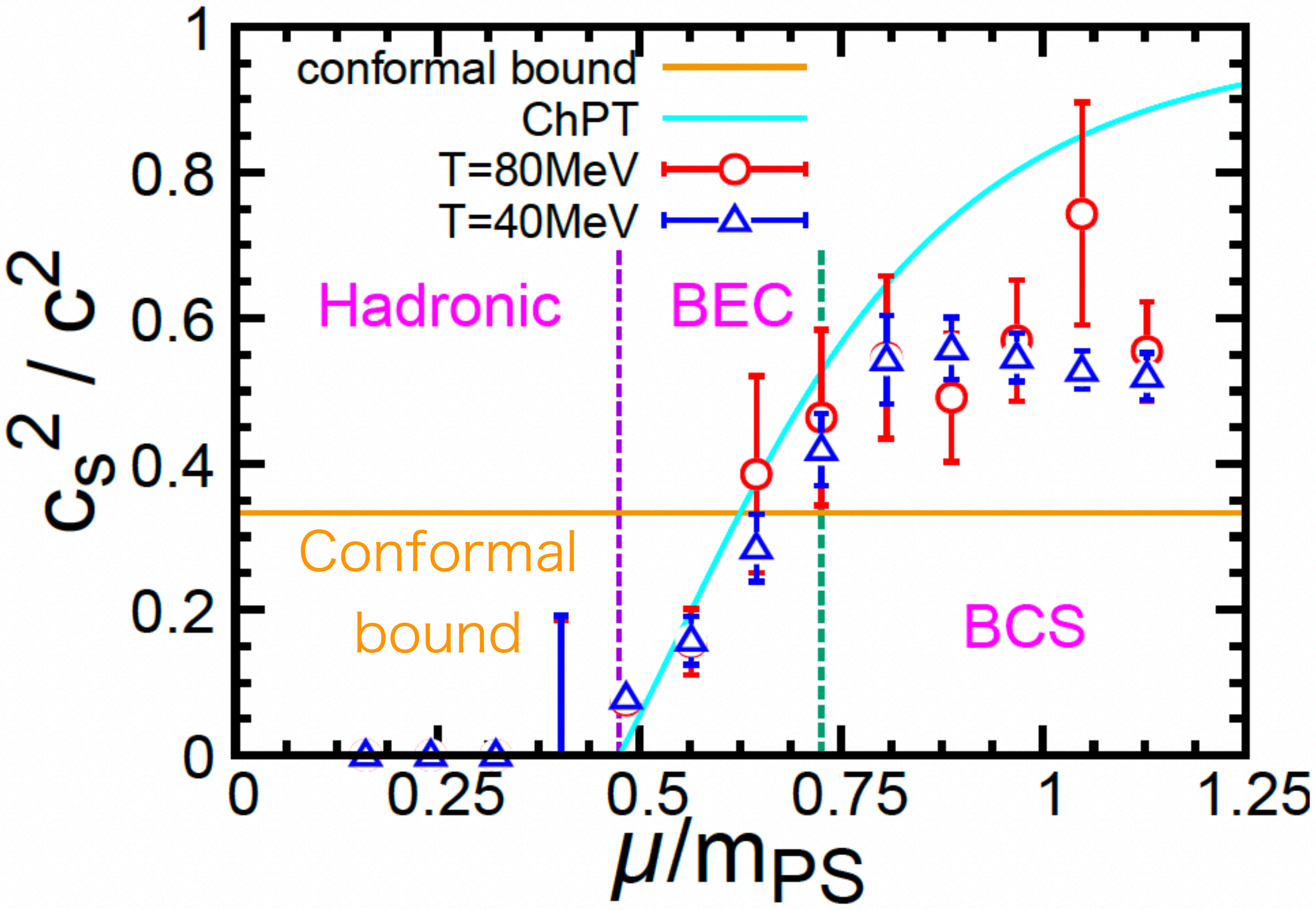
Diquark condensate



Equation of state

K.Iida and EI, PTEP 2022 (2022) 11, 111B01
K.Iida, EI, K.Murakami, D.Suenaga, JHEP 10 (2024) 022

Square of sound velocity ($c_s^2/c^2 = \Delta p/\Delta e$)



- T-dependence of the sound velocity is negligible!

- In BEC phase, result is consistent with ChPT

  Chiral Perturbation Theory (ChPT)

  $$c_s^2/c^2 = \frac{1 - \mu_c^4/\mu^4}{1 + 3\mu_c^4/\mu^4}$$: no free parameter!

  Son and Stephanov (2001) : 3color QCD with isospin μ

  Hands, Kim, Skullerud (2006) : 2color QCD with real μ

- c_s^2/c^2 exceeds the conformal bound

Meson-baryon mixing in SF phase

- In SF phase, $U(1)_B$ is broken, no difference bw mesons and baryons.

  2pt fn. $C(\mu;\tau) = \sum_n |\langle 0|\hat{O}(0)|n\rangle|^2 e^{-(E_n - \mu n_O)\tau}$

  here $|n\rangle_{SF} \propto c_0|$meson-like state $(\bar{q}\Gamma q)\rangle + c_1|$baryon-like state$(q^T\Gamma q)\rangle$

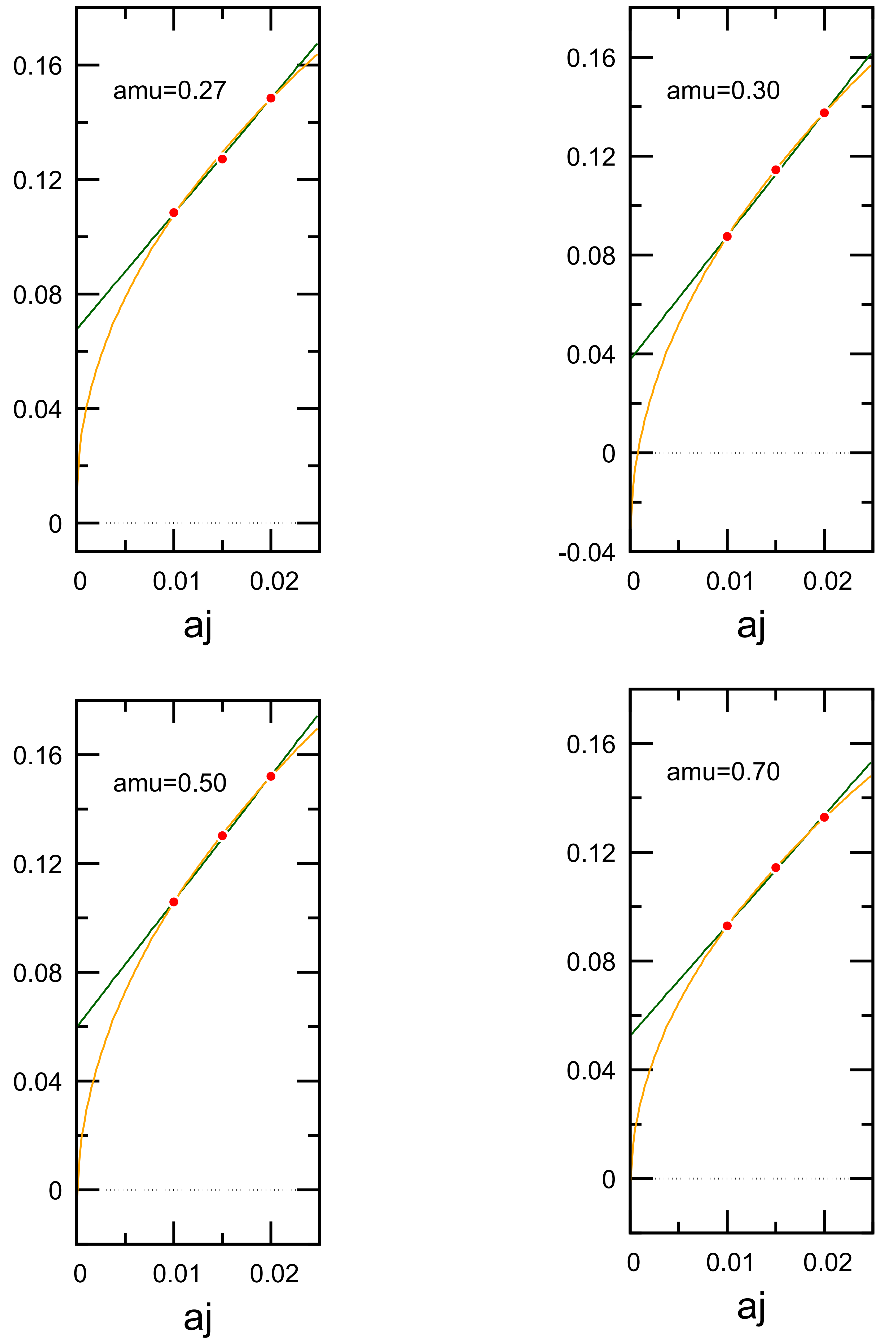
  Thus, consider all possible contractions of $\bar{q}$-q and q-q.

- Normal quark propagator ($\bar{q}$-q): $S_N = Q^{-1}(\mu)\Delta^\dagger(\mu)$

  Anomalous quark propagator (q-q):$S_A = JQ^{-1}(\mu)K$

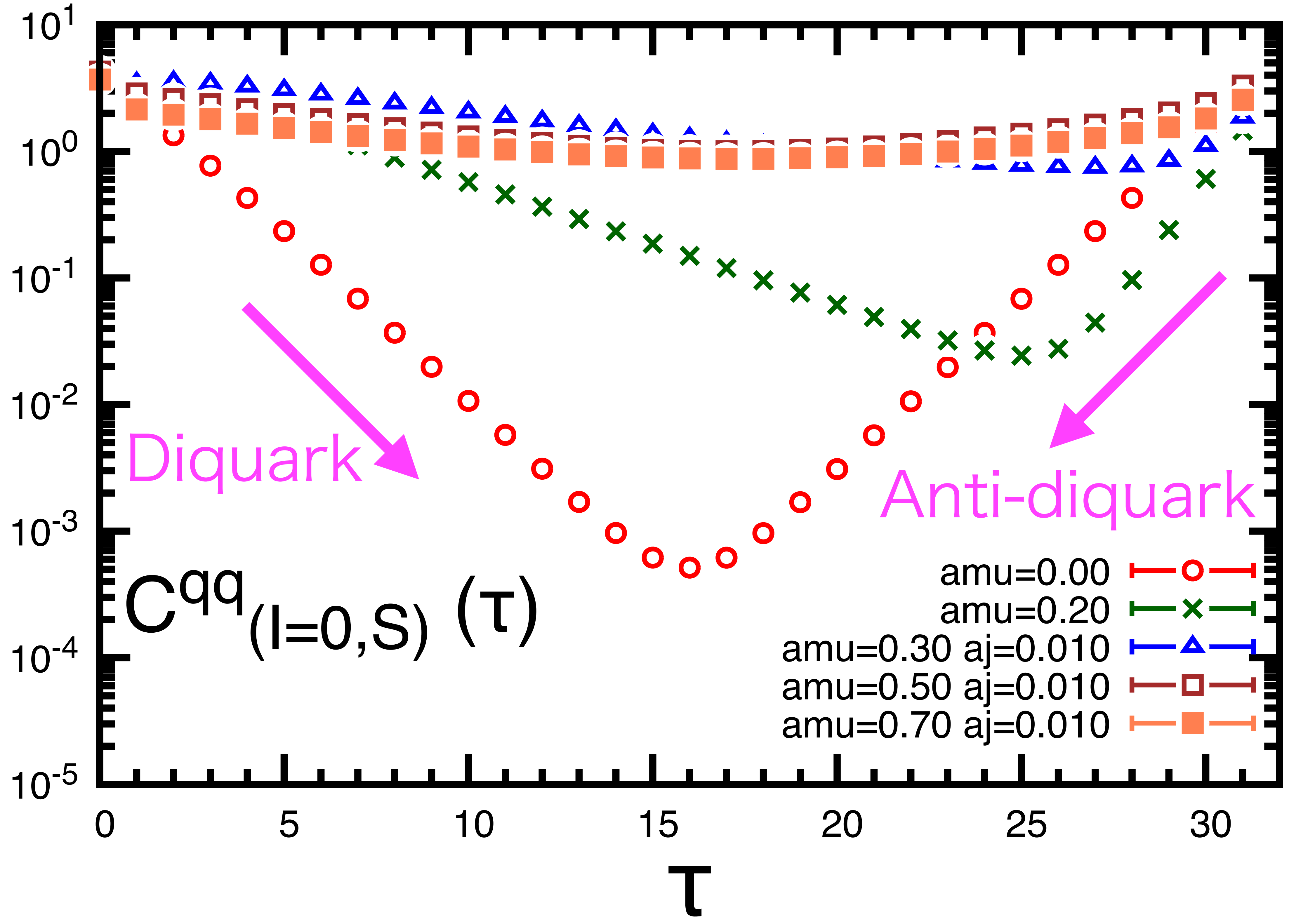
  $Q(\mu) = \Delta^\dagger(\mu)\Delta(\mu) + J^2$ (this is $DD^\dagger$ in standard QCD vacua)

j->0 extrapolation of NG boson mass

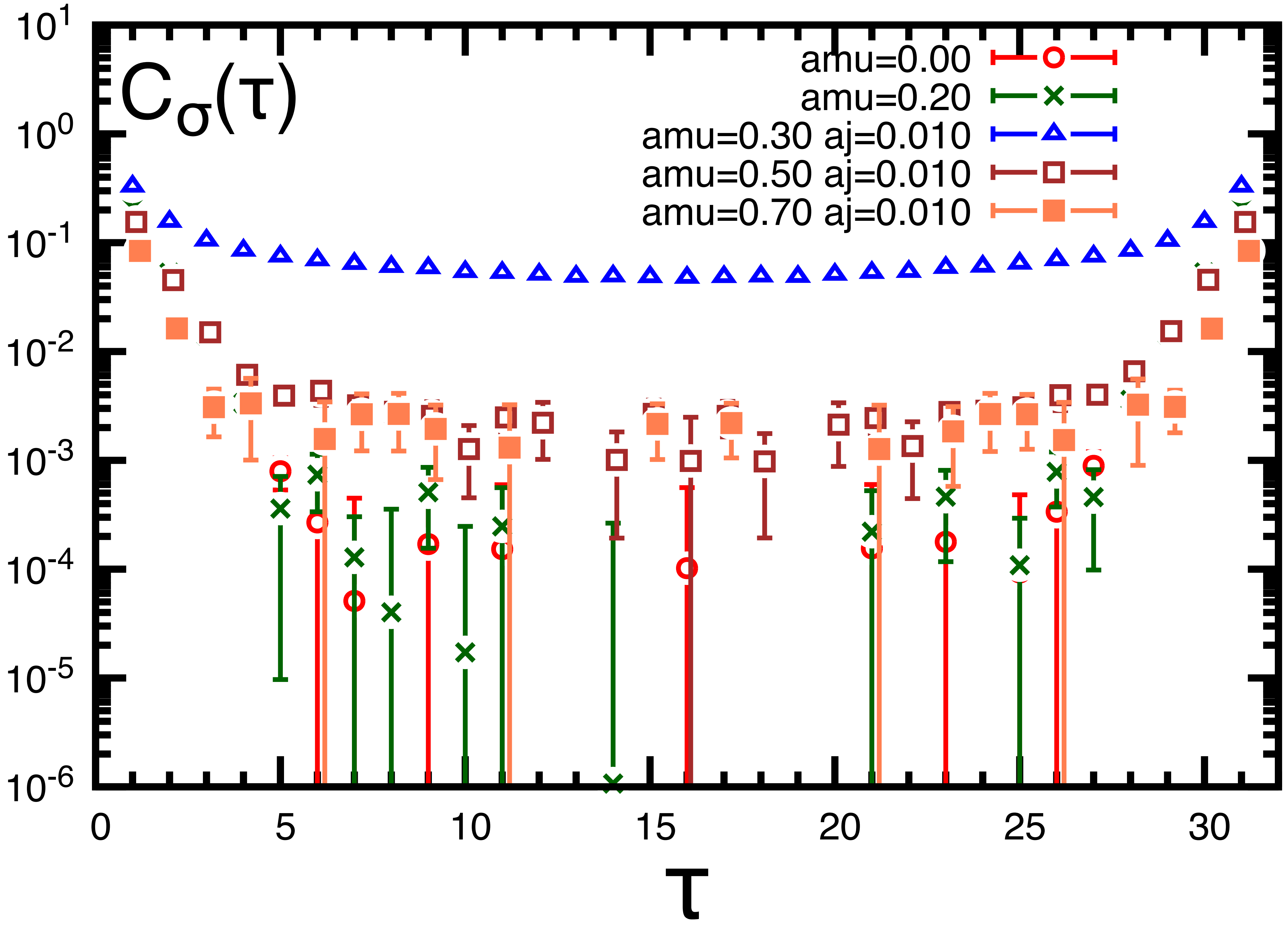


Correlation fn.: I=0 scalar diquark and meson

Diquark/Anti-diquark



Meson (sigma meson)

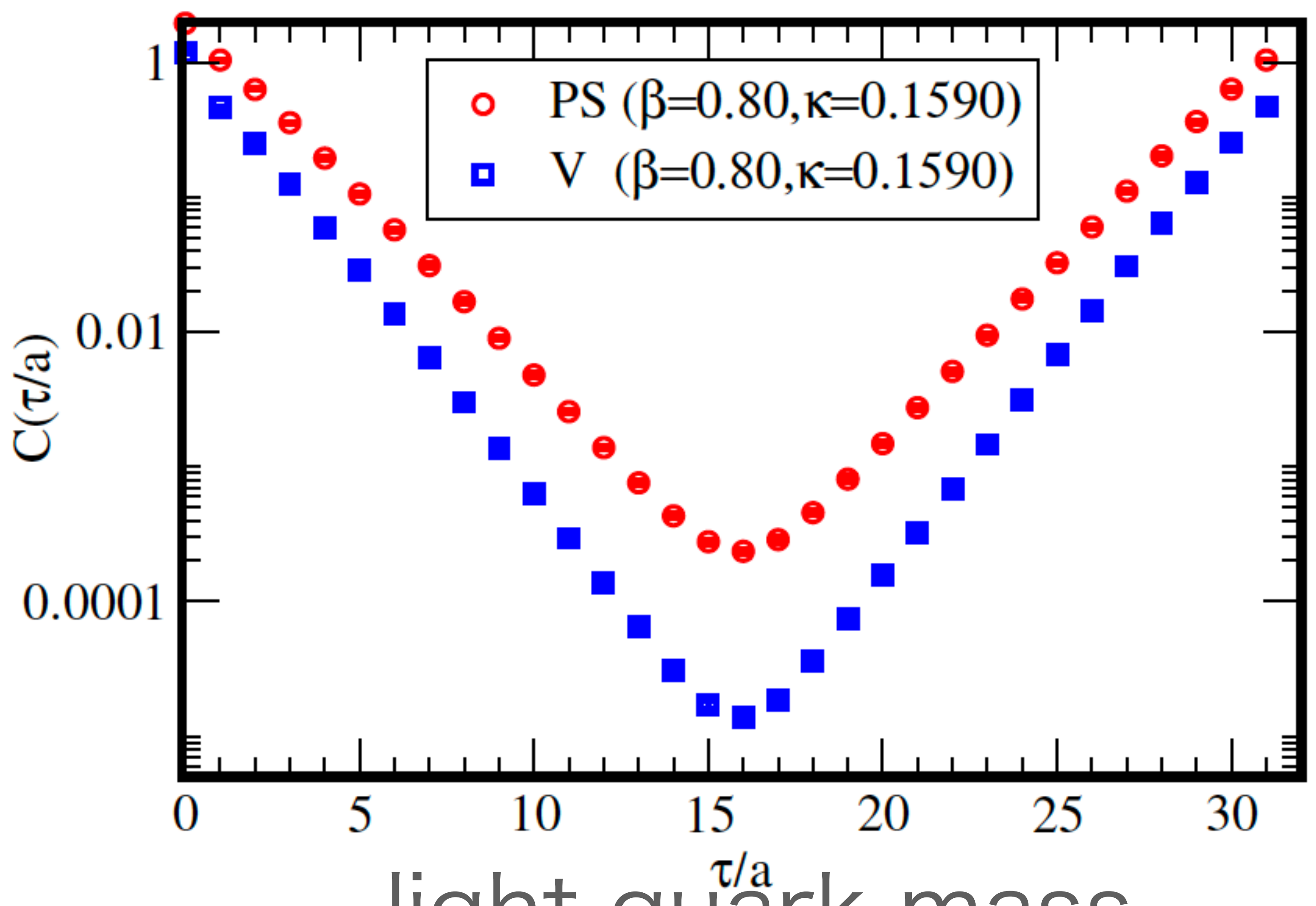


At $\mu \neq 0$ in hadronic phase (green), time-reversal sym. of C(tau) is broken. In SF phase, C(tau) takes larger values.

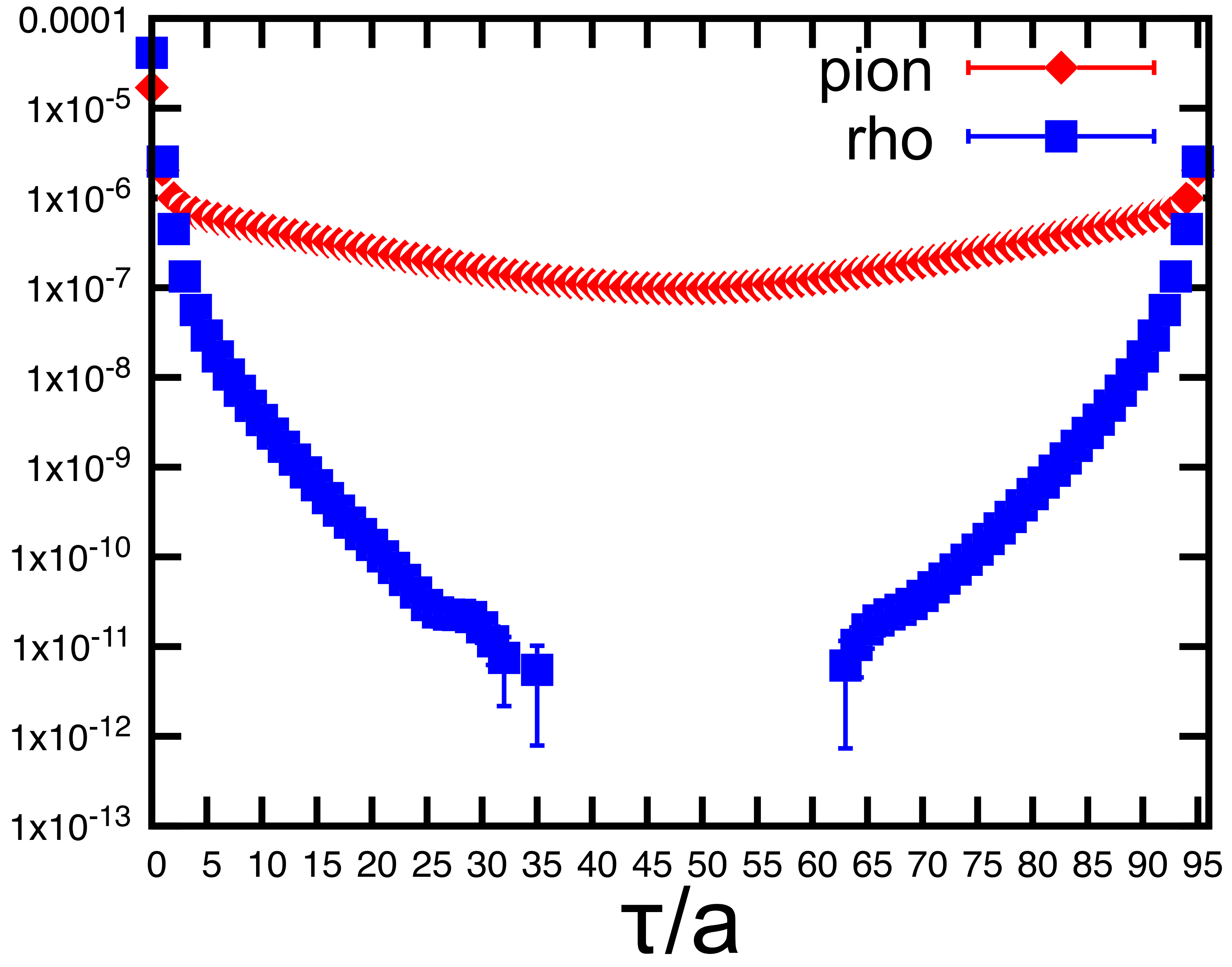
Meson channel has a large contribution from disconnected terms. Noisy in the both phases

Quark mass dependence of correlation fns.

heavy quark mass



light quark mass



- Top panel: $m_\pi/m_\rho \sim 0.8$ (heavy quark mass)

  Bottom panel: $m_\pi/m_\rho \sim 0.2$ (light quark mass)

  (physical point)

  By changing lattice bare mass (κ),

  this ratio is a result of.

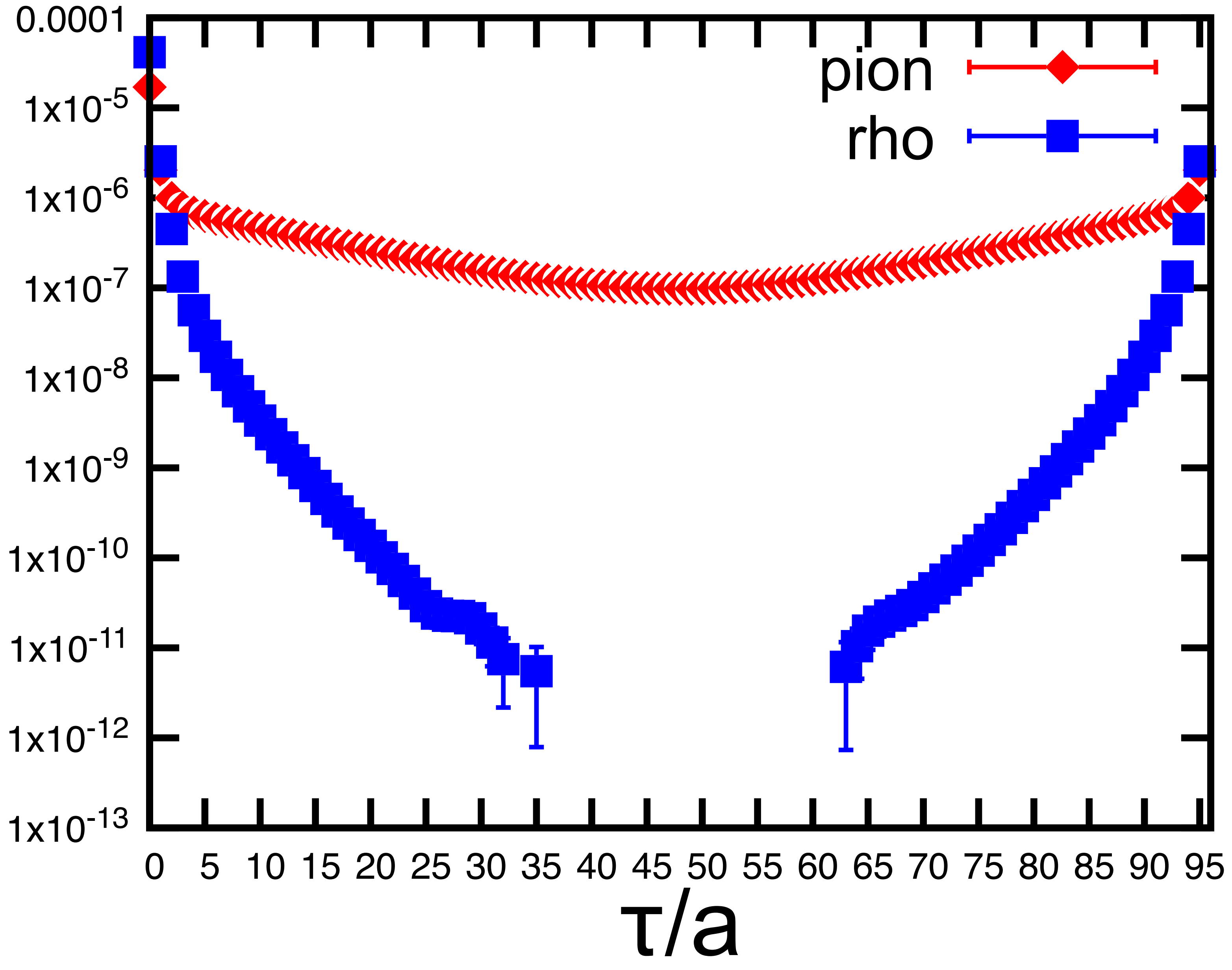
- In lighter quark mass, ρ can decay to 2 pion.

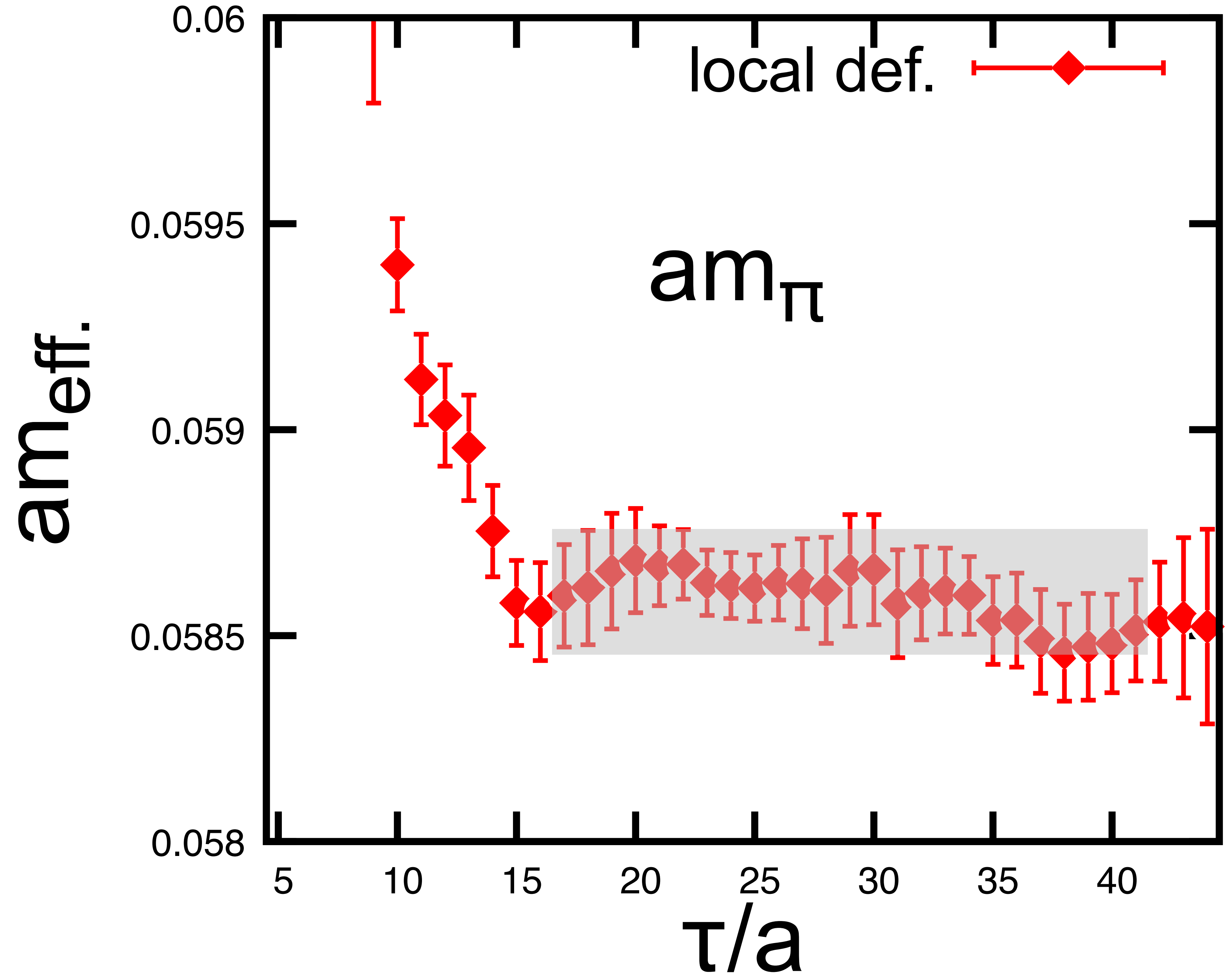
  ρ cannot propagator long time.

  The signal of the correlation fn. becomes noisy in long τ regime.

  Hard to obtain precise data for heavier hadron mass

Calculate mass from the long-time range of correlations

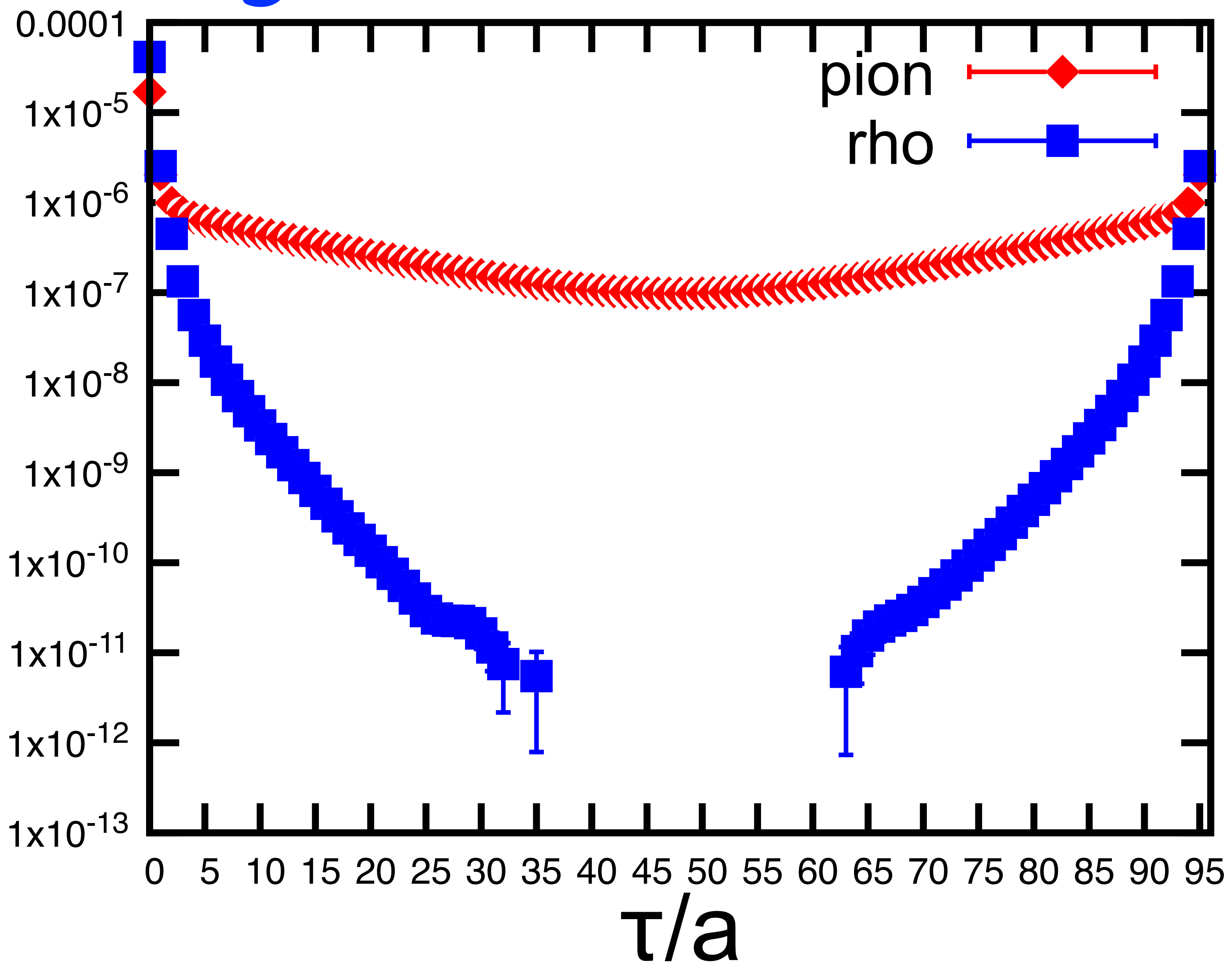


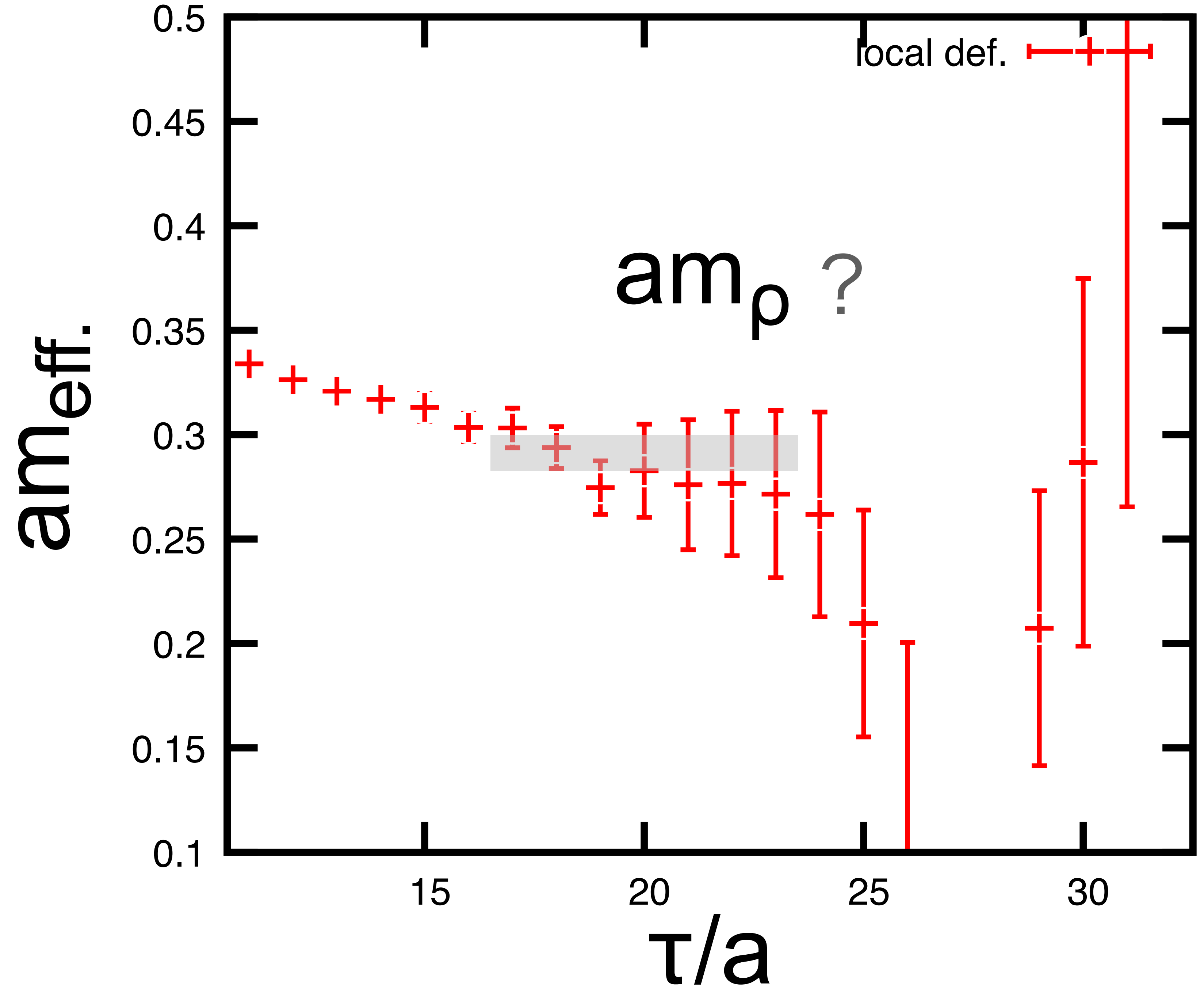


1. Fit the data in the appropriate τ region with $f(\tau) = c_0 + c_1 e^{-c_2\tau}$ the best-fit value of $c_2 = m$

2. Calc. effective mass $m_{eff}(\tau) = -\log[C(\tau+1)/C(\tau)]$ In long τ, it should converge with the lowest state mass Find a plateau of m_{eff}

Heavy (unstable) meson case





- Calc. effective mass

  $$m_{eff}(\tau) = -\log[C(\tau+1)/C(\tau)]$$

  In long τ, it should converge with the lowest state mass
  Find a plateau of m_{eff}

- It is difficult to find a plateau for heavy (unstable) case