χSB for lattice QED in a strong external magnetic field – eB dependence of the chiral condensate

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Introduction

QED in (strong) external magnetic fields breaks chiral symmetry non-perturbatively.

This had been predicted by Schwinger-Dyson methods, and confirmed by us (J. B. Kogut and D. K. Sinclair, Phys. Rev. 109, 034511 (2024)), using lattice QED (RHMC) simulations. These simulations used $eB = 2\pi \times 100/36^2 = 0.4848...$ at bare coupling $\alpha = 1/5$ and a range of bare masses m which allowed us to estimate the chiral condensate in the limit $m \to 0$.

Simulations at a much smaller B – $eB = 2\pi \times 24/36^2 = 0.1163...$, while suggesting that chiral symmetry was still broken at $m = 0$, indicated that to get a numerical estimate for the $m = 0$ chiral condensate would require lattices much larger than present resources would allow (D. K. Sinclair and J. B. Kogut PoS LATTICE2024 (2025), 158).

We are currently simulating with an intermediate B value $eB = 2\pi \times 64/36^2 = 0.3102...$. Preliminary indications are that the

$m = 0$ chiral condensate at this eB is consistent with the prediction that $\langle\bar{\psi}\psi\rangle \propto (eB)^{3/2}$.

Electrons in an external magnetic field

Consider electrons in an applied constant homogeneous magnetic field B in the z direction.

Classically the electrons traverse helical orbits around magnetic field lines. The projection of these orbits on the xy plane is a circle, while the motion in the z direction is free.

Quantum mechanically, since the motion in the xy plane is compact, it is quantized.

These quantized energy levels – the Landau levels – have energies

$$E_n(p_z) = \pm\sqrt{m^2 + 2eBn + p_z^2}$$

where $n = 0, 1, 2, ...$

Without the effects of QED the chiral condensate $\langle\bar{\psi}\psi\rangle$ vanishes in the limit $m \to 0$.

QED in an external magnetic field

We now consider what happens when we add the effects of QED to the dynamics of electrons in a strong external magnetic field.

The dynamics is expected to be dominated by the lowest Landau levels (LLL) i.e. those with $n = 0$.

The attractive forces between electrons and positrons confined to the LLLs with radii $\approx 1/\sqrt{eB} \approx 1.8$ are expected to yield a finite chiral condensate in the $m \to 0$ limit.

We employ staggered fermions for our RHMC simulations, using rational approximations to the fractional powers of the quadratic fermion operators, required to restrict our simulations to a single electron "flavour".

As $m \to 0$, chiral symmetry breaking, if present, is expected to be associated with near-zero modes.

This means that we will need to perform finite-size scaling analyses.

However, domination by LLLs with radii $\approx 1/\sqrt{eB} \approx 1.8$, means that our choice of $N_x = N_y = 18$ or $N_x = N_y = 36$ should be adequate and not need to be increased.

Hence we only need to increase $N_z = N_t$ for finite-size scaling analyses.

Preliminary results of RHMC simulations with $\alpha = 1/5$ and $eB = 2\pi \times 16/18^2 = 2\pi \times 64/36^2$

We perform RHMC simulations on lattices with either $N_x = N_y = 36$ or $N_x = N_y = 18$ with $\alpha = 1/5$ and $eB = 2\pi \times 64/36^2 = 2\pi \times 16/18^2$.

We simulate over a range of (small) electron masses $0.001 \leq m \leq 0.0075$. At each mass we start with $N_z = N_t = 36$ and increase it until the chiral condensate ceases to increase with increasing $N_z = N_t$ (finite size scaling).

At the smallest masses this requires $N_z = N_t \geq 160$. At $m = 0.001$ we actually use lattices as large as $18^2 \times 192^2$.

Figure 1, shows the preliminary results. Also shown is the prediction of the $m = 0$ limit obtained from our simulations at $eB = 2\pi \times 100/36^2 = 2\pi \times 25/18^2$ assuming $\langle\bar{\psi}\psi\rangle \propto (eB)^{3/2}$, at $m = 0$.

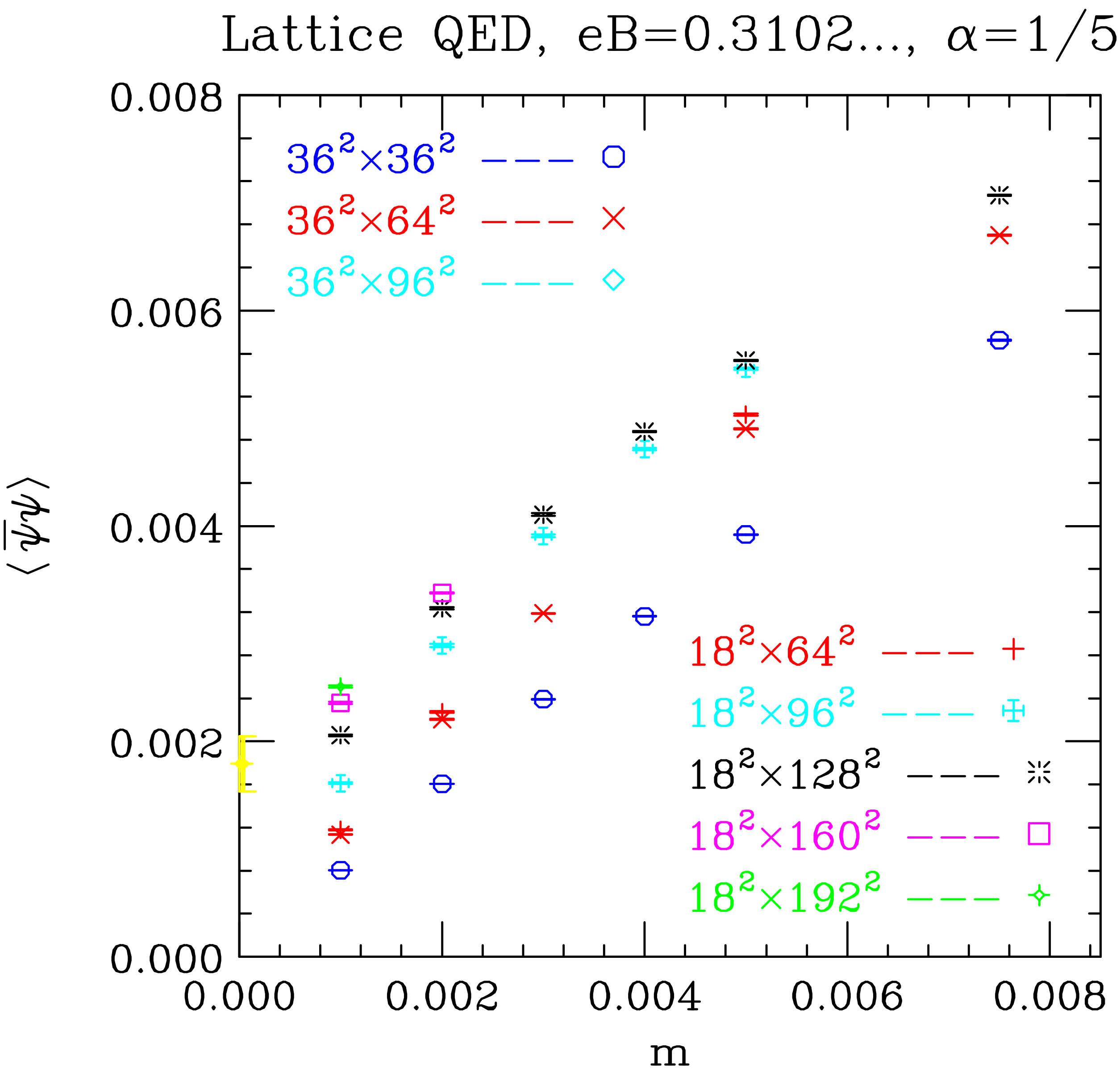

Figure 1: $\langle\bar{\psi}\psi\rangle$ as a function of mass at $eB = 2\pi \times 64/36^2 = 2\pi \times 16/18^2$, showing dependence on lattice size in the z and t directions. The yellow datum on the left axis is the prediction from the simulations at $eB = 2\pi \times 100/36^2 = 2\pi \times 25/18^2$.

What this suggests is that the measured values of the chiral condensate are consistent with the prediction based on the extrapolation from the larger magnetic fields.

How good the prediction of $(eB)^{3/2}$ scaling of the $m = 0$ chiral condensate is, depends on the reliability of our extrapolations from $m \geq 0.001$ to $m = 0$.

To improve our test of the scaling of the $m = 0$ chiral condensate, we should perform simulations at $m = 0.0005$ on lattices of size $18^2 \times 160^2$ or larger.

Discussions and Conclusions

- We continue our studies of chiral symmetry breaking in (lattice) QED catalyzed by strong magnetic fields.
- For simplicity we consider the effect of constant, strong homogeneous magnetic fields oriented in the z direction.
- We simulate on lattices with bare lattice fine structure constant $\alpha = 1/5$, so that $\alpha(\pi^2) = 1/5$ compared with $\alpha(m_e^2) \approx 1/137$.
- One loop running of the electric charge gives a crude estimate of the momentum cutoff

$$\log(\Lambda_{1/5}/m_e)/\log(\Lambda_\infty/m_e) \approx (137-5)/137,$$

where Λ_∞ is the momentum cutoff at the Landau pole. So, the momentum scales and hence the magnetic fields we consider, are so large as to be of only theoretical interest.

- So far, our runs at the smaller $eB = 0.3102...$ (our previous project was at $eB = 0.4848...$) show direct evidence of a non-zero chiral condensate in the $m \to 0$ limit. From the graph of figure 1, this would appear to be consistent with this condensate scaling $\propto (eB)^{3/2}$ as expected.

- If chiral symmetry were unbroken as $m \to 0$, one would expect little dependence of the chiral condensate on lattice size since chiral symmetry breaking would be an ultraviolet effect with $\langle\bar\psi\psi\rangle \propto m\Lambda^2$ up to logarithms.
- The fact that for small input electron masses m the chiral condensate shows significant increase with increasing $N_z = N_t$ can be considered as indirect evidence that the chiral condensate remains finite as $m \to 0$.
- The trend of the limiting values of $\langle\bar\psi\psi\rangle$ as $N_z = N_t$ is increased, as functions of m, when $m \to 0$, see figure 1, strongly suggests that the chiral condensate at $m = 0$ is non-zero. It would also appear that this $m = 0$ condensate is consistent with the prediction from its value at the larger eB assuming $(eB)^{3/2}$ scaling. To check this we should perform simulations at $m = 0.0005$.

- We are storing configurations at regular intervals during our simulations for further analysis.
- One such analysis will be to measure the changes to the coulomb field of a point charge placed in an external magnetic field.
- We are also considering measuring spontaneous chiral symmetry breaking of multi-flavoured QED in an external magnetic field also using RHMC simulations.

These simulations are being performed on the Improv and Bebop clusters at Argonne's LCRC, on Stampede3 at TACC, Expanse at SDSC and Bridges2 at PSC under an ACCESS(NSF) allocation and on Perlmutter at NERSC under an ERCAP(DOE) allocation.