

Isospin breaking corrections to the hadronic vacuum polarization contribution to the muon $g-2$

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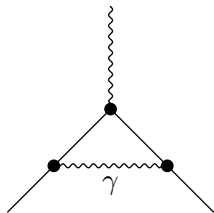
Johannes Gutenberg-University, Institute of Nuclear Physics

29.07.2026

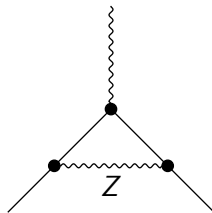
Lattice 2026, University of Maryland, College Park

The anomalous magnetic moment of the muon ($g-2$)

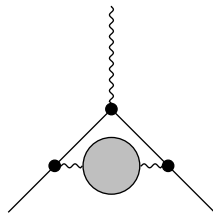
- $a_\mu = \frac{1}{2}(g - 2)$
- classical result from Dirac equation: $a_\mu = 0$
- correction from higher order terms



QED corrections

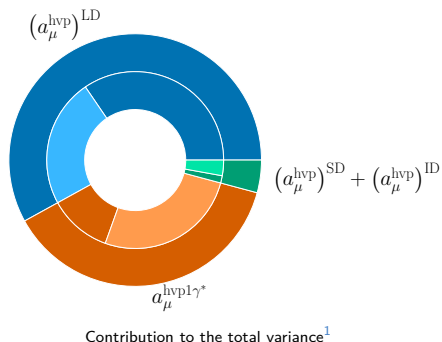


Electroweak corrections



Hadronic corrections

Error contributions of the latest Mainz result



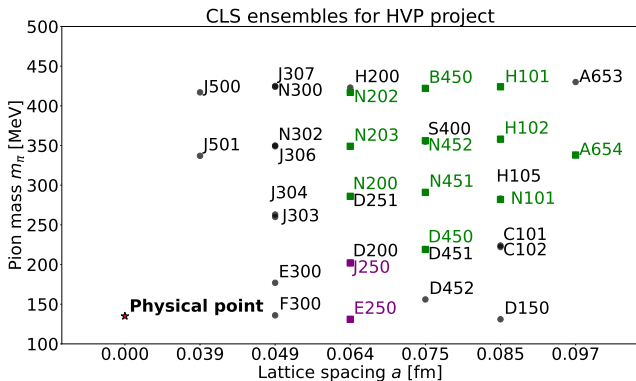
- short and intermediate distance windows are well under control
- major error contributions are the long distance window and the isospin breaking corrections
 - 1 long distance window: statistical error slightly dominates
 - 2 isospin breaking corrections: systematic error dominates

¹arXiv: 2411.07969 (hep-lat).

CLS ensembles

- isoQCD ensembles of the CLS collaboration¹
- six different lattice spacings and large range of pion masses down to the physical point used in isoQCD calculation²

- green ones are finished or in production, purple ones planned for this project



¹arXiv: 1411.3982 (hep-lat).

²arXiv: 2312.13753.

- isoQCD ensembles ($m_u = m_d = m_{ud}$ and no QED effects)
 - treat these additional effects as small perturbation and expand in Δm_d , Δm_u , Δm_s and in e ¹
- ⇒ many more new diagrams ($\mathcal{O}(20)$ diagrams in total)
- time momentum representation and euclidean windows
 - using QED_L to put photon on the lattice
 - complementary approach to Dominik Erb (previous talk)

¹arXiv: 1303.4896 (hep-lat); arXiv: 1110.6294 (hep-lat).

Set of local diagrams



(1)



(2)



(3)



(4)



(5)



(6)



(7)



(8)



(9)



(10)



(11)



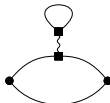
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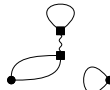
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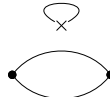
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(15)



(16)

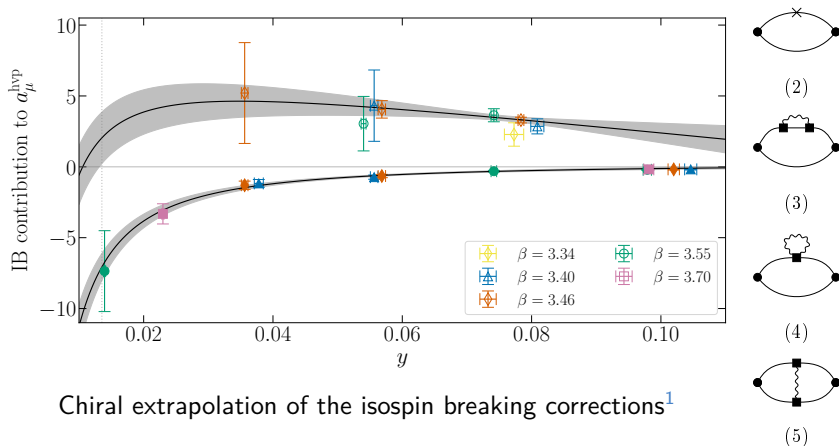


(17)



(18)

Current status of isospin breaking computations

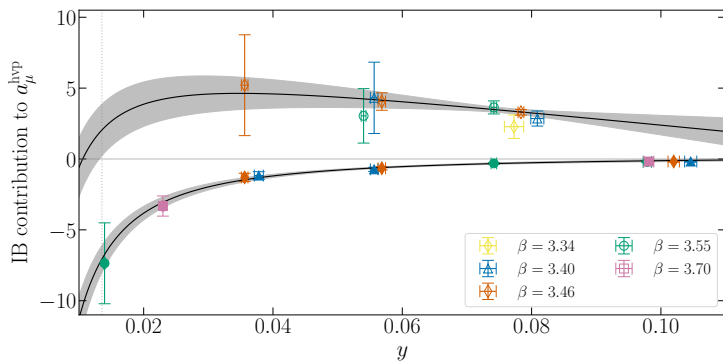


Chiral extrapolation of the isospin breaking corrections¹

- connected part not well constrained, in particular in the physical pion mass region

¹arXiv: 2411.07969 (hep-lat).

Current status of isospin breaking computations



(6)

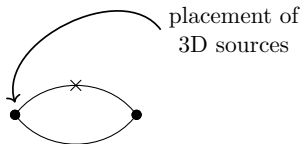
Chiral extrapolation of the isospin breaking corrections¹

- missing disconnected mass insertion diagram and self-energy
- methods for conn. and disc. QED diagrams differ substantially (QED_L vs. QED_{∞})

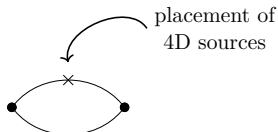
¹arXiv: 2411.07969 (hep-lat).

Computational setup

- current setup produces large errors at small pion mass
- idea: employ different noise estimator to improve errors
- current setup of computation: placing a three dimensional stochastic source at the source time of the diagram
- new setup: 4D volume sources at the mass insertion vertex

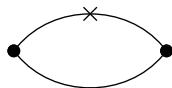


1: Connected mass insertion diagram for the valence quark



1: Connected mass insertion diagram for the valence quark

Computational setup



1: Disconnected mass insertion

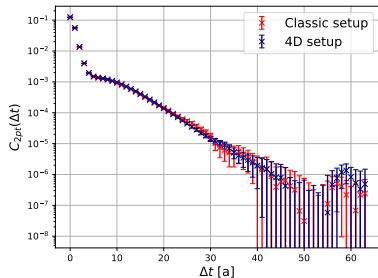
2: Connected mass insertion

- allows for computation of disconnected diagrams (split even estimator)¹
- small cost for disconnected diagram (lower number of sources and shared solves)

¹arXiv: 1903.10447 (hep-lat).

Improvement mass insertion correlation function

- reduction of statistical noise at equal computational cost $\Rightarrow$ reduced errors
- Test on N451 ($a = 0.076\text{fm}$, $m_\pi = 291\text{MeV}$)



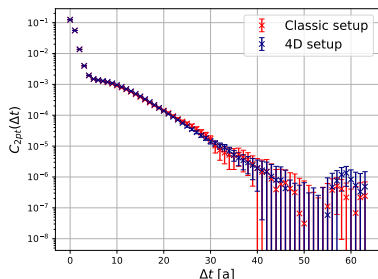
Two-point connected correlation
function for N451 (1000
configurations)



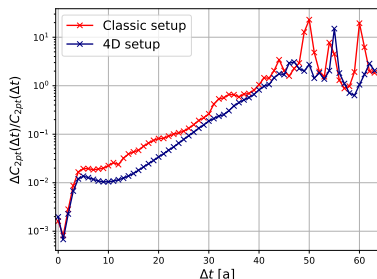
Diagram 1: Connected mass insertion
diagram for the valence
quark

Improvement mass insertion correlation function

- reduction of statistical noise at equal computational cost
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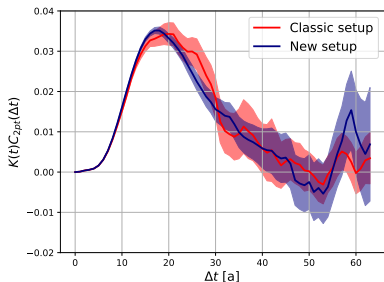
Two-point connected correlation function for N451 (1000 configurations)



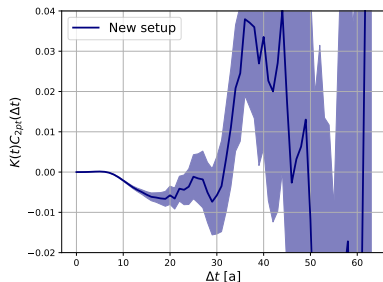
Rel. error of the two-point connected correlation function for N451 (1000 configurations)

Comparison Connected and disconnected mass insertion at $m_\pi \approx 280\text{MeV}$

- connected diagram more important
 - disconnected diagram dominates uncertainty
- ⇒ improvement for strong isospin-breaking contributions due to new setup around 30%



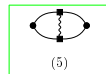
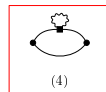
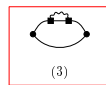
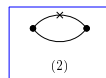
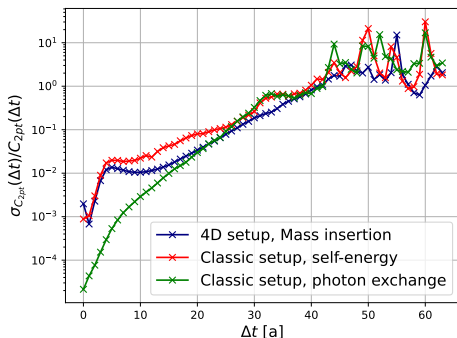
Integrand of connected mass insertion for N451



Integrand of disconnected mass insertion for N451

Photon connected contributions

- no direct improvement on connected diagrams including QED
- mass insertion and self-energy with largest contributions
- photon self-energy has highest relative error

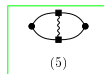
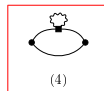
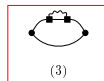
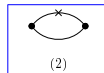
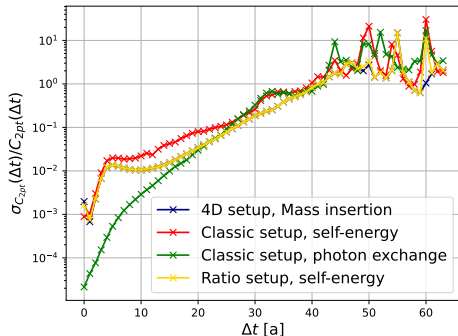


Relative error of connected diagrams

Photon connected contributions

- perform ratios to reduce the error on the self-energy:

$$C_{se}^{(new)} = \frac{C_{se}^{(classic)}}{C_{mi}^{(classic)}} C_{mi}^{(new)}$$



Analysis procedure

- rewrite $J_\mu = J_\mu^3 + \frac{1}{\sqrt{3}} J_\mu^8$ ¹

$\Rightarrow G^{ij} = \langle J_\mu^i J_\mu^j \rangle$, with $i, j \in [0, 3, 8]$

- build G^{ij} for every expansion parameter from the single correlators
- operator mixing between G^{ij} , with $i, j \in [0, 3, 8]$ ²

$$G_R^{EM} = \sum_{i,j} \left(Z_{3,i} + \frac{1}{\sqrt{3}} Z_{8,i} \right) \left(Z_{3,j} + \frac{1}{\sqrt{3}} Z_{8,j} \right) G_B^{ij}$$

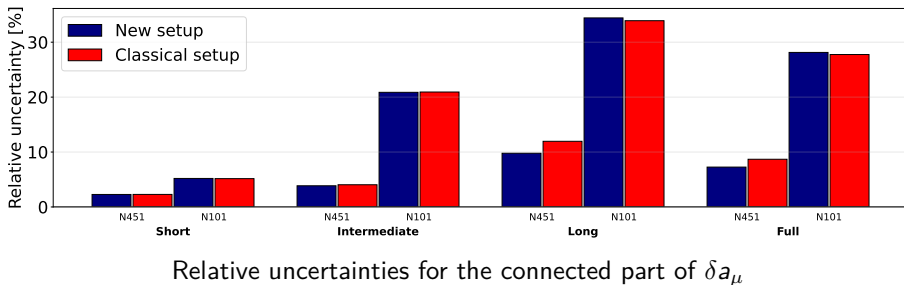
- $Z_{i,j}$ are currently from connected diagrams only
- multiply G_R^{EM} by kernel-function and then integrate up
- sum up contributions to get final result
- divergences of QED diagrams are canceled by the strong isospin-breaking diagrams via expansion coefficients

¹arXiv: 2206.06582 (hep-lat).

²arXiv: 2112.00878 (hep-lat).

Improvement on connected diagrams?

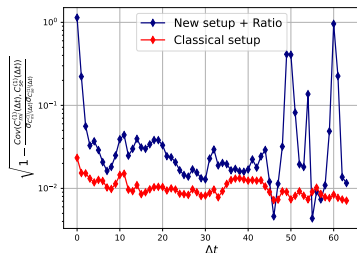
- comparison for two ensembles at equal pion mass (N451: $a = 0.076\text{fm}$, $m_\pi = 291\text{MeV}$ and N101: $a = 0.085\text{fm}$, $m_\pi = 282\text{MeV}$)
- new setup does not improve the overall uncertainty for the connected part alone



Improvement on connected diagrams?

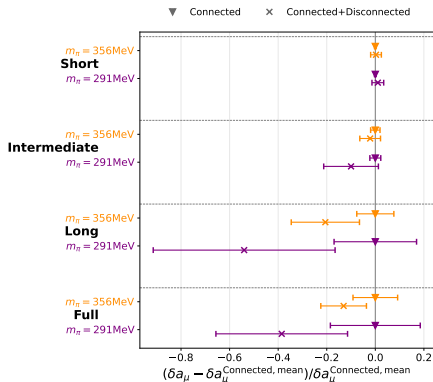
- reason: reduced correlation $\Rightarrow$ smaller cancellation of errors
 - error of mass insertion and self-energy of same size
- $\Rightarrow$ if combining strong isospin-breaking with QED effects, new setup is not beneficial anymore

$$\Rightarrow \sigma_{mi-se} \approx \sigma_{mi} \cdot \sqrt{1 - \frac{\text{Cov}(C_{mi}^{(1)}(\Delta t), C_{se}^{(1)}(\Delta t))}{\sigma_{C_{mi}^{(1)}(\Delta t)} \sigma_{C_{se}^{(1)}(\Delta t)}}$$



Inclusion of disconnected diagrams

- QCD+QED valence sector at $a = 0.076\text{fm}$: connected diagrams and disconnected mass insertion as well as disconnected self-energy
- higher statistics at $m_\pi = 356\text{MeV}$
- negative shift, increase with decreasing pion mass
- shift mainly from long distance regime



Effect of the disconnected mass insertion and self-energy

Strong isospin breaking only - R_{38K}

- separation of QED effects and strong isospin-breaking effects introduced by Erb et al.¹:

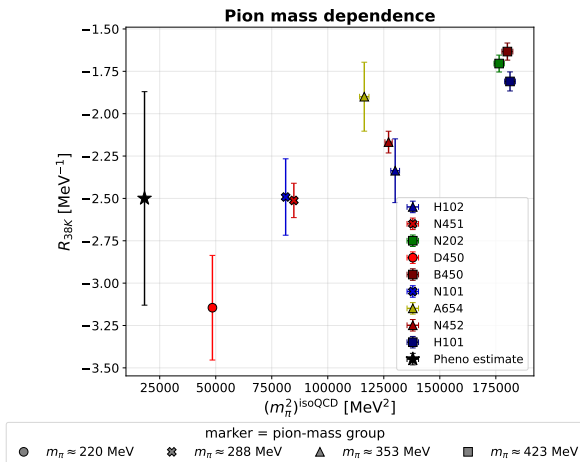
$$a_{\mu}^{HVP,38} = a_{\mu,em}^{HVP,38} + \left(\Delta M_K^{phys} - \Delta M_K^{lat} \right) \cdot R_{38K}$$
$$R_{38K} = \frac{\frac{\partial a_{\mu}^{(3,8)}}{\partial(m_u - m_d)}}{\frac{\partial M_K}{\partial(m_u - m_d)}}$$

- $a_{\mu}^{HVP,38}$ zero at leading order $\Rightarrow$ "isospin-violating part of the HVP"

¹arXiv: 2505.24344 (hep-lat).

Pion mass dependency of R_{38K}

- R_{38K} shows strong pion mass dependency
- mild dependency on the lattice spacing



Pion mass dependency of R_{38K}

Summary and Outlook

Summary:

- ① isospin breaking corrections are essential for precision calculations
- ② disconnected contributions produce negative shift in the case of HVP
- ③ new noise estimator reduces error for strong isospin-breaking, however correlations matter

Outlook:

- ① investigate correlation further
- ② pure strong isospin-breaking prediction
- ③ extend calculations to disconnected exchange diagram
- ④ include disconnected diagrams in renormalization and matching
- ⑤ improve statistics on existing ensembles and extend ensemble space

Thank you for your attention!

More talks from Mainz

H. Meyer	Tue	4:50pm
A. De Santis	Wed	9:00am
A. Conigli	Wed	9:00am
A. Beltran	Wed	9:20am
D. Erb	Wed	11:10am
D. Albanea	Wed	11:50am
A. De Santis	Fri	5:10pm