Confining Flux Tube in the Trace-Deformed $(2+1)$-Dimensional $SU(2)$ Gauge Theory

Dario Panfalone

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C. Bonati, M. Caselle, A. Negro, D. Panfalone and L. Verzichelli

University of Turin

Trace-Deformed $SU(2)$ YM Theory

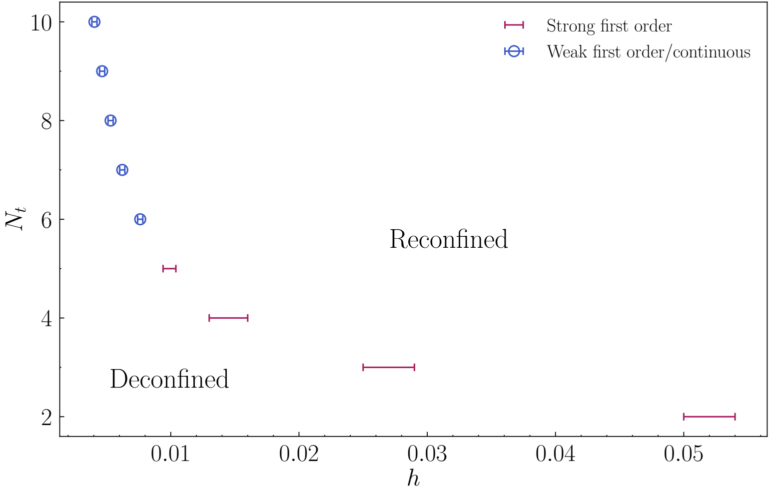
$$S^{def} = S_W + h \sum_x |\operatorname{Tr} P(x)|^2$$

- The trace deformation suppresses gauge configurations where $\langle P \rangle \neq 0$.
- Confinement is preserved even at $T > T_c$.
- For small compactification radius N_t the system is in the weak-coupling regime.
- At fixed β and $N_t = \frac{1}{T}$ in the **deconfined phase** $(T > T_c)$ of ordinary $SU(2)$ YM at large enough h we find a **reconfined phase**.

What does the (N_t, h) phase diagram look like?

Phase Diagram

Small $h \rightarrow$ **Continuous** transition
Large $h \rightarrow$ **First order** transition

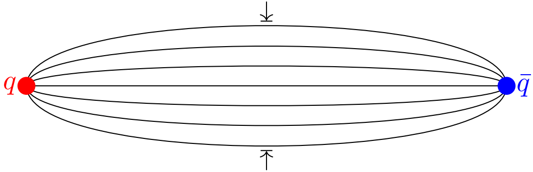


Confining String in the Trace-Deformed Phase

Does the reconfined phase share the same properties as the original confining phase?

- The glueball spectrum, the θ dependence of the free energy and the properties related to the condensation of monopoles appear to be similar in both phases [hep-lat/2010.03618, 2012.13246, 1807.06558]

What about the confining flux tube?



Flux Tube as a String

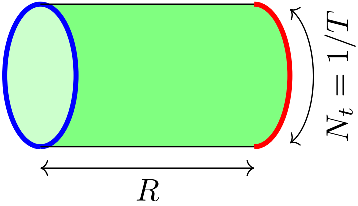
Effective String Theory provides an accurate description of confining strings in Yang-Mills theories

Polyakov loop correlator:

$$G(R) \sim \int DXe^{-S_{EFT}(X)} \equiv Z_{EST}$$

Nambu-Gotō (NG) string model:

$$S_{NG} = \sigma \int_{\Sigma} d^2\xi \sqrt{g}$$



At large distances:

$$G(R) \sim e^{E_0(N_t)R}, \qquad E_0 = \sigma N_t \sqrt{1 - \frac{\pi}{3\sigma N_t^2}}$$

EST Corrections Beyond Nambu-Gotō

In this approximation:

- The **deconfinement critical temperature** $T_c/\sqrt{\sigma}$ does not depend on the gauge group.
- The predicted value is close to the correct one, but still incorrect.

Beyond the NG approximation...

$$S_{EST} = \int d^2\xi \sqrt{g}\Big[\sigma + \gamma_1 \mathcal{R} + \gamma_2 \mathcal{K}^2 + \gamma_4 \mathcal{K}^4 + \dots\Big]$$

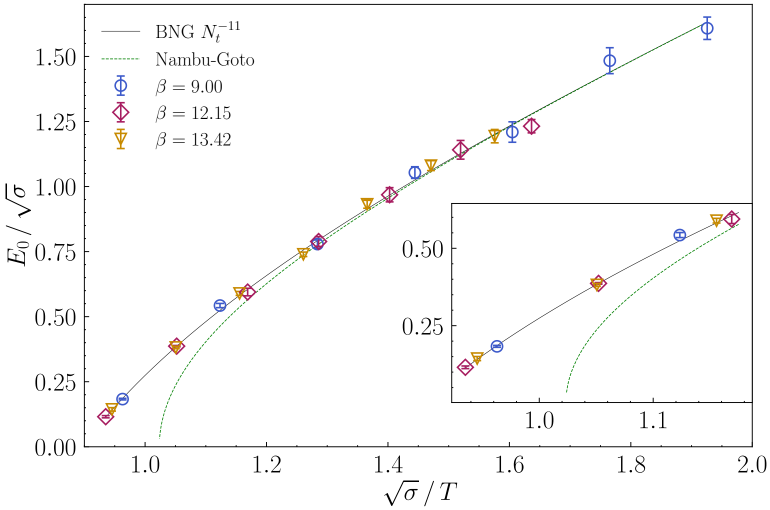
- $\mathcal{R}$ is a topological invariant
- $\mathcal{K}^2$ is proportional to EoM $\rightarrow$ can be eliminated

Low-energy universality of the EST $\Longrightarrow$ First correction BNG can only appear at the order $1/N_t^7$ (Constrained by S-matrix bootstrap analysis)

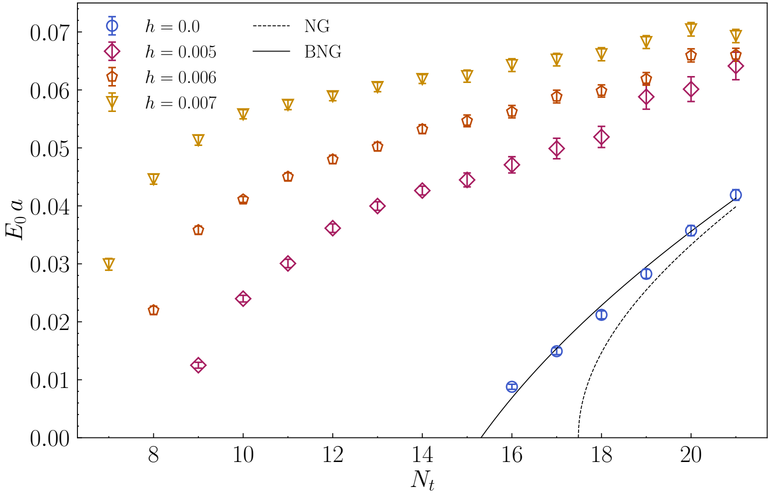
[O. Aharony, Z. Komargodski, 1906.08098]

Ordinary Confining Phase of SU(2)

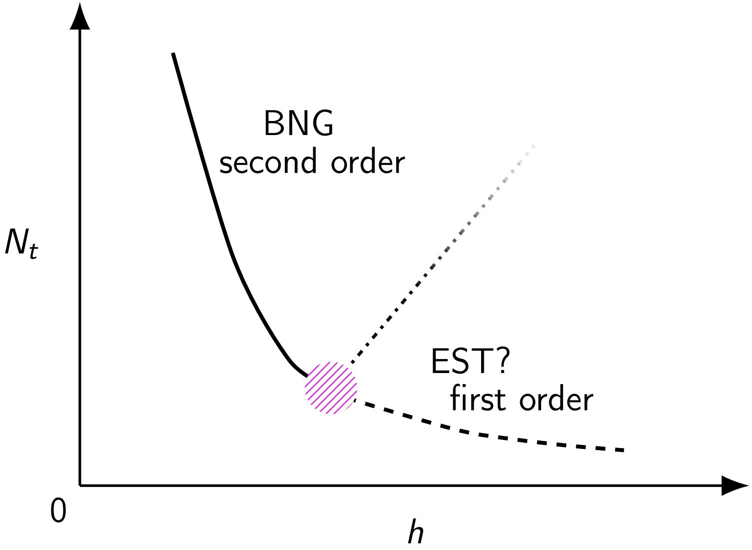
[hep-lat/2407.10678]



Data Deep in the Reconfined Phase

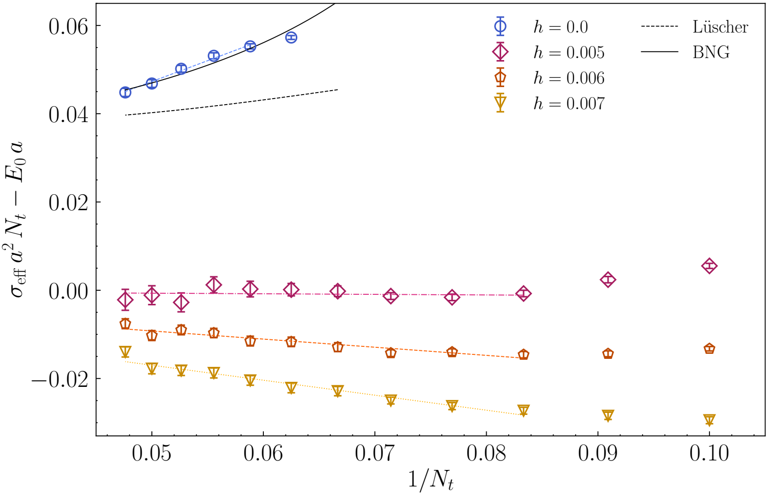


Sketch of the Phase Diagram



Interaction with the Bulk Degrees of Freedom

In $3d$ U(1), when $m \sim \sqrt{\sigma} \longrightarrow E_0(N_t) = \sigma_{eff} N_t - c\frac{\pi}{6N_t}$



A Different Kind of String? The Rigid String

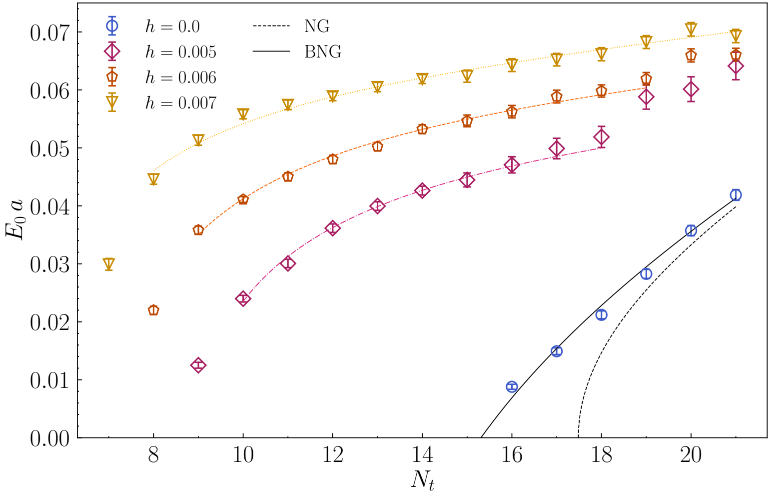
Obtained by adding to the Nambu–Goto action a term proportional to $\mathcal{K}^2$:

$$S_{EST} = \int d^2\xi \sqrt{g}[\sigma + \gamma_2 \mathcal{K}^2 + \dots]$$

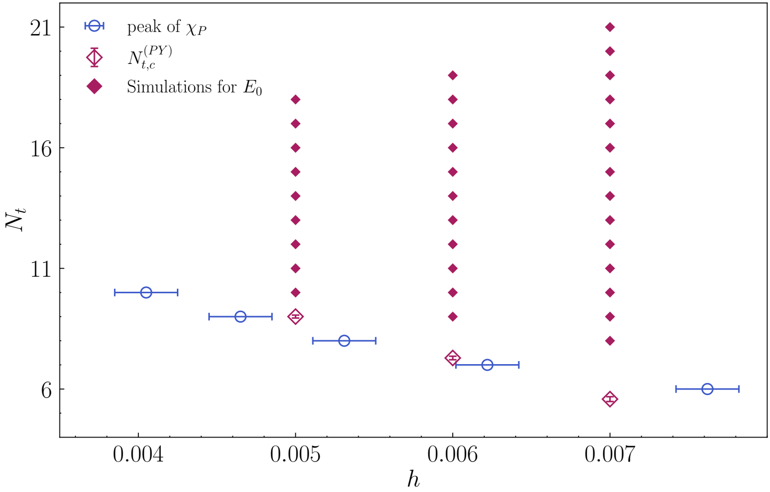
Assuming the $\mathcal{K}^2$ term as dominant and the NG term as a perturbation:

- **Polchinski-Yang solution**: valid for $T \gg \sqrt{\sigma}$ [ArXiv:9205043]
- **Unphysical for ordinary YM theory**:
  $T_c/\sqrt{\sigma} \sim 1 \longrightarrow$ deconfined in that regime
- **Can be realized deep inside the reconfined phase**
  (At large $h \longrightarrow T_c$ is higher)

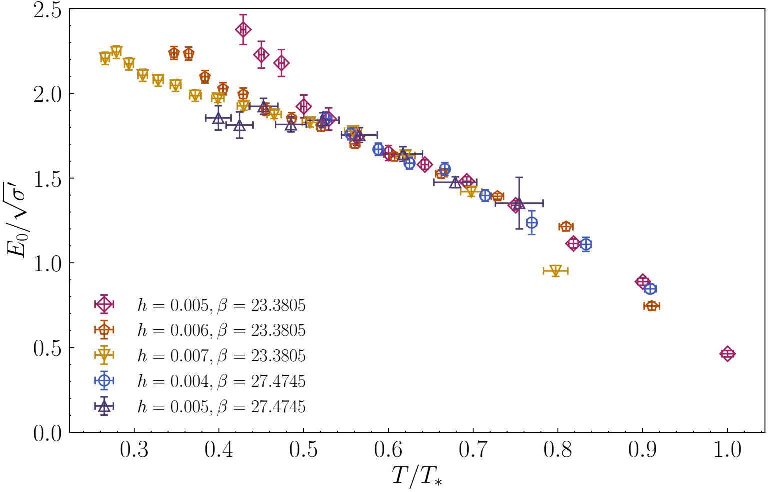
Fitting the Data with the PY Solution



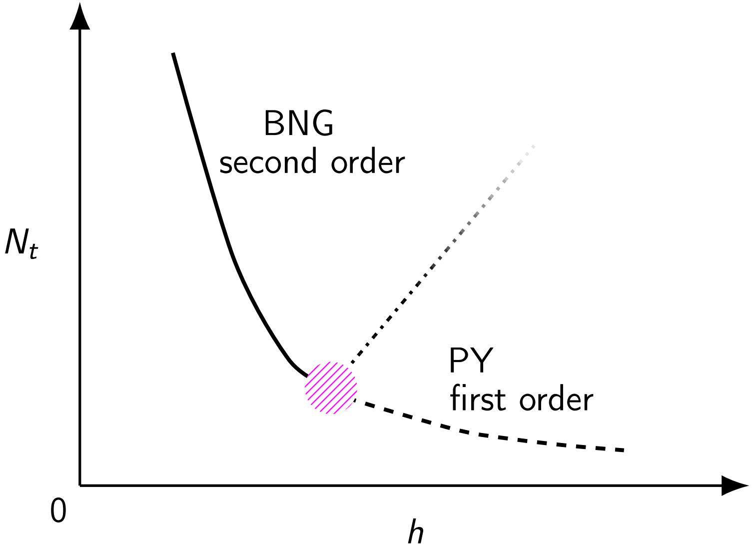
Critical Temperature



Data Collapse



Sketch of the Phase Diagram



Conclusion

- The critical line is **continuous** at small-h and **discontinuous** at large-h.
- As the deformation grows, Beyond Nambu-Goto fails to describe the data.
- At large-h the **Polchinski-Yang rigid string solutions** accurately describe the data and predicts compatible critical temperatures.

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Thank you for your attention!

Backup

EST corrections beyond Nambu-Gotō

In the high-temperature regime, the N_t dependence of the ground state E_0 up to $1/N_t^{11}$ order can be parameterized as:

$$E_0(N_t) = N_t\sigma\sqrt{1 - \frac{\pi}{3N_t^2\,\sigma}} + \frac{k_4}{\sigma^3 N_t^7} + \frac{2\pi k_4}{3\sigma^4 N_t^9} + \frac{5\pi^2 k_4}{16\sigma^5 N_t^{11}} + \frac{k_5}{\sigma^5 N_t^{11}} + \dots$$

This expression is used for comparison with the results of our numerical simulations: [ArXiv:2407.10678]

The Polchinski-Yang solution

$$\text{Valid when: } \gamma_2 \gg N_t^2\sigma, \qquad N_t^2\sigma \ll 1$$

The ground state has two fit parameters:

$$E_0 = w\lambda, \qquad w = \sqrt{N_t^2 - \frac{b(d-2)N_t}{2\sqrt{\lambda}}}$$

$$\sqrt{\lambda} = \frac{3b}{8}\frac{d-2}{N_t} + \sqrt{\frac{9b^2}{64}\frac{(d-2)^2}{N_t^2} + \sigma' - \frac{\pi(d-2)}{3N_t^2}}$$

$$b = \frac{1}{\sqrt{2\gamma_2}}, \qquad N_{t,c}^2\sigma = \frac{\pi(d-2)}{3} - \frac{b(d-2)^2}{8}$$

Metastable phases at large-h