The onset of conformal window from chiral phase boundaries

Comparison of 7 and 8 flavour QCD

in collaboration with
Owe Philipsen, Reinhold Kaiser, Jonas Schaible

Jan Philipp Klinger
Lattice 2026
University of Maryland
31.07.2026

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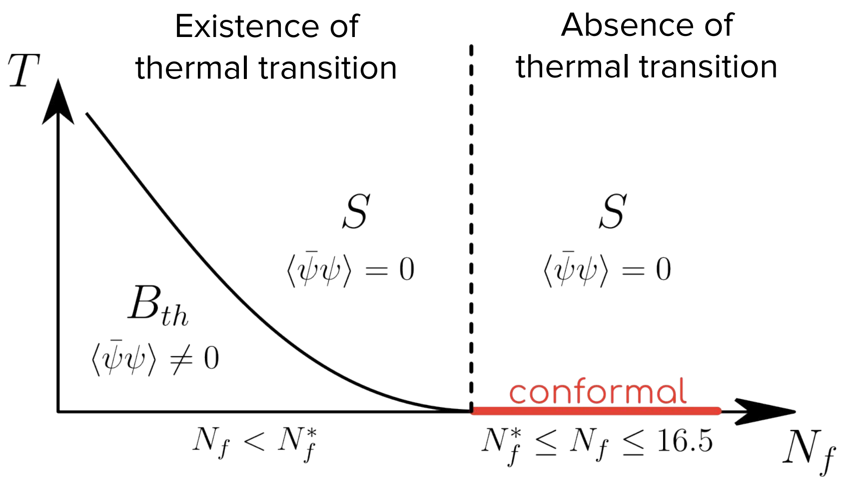
CRC-TR 211 Strong-interaction matter under extreme conditions

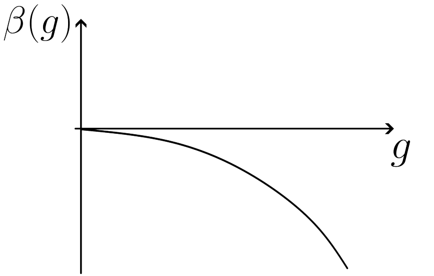
Massless many-flavour QCD

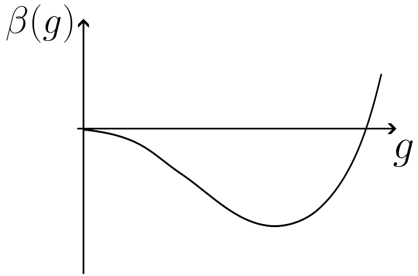
Chiral phase diagram for degenerate quarks:



Beta-function of gauge coupling:

- For $N_f < N_f^*$: $\beta(g) < 0$ ➡ QCD-like
- For $N_f \geq N_f^*$: $\beta(g^\star) = 0$ ➡ Banks-Zaks FP





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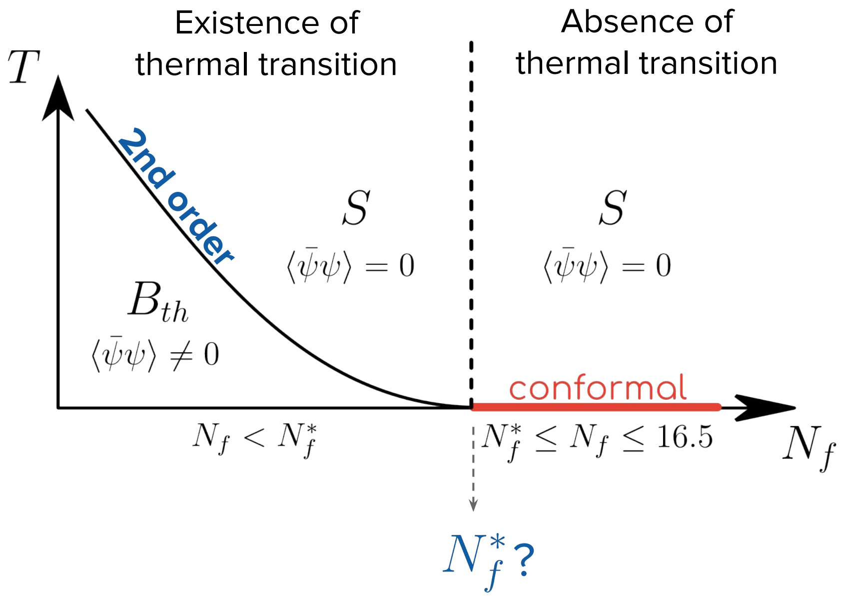

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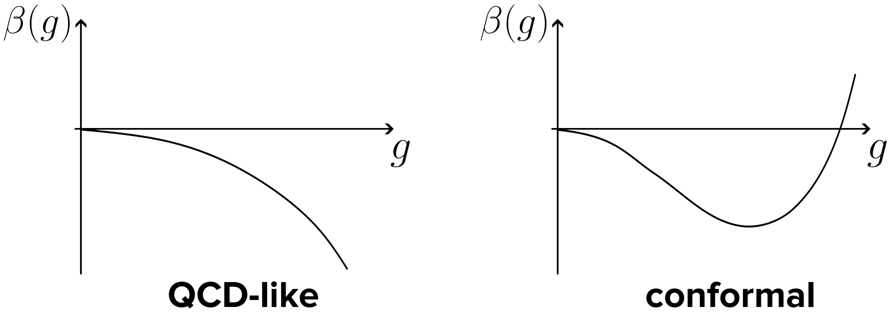


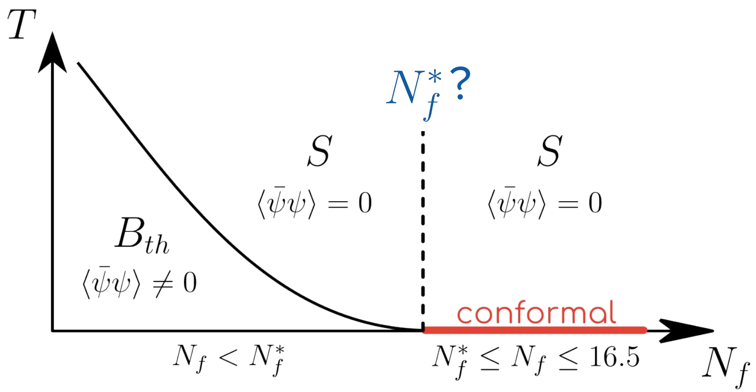
Related studies:

- Evidence for 2nd order for all $N_f < N_f^*$:
  ➡ Cuteri et al. (2021), Our Proceedings (2025, 2026)
- Promising onset candidate: $N_f^* = 8$
  ➡ Hasenfratz (2022); D'Elia et al. (2026)

Our Lattice Setup

QCD with N_f degenerate quarks with mass m:

- unimproved staggered fermions
- unimproved Wilson gauge action
- bare parameters: $\beta = 6/g^2,\ am,\ N_f,\ N_\tau$
- continuum limit: $N_\tau = 1/aT \to \infty,\ \beta \to \infty$



Approach:

- study chiral phase boundaries in $\{\beta,\ am,\ N_\tau,\ N_f\}$

Imprints of conformal phase at finite am and N_τ?

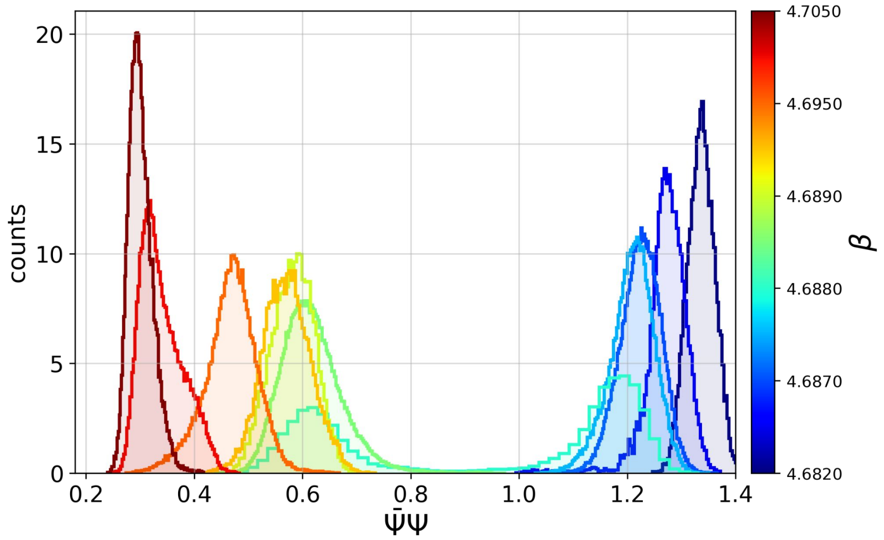
Locating phase transitions:

- order parameter $\mathcal{O}$: $\langle\bar{\psi}\psi\rangle$
- standardized moments:

$$B_n = \frac{\langle(\mathcal{O}-\langle\mathcal{O}\rangle)^n\rangle}{\left\langle(\mathcal{O}-\langle\mathcal{O}\rangle)^2\right\rangle^{n/2}}$$

Lattice QCD phases

For fixed N_f, am, N_τ: β - scan



Thermal transition

Bulk transition

Lattice QCD phases

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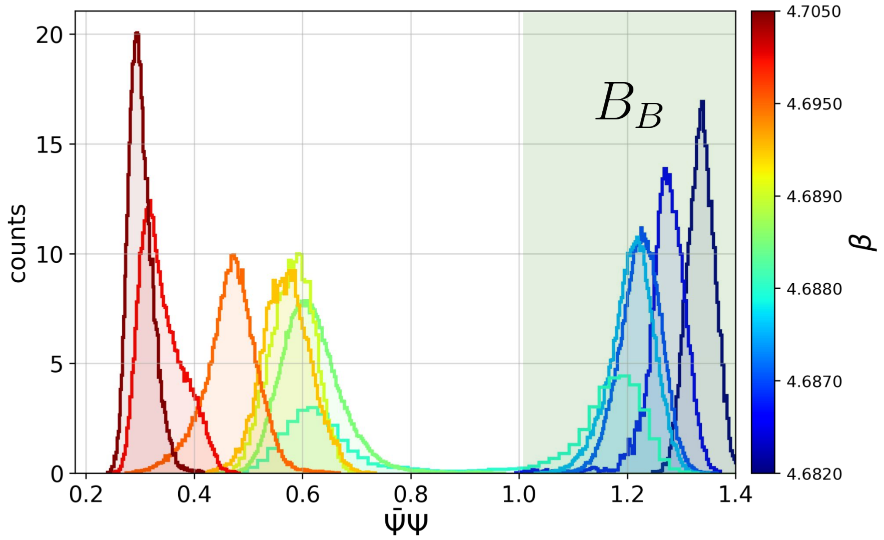

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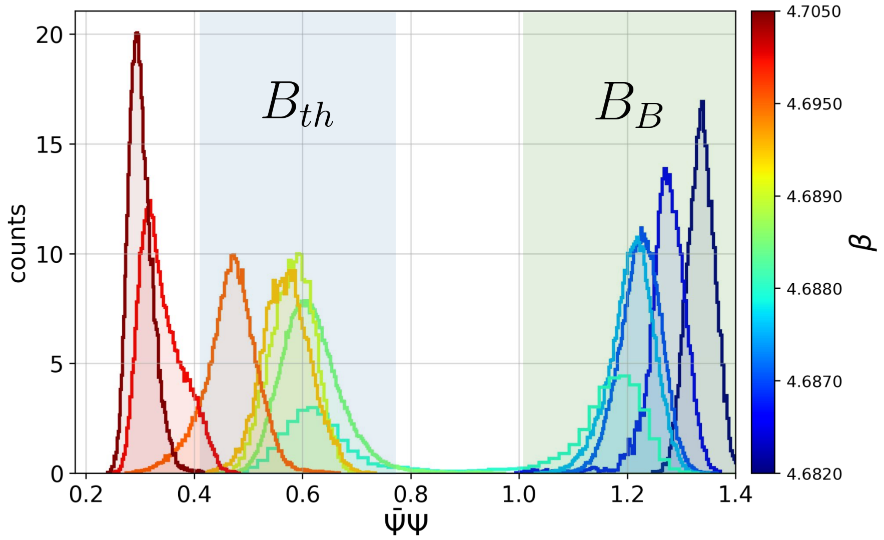

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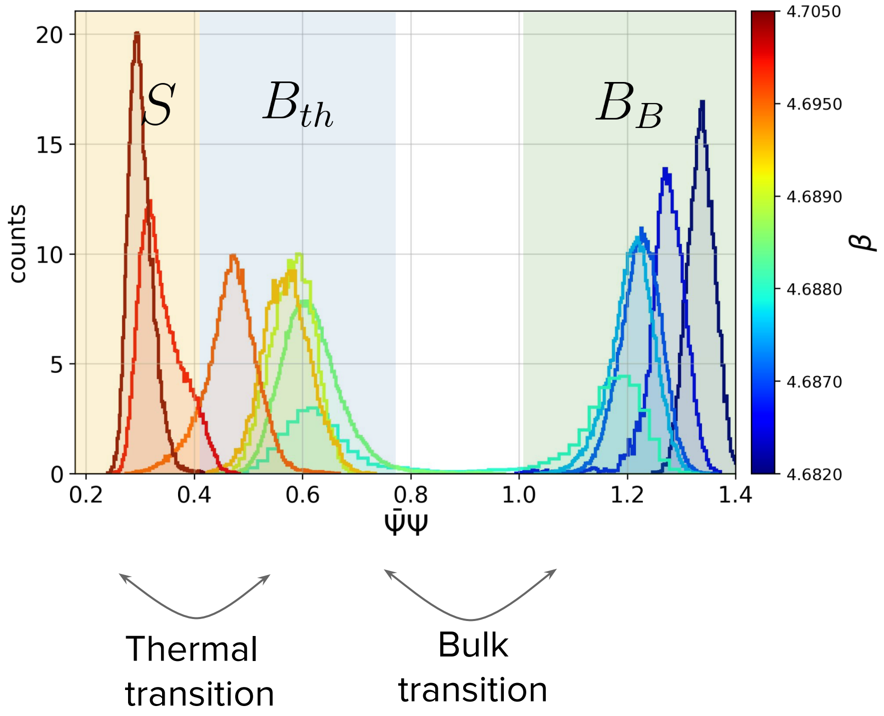

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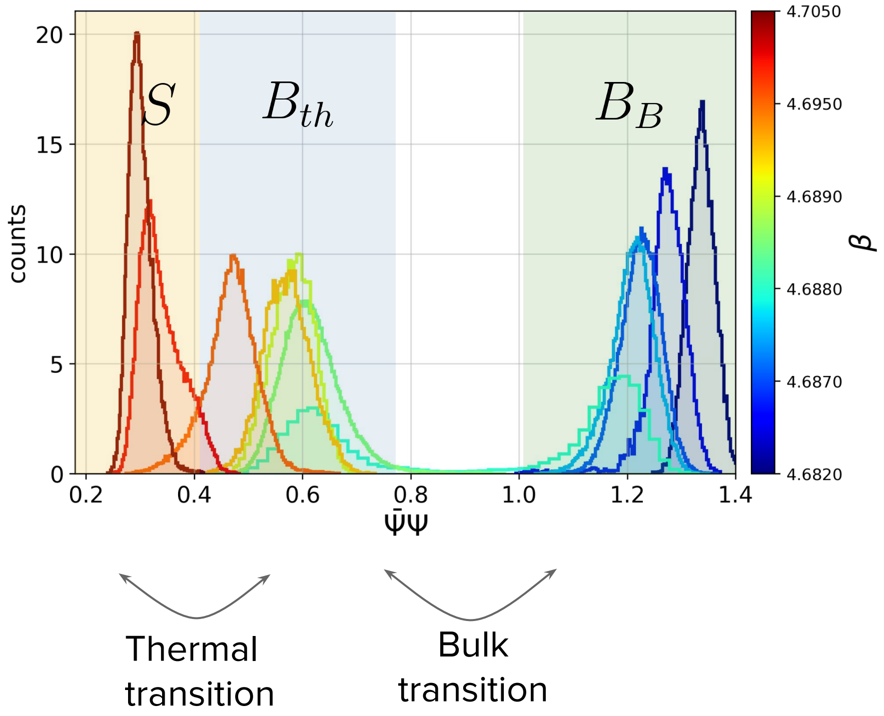
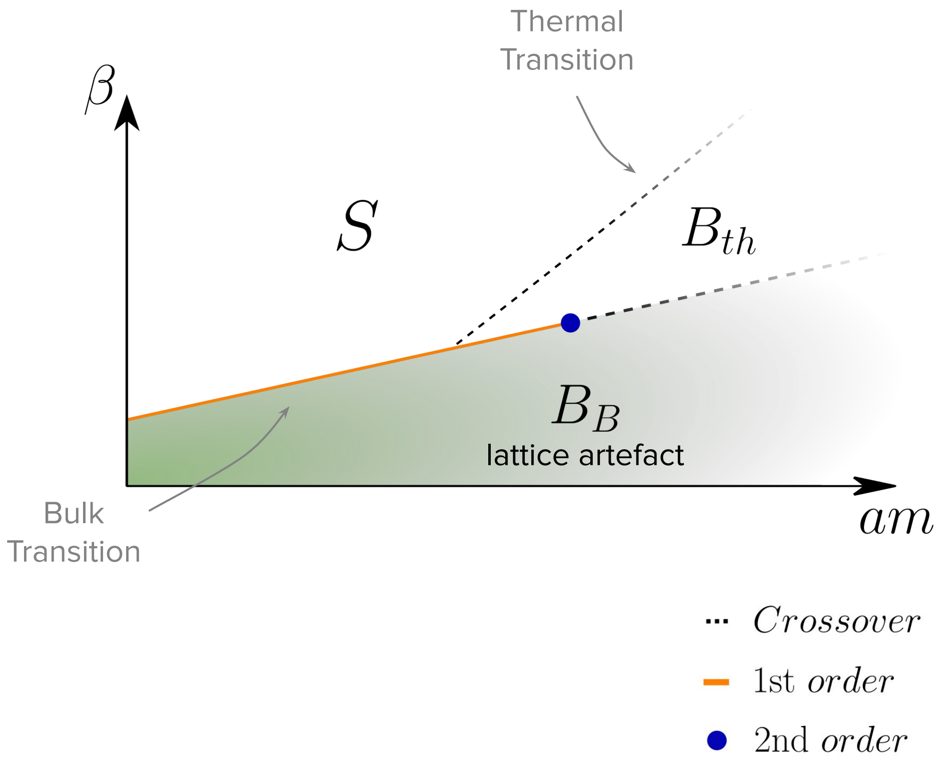


For fixed N_f and N_τ:



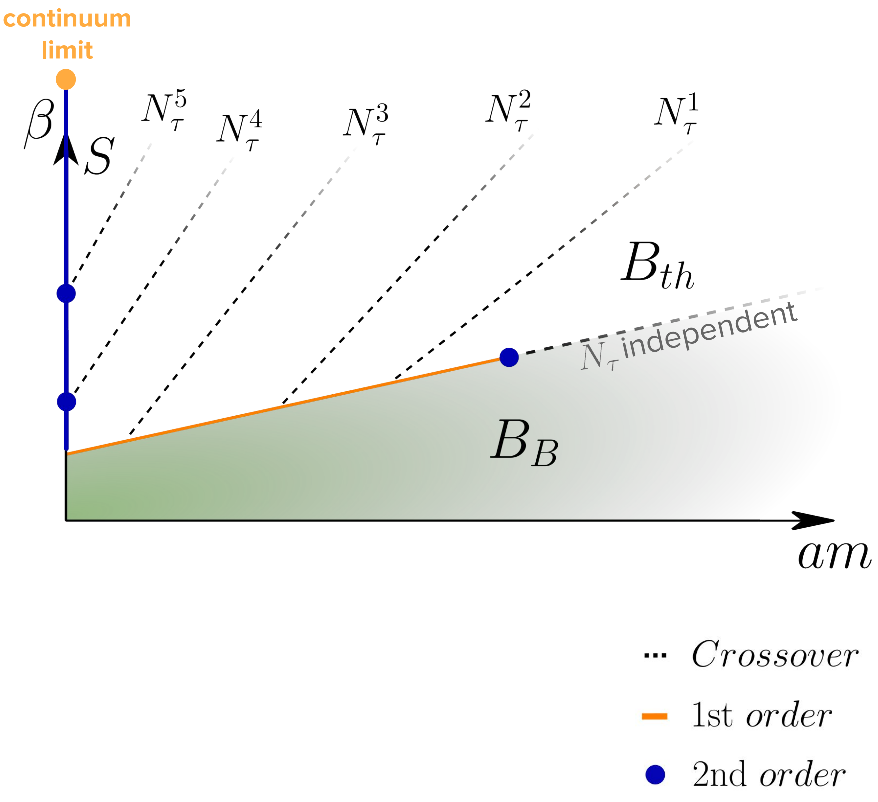
QCD-like continuum limit

Increasing N_τ :

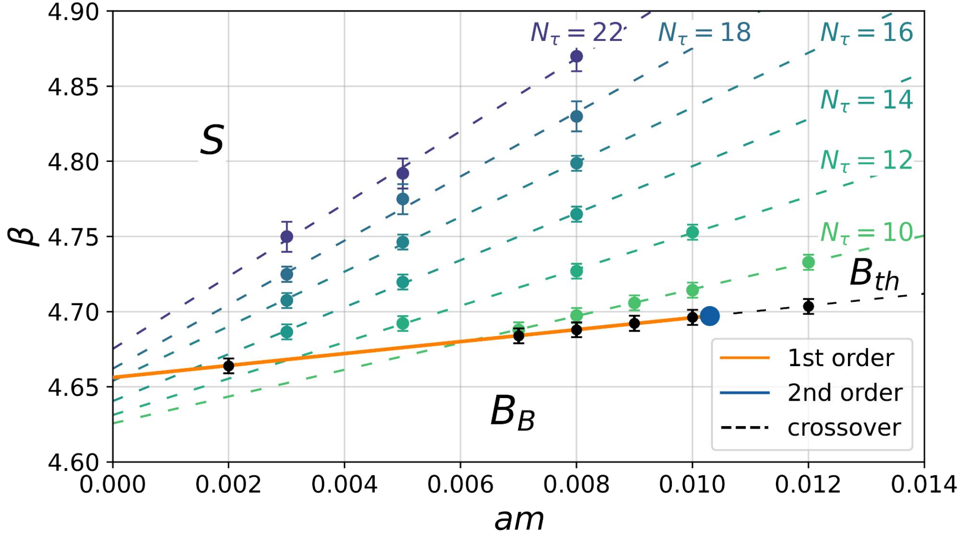
- Bulk transition is N_τ-independent
- $\beta(am)$ of thermal transition increases
- continuum limit: $\beta \to \infty$ for $N_\tau \to \infty$

Chiral limit $am = 0$:

- $\beta_c(am = 0) \to \infty$ for increasing N_τ
- Coexistence of S and B_{th} in continuum



Chiral phase boundary for $N_f = 7$



Dashed lines: Linear fits for fixed N_τ

Simulation details:

- $N_f = 7$
- $N_\tau : \{10,\ 12,\ 14,\ 16,\ 18,\ 22\}$
- Per N_τ: 3-5 masses $\in [0.003, 0.02]$
- $N_s/N_\tau \in \{1,\ 2\}$

Chiral limit $am = 0$:

$\beta_c(am = 0) \to \infty$ for increasing N_τ

QCD-like:

Thermal transition is connected to continuum

$$T(m = 0) > 0$$

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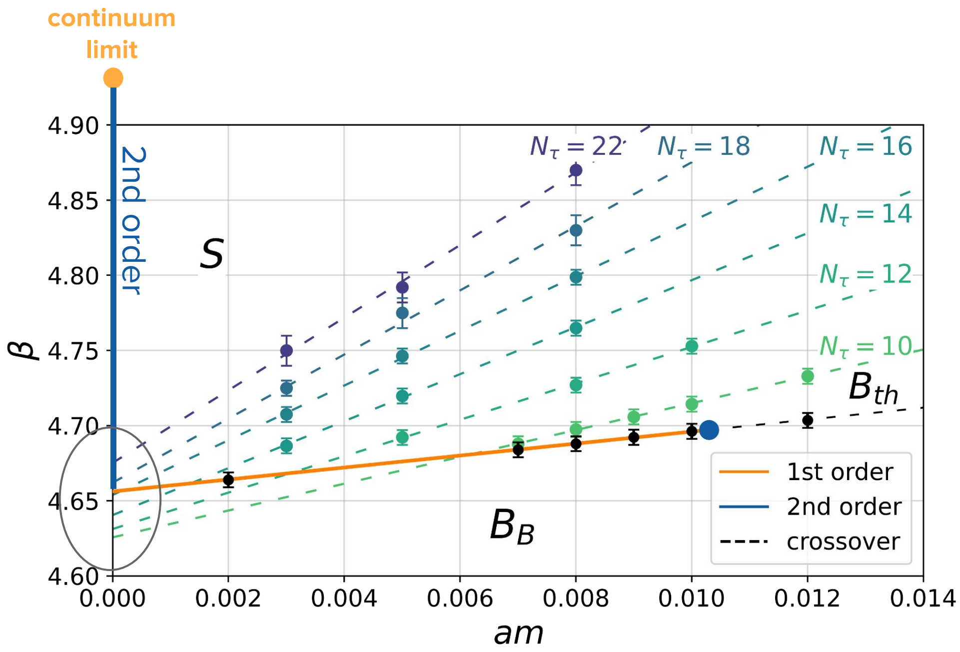

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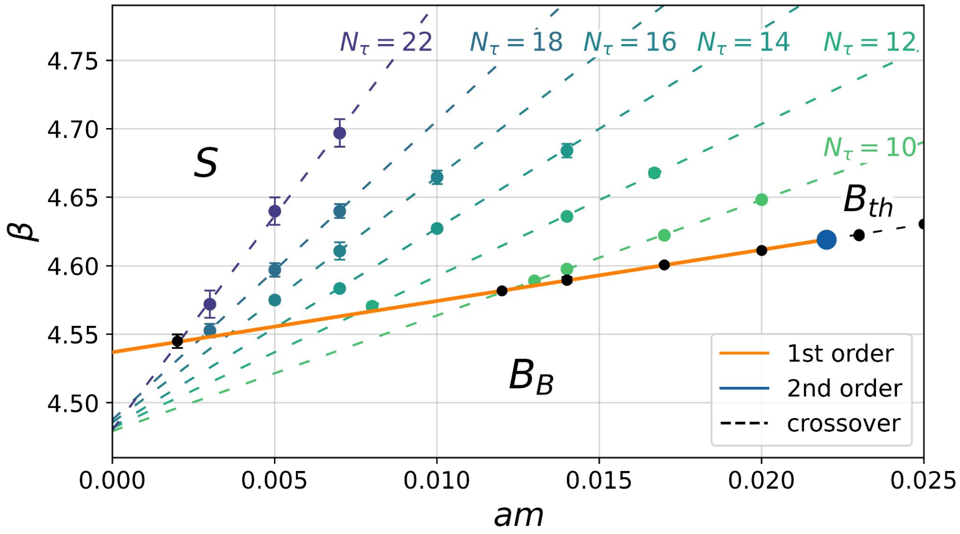

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Chiral limit $am = 0$:

✗ $\beta_c(am=0) \to \infty$ for increasing N_τ

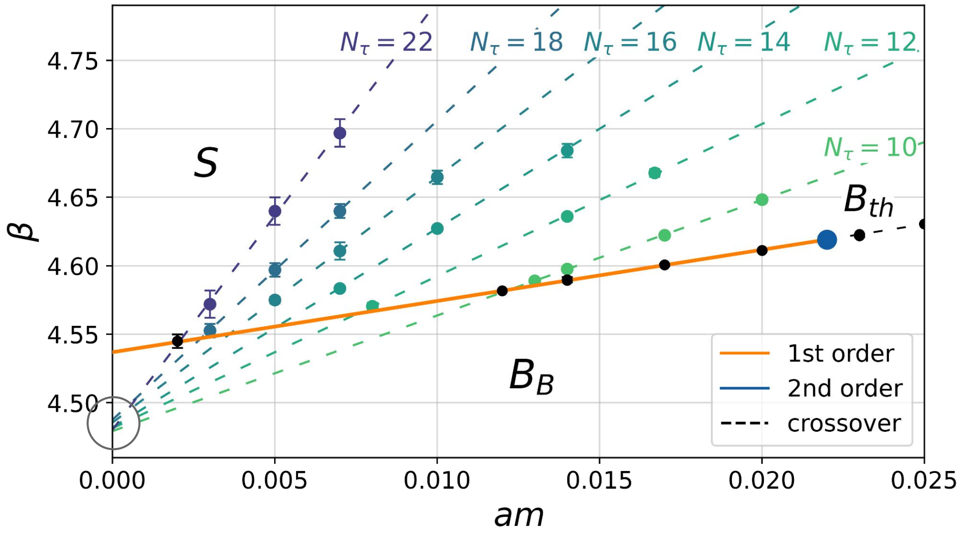
✓ $\beta_c(am=0) = const$ for increasing N_τ

Conformal:

Thermal transition is ***not*** connected to continuum

$\Rightarrow T(m=0) = 0$

Chiral phase boundary for $\mathrm{N_f} = 8$



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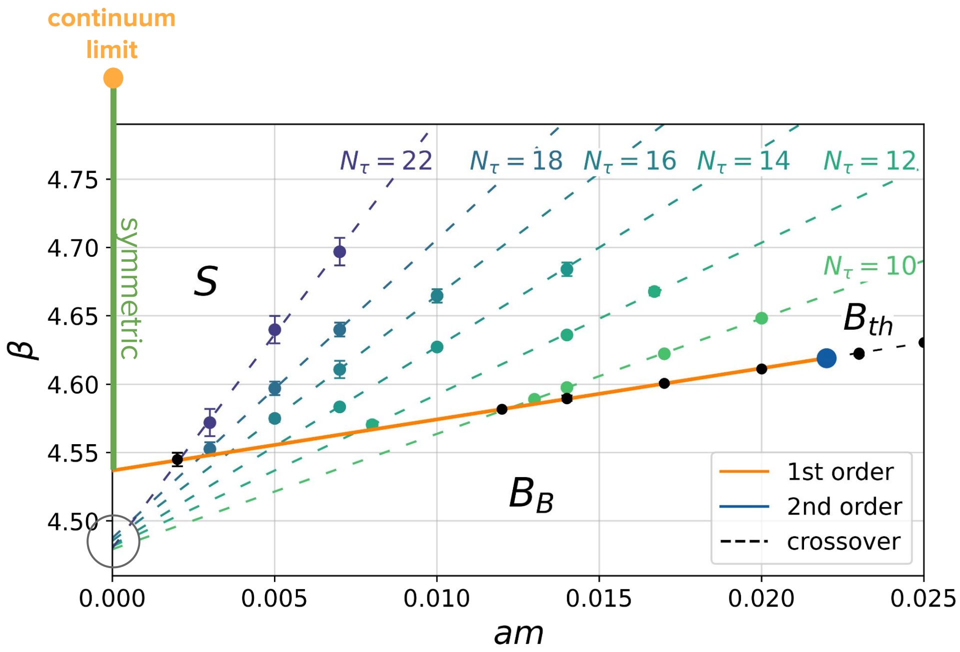

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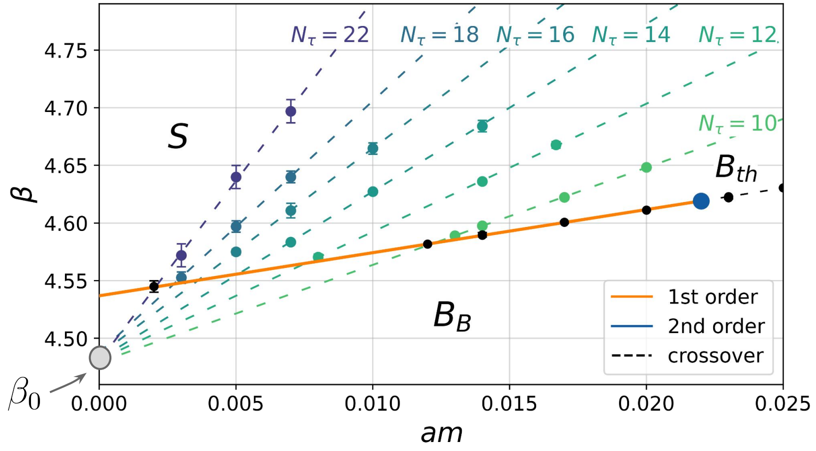
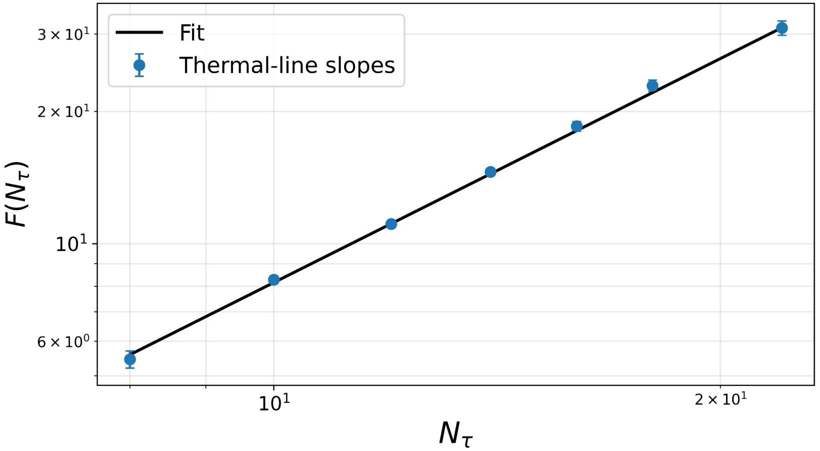
Analytic form of phase boundary for $\mathrm{N_f} = 8$



Parameterisation of phase boundary:

➡ $\beta_{pc}(am, N_\tau) = \beta_0 + F(N_\tau)am$



Slopes follow power law:

➡ $F(N_\tau) = CN_\tau^p$

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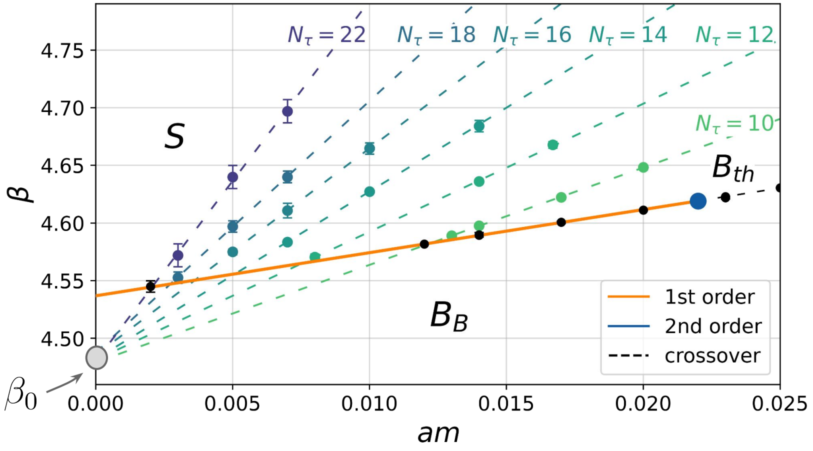

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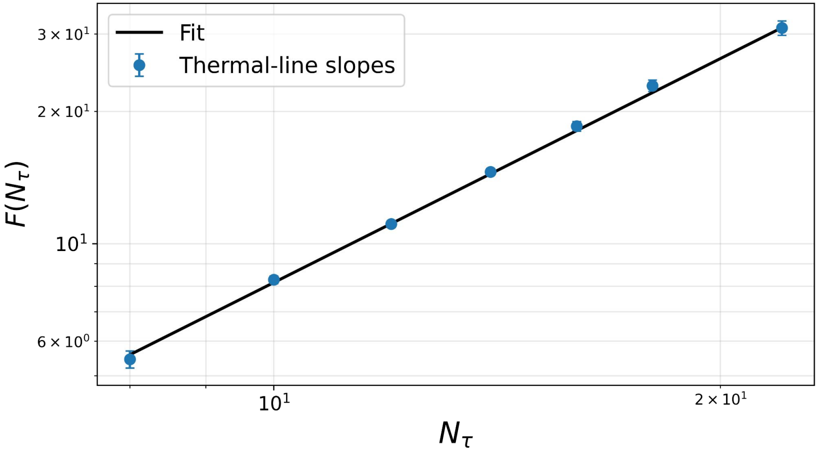

Slopes follow power law:

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Fit: $\beta_{pc}(am, N_\tau) = \beta_0 + C\, N_\tau^p\, am$

- 3 fit parameters for 19 data points!
- chi2/dof = 5.5/16 = 0.34

$\beta_0 \approx 4.48$
$C \approx 0.16$
$p \approx 1.69$

Conformal hyperscaling

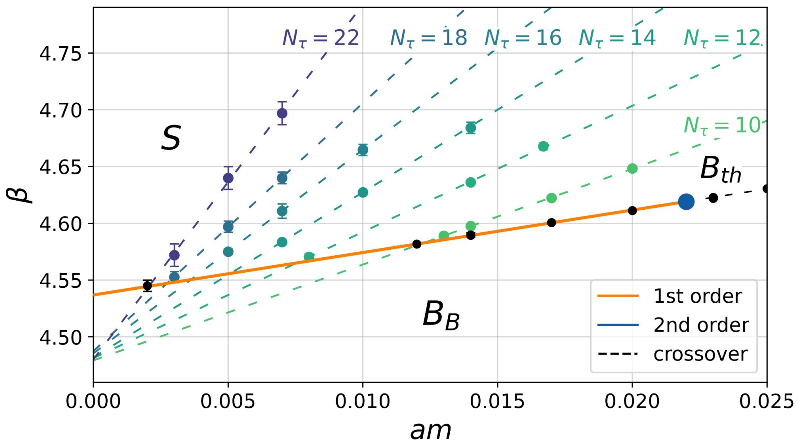
- $m > 0$: bare quark mass sets the only IR scale
  - ➡ $T \sim m^{1/(1+\gamma_m^*)}$

Hypersurface: $\beta(am, N_\tau) = \beta_0 + C\, N_\tau^p\, am$

rearrange

$$aT = \left(\frac{C}{\beta - \beta_0}\right)^{1/p} (am)^{1/p}$$

$$\boxed{aT = K(\beta)(am)^{1/p}}$$



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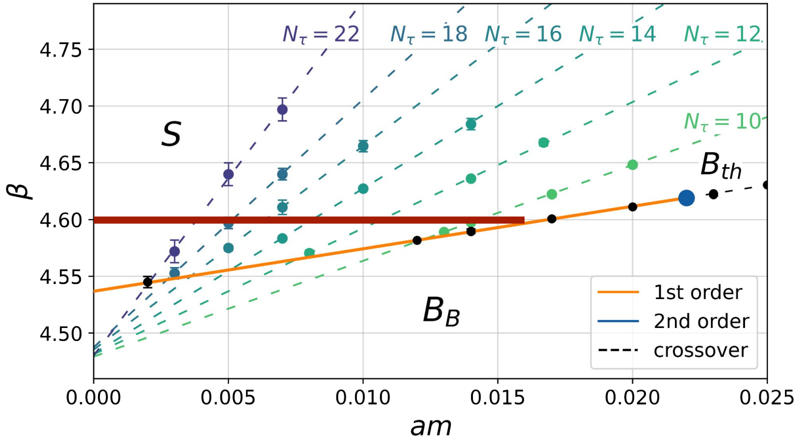

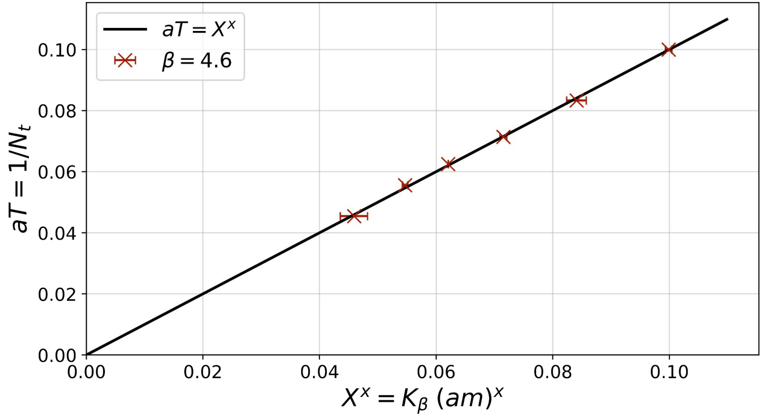

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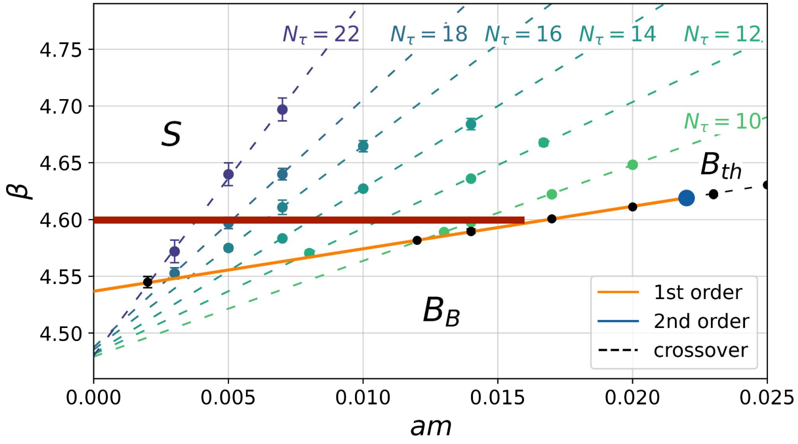
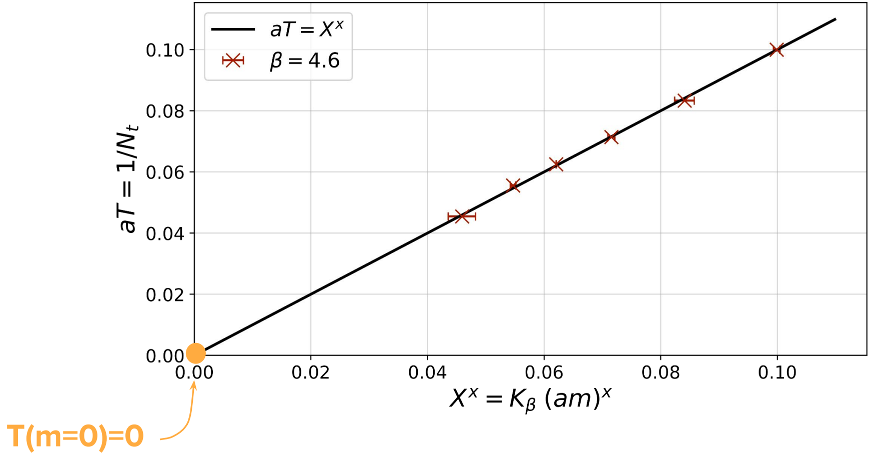
Chiral limit:

$$aT(am = 0) = 0 \quad \forall \beta = const$$

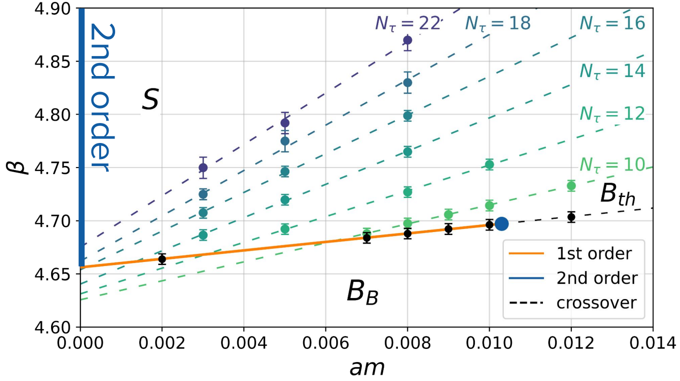
$\Rightarrow T = 0$ because $a \neq 0$ due to $\beta \neq \infty$



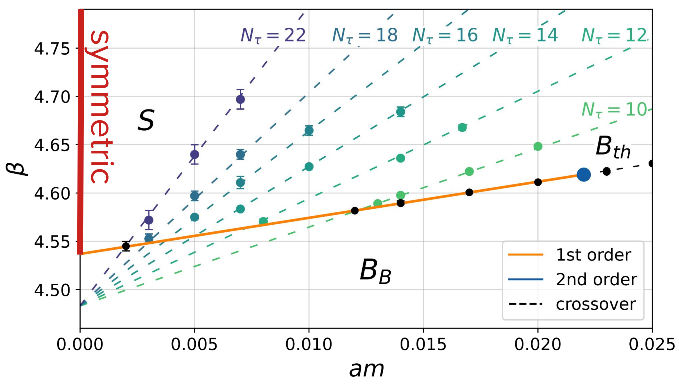


Conclusion

7 Flavour



8 Flavour

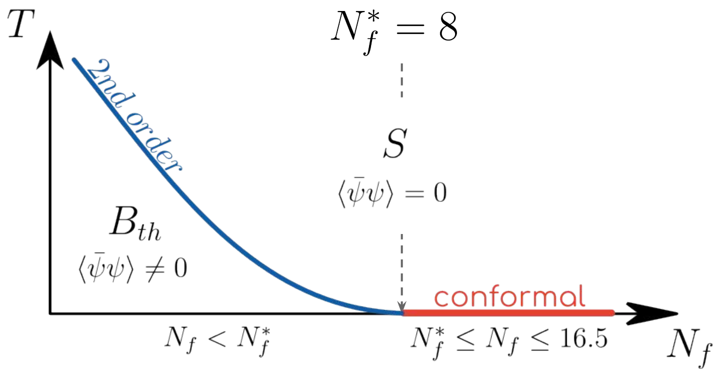


7 Flavour: QCD-like

- Thermal transition exists in chiral continuum limit
- $\beta_c(am = 0) \to \infty$ for $N_\tau \to \infty$

8 Flavour: Onset of the conformal window

- Chirally symmetric for all N_τ in chiral limit
- Conformal hyperscaling: $T \sim m^{1/(1+\gamma_m^*)}$
  - ➔ $aT = K(\beta)(am)^{1/p}$



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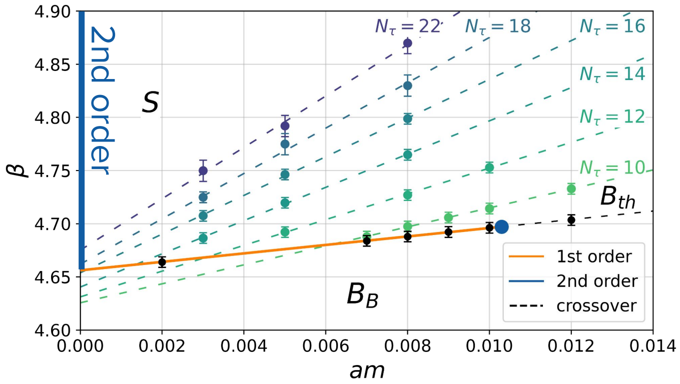

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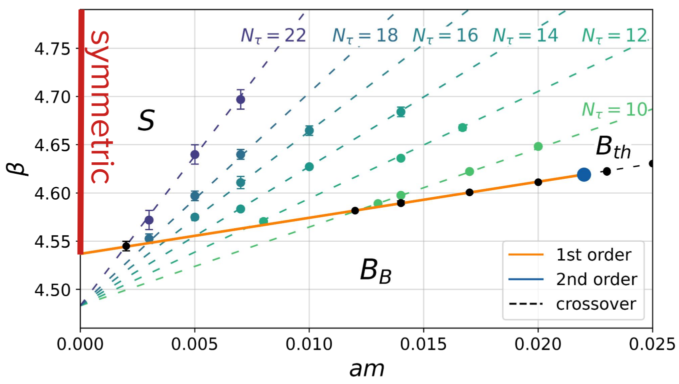

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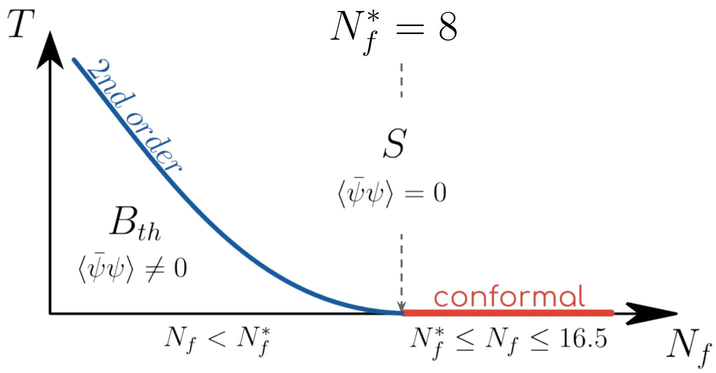

Thank You

Backup Slides

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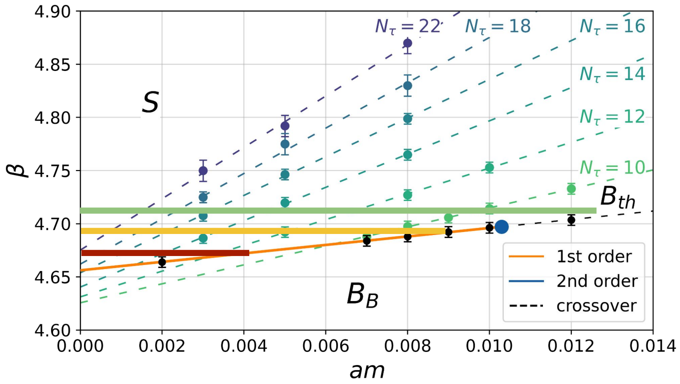
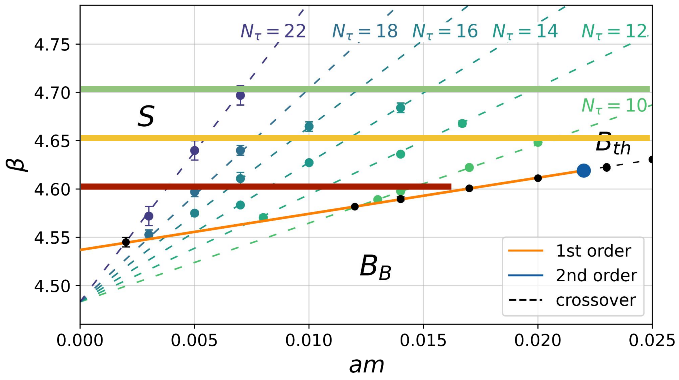
CRC-TR 211
Strong-interaction matter under extreme conditions

Comparison: Scaling with mass

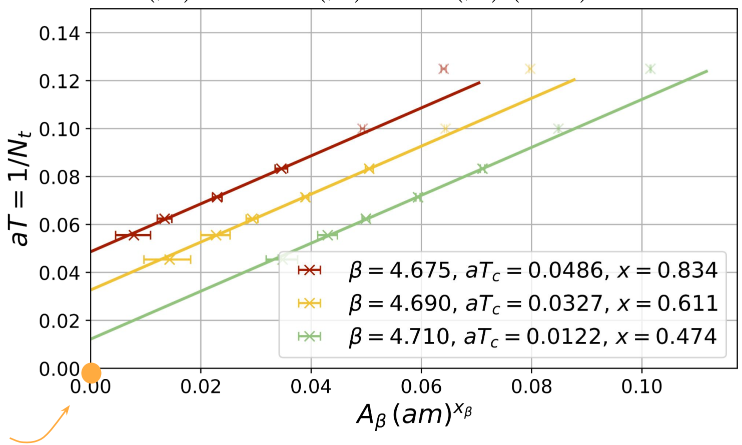
<u>7 Flavour</u>



<u>8 Flavour</u>



$$aT(\beta) = aT_c(\beta) + A(\beta)(am)^{x_\beta}$$



$T \neq 0$
$a = 0$

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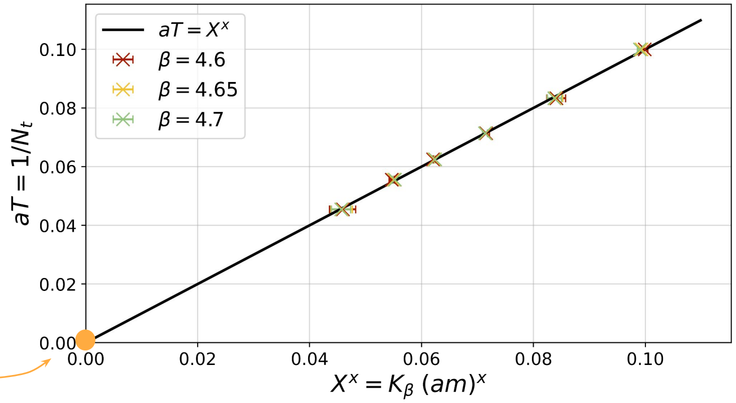

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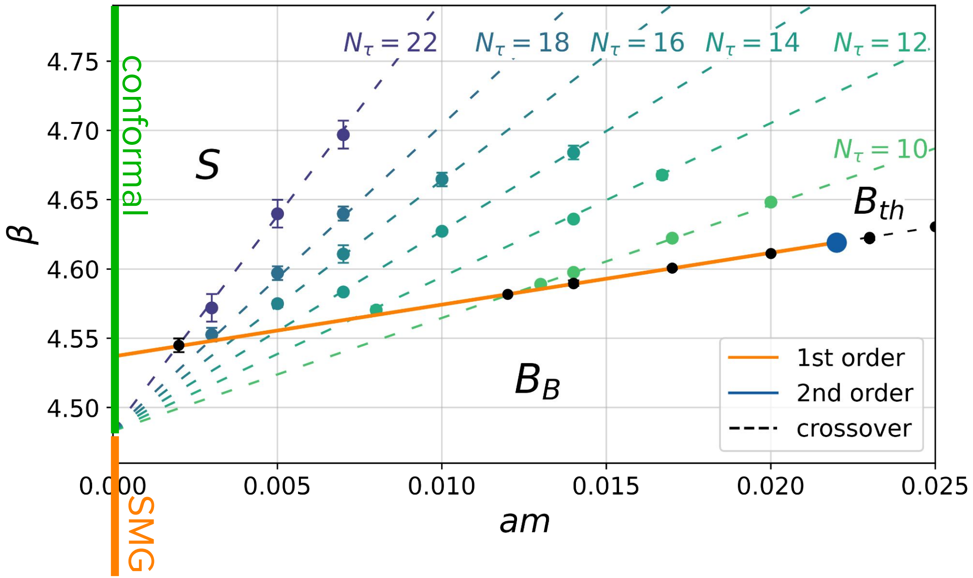
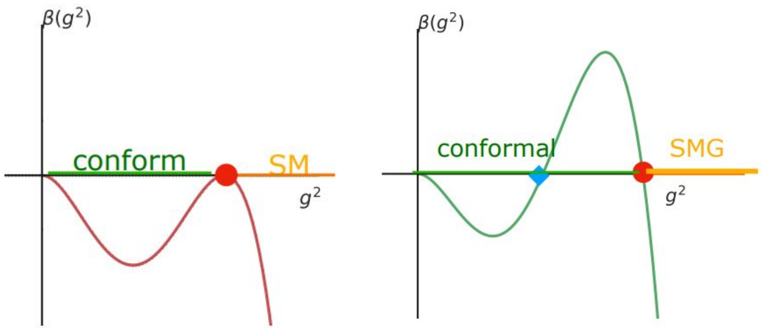
Symmetric mass generation



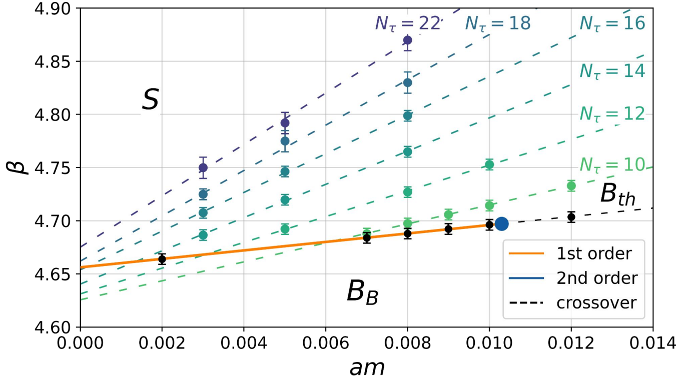
- β_0 as an UVFP ?
- continuum limit $\beta_c(am = 0) \to \beta_0$?
  - for increasing Nt



[Hasenfratz]

Comparison: Chiral limit

7 Flavour



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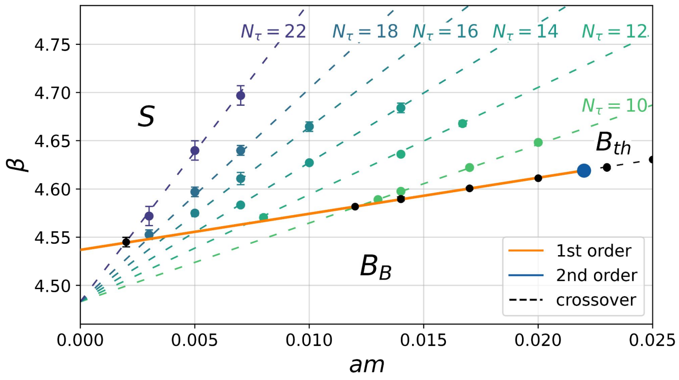
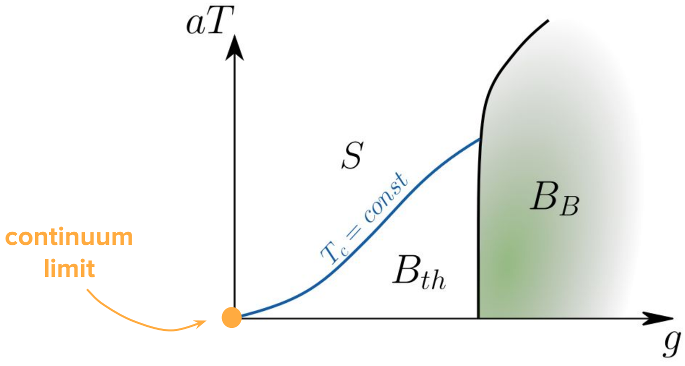
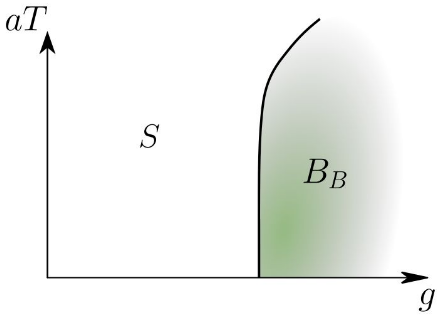


QCD-like: $\boxed{N_f < N_f^*}$ at am=0

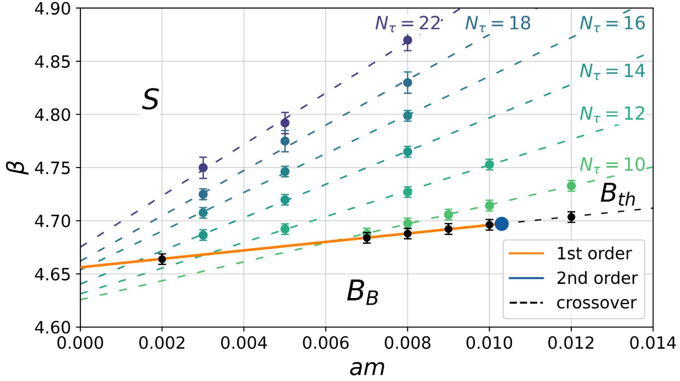


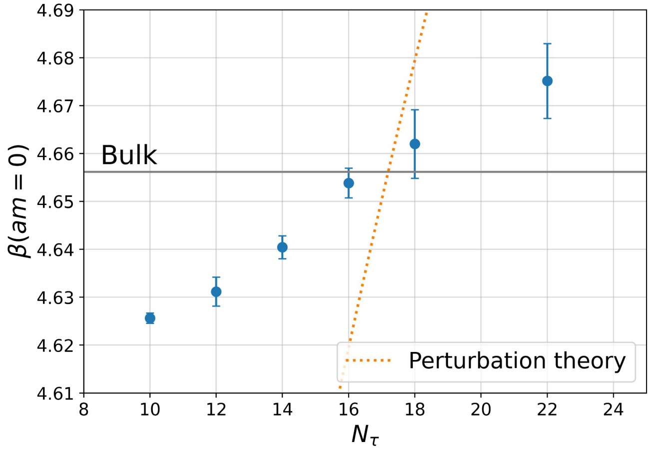
Conformal: $\boxed{N_f \geq N_f^*}$ at am=0



Comparison: $\beta(am = 0, N_\tau)$

7 Flavour





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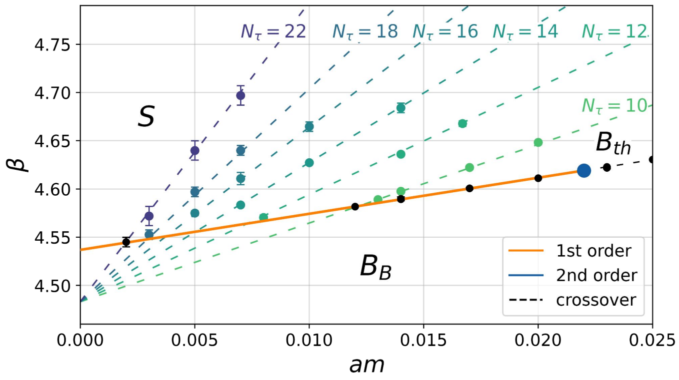
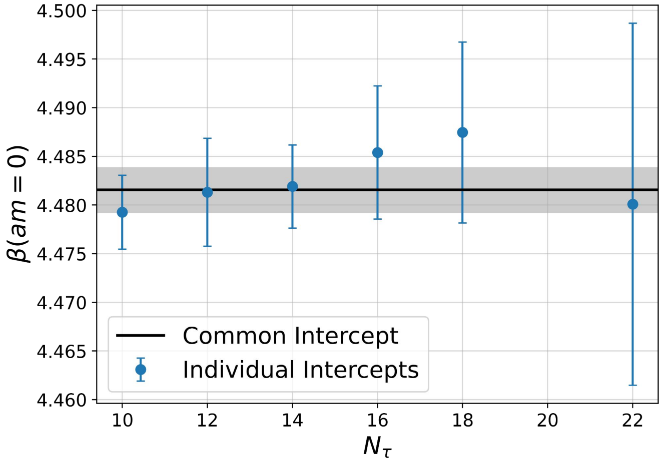
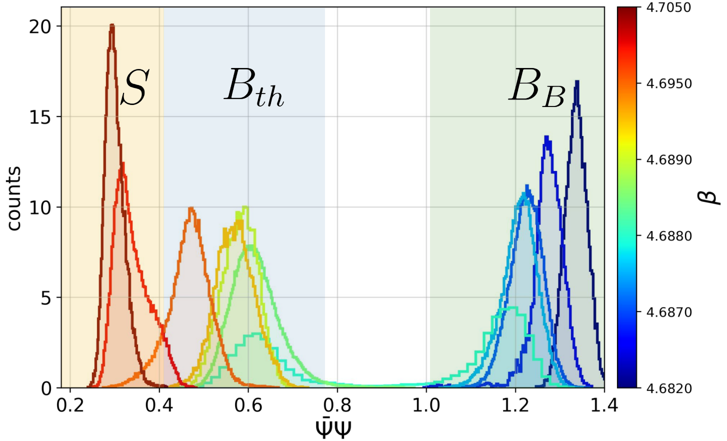


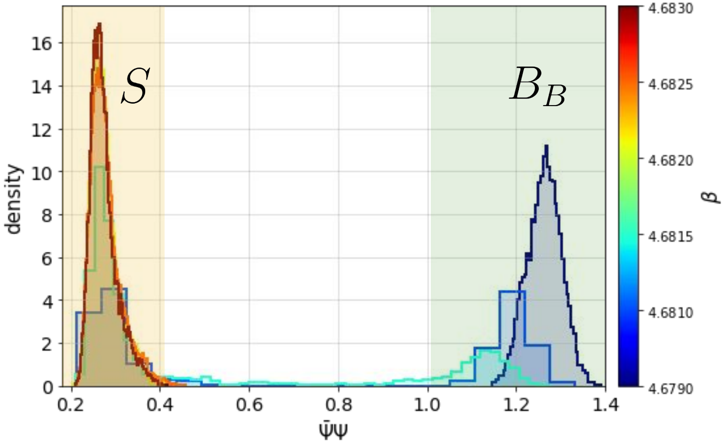


Lattice QCD phases for large Nf

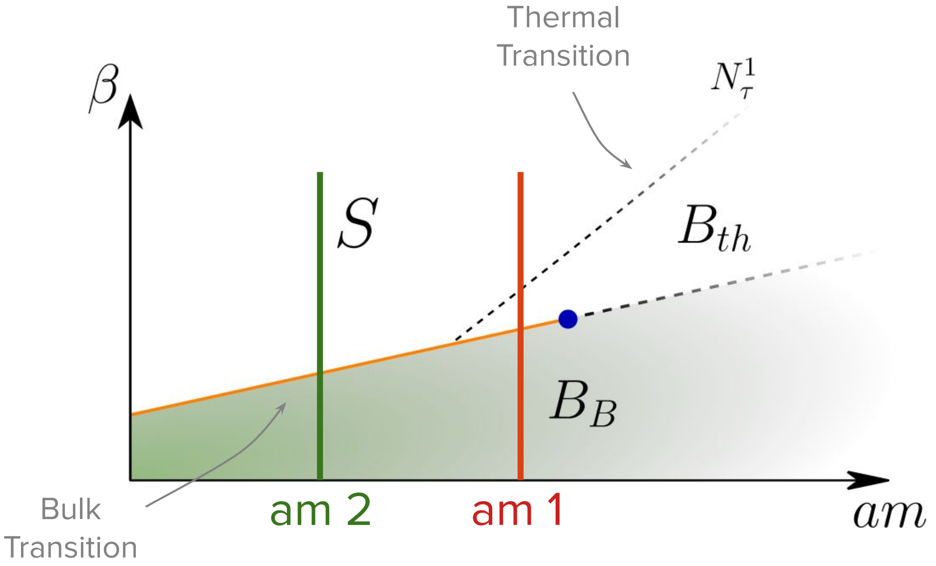
am 1:



am 2:



For fixed Nf and Ntau:



··· $Crossover$

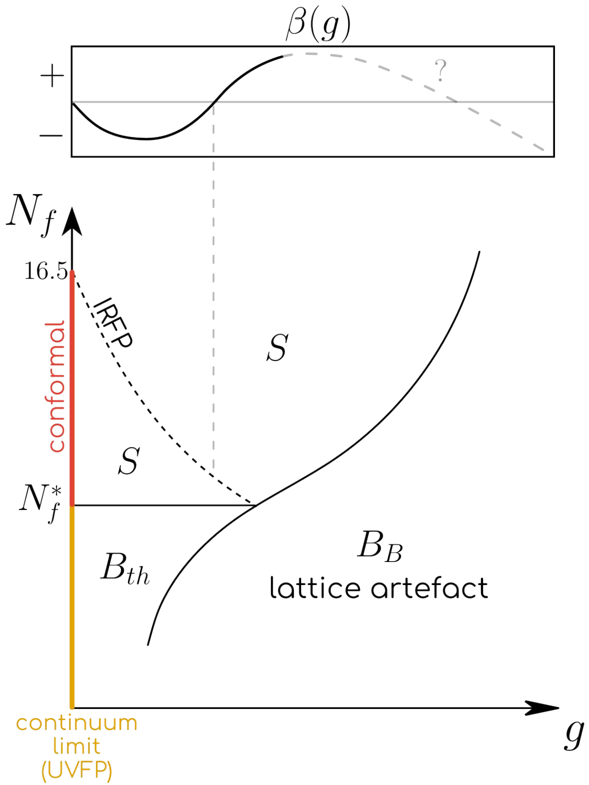
— 1st $order$

● 2nd $order$

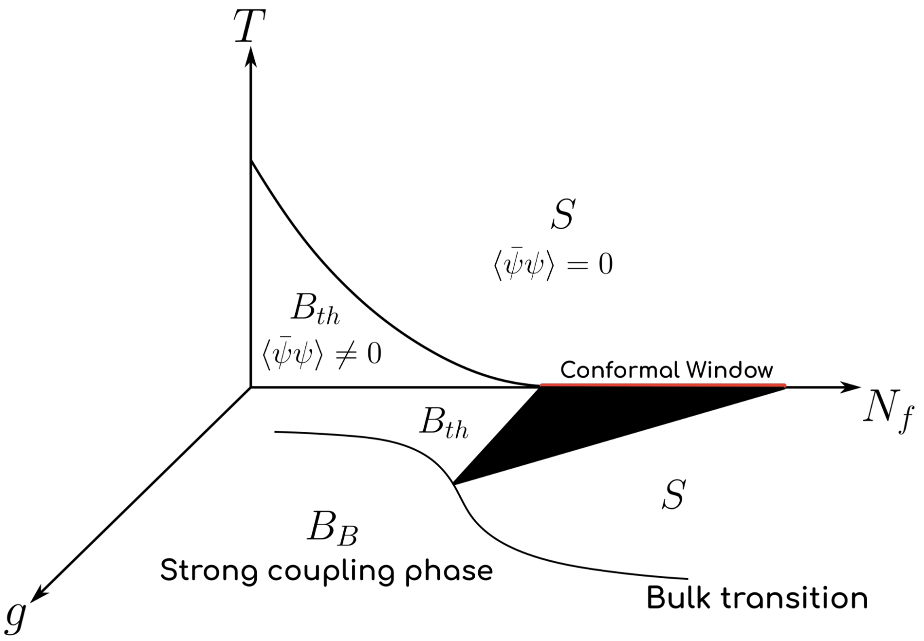
Lattice QCD at aT=0

$S : \langle\bar{\Psi}\Psi\rangle = 0$ chirally symmetric
$B_{th} : \langle\bar{\Psi}\Psi\rangle \neq 0$ chirally broken (thermal transition)
$B_B : \langle\bar{\Psi}\Psi\rangle \neq 0$ chirally broken (bulk transition)

aT=0
am=0



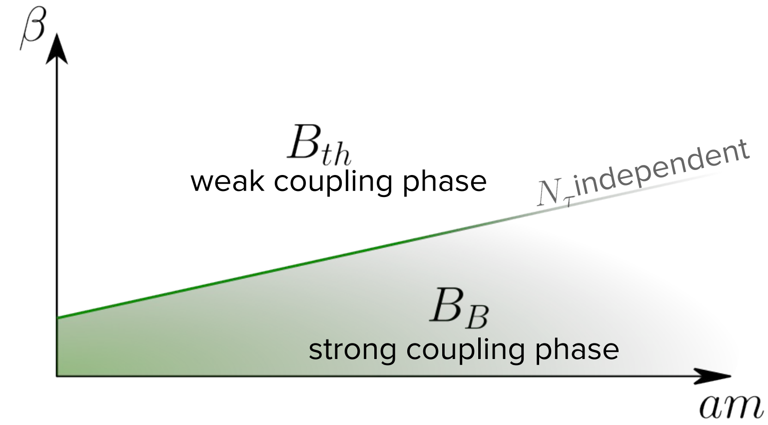
Miransky, Yamawaki 1997

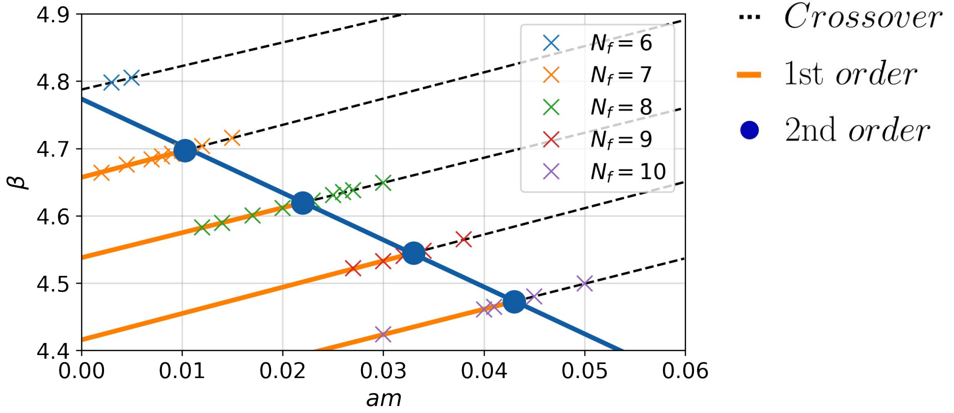


Order of Bulk Transition

For fixed N_f:

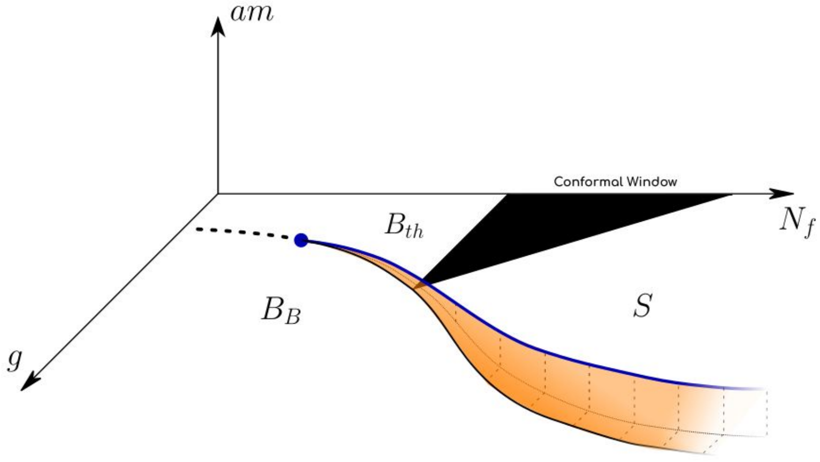
aT=0



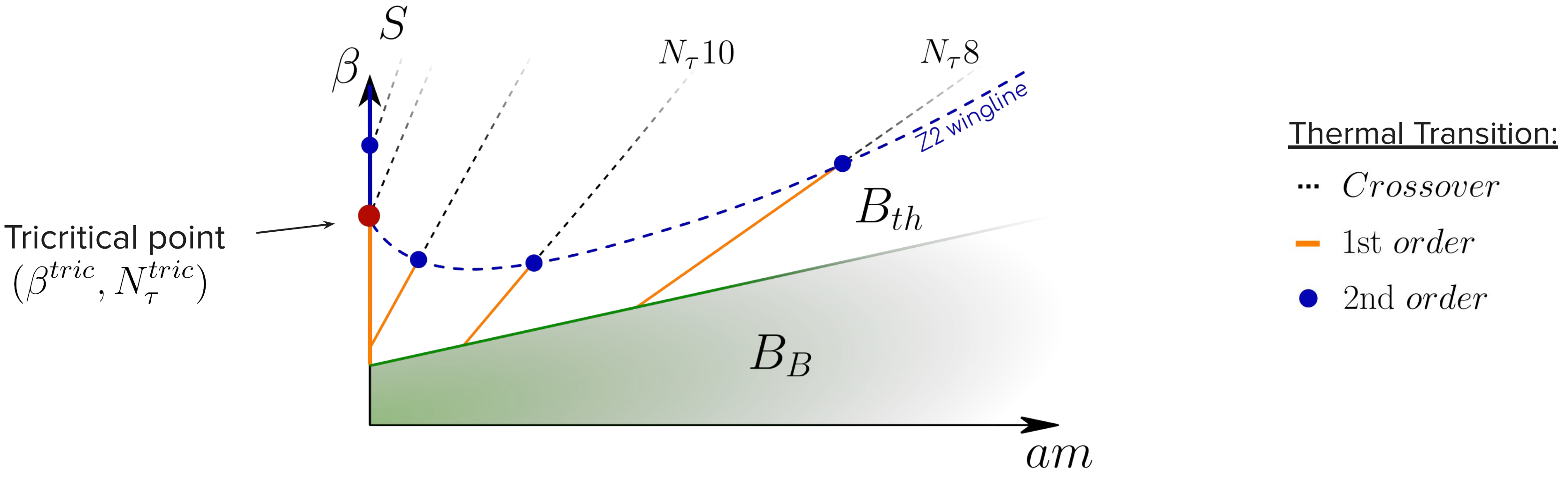


Our findings for Bulk transition:

- 2nd order wingline at am(Nf) > 0
- chiral limit am = 0:
  - 1st order for Nf>6
  - 2nd order endpoint at Nf6



Lattice QCD at Nf<=6



Our findings for thermal transition:

- We found a 1st order transition! ***But***: the order changes with N_τ
- Thermal 2nd order line terminates at $am = 0$ in a tricritical point $(\beta^{tric}, N_\tau^{tric})$

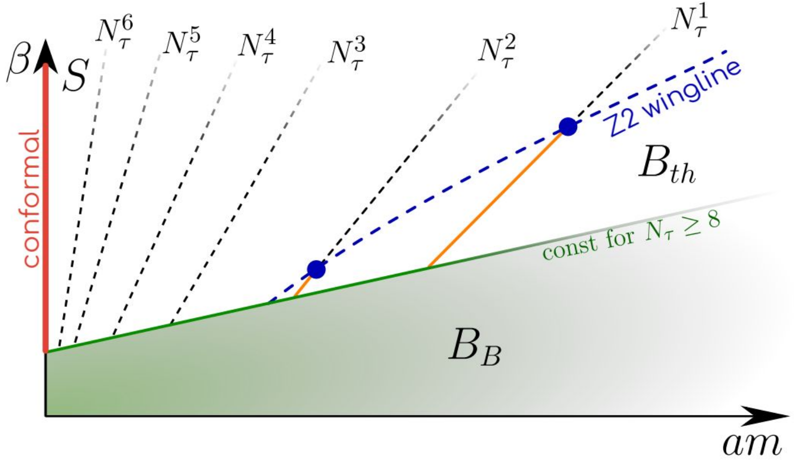
1st order is cutoff effect

... $Crossover$

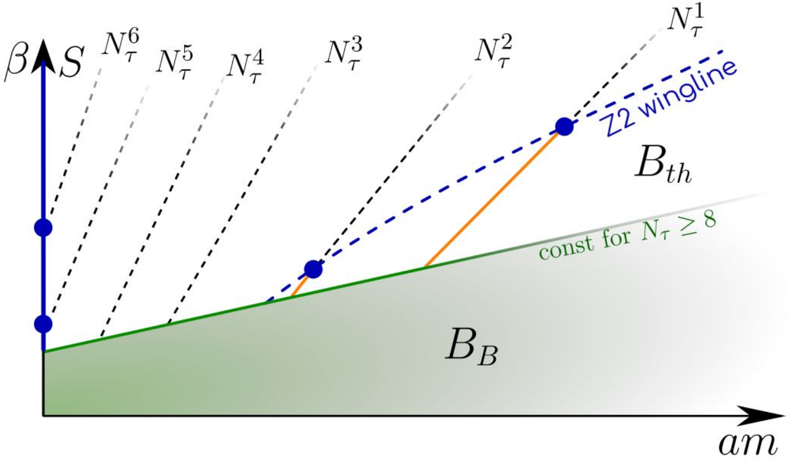
— $1st\ order$

● $2nd\ order$

conformal



QCD-like



Thermal 1st order transition vanishes in Lattice bulk phase

➔ 1st order transition is not connected to continuum

Leading order tricritical scaling

- Continuum limit $\Leftrightarrow$ origin of plot:

  $a \to 0 \quad \Leftrightarrow \quad N_\tau = 1/aT \to \infty$

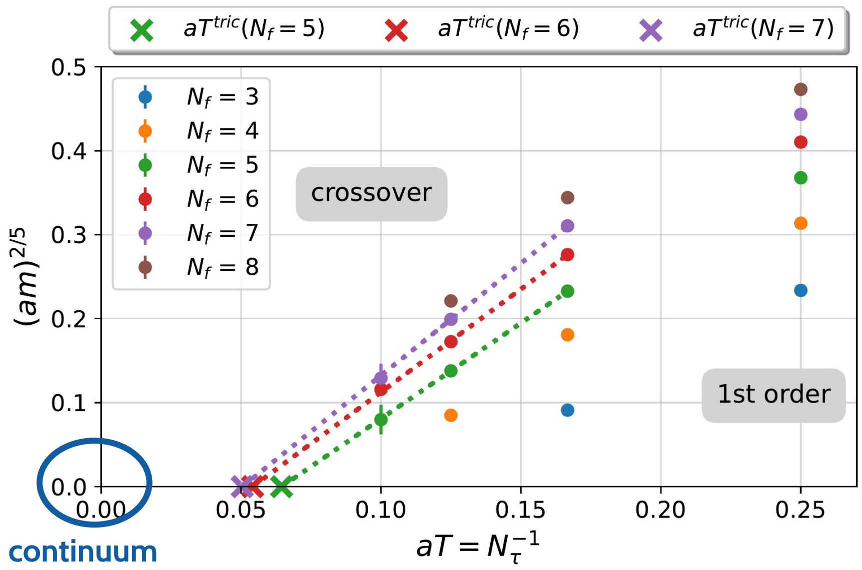
- Massless limit $\Leftrightarrow$ x-axis

- Z2 phase boundary:

  $aT_c(am) = aT_{tric} + A \cdot (am)^{2/5} + B \cdot (am)^{4/5}$

- 1st order ends at lattice spacing $N_\tau^{tric}(N_f)$

➡ 1st order is a lattice cutoff effect



QCD in chiral limit

The chiral transition is of 2nd order for $N_f \leq 7$