Probing magic and entanglement in 1+1-dimensional SU(2) Hamiltonian lattice gauge theory

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Based on arXiv: 2606.09971

Magic and entanglement in 1+1-dimensional SU(2) lattice gauge theory

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Entanglement and non-stabilizerness (magic) quantify two distinct departures of quantum systems from classical description: the former measures non-local correlations, while the latter measures the deviation from stabilizer states that can be efficiently simulated classically. Understanding magic in physically relevant quantum field theories is essential for identifying where quantum advantage may be realized in the early fault-tolerant quantum computing era. We calculate the gauge-invariant entanglement entropy and stabilizer Rényi entropy of the ground state of the (1+1)-dimensional SU(2) lattice gauge theory formulated in a dressed-site basis that enforces Gauss's law exactly. Using tensor networks, we obtain results for system sizes up to $L = 100$ (300 qubits). We find a crossover denoted by $g_\star$ where the ground state passes from a more magic-rich regime into a regime with less magic; this is also tracked by the sharpest change of both the entanglement entropy and lattice particle density. Our large-scale study of non-stabilizerness and entanglement entropy in a non-Abelian lattice gauge theory with matter provides new insight into the interplay of magic and entanglement in gauge theories, both of which are relevant for classical and early fault-tolerant quantum simulations.

July 28, 2026

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Outline

- What is magic? Why is it important?
- How do we truncate the SU(2) gauge theory with matter in 1+1-dimensions keeping gauge-singlet states?
- How do we measure magic and entanglement entropy for these LGTs?
- Results

Clifford group and GK theorem

- {H, S, CX} are generators of the Clifford group which is a normaliser of the Pauli group i.e., $\mathbf{C}_n = \{V \in U_{2^n} \mid V\mathbf{P}_n V^\dagger = \mathbf{P}_n\}$.
- Gottesman-Knill (GK) theorem says that any quantum computation — 1) where the qubits are prepared in the computational basis, 2) acted on by Clifford gates, 3) and measured in the computational basis can be simulated efficiently on a classical computer.

Example: Bell state

- $(|00\rangle + |11\rangle)/\sqrt{2}$ i.e., maximally entangled state of two qubits. Can be prepared from $|00\rangle$ by acting with H and CX gate.

- Even though it is maximally entangled, it is a stabilizer state (any state reachable by Clifford gates only starting with state in computational basis). Using GK theorem, we know that this can be efficiently (polynomial) simulated on classical computers.

- We thought that more entanglement leads to classical non-simulability. This is not true! This is stabilizer entanglement

- There must be some other different/observable that captures this classical non-simulability and makes quantum computers powerful.

Magic (or non-stabilizerness)

- We ought to look for non-stabilizerness as providing the departure of what can and what cannot be simulated efficiently. This has been termed as `magic'.
- In very simple terms, more magic means harder for classical computers.
- Also needed for any universal quantum gate set. For example, Clifford + T-gate is universal since T-gate supplies the magic!
- Connecting magic to an observable that can be efficiently computed is not straightforward.
- In recent years, based on work of *Leone et al.*, we have a measure of magic in terms of what is called `Stabilizer Renyi Entropy' (SRE).
- SRE is zero for product and stabilizer states, while it is non-zero for say $T^{\otimes n} |0\rangle^{\otimes n}$.

Resource plane

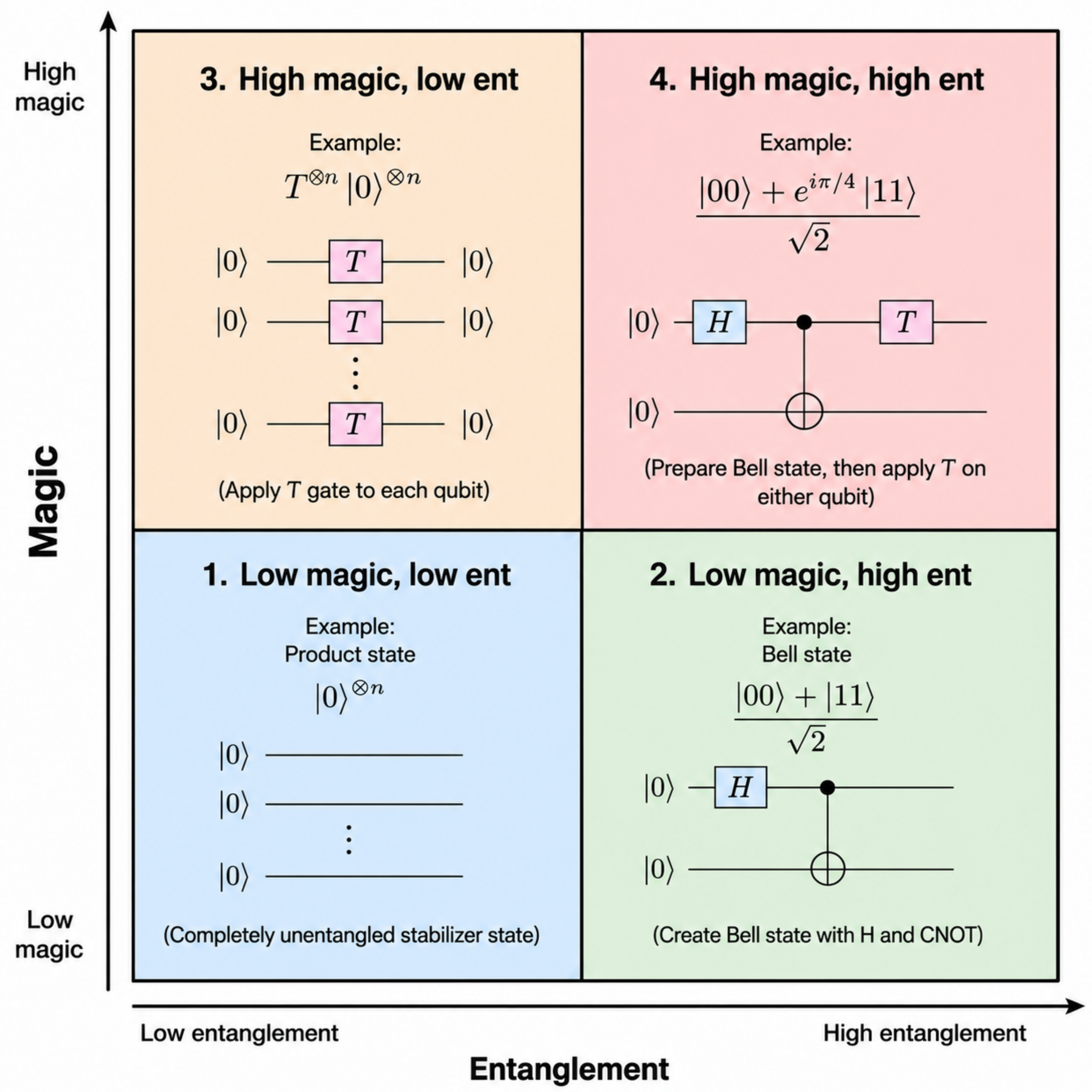


Figure credits: GPT 5.5

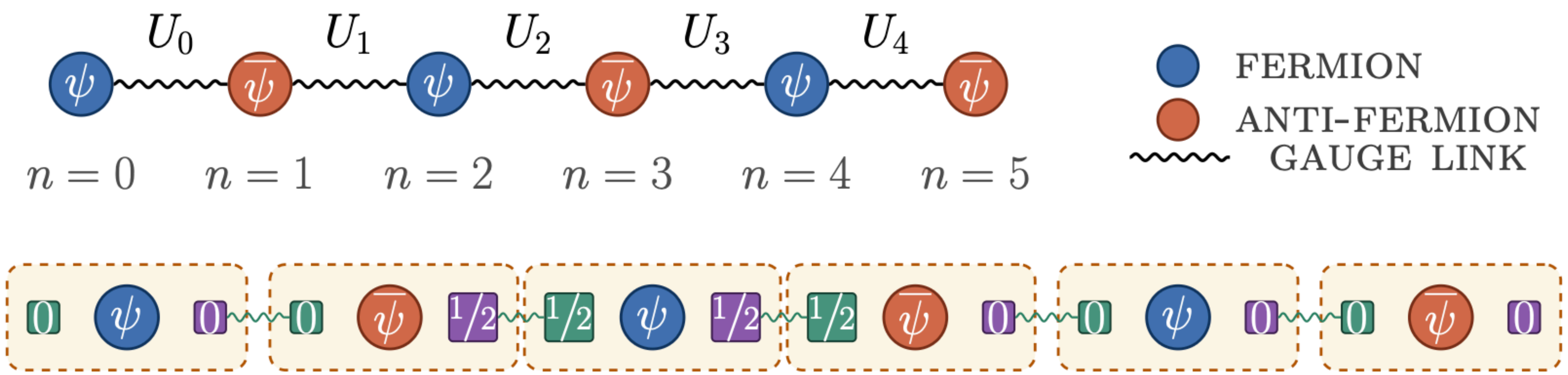
1+1 SU(2) gauge theory

- We consider simplest example of non-Abelian gauge theory in 1+1-dimensions. There are different approaches for truncating the infinite-dimensional Hilbert space. We follow the dressed-site/electric basis approach where at each lattice site, we collect the semi- gauge links on the left and right of the site into a dressed site. Then the physical lattice with matter and gauge fields restricted to physical gauge-singlet states are dressed sites connected by Abelian conservation law.

- We restrict to $j_{\text{max}} = 1/2$, extensions to higher j_{max} can be done. We did not consider this, however provide some details about construction of Hamiltonian and gauge-invariant space in the Appendix of the paper.

1+1 SU(2) gauge theory

- The setup is as follows:



- We restrict to $j_{\text{max}} = 1/2$, so each link (or semi-link) can either values 0, 1/2. The matter fields (at physical sites) are represented as $\psi, \overline{\psi}$ in the standard staggered fashion. The vacuum and doubly occupied matter sites are singlets, while we have one doublet.

1+1 SU(2) gauge theory

- $|0\rangle = |j, m = 0\rangle$, and $|d\rangle = (\psi_n^r)^\dagger (\psi_n^g)^\dagger |0\rangle$ are singlets while $(\psi_n^r)^\dagger |0\rangle\,,\ (\psi_n^g)^\dagger |0\rangle$ are doublets. In short, for matter sites, we have **1** $\oplus$ **2** $\oplus$ **1**. For semi-links, we have **1** $\oplus$ **2** for $j_{\text{max}} = 1/2$.

- When we combine these in a dressed site which is gauge-singlet, we have to find singlets in following expansion: $(\mathbf{1} \oplus \mathbf{2}) \otimes (\mathbf{1} \oplus \mathbf{2} \oplus \mathbf{1}) \otimes (\mathbf{1} \oplus \mathbf{2})$ which is 36-dimensional local space.

- One can show that there are 6 gauge singlets, i.e., $6/36$ states are physical! We therefore build a Hamiltonian acting on these states of size $6^L \times 6^L$, where L is the number of lattice sites. We need $3L$ qubits.

1+1 SU(2) gauge theory

- The Hamiltonian is given by: $\sum_n \left[g^2 \hat{C}_n + m(-1)^n \hat{M}_n + \hat{A}^1_n \hat{B}^1_{n+1} + \hat{A}^2_n \hat{B}^2_{n+1} \right]$, where the operators $\hat{C}, \hat{M}$ are diagonal and the hopping operators, $\hat{A}, \hat{B}$ raise or lower the flux by 1/2. More details in the paper.

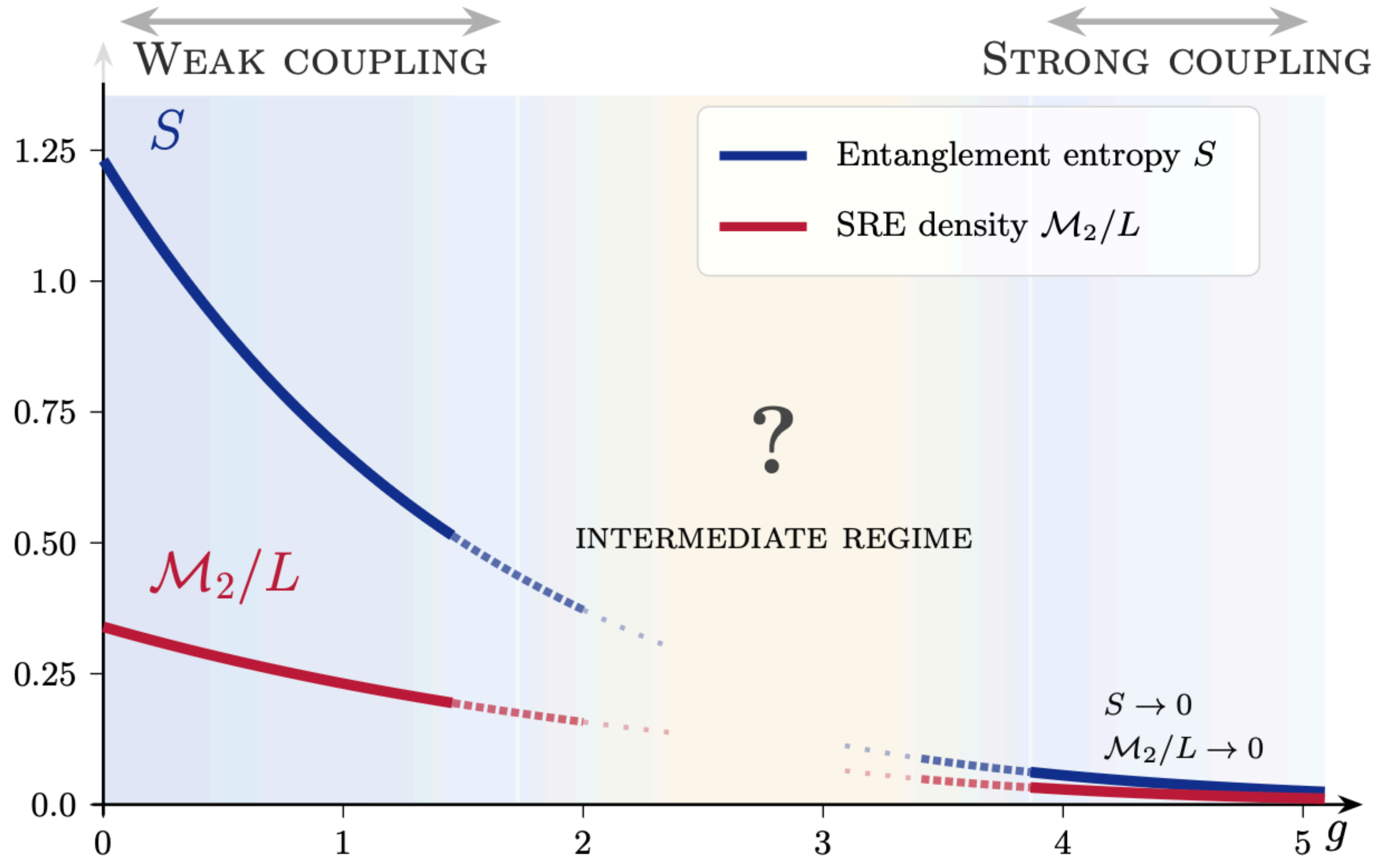
- We will probe entanglement entropy (EE) and magic in the $m - g$ plane. Note that these are dimensionless lattice parameters converted from dimensionful parameters by using powers of a.

1+1 SU(2) gauge theory - expectations!

- Following Dirac, *we do not start a sentence without knowing its end*. How should the results look like?

- CFTs are magical and possess high magic [White *et al.*, arXiv: 2007.01303] so we should find maximum magic around $g \to 0$. In that limit, we also have high entanglement (Calabrese-Cardy, 2004). As $g \to \infty$, we know from the structure of ground state that it should have zero magic (product state) and no entanglement (Donnelly, 2011).

- So, we have:

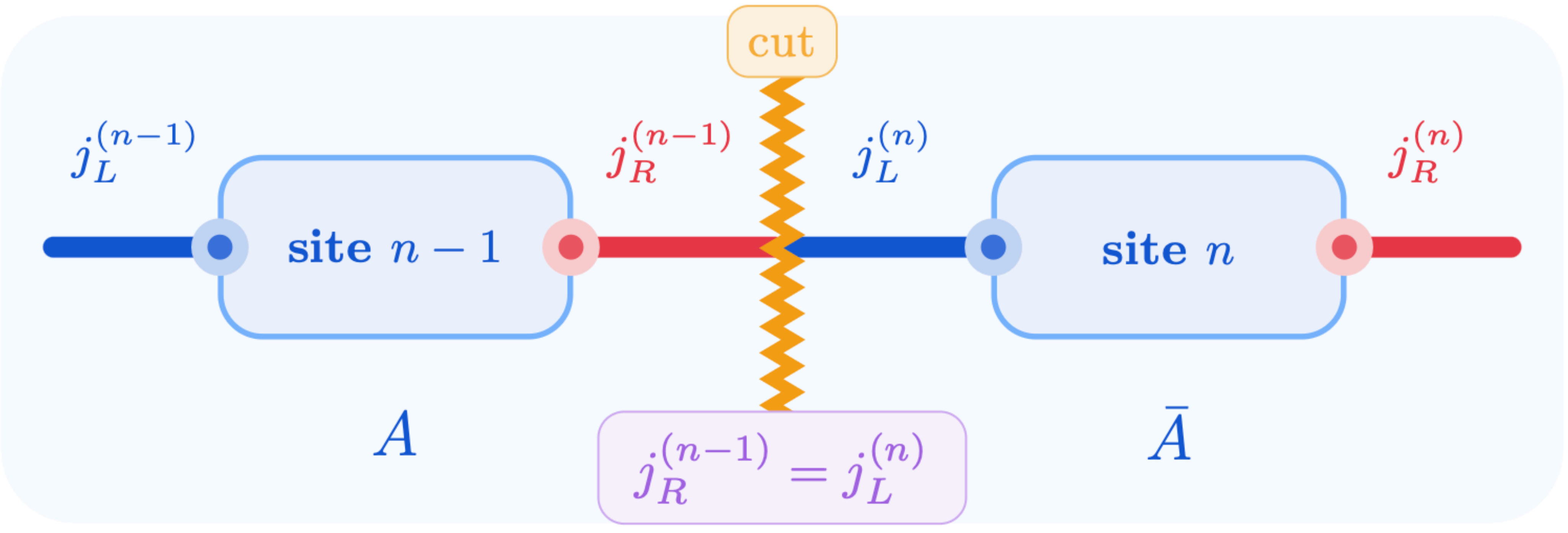


EE and SRE

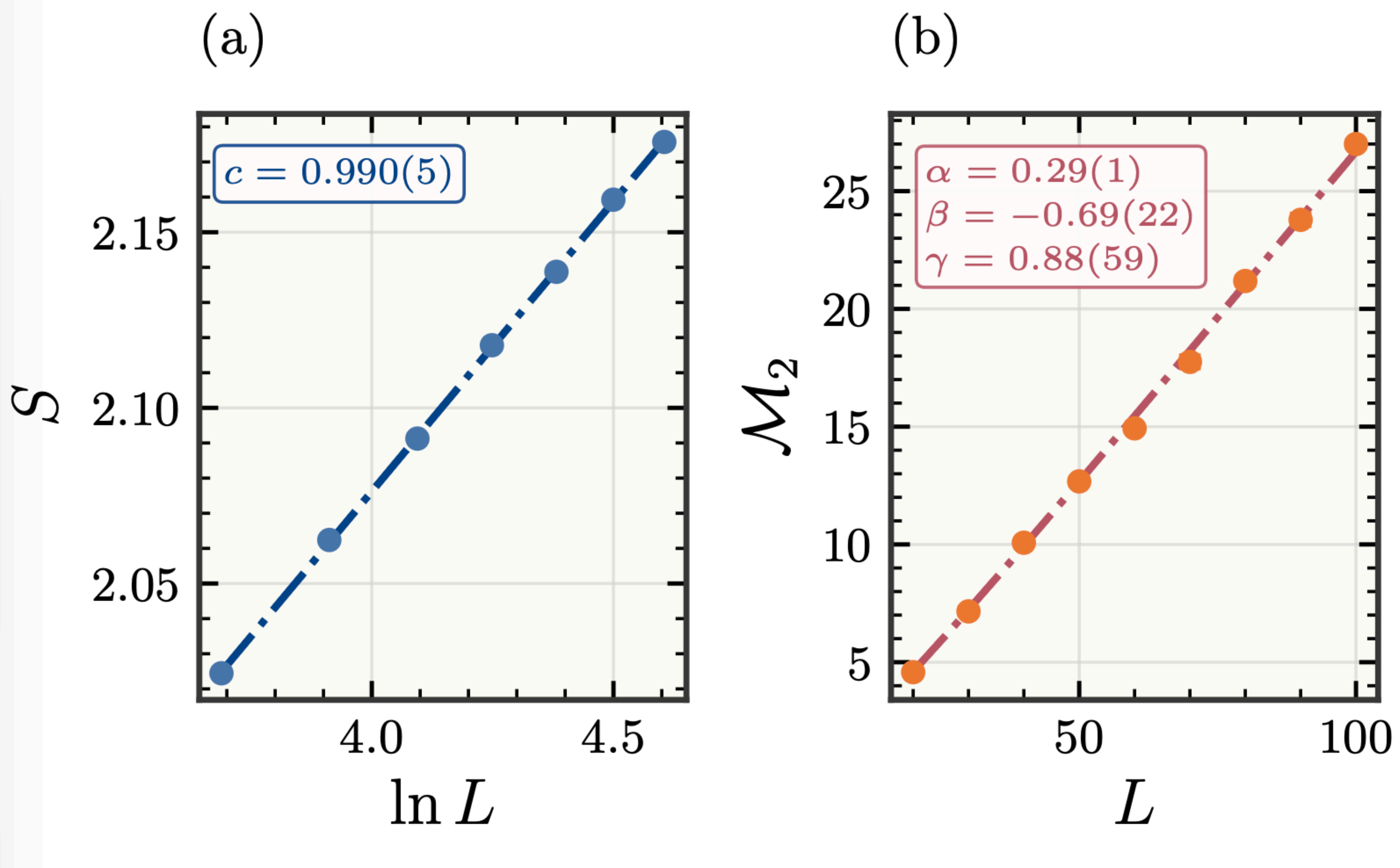
- For magic, we compute the stabilizer Renyi entropy (SRE) defined as: $\mathscr{M}_n = \frac{1}{1-n}\log_2\left(\frac{1}{2^N}\sum_{P\in\mathscr{P}_N}\left|\langle P\rangle\right|^{2n}\right)$,

  where n is the order of the SRE (we consider n=2), N is the total number of qubits of the system. Computing this is exponentially hard but many MCMC based methods have been developed which can help us compute these for large system sizes.

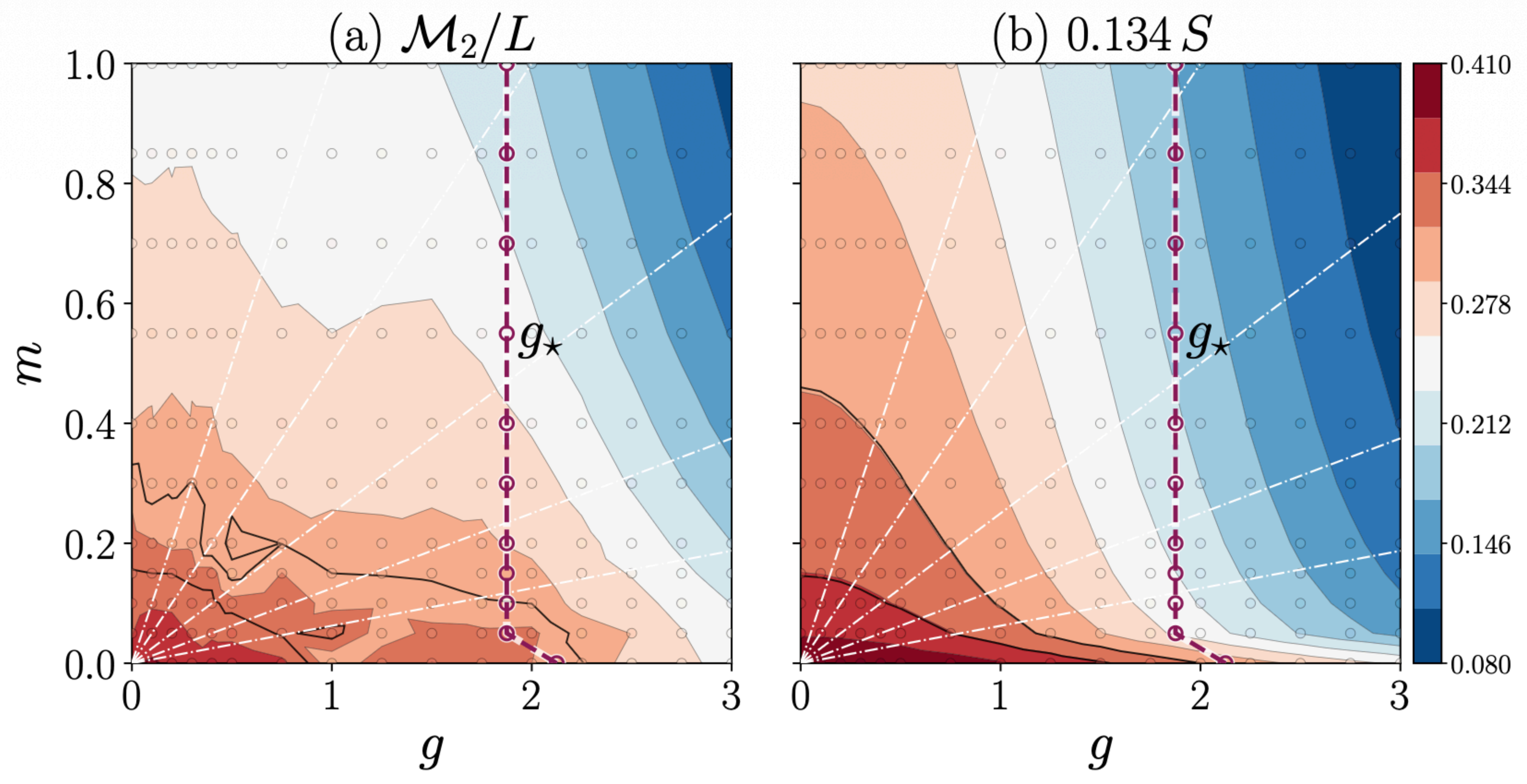
- For entanglement entropy, we use the extended-Hilbert space formulation [Donnelly, 2011]. EE can then be written as sum of three terms. See paper for more details.



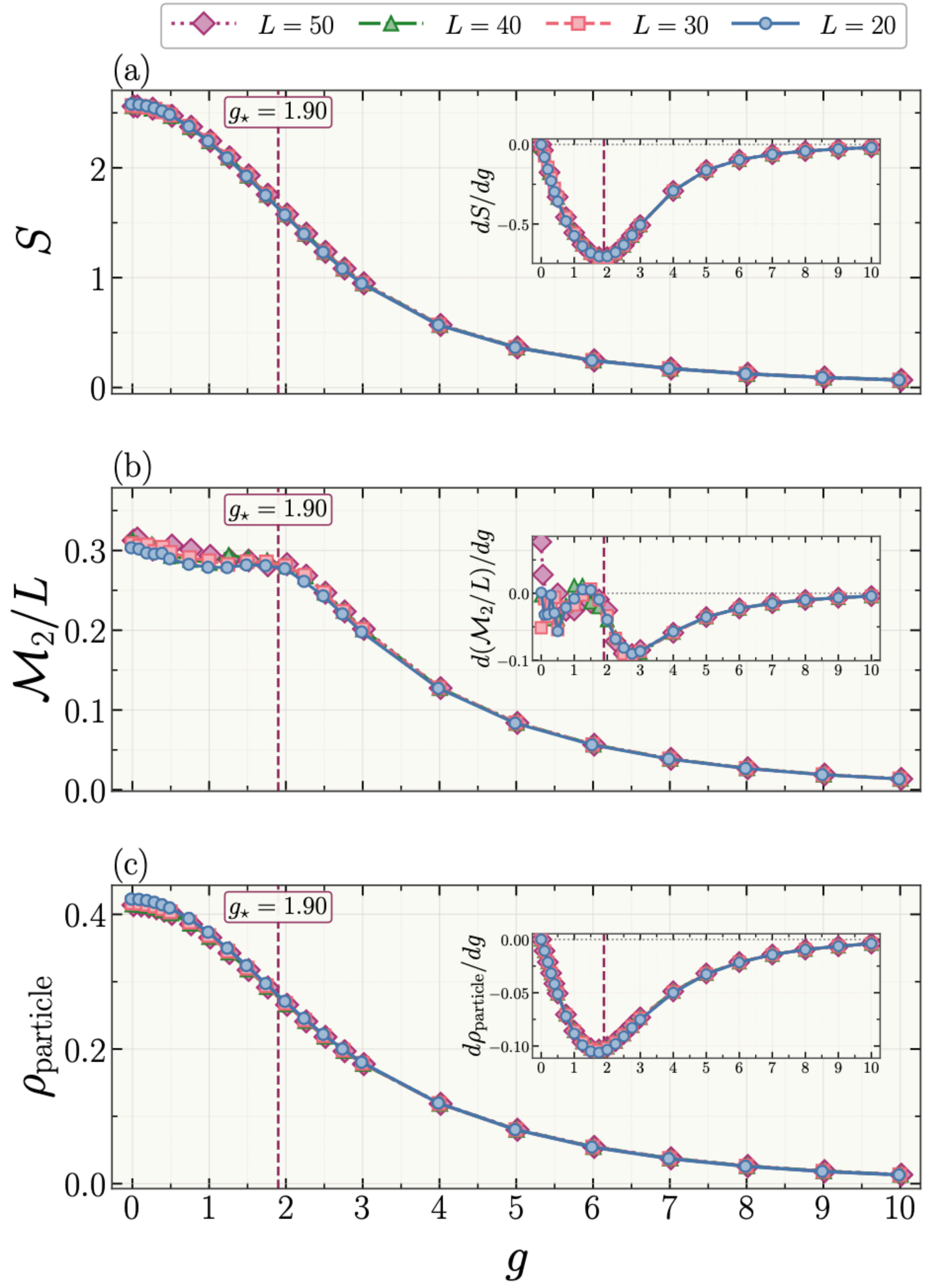
Results



(a)
(b)
$c = 0.990(5)$
$\alpha = 0.29(1)$
$\beta = -0.69(22)$
$\gamma = 0.88(59)$
S
$\ln L$
$\mathcal{M}_2$
L



(a) $\mathcal{M}_2/L$
(b) $0.134\,S$
$g_\star$
m
g
0.410
0.344
0.278
0.212
0.146
0.080



$L = 50$
$L = 40$
$L = 30$
$L = 20$
(a)
(b)
(c)
$g_\star = 1.90$
S
dS/dg
$\mathcal{M}_2/L$
$d(\mathcal{M}_2/L)/dg$
ρ_{particle}
$d\rho_{\mathrm{particle}}/dg$
g

Summary

- We find that continuum limit of LGTs correspond to QFT states that have high magic. This has drastic implications for future quantum simulations of lattice gauge theories. It sort of implies that reaching continuum limit of Hamiltonian LGTs needs substantial *(non-classical)* quantum resources . One can think of this in terms of T-gates required.

- In this truncated 1+1d, SU(2), magic and entanglement need not necessarily track each other (i.e., they act as different quantum resource measure) since while EE can be *consistently* decreasing, magic can stay constant just before weak-strong crossover. In gauge theories with proper phase transitions, it will be even more interesting to study this. Work in progress!

- We are following up along several directions and exploring other interesting lattice counterparts of quantum field theories.

Thank You

8 x 8 2d anti-ferromagnetic Heisenberg model to all times from short times!

To appear in 4-5 hours on arXiv, category:quant-ph

Efficient computation of real-time correlators using Pauli Propagation

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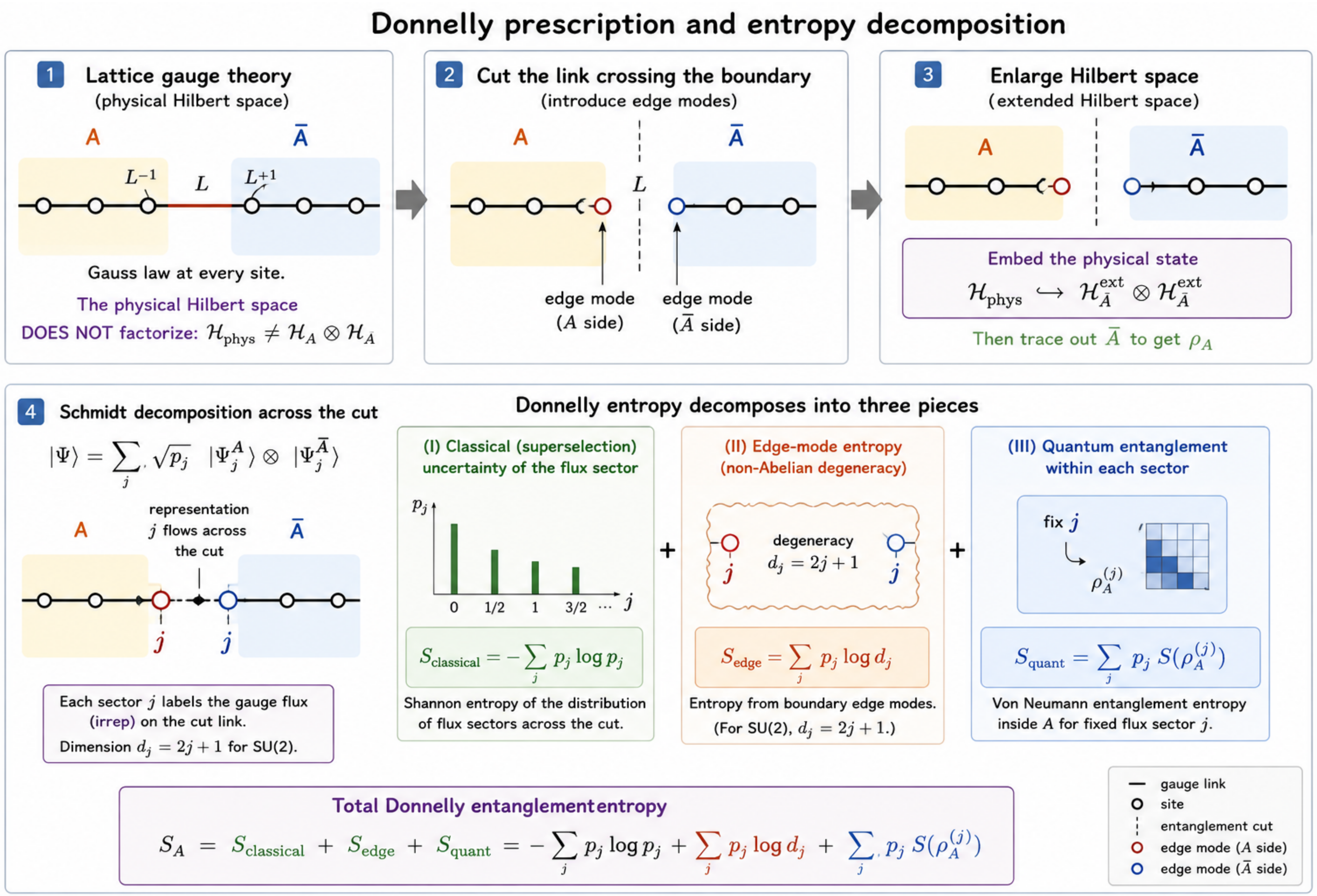
(Dated: July 27, 2026)

Pauli propagation has shown promise for classically simulating quantum dynamics by evolving observables directly in the Heisenberg picture. In this work, we investigate its use for computing real-time two-point time-ordered correlators in one- and two-dimensional quantum systems. One major limitation of Pauli propagation is the rapid growth in the number of Pauli strings beyond short times. We overcome this limitation by combining accurate short-time Pauli-propagation data with time extension methods based on a positivity condition and the observation that the dynamics are often dominated by a small number of characteristic frequencies. This combined approach extends correlation functions far beyond the directly accessible time window while avoiding the exponential proliferation of Pauli operators. We demonstrate that the resulting correlators retain the relevant dynamical and spectral information. Our results broaden the regime in which classical methods can reliably probe the real-time dynamics of interacting quantum many-body systems.

Backup



Donnelly prescription and entropy decomposition
1 Lattice gauge theory
(physical Hilbert space)
A
$\bar{A}$
$L-1$
L
$L+1$
Gauss law at every site.
The physical Hilbert space
DOES NOT factorize: $\mathcal{H}_{\text{phys}} \neq \mathcal{H}_A \otimes \mathcal{H}_{\bar{A}}$
2 Cut the link crossing the boundary
(introduce edge modes)
A
$\bar{A}$
L
edge mode
(A side)
edge mode
($\bar{A}$ side)
3 Enlarge Hilbert space
(extended Hilbert space)
A
$\bar{A}$
Embed the physical state
$\mathcal{H}_{\text{phys}} \hookrightarrow \mathcal{H}_A^{\text{ext}} \otimes \mathcal{H}_{\bar{A}}^{\text{ext}}$
Then trace out $\bar{A}$ to get ρ_A
4 Schmidt decomposition across the cut
$|\Psi\rangle = \sum_j \sqrt{p_j}\ |\Psi_j^A\rangle \otimes |\Psi_j^{\bar{A}}\rangle$
representation
j flows across
the cut
A
$\bar{A}$
j
j
Each sector j labels the gauge flux
(irrep) on the cut link.
Dimension $d_j = 2j+1$ for SU(2).
Donnelly entropy decomposes into three pieces
(I) Classical (superselection)
uncertainty of the flux sector
p_j
0
1/2
1
3/2
⋯
j
$S_{\text{classical}} = -\sum_j p_j \log p_j$
Shannon entropy of the distribution
of flux sectors across the cut.
+
(II) Edge-mode entropy
(non-Abelian degeneracy)
degeneracy
$d_j = 2j+1$
j
j
$S_{\text{edge}} = \sum_j p_j \log d_j$
Entropy from boundary edge modes.
(For SU(2), $d_j = 2j+1$.)
+
(III) Quantum entanglement
within each sector
fix j
$\rho_A^{(j)}$
$S_{\text{quant}} = \sum_j p_j\ S(\rho_A^{(j)})$
Von Neumann entanglement entropy
inside A for fixed flux sector j.
Total Donnelly entanglemententropy
$S_A = S_{\text{classical}} + S_{\text{edge}} + S_{\text{quant}} = -\sum_j p_j \log p_j + \sum_j p_j \log d_j + \sum_j p_j\ S(\rho_A^{(j)})$
gauge link
site
entanglement cut
edge mode (A side)
edge mode ($\bar{A}$ side)

Backup

$$|1\rangle = |0,0,0\rangle,$$

$$|2\rangle = \frac{|r,0,g\rangle - |g,0,r\rangle}{\sqrt{2}},$$

$$|3\rangle = \frac{|g,r,0\rangle - |r,g,0\rangle}{\sqrt{2}},$$

$$|4\rangle = \frac{|0,r,g\rangle - |0,g,r\rangle}{\sqrt{2}},$$

$$|5\rangle = |0,d,0\rangle,$$

$$|6\rangle = \frac{|r,d,g\rangle - |g,d,r\rangle}{\sqrt{2}},$$