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LATTICE 2026

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UNIVERSITY OF MARYLAND

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Lattice 2026 @ University of Maryland

Proton TMDPDFs from Lattice QCD

Spin Dependence of the Non-perturbative Transverse Structure

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In collaboration with ANL-BNL group

U.S. DEPARTMENT of ENERGY

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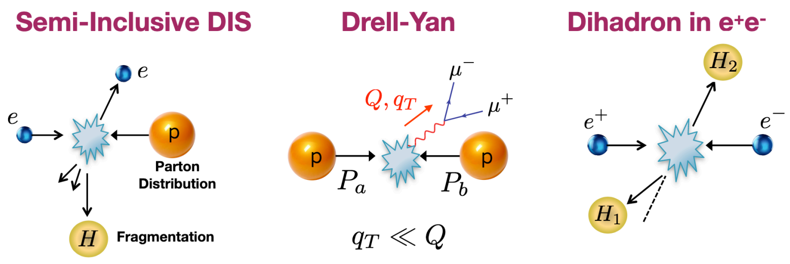
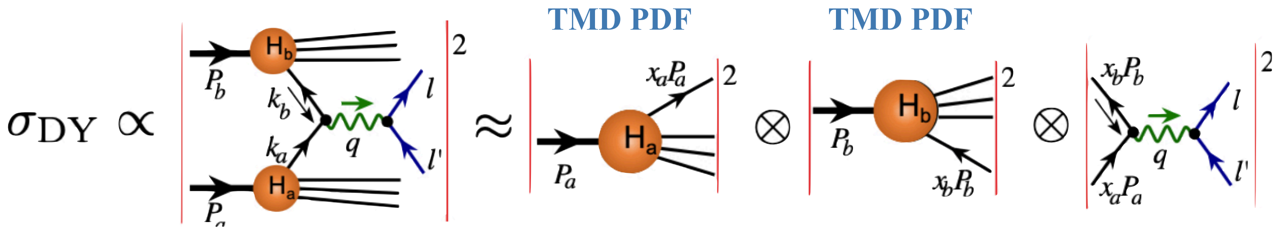
Parton Physics: from PDFs to TMDs

TMD processes and TMD factorization

- TMD processes are important processes in high energy collisions, like Drell-Yan process on LHC and Semi-Inclusive DIS on EIC;





- Why we need TMD factorization?

Scale separation
Large log: $\ln(Q/q_T)$
Soft mode

  - When $q_T \sim Q$, collinear factorization is enough

PDF

$$\frac{d\sigma}{d^4q} = \sum_{i,j} \int_{x_a}^{1} d\xi_a \int_{x_b}^{1} d\xi_b \boxed{f_{i/H_a}(\xi_a)} f_{j/H_b}(\xi_B) \frac{d\hat{\sigma}_{ij}(\xi_a, \xi_b)}{d^4q} \left[1 + O\left(\frac{\Lambda_{QCD}^2}{q_T^2}, \frac{\Lambda_{QCD}^2}{Q^2}\right)\right]$$

  - When $\Lambda_{QCD} \lesssim q_T \ll Q$, TMD factorization is needed

TMD PDF

$$\frac{d\sigma}{d^4q} = \frac{1}{s} \sum_{i \in \text{flavors}} \hat{\sigma}_{\bar{i}}^{TMD}(Q) \int d^2\mathbf{k}_T \boxed{f_{i/H_a}(x_a, \mathbf{k}_T)} f_{\bar{i}/H_b}(x_b, \mathbf{q}_T - \mathbf{k}_T) \left[1 + O\left(\frac{q_T^2}{Q^2}, \frac{\Lambda_{QCD}^2}{Q^2}\right)\right]$$

Leading Quark TMDPDFs — Nucleon Spin — Quark Spin

Nucleon Polarization		Quark Polarization: Un-Polarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
	U	f_1 = Unpolarized		$h_1^\perp$ = − Boer-Mulders
	L		g_1 = − Helicity	$h_{1L}^\perp$ = − Worm-gear
	T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = − Worm-gear	h_1 = − Transversity; $h_{1T}^\perp$ = − Pretzelosity

R. Boussarie, et al., 2304.03302 (2023)

Phenomenological Extractions of TMDs

Significant progress has been made in the phenomenological parameterizations of TMDs

- Collins-Soper kernel (CS kernel): rapidity evolution kernel of TMDs;

  A. Bacchetta, et al. (MAP) JHEP 08 (2024); V. Moos, et al., JHEP 11 (2025) …

- Pion TMDs
  - Unpolarized *A. Vladimirov, JHEP 10 (2019); M. Cerutti, et al. (MAP), Phys.Rev.D 107 (2023); P. C. Barry, et al. (JAM) Phys.Rev.D 108 (2023)*
- Nucleon TMDs
  - Unpolarized

    M. Bury, et al., JHEP 10 (2022); A. Bacchetta, et al., JHEP 10 (2022); V. Moos, et al., JHEP 05 (2024); A. Bacchetta, et al., JHEP 08 (2024); V. Moos, et al., JHEP 11 (2025); P. Barry et al., 2510.13771 …
  - Helicity and Transversity

    L. Gamberg, et al., PRD 106 (2022); A. Bacchetta, et al., PRL 134 (2025) …
  - Sivers

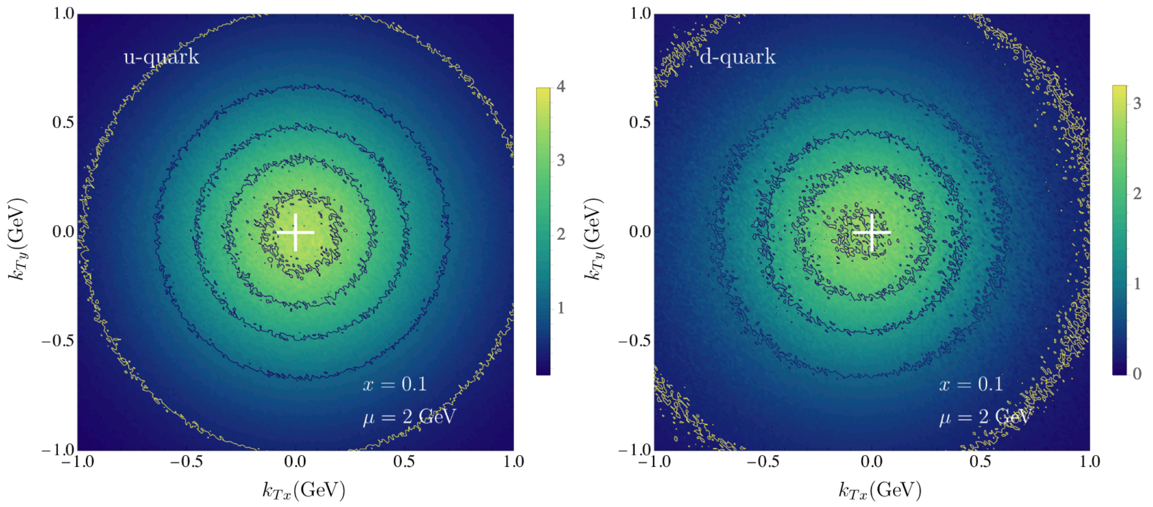
    M. Bury, et al., PRL 126 (2021); I. P. Fernando, et al., Phys.Rev.D 108 (2023) …
  - Boer-Mulders

    Z. Lu, et al., Phys.Rev.D 81 (2010); X. Liu, et al., Eur.Phys.J.C 81 (2021) …

Some of groups incorporate neural network;

Some of groups include lattice data;

Tomographic scan of the nucleon via quark density function



Lattice Calculations of TMDs

Lattice QCD provides first-principles predictions of TMDs

- Mellin Moments; *B.U. Musch, et al., Phys.Rev.D 85 (2012); B. Yoon, et al., 1601.05717; B. Yoon, et al., Phys. Rev. D 96 (2017) …*
- Large Momentum Effective Theory (LaMET)
  - CS kernel *M. H. Chu, et al. (LPC), JHEP 08 (2023); A. Avkhadiev et al., PRL 132 (2024); D. Bollweg, et al., Phys. Lett. B 852 (2024); J-X Tan, et al. (LPC), 2511.22547; A. Avkhadiev et al., 2510.26489, A. Francis et al., 2606.19221 …*
  - Intrinsic soft functions *Q. A. Zhang, et al. (LPC), PRL 125 (2020); M. H. Chu, et al. (LPC), JHEP 08 (2023)*
  - Unpolarized TMD *JH, et al. (LPC), Phys.Rev.D 109 (2024); D. Bollweg, JH, et al., Phys.Rev.D 106 (2022)*
  - Helicity TMD *D. Bollweg, X. Gao, S. Mukherjee and Y. Zhao, PRL 135 (2025)*
  - Boer-Mulders TMD *L. Walter, et al. (LPC), PRD 111 (2025); L. Ma, et al. (LPC), JHEP 08 (2025)*

Large-Momentum Effective Theory (LaMET)

TMDPDF is defined from a light-cone correlator in a hadron, which is boost invariant

$$f(x, b_\perp, \ldots) = \int_{-\infty}^{\infty} \frac{db^-}{2\pi} e^{-ib^-(xP^+)} \left\langle P \left| \bar{\psi}\left(b^\mu\right) W_\sqsubset \left(b^\mu, 0\right) \frac{\gamma^+}{2} \psi(0) \right| P \right\rangle \longleftrightarrow \left\langle |\vec{P}| = \infty \left| O(t=0) \right| |\vec{P}| = \infty \right\rangle$$

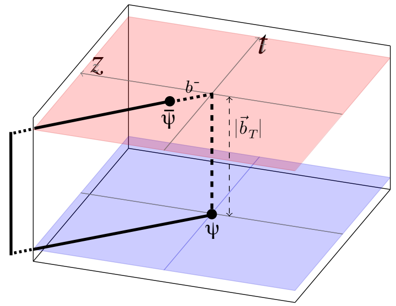
Equal-time correlator! Parton model!

Define a quasi distribution with large-momentum states and time-independent operators

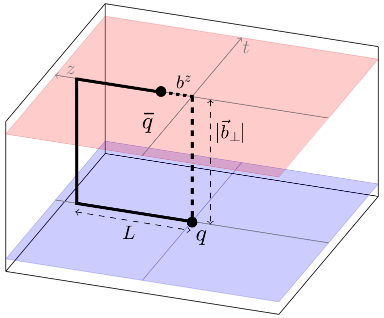
Wilson link

$$\tilde{f}^0_\Gamma(x, b_\perp, P^z, \mu) = P^z \int \frac{dz}{2\pi} e^{iz(xP^z)} \frac{1}{2P^t} \langle P | \bar{\psi}_0 \left(z, b_\perp\right) W_\sqsubset \left(z, b_\perp; 0\right) \Gamma \psi_0(0) | P \rangle$$

Differ by UV structures



Lorentz boost & Matching



LaMET enables us to obtain the precision-controlled TMD PDFs in moderate x region.

$$\sqrt{S_I\left(b_\perp; \mu\right)} \cdot \tilde{f}_\Gamma\left(x, b_\perp, P^z; \mu\right) = f\left(x, b_\perp; \mu, \zeta\right) H_f\left(x, P^z; \mu\right) \exp\left[\frac{1}{2} \ln \frac{(2xP^z)^2}{\zeta} \gamma^{\overline{\mathrm{MS}}}\left(b_\perp; \mu\right)\right] + \text{Power corrections}$$

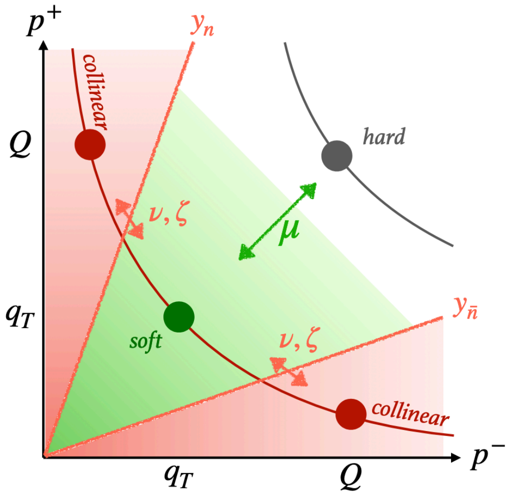
Soft Functions

Soft gluon radiation will lead to existence of soft functions

$$\sqrt{S_I\left(b_\perp;\mu\right)}\cdot\tilde{f}_\Gamma\left(x,b_\perp,P^z;\mu\right)=f\left(x,b_\perp;\mu,\zeta\right)H_f\left(x,P^z;\mu\right)\exp\left[\frac{1}{2}\ln\frac{(2xP^z)^2}{\zeta}\gamma^{\overline{\mathrm{MS}}}\left(b_\perp;\mu\right)\right]$$

After regularization, the rapidity evolution is controlled by Collins-Soper scale: $\zeta=2(xP^+)^2e^{-2y_n}$

- Due to the factorization of the soft mode, the soft function contains the well-known rapidity divergence;
- The soft function can be separated into two parts:
  - Rapidity evolution kernel: CS kernel $\gamma^{\overline{\mathrm{MS}}}\left(b_\perp;\mu\right)$
  - Rapidity independent part: intrinsic soft function $\sqrt{S_I\left(b_\perp;\mu\right)}$



R. Boussarie, et al., 2304.03302 (2023)

- CS kernel can be extracted from the rapidity evolution of TMDs

$$\tilde{\phi}_\Gamma(x,b_T,P_z;\mu)=A(x,b_T)\ H_\phi(x,\bar{x},P_z;\mu)\ \exp\left[\ln\frac{P_z}{P_{\mathrm{fix}}}\tilde{\gamma}(x,b_T)\right]$$

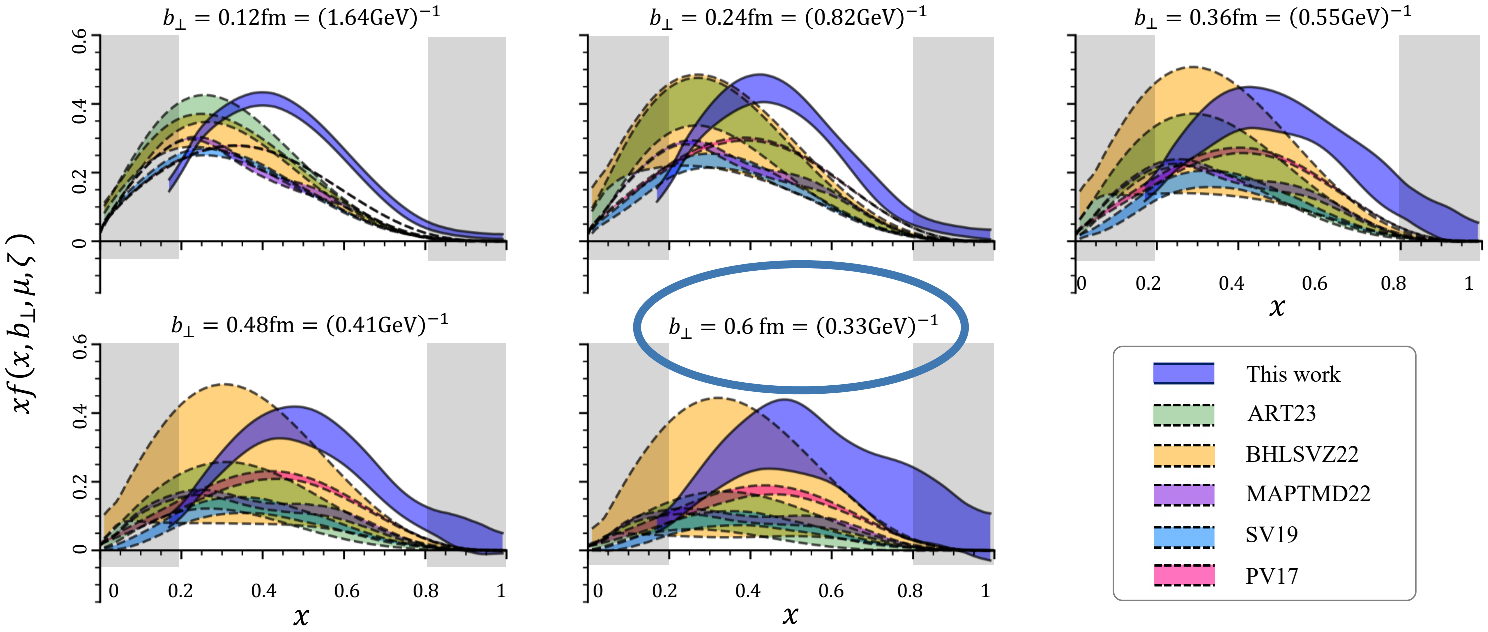
- Intrinsic soft function can be extracted from the meson form factor

$$S_I(b_\perp;\mu)=\frac{F(b_\perp,P^z)}{\int dx_1dx_2H_F(x_1,x_2,P^z;\mu)\tilde{\Phi}^\dagger(x_1)\tilde{\Phi}(x_2)}\ \text{with}\ F\left(b_\perp,P_1,P_2,\Gamma,\Gamma'\right)\equiv-4N_c\frac{\left\langle P_2\right|\bar{q}\left(b_\perp\right)\Gamma q\left(b_\perp\right)\bar{q}\left(0\right)\Gamma'q(0)\left|P_1\right\rangle}{f_\pi^2(P_1\cdot P_2)}$$

X. Ji, Y. Liu and Y. S. Liu, Nucl. Phys. B 955 (2020)

Unpolarized Proton TMD PDFs

First lattice QCD calculation of x-dependent proton TMD PDFs

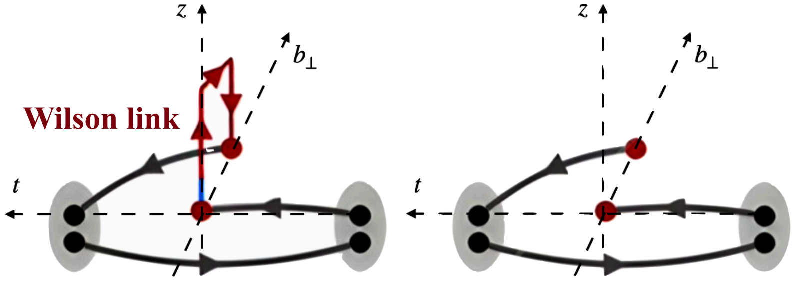


- ✦ $a = 0.12$ fm
- ✦ $P^z_{\text{max}} = 2.58$ GeV
- ✦ **Physical limit of m_π^{val}**
- ✦ **N3LL matching**
  (Next-to-next-to-next-to-leading logarithmic)
- ✦ **NLO soft function**
  (Next-to-leading order)

JH, et al. (LPC), Phys.Rev.D 109 (2024)

- Systematics are estimated and included in the final results;
- Constrained by the limited computing resources, it is hard to implement on a finer lattice;
- Due to the signal-to-noise ratio (SNR), it is hard to probe the large b_T region.

Coulomb Gauge Method



Define a quasi distribution in CG without Wilson line

$$\tilde{f}^{0}_{\mathrm{CG}}(x, b_\perp, P^z, \mu) = P^z \int \frac{dz}{2\pi} e^{iz(xP^z)} \frac{1}{2P^t} \langle P | \bar{\psi}_0\left(z, b_\perp\right) \Gamma \psi_0(0) \Big|_{\vec{\nabla}\cdot\vec{A}=0} | P \rangle$$

Y. Zhao, PRL 133 (2024)

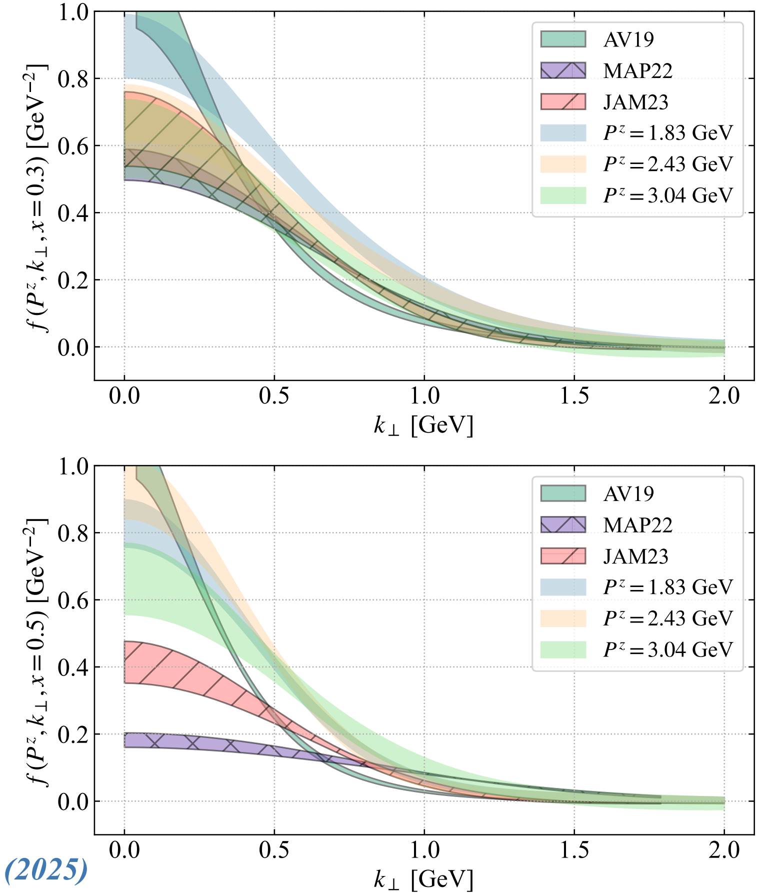
X. Gao, W. Y. Liu and Y. Zhao, PRD 109 (2024)

Why choose CG?

- The quasi distribution in CG belongs to the universality class in LaMET;
- No self-energy of the Wilson link, improving the SNR significantly;
- Simplified renormalization: $\bar{\psi}_0(z, b_\perp)\Gamma\psi_0(0) = Z_\psi(a)\left[\bar{\psi}(z, b_\perp)\Gamma\psi(0)\right]$;
- Larger off-axis momenta (3D rotational symmetry);
- A growing number of successful applications.

See also the talks by Qi (7/29 9:20am), Rui (7/29 9:40am) and Yang (last talk)

Pion unpolarized TMD PDFs



D. Bollweg, X. Gao, JH, S. Mukherjee and Y. Zhao, PRD 112 (2025)

Lattice Setup

Extract intrinsic soft functions from large-momentum form factors

- 2+1-flavor domain-wall fermion (for both sea and valence quark) with Iwasaki gauge action;
- Physical pion mass (for both sea and valence quark);
- Lattice spacing is $a = 0.0836$ fm;
- The proton quasi-TMD beam functions computed at proton momentum $P_z = 1.62$ GeV;
- The pion quasi-TMD wave functions with pion momenta up to $P_z = 1.85$ GeV;
- Pion form factors at momentum transfers up to $Q^2 = 13.7$ GeV^2.

Larger P_z in subsequent works

Quasi-TMD

TMDPDF

Hard kernel

$$\sqrt{S_I(b_\perp;\mu)} \cdot \tilde{f}_\Gamma(x, b_\perp, P^z;\mu) = f(x, b_\perp;\mu,\zeta) H_f(x, P^z;\mu) \exp\left[\frac{1}{2}\ln\frac{(2xP^z)^2}{\zeta}\gamma^{\overline{\mathrm{MS}}}(b_\perp;\mu)\right] + \text{Power corrections}$$

Intrinsic soft function ← **Quasi-TMDWF** / **Pion Form Factor** → **CS kernel**

Renormalization & Extrapolation

Quark field renormalization in CG: the scheme dependence is cancelled in LaMET matching

- The bare matrix elements of quasi-TMD beam functions and quasi-TMDWFs are renormalized as:

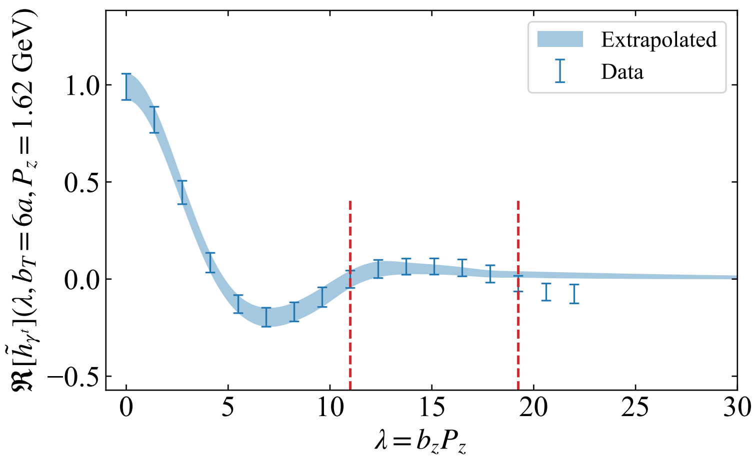
$$\tilde{h}_{\tilde{\Gamma}}(b_z, b_T, P_z; \mu) = \frac{\tilde{h}^0_{\tilde{\Gamma}}(b_z, b_T, P_z; a)}{\tilde{\varphi}^0_{\gamma^t\gamma^5}(0, b_T, 0; a)} \text{ and } \tilde{\varphi}_{\tilde{\Gamma}}(b_z, b_T, P_z; \mu) = \frac{\tilde{\varphi}^0_{\tilde{\Gamma}}(b_z, b_T, P_z; a)}{\tilde{\varphi}^0_{\gamma^t\gamma^5}(0, b_T, 0; a)};$$

Renormalized by zero-momentum matrix elements

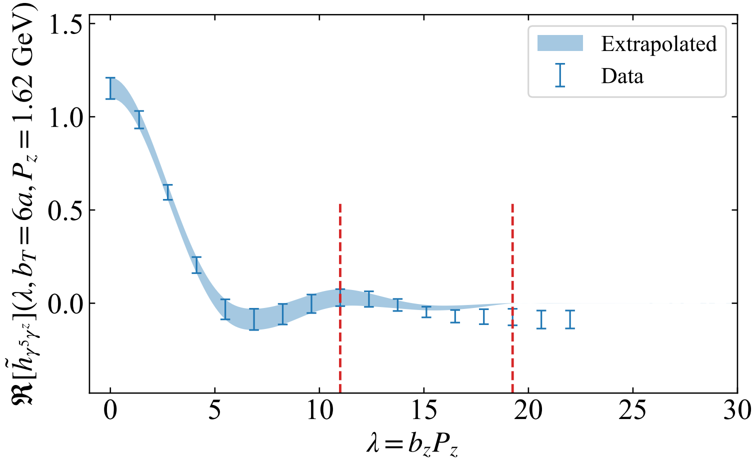
- The next-to-leading asymptotic form is used for extrapolation: $\tilde{h}(\lambda) = \left[A_2 \cdot e^{i\mathrm{sign}(\lambda)\phi_2} + \frac{A_2'}{|\lambda|} \cdot e^{i\mathrm{sign}(\lambda)\phi_2'}\right] \frac{e^{-m|\lambda|}}{|\lambda|^n}$

X. Ji, Y. Liu and Y. Su, 2601.12189

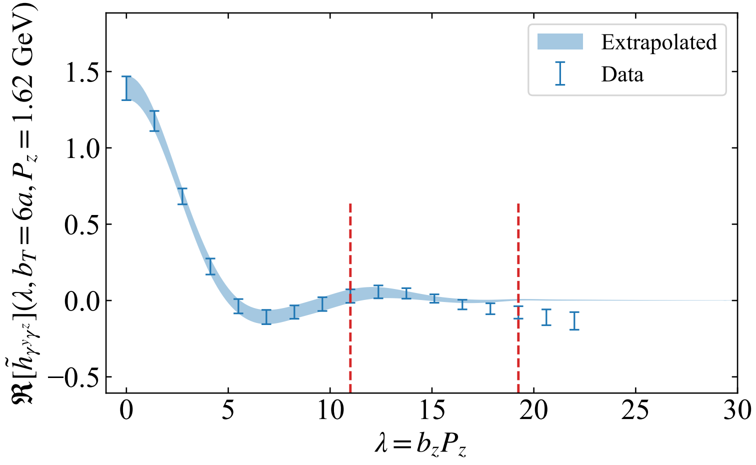
Unpolarized quasi-TMD



Helicity quasi-TMD



Transversity quasi-TMD



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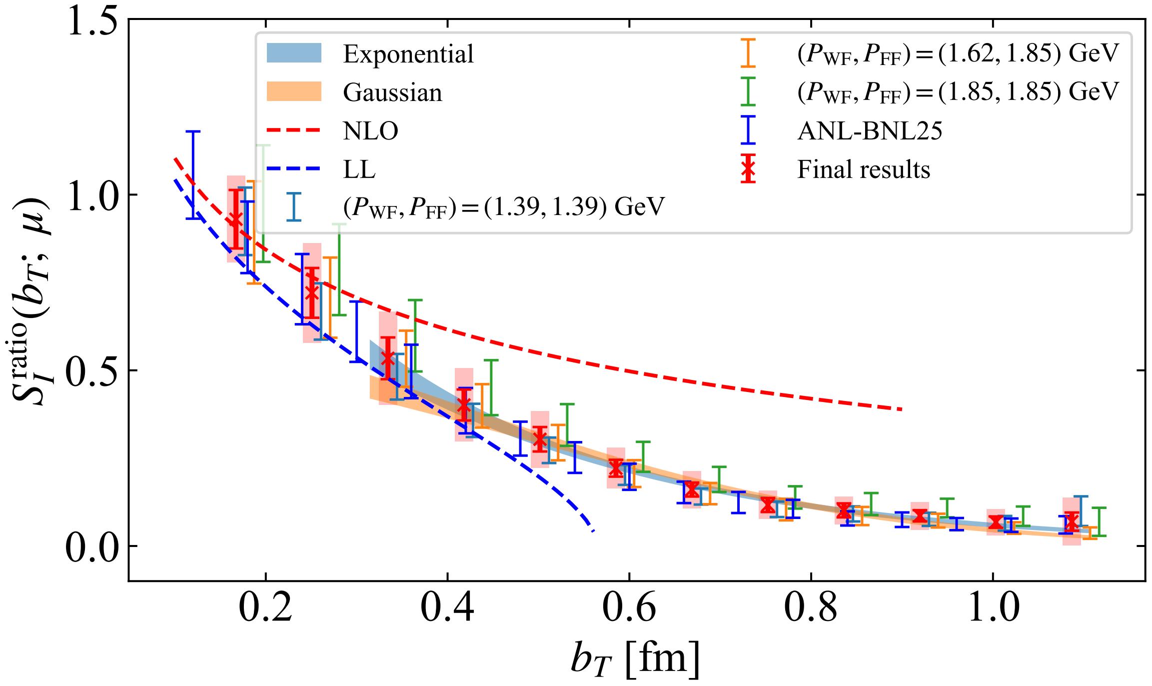
Intrinsic Soft Functions

Extract intrinsic soft functions at NLL from quasi-TMDWF and pion form factors

$$S_I(b_\perp;\mu) = \frac{F(b_\perp, P^z)}{\int dx_1 dx_2 H_F(x_1, x_2, P^z;\mu)\tilde{\Phi}^\dagger(x_1)\tilde{\Phi}(x_2)}$$ where $$F\left(b_\perp, P_1, P_2, \Gamma, \Gamma'\right) \equiv -4N_c \frac{\left\langle P_2 \middle| \bar{q}\left(b_\perp\right)\Gamma q\left(b_\perp\right)\bar{q}(0)\Gamma' q(0) \middle| P_1 \right\rangle}{f_\pi^2 (P_1 \cdot P_2)}$$

- Three momentum pairs are considered for the estimation of systematic uncertainties;
- At small b_T, lattice results are consistent with perturbative expectations.



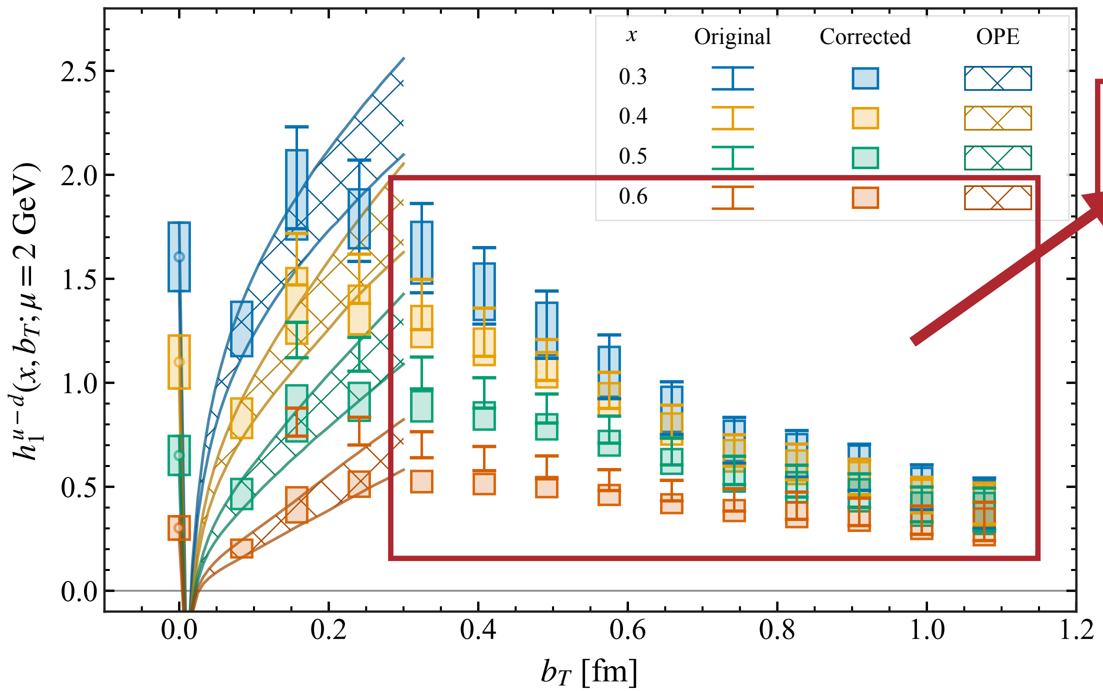
Transversity TMD PDFs

Complete distribution in the coordinate space for Fourier transform

$$\sqrt{S_I(b_\perp;\mu)} \cdot \tilde{f}_\Gamma(x, b_\perp, P^z; \mu) = f(x, b_\perp; \mu, \zeta)\, H_f(x, P^z; \mu) \exp\left[\frac{1}{2}\ln\frac{(2xP^z)^2}{\zeta}\gamma^{\overline{\mathrm{MS}}}(b_\perp;\mu)\right]$$

- At short b_T, TMD factorization requires $b_T \gg (2xP_z)^{-1}$, but we can use OPE as long as $b_T \ll \Lambda_{\mathrm{QCD}}^{-1}$;

- An extrapolation to get $b_T \to \infty$ is necessary to complete the Fourier transform.



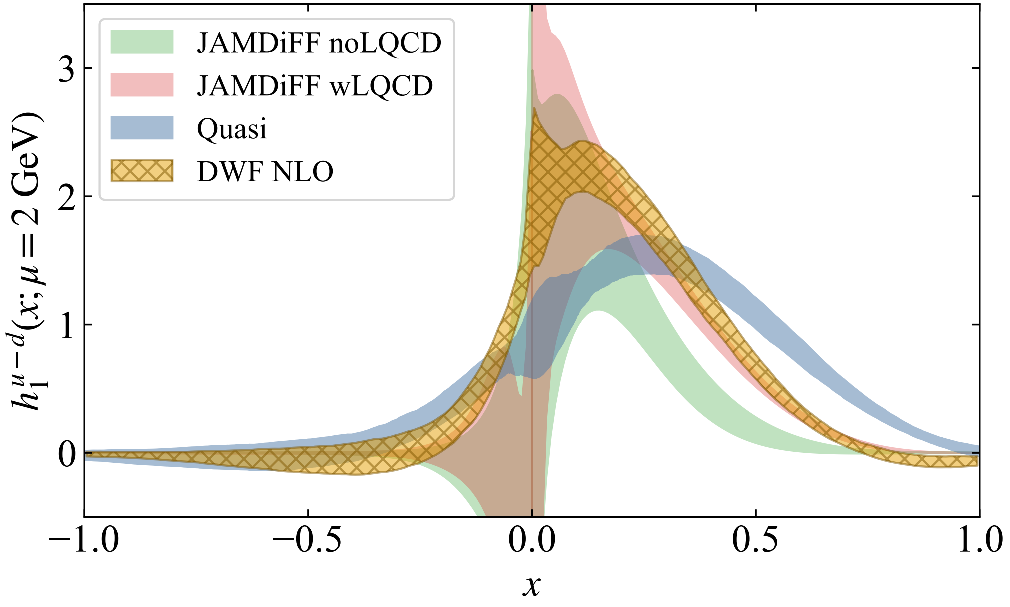
Moderate region: $1/(2xP_z) \ll b_T < b_{\max}$

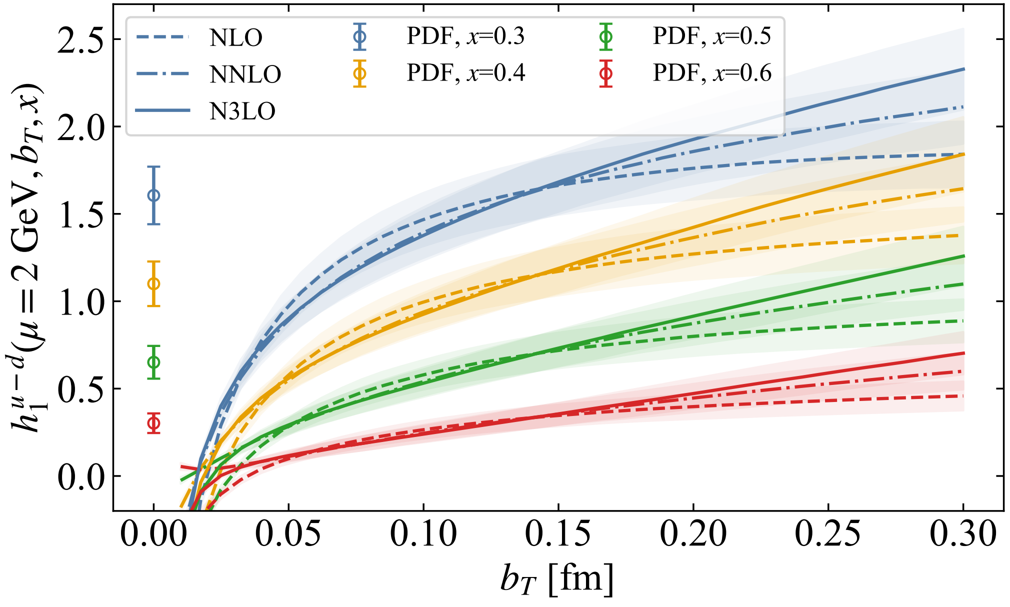
Short Distance OPE

TMD PDFs admit an OPE onto the corresponding collinear PDFs at perturbative b_T

- The collinear PDFs can be extracted from correlators at $b_T = 0$, renormalized in hybrid scheme;
- OPE prediction is given as $f_\Gamma(x, b_T; \mu, \zeta) = C_\Gamma(x, b_T; \mu, \zeta) \otimes f_\Gamma^{\rm coll}(x, \mu) + \mathcal{O}(b_T^2 \Lambda_{\rm QCD}^2)$;
- The short distance matching kernels are available up to N3LO, showing a good convergence.

M.-X. Luo et al., JHEP 06 (2021) 115; Y. Zhu, JHEP 02 (2026) 115; Y. Zhu, JHEP 03 (2026)

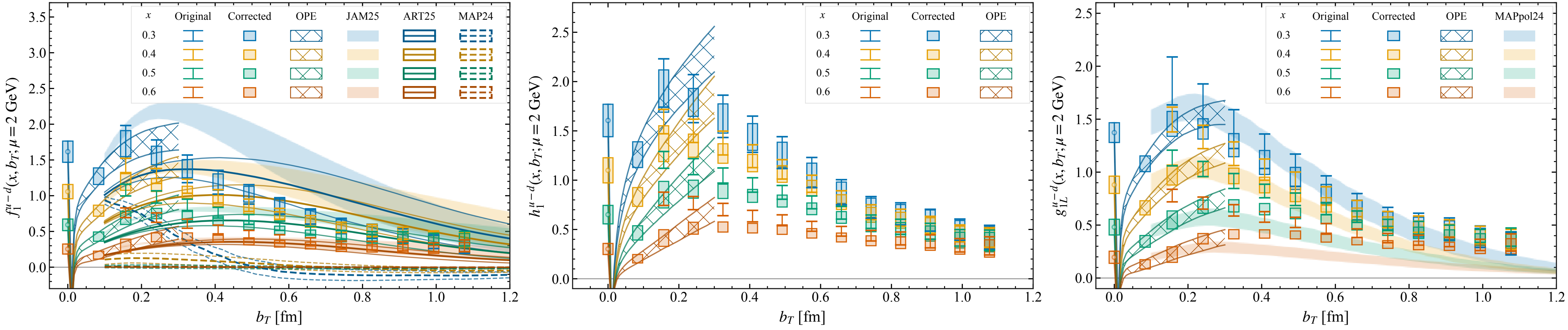




TMD PDFs in the Coordinate Space

Results for unpolarized, helicity and transversity TMD PDFs

- At $b_T = 0$, we have collinear PDFs; at $b_T = a$, we use the OPE results;
- The short b_T region is corrected by a $\delta C(x, b_T, P_z) = 1 + c/(2xb_TP_z)$ to match OPE results;
- Tensions among global analyses highlight the importance of first-principles lattice calculations as an independent determination of TMDPDFs.



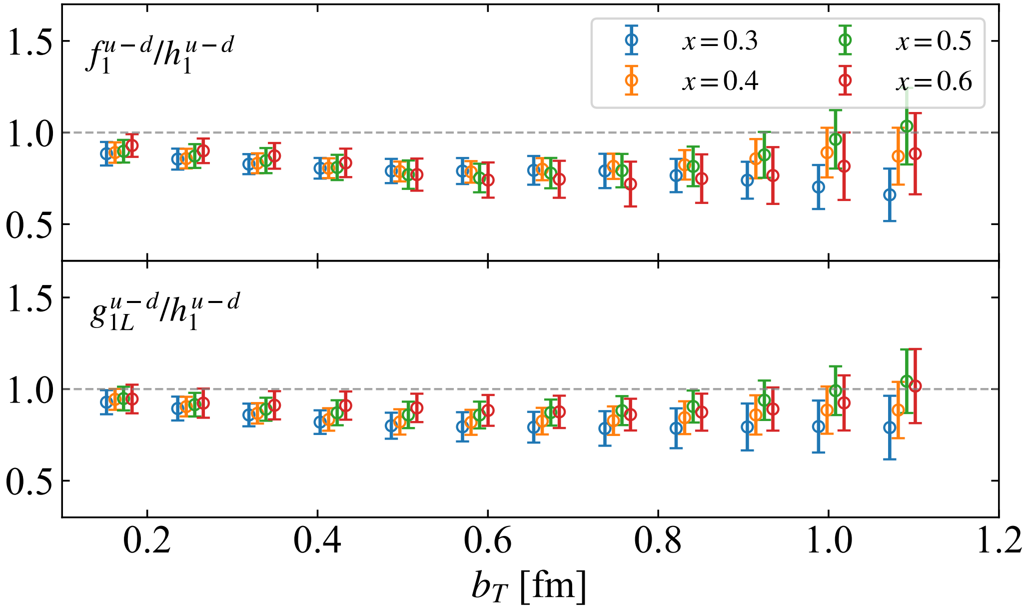
RGI Ratio of TMD PDFs

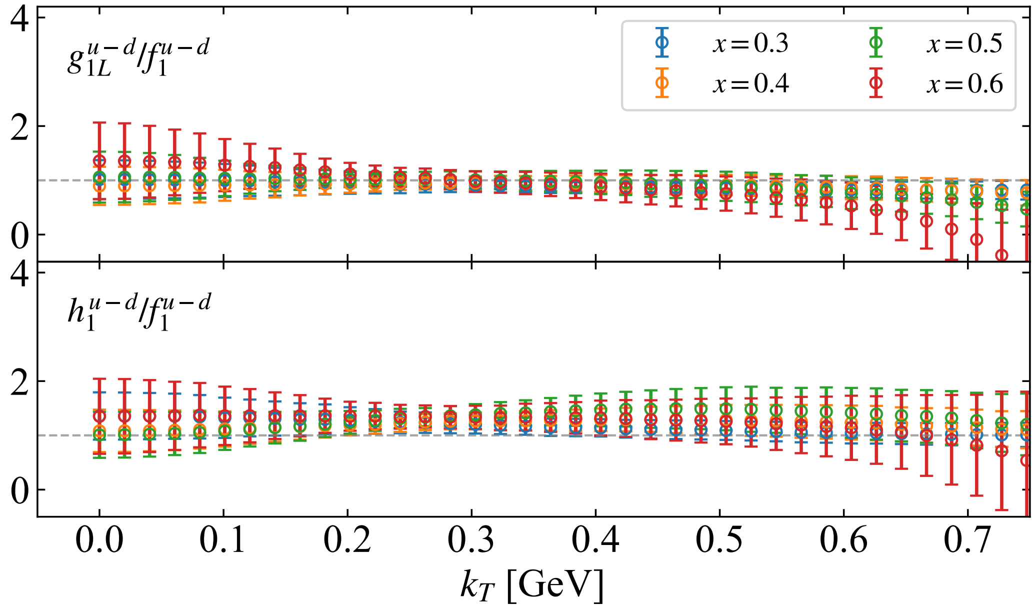
TMD PDFs with different polarizations share the common transverse structure

$$\sqrt{S_I\left(b_\perp;\mu\right)}\cdot\tilde{f}_\Gamma\left(x,b_\perp,P^z;\mu\right)=f\left(x,b_\perp;\mu,\zeta\right)H_f\left(x,P^z;\mu\right)\exp\left[\frac{1}{2}\ln\frac{(2xP^z)^2}{\zeta}\gamma^{\overline{\mathrm{MS}}}\left(b_\perp;\mu\right)\right]$$

- This ratio is free from renormalization scheme and scale dependence;
- This ratio is independent from CS kernel, intrinsic soft functions;
- The non-perturbative transverse structure is largely insensitive to spin polarizations;

Consistent with the results in D. Bollweg et al., PRL 135 (2025)





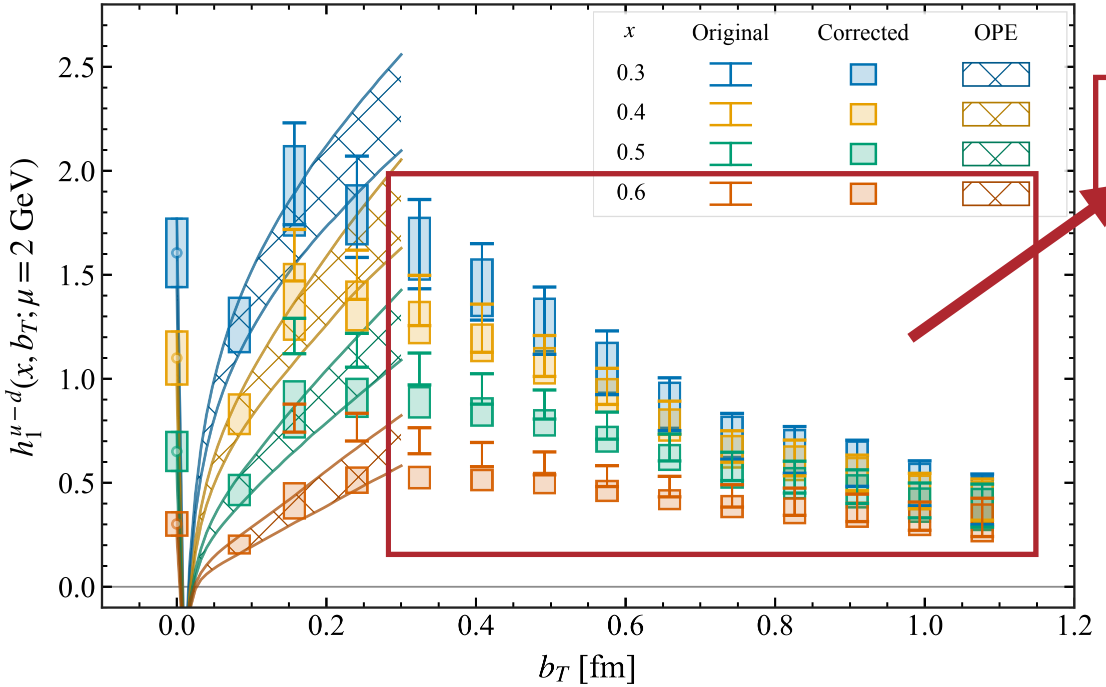
Transversity TMD PDFs

Complete distribution in the coordinate space for Fourier transform

$$\sqrt{S_I\left(b_\perp;\mu\right)} \cdot \tilde{f}_\Gamma\left(x,b_\perp,P^z;\mu\right) = f\left(x,b_\perp;\mu,\zeta\right) H_f\left(x,P^z;\mu\right) \exp\left[\frac{1}{2}\ln\frac{(2xP^z)^2}{\zeta}\gamma^{\overline{\mathrm{MS}}}\left(b_\perp;\mu\right)\right]$$

- At short b_T, TMD factorization requires $b_T \gg (2xP_z)^{-1}$, but we can use OPE as long as $b_T \ll \Lambda_{\mathrm{QCD}}^{-1}$;

- An extrapolation to get $b_T \to \infty$ is necessary to complete the Fourier transform.



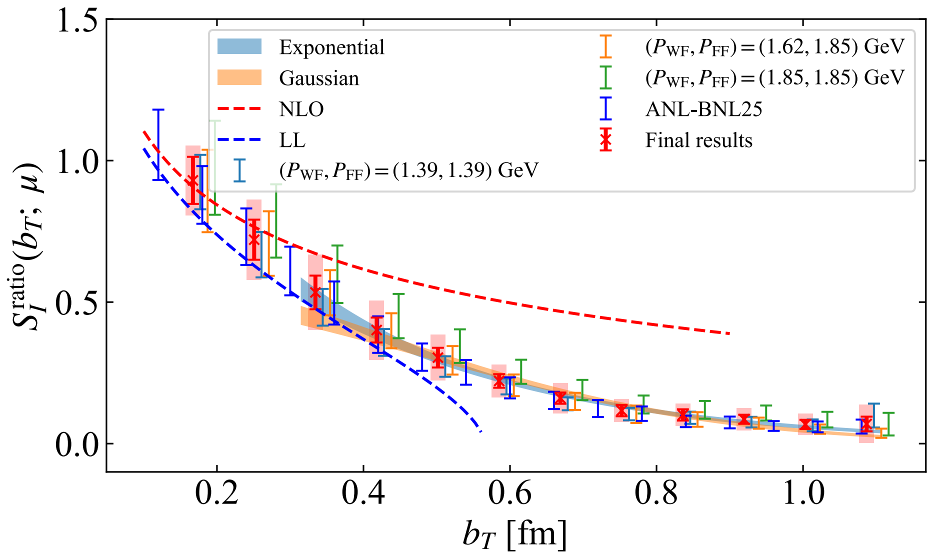
Moderate region: $1/(2xP_z) \ll b_T < b_{\max}$

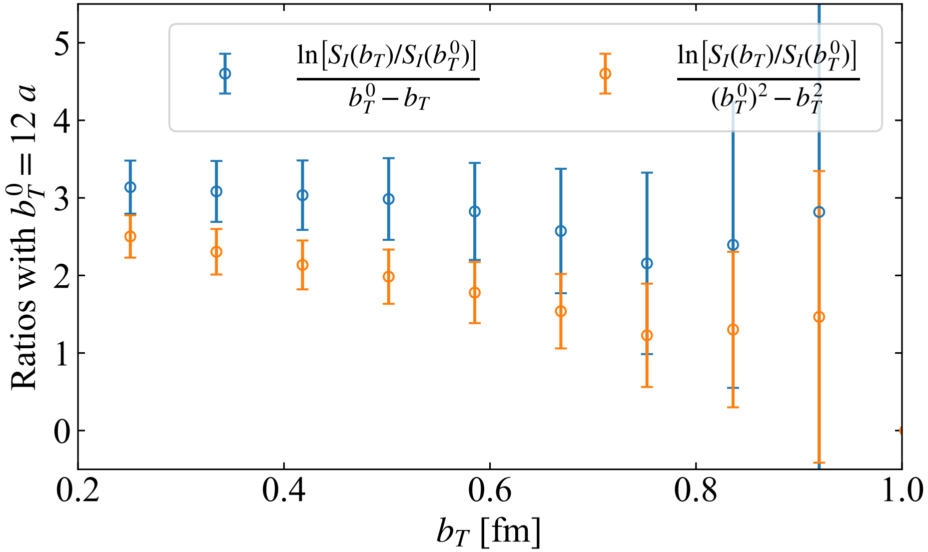
Intrinsic Soft Functions

Extract intrinsic soft functions at NLL from quasi-TMDWF and pion form factors

$$S_I(b_\perp;\mu) = \frac{F(b_\perp, P^z)}{\int dx_1 dx_2 H_F(x_1,x_2,P^z;\mu)\tilde{\Phi}^\dagger(x_1)\tilde{\Phi}(x_2)}$$

- Large-b_T behavior: exponential (Ae^{-mb_T}) or gaussian ($Ae^{-m^2b_T^2}$)?
- Fit results give χ^2/d.o.f. = 0.3 v.s. 1.4 and logGBF = 20.2 v.s. 14.2 (Bayes Factor of 400).



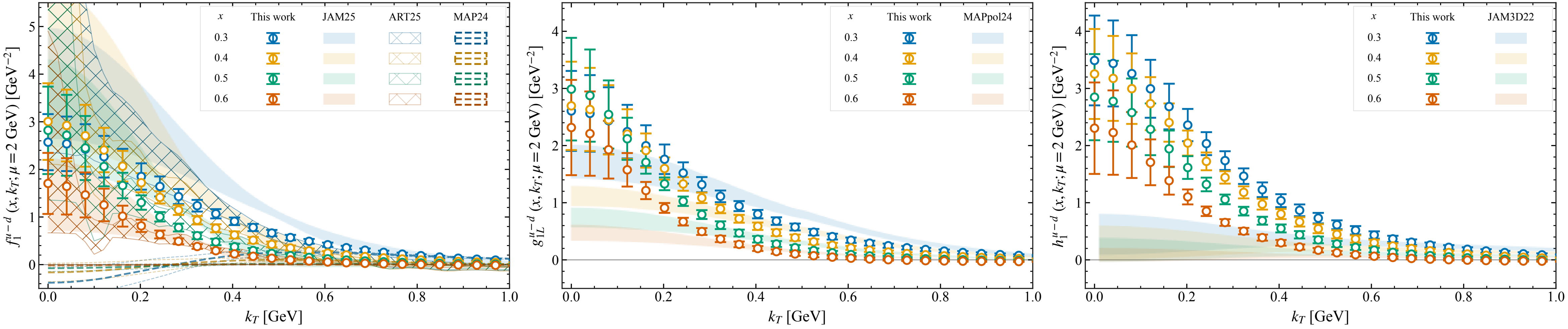


Effective mass plot also favors the exponential form

TMD PDFs in the Momentum Space

Results for unpolarized, helicity and transversity TMD PDFs

- The large b_T region is extrapolated using exponential form (Ae^{-mb_T}) with loose prior from soft functions;
- Tensions among global analyses highlight the importance of first-principles lattice calculations as an independent determination of TMDPDFs.



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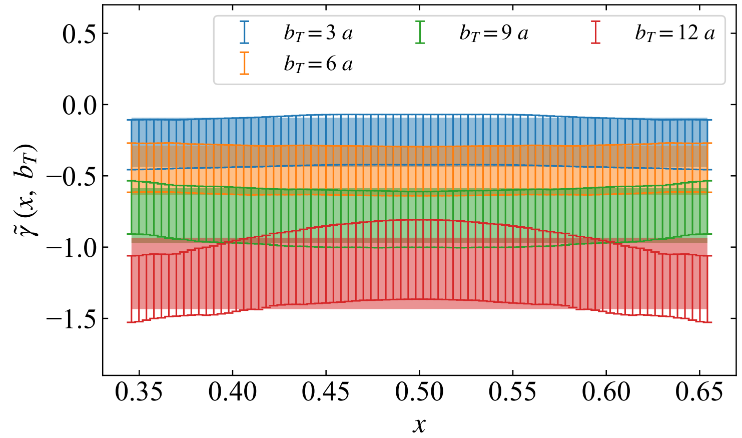
CS kernel

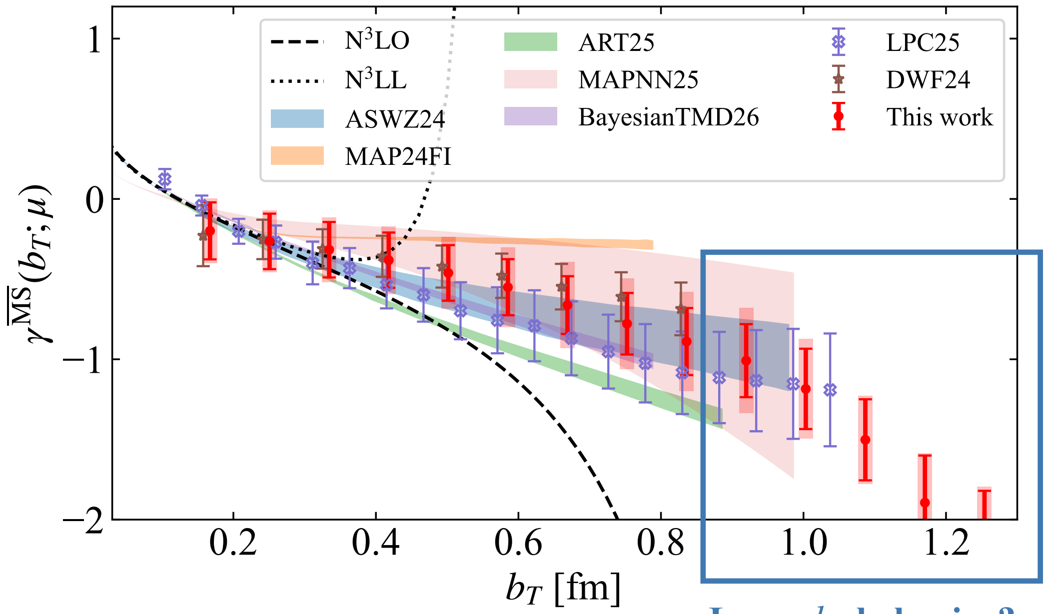
Extract CS kernel from quasi-TMDWF at different P_z

$$\sqrt{S_I(b_\perp;\mu)}\cdot\tilde{f}_\Gamma\left(x,b_\perp,P^z;\mu\right)=f\left(x,b_\perp;\mu,\zeta\right)H_f\left(x,P^z;\mu\right)\exp\left[\frac{1}{2}\ln\frac{(2xP^z)^2}{\zeta}\gamma^{\overline{\mathrm{MS}}}\left(b_\perp;\mu\right)\right]$$

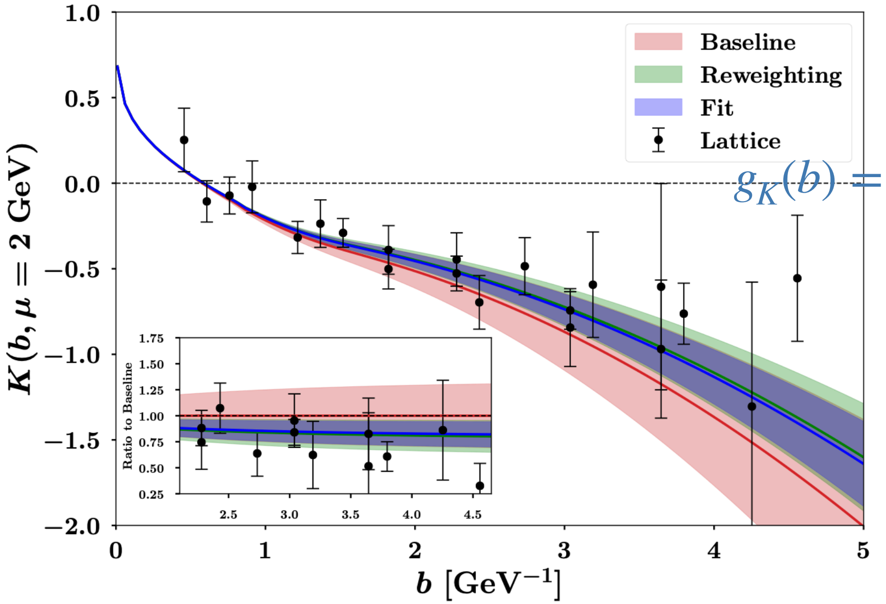
- Fit $\tilde{\phi}_{\tilde{\Gamma}}(x,b_T,P_z;\mu)=A(x,b_T)\;H_\phi(x,\bar{x},P_z;\mu)\;\exp\left[\ln\frac{P_z}{P_{\mathrm{fix}}}\tilde{\gamma}(x,b_T)\right]$
- Constant fits on $\tilde{\gamma}(x,b_T)$ for each b_T to get $\gamma^{\overline{\mathrm{MS}}}$.



Good consistency with DWF24



Large b_T behavior?



$g_K(b)=2g_2^2b^2$

A. Avkhadiev et al., Phys.Rev.Lett. 136 (2026)

Conclusion

Proton TMDPDFs from Lattice QCD

- We have presented the first simultaneous lattice-QCD determination of full isovector proton light-cone TMDPDFs in the unpolarized, helicity, and transversity channels;
- Although the three channels have distinct spin structures and different normalizations, their non-perturbative transverse structures are similar within the current uncertainties;
- After renormalization, the large-b_T behavior of TMD PDFs is dominated by the soft function;
- After combining the short distance OPE and large distance extrapolation, the k_T-space TMDPDFs provides a spin-dependent three-dimensional proton tomography;
- In future work, we will generalize to larger P_z and multi-ensemble analysis.

LaMET-Agent: For Data Analysis in LaMET

Why we need Agents?

- How to promote a method / framework? - Make it as user-friendly as possible!
- Agent is a new format to promote and present a method / framework.

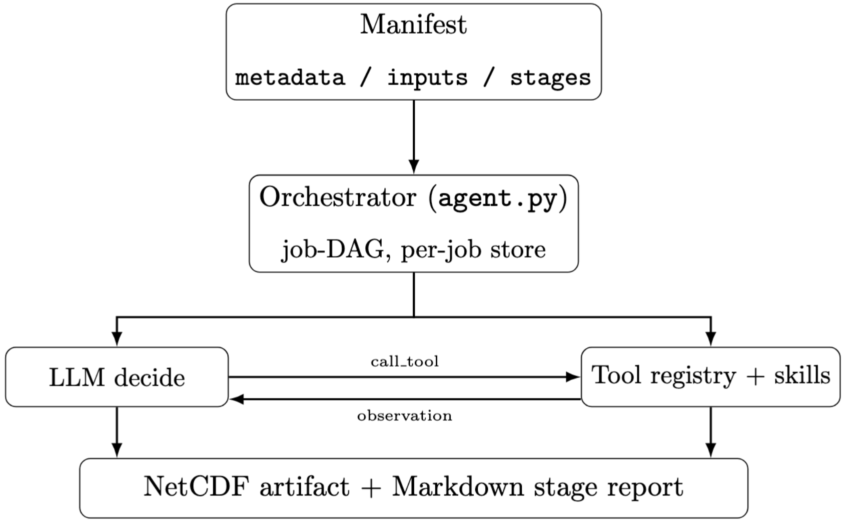
Why we need LaMET-Agent?

- **For researchers new to LaMET**: the agent makes LaMET data analysis accessible and helps users understand the correct analysis workflow and methodology;
- **For experienced LaMET users**: the agent automates routine analysis tasks, accelerates the workflow, and allowing researchers to focus on physics instead of implementation;
- LaMET-Agent provides a standard, well-organized, understandable workflow.

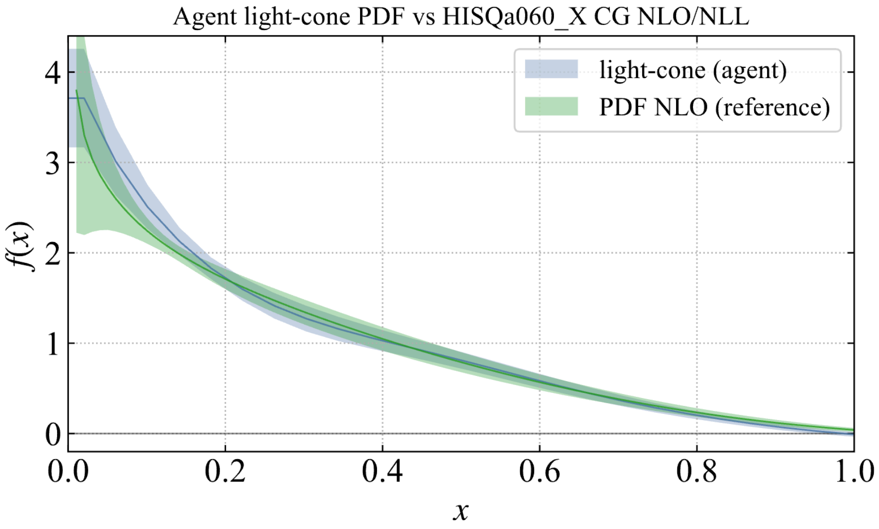
LaMET-Agent: For Data Analysis in LaMET

A delicate combination of deterministic pipelines and LLM decisions

- Six stages: Correlator Analysis, Renormalization, Fourier Transform, Perturbative Matching, Physical Extrapolation, Review;
- Single-run workflow for multi-ensemble analyses; unified NetCDF-based data management; stage reports for transparency; easily extensible to new observables and analysis techniques.



Reproducing the Pion PDF with LaMET Agent



X. Gao, W. Y. Liu and Y. Zhao, PRD 109 (2024)

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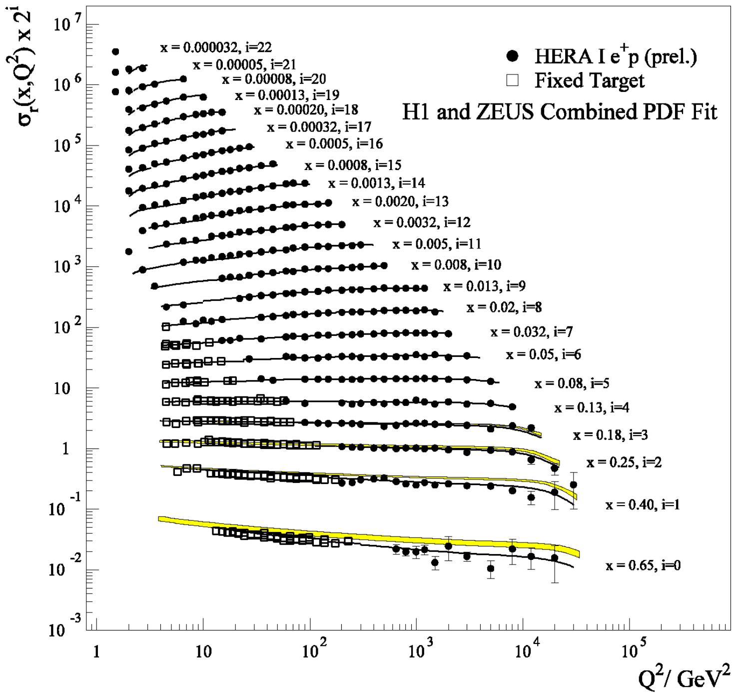
Thank you!

Jinchen He @ Lattice 2026

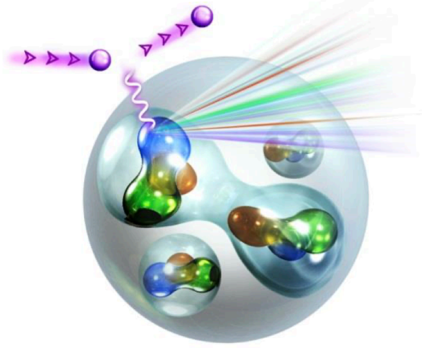
Parton Physics: Collinear structures

Bjorken scaling in 1968

- The structure functions are approximately independent of Q^2
- Evidence for point-like structures
- Parton model! *R. P. Feynman, Conf. Proc. C 690905 (1969)*
- Scaling violation at large Q^2 → running coupling in QCD

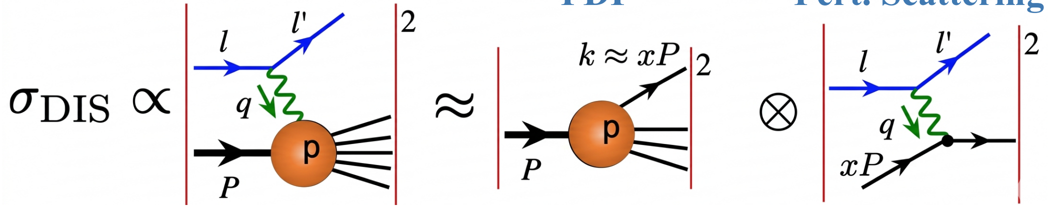


Collinear factorization in QCD



Cr. Dave Gaskell

PDF Pert. Scattering



R. Boussarie, et al., ``TMD Handbook", [arXiv:2304.03302 [hep-ph]]

Soft Functions

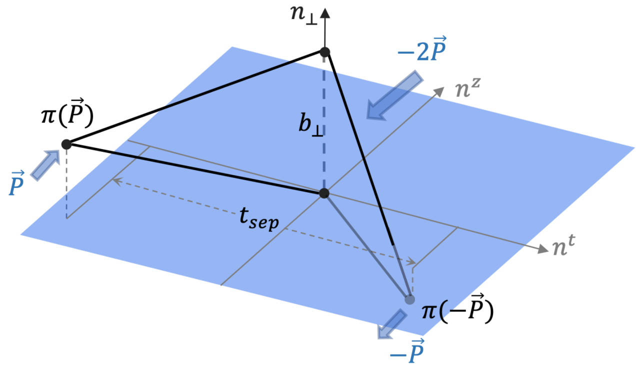
Extract intrinsic soft functions from large-momentum form factors

- The intrinsic soft function cannot be directly calculated on lattice because of two light-like Wilson lines in different directions;

- Fortunately, it can be extracted from the meson form factor

$$F\left(b_\perp, P_1, P_2, \Gamma, \Gamma'\right) \equiv -4N_c \frac{\left\langle P_2 \middle| \bar{q}\left(b_\perp\right)\Gamma q\left(b_\perp\right)\bar{q}\left(0\right)\Gamma' q(0) \middle| P_1 \right\rangle}{f_\pi^2 (P_1 \cdot P_2)}$$

X. Ji, Y. Liu and Y. S. Liu, Nucl.Phys.B **955 (2020)**



Q. A. Zhang, et al., Phys. Rev. Lett. 125 (2020)

  - The form factor satisfies the factorization formula

$$F(b_\perp, P^z) = \int dx_1 dx_2\ H_F(x_1, x_2, P^z; \mu)\ \phi^\dagger(x_1, b_\perp, y_n; \mu, \zeta_1, \bar{\zeta}_1)\ \phi(x_2, b_\perp, -y_n; \mu, \zeta_2, \bar{\zeta}_2)$$

  - Therefore, the intrinsic soft function can be extracted via

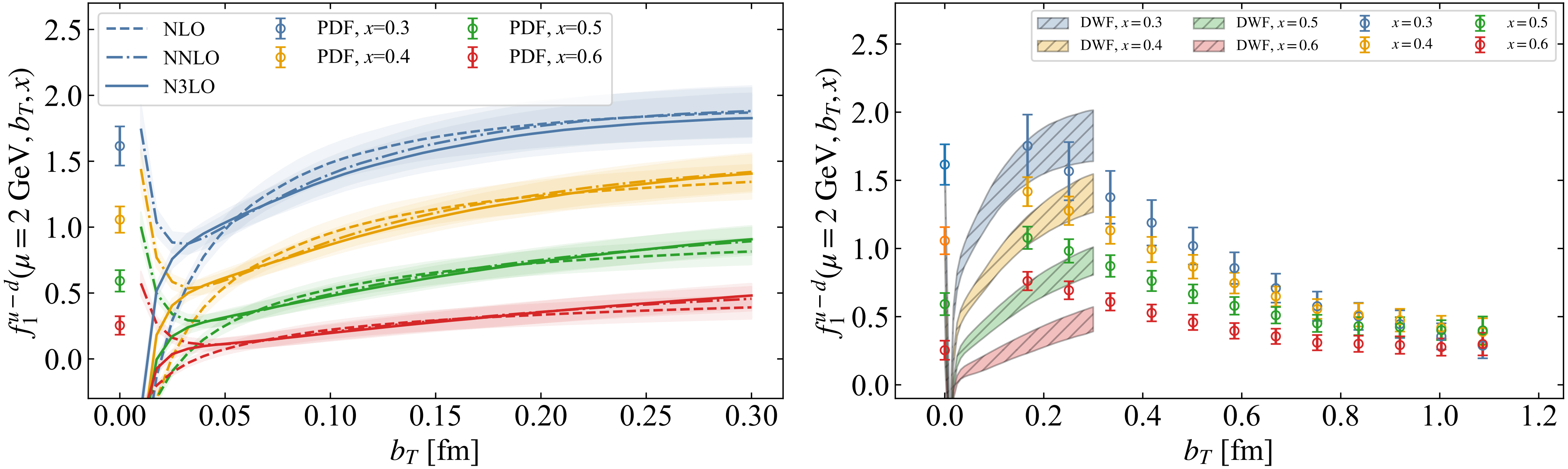
$$S_I(b_\perp; \mu) = \frac{F(b_\perp, P^z)}{\int dx_1 dx_2 H_F(x_1, x_2, P^z; \mu)\tilde{\Phi}^\dagger(x_1)\tilde{\Phi}(x_2)} \quad \text{where } \tilde{\Phi}(x) \equiv \frac{\tilde{\phi}_\Gamma\left(x, b_\perp, P^z; \mu\right)}{H_\phi\left(x, \bar{x}, P^z; \mu\right)}$$

Short distance OPE

TMD PDFs admit an OPE onto the corresponding collinear PDFs at perturbative b_T

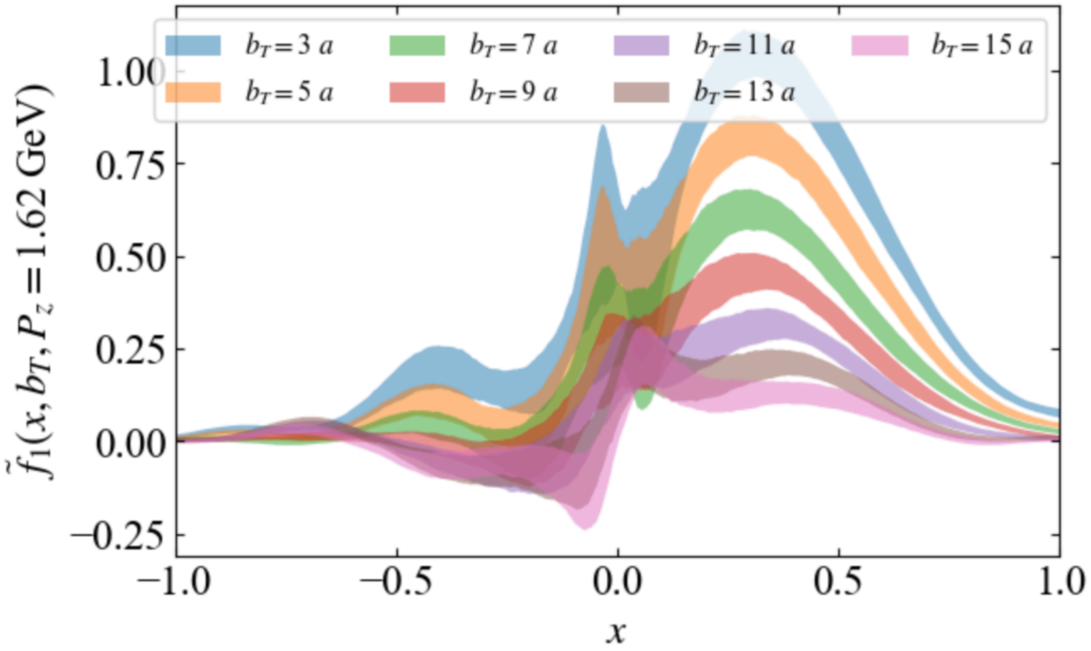
- The intrinsic soft function cannot be directly calculated on lattice because of two light-like

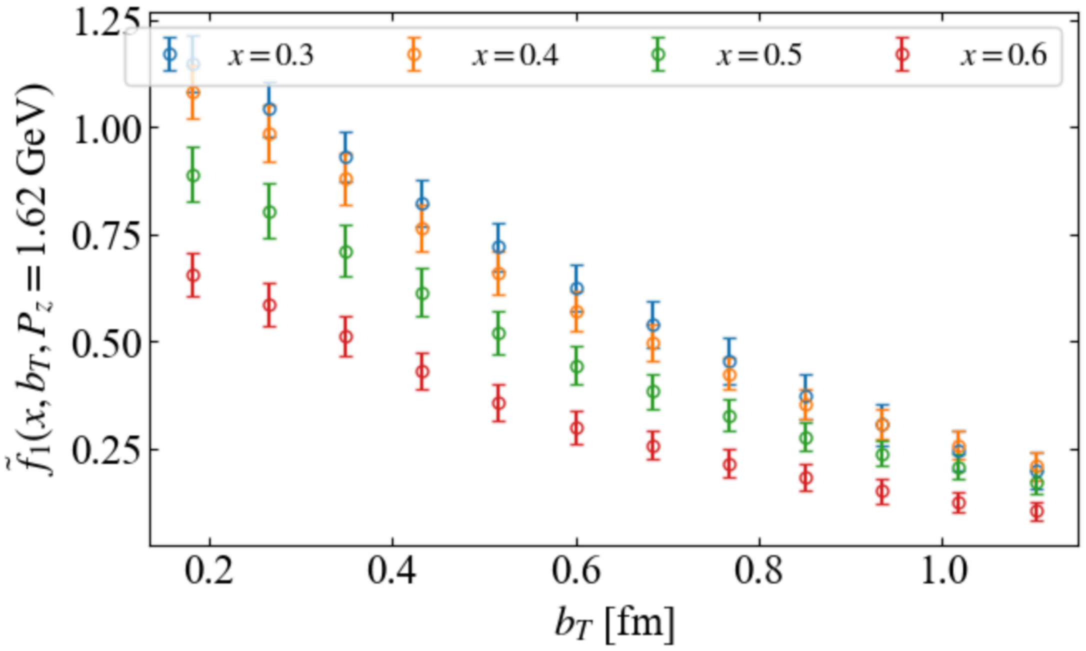
- $f_\Gamma(x, b_T; \mu, \zeta) = C_\Gamma(x, b_T; \mu, \zeta) \otimes f_\Gamma^{\text{coll}}(x, \mu) + \mathcal{O}(b_T^2 \Lambda_{\text{QCD}}^2)$



bT Dependence of Bare qTMD

What if before renormalization?





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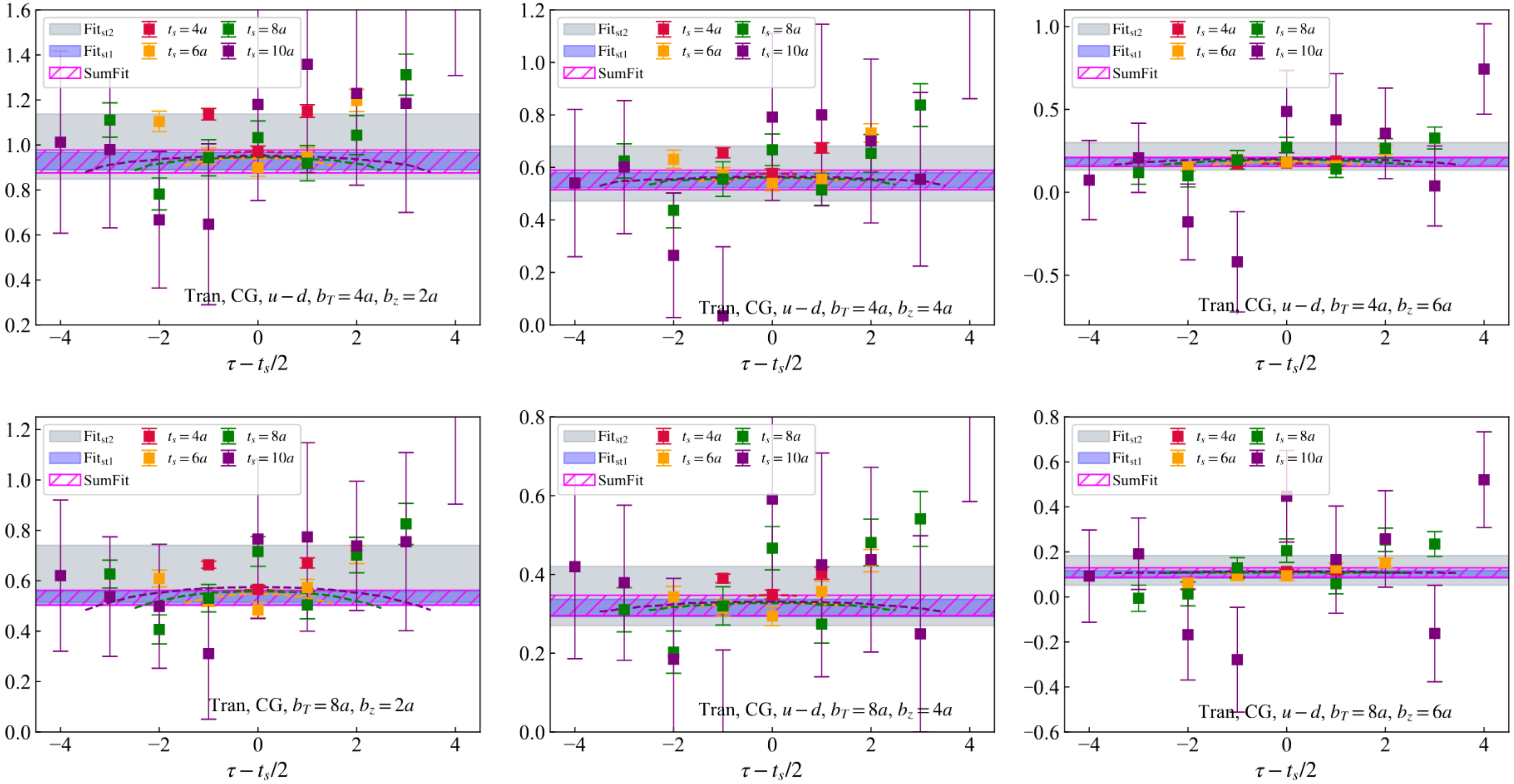
Bare Matrix Elements

The ratios of three-point to two-point functions



Renormalization

After renormalization, the large b_T dependence is mainly determined by the soft function

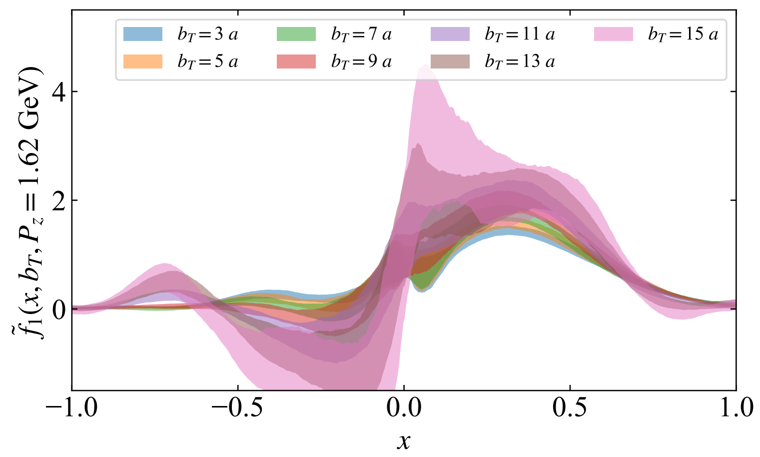
- The bare matrix elements of quasi-TMD beam functions and quasi-TMDWFs are renormalized as:

$$\tilde{h}_{\tilde{\Gamma}}(b_z, b_T, P_z; \mu) = \frac{\tilde{h}^0_{\tilde{\Gamma}}(b_z, b_T, P_z; a)}{\tilde{\varphi}^0_{\gamma^t\gamma^5}(0, b_T, 0; a)} \text{ and } \tilde{\varphi}_{\tilde{\Gamma}}(b_z, b_T, P_z; \mu) = \frac{\tilde{\varphi}^0_{\tilde{\Gamma}}(b_z, b_T, P_z; a)}{\tilde{\varphi}^0_{\gamma^t\gamma^5}(0, b_T, 0; a)};$$

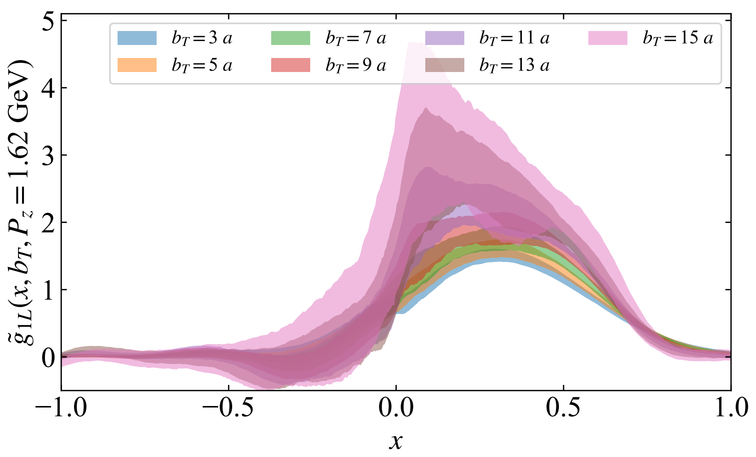
Renormalized by zero-momentum matrix elements

- In the moderate x region, quasi-TMD beam functions in all three polarizations show minor b_T-dependence;

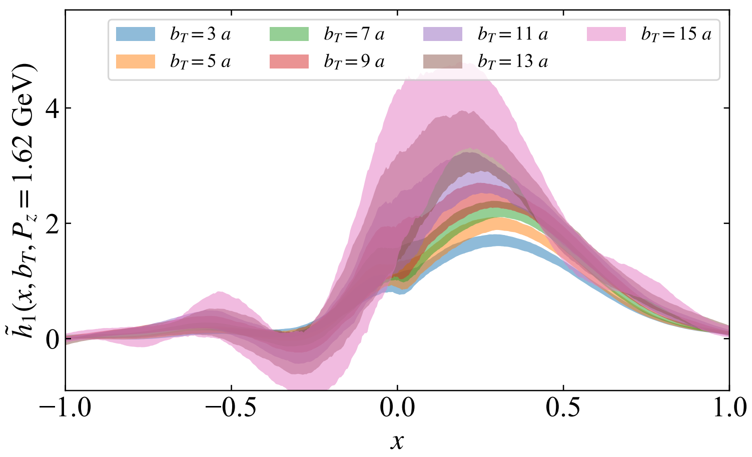
Unpolarized quasi-TMD



Helicity quasi-TMD



Transversity quasi-TMD



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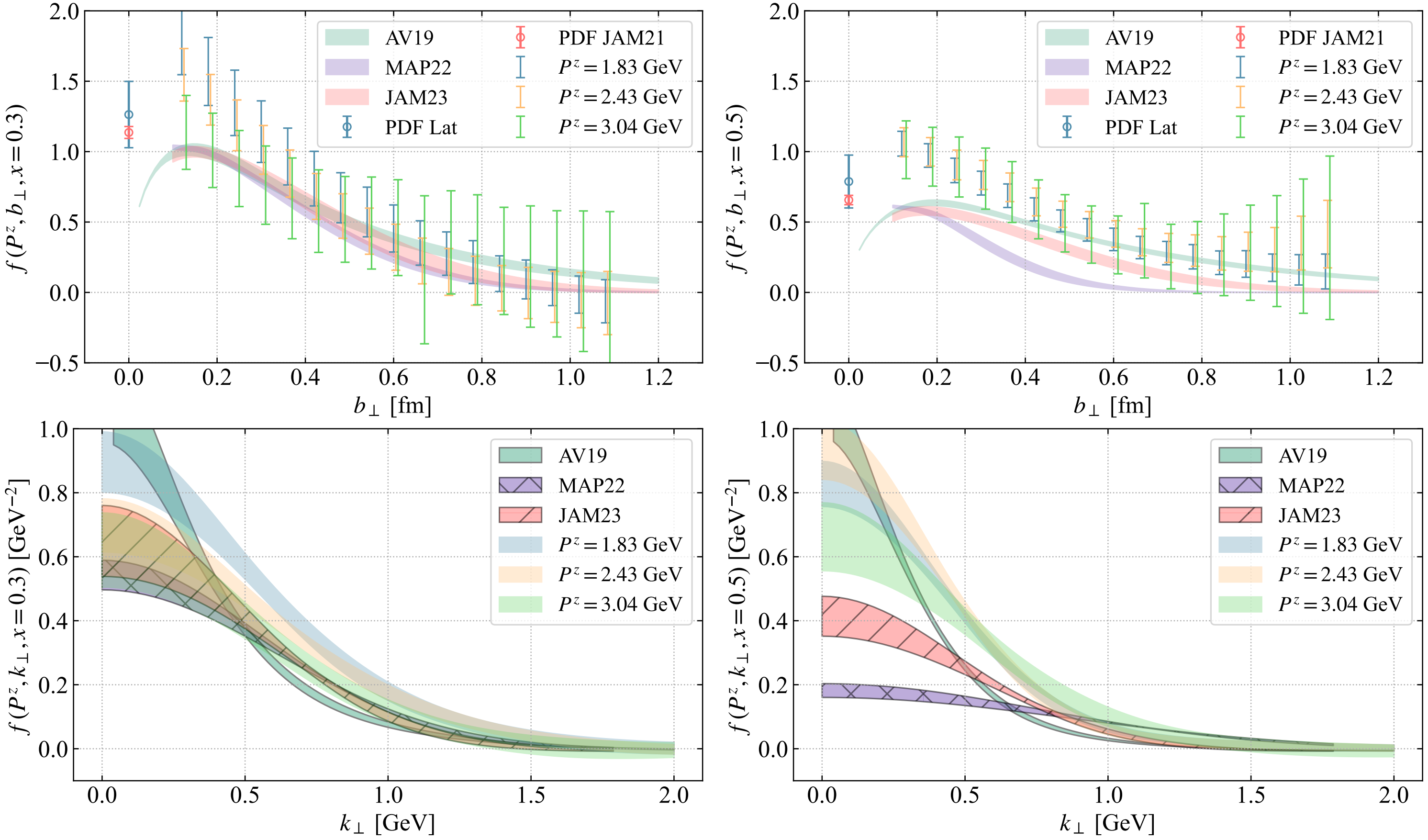
Pion TMDs from Coulomb Gauge Method

Pion momentum larger than 3 GeV on the lattice with $a = 0.06$ fm;

- Different phenomenological collaborations agree well at relative small x, where we saw consistency;
- Lattice provides predictions at relative large x, where phenomenological results exhibit discrepancies.



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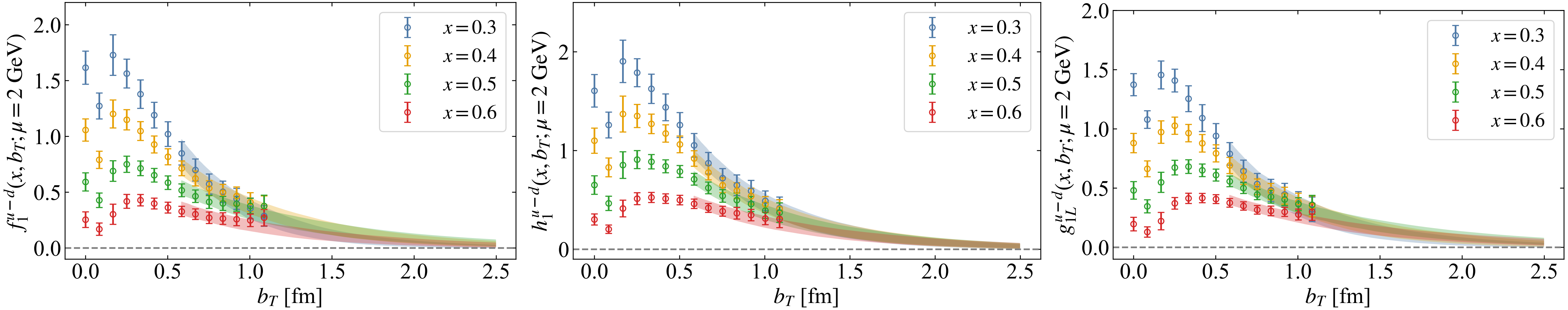
Extrapolation in Larger bT Region

Extrapolation using the exponential form and loose prior from soft functions



Soft Functions in TMD Factorization

$$\frac{d\sigma}{dQdYd^2\mathbf{q}_T} = \left(\frac{d\sigma^W}{dQdYd^2\mathbf{q}_T} + \frac{d\sigma^Y}{dQdYd^2\mathbf{q}_T}\right)\left[1 + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^2}\right)\right].$$

$$\frac{d\sigma^W}{dQdYd^2\mathbf{q}_T} = \sum_{\text{flavors } i} H_{i\bar{i}}(Q^2,\mu)\int d^2\mathbf{b}_T\, e^{i\mathbf{b}_T\cdot\mathbf{q}_T}\, \tilde{f}_{i/p}(x_a,\mathbf{b}_T,\mu,\zeta_a)\, \tilde{f}_{\bar{i}/p}(x_b,\mathbf{b}_T,\mu,\zeta_b) \tag{2.29a}$$

$$= \sum_{\text{flavors } i} H_{i\bar{i}}(Q^2,\mu)\int d^2\mathbf{b}_T\, e^{i\mathbf{b}_T\cdot\mathbf{q}_T}\, \tilde{B}_{i/p}(x_a,\mathbf{b}_T,\mu,\zeta_a/\nu^2)\, \tilde{B}_{\bar{i}/p}(x_b,\mathbf{b}_T,\mu,\zeta_b/\nu^2) \times \tilde{S}_{n_a n_b}(b_T,\mu,\nu)\,. \tag{2.29b}$$

$$\tilde{f}_{i/p}(x,\mathbf{b}_T,\mu,\zeta) = \tilde{B}_{i/p}(x,\mathbf{b}_T,\mu,\zeta/\nu^2)\sqrt{\tilde{S}_{n_a n_b}(b_T,\mu,\nu)}\,, \tag{2.32}$$

$$\tilde{B}_{i/p}(x,\mathbf{b}_T,\mu,\zeta/\nu^2) = \lim_{\substack{\epsilon\to 0\\ \tau\to 0}} \tilde{Z}^i_B(b_T,\mu,\nu,\epsilon,\tau,xP^+)\, \tilde{B}^0_{i/p}(x,\mathbf{b}_T,\epsilon,\tau,xP^+)$$

$$= \lim_{\substack{\epsilon\to 0\\ \tau\to 0}} \tilde{Z}^i_B(b_T,\mu,\nu,\epsilon,\tau,xP^+)\, \frac{\tilde{f}^{0\,(\mathrm{u})}_{i/p}(x,\mathbf{b}_T,\epsilon,\tau,xP^+)}{\tilde{S}^{0\,\mathrm{subt}}_{n_a n_b}(b_T,\epsilon,\tau)}\,, \tag{2.34}$$

$$\tilde{S}_{n_a n_b}(b_T,\mu,\nu) = \lim_{\substack{\epsilon\to 0\\ \tau\to 0}} \tilde{Z}_S(b_T,\mu,\nu,\epsilon,\tau)\, \tilde{S}^0_{n_a n_b}(b_T,\epsilon,\tau)\,. \tag{2.35}$$

$$\tilde{f}_\Gamma(x,b_\perp,P^z;\mu) = H_f(x,P^z;\mu)\, B(x,b_\perp,xP^+;\mu,\nu) \times S^0_C(b_\perp;\mu,\nu)\,, \tag{3}$$

$$\sqrt{S_I(b_\perp,y_n;\mu)}\cdot \tilde{f}_\Gamma(x,b_\perp,P^z;\mu) = f(x,b_\perp;\mu,\zeta)H_f(x,P^z;\mu) + \text{p.c.}\,, \tag{5}$$

- **Zero-bin subtracted beam function has no soft contribution;**
- S^0_C **is the quasi-soft function with zero-bin contribution;**
- **The intrinsic soft function S_I is the combination of subtracted soft function and the quasi-soft function**

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