Lattice QCD on the 16-cell honeycomb

Sandor Katz and Daniel Nogradi

Eotvos University, Budapest

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Context and motivation

Symmetry important, lattice breaks it

Lorentz, Poincarè, chiral, ...

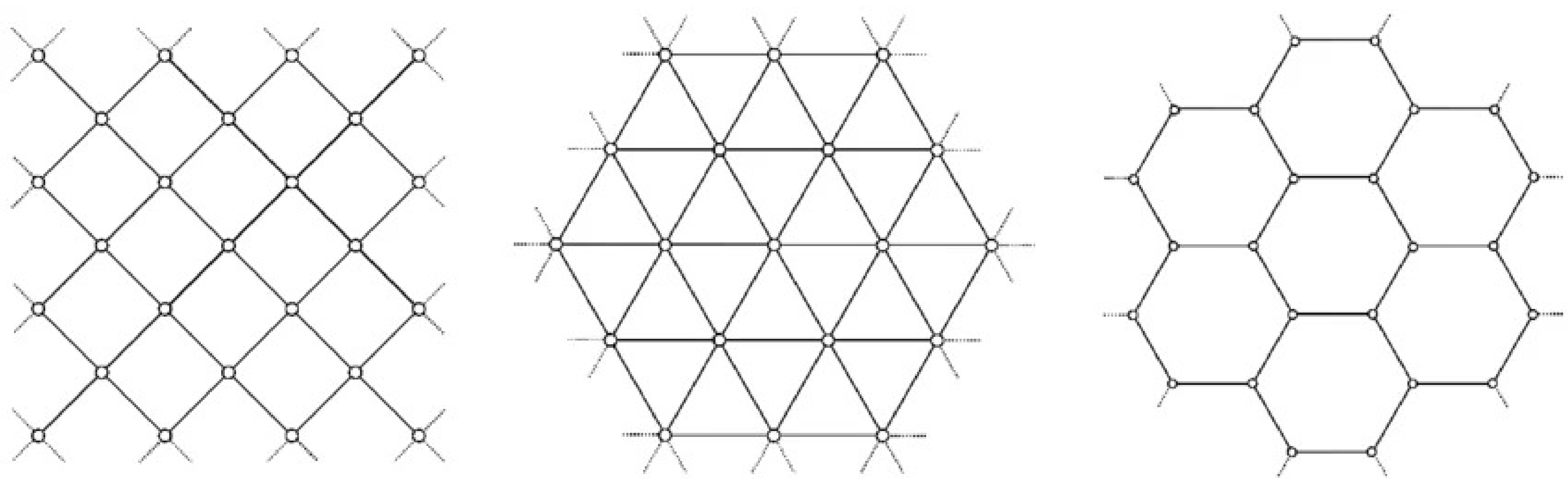
Lots of effort: break as little as possible

Lattice: almost always 4D cubic

Not most symmetric!

In 4D, 3 different regular space-filling tesselations

2D analogue



Context and motivation

4D: also 3 regular space-filling tesselations

Cubic, 16-cell, 24-cell, like in 2D, but not 3D

Order of symmetry group:

cubic: 384 , 16-cell and 24-cell: 1152

Very similar again to 2D

Context and motivation

Let's do lattice QCD on the 16-cell honeycomb

Larger symmetry $\rightarrow$ closer to continuum $\rightarrow$

$\rightarrow$ larger lattice spacing $\rightarrow$ computational gain

Old idea, 1980s, William Celmaster

Old idea

- William Celmaster: gauge theory + naive fermions
- Lorentz violating naive fermion doublers
- Abandoned ...
- Bhanot/Heller/Neuberger: scalar field theory

Many ways to define, sites:

- **Cubic lattice** $\cup$ $(1/2,\ 1/2,\ 1/2,\ 1/2)$ **shifted cubic lattice** $\rightarrow$

  $\rightarrow$ **body centered cubic lattice**

- (n_1, n_2, n_3, n_4) **with** $n_1 + n_2 + n_3 + n_4$ **even** $\rightarrow$ D_4 **lattice**

Links: nearest neighbors, 24 total (8 for cubic)

The lattice

Nearest neighbor directions, 24 total

$$(\pm 1, 0, 0, 0), \quad (0, \pm 1, 0, 0), \quad (0, 0, \pm 1, 0), \quad (0, 0, 0, \pm 1)$$

Usual cubic neighbors: 8

$$(\pm 1/2, \pm 1/2, \pm 1/2, \pm 1/2)$$

16 more

Gauge theory

12 links per site x (4 for cubic)

Shortest closed loop: triangle, $P(x) = U_3\, U_2\, U_1$

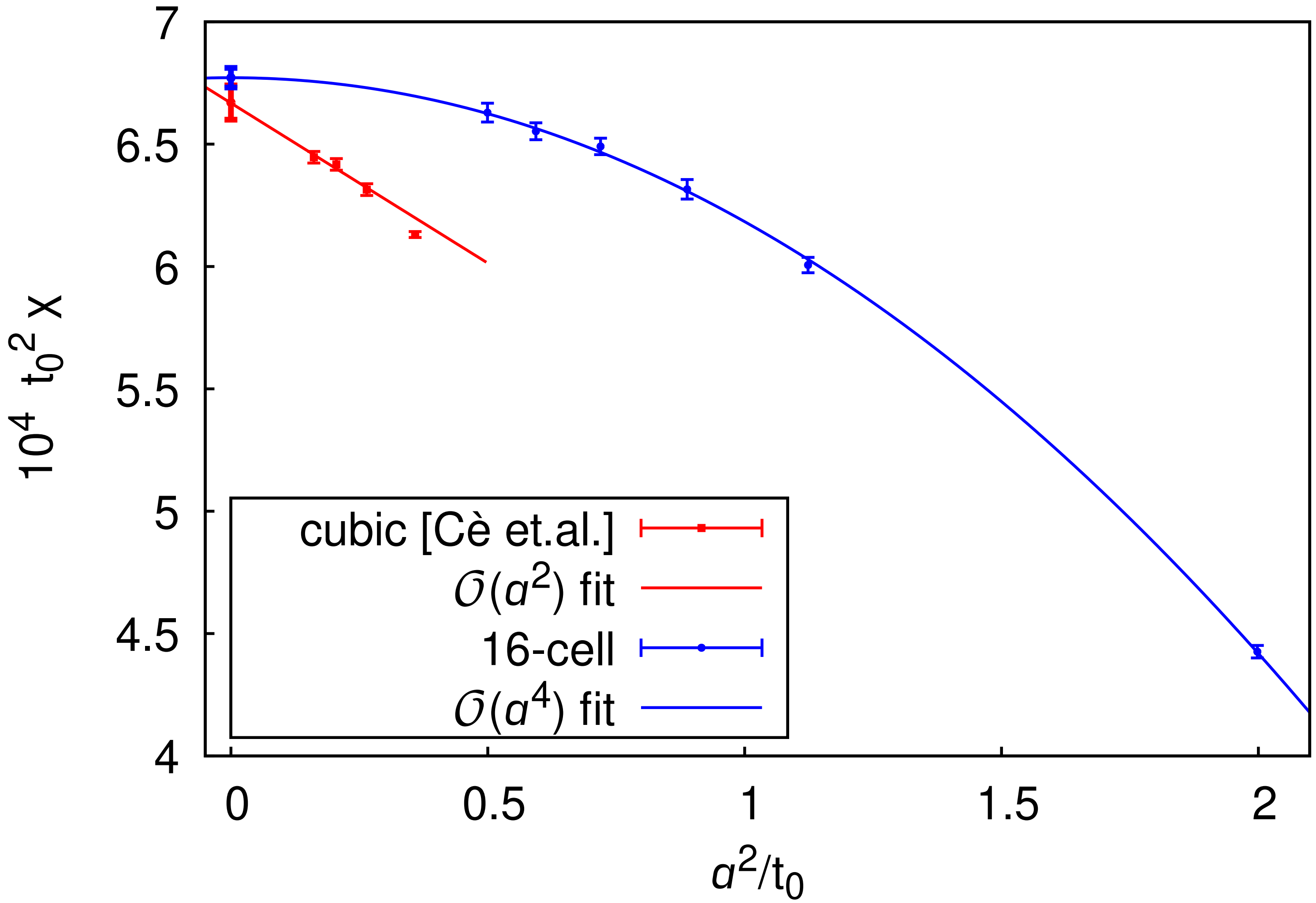
$$S = \frac{\beta}{2} \sum_x \sum_{i=1}^{32} \left(1 - \frac{1}{N} \mathrm{Re}\,\mathrm{Tr}\, P_i(x) \right)$$

$\beta = 2N/g^2$ **just as before**

Gradient flow straightforward

Topological charge $\to$ clover-style hexagons from triangles

Topological susceptibility, $SU(3)$



$O(a^2)$ consistent with zero $\rightarrow$ probably very small

Fermions

Naive discretization, n_i the 24 neighbor vectors

$$D_0 = \frac{1}{6}\sum_{i=1}^{24} \gamma_\mu n_{\mu i} \nabla_i$$

Many doublers, some break Lorentz invariance Celmaster

Introduce Wilson term

$$D = D_0 + a\frac{r}{6}\sum_{i=1}^{24} \nabla_i^* \nabla_i$$

Only 1 physical mode

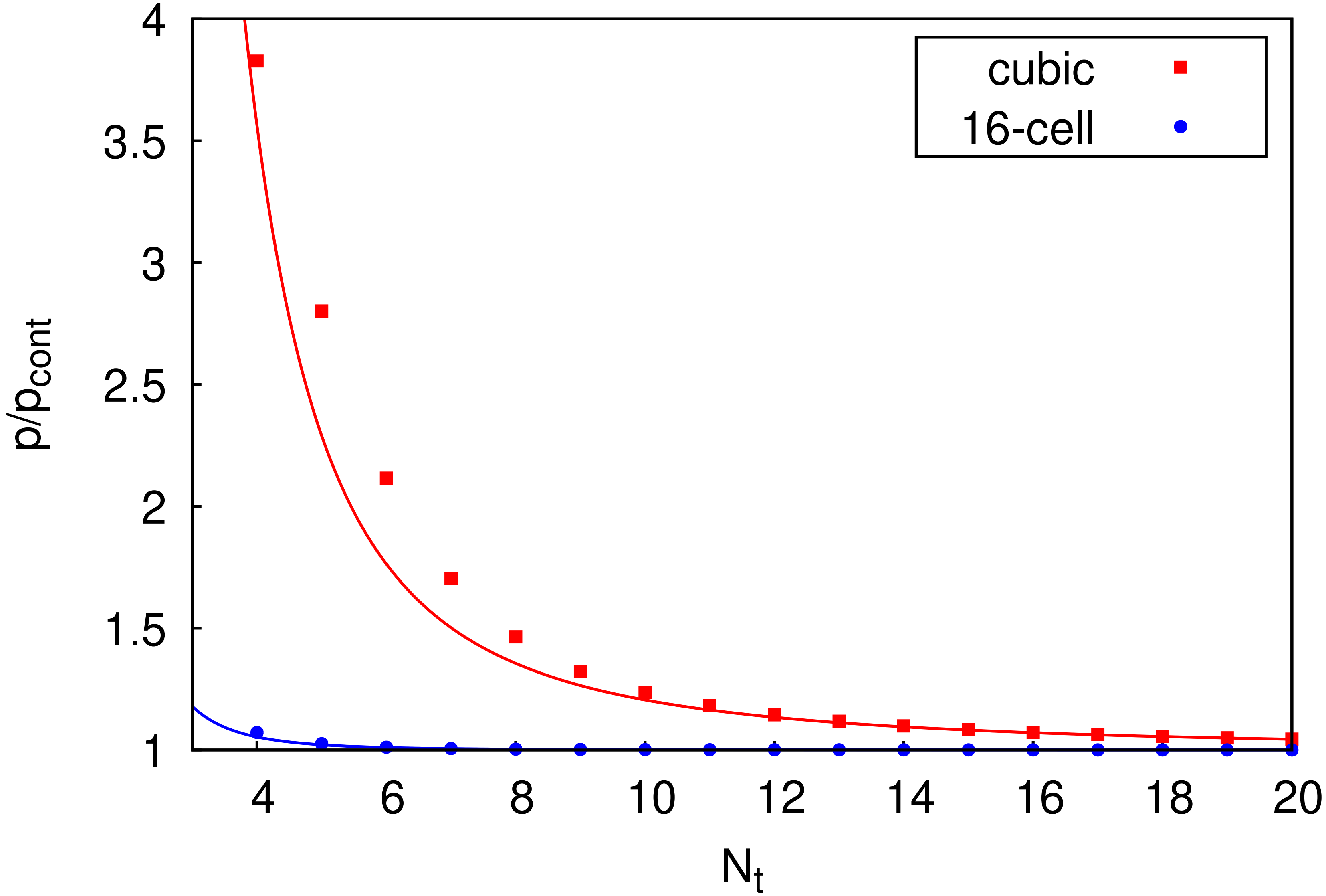
Fermions, free pressure

$$\mathcal{O} = \frac{p(T) - p(0)}{T^4}$$

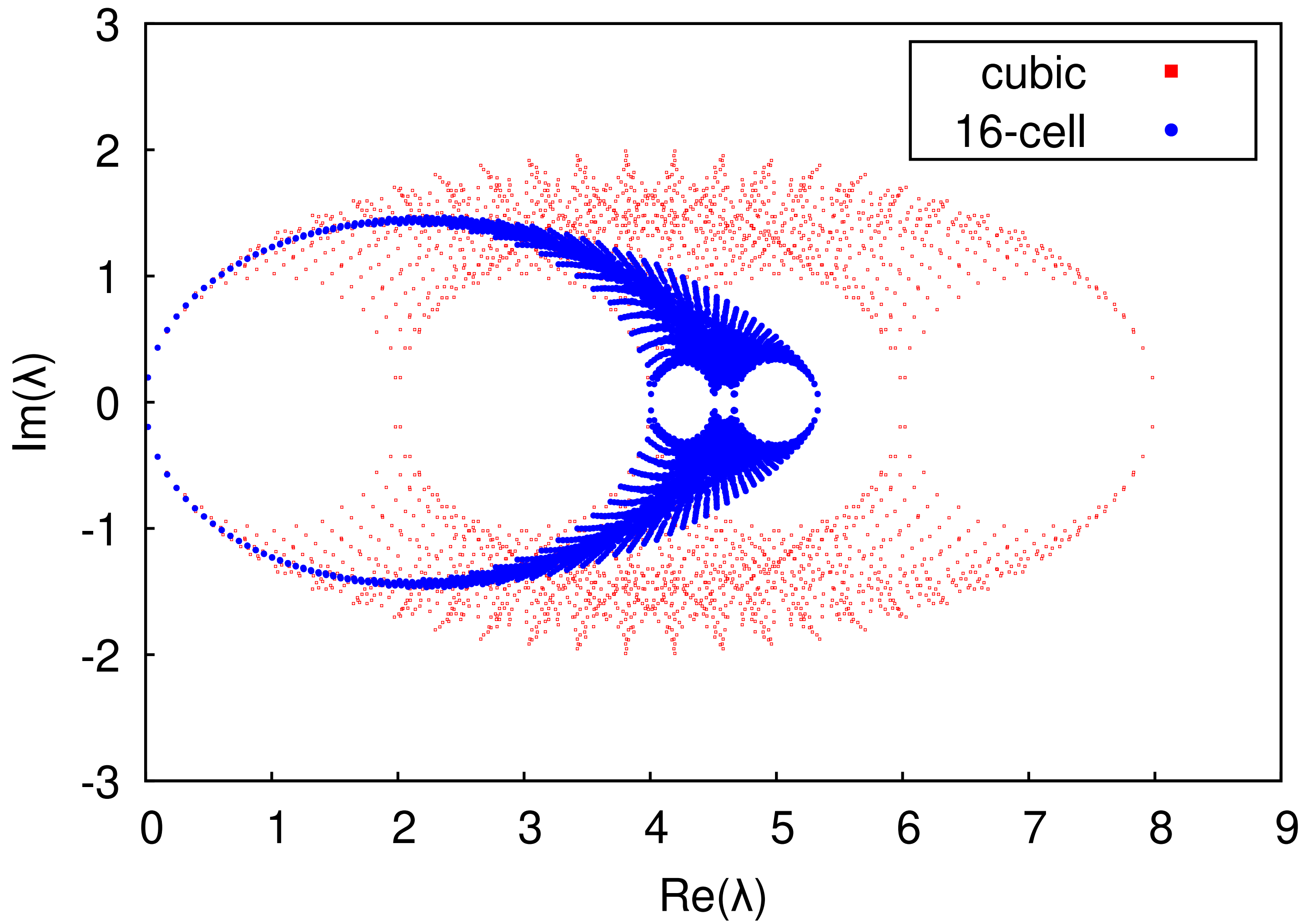
$$\begin{aligned}
\mathcal{O}_{cubic} &= \frac{7\pi^2}{720}\left(1 + \frac{248}{147}\frac{\pi^2}{N_t^2} + \frac{635}{147}\frac{\pi^4}{N_t^4}\dots\right) \\
\mathcal{O}_{16-cell} &= \frac{7\pi^2}{720}\left(1 + \frac{127}{980}\frac{\pi^4}{N_t^4} + \frac{73}{4158}\frac{\pi^6}{N_t^6} + \dots\right).
\end{aligned}$$

16-cell: leading correction $O(a^4)$

Fermions, free pressure



Fermions, free spectrum

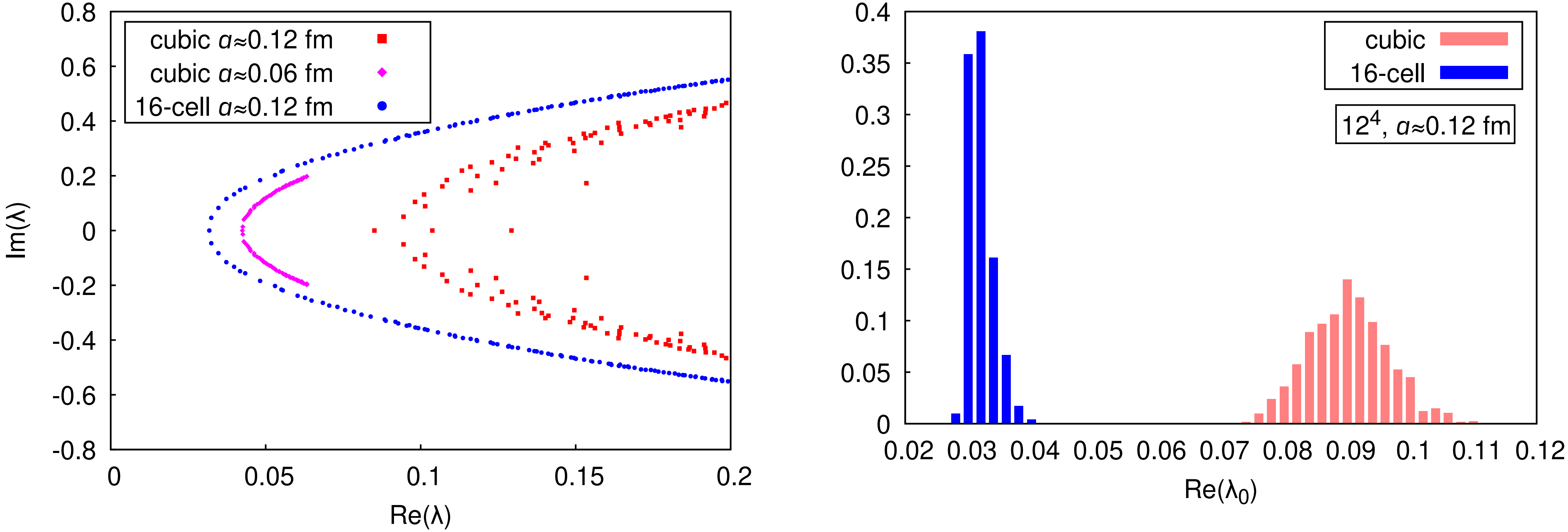


Close to a circle (ellipse) $\rightarrow$ Ginsparg-Wilson ... ???

Free improvement amazing, what about interacting case?

- **Dirac-spectrum, additive mass normalization (Wilson term)**
- **Index theorem, topological charge, chirality**

Additive mass normalization (Wilson term)



Fluctuates much less → real gain in dynamical simulations

Index theorem, topology, chirality

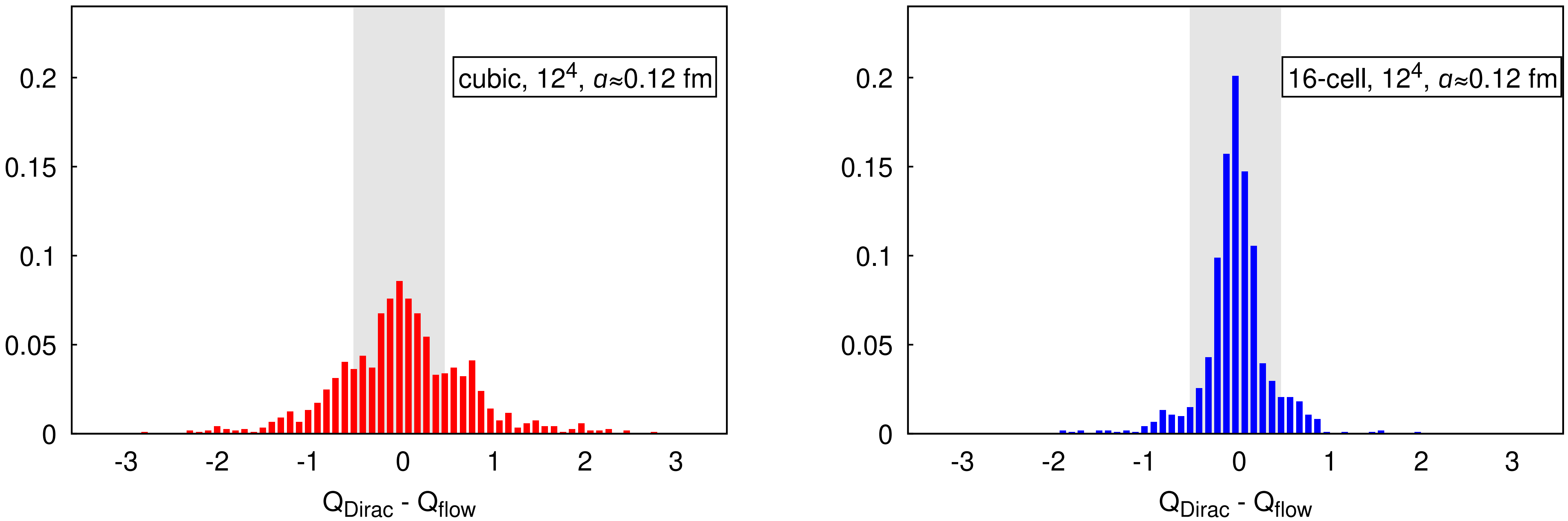
Wilson-term: no chiral symmetry

$n_{\pm}$: number of real low-lying eigenvalues with non-zero chirality

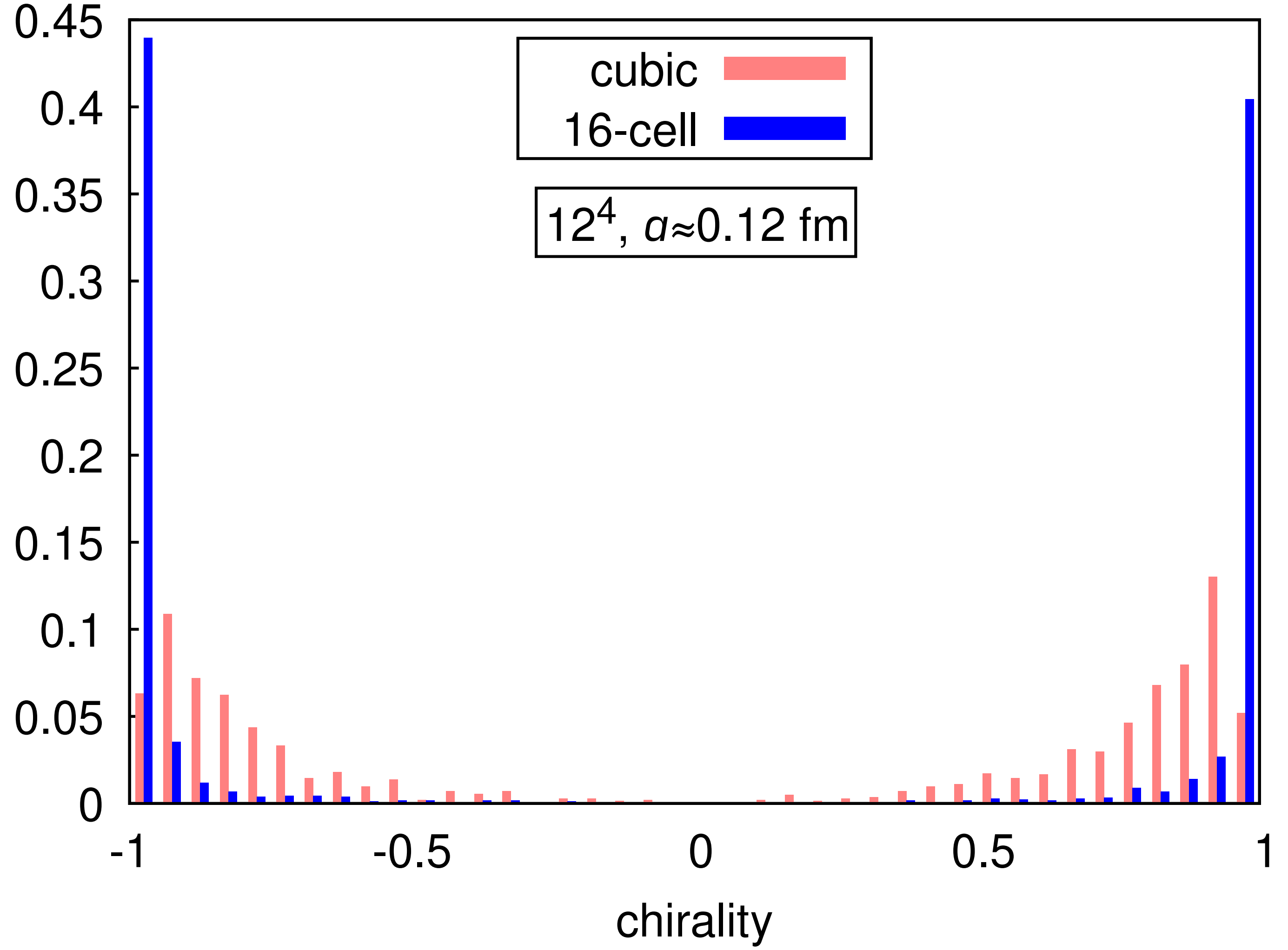
$$Q_{Dirac} = n_{+} - n_{-} \qquad\qquad Q_{flow} = Q_{flow}(t = t_0)$$

Of course $Q_{Dirac} \neq Q_{flow}$ and chirality $\neq \pm 1$

Index theorem, topology, chirality



Distribution of $Q_{Dirac} - Q_{flow}$



Distribution of real low-mode chiralities

Summary and outlook

- Cubic $\rightarrow$ 16-cell lattice: larger space-time symmetry
- Expect better scaling
- Confirmed in both gauge and fermion sector
- Currently: dynamical simulations – very promising

Summary and outlook

$3\times$ **more neighbors,** $2\times$ **more sites:** $6\times$ **cost**

HMC $\sim V^{3/2}$

$2\times$ **gain in lattice spacing** $\rightarrow$ $64\times$ **gain in HMC** $\rightarrow$

$\rightarrow$ **fixed** $6\times$ **cost penalty** $\rightarrow$

$\rightarrow$ **order of magnitude cheaper than cubic**

Plus more from better chiral symmetry

Thank you for your attention!