# Tensor networks for lattice field theories

From statistical mechanics to deconfinement in **gauge theories**

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arXiv:2602.13124

GHENT UNIVERSITY

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Setup — YOU KNOW THIS ONE

# Lattice field theory, in one slide

- Discretize spacetime: fields variables live on a lattice, and the path integral becomes a partition function.
- Every configuration carries a Boltzmann weight $e^{-S[\varphi]}$
- **Monte Carlo** performs importance sampling with these weights.

$$Z = \int D\varphi \, e^{-S[\varphi]}$$

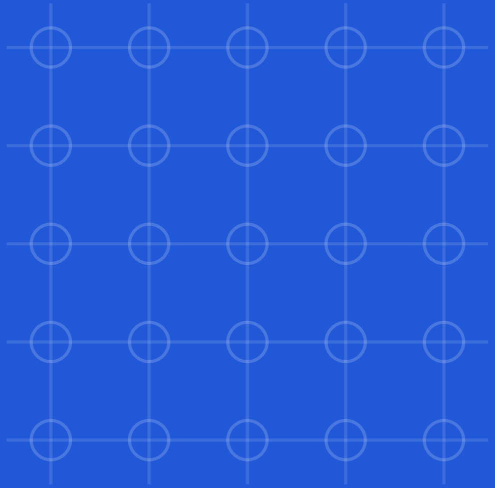
$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int D\varphi \, \mathcal{O} \, e^{-S[\varphi]}$$

SECTION 01

# 01

# Tensor networks

A diagrammatic language for sums over many indices.



# Tensors and diagrams


## CONTRACTION

A leg is an index; a connected leg is a summed index.

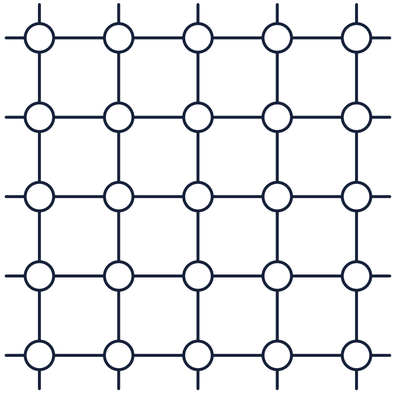

$$C_{ik} = \sum_j A_{ij} B_{jk}$$

01 — KEY IDEA

# A partition function is a tensor network

- The Boltzmann weight factorizes over local terms – one per bond or plaquette.
- Each local term $\to$ a local **tensor**; a shared variable $\to$ a connected **leg**.
- $Z$ = the entire network contracted to one number (a tensor-trace).

$$Z = \sum_{\{s\}} \prod_{\text{local}} W_{\text{local}} = \mathrm{tTr} \prod_{n} T^{(n)}$$

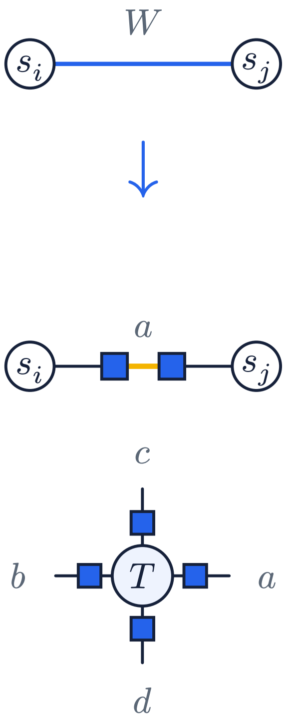


01 — WORKED EXAMPLE

# The 2D Ising model in one slide

- A weight $W(s_i, s_j) = e^{\beta s_i s_j}$ on every bond; split it: $W = \sqrt{W}\,\sqrt{W}$.
- Four half-weights meet at a site; summing the spin gives one rank-4 tensor per site.
- The result: a uniform network of identical $T$'s – contract it and you have $Z$.

$$T_{abcd} = \sum_s \left(\sqrt{W}\right)_{sa}\left(\sqrt{W}\right)_{sb}\left(\sqrt{W}\right)_{sc}\left(\sqrt{W}\right)_{sd}$$



# Gauge fields and fermions

$$e^{\beta\,\mathrm{Re}\,\chi(U)} = \sum_r F_r(\beta)\,\chi_r(U), \qquad U \in \mathrm{U}(1), \mathrm{SU}(N), \ldots$$

## CONTINUOUS GROUPS

### Irreps live on the bonds

Group integrals become discrete sums over irreps $r$ which live on the legs. The tensors are **block-sparse**.

## FERMIONS

### Grassmann tensor networks

$\overline{\psi}, \psi$ replaced with auxillary grassman variables; the tensors have a Fermionic $\mathbb{Z}_2$ symmetry. **No sign problem**.

01 — WHY BOTHER

# Why tensor networks?

### MONTE CARLO

**Sample configurations**

Needs a positive weight – the sign problem blocks finite density, real time, $\theta$-terms.

### TENSOR NETWORKS

**Contract the weight**

Evaluate $Z$ directly – no sampling, no sign problem. Direct access to free energy and universal data.

### TOGETHER

**Complementary**

Not a replacement for MC – a second probe that reaches where MC cannot.

SECTION 02

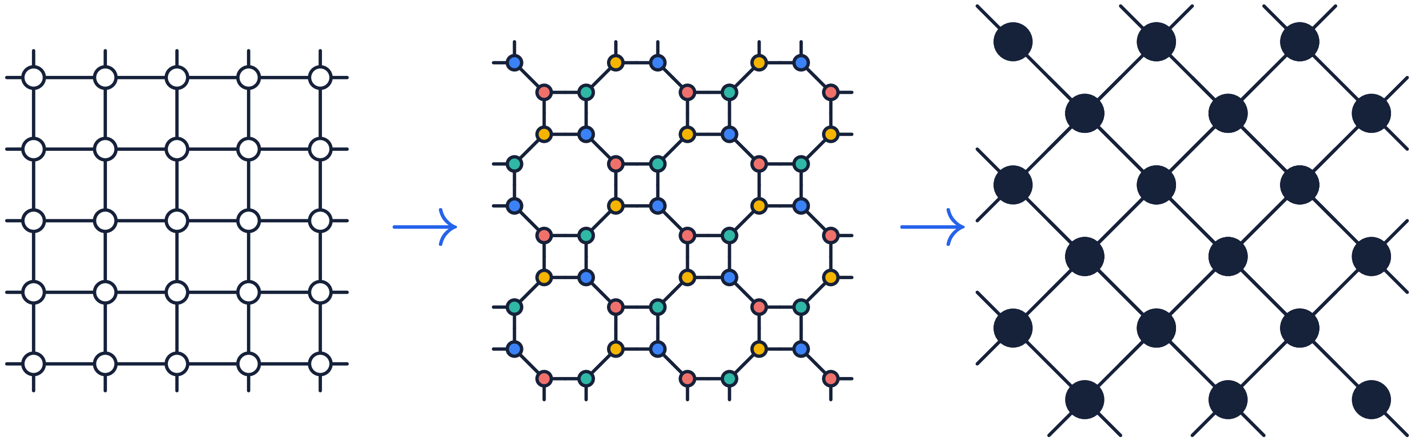
02

# Tensor network renormalization

Contract by coarse-graining – and read off universal physics.

# Tensor renormalization group (TRG)

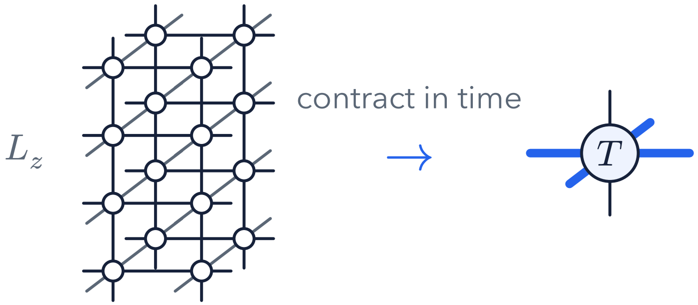
- Exact contraction is exponential – so coarse-grain instead.
- **TRG:** split each tensor by SVD, truncating to the $\chi$ largest values.
- Fuse the pieces into one coarser tensor per plaquette.
- Iterate: $2^s$ tensors collapse to one after $s$ steps.



# CFT data from the transfer matrix

- Tracing one direction of the fixed-point tensor builds a **renormalized transfer matrix** – a cylinder partition function.
- At criticality its spectrum is the CFT data: **central charge** $c$ and **scaling dimensions** $\Delta_i$ (PhysRevB.80.155131).
- Ratios of partition functions count ground state degeneracies:

$$X = \frac{Z_{L\times L}^2}{Z_{L\times 2L}} = \text{number of ground states}$$




**Open source: TNRKit.jl**

SciPost Phys. Codebases 77 (2026)

SECTION 03 · THIS WORK

# 03

# Thermal TNR for $\mathbb{Z}_N$ gauge theories

Finite-temperature deconfinement in (2+1)D · arXiv:2602.13124

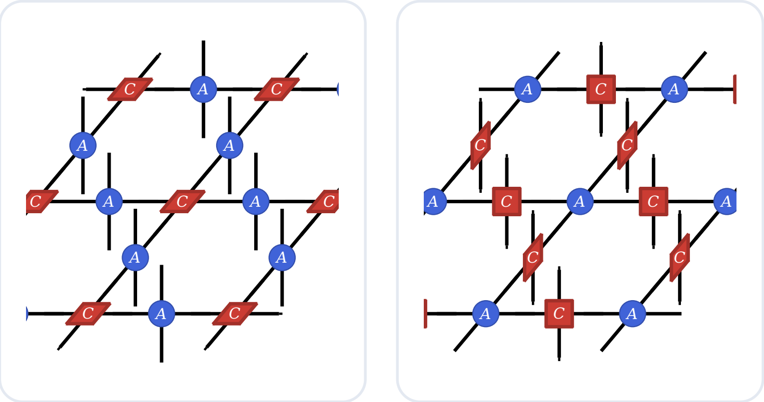
# $\mathbb{Z}_N$ lattice gauge theory as a 3D network

- Wilson action on a $(2+1)$D lattice; links $U_{n,\mu} \in \mathbb{Z}_N$, temporal extent $L_z = 1/T$.

$$S = \beta \sum_{n,\mu<\nu} \left[1 - \mathrm{Re}\, U_{n,\mu\nu}\right]$$

- Character expansion $e^{\beta\,\mathrm{Re}\,U} = \sum_r F_r(\beta)\chi_r(U) \to$ link tensors $A$ (Gauss law, a $\delta$) + plaquette tensors $C$ (the weight).
- Stack of **electric** (Gauss projector) and **magnetic** (plaquette) PEPO layers – bond dimension $N$

MAGNETIC + ELECTRIC PEPOS

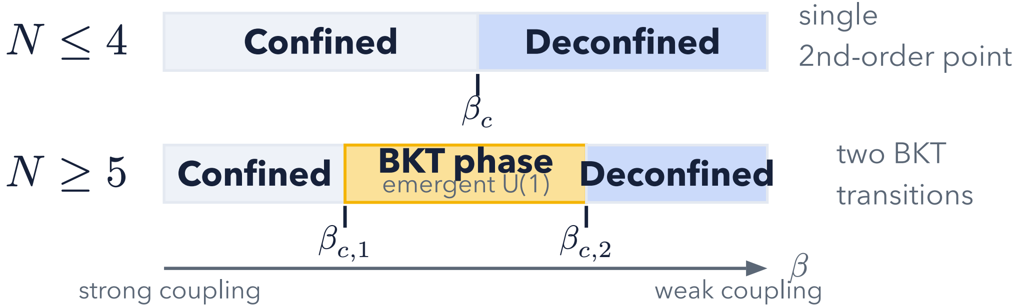



blue: link tensors $A$ · red: plaquette tensors $C$

**SVETITSKY-YAFFE CONJECTURE**

The finite-$T$ transition of a $(d+1)$-D gauge theory sits in the universality class of a $d$-D spin model whose global symmetry is the **center** of the gauge group.

$\mathbb{Z}_N$ gauge in (2+1)D $\Leftrightarrow$ 2D $N$-state clock model:



## GAUGE

### $\mathbb{Z}_N$ gauge theory

Links on the lattice, Gauss law enforced by $A$-tensors. Temporal gauge $U_{n,z} = 1$ removes the electric layer up to one projector.

## DUAL SPIN

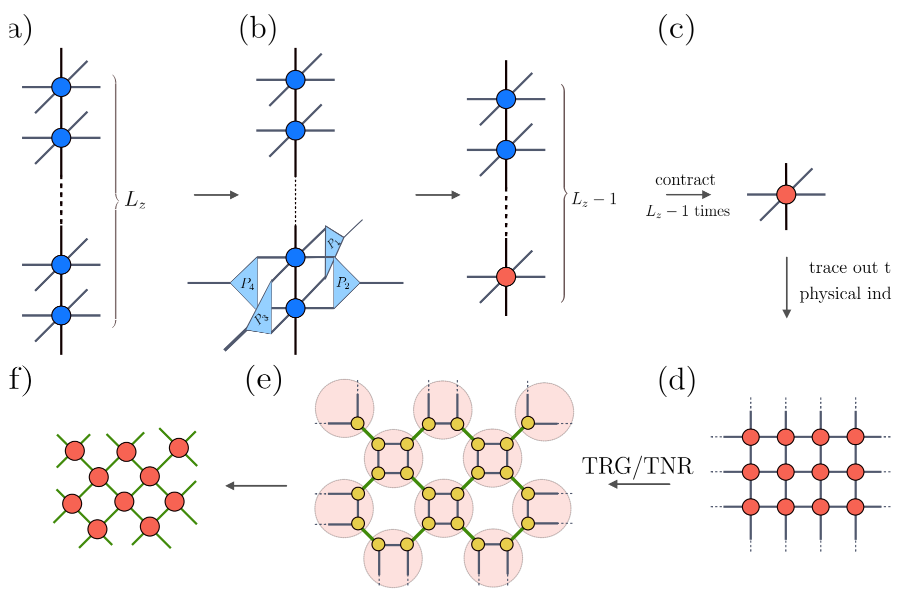
### $N$-state clock model

Dual spins solve the Gauss constraint **exactly** – smaller tensors, better accuracy at the same $\chi$. Strong and weak coupling swap roles.

Confined phase $\leftrightarrow$ $\mathbb{Z}_N$-symmetric spins · deconfined phase $\leftrightarrow$ spontaneously broken $\mathbb{Z}_N$ – we simulate **both** and cross-check.

# The thermal TNR algorithm

- Contract the **temporal** direction first, with optimal loop projectors – cost grows linearly in $L_z$, not exponentially.
- The 3D network collapses to an effective 2D one; coarse-grain it with **Loop-TNR** / **BW-TRG**.
- ★ Renormalized transfer matrix $\rightarrow c$, $\Delta_i$, Gu–Wen ratio $X$.



SECTION 04

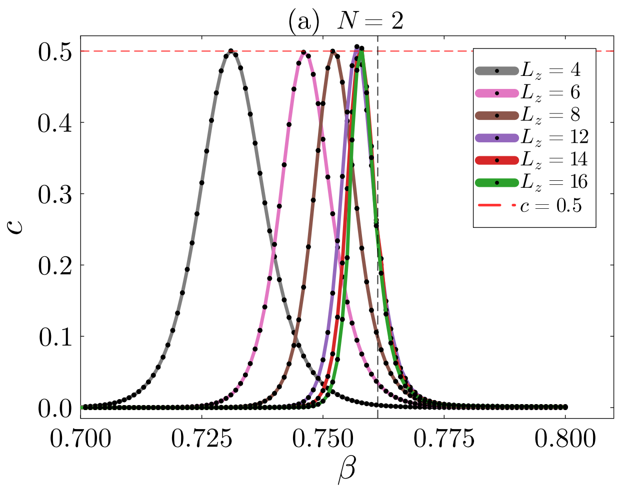
# 04

# Results

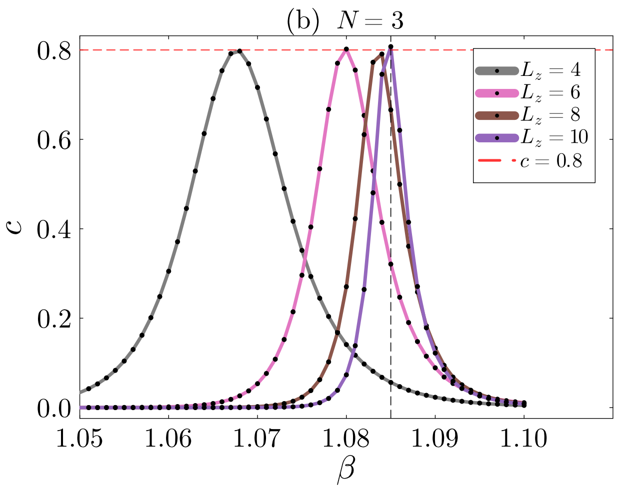
Svetitsky–Yaffe verified · emergent U(1) · zero-temperature extrapolation

# Central charge

## N = 2 · C → 1/2 (2D ISING)




## N = 3 · C → 4/5 (3-STATE POTTS)

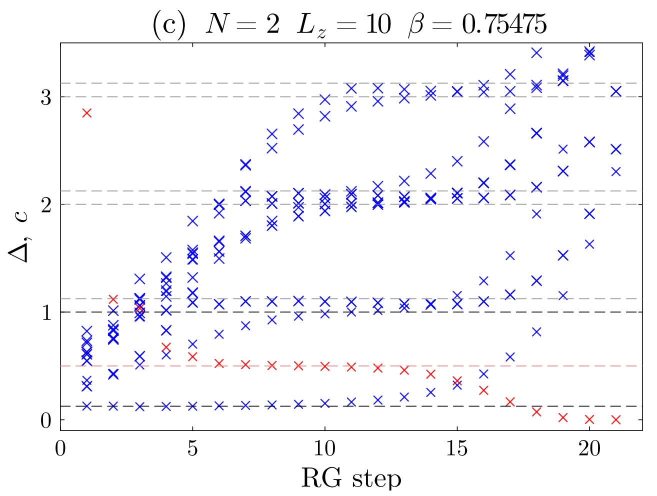



$c$ is nonzero only at criticality – its peak pins down $\beta_c(L_z)$; dashed lines: $T = 0$ Monte Carlo values.

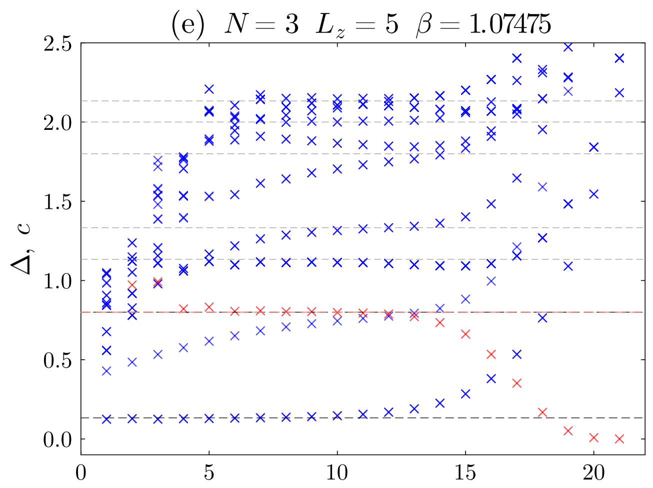
04 — RESULTS – UNIVERSALITY

# Scaling dimensions

**N = 2, $L_z$ = 10 · ISING TOWER**




**N = 3, $L_z$ = 5 · POTTS TOWER**




The full spectrum $\Delta_i$ (blue) and $c$ (red) sit on the known CFT values (dashed) across RG steps.

# An emergent U(1) intermediate phase

- $c = 1$ plateau between two BKT transitions – a Tomonaga-Luttinger liquid, as for the 5-state clock model.
- Compactification radius ($1/K = 2R^2$) crosses $R = 2/5$ ($\beta_{c,1}$) and $R = 1/2$ ($\beta_{c,2}$) – both BKT points located.

**$L_z = 2$**

**1.624(2)**

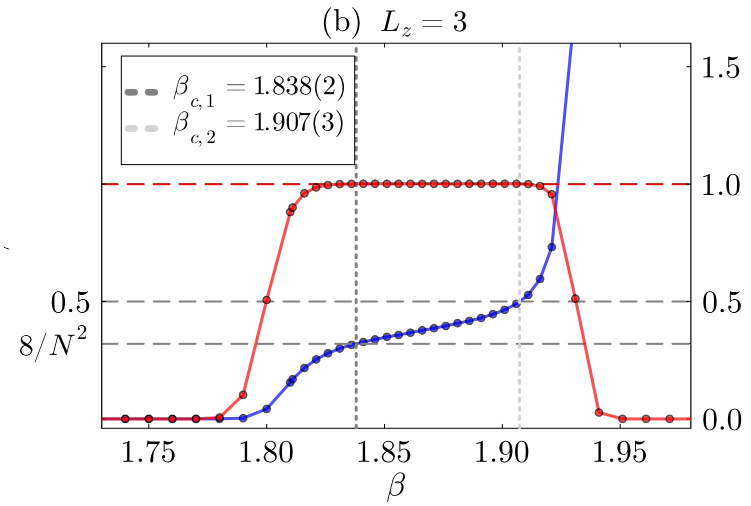
$\beta_{c,1}$ · MC: 1.617(2)

**$L_z = 4$**

**1.944(2)**

$\beta_{c,1}$ · MC: 1.943(2)

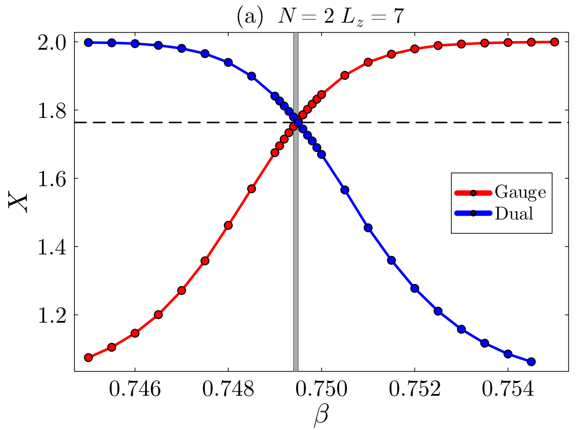
**C AND $1/K = 2R^2$ ACROSS THE WINDOW, $L_z = 3$**



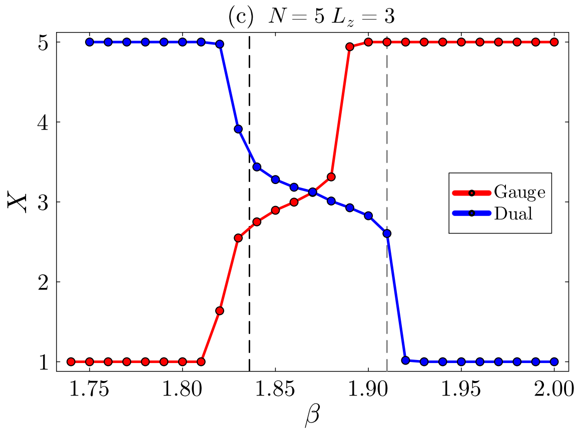
04 — RESULTS – DEGENERACY

# Duality

### N = 2 · A SHARP STEP



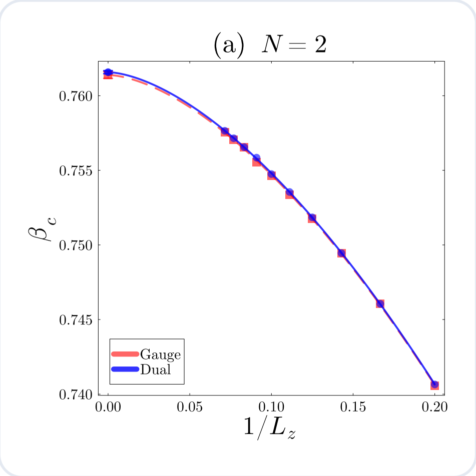

### N = 5 · CONTINUOUS (BKT)




$X$ counts ground states: 1 in the confined phase, $N$ in the deconfined phase – mirrored between gauge (red) and dual spin (blue) representations, exactly as duality demands.

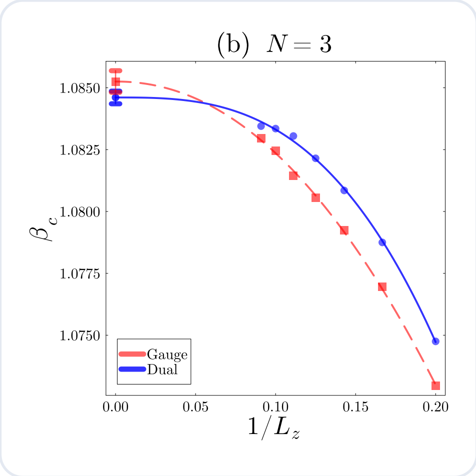
# Extrapolating to zero temperature

**N = 2**




$\beta_c^* = 0.76139(13)$ · MC: 0.761395(4) – 4-digit agreement

**N = 3**



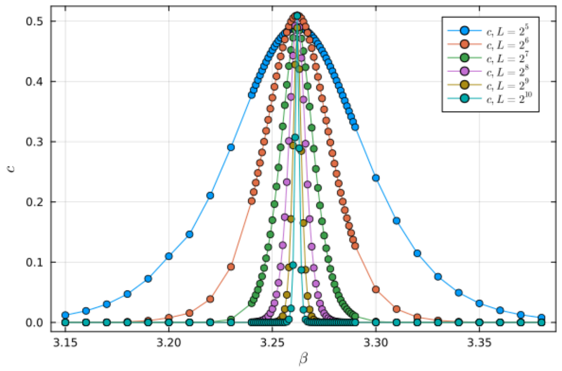

$\beta_c^* = 1.08455(8)$ · MC: 1.084314(8) · $\nu \approx 1/3$ (first-order Potts)

Fit $\beta_c(L_z) = \beta_c^* + c\,L_z^{-1/\nu}$ – the **first** $T = 0$ point from TNR methods.

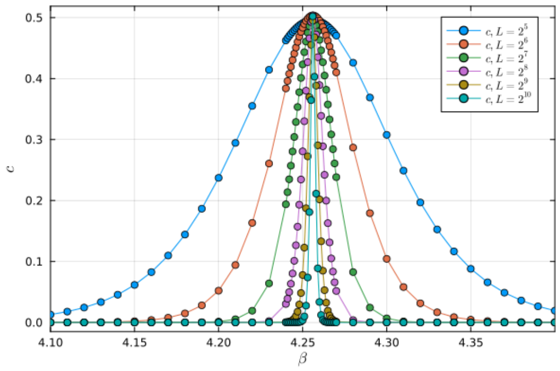
04 — SU(2) · IN PROGRESS

# Toward non-Abelian: SU(2) in (2+1)D

**Central charge $c$ · peak at $\beta_c \approx 3.26$**




**Central charge $c$ · peak at $\beta_c \approx 4.25$**




- SY conjecture: SU(2) in (2+1)D $\to$ 2D Ising – $c = 1/2$ at the transition.
- Critical couplings $\beta_c \approx 3.26, 4.25$ roughly match Monte Carlo. *Preliminary.*

Wrap-up — WHERE THIS LEAVES US

# Summary & outlook

### TAKEAWAYS

- LFT $\to$ a tensor network $\to$ coarse-grain. Sign-problem-free, with **direct access to CFT data**.
- TTNR: contract time first with optimal projectors – finite-$T$ gauge theories at linear cost in $L_z$.
- $\mathbb{Z}_N$ in (2+1)D: Svetitsky–Yaffe verified for $N = 2, 3, 5$; emergent U(1) at $N = 5$; $T = 0$ points to MC precision.

### NEXT

- Entanglement filtering for 3D networks – larger $L_z$, smaller $\chi$.
- $\mathrm{SU}(N)$ gauge theories at finite temperature via the same character expansion.
- Lattice gauge theories with fermions

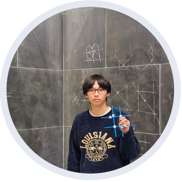
**Team** — ACKNOWLEDGEMENTS

# The people behind this work

## THERMAL TENSOR NETWORKS

Deconfinement in (2+1)D $\mathbb{Z}_N$ lattice gauge theories

arXiv:2602.13124



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## TNRKIT.JL

A practical introduction to tensor network renormalization

SciPost Phys. Codebases 77 (2026)

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LATTICE 2026

TNRKit.jl

# Thank you

From statistical mechanics to deconfinement in gauge theories

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