Simulating lattice gauge theories on hybrid qubit-qumode quantum platforms

Lattice 2026, University of Maryland, College Park, MD

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Stony Brook University

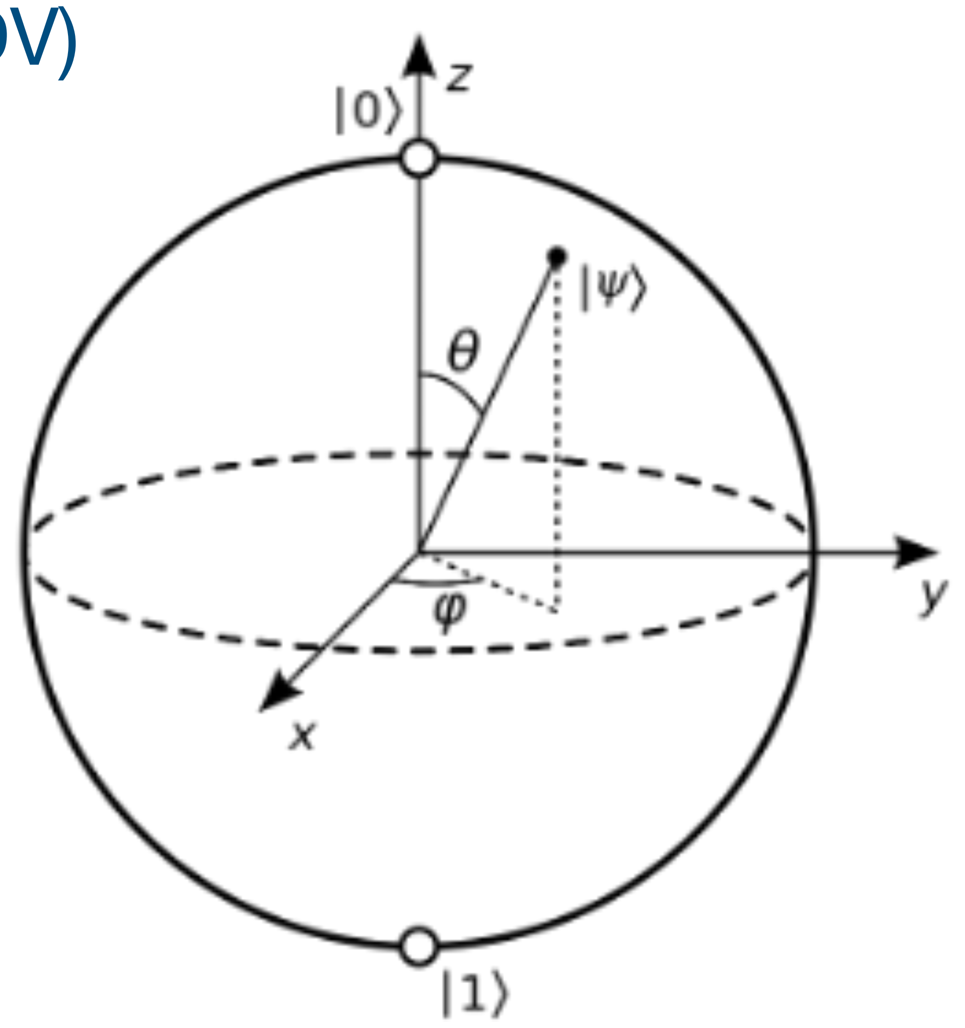
Quantum resources

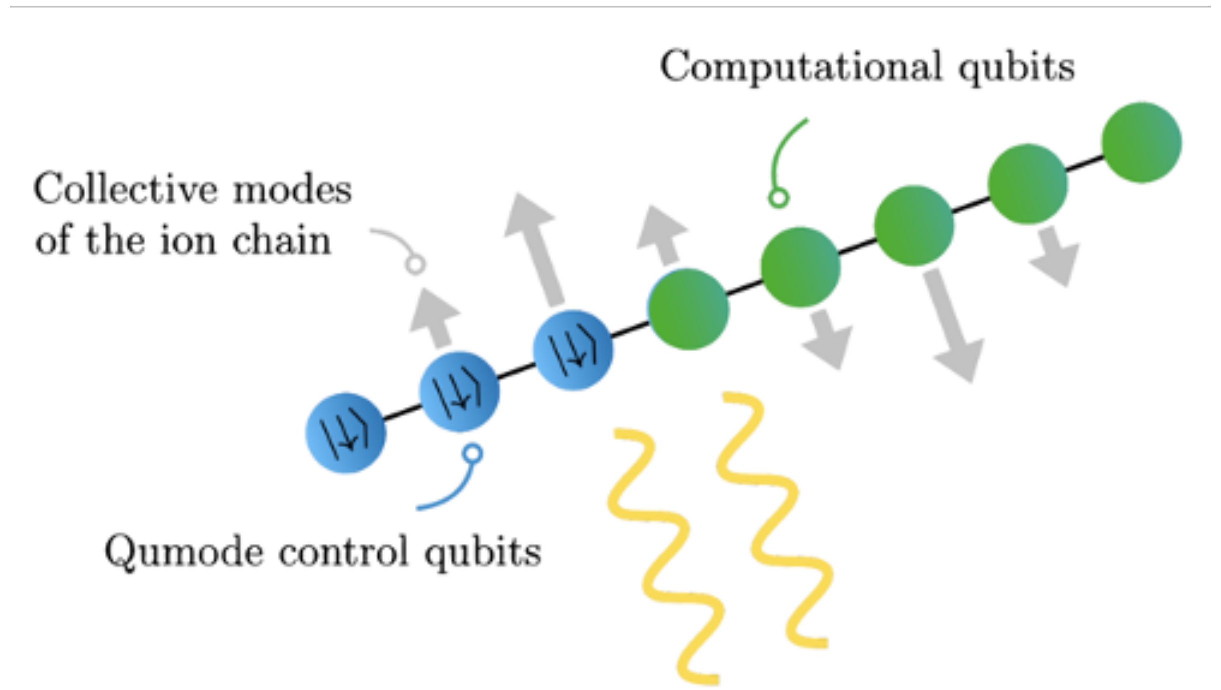
The elementary units for quantum computing

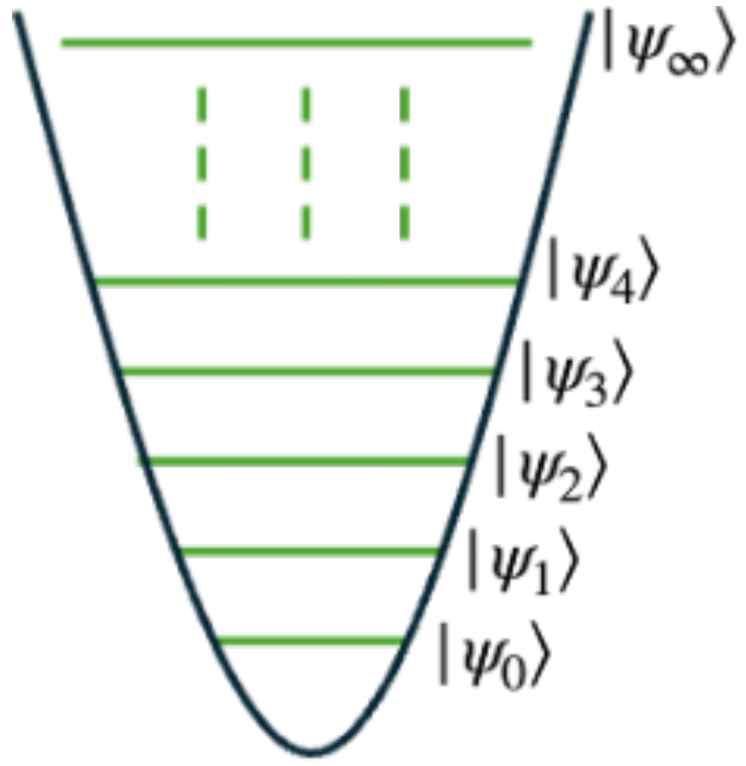
Gate based digital quantum computing

- Qubits (Qudits $2 \to d$)
  - Realization with: superconducting circuits, cold atoms, trapped ions, topological qubits,…
  - **Two-dimensional** Hilbert space per qubit
  - Digital gate-based computing with **discrete variables** (DV)

- Qumodes ($d \to \infty$)
  - Realization with: photonics, trapped ions, superconducting circuits,…
  - **Infinite-dimensional** Hilbert space per qumode
  - Gate based with **continuous variables** (CV)



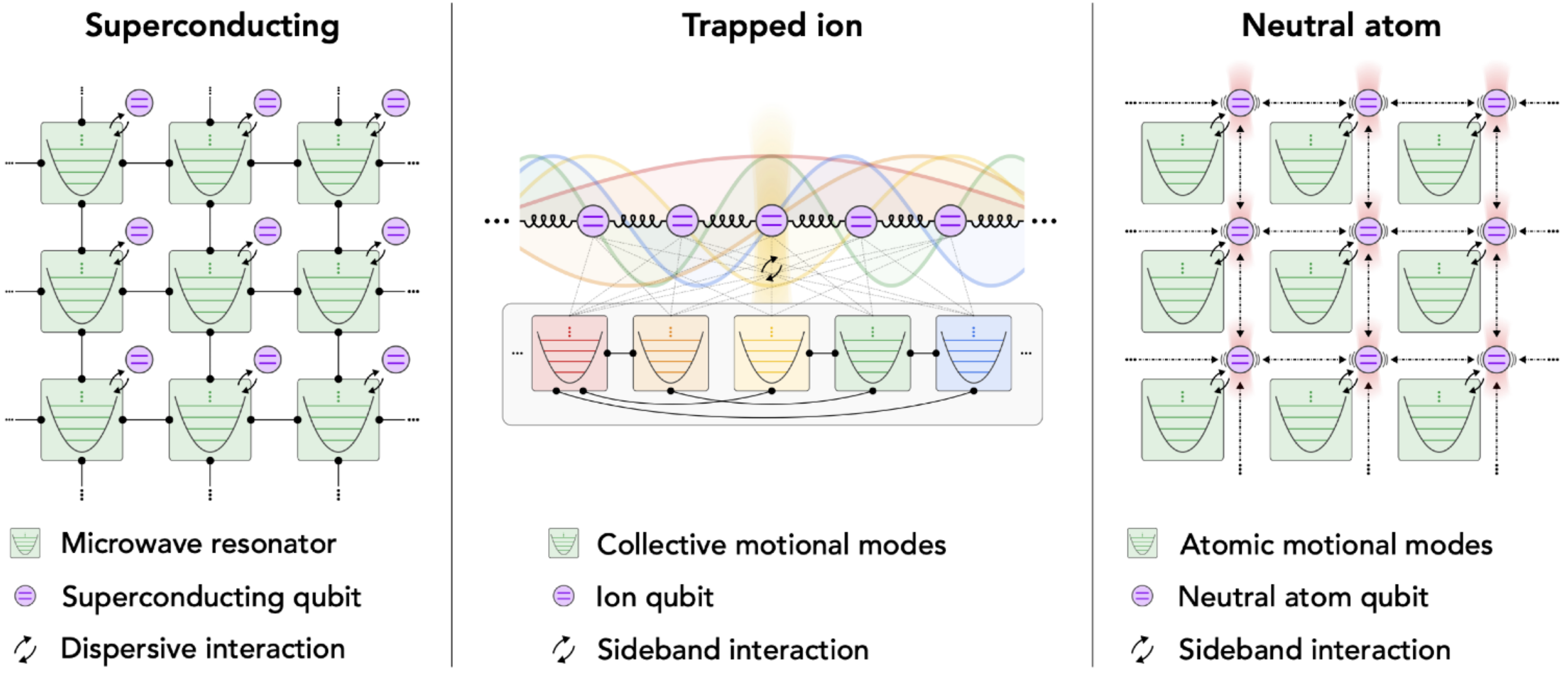




Araz, Grau, Montgomery, Ringer `24

New paradigm: Hybrid architectures

Girvin, Wiebe et al `24

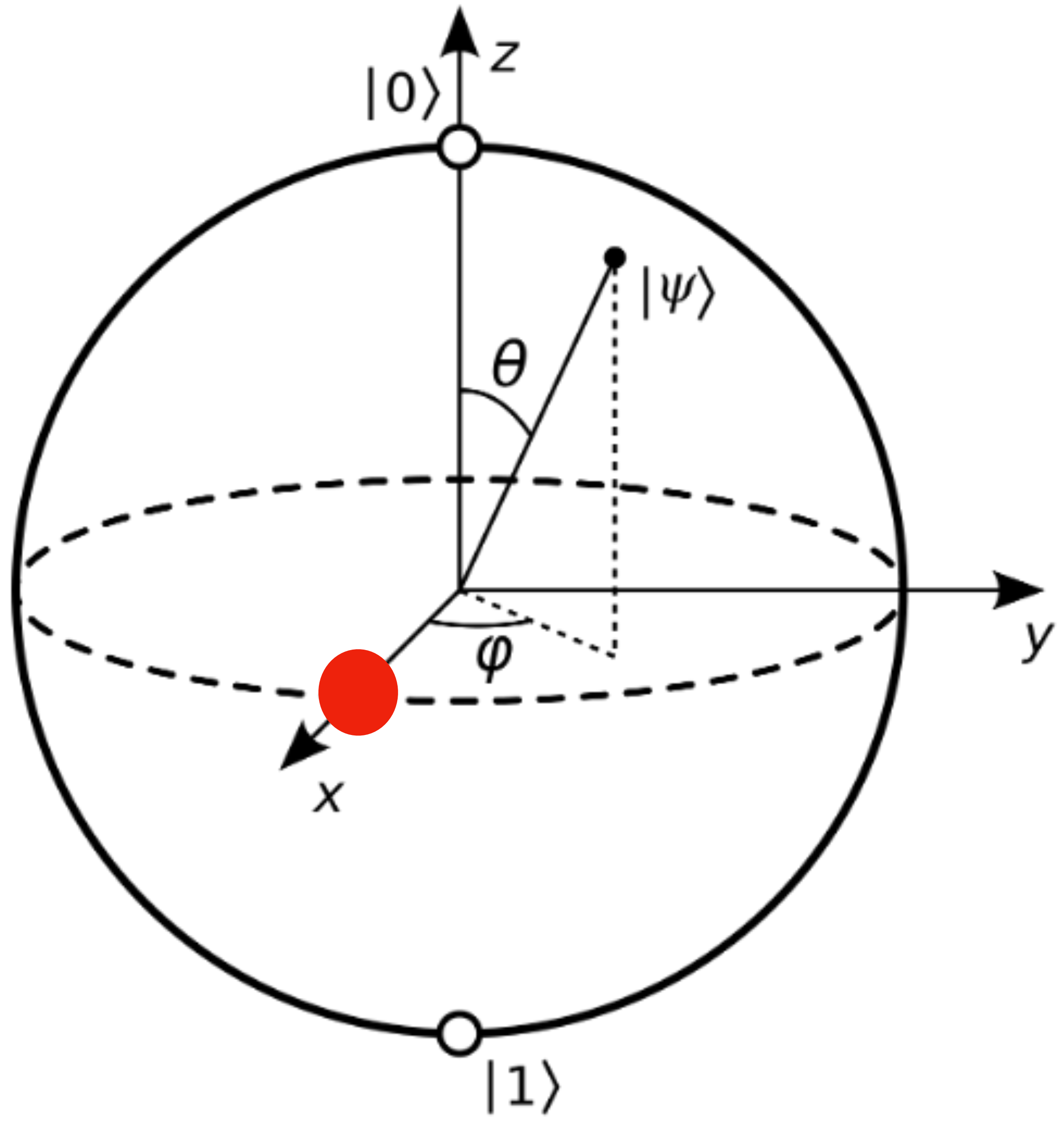


Platform dependence: Native gates, Coherence time, Connectivity, …

Qubit states and operations

Gate type	Operation	Short	Operator
Qubit	Pauli operators		σ^i
	Rotation	$\mathrm{R}_i(\theta)$	$e^{i\theta\sigma^i/2}$
	Controlled NOT	CNOT	$e^{i\frac{\pi}{4}(\mathbb{I}_1-Z_1)(\mathbb{I}_2-X_2)}$
Qumode	Rotation	$\mathrm{R}(\theta)$	$e^{i\theta\hat{a}^\dagger\hat{a}}$
	Fourier	F	$e^{i\frac{\pi}{2}\hat{a}^\dagger\hat{a}}$
	Displacement	$\mathrm{D}(z)$	$e^{z\hat{a}^\dagger-z^*\hat{a}}$
	Single-mode squeezing	$\mathrm{S}(z)$	$e^{(z^*\hat{a}\hat{a}-z\hat{a}^\dagger\hat{a}^\dagger)/2}$
	Beam splitter	$\mathrm{BS}(z)$	$e^{z\hat{a}^\dagger\hat{b}-z^*\hat{a}\hat{b}^\dagger}$
	Kerr	$\mathrm{K}(z)$	$e^{iz(\hat{a}^\dagger\hat{a})^2}$
	Cross-Kerr	$\mathrm{CK}(z)$	$e^{iz\hat{a}^\dagger\hat{a}\hat{b}^\dagger\hat{b}}$
	Quadratic phase	$\mathrm{P}(\theta)$	$e^{i\frac{\theta}{2}\hat{q}^2}$
	Cubic phase	$\mathrm{V}(\theta)$	$e^{i\frac{\theta}{3}\hat{q}^3}$
Hybrid	Red sideband/Jaynes Cummings	$\mathrm{RSB}(z)$	$e^{iz\hat{a}X^++iz^*\hat{a}^\dagger X^-}$
	Blue sideband/Anti-Jaynes Cummings	$\mathrm{BSB}(z)$	$e^{iz\hat{a}^\dagger X^++iz^*\hat{a}X^-}$
	Controlled rotation	$\mathrm{CR}(\theta)$	$e^{i\theta Z\hat{a}^\dagger\hat{a}}$
	Controlled displacement	$\mathrm{CD}(z)$	$e^{Z(z\hat{a}^\dagger-z^*\hat{a})}$
	Controlled squeezing	$\mathrm{CS}(z)$	$e^{Z(z^*\hat{a}\hat{a}-z\hat{a}^\dagger\hat{a}^\dagger)/2}$
	Controlled beam splitter	$\mathrm{CBS}(z)$	$e^{Z(z\hat{a}^\dagger\hat{b}-z^*\hat{a}\hat{b}^\dagger)}$

Gate = Unitary operation



Superposition

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Entanglement

$$|\mathrm{Bell}\rangle = \frac{|00\rangle + |1\rangle}{\sqrt{2}}$$

Lloyd and Braunstein `99

Qumode states and operations

Gate type	Operation	Short	Operator
Qubit	Pauli operators		σ^i
	Rotation	$\mathrm{R}_i(\theta)$	$e^{i\theta\sigma^i/2}$
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Qumode	Rotation	$\mathrm{R}(\theta)$	$e^{i\theta\hat{a}^\dagger\hat{a}}$
	Fourier	F	$e^{i\frac{\pi}{2}\hat{a}^\dagger\hat{a}}$
	Displacement	$\mathrm{D}(z)$	$e^{z\hat{a}^\dagger - z^*\hat{a}}$
	Single-mode squeezing	$\mathrm{S}(z)$	$e^{(z^*\hat{a}\hat{a} - z\hat{a}^\dagger\hat{a}^\dagger)/2}$
	Beam splitter	$\mathrm{BS}(z)$	$e^{z\hat{a}^\dagger\hat{b} - z^*\hat{a}\hat{b}^\dagger}$
	Kerr	$\mathrm{K}(z)$	$e^{iz(\hat{a}^\dagger\hat{a})^2}$
	Cross-Kerr	$\mathrm{CK}(z)$	$e^{iz\hat{a}^\dagger\hat{a}\,\hat{b}^\dagger\hat{b}}$
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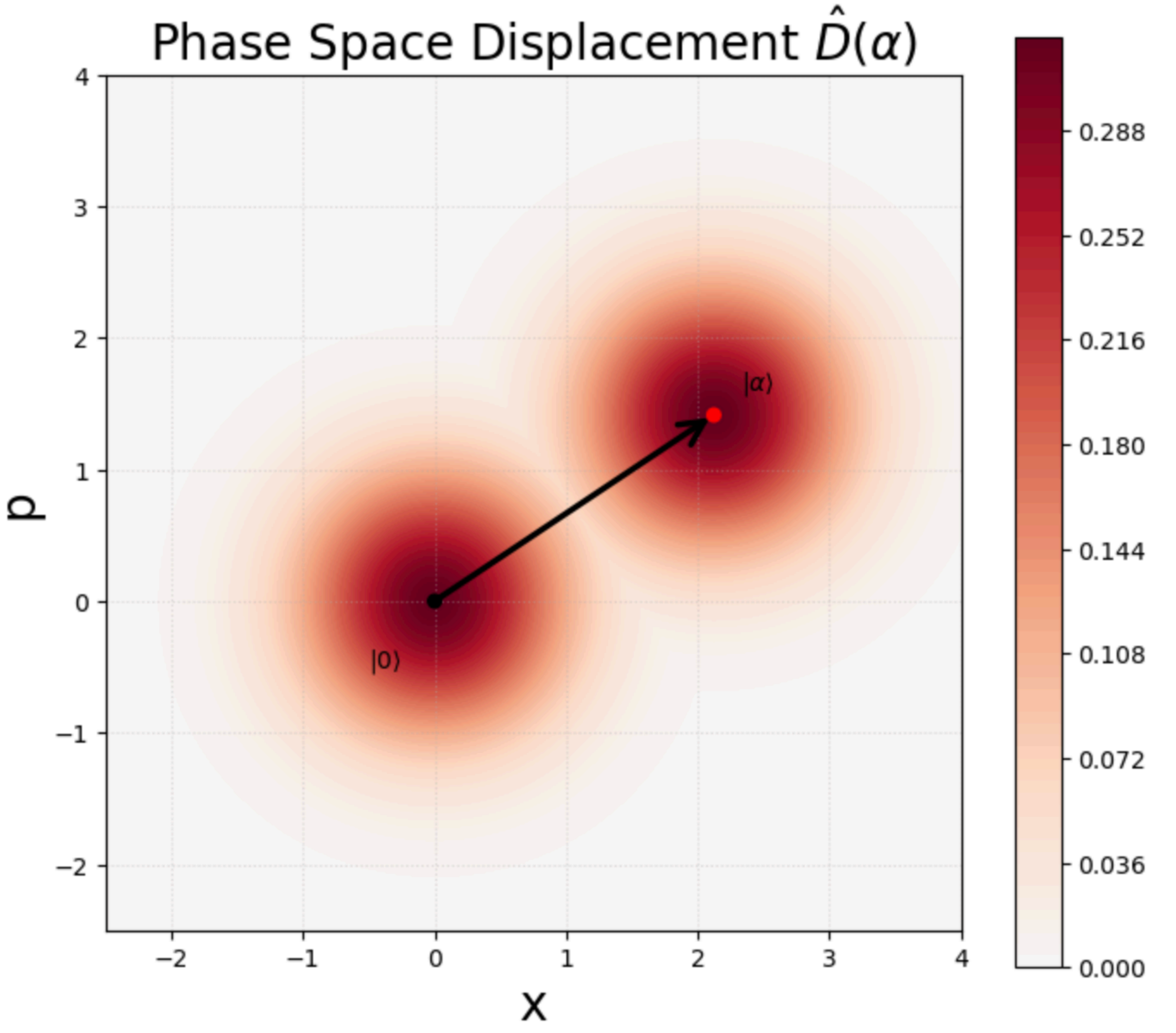
Wigner function



$$a, a^\dagger \Longleftrightarrow \hat{q}, \hat{p}$$

Ladder operators **Quadratures**

$$|n\rangle \propto \left(a^\dagger\right)^n |0\rangle$$

Hybrid states and operations

Gate type	Operation	Short	Operator
Qubit	Pauli operators		σ^i
	Rotation	$\mathrm{R}_i(\theta)$	$e^{i\theta\sigma^i/2}$
	Controlled NOT	CNOT	$e^{i\frac{\pi}{4}(\mathbb{1}_1-Z_1)(\mathbb{1}_2-X_2)}$
Qumode	Rotation	$\mathrm{R}(\theta)$	$e^{i\theta\hat{a}^\dagger\hat{a}}$
	Fourier	F	$e^{i\frac{\pi}{2}\hat{a}^\dagger\hat{a}}$
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	Kerr	$\mathrm{K}(z)$	$e^{iz(\hat{a}^\dagger\hat{a})^2}$
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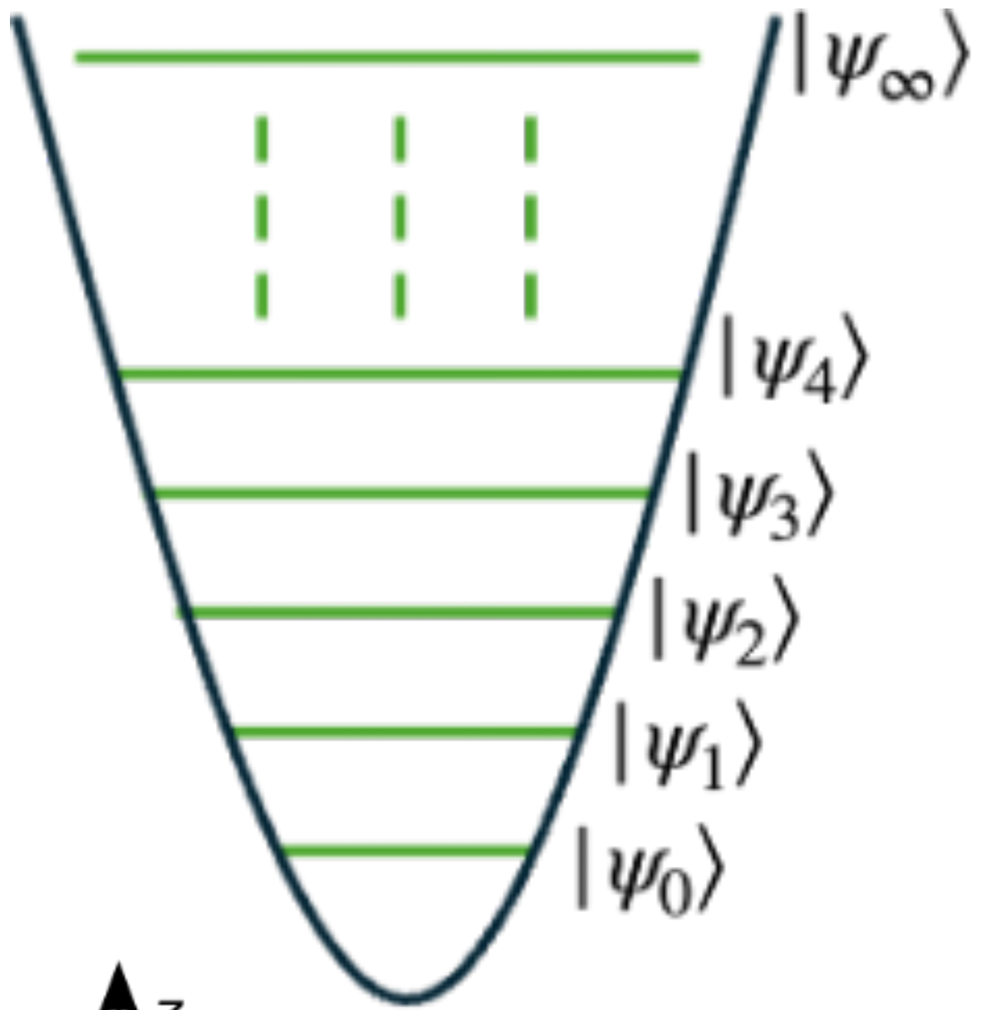
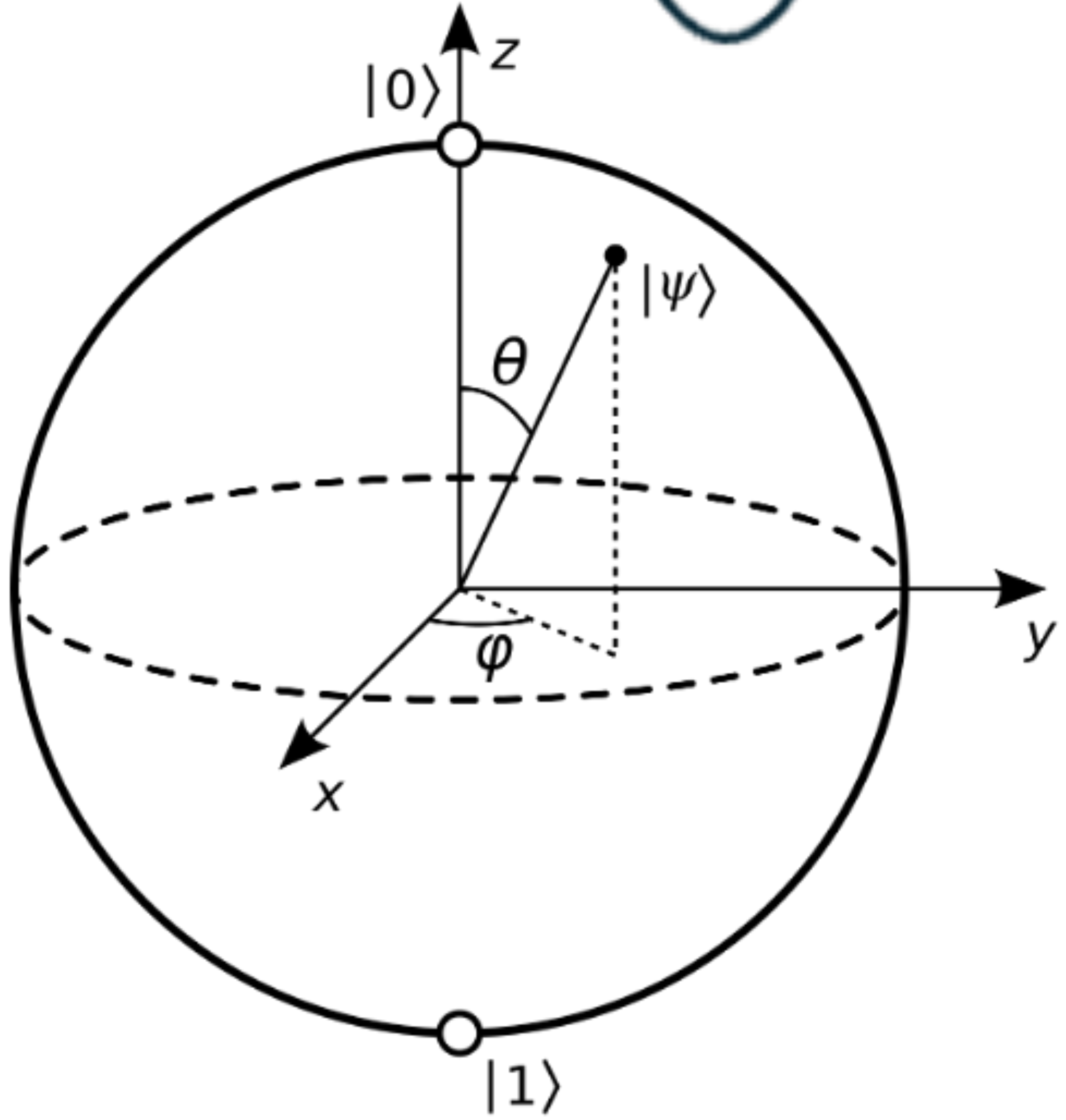
Lloyd and Braunstein `99





Qubit and qumode universality

There exists a minimal set of gates from which any other gate (unitary) can be built

Qubit only	Hybrid	Qumode only
$\text{Universal}\begin{cases}\text{Clifford}\begin{cases}S = Z^{1/2}\\ H\\ \text{CNOT}\end{cases}\\ \text{Magic}\left\{T = e^{-i\frac{\pi}{8}Z}\right.\end{cases}$	$\text{Universal}\begin{cases}\mathrm{CD}(\alpha) = \mathrm{e}^{Z\otimes(\alpha a^\dagger - \alpha^* a)}\\ R_{\sigma_\varphi}(\theta)\ \text{(qubit rot.)}\\ \mathrm{BS}(\theta,\phi)\end{cases}$ NEW $e^{-i\theta\cos(P(\hat{x},\hat{p}))\otimes\sigma_j}$	$\text{Universal}\begin{cases}\text{Gaussian}\begin{cases}D(\alpha)\\ R(\theta)\\ S(\zeta)\\ \mathrm{BS}(\theta,\phi)\end{cases}\\ \text{Non}-\text{Gaussian}\left\{V = e^{-i\theta\hat{q}^3}\right.\end{cases}$
Pauli string σ_i	$\sigma_i \otimes P(\hat{x},\hat{p})$	Polynomial $P(\hat{x},\hat{p})$
Barenco et al, Gottesman, Knill	Girvin, Wiebe et al. \`24	Lloyd, Braunstein \`97

Non-polynomial (trigonometric) gates

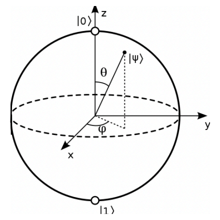
RT, Rico, Ringer et al. \`25

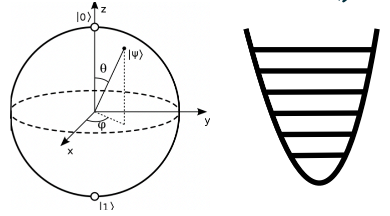
Mapping physical models to quantum resources

Gauge theories with hybrid platforms

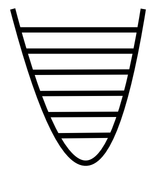
Simulate quarks and gluons with different quantum resources

QCD Lagrangian $\mathcal{L} = \bar{\psi}\left(i\partial^{\mu}\gamma_{\mu} - m\right)\psi + g_s \bar{\psi}\gamma^{\mu}T_a\psi A_{\mu}^{a} - \frac{1}{4}F_{a}^{\mu\nu}F_{\mu\nu}^{a}$

Qubits 



Qubit-qumode interaction

 Qumodes

$$\mathcal{H}_{\text{qumode}}^{m} \otimes \mathcal{H}_{\text{qubit}}^{n}$$

Hybrid qubit/qumode approach

Algebra representations with quantum resources

Bosonic CCR

* plus gauge group algebra

$$\left[\hat{\pi}_n, \hat{\phi}_{n'}\right] = -i\delta_{nn'}$$

There is no finite dimensional faithful representation of the Weyl/CCR algebra

Fermionic CAR

$$\left\{\psi_n, \psi^{\dagger}_{n'}\right\} = \delta_{nn'}$$

There exists finite dimensional faithful representations of the CAR algebra

QED in 2+1 dimensions

Staggered QED in 2+1 dimensions

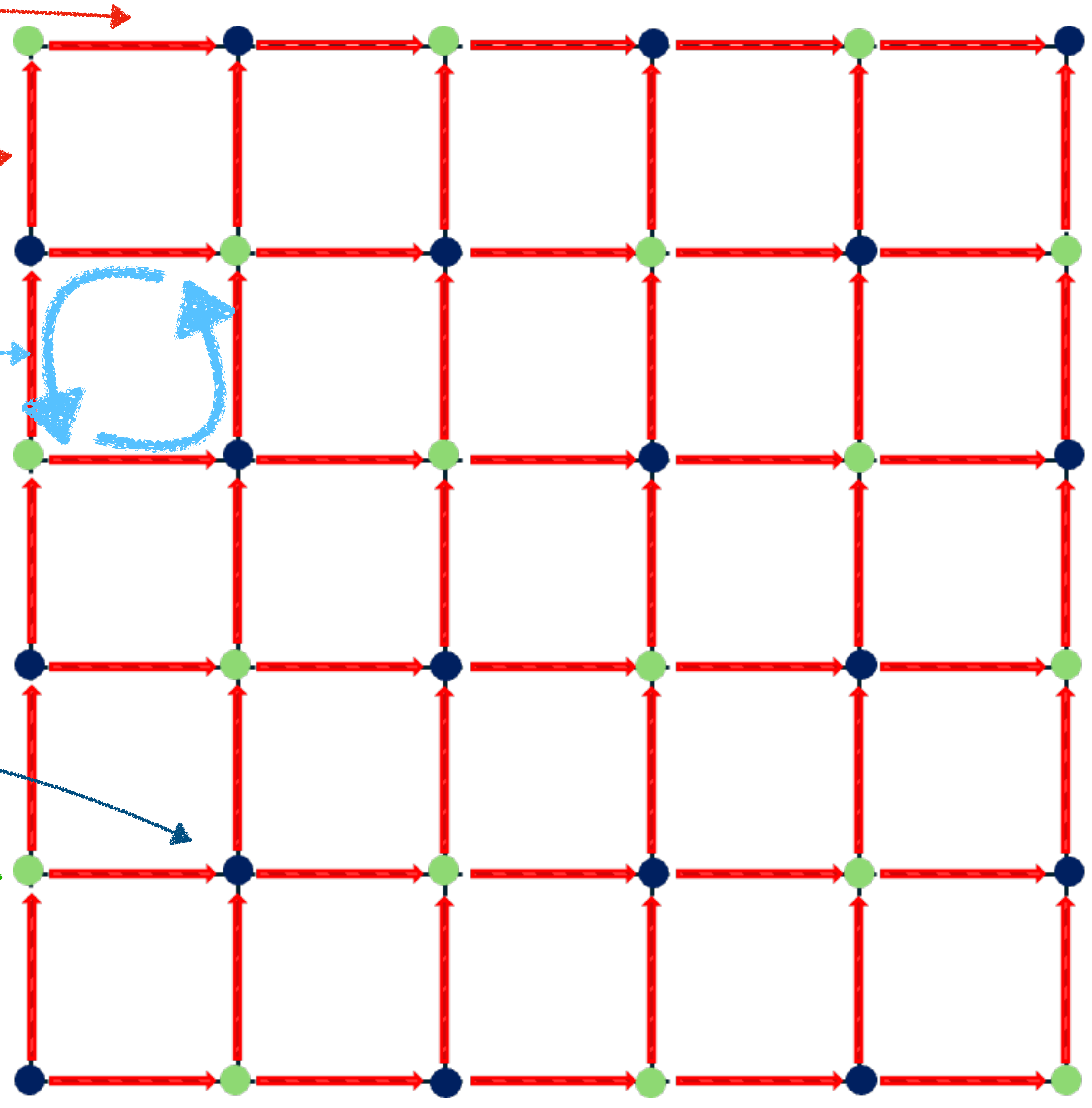
Hamiltonian for a general lattice

$$H_E = \frac{g^2}{2} \sum_{\mathrm{n}} \left(E^2_{\mathrm{n,e_x}} + E^2_{\mathrm{n,e_y}} \right)$$

$$H_B = -\frac{1}{2g^2a^2} \sum_n \left(P_n + P_n^\dagger - 2 \right)$$

$$H_M = m_0 \sum_n (-)^{n_x+n_y} \Psi_n^\dagger \Psi_n$$

$$H_K = \frac{1}{2} \sum_n \left[i\Psi_n^\dagger U^\dagger_{n,e_x} \Psi_{n+e_x} - (-)^{n_x+n_y} \Psi_n^\dagger U^\dagger_{n,e_y} \Psi_{n+e_y} \right] + \text{h.c.}$$



$$P_n \equiv U_{n,x} U_{n+e_x,y} U^\dagger_{n+e_y,x} U^\dagger_{n,y}$$

Two ways of encoding U(1) fields

The correct theory has $\hat{U} = e^{i\hat{\chi}}$, $\quad \mathrm{Spec}(\hat{\chi}) = S^1$, $\mathrm{Spec}(\hat{E}) = \mathbb{Z}$

$$L^2(\mathbb{R}^2) \mapsto L^2(S^1)$$

Two modes per link

All polynomial gates

$$U \mapsto \hat{q}^0 + i\hat{q}^1$$

$$E \mapsto J \equiv \hat{q}^0\hat{p}^1 - \hat{q}^1\hat{p}^0$$

Ale, RT, Rico, Ringer, Siopsis
JHEP 04 (2026) 122

Valid representations of

$$[\hat{E}, \hat{U}] = \hat{U}$$

$$\Updownarrow$$

$$[\hat{\chi}, \hat{E}] = i$$

but NOT isospectral

$$L^2(\mathbb{R}) \mapsto L^2(S^1)$$

One mode per link

Non-polynomial gates

$$U \mapsto e^{i\alpha\hat{x}}, \frac{\hat{U} + \hat{U}^\dagger}{2} = \cos(\alpha\hat{x})$$

$$E \mapsto \frac{\hat{p}}{\alpha}$$

Ale, RT, Rico, Ringer, Siopsis
(in preparation)

Unit circle in the complex plane

The correct theory has $\hat{U} = e^{i\hat{\chi}}$, $\text{Spec}(\hat{\chi}) = S^1$, $\text{Spec}(\hat{E}) = \mathbb{Z}$

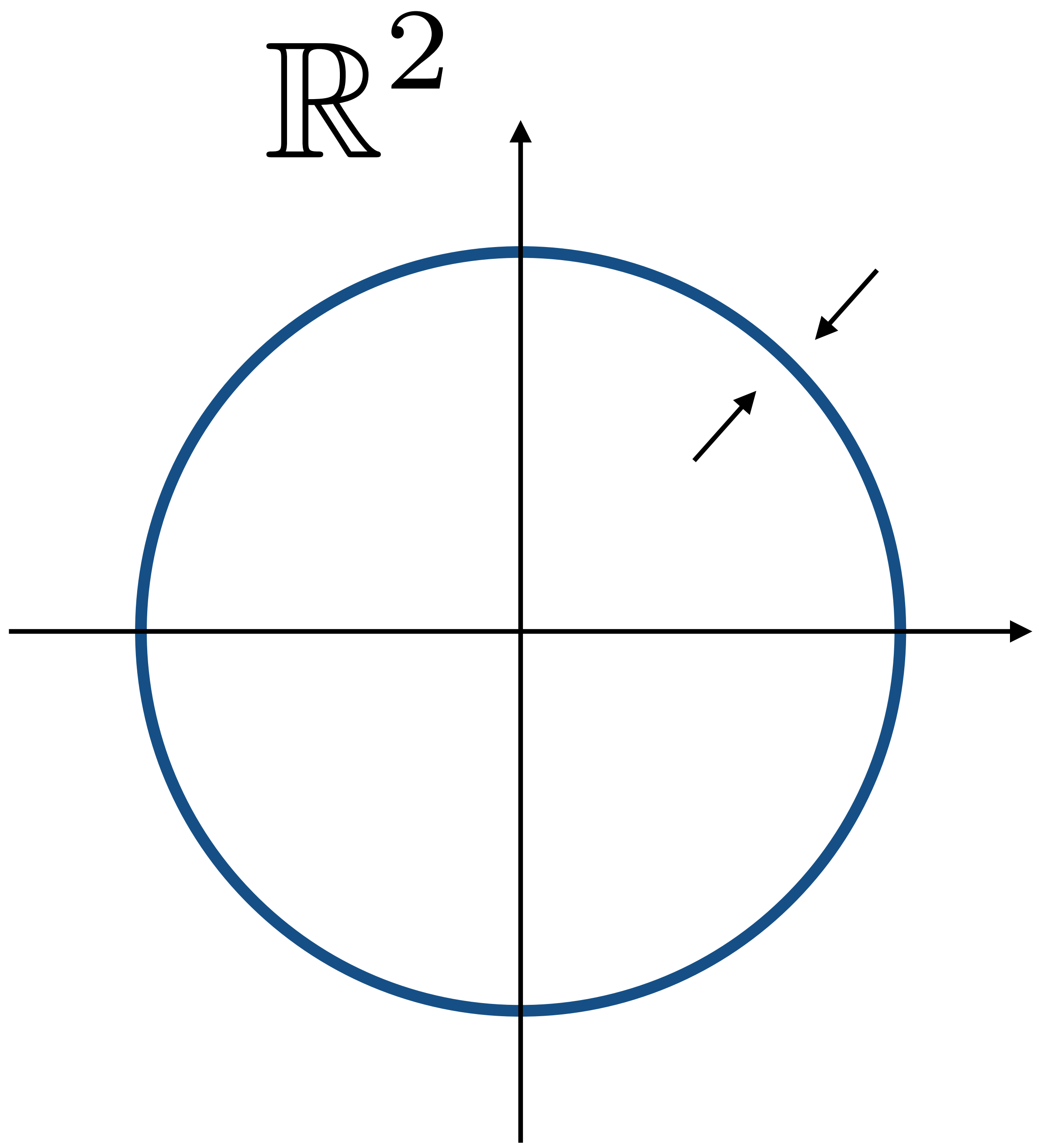
$$L^2(\mathbb{R}^2) \mapsto L^2(S^1)$$

Two modes per link

All polynomial gates

$$U \mapsto \hat{q}^0 + i\hat{q}^1$$

$$E \mapsto J \equiv \hat{q}^0\hat{p}^1 - \hat{q}^1\hat{p}^0$$



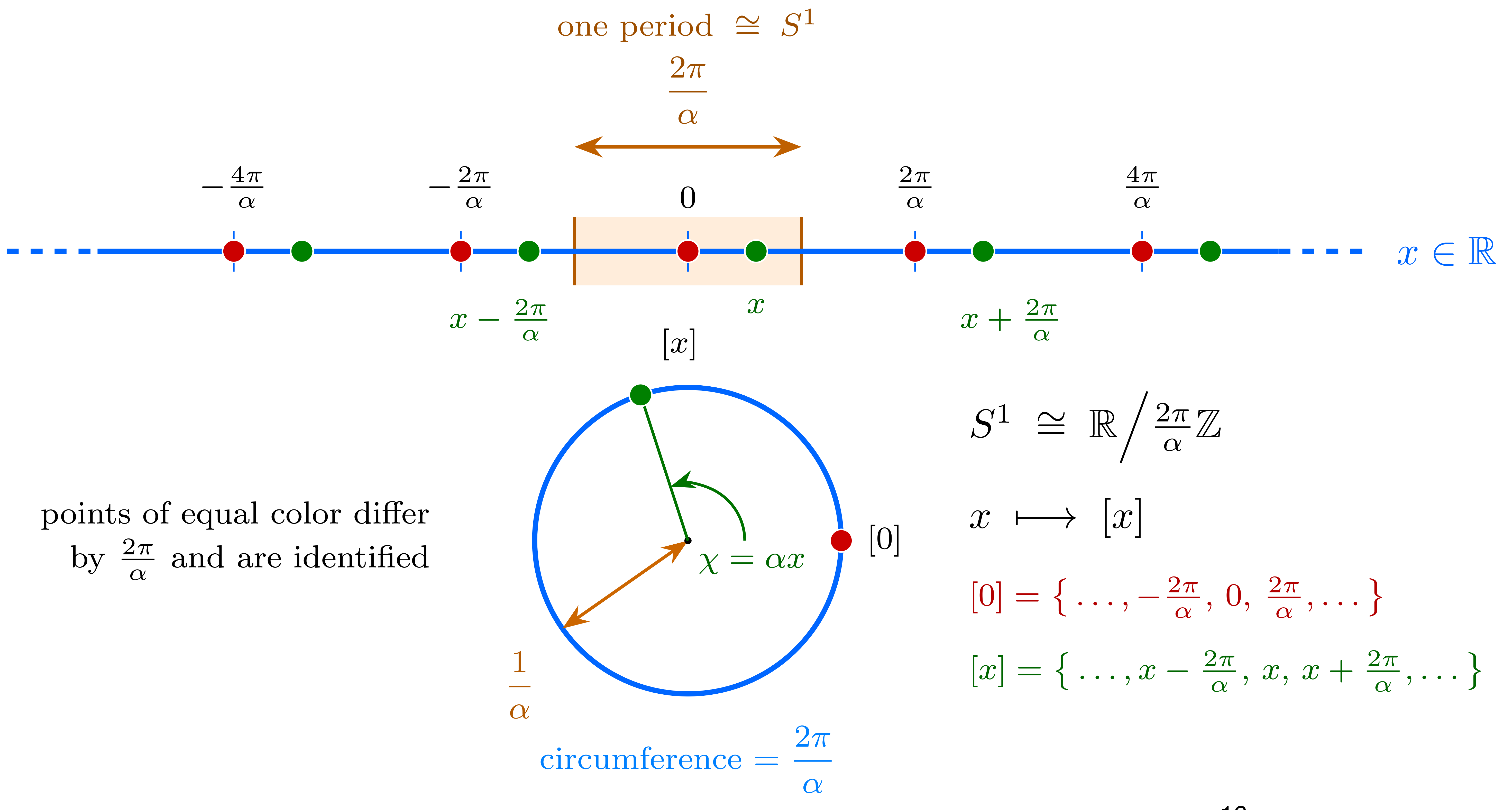
With the operator constraint

$$(\hat{q}^0)^2 + (\hat{q}^1)^2 = \mathbb{1}$$

Ale, RT, Rico, Ringer, Siopsis
JHEP 04 (2026) 122

Wrapping the real line

The correct theory has $\hat{U} = e^{i\hat{\chi}}, \quad \mathrm{Spec}(\hat{\chi}) = S^1, \ \mathrm{Spec}(\hat{E}) = \mathbb{Z}$



$$L^2(\mathbb{R}) \mapsto L^2(S^1)$$

One mode per link

Non-polynomial gates

$$U \mapsto e^{i\alpha\hat{x}}, \quad \frac{\hat{U} + \hat{U}^\dagger}{2} = \cos(\alpha\hat{x})$$

$$E \mapsto \frac{\hat{p}}{\alpha}$$

In both cases:

No discretization of the gauge group

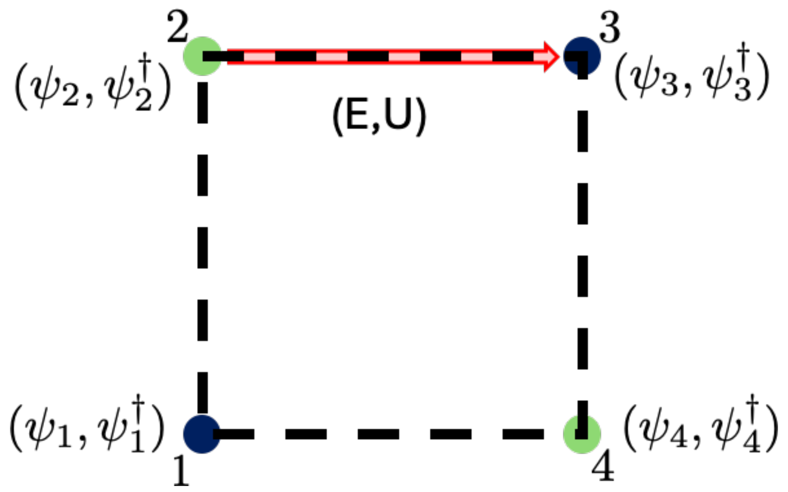
$$U(1) \cancel{\mapsto} Z_N \quad \text{then} \quad N \to \infty$$

No discretization of the algebra

$$[\hat{E}, \hat{U}] \cancel{\approx} \hat{U}$$

Direct extension to SU(2) done

Ringer, Siopsis et al JHEP06(2025)084



$$H_{U(1)} = 2g^2\hat{E}^2 + \frac{1}{g^2}\left[1 - \frac{\hat{U} + \hat{U}^\dagger}{2}\right]$$

Two-mode encoding

$$H_{U(1)} = 2g^2(\hat{q}^0\hat{p}^1 - \hat{q}^1\hat{p}^0)^2 + \frac{1}{g^2}\left[1 - \hat{q}^0\right)]$$

One-mode encoding

$$H_{U(1)} = 2g^2\frac{\hat{p}^2}{\alpha^2} + \frac{1}{g^2}\left[1 - \cos(\hat{\alpha}\hat{x})\right]$$

Enforcing the constraint

$$\left[H, (\hat{q}_n^0)^2 + (\hat{q}_n^1)^2\right] = 0 \, \forall n$$

Enlarged phase space but
Gauge invariance is preserved
Physical sector is protected

Hamiltonian penalty term

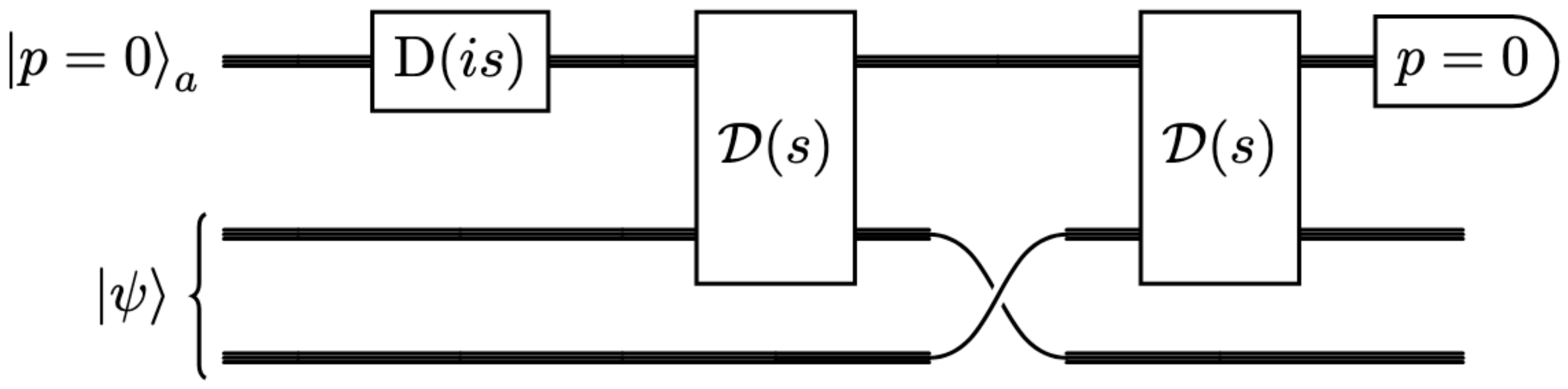
$$H \mapsto H + H_\mu$$

$$H_\mu = \frac{\mu}{2} \sum_{\mathbf{n},i} (\mathbf{q}_{\mathbf{n},i}^2 - 1)^2$$

Disfavors non-physical configurations

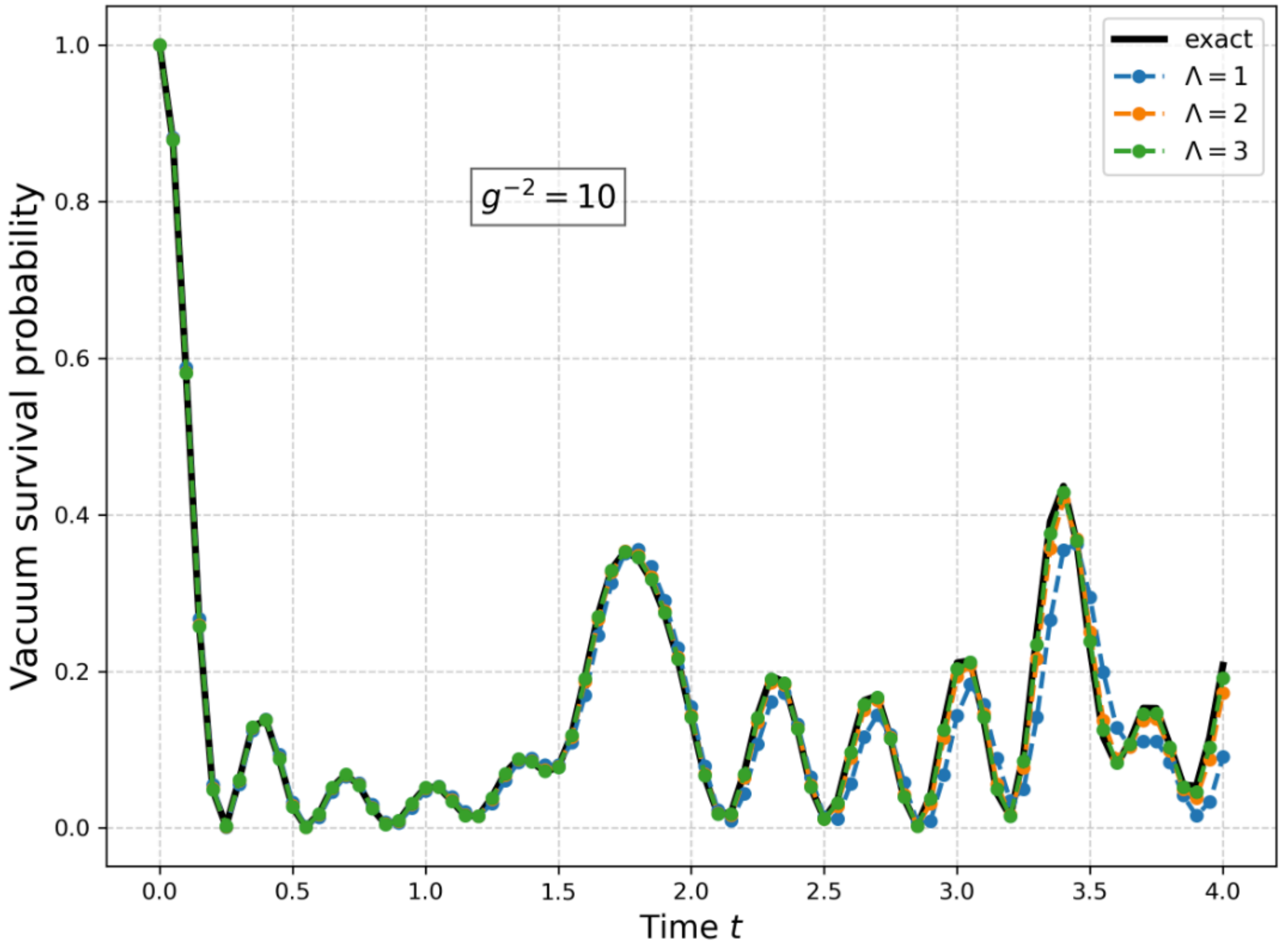
Inner product modification

$$|\psi\rangle \mapsto |\psi(\Lambda)\rangle \propto \int \mathrm{d}^2 q \, e^{-\frac{\Lambda^2}{g^2}(\mathbf{q}^2-1)^2} \psi(\mathbf{q}) \, |\mathbf{q}\rangle$$



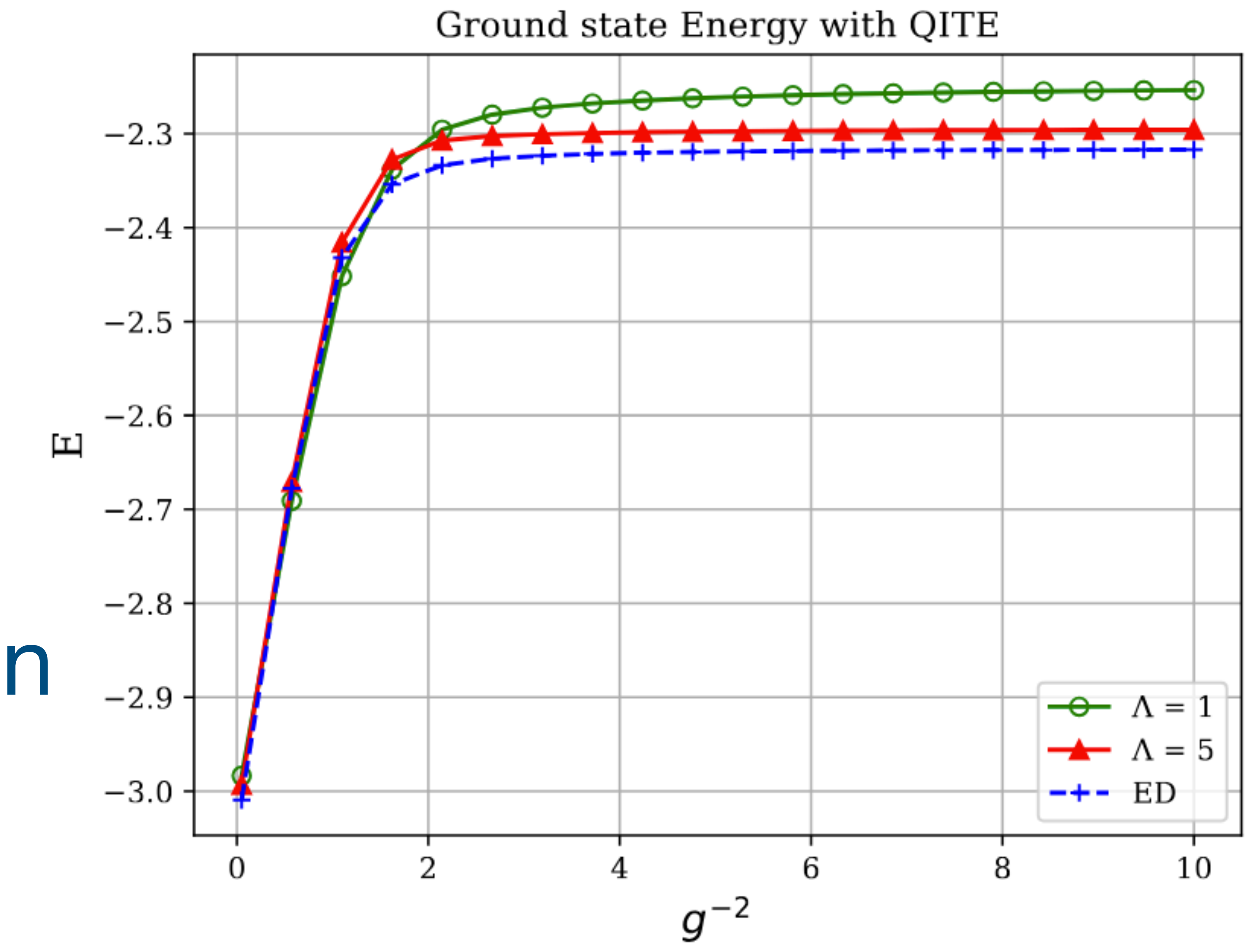
$\mathscr{D}_\mu(s) = e^{-isq_\mu^2 q_a}$ $\quad$ $\Lambda = \frac{g}{2} s e^r$

Testing the two-mode case

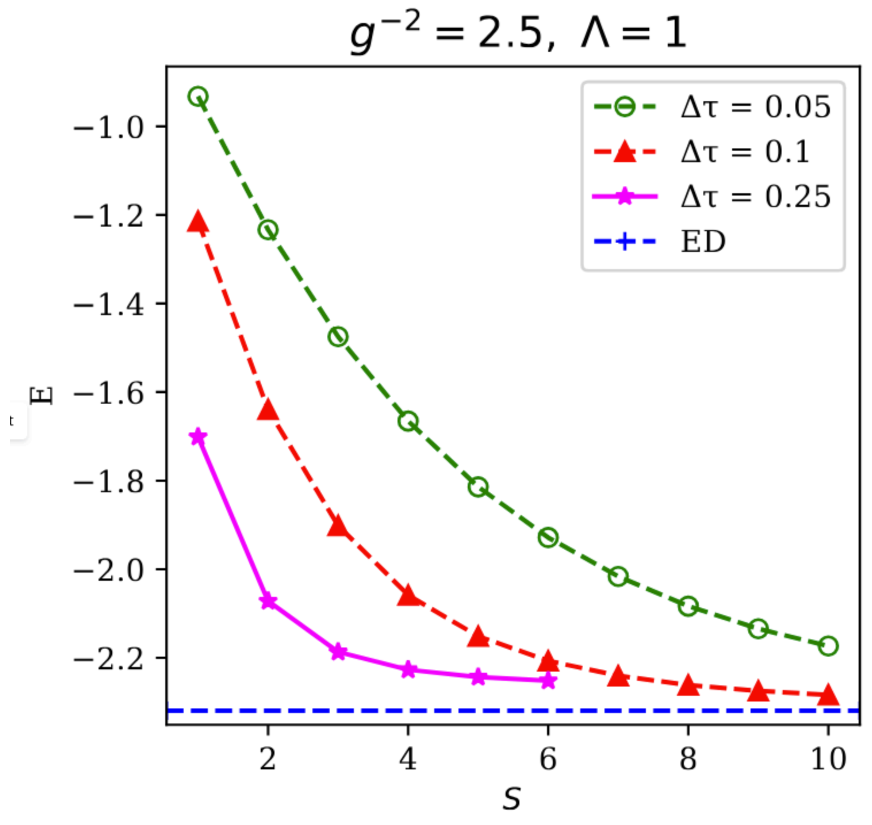


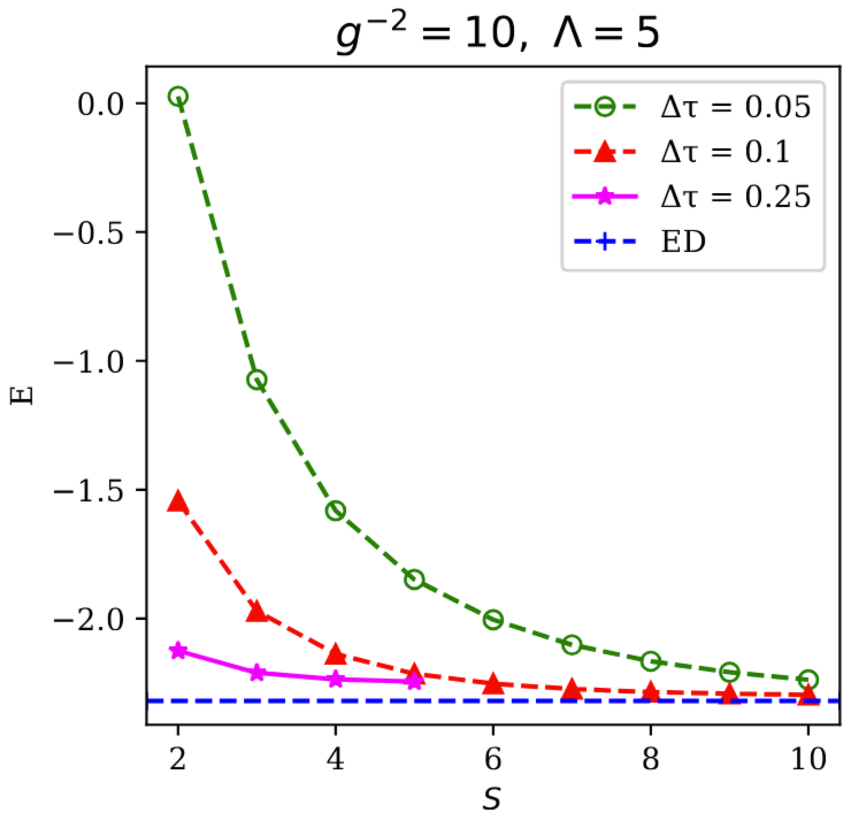
Ground state preparation

(QITE)



Time evolution





Ground state energy

Trigonometric gates representation

Encoding compactness in a mode and error correction

Inspired by Drell, Quinn et al `79

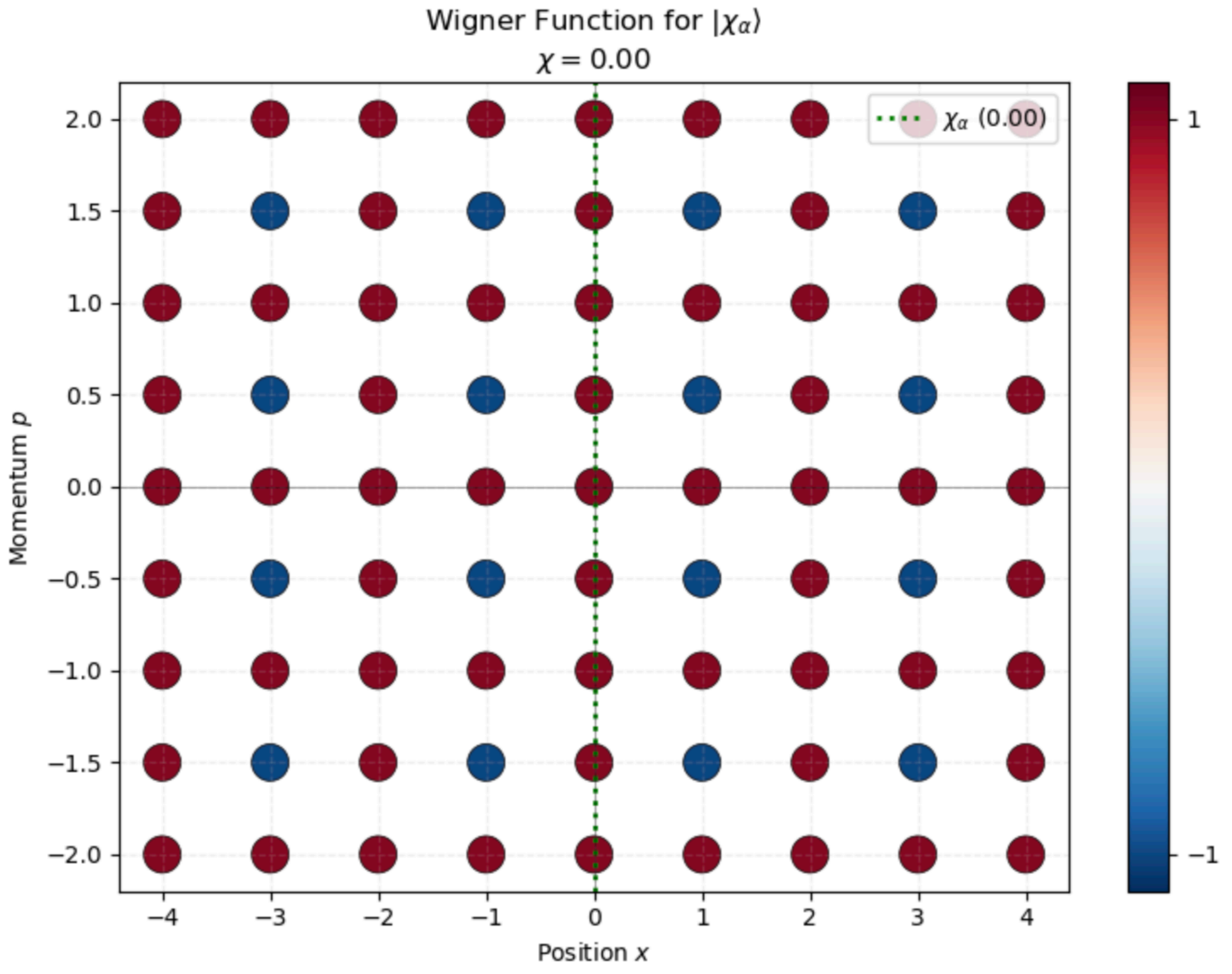
$L^2(\mathbb{R}) \to L^2(S^1)$ via stabilization operations

Natural error correction to protect the S^1 topology

$$H_{U(1)} = 2g^2\hat{E}^2 + \frac{1}{g^2}\left[1 - \cos(\hat{\chi})\right]$$

$$H_{U(1)} \mapsto 2g^2\left(\frac{\hat{p}}{\alpha}\right)^2 + \frac{1}{g^2}\left[1 - \cos(\alpha\hat{x})\right] - J\text{Penalty}$$

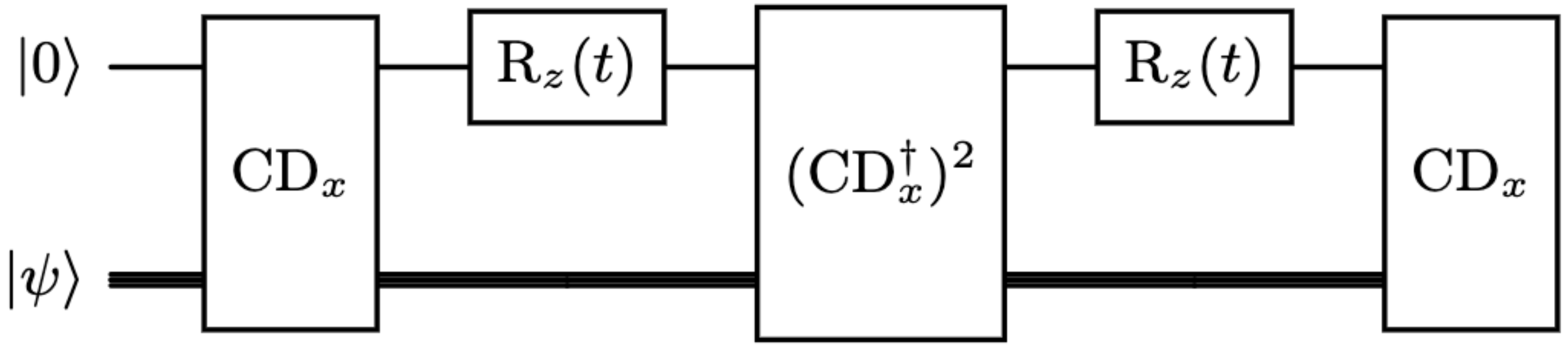
$$\text{Penalty} = \cos\left(\frac{2\pi}{\alpha}\hat{p}\right) - 1$$



Like a qunaught GKP state

Experimental realization

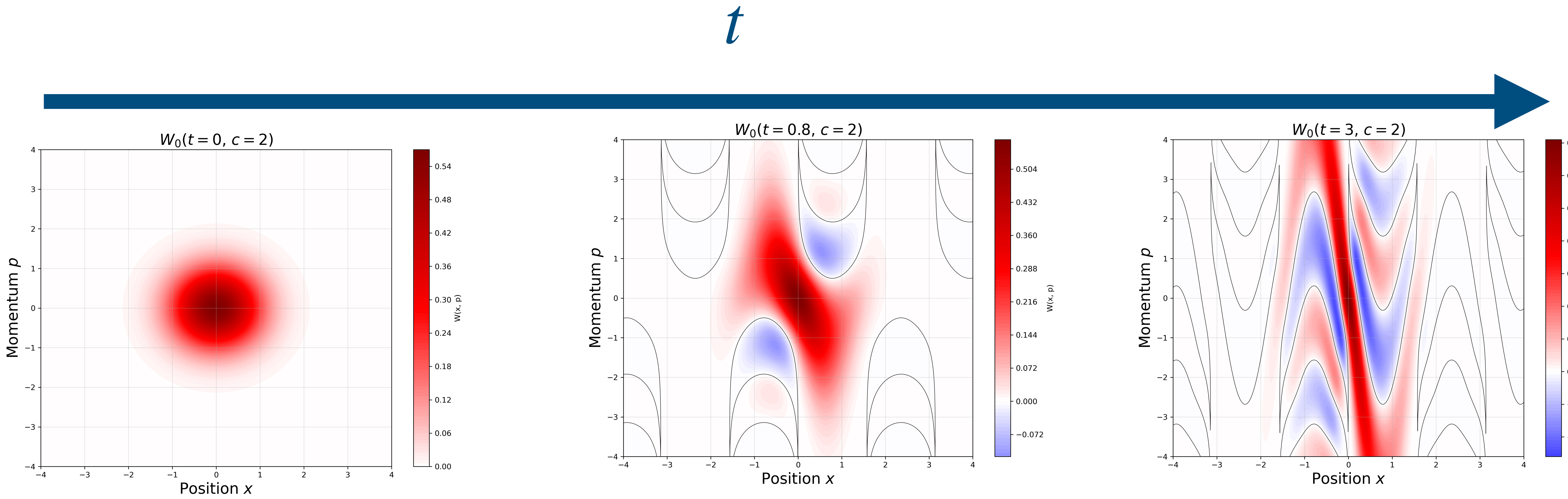
with QSCOUT at Sandia National Lab



$$\approx e^{-it\cos(c\hat{x})}$$

RT, Ringer, Grau, Montgomery, S. Clark et al

ArXiv:2607.14085



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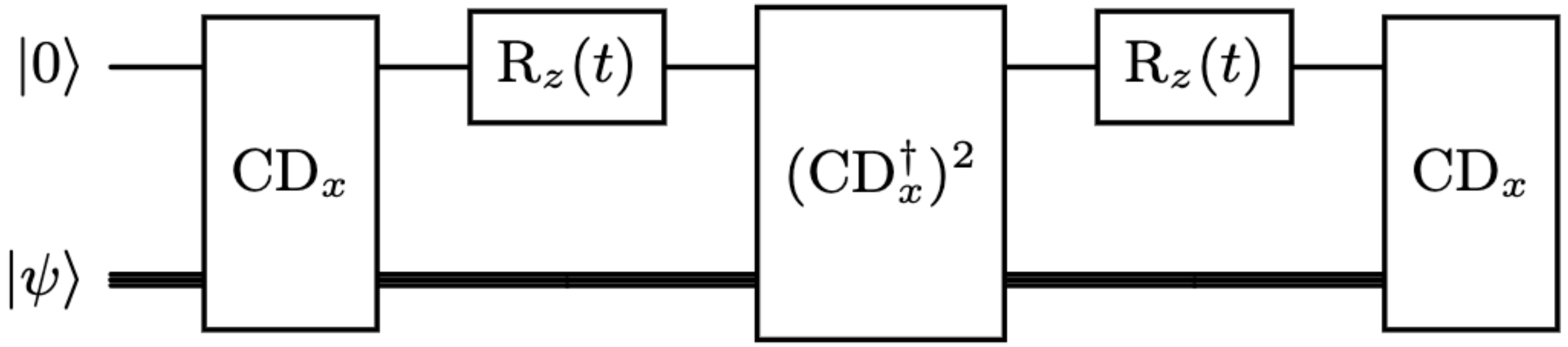

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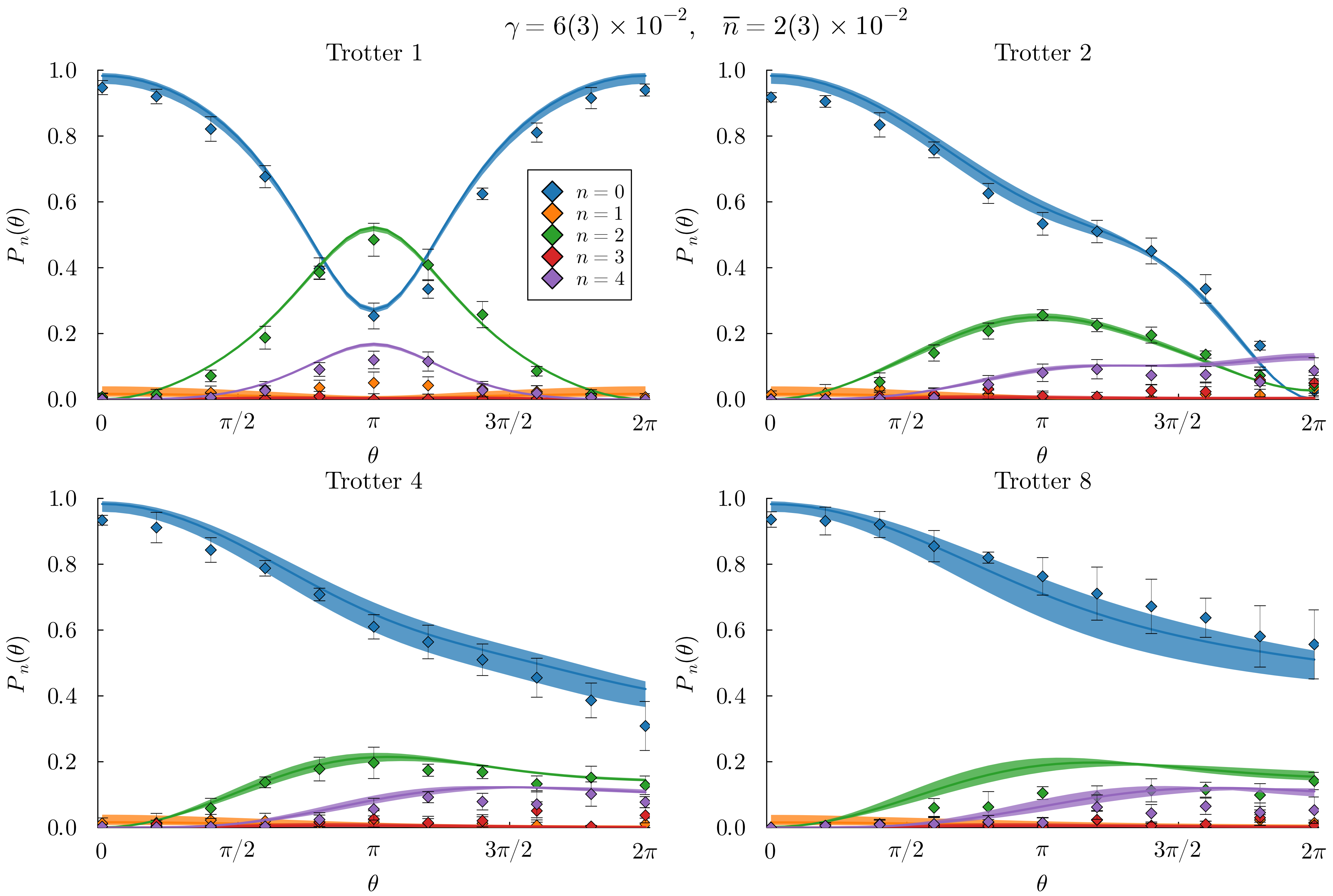
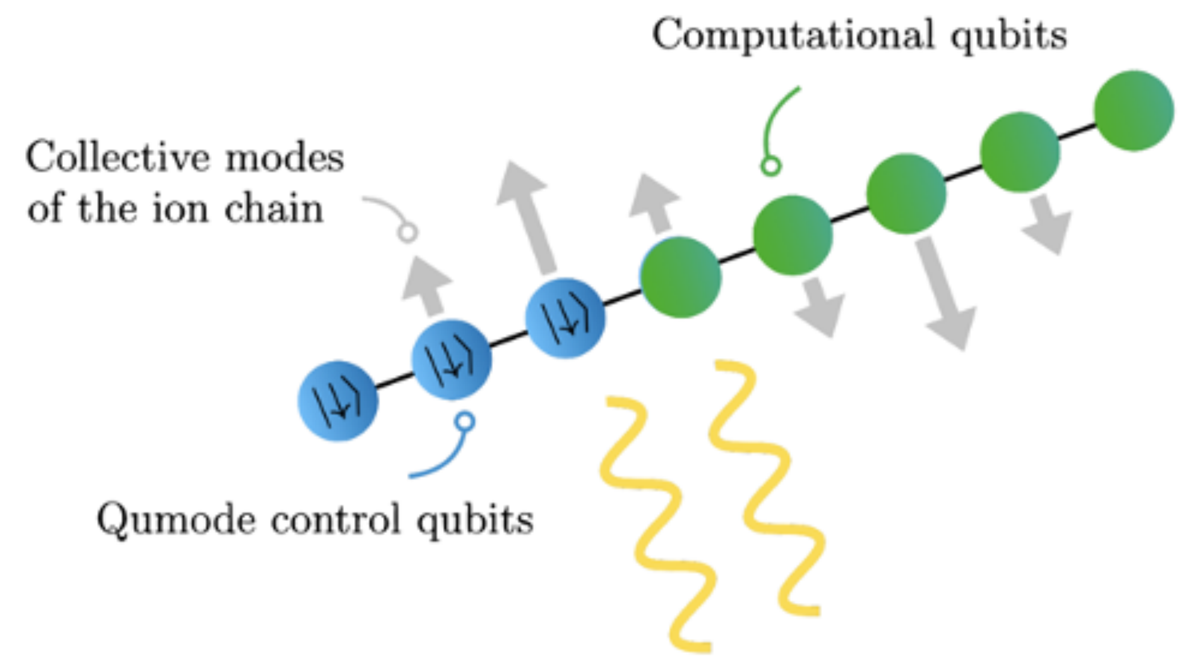
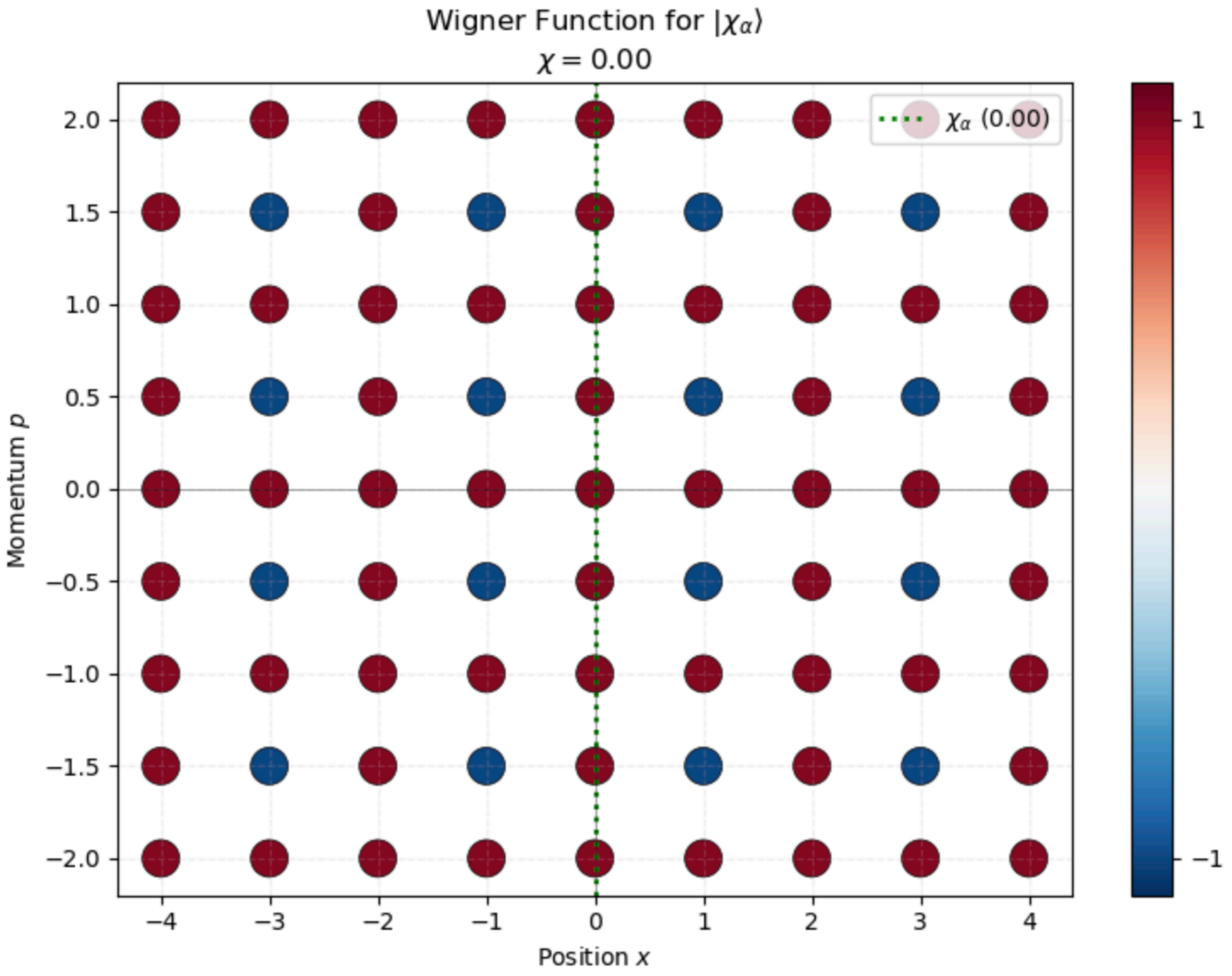




Repeated iterations approximate the gate for any t (θ)

Stabilizing the circle S^1



Example: Ideal state $|\chi = 0\rangle = \sum_{n\in\mathbb{Z}} |p = \alpha n\rangle_p$

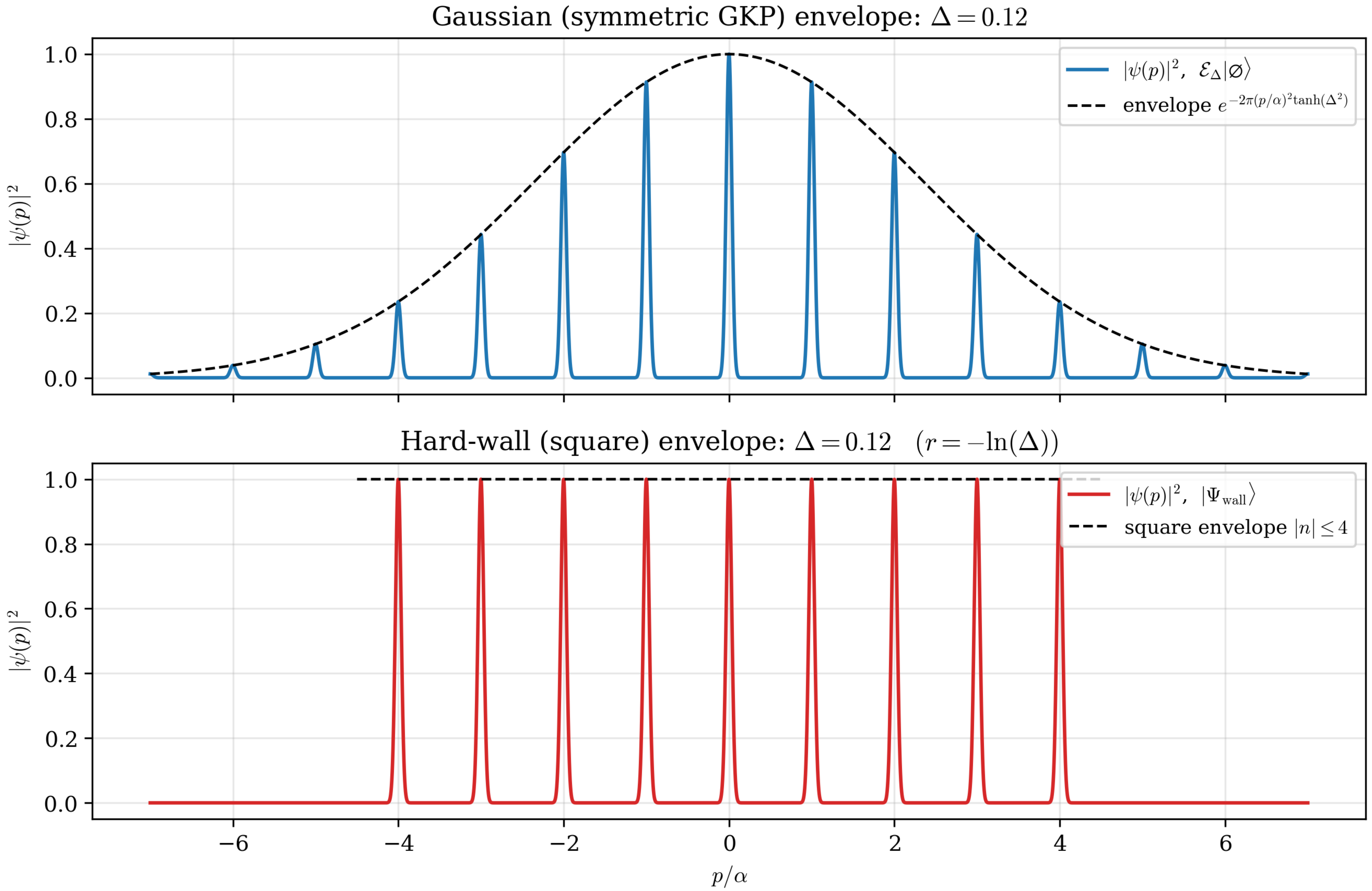
$$|\chi = 0; \delta\rangle = \sum_{n\in\mathbb{Z}} |p = \alpha n + \delta\rangle_p$$

Correctable and detectable error

$$|\chi = \delta\rangle = \sum_{n\in\mathbb{Z}} e^{-i\delta} |p = \alpha n\rangle_p$$

Logical rotation:
non-correctable
non-detectable

The only allowed states

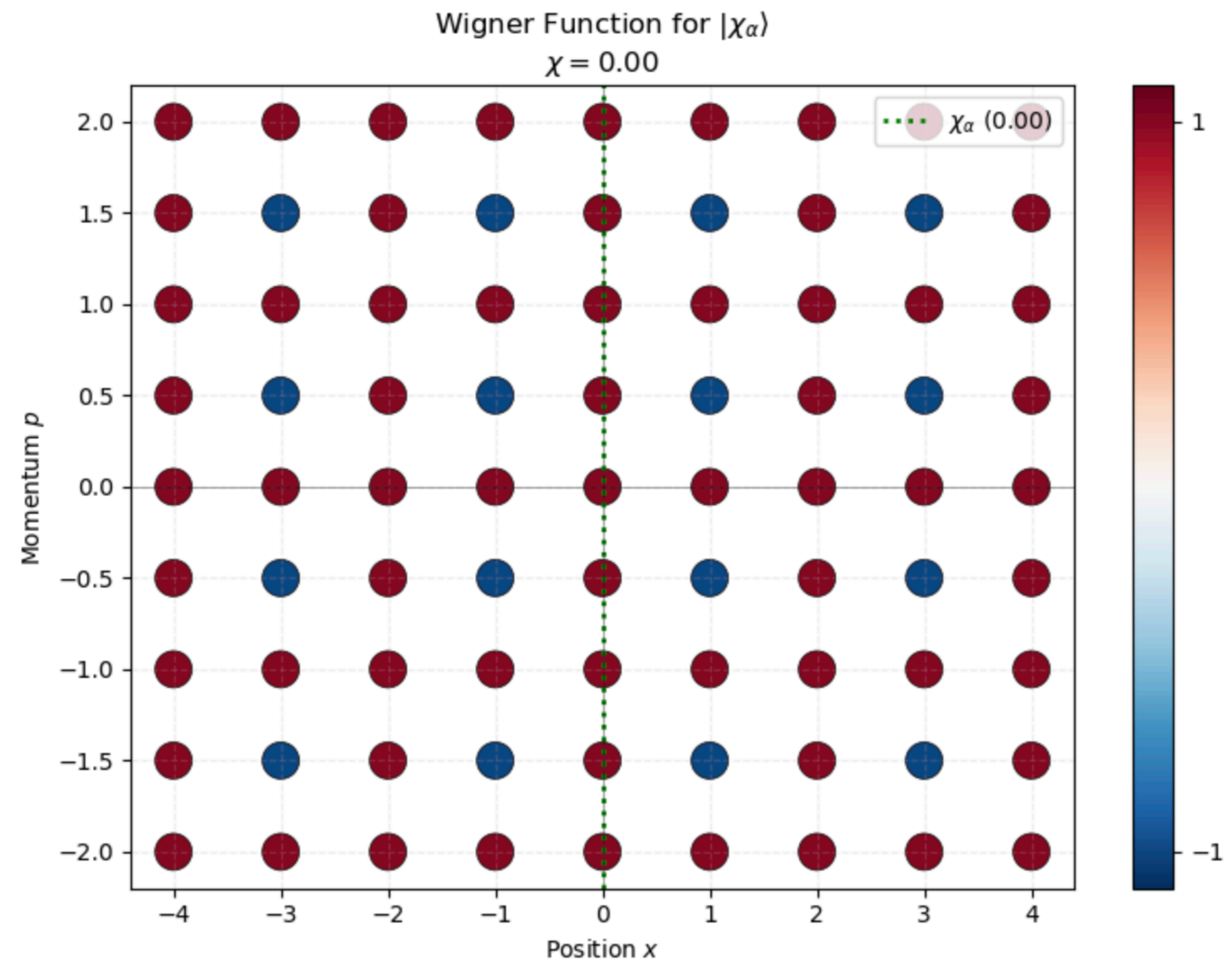


Only the states stabilized by

$$S_\alpha = e^{2\pi i \frac{\hat{p}}{\alpha}}$$

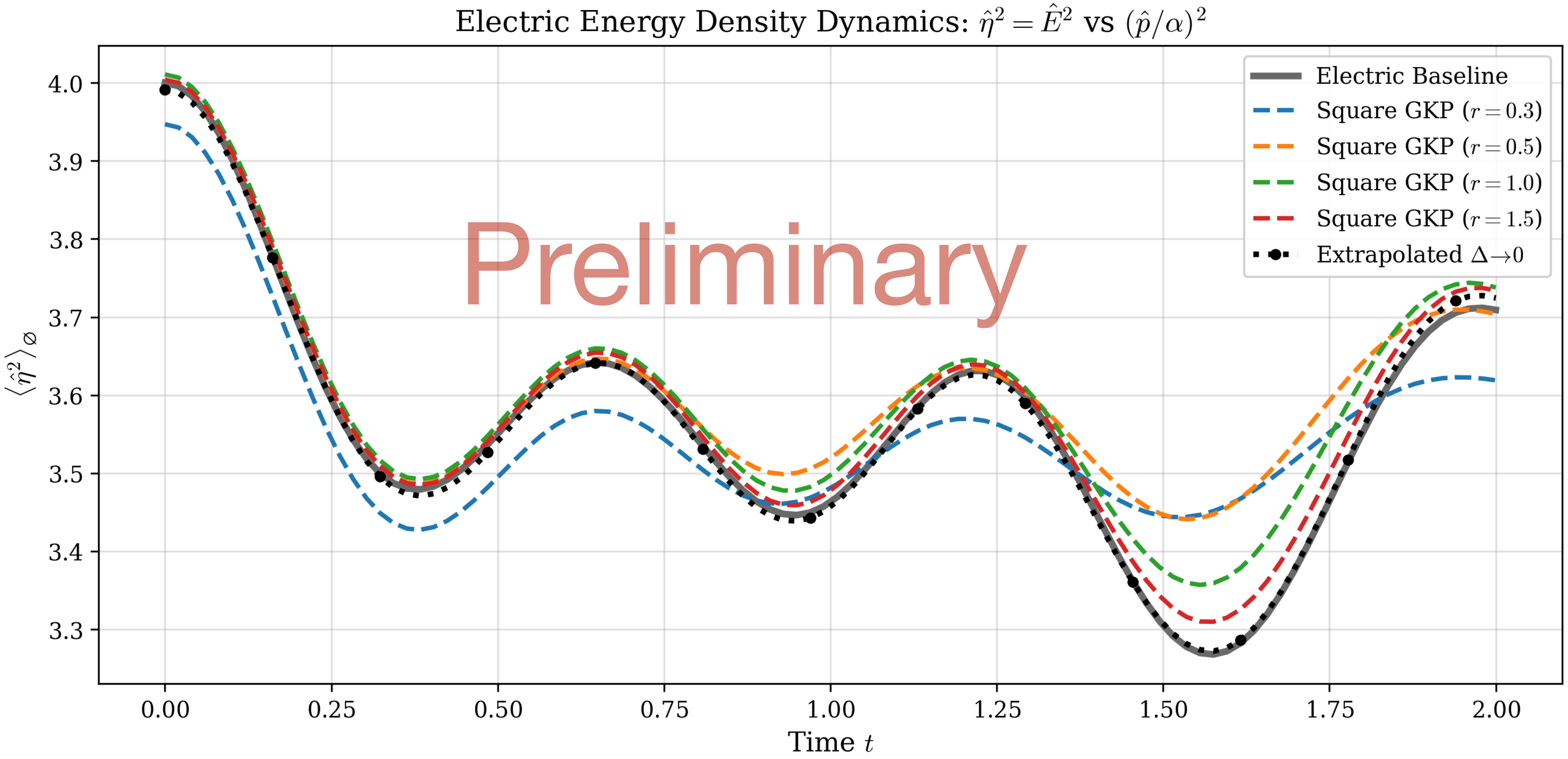
$$|\Psi\rangle = \sum_{n\in\mathbb{Z}} c_n |p = \alpha n\rangle_p \in L^2(\mathbb{R})$$

GKP family (no logical info)



Error correction and mitigation

Electric energy extrapolation

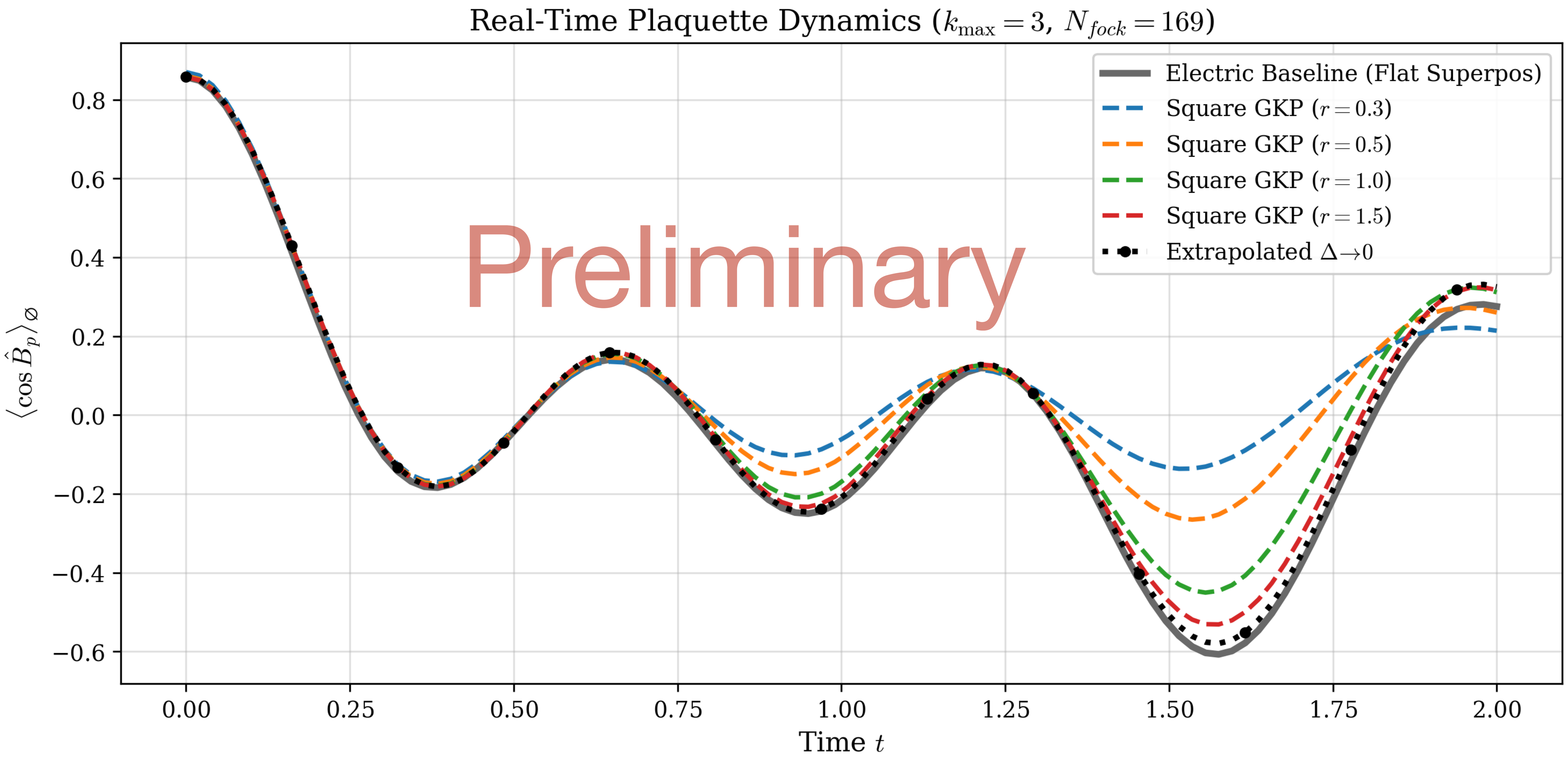


Error is $\mathcal{O}(\Delta^2)$

Analytic extrapolation

Error correction and mitigation

Plaquette operator extrapolation

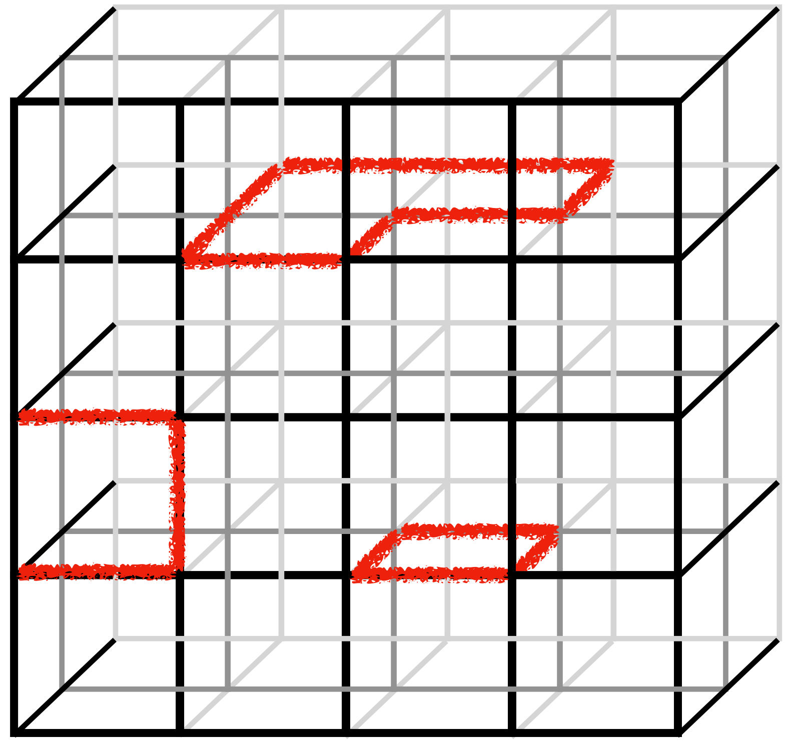


Error is $\mathcal{O}(\Delta^2)$

Analytic extrapolation

Summary

- **Hybrid formalism to map bosons to Qumodes and fermions to Qubits for lattice gauge theories**
  - Feasibility with different existing hybrid platforms
  - Ground state preparation, Time evolution, etc…
  - Future: Scattering process, hadronization, structure, etc…



- **Explicit example for Abelian U(1) gauge theory with fermions in any dimension**
  - Extensions to non-Abelian theories are possible (SU(2) Ringer et al JHEP06(2025)084)
- **New class of gates: non-polynomial (trigonometric) gates (important for physics and much more)**

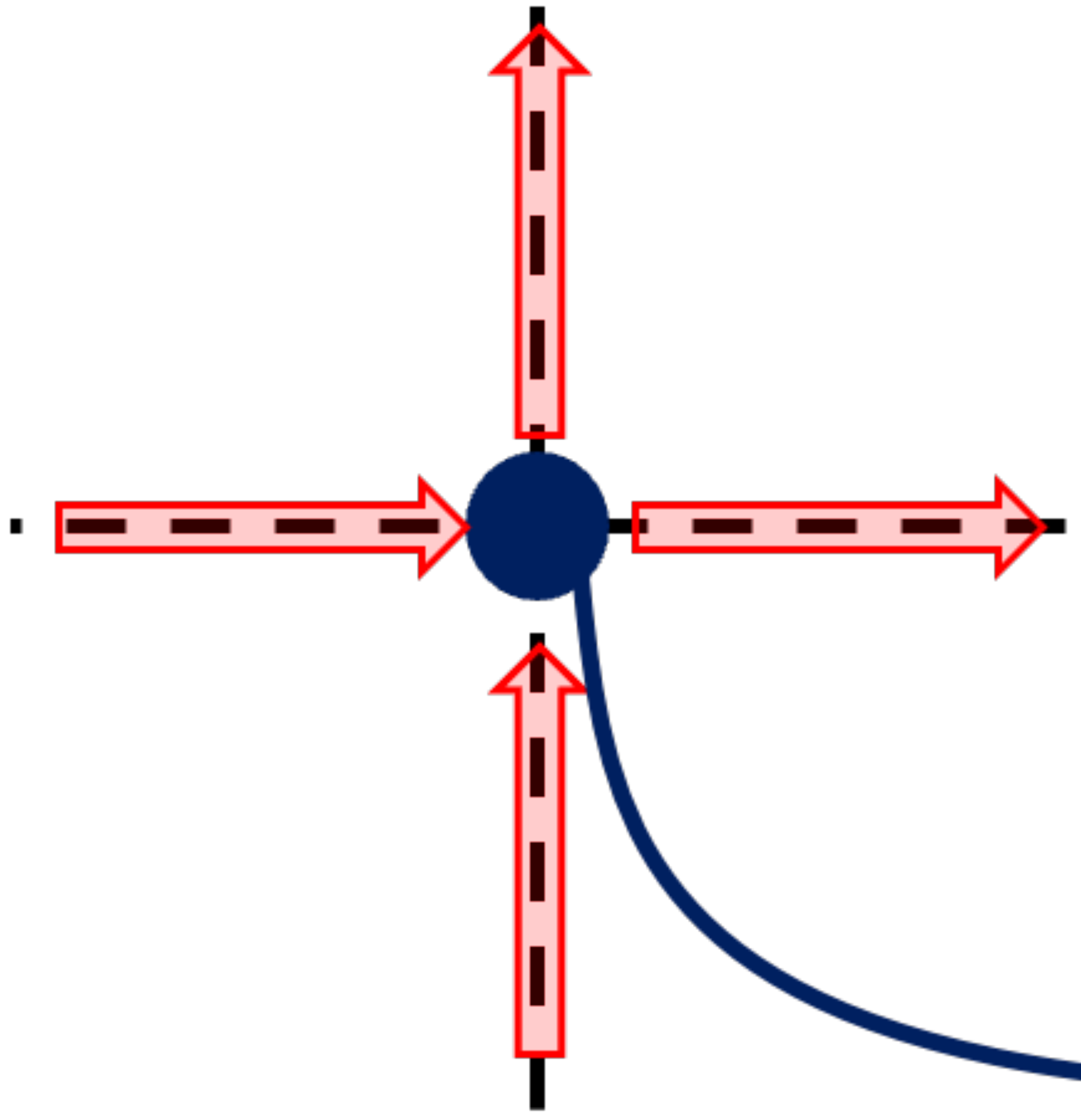
Thank you

Extra slides

Gauss Law

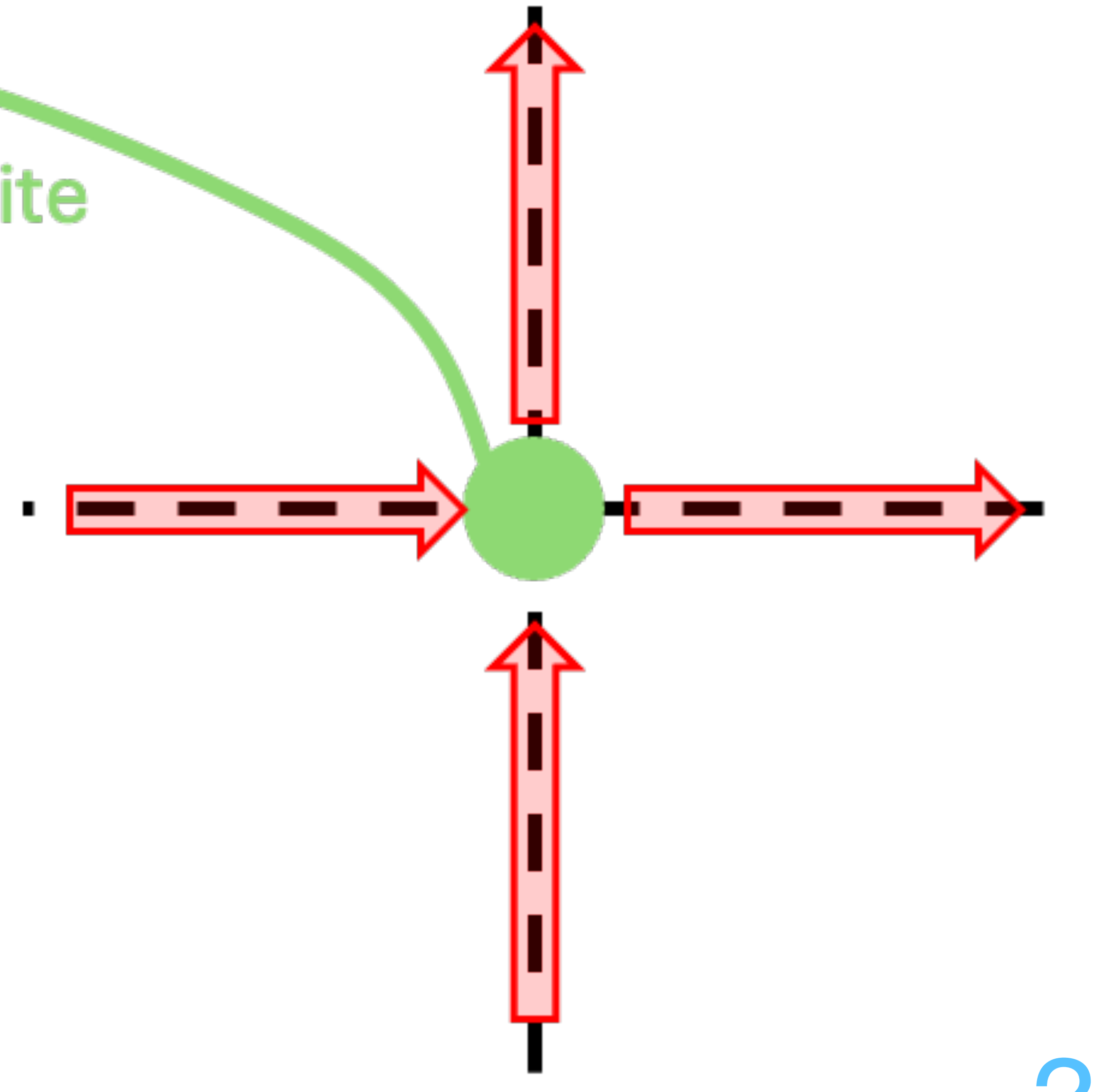
$$\vec{\nabla} \cdot \vec{E} = \rho$$

$$\boxed{E_{\mathbf{n},\hat{e}_x} + E_{\mathbf{n},\hat{e}_y} - E_{\mathbf{n},-\hat{e}_x} - E_{\mathbf{n},-\hat{e}_y} = Q_{\mathbf{n}}}$$



Odd site

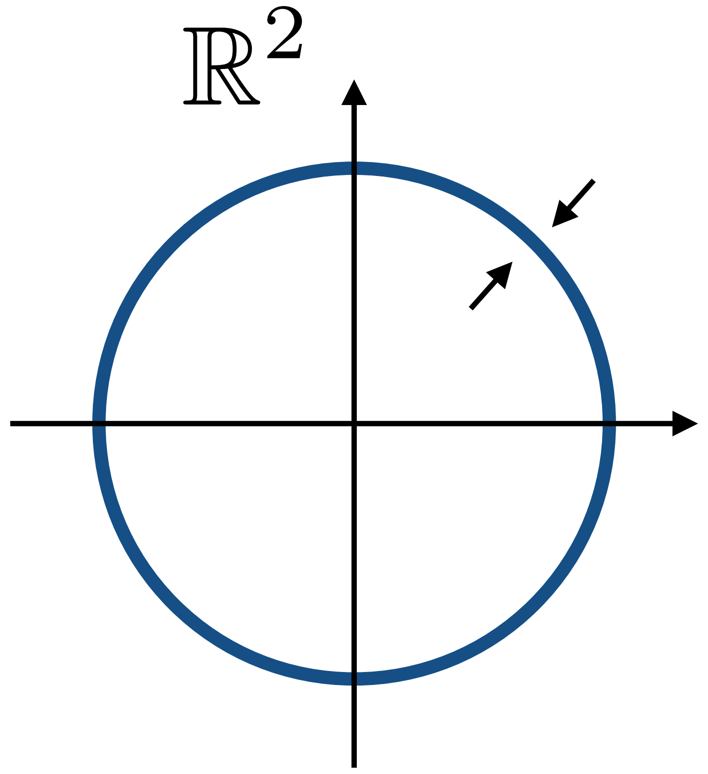
$$Q_{\mathbf{n}} = \psi^{\dagger}_{\mathbf{n}}\psi_{\mathbf{n}} - \frac{\mathbb{I}}{2}\left[1 - (-1)^{n_x+n_y}\right]$$



Even site

The full Hamiltonian

Ale, RT, Rico, Ringer, Siopsis `25



Due to Gauss's law

$$H_E^{(Q)} = g^2 \sum_{\boldsymbol{n},\boldsymbol{n}'} \left(\mathcal{H}^{(2)}_{\boldsymbol{n}\boldsymbol{n}'} J_{\boldsymbol{n}} J_{\boldsymbol{n}'} + \mathcal{H}^{(1)}_{\boldsymbol{n}\boldsymbol{n}'} Q_{\boldsymbol{n}} J_{\boldsymbol{n}'} + \mathcal{H}^{(0)}_{\boldsymbol{n}\boldsymbol{n}'} Q_{\boldsymbol{n}} Q_{\boldsymbol{n}'} \right),$$

$$H_B^{(Q)} = \frac{1}{2g^2} \sum_{\boldsymbol{n}} \left(\boldsymbol{q}_{\boldsymbol{n}+e_y} - \boldsymbol{q}_{\boldsymbol{n}} \right)^2,$$

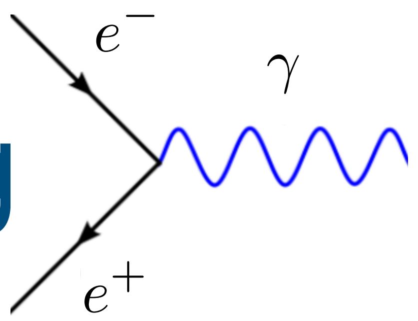
$$H_M^{(Q)} = m_0 \sum_{\boldsymbol{n}} (-)^{n_x+n_y} Q_{\boldsymbol{n}},$$

$$H_K^{(Q)} = \frac{1}{2} \sum_{\boldsymbol{n}} \left(q^0_{\boldsymbol{n}} - i q^1_{\boldsymbol{n}} \right) X^+_{\boldsymbol{n}} X^-_{\boldsymbol{n}+e_x} + \frac{1}{2} \sum_{\boldsymbol{n}} P_{L(\boldsymbol{n},\boldsymbol{n}+e_y)} X^+_{\boldsymbol{n}} X^-_{\boldsymbol{n}+e_y} + \text{h.c.}$$

$$Q_{\boldsymbol{n}} = \Psi^\dagger_{\boldsymbol{n}} \Psi_{\boldsymbol{n}} - \frac{1-(-)^{n_x+n_y}}{2} \mathbb{1}$$

$$J_{\boldsymbol{n}} = q^0_{\boldsymbol{n}} p^1_{\boldsymbol{n}} - q^1_{\boldsymbol{n}} p^0_{\boldsymbol{n}}$$

Electron/photon coupling



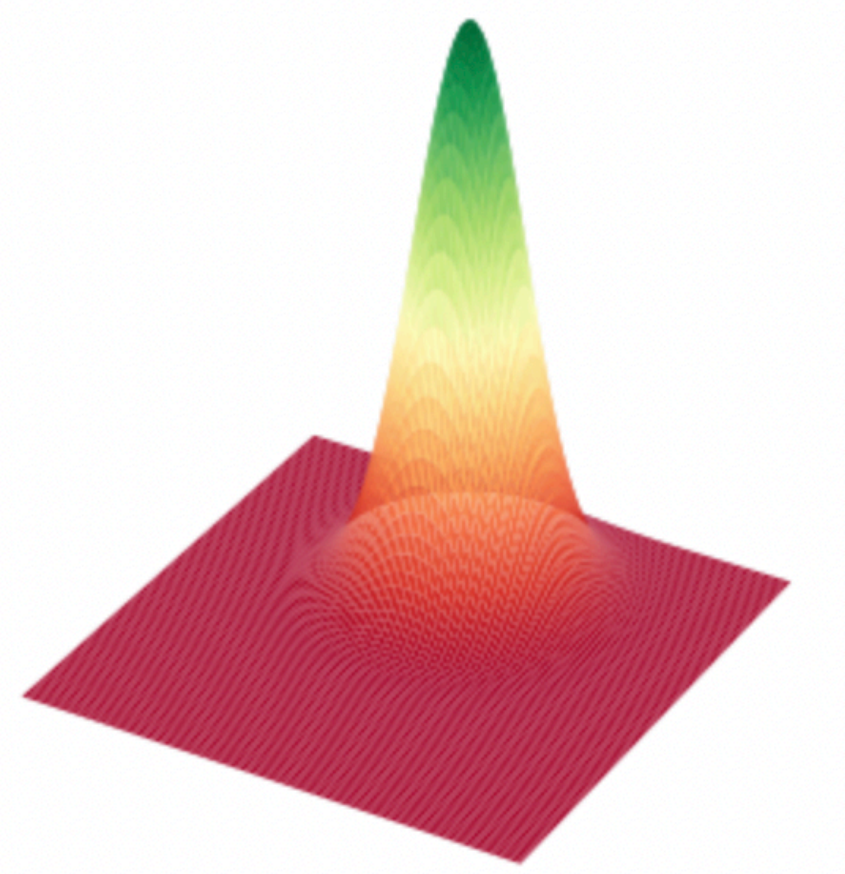
Qubit/qumode Qumode

Universal gate set

see Lloyd, Braunstein `99

$$W(q,p)$$

- Displacement $D(z) = e^{z\hat{a}^\dagger - z^*\hat{a}}$

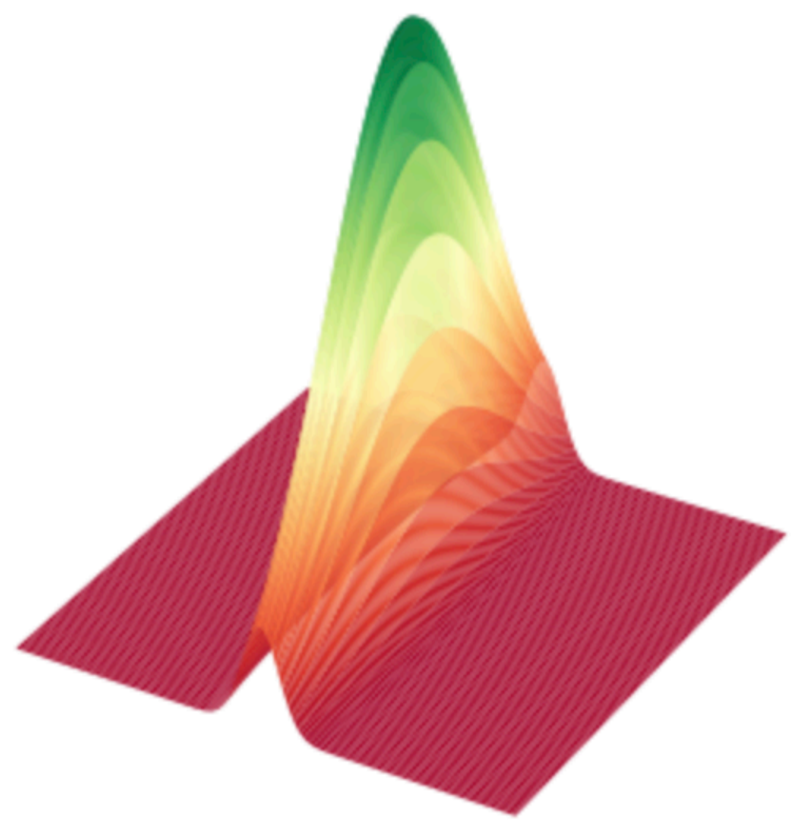


- Two-qumode beam splitter

$$U_{\mathrm{bs}}(z) = e^{z\hat{a}\hat{b}^\dagger - z^*\hat{a}^\dagger\hat{b}} \qquad z = \theta e^{i\phi}$$

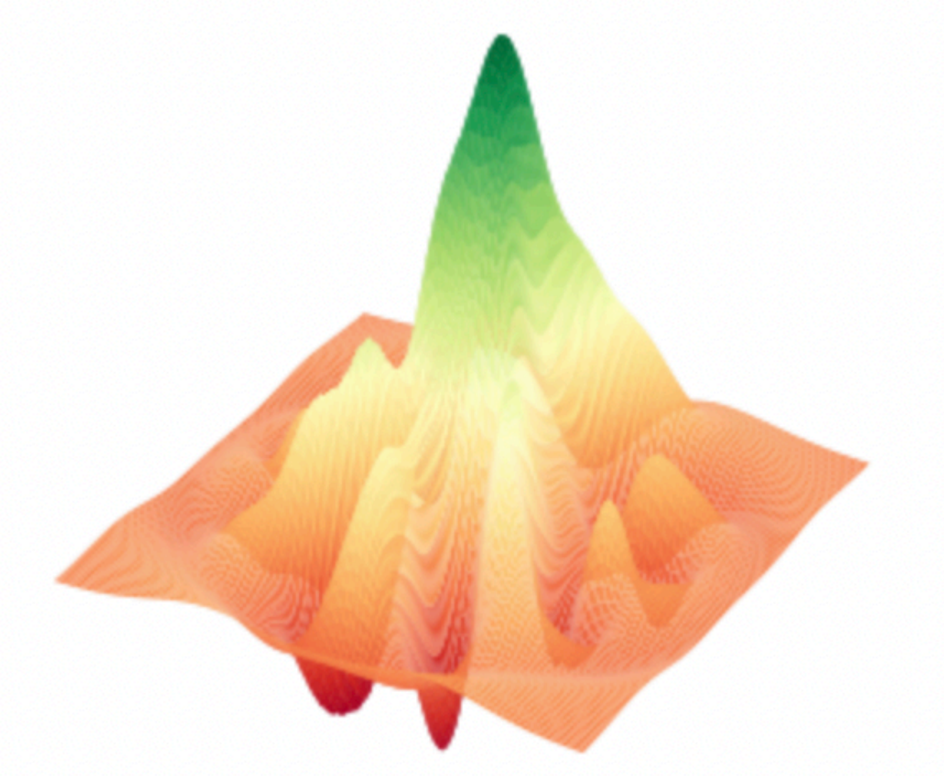
- Squeezing

$$S(z) = e^{\frac{1}{2}(z^*\hat{a}^2 - z\hat{a}^{\dagger 2})}$$



- Non-Gaussian operations

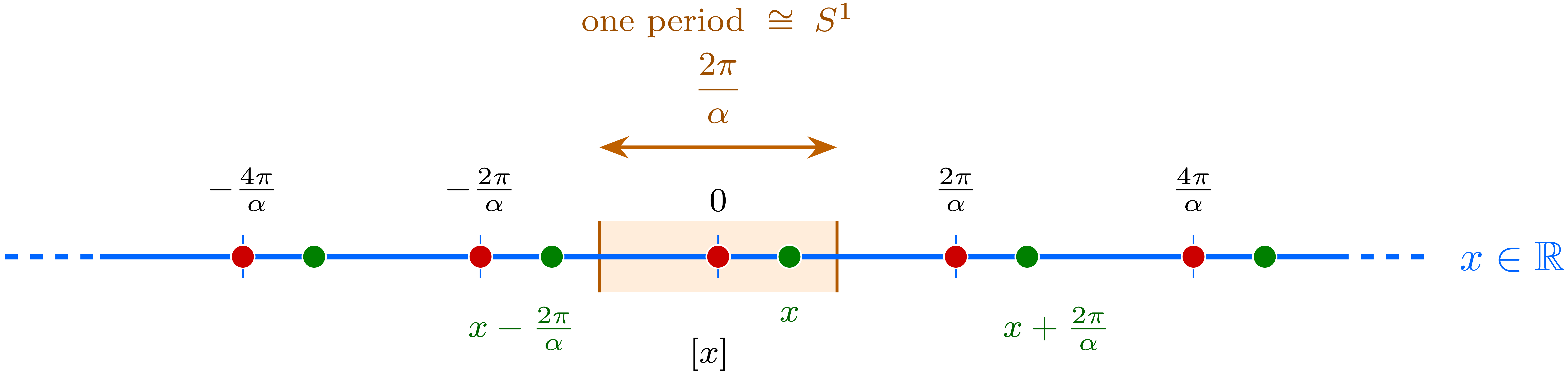
  e.g. Kerr gate or cubic gate



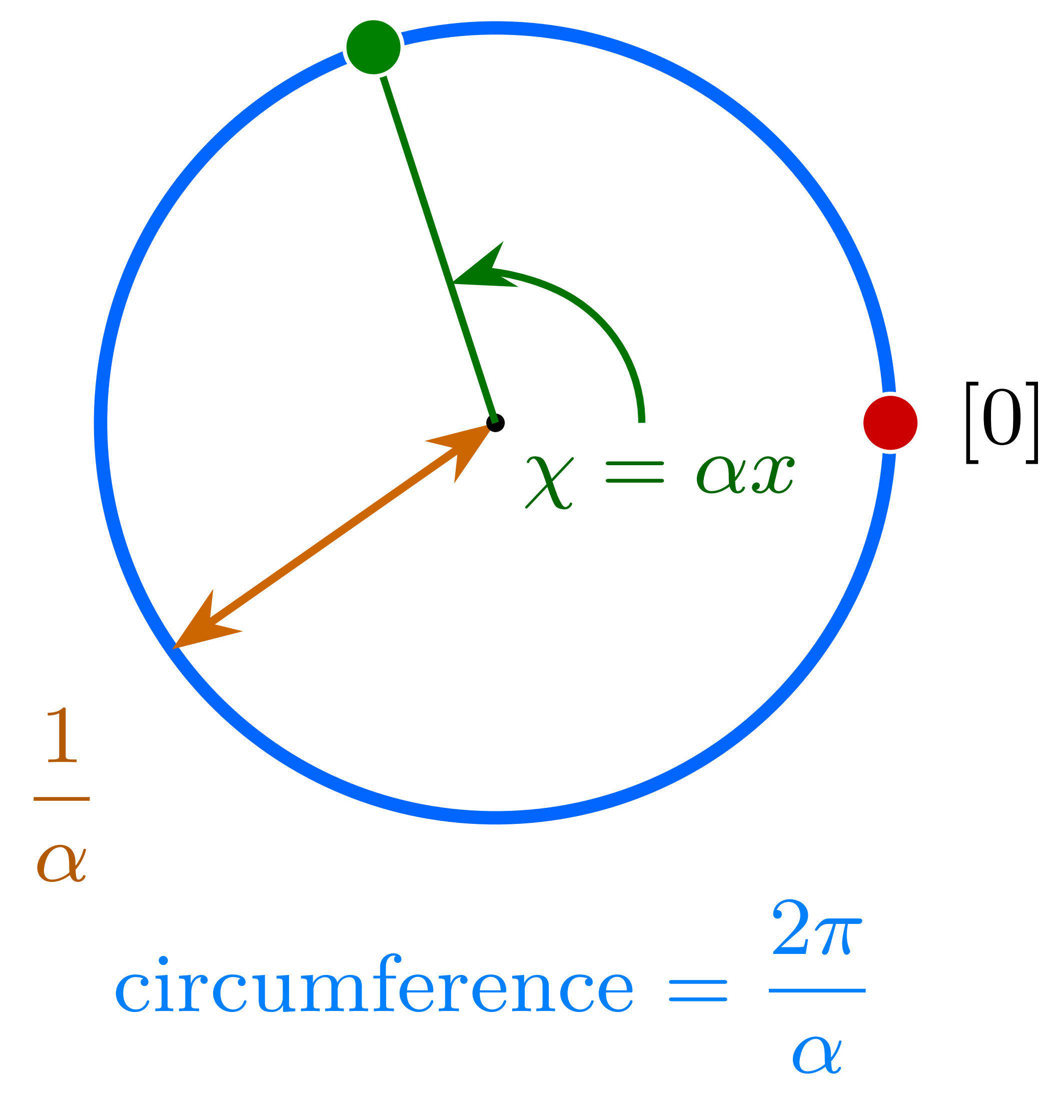
One mode GKP inspired encoding

$$\hat{E} \mapsto \frac{\hat{p}}{\alpha}$$

$$\hat{\chi} \mapsto \alpha \hat{x}$$



points of equal color differ by $\frac{2\pi}{\alpha}$ and are identified



$$S^1 \cong \mathbb{R} \big/ \tfrac{2\pi}{\alpha}\mathbb{Z}$$

$$x \longmapsto [x]$$

$$[0] = \left\{ \ldots, -\tfrac{2\pi}{\alpha}, 0, \tfrac{2\pi}{\alpha}, \ldots \right\}$$

$$[x] = \left\{ \ldots, x - \tfrac{2\pi}{\alpha}, x, x + \tfrac{2\pi}{\alpha}, \ldots \right\}$$