“First Overlap Formulation and Perturbative Results for $\mathcal{N} = 1$ Supersymmetric QCD”

Marios Costa

marios.costa@cut.ac.cy

Cyprus University of Technology

Department of Mechanical Engineering and Materials Science and Engineering

In collaboration with:
Prof. H. Panagopoulos, Dr. G. Spanoudes, Mrs. E. Ioannou, Mr. C. Kartoudes (University of Cyprus), Dr. H. Herodotou, Mr. E. Antoniou (Rinnoco Ltd).

July 28, 2026

General Outline

For a more detailed description, see:

M. Costa, E. Ioannou and H. Panagopoulos, "N=1 supersymmetric QCD on the lattice using overlap fermions," Phys. Rev. D **112** (2025) no.9, 094506 doi:10.1103/k5q5-2f57 [arXiv:2507.16651 [hep-lat]].

Action of SQCD in the continuum

The continuum action for massive SQCD in the WZ gauge is given by:

$$\begin{aligned}\mathcal{S}_{\rm SQCD} \quad = \quad & \int d^4x \Big\{ -\frac{1}{4}u^{\alpha}_{\mu\nu}u^{\alpha}_{\mu\nu} + \frac{i}{2}\bar{\lambda}^{\alpha}\gamma^{\mu}\mathcal{D}_{\mu}\lambda^{\alpha} + i\bar{\psi}\gamma^{\mu}\mathcal{D}_{\mu}\psi \\ & +(\mathcal{D}_{\mu}A_{+})^{\dagger}(\mathcal{D}_{\mu}A_{+}) + (\mathcal{D}_{\mu}A_{-})(\mathcal{D}_{\mu}A_{-})^{\dagger} \\ & \underbrace{-\sqrt{2}g\left(A^{\dagger}_{+}\bar{\lambda}^{\alpha}T^{\alpha}P_{+}\psi - \bar{\psi}P_{-}\lambda^{\alpha}T^{\alpha}A_{+} + A_{-}\bar{\lambda}^{\alpha}T^{\alpha}P_{-}\psi - \bar{\psi}P_{+}\lambda^{\alpha}T^{\alpha}A^{\dagger}_{-}\right)}_{\text{Yukawa interactions}} \\ & \underbrace{-\frac{1}{2}g^2\left(A^{\dagger}_{+}T^{\alpha}A_{+} - A_{-}T^{\alpha}A^{\dagger}_{-}\right)^2}_{\text{quartic squark interactions}} \underbrace{-m\bar{\psi}\psi}_{\text{quark mass term}} \underbrace{-m^2\left(A^{\dagger}_{+}A_{+} + A_{-}A^{\dagger}_{-}\right)}_{\text{squark mass terms}} \Big\}.\end{aligned}$$

Here, m is the common quark–squark mass required by supersymmetry. The gauge field strength and covariant derivatives are:

$$\begin{aligned} u_{\mu\nu} &= \partial_{\mu}u_{\nu} - \partial_{\nu}u_{\mu} + ig[u_{\mu},u_{\nu}], \quad \mathcal{D}_{\mu}\lambda = \partial_{\mu}\lambda + ig[u_{\mu},\lambda], \\ \mathcal{D}_{\mu}\psi &= \partial_{\mu}\psi + igu^{\alpha}_{\mu}T^{\alpha}\psi, \quad \mathcal{D}_{\mu}A_{\pm} = \partial_{\mu}A_{\pm} \pm ig\, u^{\alpha}_{\mu}T^{\alpha}A_{\pm}. \end{aligned}$$

Main challenge: How can we formulate this continuum theory on the lattice while preserving chiral symmetry?

Overlap Quark Action (well-established in LQCD)

The construction of the quark action is straightforward, including the extension to nonzero quark masses:

$$\int dx\, \bar{\psi}(x)\,(\gamma_\mu \mathcal{D}_\mu + m)\,\psi(x) \to a^8 \sum_{x,y} \bar{\psi}(x)\left[\left(1-\frac{a\,m}{2}\right)\mathcal{D}_{\rm ov}(x,y) + m\,\frac{1}{a^4}\delta_{x,y}\right]\psi(y),$$

$$\mathcal{D}_{\rm ov}(x,y) = \frac{\rho^\psi}{a}\left[\frac{\delta_{x,y}}{a^4} + \left(X(X^\dagger X)^{-1/2}\right)_{x,y}\right],$$

$$X = -\frac{\rho^\psi}{a}\,\frac{\delta_{x,y}}{a^4} + \frac{1}{2}\sum_\mu \left[\gamma_\mu(\nabla_\mu(x,y) + \nabla^*_\mu(x,y)) - a\,r^\psi\,a^4 \sum_z \nabla^*_\mu(x,z)\nabla_\mu(z,y)\right],$$

$$\nabla_\mu(x,y) = \frac{1}{a}\left(U_\mu(x)\,\frac{1}{a^4}\,\delta_{x+a\hat{\mu},y} - 1\!\!1\,\frac{1}{a^4}\,\delta_{x,y}\right),$$

$$\nabla^*_\mu(x,y) = \frac{1}{a}\left(1\!\!1\,\frac{1}{a^4}\,\delta_{x,y} - U^\dagger_\mu(x-a\hat{\mu})\,\frac{1}{a^4}\,\delta_{x-a\hat{\mu},y}\right).$$

Overlap Gluino Action

By analogy with the quark case, the gluino overlap action is:

$$\int dx\, \mathrm{Tr}\left[\bar{\lambda}(x)\gamma_\mu \mathcal{D}_\mu \lambda(x)\right] \to -a^8 \sum_{x,y} \mathrm{Tr}\Big[\lambda^T(x)\, \mathcal{C}\, \mathcal{D}_{\rm ov}^{\rm adj}(x,y)\lambda(y)\Big]\,,$$

- Gluino is a Majorana massless fermion.
- It belongs to the adjoint representation.

$$\mathcal{D}_{\rm ov}^{\rm adj}(x,y) = \frac{\rho^\lambda}{2a}\left[\frac{\delta_{x,y}}{a^4} + \left(X^{\rm adj}(X^{\rm adj\dagger}X^{\rm adj})^{-1/2}\right)_{x,y}\right],$$

$$X^{\rm adj}(x,y) = -\frac{\rho^\lambda}{a}\frac{\delta_{x,y}}{a^4} + \frac{1}{2}\sum_\mu \left[\gamma_\mu(\nabla_\mu^{\rm adj}(x,y) + \nabla_\mu^{*\rm adj}(x,y)) - a\, r^\lambda a^4 \sum_z \nabla_\mu^{*\rm adj}(x,z)\nabla_\mu^{\rm adj}(z,y)\right]$$

$$\nabla_\mu^{\rm adj}(x,y) = \frac{1}{a}\left(U_\mu^{\rm adj}(x)\,\frac{1}{a^4}\,\delta_{x+a\hat{\mu},y} - \mathbb{1}\,\frac{1}{a^4}\,\delta_{x,y}\right),$$

$$\nabla_\mu^{*\,\rm adj}(x,y) = \frac{1}{a}\left(\mathbb{1}\,\frac{1}{a^4}\,\delta_{x,y} - U_\mu^{\rm adj\,\dagger}(x-a\hat{\mu})\,\frac{1}{a^4}\,\delta_{x-a\hat{\mu},y}\right),$$

$$\left[U_\mu^{\rm adj}(x)\right]^{\alpha\beta} = 2\,\mathrm{Tr}\left(T^\alpha\, U_\mu(x)\, T^\beta\, U_\mu^\dagger(x)\right).$$

Remaining pieces of the SQCD Action on the lattice

1. Gauge sector: Wilson plaquette action,

$$\frac{1}{4} u^{\alpha}_{\mu\nu} u^{\alpha}_{\mu\nu} \rightarrow \frac{N_c}{g^2} \sum_{x,\mu,\nu} \left(1 - \frac{1}{N_c} \mathrm{Tr}\, U_{\mu\nu}(x)\right).$$

2. Squark kinetic terms: Nearest-neighbor covariant derivatives,

$$\mathcal{D}_\mu A_+(x) \rightarrow \frac{1}{a} \left[U_\mu(x) A_+(x + a\hat{\mu}) - A_+(x)\right],$$

$$\mathcal{D}_\mu A_-(x) \rightarrow \frac{1}{a} \left[A_-(x + a\hat{\mu})\, U^\dagger_\mu(x) - A_-(x)\right]$$

3. Squark mass and quartic scalar interactions: Standard discretizations (ultra-local),

$$\int d^4x \rightarrow a^4 \sum_x .$$

4. Yukawa interaction: Requirement of careful construction on the lattice.

Main nontrivial issue

The Yukawa terms are not invariant under the lattice chiral transformations.

Chiral Lattice Transformations

Yukawa terms: $A_+^\dagger \bar\lambda P_+ \psi - \bar\psi P_- \lambda A_+ + A_- \bar\lambda P_- \psi - \bar\psi P_+ \lambda A_-^\dagger$

The chiral transformations for quark and gluino fields are defined as:

$$\delta\psi = \gamma_5 \left(1 - \frac{a}{2}\mathcal{D}_{\rm ov}\right)\psi, \qquad \delta\bar\psi = \bar\psi\left(1 - \frac{a}{2}\mathcal{D}_{\rm ov}\right)\gamma_5,$$

$$\delta\lambda = \gamma_5 \left(1 - \frac{a}{2}\mathcal{D}_{\rm ov}^{\rm adj}\right)\lambda, \qquad \delta\bar\lambda = \bar\lambda\left(1 - \frac{a}{2}\mathcal{D}_{\rm ov}^{\rm adj}\right)\gamma_5.$$

$\mathcal{D}_{\rm ov}$ and $\mathcal{D}_{\rm ov}^{\rm adj}$ are the overlap Dirac operators in the fundamental and adjoint representations, respectively. Further, the transformation depends on the parameter (θ), with the quark and gluino fields transforming with opposite signs, and:

$\{\gamma_5, \mathcal{D}_{\rm ov}\} = a\,\mathcal{D}_{\rm ov}\,\gamma_5\,\mathcal{D}_{\rm ov}$, $\left\{\gamma_5, \mathcal{D}_{\rm ov}^{\rm adj}\right\} = a\,\mathcal{D}_{\rm ov}^{\rm adj}\,\gamma_5\,\mathcal{D}_{\rm ov}^{\rm adj}$.

$$\begin{aligned} \delta\left(A_+^\dagger \bar\lambda P_+ \psi\right) &= A_+^\dagger(-\delta\bar\lambda)P_+\psi + A_+^\dagger\bar\lambda P_+(\delta\psi) \\ &= \frac{a}{2}A_+^\dagger\bar\lambda\left(\mathcal{D}_{\rm ov}^{\rm adj}P_+ - P_+\mathcal{D}_{\rm ov}\right)\psi + \mathcal{O}(a^2). \end{aligned}$$

Chirally Invariant Yukawa Term: Fermionic Auxiliary Fields

Introduce two non-dynamical fermionic auxiliary fields:

M. Luscher, "Exact chiral symmetry on the lattice and the Ginsparg-Wilson relation," Phys. Lett. B **428** (1998), 342–345, doi:10.1016/S0370-2693(98)00423-7 [arXiv:hep-lat/9802011].

- χ_ψ: Dirac auxiliary quark field in the fundamental representation.
- χ_λ^α: Majorana auxiliary gluino field in the adjoint representation.

$$\mathcal{S}_{\rm aux} = a^4 \sum_x \left[-\frac{\rho^\lambda}{a}\, \bar\chi_\lambda^\alpha \chi_\lambda^\alpha - \frac{2\rho^\psi}{a}\, \bar\chi_\psi \chi_\psi \right].$$

$$\mathcal{S}_{\rm Yuk} = a^4 \sum_x i\sqrt{2}\, g \Big[A_+^\dagger(\bar\lambda^\alpha + \bar\chi_\lambda^\alpha) T^\alpha P_+(\psi + \chi_\psi) - (\bar\psi + \bar\chi_\psi) P_-(\lambda^\alpha + \chi_\lambda^\alpha) T^\alpha A_+$$
$$+ A_-(\bar\lambda^\alpha + \bar\chi_\lambda^\alpha) T^\alpha P_-(\psi + \chi_\psi) - (\bar\psi + \bar\chi_\psi) P_+(\lambda^\alpha + \chi_\lambda^\alpha) T^\alpha A_-^\dagger \Big].$$

Modification of Chiral Transformations

For ψ (quarks) and χ_ψ:

$$\delta\psi = \gamma_5 \left(1 - \frac{a\mathcal{D}_{\rm ov}}{2}\right)\psi + \gamma_5 \chi_\psi, \qquad \delta\chi_\psi = \gamma_5 \frac{a\mathcal{D}_{\rm ov}}{2}\psi,$$

$$\Rightarrow \delta(\psi + \chi_\psi) = \gamma_5(\psi + \chi_\psi).$$

For λ (gluinos) and χ_λ:

$$\delta\lambda = \gamma_5 \left(1 - \frac{\mathcal{D}^{\rm adj}_{\rm ov}}{2}\right)\lambda + \gamma_5 \chi_\lambda, \qquad \delta\chi_\lambda = \gamma_5 \frac{\mathcal{D}^{\rm adj}_{\rm ov}}{2}\lambda,$$

$$\Rightarrow \delta(\lambda + \chi_\lambda) = \gamma_5(\lambda + \chi_\lambda).$$

Chiral Invariance of the Modified Action

The combinations $\psi + \chi_\psi$ and $\lambda + \chi_\lambda$ transform as ordinary continuum chiral fields, ensuring that both the modified Yukawa terms and the complete lattice action remain chirally invariant.

Integrating out the Auxiliary Fields

- The auxiliary fields appear quadratically and linearly.
- They can be functionally integrated out exactly, their integration produces effective interactions involving quarks, gluinos and squarks.
- Integration over χ_ψ gives an effective gluino-squark interaction.
- Integration over χ_λ gives a squark-field-dependent Pfaffian and an additional effective action.

As a result, the final lattice SQCD action contains three new contributions:

$$S^{\rm Total}_{\rm SQCD} = S^{L}_{\rm SQCD} + S_{\lambda\lambda} + S_{\rm Pf} + S_{\Psi\Psi}.$$

Important point

The new terms are higher order in the coupling constant, ultra-local, non-polynomial in the squark fields, and vanish in the continuum limit.

Three New Effective Contributions - Computational Setup

$$S_{\lambda\lambda} \equiv \frac{ag^2}{2}\bar{\lambda}^\alpha \left[(A_+^\dagger\{T^\alpha, T^\beta\}A_-^\dagger)P_+ + (A_-\{T^\alpha, T^\beta\}A_+)P_-\right]\lambda^\beta.$$

Already in a form suitable for perturbation theory.

$$S_{\rm Pf} \equiv -\sum_x \Big(\mathrm{Tr}\log(a\,\Xi_+(x)) + \mathrm{Tr}\log(a\,\Xi_-(x))\Big)$$

$$S_{\rm Pf} = \sum_x \frac{a^2 g^2}{2\rho^\lambda}\left(\frac{N_c^2-1}{N_c}\right)\left[(A_+^{\dagger f}A_-^{\dagger f}) + (A_-^f A_+^f)\right]$$
$$+\sum_x \frac{a^4 g^4}{16(\rho^\lambda)^2}\Bigg[\Big((A_+^{\dagger f}A_-^{\dagger f})(A_+^{\dagger f'}A_-^{\dagger f'}) + (A_-^f A_+^f)(A_-^{f'}A_+^{f'})\Big)\left(\frac{N_c^2+2}{N_c^2}\right)$$
$$+\Big((A_+^{\dagger f}A_-^{\dagger f'})(A_+^{\dagger f'}A_-^{\dagger f}) + (A_-^f A_+^{f'})(A_-^{f'}A_+^f)\Big)\left(\frac{N_c^2-4}{N_c}\right)\Bigg] + \mathcal{O}(g^6)\,.$$

$$\Xi \equiv \Xi_+\,P_+ + \Xi_-\,P_-\,,\quad \Xi^{\alpha\beta} = -\frac{\rho^\lambda}{a}\left(\delta^{\alpha\beta} - \frac{a^2\,g^2}{2\,\rho^\lambda}\,Q^{\alpha\beta}\right),\quad Q^{\alpha\beta} = (A_+^\dagger\{T^\alpha, T^\beta\}A_-^\dagger)P_+ + (A_-\{T^\alpha, T^\beta\}A_+)P_-\,.$$

Three New Effective Contributions -Computational Setup

$S_{\Psi\Psi} \equiv \frac{1}{4} a^4 \sum_x (\bar{\Psi}^\alpha - (\Psi^T)^\alpha \mathcal{C}) \left[(\mathcal{C}^T \Xi)^{-1} \right]^{\alpha\beta} ((\bar{\Psi}^T)^\beta + \mathcal{C}\Psi^\beta)$

To obtain its perturbative form (up to terms of $\mathcal{O}(g^6)$), it suffices to insert $\Xi^{-1} = (\Xi_+)^{-1} P_+ + (\Xi_-)^{-1} P_-$ into $\mathcal{S}_{\Psi\Psi}$ and expand to first subleading order:

$$\left((\Xi_+)^{-1}\right)^{\alpha\beta} = -\frac{a}{\rho^\lambda}\,\delta^{\alpha\beta} - \frac{a^3\,g^2}{2(\rho^\lambda)^2}\,A_+^\dagger \{T^\alpha, T^\beta\} A_-^\dagger + \mathcal{O}(g^4)\,,$$

$$\left((\Xi_-)^{-1}\right)^{\alpha\beta} = -\frac{a}{\rho^\lambda}\,\delta^{\alpha\beta} - \frac{a^3\,g^2}{2(\rho^\lambda)^2}\,A_- \{T^\alpha, T^\beta\} A_+ + \mathcal{O}(g^4)\,,$$

$$\Psi^\alpha = i\sqrt{2} g (A_+^\dagger P_+ + A_- P_-) T^\alpha \psi + \frac{a\,g^2}{2} Q^{\alpha\beta} \lambda^\beta,$$

$$\bar{\Psi}^\alpha = -i\sqrt{2} g\,\bar{\psi}\, T^\alpha (A_+ P_- + A_-^\dagger P_+) + \frac{a\,g^2}{2} \bar{\lambda}^\beta Q^{\beta\alpha}$$

$Q^{\alpha\beta} = (A_+^\dagger \{T^\alpha, T^\beta\} A_-^\dagger) P_+ + (A_- \{T^\alpha, T^\beta\} A_+) P_-\,,\quad \Xi \equiv \Xi_+ P_+ + \Xi_- P_-\,,\quad \Xi^{\alpha\beta} = -\frac{\rho^\lambda}{a} \left(\delta^{\alpha\beta} - \frac{a^2\,g^2}{2\,\rho^\lambda} Q^{\alpha\beta} \right).$

Perturbative Setup - Calculations

Computational Setup

$\boxed{\text{Perturbative expansion}} \Longrightarrow \boxed{\text{Interaction vertices}}$

$\overset{\text{Wick contractions}}{\Longrightarrow} \boxed{\text{Feynman diagrams}} \Longrightarrow \boxed{\text{Green's functions}}$

See the poster by Constantinos Costa, "A Software Package for Multiloop Feynman Diagram Calculations on the Lattice."

Goal

Use this perturbative framework to compute the critical masses and the renormalization factors for all fields, masses, and (gauge, Yukawa, quartic) couplings.

Results obtained so far

We have computed the self-energies of all fields, namely gluons, gluinos, quarks, squarks, and ghosts, including nonzero quark and squark masses, m_ψ and m_A, respectively.

One-Loop Gluon Propagator

The gluon inverse propagator is obtained at one-loop order by summing the diagrams contributing to the 2-pt Green's function $\langle u_\mu \, u_\nu \rangle$.

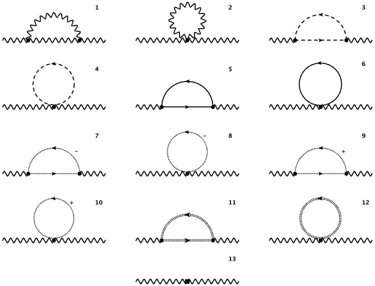

Figure 1: One-loop diagrams for the gluon two-point Green's function. A wavy line denotes gluons, a dashed line denotes gluinos, a dotted line denotes squarks, a solid line denotes quarks, a double dotted line denotes the ghost field. The square vertex denotes contributions from the integration measure.

Lattice One-Loop Result

For $r^\lambda = r^\psi = 1$, the gluon inverse propagator takes the form:

$$\langle \tilde{u}_\mu^\alpha(q)\, \tilde{u}_\nu^\beta(q')\rangle_{\rm inv}^{LR} = (2\pi)^4 \delta(q+q')\ \frac{\delta^{\alpha\beta}}{2}\left[\frac{1}{\alpha} q_\mu q_\nu + \left(q^2\delta_{\mu\nu} - q_\mu q_\nu\right)\left(1 - \Sigma_u^{\rm latt}(q, m_A, m_\psi)\right)\right],$$

where the gluon lattice self-energy is:

$$\begin{aligned}
\Sigma_u^{\rm latt}(q, m_A, m_\psi) = \frac{g^2}{16\pi^2}\Bigg[& N_f \Bigg(c_u^{N_f} - \frac{4}{3q^3}(4m_A^2 + q^2)^{\frac{3}{2}}\ \mathrm{arctanh}\left(\frac{q}{\sqrt{4m_A^2 + q^2}}\right) \\
& + \frac{8}{3q^3}\sqrt{4m_\psi^2 + q^2}\,(2m_\psi^2 - q^2)\,\mathrm{arctanh}\left(\frac{q}{\sqrt{4m_\psi^2 + q^2}}\right) \\
& + \frac{16}{3q^2}(m_A^2 - m_\psi^2) - \frac{2}{3}\log\left(a^2 m_A^2\right) - \frac{4}{3}\log\left(a^2 m_\psi^2\right)\Bigg) \\
& + \frac{39.4784}{N_c} + N_c\left(c_u^{N_c} + 1.7726\alpha - \frac{1}{2}\alpha^2 + (3-\alpha)\log\left(a^2 q^2\right)\right)\Bigg].
\end{aligned}$$

$c_u^{N_f}$ ($c_u^{N_c}$) depends on the parameters ρ^ψ (ρ^λ). We have obtained results for parameter values ranging from 0.1 to 1.9, in steps of 0.1.

Gluon Field Renormalization

The gluon field renormalization factor is determine by:

$$\langle \tilde{u}^{\alpha}_{\mu}(q)\, \tilde{u}^{\beta}_{\nu}(q')\rangle^{\overline{\mathrm{MS}}}_{\mathrm{inv}} = \left[Z_u^{LR,\overline{\mathrm{MS}}}\right]^{-1} \langle \tilde{u}^{\alpha}_{\mu}(q)\, \tilde{u}^{\beta}_{\nu}(q')\rangle^{LR}_{\mathrm{inv}}.$$

For $r^{\psi} = \rho^{\psi} = r^{\lambda} = \rho^{\lambda} = 1$, at one loop:

$$Z_u^{LR,\overline{\mathrm{MS}}} = 1 - \frac{g^2}{16\pi^2}\Big[N_c\left(43.7766 - 2.7726\alpha - (3-\alpha)\log\left(a^2\bar{\mu}^2\right)\right) + \frac{39.4784}{N_c} - N_f\left(4.7247 + 2\log\left(a^2\bar{\mu}^2\right)\right)\Big].$$

One-Loop Gluino Propagator

The gluino inverse propagator is obtained at one-loop order by summing the diagrams contributing to the 2-pt Green's function $\langle\lambda\bar{\lambda}\rangle$.

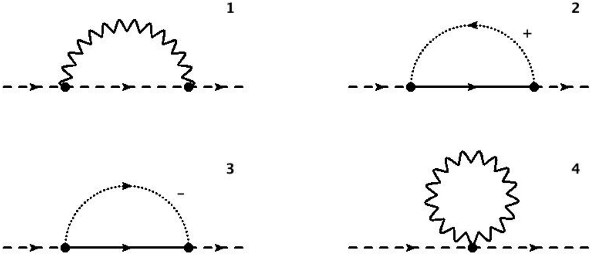

Figure 2: One-loop diagrams for the gluino two-point Green's function.

Lattice One-Loop Result

The one-loop inverse gluino propagator takes the form:

$$\left\langle \tilde{\lambda}^{\alpha}(q)\bar{\tilde{\lambda}}^{\beta}(q')\right\rangle^{LR}_{\rm inv} = (2\pi)^4\delta(q-q')\, i\not{q}\,\frac{\delta^{\alpha\beta}}{4}\left[1-\Sigma^{\rm latt}_{\lambda}(q,m_A,m_\psi)\right].$$

where:

$$\begin{aligned}\Sigma^{\rm latt}_{\lambda}(q,m_A,m_\psi) = \frac{g^2}{16\pi^2}\Bigg[& N_f\Bigg(c_\lambda^{N_f} + \log\left(a^2q^2\right) - \frac{1}{q^2}\left(m_A^2 - m_\psi^2\right) \\ & + \frac{1}{2(q^2)^2}\left((m_A^2-m_\psi^2)^2 + 2m_A^2q^2\right)\log\left(\frac{m_A^2}{m_\psi^2}\right) - \log\left(\frac{q^2}{m_A m_\psi}\right) \\ & + \frac{m_A^2 - m_\psi^2 + q^2}{2(q^2)^2}\sqrt{(m_A^2-m_\psi^2)^2 + 2(m_A^2+m_\psi^2)q^2 + (q^2)^2}\,\times \\ & \log\left(\frac{m_A^2+m_\psi^2+q^2+\sqrt{(m_A^2-m_\psi^2)^2+2(m_A^2+m_\psi^2)q^2+(q^2)^2}}{m_A^2+m_\psi^2+q^2-\sqrt{(m_A^2-m_\psi^2)^2+2(m_A^2+m_\psi^2)q^2+(q^2)^2}}\right)\Bigg) \\ & + N_c\left(\alpha\log\left(a^2q^2\right) + c_\lambda^{N_c} - 4.79201\,\alpha\right)\Bigg].\end{aligned}$$

$c_\lambda^{N_f}$ and $c_\lambda^{N_c}$ are numerical coefficients that depend on the parameters $(r^\psi,\, \rho^\psi)$ and $(r^\lambda,\, \rho^\lambda)$, respectively.

Gluino Field Renormalization

The gluino field renormalization factor is determined from the following condition:

$$\left\langle \tilde{\lambda}^{\alpha}(q)\bar{\tilde{\lambda}}^{\beta}(q')\right\rangle_{\rm inv}^{\overline{\rm MS}} = \left[Z_{\lambda}^{LR,\overline{\rm MS}}\right]^{-1}\left\langle \tilde{\lambda}^{\alpha}(q)\bar{\tilde{\lambda}}^{\beta}(q')\right\rangle_{\rm inv}^{LR}.$$

At one loop, for $r^{\psi} = \rho^{\psi} = r^{\lambda} = \rho^{\lambda} = 1$:

$$Z_{\lambda}^{LR,\overline{\rm MS}} = 1 - \frac{g^2 N_c}{16\pi^2}\left[\alpha\left(\log\left(a^2\bar{\mu}^2\right) - 3.79201\right) - 32.8386\right]$$

$$-\frac{g^2 N_f}{16\pi^2}\left[-0.07534 + \log\left(a^2\bar{\mu}^2\right)\right].$$

One-Loop Quark Propagator

The quark inverse propagator is obtained at one-loop order by summing the diagrams contributing to the 2-pt Green's function $\langle \psi \bar{\psi} \rangle$.

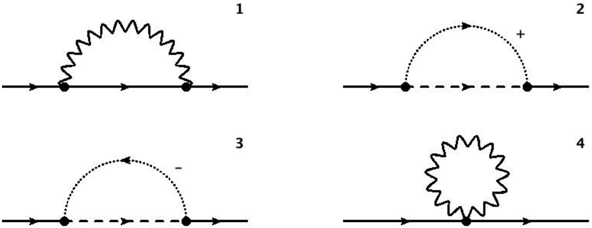

Figure 3: One-loop diagrams for the quark two-point Green's function.

Lattice One-Loop Result

The inverse quark propagator takes the form:

$$\langle\tilde{\psi}(q)\tilde{\bar{\psi}}(q')\rangle^{LR}_{\rm inv} = (2\pi)^4\delta(q-q')\left[(i\not{q} - m_\psi) - \Sigma^{\rm latt}_\psi(q, m_A, m_\psi)\right],$$

where:

$$\begin{aligned}
\Sigma^{\rm latt}_\psi(q, m_A, m_\psi) \quad = \quad & \frac{g^2 C_F}{16\pi^2}\Bigg[i\not{q}\Bigg(c^{\not{q}}_\psi + 4.7920\,\alpha + \frac{m_A^2}{q^2} - \alpha\,\frac{m_\psi^2}{q^2} \\
& - \left(\frac{m_A^4}{(q^2)^2} + 2\,\frac{m_A^2}{q^2}\right)\log\left(1 + \frac{q^2}{m_A^2}\right) + \log\left(\frac{1}{a^2(m_A^2 + q^2)}\right) \\
& + \alpha\,\log\left(\frac{1}{a^2(m_\psi^2 + q^2)}\right) - \alpha\,\frac{m_\psi^4}{(q^2)^2}\,\log\left(1 + \frac{q^2}{m_\psi^2}\right)\Bigg) \\
& - m_\psi\left(c^{m_\psi}_\psi + 5.7920\,\alpha + (3+\alpha)\log\left(\frac{1}{a^2(m_\psi^2 + q^2)}\right)\right. \\
& \left. - (3+\alpha)\,\frac{m_\psi^2}{q^2}\,\log\left(1 + \frac{q^2}{m_\psi^2}\right)\right)\Bigg].
\end{aligned}$$

$C_F = (N_c^2 - 1)/(2\,N_c)$ is the quadratic Casimir operator in the fundamental representation. The finite lattice coefficients ($c^{\not{q}}_\psi$, $c^{m_\psi}_\psi$) depend on the overlap parameters.

Quark Field and Mass Renormalization

The quark field renormalization factor is determined by:

$$\langle\tilde{\psi}(q)\tilde{\bar{\psi}}(q')\rangle^{\overline{\rm MS}}_{\rm inv} = \left[Z_\psi^{LR,\overline{\rm MS}}\right]^{-1}\langle\tilde{\psi}(q)\tilde{\bar{\psi}}(q')\rangle^{LR}_{\rm inv}.$$

From the above equation it follows that:

$$\left[Z_\psi^{LR,\overline{\rm MS}}\right]^{-1} i\not{q} - \left(\Sigma_\psi^{\rm latt} - \Sigma_\psi^B\right)\Big|_{\propto\ i\not{q}} = i\not{q},$$

$$\left[Z_\psi^{LR,\overline{\rm MS}}\right]^{-1}\left[Z_{m_\psi}^{LR,\overline{\rm MS}}\right]^{-1} m_\psi^R + \left(\Sigma_\psi^{\rm latt} - \Sigma_\psi^B\right)\Big|_{\propto\ m_\psi} = m_\psi^R.$$

For $r^\psi = \rho^\psi = r^\lambda = \rho^\lambda = 1$, at one loop:

$$Z_\psi^{LR,\overline{\rm MS}} = 1 - \frac{g^2 C_F}{16\pi^2}\left[32.9140 + 3.7920\,\alpha - (1+\alpha)\log\left(a^2\bar{\mu}^2\right)\right],$$

$$Z_{m_\psi}^{LR,\overline{\rm MS}} = 1 - \frac{g^2 C_F}{16\pi^2}\left[-32.4440 - \log\left(a^2\bar{\mu}^2\right)\right].$$

One-Loop Squark Propagator

The squark field components are arranged into a doublet:

$$A \equiv \begin{pmatrix} A_+ \\ A_-^\dagger \end{pmatrix} .$$

The diagonal part of the 2-pt Green's function $\langle \tilde{A}(q)\, \tilde{A}^\dagger(q') \rangle_{\rm inv}^{LR}$ is given by the sum of diagrams in Fig. 4, while the nondiagonal part is given by the diagrams in Fig. 5.

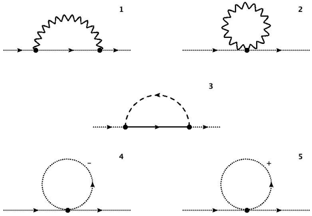

Figure 4: One-loop Feynman diagrams contributing to the two-point Green's functions $\langle A_+ A_+^\dagger \rangle$ and $\langle A_-^\dagger A_- \rangle$.

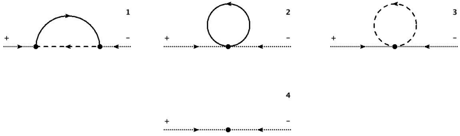

Figure 5: One-loop Feynman diagrams contributing to the two-point Green's functions $\langle A_-^\dagger A_+^\dagger \rangle$ and $\langle A_+ A_- \rangle$.

Lattice One-Loop Result

The inverse squark propagator takes the form:

$$\left\langle \tilde{A}(q)\tilde{A}^\dagger(q')\right\rangle^{LR}_{\rm inv} = (2\pi)^4\delta(q-q')\left[(q^2+m_A^2) - \Sigma_A^{\rm latt}(q,m_A,m_\psi)\right]\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix},$$

where:

$$\begin{aligned}\Sigma_A^{\rm latt}(q,m_A,m_\psi) = \frac{g^2 C_F}{16\pi^2 a^2}\Bigg[& c_{-2} + c_{-1}\, a\, m_\psi + c_{m_\psi^2}\, a^2 m_\psi^2 \\ & + a^2 m_A^2\Big(1.7920 - 3.7920\,\alpha\Big) + a^2 q^2\Big(c_{q^2} - 3.7920\,\alpha\Big) \\ & - a^2\Big((1-\alpha)m_A^2 + (3-\alpha)q^2\Big)\log\big(a^2 m_A^2\big) + a^2\Big(4m_\psi^2 + 2q^2\Big)\log\big(a^2 m_\psi^2\big) \\ & + a^2(3-\alpha)\Big(\frac{m_A^4}{q^2} - q^2\Big)\log\left(1+\frac{q^2}{m_A^2}\right) + a^2\Big(\frac{2m_\psi^4}{q^2} + 4m_\psi^2 + 2q^2\Big)\log\left(1+\frac{q^2}{m_\psi^2}\right)\Bigg]\end{aligned}$$

The coefficients c_{-2}, c_{-1}, $c_{m_\psi^2}$, and c_{q^2} depend on the overlap parameters.

Critical Mass – Squark Field and Mass Renormalization

There are terms proportional to inverse powers of the lattice spacing give rise to a critical mass for the squark fields (for $r^\psi = \rho^\psi = r^\lambda = \rho^\lambda = 1$):

$$m^{2\,\text{squark}}_{\text{crit.}} = -\frac{g^2\, C_F}{16\,\pi^2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \left[30.8596\,\frac{1}{a}\,m_\psi\; - 77.2913\,\frac{1}{a^2}\right].$$

The squark field renormalization factor is determined by:

$$\left\langle \tilde{A}(q)\,\tilde{A}^\dagger(q')\right\rangle^{\overline{\text{MS}}}_{\text{inv}} = \left[Z_A^{LR,\overline{\text{MS}}}\right]^{-1} \left\langle \tilde{A}(q)\,\tilde{A}^\dagger(q')\right\rangle^{LR}_{\text{inv}},$$

and the corresponding mass renormalization factor is defined by: $Z^{LR,\overline{\text{MS}}}_{m_A}\, m_A^2 = (m_A^R)^2$. The renormalization factors of the squark field and mass in the $\overline{\text{MS}}$ scheme for $r^\psi = \rho^\psi = r^\lambda = \rho^\lambda = 1$ are:

$$Z_A^{LR,\overline{\text{MS}}} = \mathbb{1} + \frac{g^2\, C_F}{16\,\pi^2}\left[\left(-16.5576 + 3.79201\,\alpha + (1-\alpha)\log\left(a^2\bar{\mu}^2\right)\right)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{2}{3}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right],$$

$$Z_{m_A}^{LR,\overline{\text{MS}}} = \mathbb{1} - \frac{g^2\, C_F}{16\,\pi^2}\left[\left(-17.1764 - 2\,\log\left(a^2\bar{\mu}^2\right)\right)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{2}{3}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right].$$

One-Loop Ghost Propagator

The ghost inverse propagator is obtained at one-loop order by summing the diagrams contributing to the 2-pt Green's function $\langle c\bar{c}\rangle$.

Figure 6: One-loop diagrams for the ghost two-point Green's function.

Lattice One-Loop Result – Ghost Field Renormalization

The inverse ghost propagator takes the form:

$$\left\langle c^{\alpha}(q)\, \bar{c}^{\beta}(q')\right\rangle^{LR}_{\rm inv} = (2\pi)^4 \delta(q-q')\, \delta^{\alpha\beta} \left[q^2 - \Sigma_c^{\rm latt}(q)\right],$$

where:

$$\Sigma_c^{\rm latt}(q) = \frac{g^2 N_c}{16\pi^2}\, q^2 \left[4.6086 - 1.2029\,\alpha - \frac{1}{4}(3-\alpha)\log\left(a^2 q^2\right)\right].$$

The ghost field renormalization factor is determined from:

$$\left\langle c\, \bar{c}\right\rangle^{\overline{\rm MS}}_{\rm inv} = \left[Z_c^{LR,\overline{\rm MS}}\right]^{-1} \left\langle c\, \bar{c}\right\rangle^{LR}_{\rm inv},$$

which yields the following expression for $Z_c^{LR,\overline{\rm MS}}$:

$$Z_c^{LR,\overline{\rm MS}} = 1 - \frac{g^2 N_c}{16\pi^2}\left[3.6086 - 1.2029\,\alpha - \frac{1}{4}(3-\alpha)\log\left(a^2\bar{\mu}^2\right)\right].$$

Summary – Conclusion

In our ongoing work, we will compute the renormalization factors and counterterms associated with the gauge, Yukawa, and quartic couplings.

We will also investigate whether alternative squark discretizations can eliminate the critical squark mass.

Furthermore, we study perturbatively the mixing of the supercurrent operator upon renormalization using improved lattice discretizations in $\mathcal{N} = 1$ SYM. See the poster by Marios Costa, "Perturbative Renormalization of Noether Currents in SYM with Improved Discretizations".

Advantages of the Overlap Formulation

- Lattice-modified chiral symmetry at finite lattice spacing.
- Absence of fermion doublers while maintaining locality.
- Reduced set of lattice counterterms (no additive quark and gluino mass) compared with Wilson fermions.
- Framework for nonperturbative simulations.

Acknowledgements

Thank you for your attention!

RESEARCH & INNOVATION FOUNDATION

Co-funded by the European Union

Republic of Cyprus

The project EXCELLENCE/0524/0208 is implemented under the programme of social cohesion "THALIA 2021-2027" co-funded by the European Union, through Research and Innovation Foundation.