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Gauge field flow for chiral gauge theories on a slab and a disk boundary

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Publications

发表文章

- This talk is based on

  1. Jinlong Dang, Rohith Karur, Srimoyee Sen. (2026). *Gauge field flow for chiral gauge theories on a slab.* arXiv:2606.05306.

  2. Jinlong Dang, Rohith Karur, Srimoyee Sen. (2026). *Gauge field flow for chiral gauge theories on a disk boundary.* arXiv:2606.05305.

- Collaborators:

  Rohith Karur, University of California, Berkeley

  Srimoyee Sen, Iowa State University

Standard Model

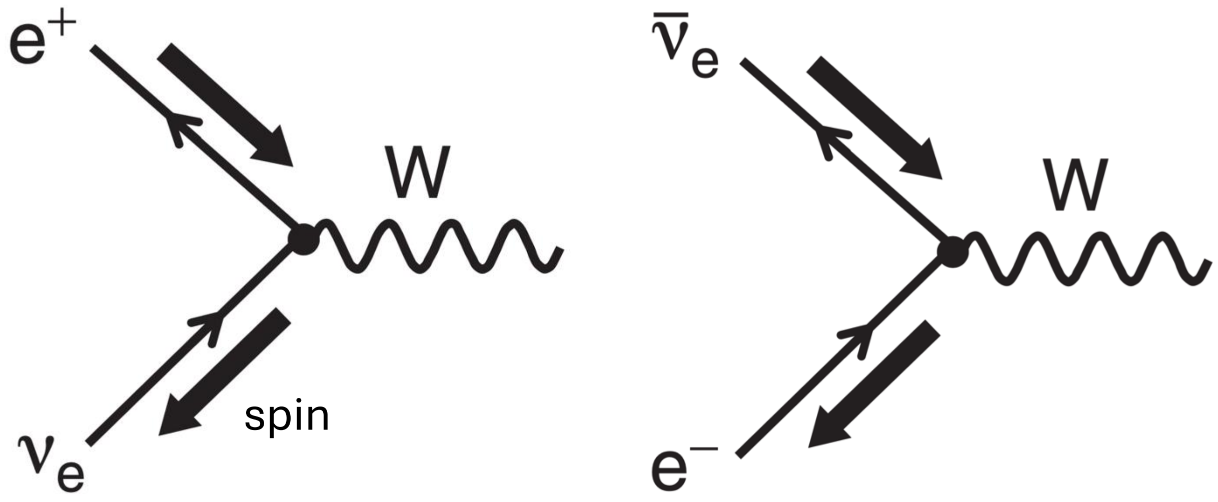
标准模型

- Standard model is a chiral gauge theory. Although it is a tremendous achievement in the last century, a non-perturbative definition of a such chiral gauge theory is notorious difficult.

Matter fields in the standard model

Field χ	$SU(3)$ representation	$SU(2)$ representation	$U(1)$ charge
$l_L = \begin{pmatrix}\nu_e\\ e\end{pmatrix}_L, \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L, \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L$	1	2	-1
$l_R = e_R,\ \mu_R,\ \tau_R$	1	1	-2
$Q_L = \begin{pmatrix}u\\ d\end{pmatrix}_L, \begin{pmatrix}c\\ s\end{pmatrix}_L, \begin{pmatrix}t\\ b\end{pmatrix}_L$	3	2	$\frac{1}{3}$
$U_R = u_R,\ c_R,\ t_R$	3	1	$\frac{4}{3}$
$D_R = d_R,\ s_R,\ b_R$	3	1	$-\frac{2}{3}$
$\phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}$	1	2	1

- In the Standard model, the gauge group of electroweak interaction has different representation for particles with different chirality.



Chiral Guage Theory

手征规范场论

- A simple example of a chiral gauge theory Lagrangian:

$$\mathcal{L} = \psi_{L_i}^{\dagger} i\bar{\sigma}\cdot \mathrm{D}\psi_{L_i} + \psi_{\mathrm{R}_i}^{\dagger} i\sigma \cdot \partial\psi_{\mathrm{R}_i}$$

- For a chiral gauge theory, a regulator typically break its chiral structure

1. Pauli-Villars Regulation

$$M\bar{\psi}\psi = \mathrm{M}\psi_{\mathrm{R}}^{\dagger}\psi_{L} + \mathrm{M}\psi_{L}^{\dagger}\psi_{\mathrm{R}}$$

2. Dimensional Regulation (γ_5-problem)

$$\{\gamma_5, \gamma^{\mu}\} = 0, \qquad \mathrm{Tr}(\gamma_5\gamma^{\mu}\gamma^{\nu}\gamma^{\rho}\gamma^{\sigma}) = -4i\epsilon^{\mu\nu\rho\sigma}, \qquad \mathrm{Tr}(\Gamma_1\Gamma_2) = \mathrm{Tr}(\Gamma_2\Gamma_1)\,.$$

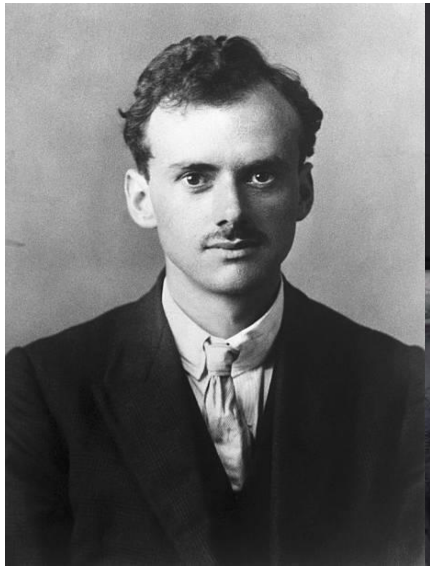
In D $\neq$ 4 dimensions, it is inconsistent to require these properties simultaneously.

- For a chiral gauge theory, break chirality $\leftrightarrow$ break gauge symmetry $\leftrightarrow$ break unitarity.

Difficulty in defining UV Physics

定义UV物理的困难

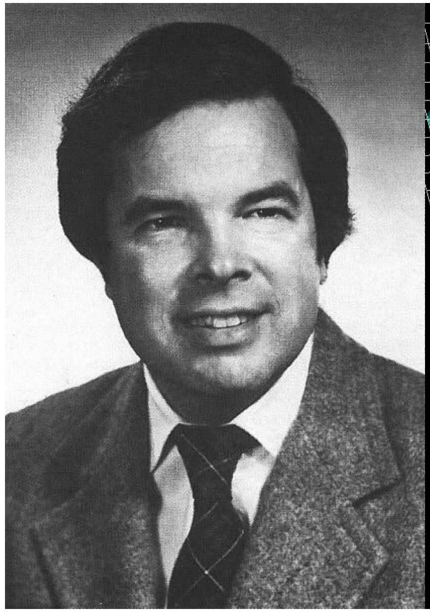
- Dirac and Feynman both made huge contribution to Quantum Field Theory, but had very different perspectives about the infinities one encounters from short distance (UV) physics.

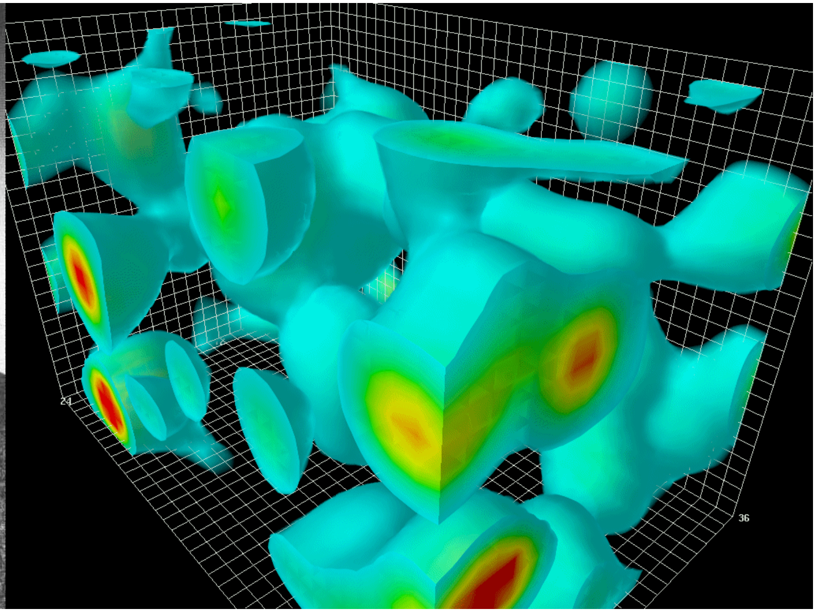


Wilson’s New Perspective

Wilson的新观点

- Wilson wanted to understand the strong interactions, where Feynman’s perturbative approach fails. So, he asked: **How can on put a quantum field theory on a computer?**





- Treat a quantum field theory as a condensed matter system
- Work in 4d Euclidian spacetime
- Approximate spacetime as a lattice of points and represent fields as interacting degrees of freedom on sites and links

- Kaplan: Specific computations aside, however, we cannot claim to thoroughly understand the theory without understanding how in principle to compute its implications in a controlled manner to arbitrary precision.

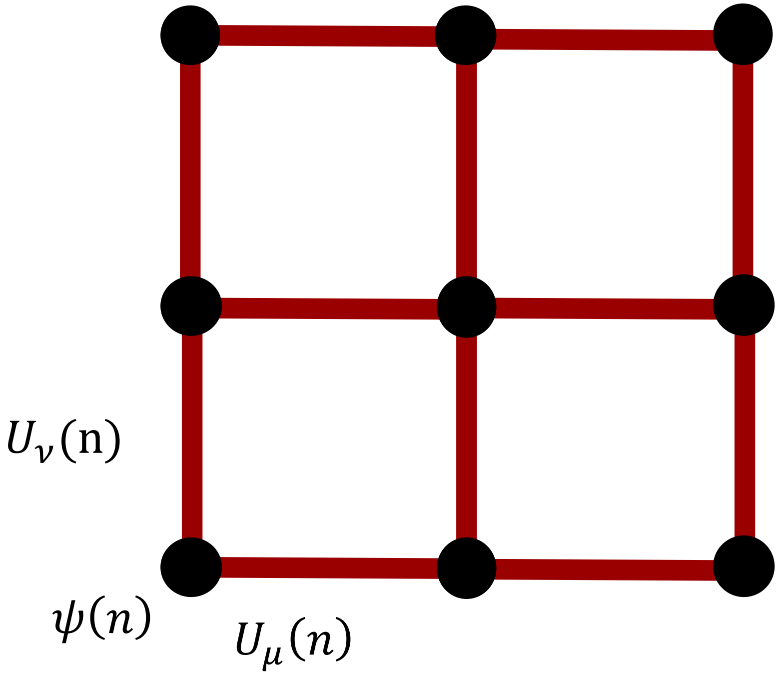
Lattice Gauge Theory

格点规范场论

- An example of N_f flavor QCD fermion action

$$S_F[\psi,\bar{\psi},U] = \sum_{f=1}^{N_f} a^4 \sum_{n,m\in\Lambda} \overline{\psi^{(f)}}(n) D^{(f)}(n \mid m)\psi^{(f)}(m)\,.$$

where a is the lattice spacing, $D^{(f)}$ is a Dirac fermion operator on lattice who depends on gauge link U_μ



$$Z = \int \mathrm{D}U\, e^{-\mathrm{S_G}} \det D^{(f)} \approx \frac{1}{\mathrm{N}}\sum \det D^{(f)}$$

$$S_G[U] = \frac{\beta}{3}\sum_{n\in\Lambda}\sum_{\mu<\nu} \mathrm{Re}\,\mathrm{tr}\left[\mathbb{1} - U_{\mu\nu}(n)\right]$$

- A Monte Carlo simulation approximates the integral by an average of the observable evaluated on N sample gauge field configurations.

Chiral lattice Gauge Theories?

手征格点规范场论?

- A Sign Problem
  - Dirac fermion

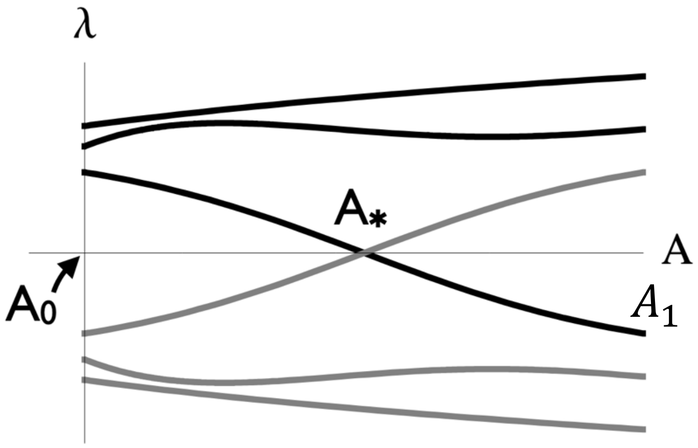
$$\det(D^{(f)}+\mathrm{m}) = \det\begin{pmatrix} \mathrm{m} & i\bar{\sigma}\cdot\mathrm{D} \\ i\sigma\cdot\mathrm{D} & \mathrm{m}\end{pmatrix} = \prod_{\lambda_n>0}(\lambda_n^2+m^2)\times\prod_{\lambda_n=0} m > 0$$

  - Chiral fermion

$$\det D_{\mathrm{L}}^{(f)} = (\det D_{\mathrm{R}}^{(f)})^{*} = \sqrt{|\det D^{(f)}|}\, e^{\mathrm{i}W[\mathrm{A}]} \in \mathbb{C}$$

Parity-violating part

- An ambiguous phase? So-called "measure problem"



Spectrum Flow
(Always Posible)

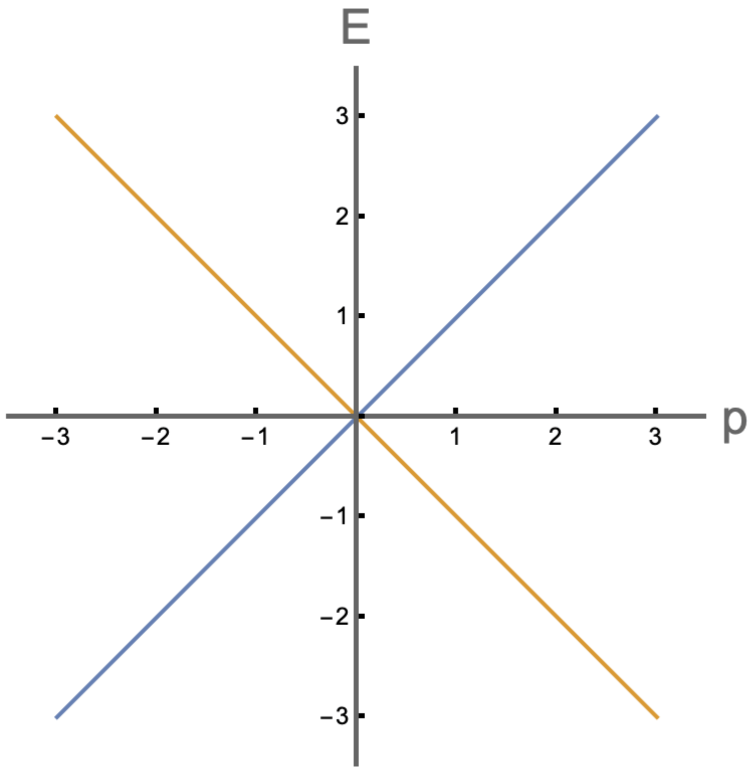
$$W[A_0] = W[A_1] \pm \pi \text{ ?}$$
$$A = (1-s)A_0 + sA_1$$
$$A_1 = g^{-1}A_0 g + g^{-1}dg$$

A crucial problem for Ginsparg-Wilson fermions formalism

Chiral lattice Gauge Theories? NO-GO!

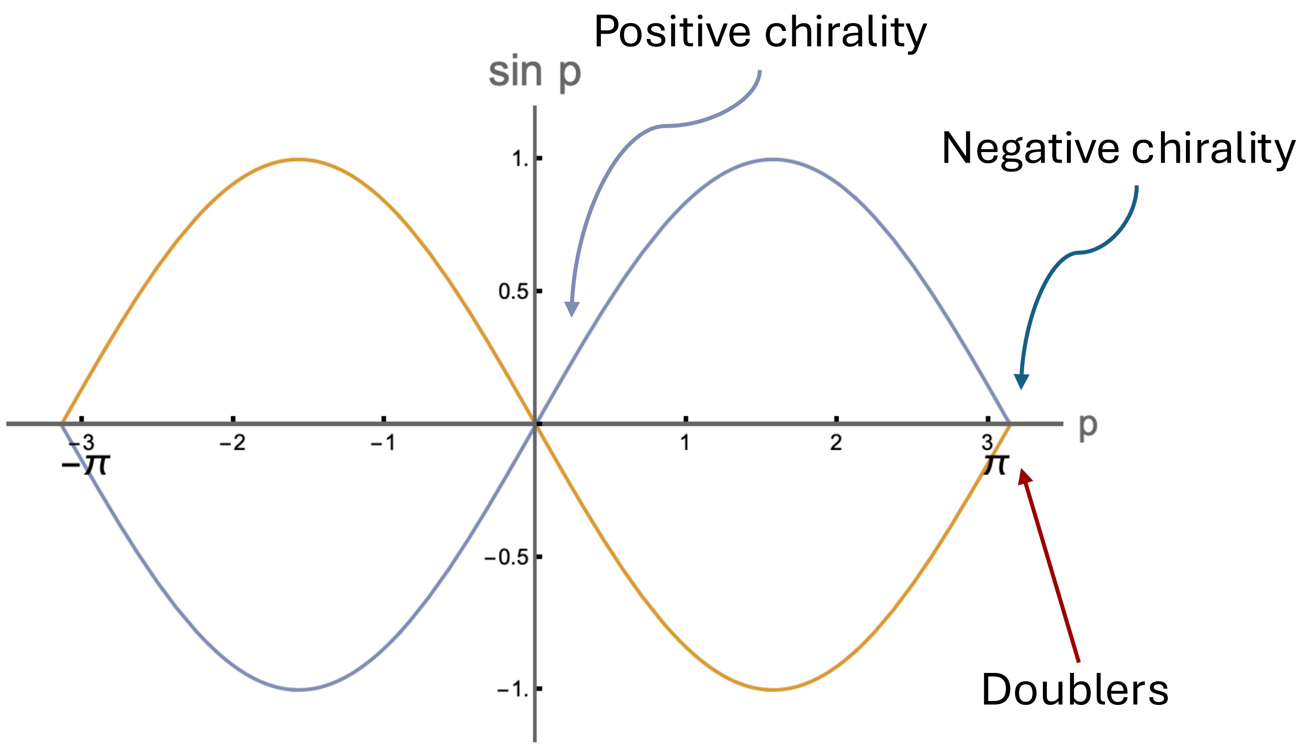
手征格点规范场论？山穷水尽

- Nilsen-Ninomiya NO-GO theorem (1981): Cannot formulate Dirac fermion with exact chiral symmetry without an unwanted doubling of all fermion species.



Continuum

$E = \pm p$



Lattice in space with continous time

$E = \pm \sin p$

Chiral lattice Gauge Theories? Ways Out…

手征格点规范场论？柳暗花明

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- Possible paths to a chiral lattice gauge theory

  - **Symmetric mass generation**, started from 1986 revived since 2011 due to Fidkowski and Kitaev

    Simulated in 1+1D using tensor networks (DMRG) to avoid sign problem, *arXiv:2202.12355*

  - **Symmetry disentanglers,** proposed in 2026 in Thorngren, Preskill, Fidkowski, *arXiv:2601.04304.*

    No simulation yet

  - **Domain Wall,** vector fermions proposed by Kaplan in 1992, chiral fermions in 2024

    Abundant application in LQCD with vector fermions by Lagrangian method.
    NO Hamiltonian method explored even for vector fermions

  - …

Domain Wall Fermion

畴壁费米子

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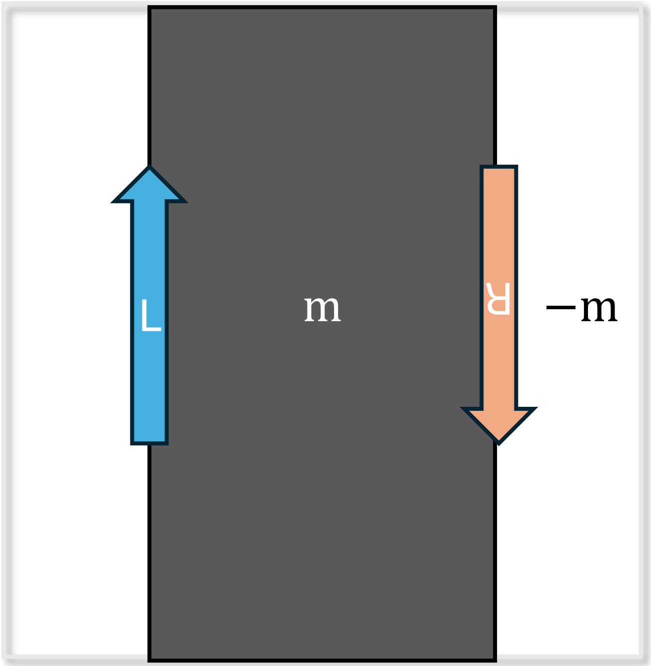
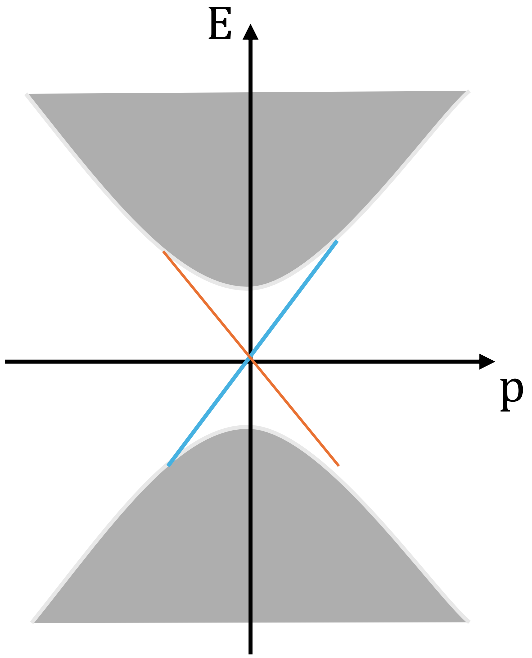
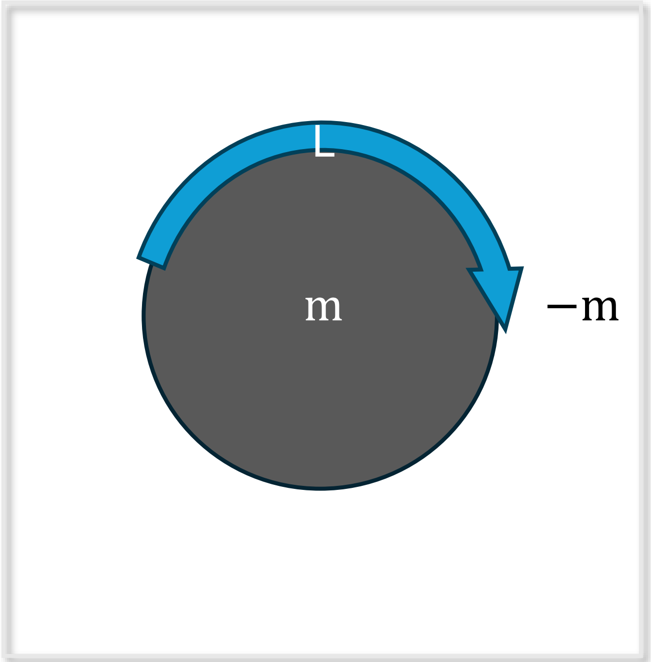
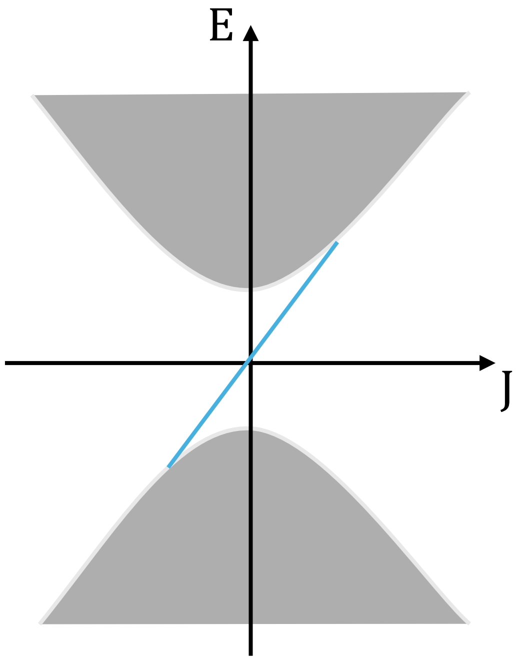
- n + 1-D fermions with mass defects ⟷ Topological protected chiral modes on n-D defects

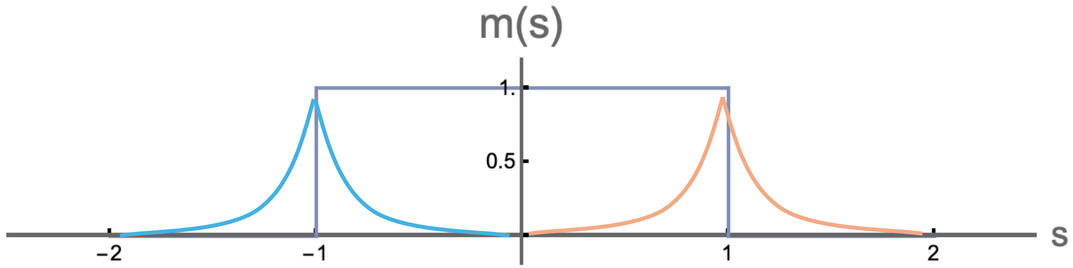
$$H = -i\gamma^i \partial_i + m(s) + \frac{w}{2}\Delta$$









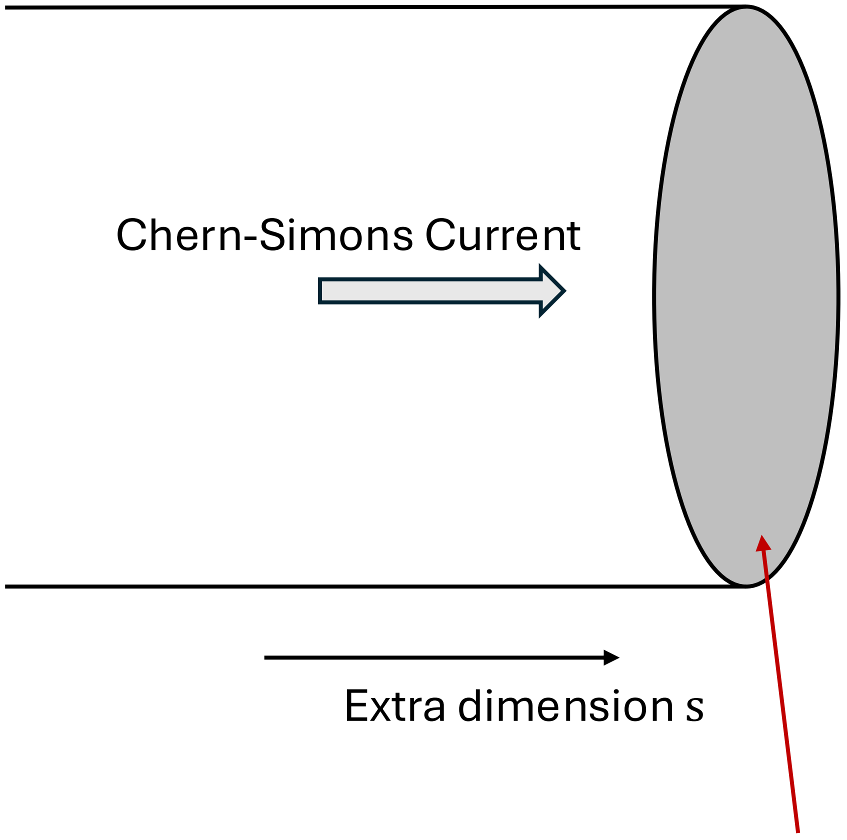


Kaplan, Phys. Lett. B. (1992)
Kaplan, Sen, Phys.Rev.Lett . 132 (2024) 14, 141604

Callan-Harvey Effect

Callan-Havey效应

➢ The anomaly of the boundary theory is provided by anomaly inflow from the bulk, like Quantum Hall.



➢ After integrating out bulk contribution, there is an effective Chern-Simons action, e.g., for $\mathrm{U}(1)$ gauge theory in 2+1D

$$S_3 = q^2 c_3 \int A \wedge dA$$

➢ Chern-Simons current provides the anomaly inflow

$$\nabla_t j_t + \nabla_x j_x = -\nabla_s j_s = q c_3 \mathrm{F}_{\mathrm{xt}}$$

➢ What Callan-Harvey missed: UV physics could make current jump

$$2\pi\, c_3 \in \pi_2(T^2) = \mathbb{Z}$$

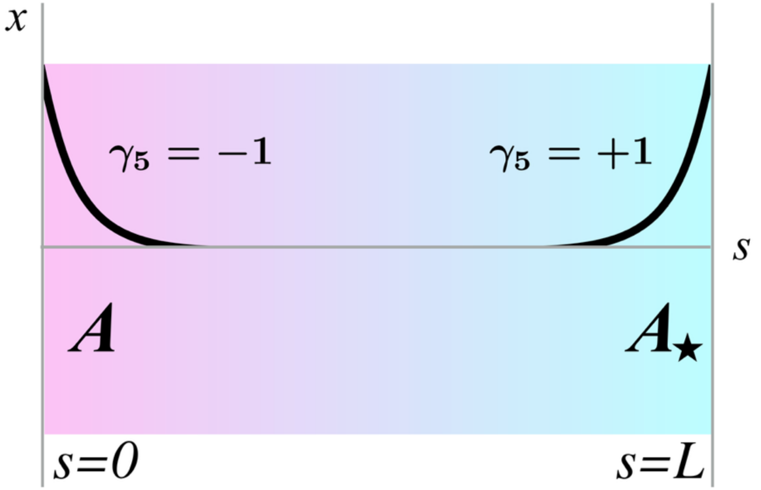
Gauge Field Flow

规范场流

➢ We use gauge field flow to extend our gauge field from the defects to the bulk.

- Gradient Flow

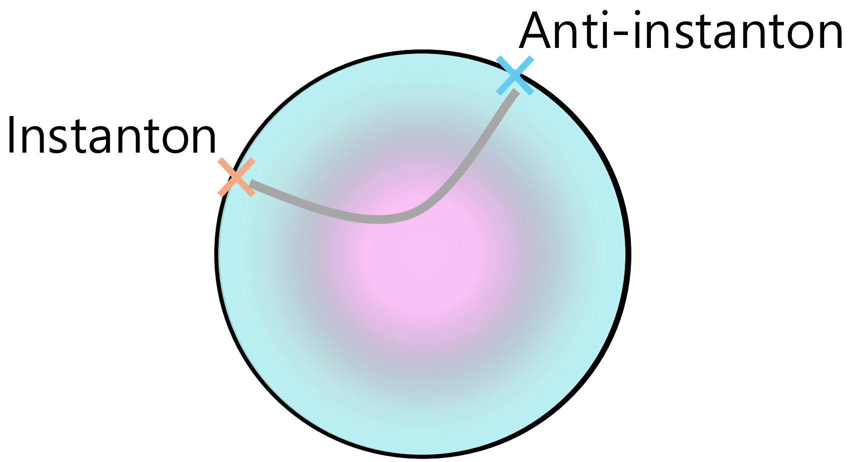
$$\frac{\partial \bar{A}}{\partial s} = -\xi \frac{\delta S_d^{YM}}{\delta \bar{A}} \longrightarrow \partial_s \bar{A}_\mu = \frac{\xi \epsilon(s)}{|\Lambda|} \partial_\lambda \bar{F}_{\lambda\mu}$$



Physical gauge field damps out $e^{-\xi p^2 |s|/m}$

- Annealing flow(Equation of motion flow)

$$\delta S_{d+1}^{YM} = 0 \xrightarrow[A_s = 0]{\text{On a slab}} \partial_s^2 \bar{A}_\mu + \partial_\nu(\partial_\nu \bar{A}_\mu - \partial_\mu \bar{A}_\nu) = 0.$$



Omitting many caveats

Grabowska, Kaplan, Phys. Rev. D. 94, 114504 (2016)
Kaplan, Sen, arXiv:2412.02024

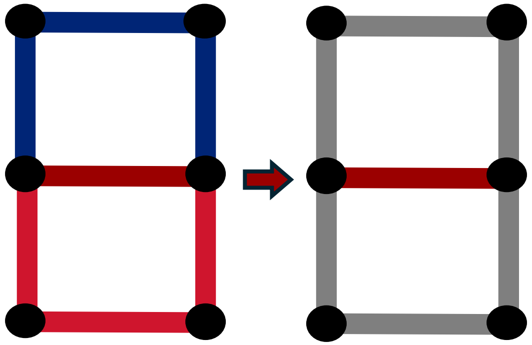
Gradient Flow on Lattice

格点上的梯度流

- In 2+1D, with a Wilson action $S_{\mathrm{W}}(s) = \frac{1}{2}\sum_{t,x} \mathrm{Re}\left\{1 - U_{\text{plaquette}}(t,x)\right\}.$

comes a 'Wilson flow' into the extra dimension as

$$\partial_s U_\mu(\mathrm{n}) = -\frac{\delta S_{\mathrm{W}}}{\delta U_\mu(\mathrm{n})} U_\mu(\mathrm{n}) \equiv -\mathrm{Z}[\mathrm{U}] U_\mu(\mathrm{n}), \qquad U_s \equiv 1$$



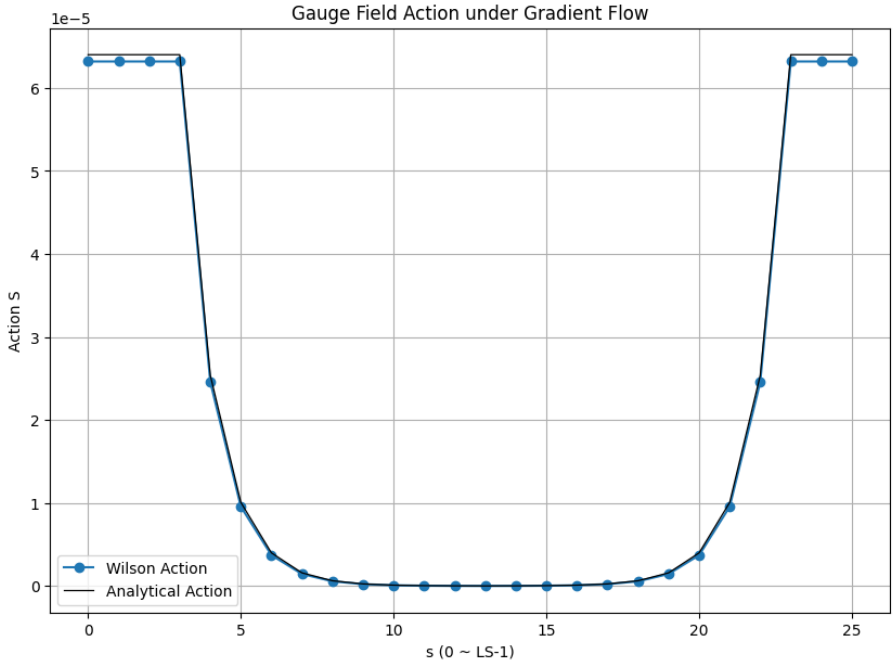
The Euler scheme for numerical calculation of the 'Wilson flow' equation

$$U_\mu(n + \epsilon\hat{s}) = e^{-\epsilon\xi Z[U]} U_\mu(n)$$

For U(1) field, the scheme only converge when $\epsilon\xi < \frac{1}{2}$.

- Set Boundary as $A_x(t) = \frac{L_t}{2\pi} E_0 \cos\left(\frac{2\pi}{L_t} t\right)$

In continuum, we expect the flow to bulk as $\times\, e^{-\xi\left(\frac{2\pi}{L_t}\right)^2 \Delta s}$



$\xi = 3, \epsilon = 1/7$

Lüscher, Commun. Math. Phys. 93, 899–919 (2010)

Equation of Motion Flow on Lattice

格点上的运动方程流

- We solve for Equation of Motion Flow by a gradient flow in an imaginary flow direction τ

$$\partial_i \bar{\mathrm{F}}_{\mathrm{i}\mu} = 0 \longrightarrow \partial_\tau \bar{\mathrm{A}}_\mu = \xi \partial_i \bar{\mathrm{F}}_{\mathrm{i}\mu}$$

On lattice

$$\partial_s U_\mu(\mathrm{n}) = -\frac{\delta S_{2+1}}{\delta U_\mu(\mathrm{n})} U_\mu(\mathrm{n}) \equiv -\mathrm{Z}_{2+1}[\mathrm{U}]U_\mu(\mathrm{n}), \qquad U_s \equiv 1$$

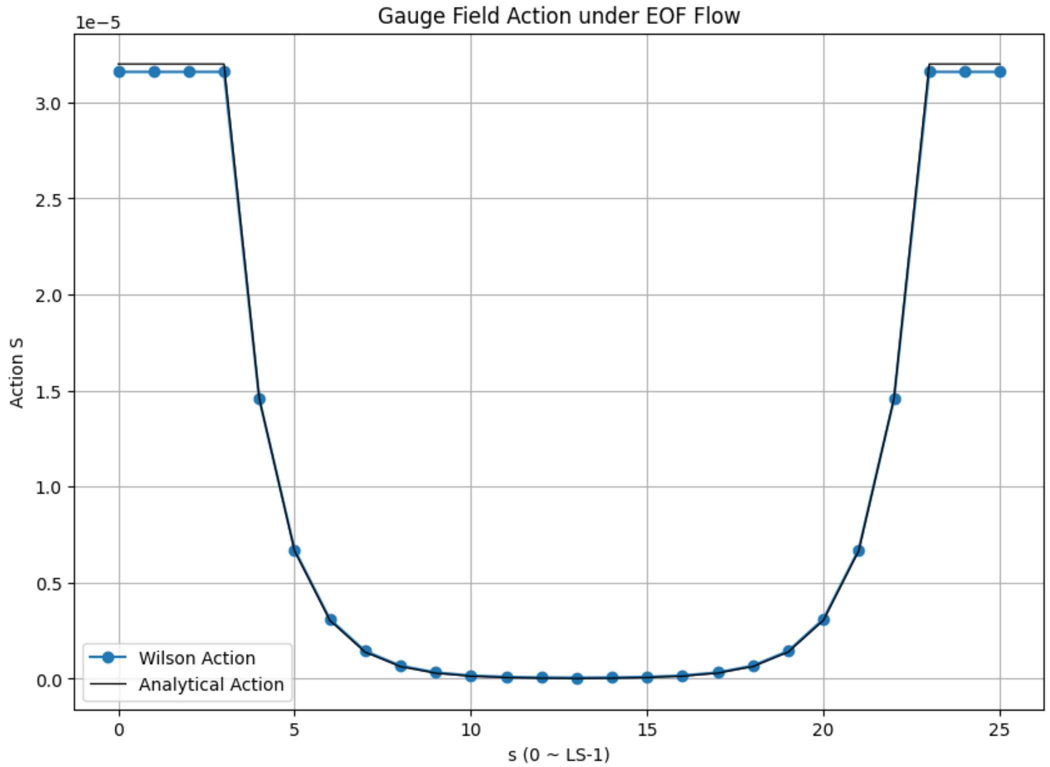
Only converging to some minimum point.

- Set the boundary as

$$A_x(t) = \frac{L_t}{2\pi} E_0 \cos\left(\frac{2\pi}{L_t} t\right)$$

In continuum, we expect

$$\times\, e^{-\xi\left(\frac{2\pi}{L_t}\right)\Delta s}$$



Lattice Construction

格点构造

- A lattice of size $L_t \times L_x \times L_s$ with $\mathrm{Q} = 3,4,5,0$ fermions coupled to a background gauge field

$$S = \sum_{n,m,Q} \overline{\psi_Q^n}\, D_{nm}^Q \psi_Q^m$$

$$= \frac{1}{2}\sum_{n,i,Q}\left(\overline{\psi_Q^n}\gamma_\mu [U_i(n)]^Q \psi_Q^{n+\hat{\imath}} - \overline{\psi_Q^{n+\hat{\imath}}}\gamma_\mu [U_i^*(n)]^Q \psi_Q^n\right) + m_Q(s)\sum_n \overline{\psi_Q^n}\,\psi_Q^n$$

$$+ w\sum_{n,i}\left(-2\overline{\psi_Q^n}\psi_Q^n + \overline{\psi_Q^n}[U_i(n)]^Q\psi_Q^{n+\hat{\imath}} + \overline{\psi_Q^{n+\hat{\imath}}}[U_i^*(n)]^Q\psi_Q^n\right)$$

If with dynamic gauge field

$$\widetilde{D^Q} = \frac{D^Q}{\sqrt{D^Q (D^Q)^\dagger + \mu^2 \delta(s) + \epsilon}}$$

Might be the first step to a Hamiltonian method

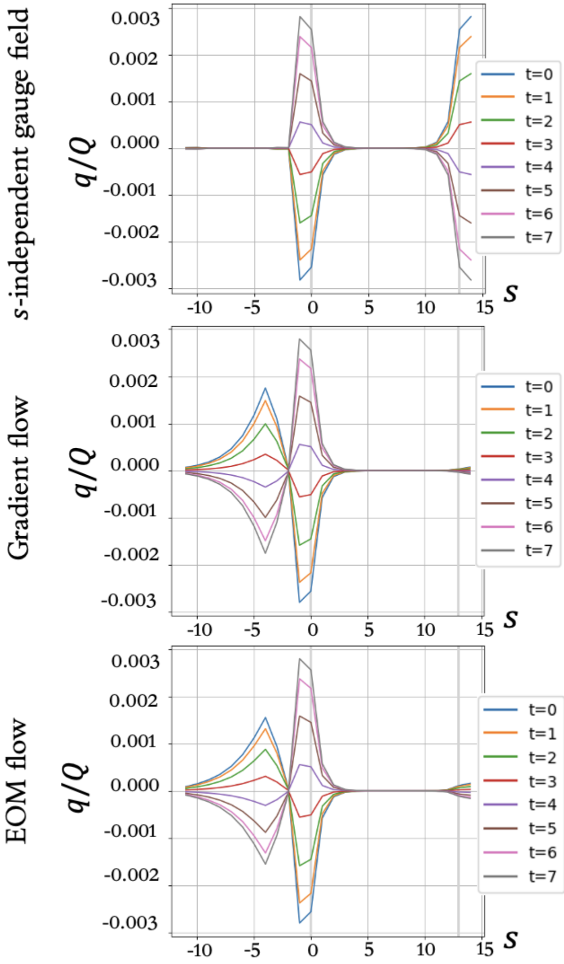
$$j_i^{Q,2+1} = \frac{1}{2}\left[\overline{\psi_Q^n}\gamma_i [U_i(n)]^Q \psi_Q^{n+\hat{\imath}} + \overline{\psi_Q^{n+\hat{\imath}}}\gamma_i [U_i^*(n)]^Q \psi_Q^n\right] + w\left[\overline{\psi_Q^n}[U_i(n)]^Q\psi_Q^{n+\hat{\imath}} - \overline{\psi_Q^{n+\hat{\imath}}}[U_i^*(n)]^Q\psi_Q^n\right]$$

$$\langle j_i^{Q,2+1}\rangle = \frac{1}{\hat{Z}}\int [D\bar{\psi}D\psi]e^{-S} j_i^{Q,2+1} = -\operatorname{Tr}\left[\hat{\mathrm{j}}_i^{Q,2+1} h^{-1}\right]$$

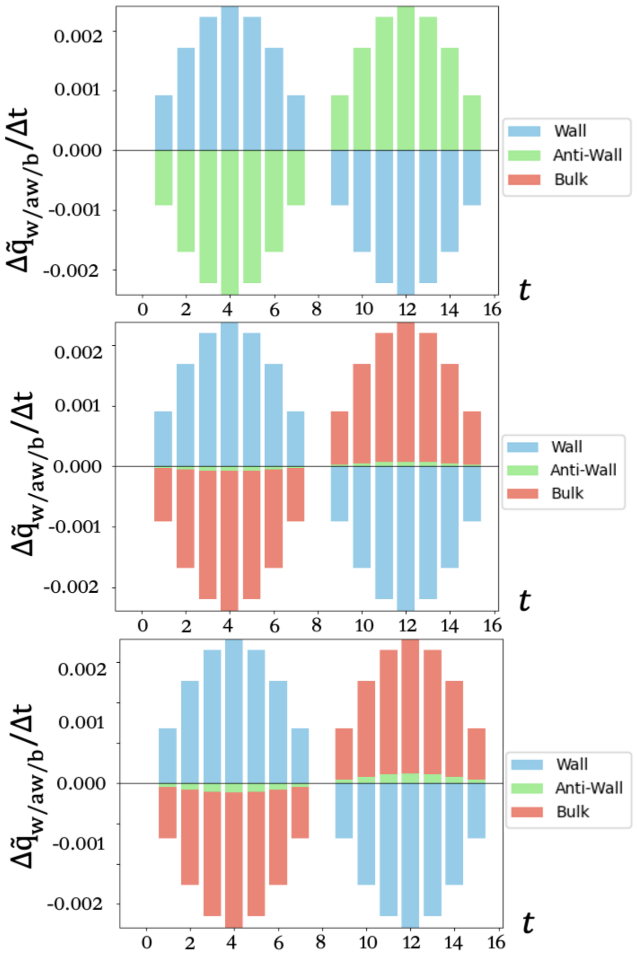
On a Slab Domain Wall

板形畴壁

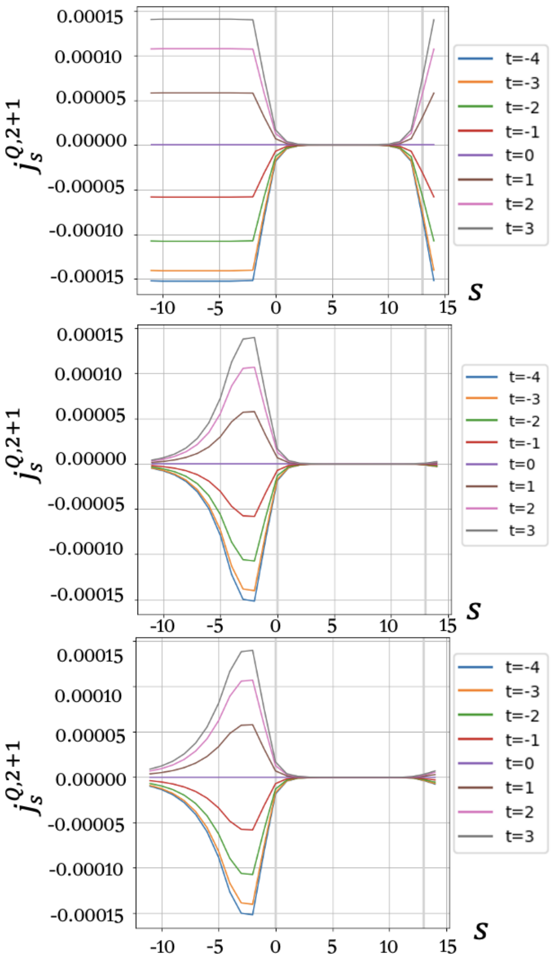
- Current density and charges



Charge per unit length in s as a function of s



Plot of rate of change of total charge on wall, anti-wall and bulk



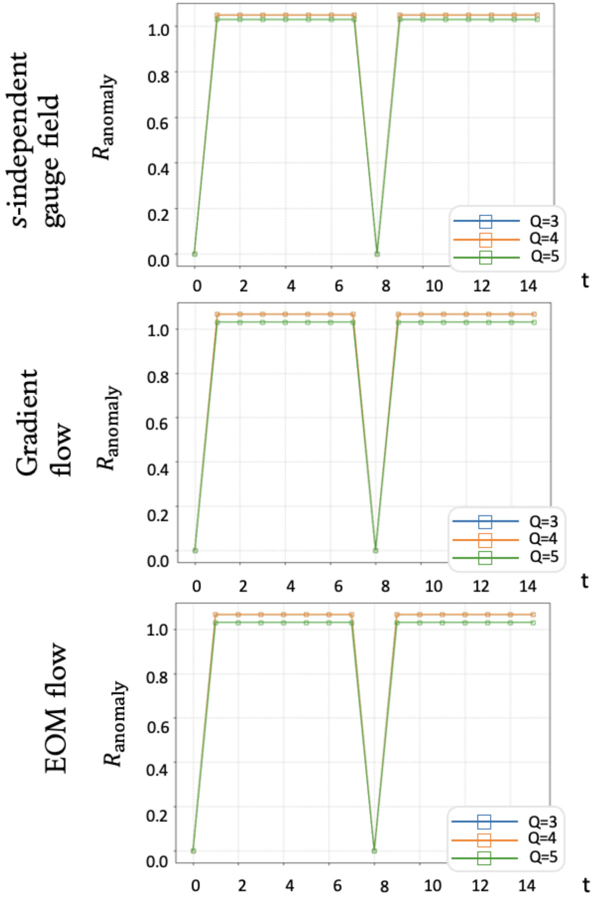
The $2+1$ dimensional current $j_s^{2+1,Q}$ for $Q=3$ normalized by $Q=3$

On a Slab Domain Wall

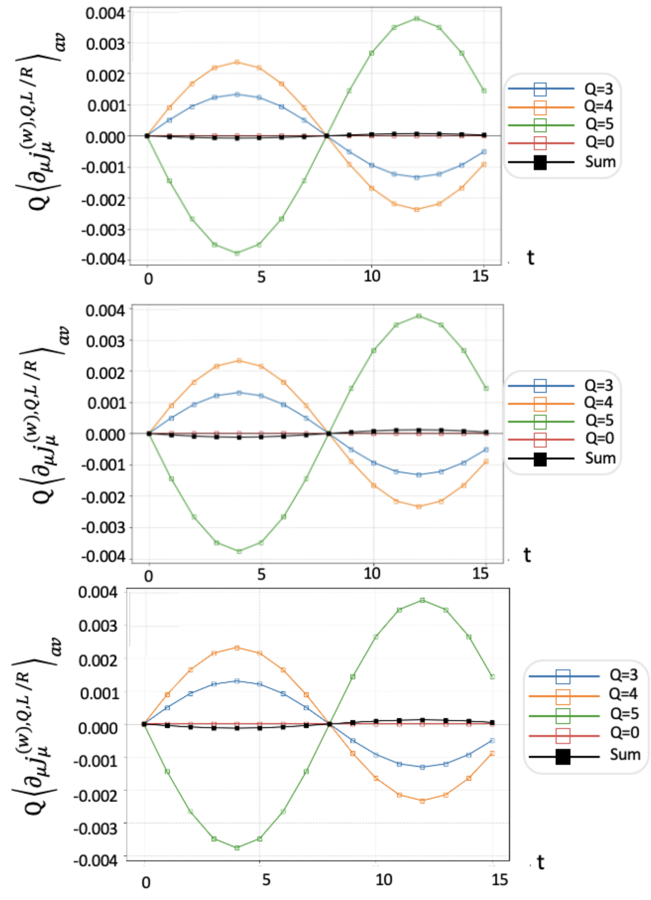
板形畴壁

$$R_{\text{Anomaly}}(t) \equiv \pm\left(QE_{\text{eff}}(t)/2\pi\right)/\left\langle \partial_\mu j_\mu^{(w),Q,L}(t)\right\rangle_{\text{av}}$$

- Anomalies



Plot of the anomaly ratio

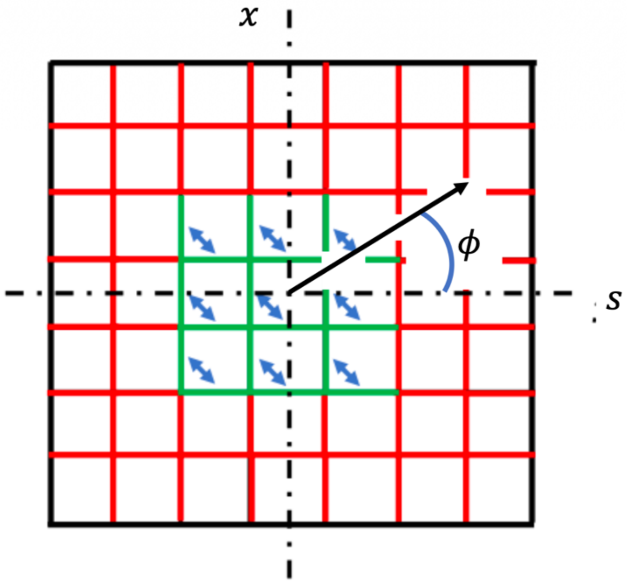


Two – dimensional divergence $Q\left\langle \partial_\mu j_\mu^{(w),Q,L/R}\right\rangle_{av}$ for all flavors and their sum

Equation of Motion Flow on Lattice

格点上的运动方程流

➢ We want solve the Equation of Motion Flow in a cylindrical way on a square lattice.

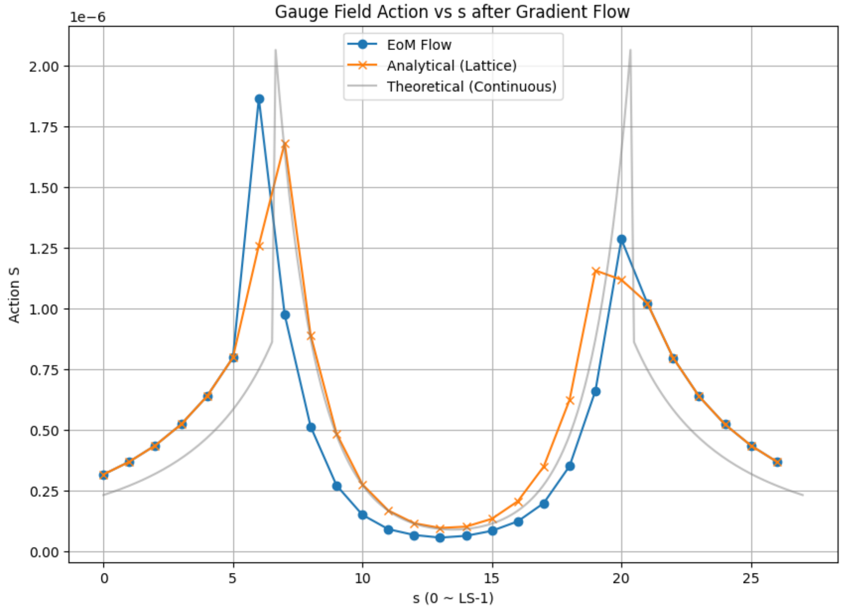


$$U_r(n) = [U_s(n)]^{\cos\phi}[U_x(n)]^{\sin\phi} \equiv 1$$
$$U_\phi(n) = [U_s(n)]^{-\sin\phi}[U_x(n)]^{\cos\phi}.$$

$$\partial_\tau U_\phi(n) = -\xi\left\{\frac{\delta\hat{S}}{\delta U_\phi(n)}\right\}U_\phi(n) \equiv \xi Z_{n,\phi}U_\phi(n)$$

$$U_s'(n) = \exp\left(-\xi\epsilon Z_{n,\phi}\sin\phi\right)U_s(n)$$
$$U_x'(n) = \exp\left(+\xi\epsilon Z_{n,\phi}\cos\phi\right)U_x(n).$$

$$Z_{n,\phi} = (-\sin\phi\, Z_{n,s} + \cos\phi\, Z_{n,x})$$



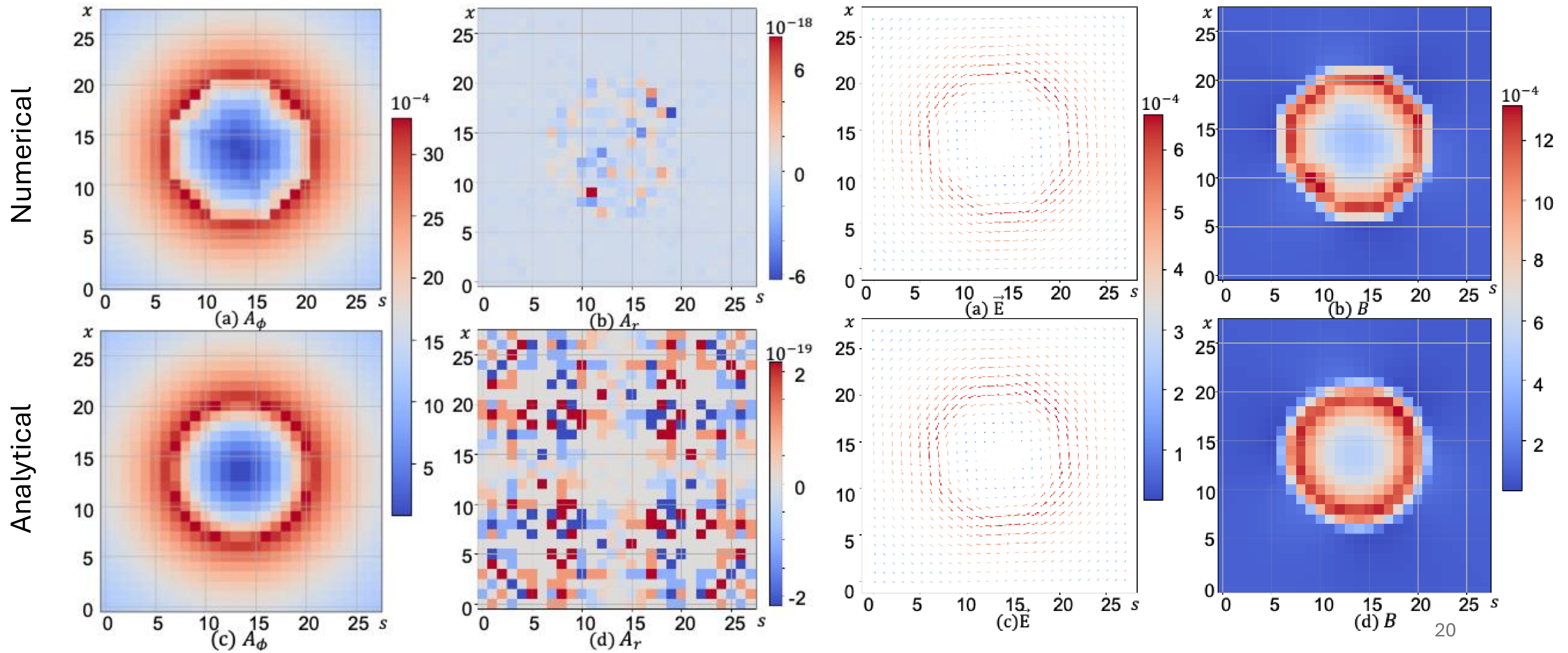
➢ Set the boundary condition as $\overline{A_\phi} = \dfrac{RL_t}{2\pi r}E_0\cos(\omega t)$.

In continuum, there will be $\times I_1(2\pi/L_t r)$

On a Disk Domain Wall

盘形畴壁

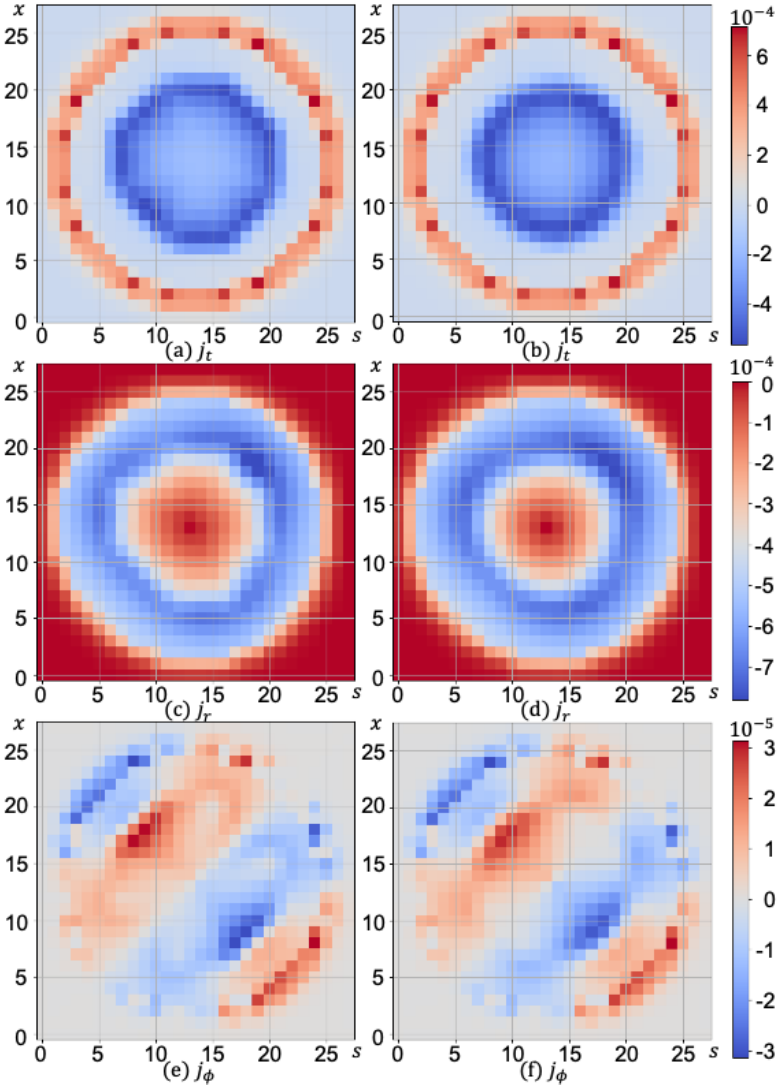
- Gauge Field and Electromagnetic field



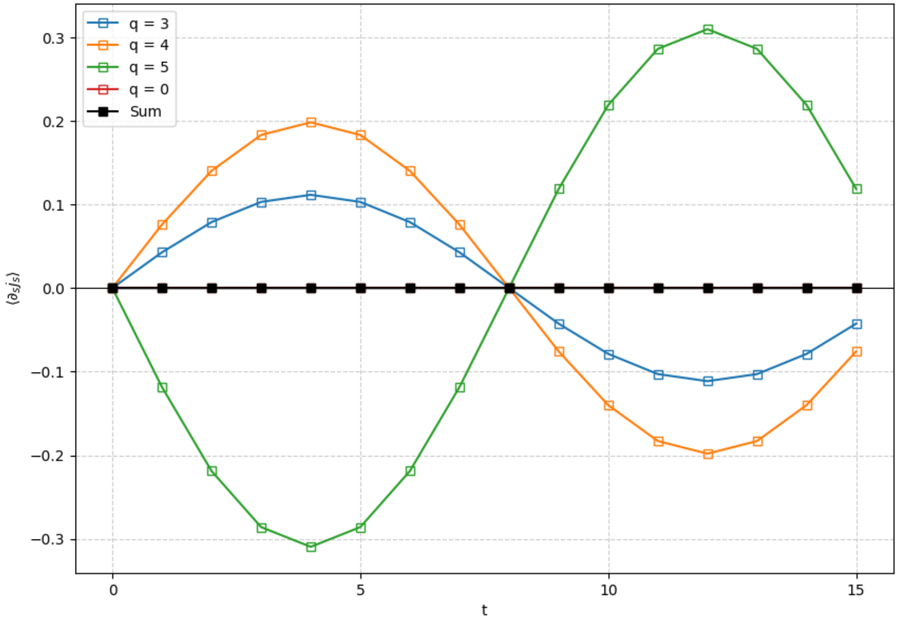
On a Disk Domain Wall

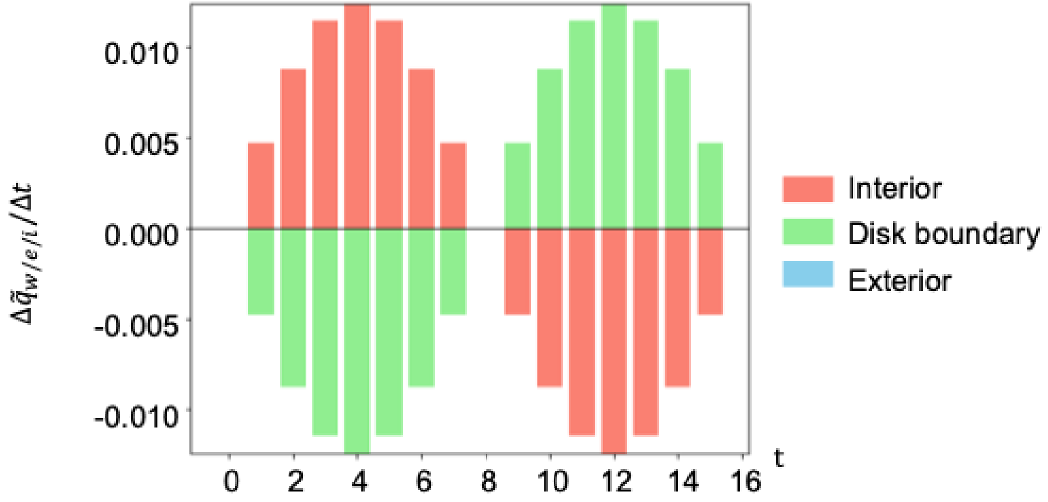
盘形畴壁

currents



Anomaly Cancellation





Charges

Conclusion & Outlook

总结

- ✓ For a slab domain wall 3-4-5-0 system, we use Gradient Flow and Equation of Motion Flow to decouple mirror fermions
- ✓ For a disk domain wall 3-4-5-0 system, , we use Equation of Motion Flow to gauge the bulk.

$\Rightarrow$ **Chiral Gauge Theory**

- ➢ A Hamiltonian Method? On quantum circuits?
- ➢ Higher dimension?
- ➢ Instantons under flow?
- ➢ CP problem and η' mass?

Thank you for your attention!

Jinlong Dang