Baryon-baryon interactions with the plane-wave quantization condition

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1. INTRODUCTION

The **plane-wave quantization condition** [1] is a recently-proposed alternative to Lüscher's two-particle formalism [2] that allows one to incorporate long-range interactions and all partial waves into the analysis of two-particle lattice results. Both of these features may play a major role in baryon-baryon systems, especially as the physical pion mass is approached. **We use the plane-wave QC to analyze lattice results for systems of two nucleons and two hypernuclei**. We find that long-range interactions and higher partial waves can have a significant impact on these systems, even at heavier-than-physical pion masses.

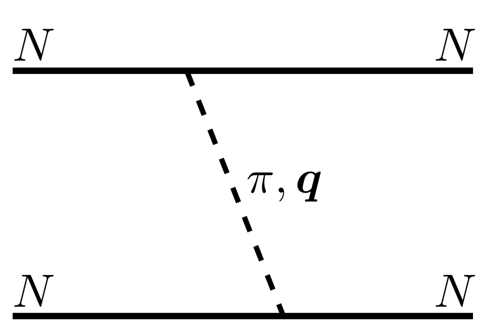
2. CHIRAL EFTs FOR BARYON-BARYON SYSTEMS

Chiral effective field theories (EFTs) provide the most successful description of nucleon-nucleon experimental data up to 300 MeV. They are based on the use of the Lippmann-Schwinger equation (LSE) to obtain a non-perturbative result for the scattering amplitude,

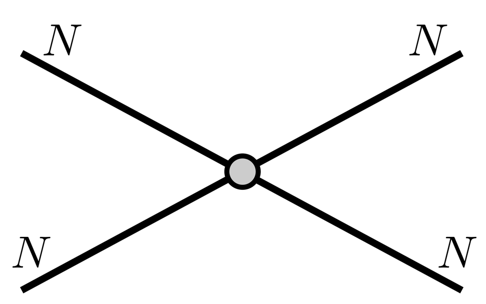
$$\mathcal{M}_2(\boldsymbol{p}_\mathrm{f},\boldsymbol{p}_\mathrm{i};k^2) = -V(\boldsymbol{p}_\mathrm{f},\boldsymbol{p}_\mathrm{i}) + \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} m_N \frac{V(\boldsymbol{p}_\mathrm{f},\boldsymbol{p})\mathcal{M}_2(\boldsymbol{p},\boldsymbol{p}_\mathrm{i};k^2)}{k^2-p^2+i\varepsilon}$$

Here $p = |\boldsymbol{p}|$ and V is an **EFT potential** that contains both **long-range interactions mediated via the exchange of one or more particles and short-range interactions**. At leading order (LO) in heavy-baryon ChPT, these are a one-pion exchange (OPE) term and two constant contact terms, respectively.

One-pion exchange (OPE)



Contact interactions



3. LIMITATIONS OF LÜSCHER'S FORMALISM

Some of the assumptions made in the derivation of Lüscher's quantization condition [2] (QC2) break down in the presence of OPE interactions, as these give rise to a **left-hand cut** (lhc) **in the partial-wave projected amplitude**:

- Lüscher's formalism breaks on the lhc, where the two-particle K-matrix becomes complex, and large exponentially-suppressed corrections may occur above the cut.
- The lhc limits the range of convergence of the effective range expansion, invalidating the partial-wave truncation needed for practical applications of the formalism.

In practice, when using the QC2 to predict finite-volume energies from chiral-EFT results for the scattering amplitude, one finds **unphysical solutions and missing energy levels, especially if higher partial waves are included**.

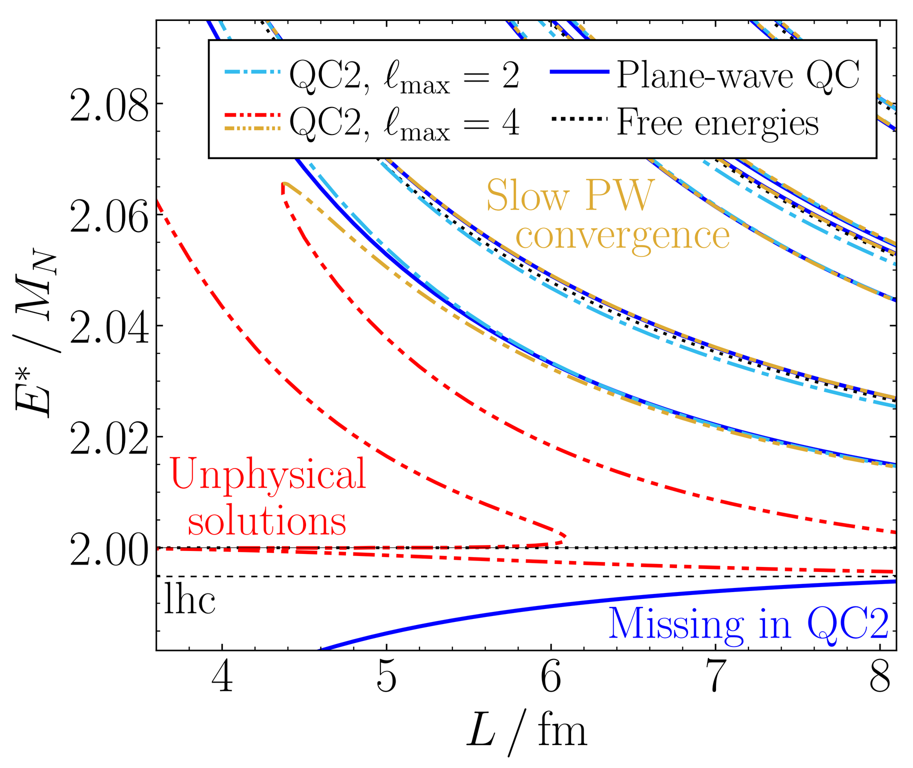


Figure 1. Predictions of the finite-volume spectrum for the NN system in the deuteron channel at physical pion mass, obtained using QC2 with a chiral-EFT amplitude, with the potential tuned to reproduce the deuteron binding energy and physical πNN coupling. Predictions from the plane-wave QC are also shown.

4. PLANE-WAVE QUANTIZATION CONDITION

The **plane-wave QC** [1] is based on solving the LSE in finite-volume in the plane-wave basis. It incorporates OPE effects from the EFT potential and avoids problems of slow partial-wave convergence by working in the basis of plane waves. For the non-relativistic baryon-baryon system, **finite-volume energies are the solution of an eigenvalue problem**,

$$\left[\frac{1}{L^3}\sqrt{\mathbb{J}}\mathbb{V}\sqrt{\mathbb{J}}+\mathbb{H}\right]v_a = \frac{k_a^2}{2M_N}v_a \qquad \text{with} \qquad \mathbb{H}_{mn} = \frac{p_n^{*2}}{2M_N}\delta_{mn}$$

Here p_n^{*2} are the free energies of the plane-wave states in the center-of-mass frame and $\mathbb{J}$ is the Jacobian of the transformation from the lattice frame to the center of mass. We use the so-called Wu boost [3], which is independent of the total energy of the system.

5. OPE EFFECTS ON THE H-DIBARYON BINDING ENERGY

We use the plane-wave QC to **assess OPE effects on the binding energy of the H dibaryon**. We reanalyze the finite-volume energy spectra from ref. [4], determined on a series of CLS ensembles at the SU(3)-symmetric point, with pseudoscalar masses $M_\mathrm{PS} \approx 420$ MeV (PS $= \pi, K, \eta$) and $a \approx 0.036-0.091$ fm.

We constrain an **EFT-inspired potential containing a LO OPE term and LO+NLO contact terms** using the plane-wave QC. Note that in this system, the OPE stands for the exchange of any pseudoscalar meson, not only pions. From the potential, the scattering amplitude is determined with the infinite-volume LSE. **The plane-wave QC provides an accurate description of the finite-volume spectra, including energy levels lying on top of the lhc and others dominated by d waves**, without including additional d-wave terms in the potential.

We compare our results to predictions from ref. [4] obtained using Lüscher's formalism truncated to s wave, observing a **significant impact from OPE interactions**. From a simultaneous fit to all ensembles, we find that **the inclusion of OPE effects leads to a smaller H-dibaryon binding energy than previously predicted**.

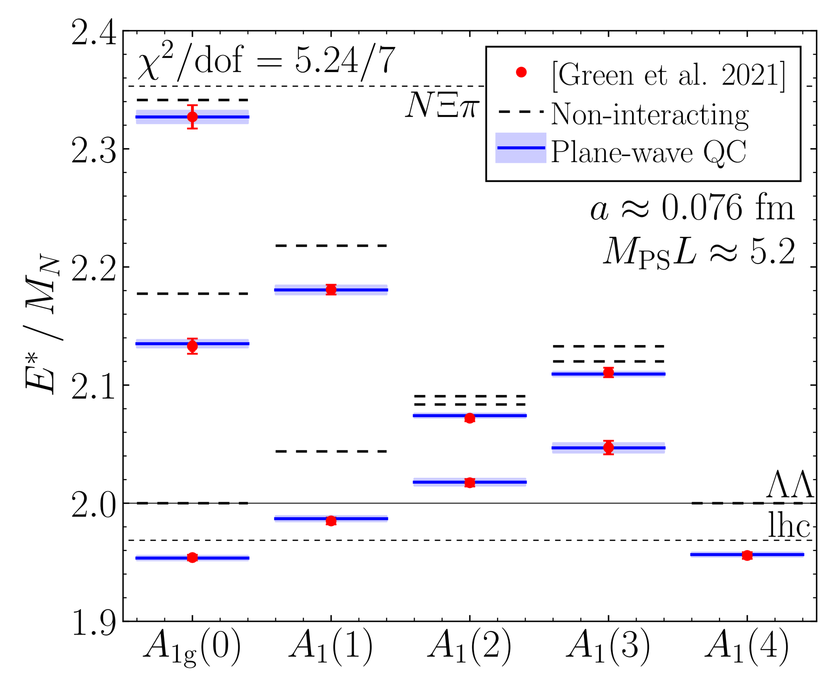


Figure 2. Finite-volume energies from the B450 CLS ensemble, together with the best-fit predictions to an EFT-inspired potential using the plane-wave QC.

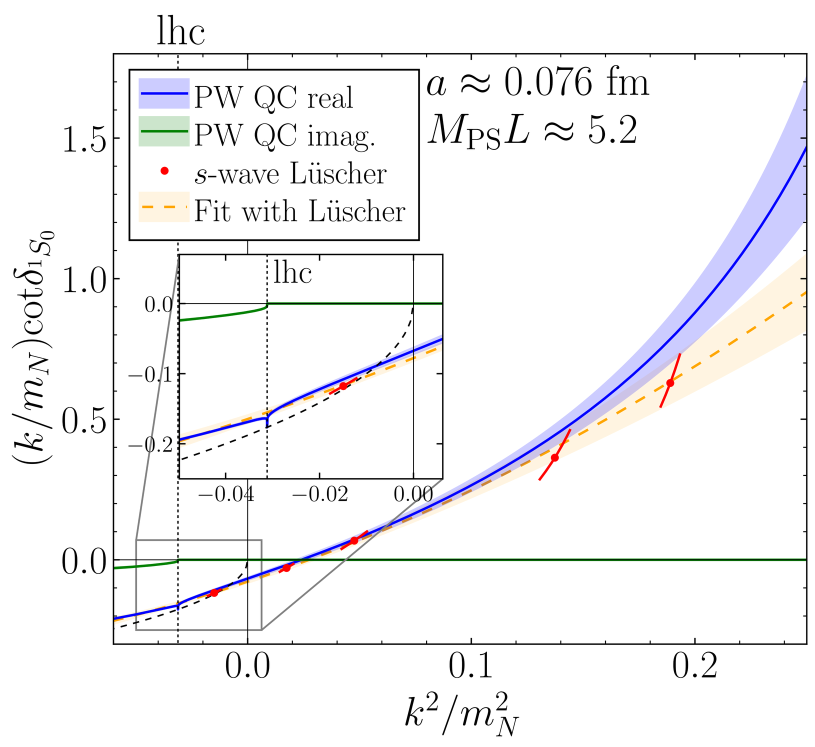


Figure 3. Scattering phase-shift obtained using the LSE with the potential determined from fitting with the plane-wave QC. For comparison, results obtained using Lüscher's QC on s-wave-dominated energy levels are also shown, as well as predictions from a fit of these points.

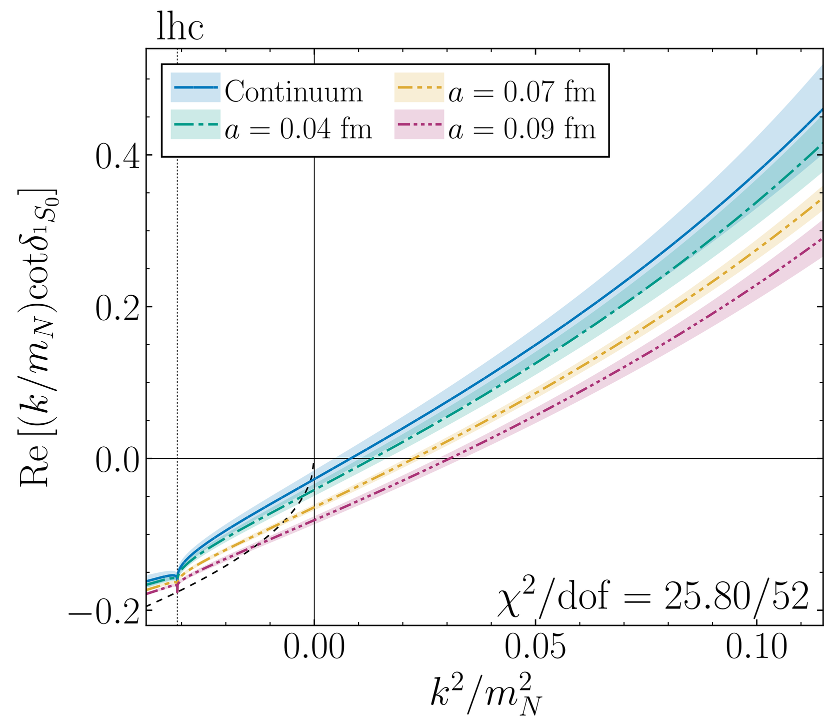


Figure 4. Scattering phase shift as a function of the lattice spacing, determined with the LSE and the results for the EFT-inspired potential from a fit with the plane-wave QC.

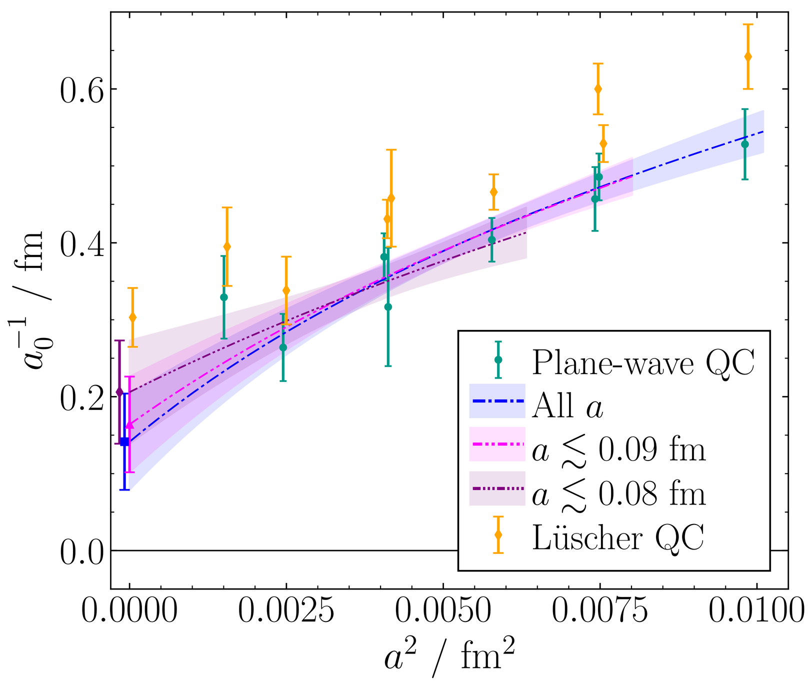


Figure 5. Continuum extrapolation of the inverse scattering length. A negative result would indicate an unbound H dibaryon.

6. PLANE-WAVE ANALYSIS OF NUCLEON-NUCLEON INTERACTIONS

As a second application, we **analyze new lattice results for nucleon-nucleon scattering at $M_\pi = 280, 200$ MeV**, and reanalyze data from ref. [5] at $M_\pi \approx 710$ MeV. We perform simultaneous fits with the plane-wave QC to the $I = 0$ (deuteron) and $I = 1$ (dineutron) channels, with a common OPE coefficient. We find that the chiral-EFT amplitude provides a successful description of the data for all M_π, with **OPE effects becoming relevant for $M_\pi \lesssim 200$ MeV**. We obtain results that are consistent with virtual bound states for all pion masses considered. **See M. Salg's talk on Wednesday at 9:20 for more details about this work**.

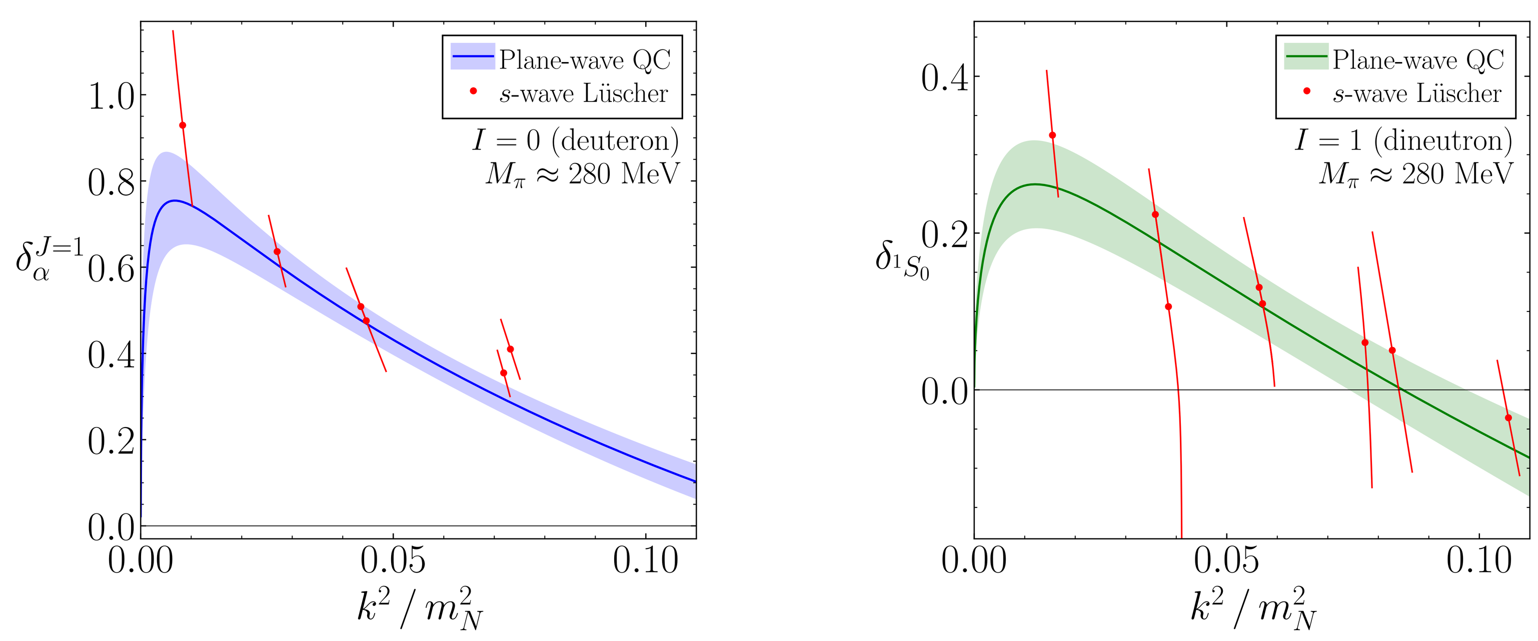


Figure 6. NN scattering phase shift for the deuteron and dineutron channels with $M_\pi \approx 280$ MeV, obtained using the plane-wave QC and the LSE (bands), together with predictions obtained using Lüscher's QC truncated to s wave (points).

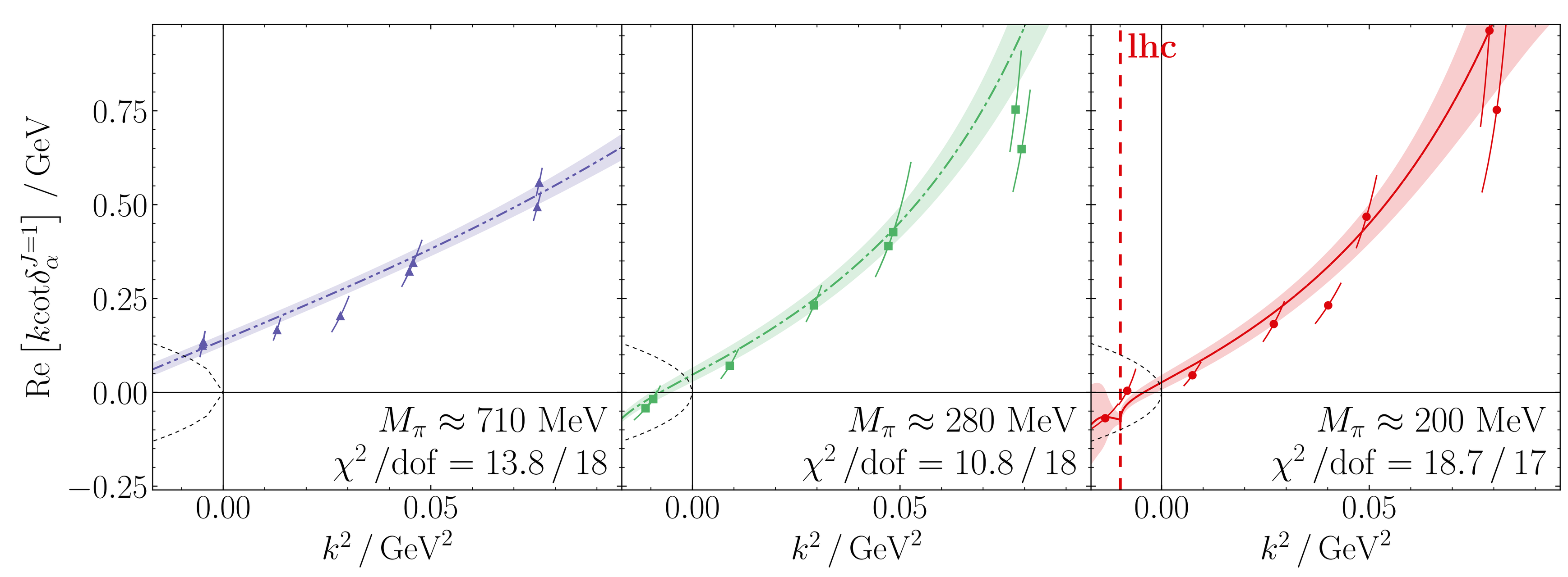


Figure 7. NN scattering phase shift in the deuteron channel obtained using the plane-wave QC and the LSE (bands), for all pion masses considered, together with results obtained using Lüscher's QC truncated to s-wave (points). Left-hand cuts for the two heavier pion masses lie to the left of the plot range.

7. CONCLUSIONS AND OUTLOOK

We have investigated **baryon-baryon systems using the plane-wave QC**:

- The plane-wave QC captures OPE effects and higher partial waves, avoiding unphysical solutions to the QC and slow partial-wave convergence.
- We find that **OPE effects significantly reduce the H-dibaryon binding energy**.
- Our results show that including **OPE interactions is essential to analyze the NN system for $M_\pi \lesssim 200$ MeV**.

Next, we plan to use the plane-wave QC to investigate higher partial waves for the NN system and the H dibaryon away from the SU(3) symmetric point.

REFERENCES

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[4] J. R. Green, D. Hanlon, P. M. Junnarkar and H. Wittig, *Phys. Rev. Lett.* 127 (2021) 24, 242003.
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