Moments of Transversity GPD from Nonlocal Operators OPE

N. Khosla[1], J. Miller[1], S. Bhattacharya[2], K.Cichy[3], M. Constantinou[1], A. Metz[1], F. Steffens[4]

[1]Temple University, [2]University of Connecticut, [3]Mickiewicz University, [4]University of Bonn

QUARK-GLUON TOMOGRAPHY COLLABORATION

Abstract

In this work we calculate Generalized Form Factors (GFFs) of the proton transversity GPDs via a short-distance expansion to matrix elements of nonlocal tensor operators. We use a Lorentz-invariant parametrization in an asymmetric kinematic frame, which substantially reduces the computational cost and provides broad momentum-transfer coverage. The calculation uses a N_f = 2+1+1 clover-improved twisted-mass ensemble at a pion mass of approximately 260 MeV. These results provide complementary information on the proton's transverse spin structure, impact-parameter distributions, and transverse quark densities.

Background and Motivation

- GPDs provide aspects of three dimensional description of hadron structure by correlating the momentum and spatial distribution of quarks and gluons.
- The four leading-twist transversity GPDs describe correlations between transverse quark polarization and proton structure. Their Mellin moments provide input for transverse quark-spin densities.
- Transversity GPDs are chiral-odd and are difficult to access experimentally.
- Use of nonlocal matrix elements, developed to access x-dependent PDFs and GPDs within a short-distance expansion to access Mellin moments.
- A Lorentz-invariant parametrization applicable to asymmetric kinematic frames [1] leading to a broad range of momentum transfers at reduced computational cost.

Methodology for OPE

- Asymmetric-frame matrix elements at zero skewness ξ:

$$F^{[i\sigma^{\mu\nu}\gamma_5]} \equiv \langle N(p_f)|\bar{\psi}(z)\, i\sigma^{\mu\nu}\mathcal{W}(0,z)\psi(0)\,|N(p_f-\Delta)\rangle \quad \textbf{(1)}$$

- Parametrize quasi-GPDs through Eqs. (2)-(3), and renormalize through single ratio (SR):

$$\mathscr{M}^{LI}_{H_T}(P\cdot z) = \frac{H_T^{LI}(z,P,\Delta)}{H_T^{LI}(z,0,0)}$$

- Relate SR to the Mellin moments (GFFs) short-distance expansion with perturbative Wilson coefficients [2,3,4]

$$\mathscr{M}(P\cdot z) = \sum_{n=0}^{\infty}\frac{(iP_3 z)^n}{n!}\left\{1+\frac{\alpha_s C_F}{2\pi}\left[2(H_{n+1}-1)\ln\left(z^2\mu^2\frac{e^{2\gamma_E+1}}{4}\right)-2\left(H_n^2+H_n^{(2)}\right)\right]\right\} A_{n,0}$$

- Test different ranges of z for the fits
- Conduct a weighted average to get the final GFFs

Lattice Setup

$N_f=2+1+1$ twisted-mass fermions & clover			
Ensemble	a [fm]	volume $L^3\times T$	m_π [MeV]
cA211.32	0.093	$32^3\times 64$	260

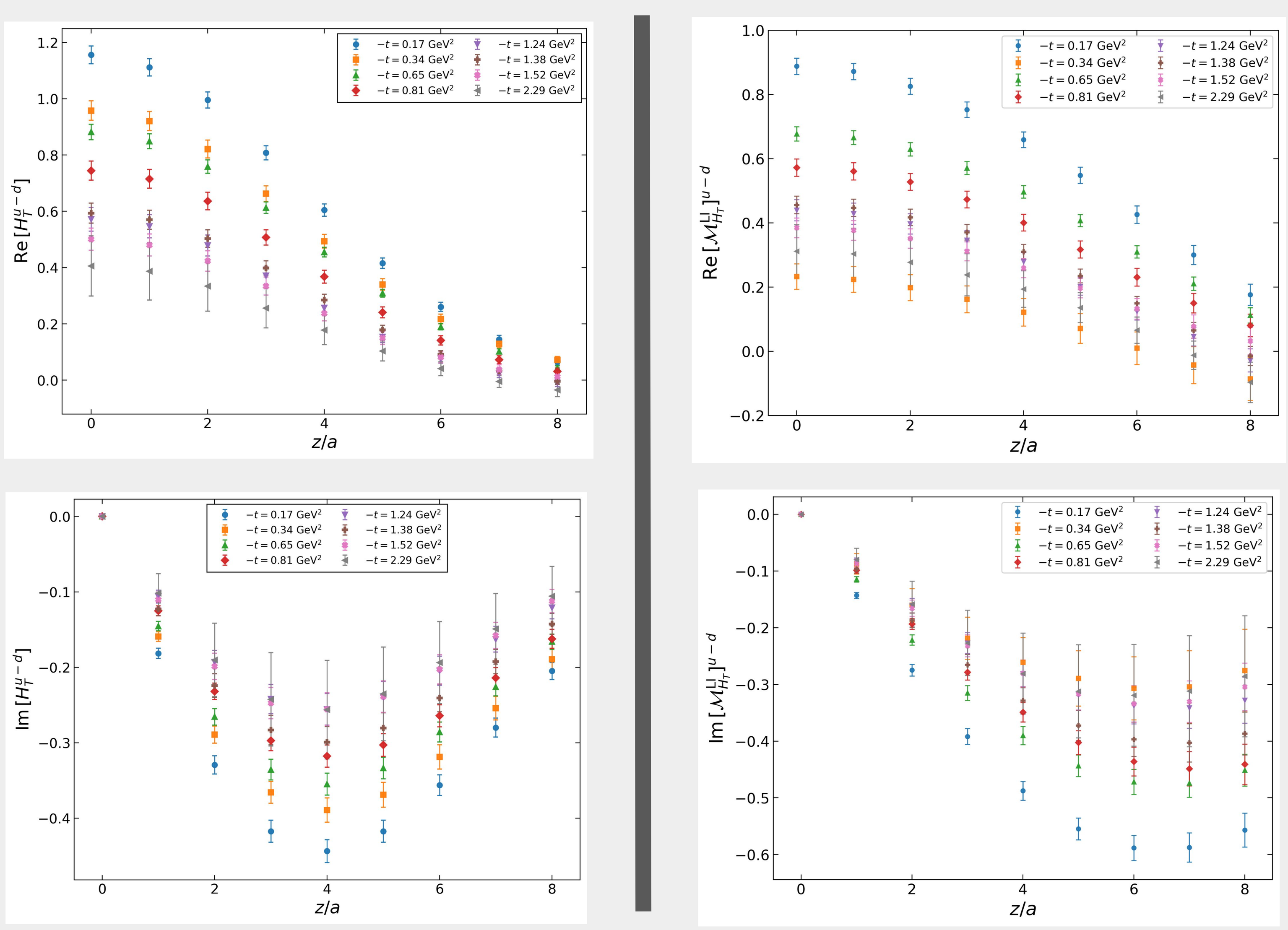
$\Delta\ [\frac{2\pi}{L}]$	$-t$ [GeV^2]	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
(±1,0,0), (0,±1,0)	0.17	8	269	8	17216
(±1, ±1, 0)	0.34	16	195	8	24960
(±2,0,0), (0,±2,0)	0.65	8	269	8	17216
(±1,±2,0), (±2,±1,0)	0.81	16	195	8	24960
(±2,±2,0)	1.24	16	195	8	24960
(±3,0,0), (0,±3,0)	1.38	8	269	8	17216
(±1,±3,0), (±3,±1,0)	1.52	16	195	8	24960
(±4,0,0), (0,±4,0)	2.29	8	269	8	17216

- P_3 =1.25 GeV
- Calculate 2pt- and 3pt- functions
- Ground-state of Eq.(1): plateau fit
- Calculate Lorentz-invariant amplitudes:

$$\begin{aligned} F^{[i\sigma^{\mu\nu}\gamma_5]}(z,P,\Delta) &\equiv \langle p_f;\lambda'|\bar{\psi}(-\tfrac{z}{2})\, i\sigma^{\mu\nu}\gamma_5\, \mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p_i;\lambda\rangle \\ &= \bar{u}(p_f,\lambda')\Big[P^{[\mu}z^{\nu]}\gamma_5 A_{T1} + \frac{P^{[\mu}\Delta^{\nu]}}{m^2}\gamma_5 A_{T2} + z^{[\mu}\Delta^{\nu]}\gamma_5 A_{T3} + \gamma^{[\mu}\left(\frac{P^{\nu]}}{m}A_{T4} + mz^{\nu]}A_{T5} + \frac{\Delta^{\nu]}}{m}A_{T6}\right)\gamma_5 \\ &+ m\not{z}\gamma_5\left(P^{[\mu}z^{\nu]}A_{T7} + \frac{P^{[\mu}\Delta^{\nu]}}{m^2}A_{T8} + z^{[\mu}\Delta^{\nu]}A_{T9}\right) + i\sigma^{\mu\nu}\gamma_5 A_{T10} + i\epsilon^{\mu\nu Pz}A_{T11} + i\epsilon^{\mu\nu z\Delta}A_{T12}\Big]u(p_i,\lambda) \end{aligned} \quad \textbf{(2)}$$

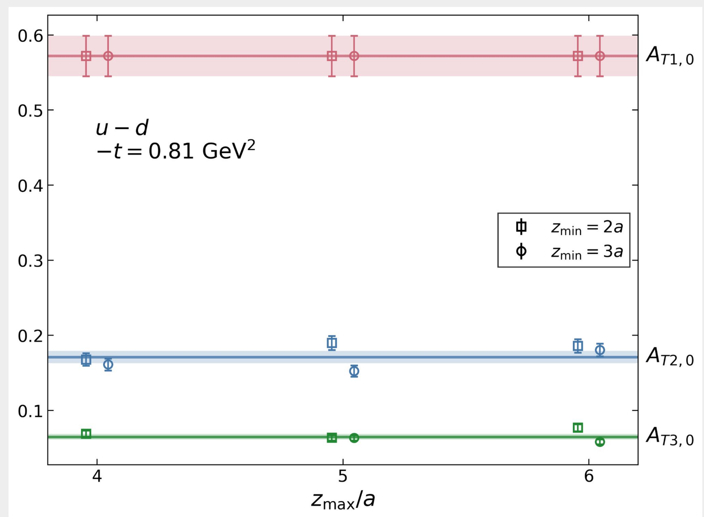
- Extract quasi-GPDs, e.g., $H_T^{u\text{-}d}$: $H_T^{LI} = -2A_{T2}\left(1-\frac{P^2}{m^2}\right) + A_{T4} - \Delta\cdot zA_{T8} + A_{T10}$ **(3)**

LI Quasi-GPD H_T and Single Ratio

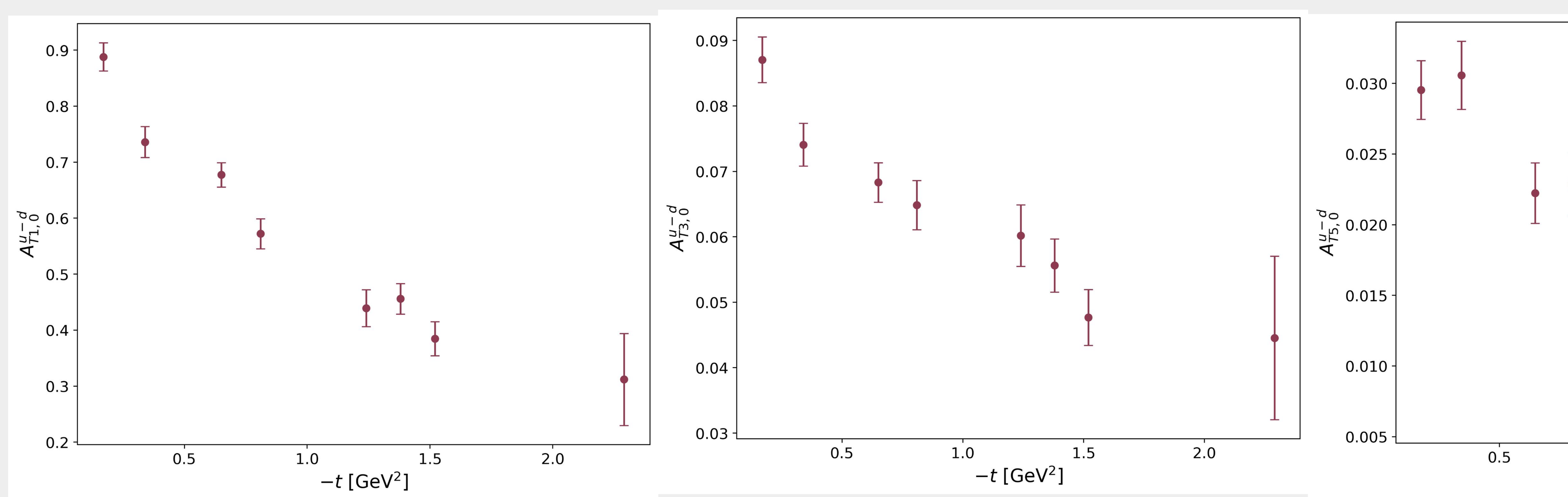


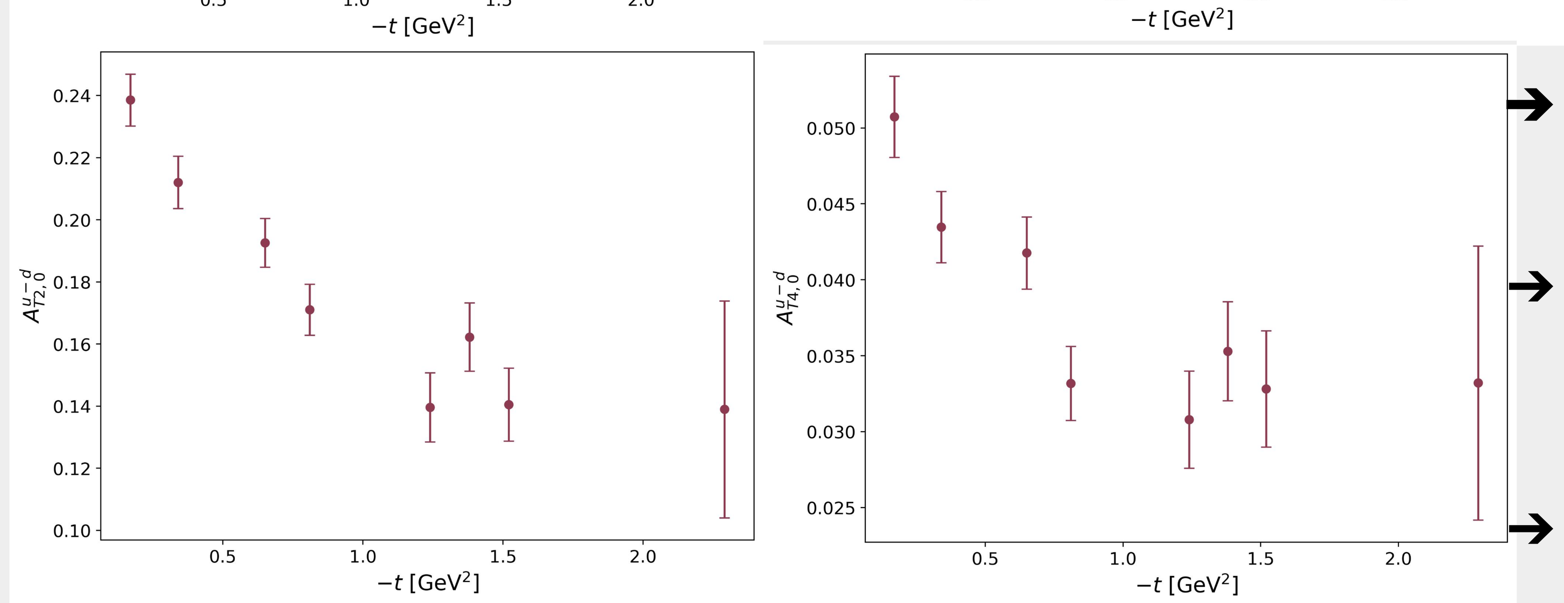
Final Results - Mellin Moments

Weighted average analysis



- Six OPE fit ranges are considered to estimate systematic effects associated with the Wilson-line separation.
- Consistent determinations across multiple fit ranges indicate the stability of the extracted moments.
- Final values are obtained through a weighted average procedure that incorporates statistical uncertainties from all fit ranges.





- **Mellin moments up to fifth order are successfully extracted from non-local matrix elements!**
- The Lorentz-invariant framework provides broad momentum-transfer coverage at reduced computational cost.
- A non-trivial (-t)-dependence is observed in the extracted moments.

Conclusions and Future Work

- Extract the isoscalar (u+d) combinations of the transversity GPDs, and perform a flavor decomposition
- Parametrize the -t dependence of the GFFs: dipole fit, z-expansion, ANN
- Obtain impact parameter space distributions to probe the spatial structure of the proton.
- Extend the analysis to all leading-twist transversity GPDs.

References and Acknowledgements

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