Progress toward the continuum limit of Mellin moments of the unpolarized gluon PDF in the proton

QUARK-GLUON TOMOGRAPHY COLLABORATION

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Abstract

We present a lattice QCD determination of the Mellin moments for the unpolarized gluon PDF in the proton across three ensembles, which allow us to explore the continuum limit. We calculate matrix elements of nonlocal gluon operators coupled to boosted proton states and apply short-distance operator product expansion with perturbatively-calculable Wilson coefficients, to relate matrix elements to Mellin moments. Our analysis is a follow-up work of **Ref.[1]**. We investigate systematic effects due to the OPE truncation, the minimum/maximum Wilson-line separations entering the fit, and assess residual uncertainties by varying the initial matching scale and performing DGLAP evolution to 2 GeV.

Motivation

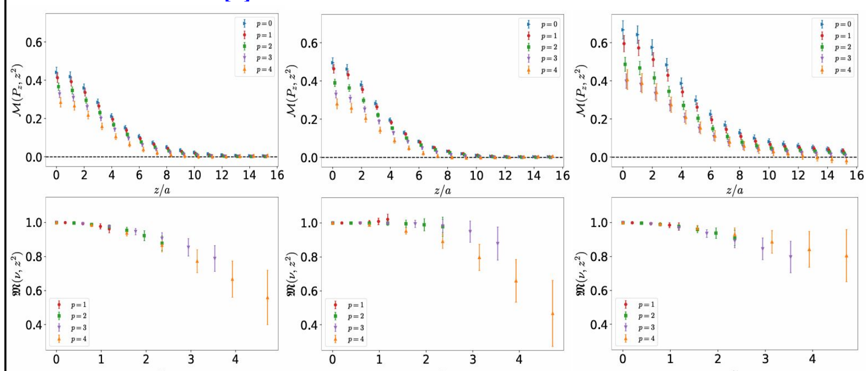
- Gluons play an important role in hadron structure, yet information is sparse and less precise than their quark counterparts.
- Mellin moments encode integral properties of the PDF, that connect to physical information. Also, different-order Mellin moments encode information of PDFs at different x-regions.
- Accessing gluon moments higher than <x> is unfeasible with traditional methods.
- Multi-ensemble calculations are important to assess certain systematic uncertainties, such as continuum limit.

Ensemble Parameters / Statistics

Ensemble	N_f	a [fm]	$L^3 \times T$	m_π [MeV]	P_3 [GeV]	$N_{\rm conf}$	$N_{\rm src}$	$N_{\rm dir}$	$N_{\rm meas}$
cA211.30.32	$2+1+1$	0.09076(54)	$32^3 \times 64$	272	0, 0.43, 0.85, 1.28, 1.71	1134	200	6	1,360,800
cB211.25.32	$2+1+1$	0.07948(11)	$32^3 \times 64$	260	0, 0.49, 0.97, 1.46, 1.95	914	400	6	2,193,600
cD211.17.48	$2+1+1$	0.05485(09)	$48^3 \times 96$	250	0, 0.46, 0.91, 1.37, 1.82	1133	400	6	2,719,200

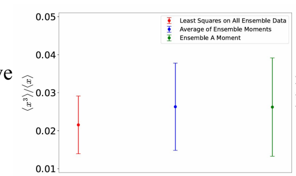
Matrix Elements and Double Ratios

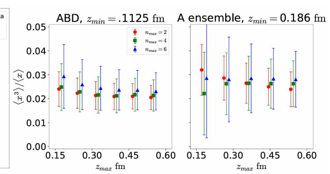
Ensemble A* Ref. [1]** ***Ensemble B ***Ensemble D***



Technique Comparison

Aggregate data fitting versus naive average of ensemble moments





Comparison of aggregate ensemble data to ensemble A alone

Methodology and Operator Product Expansion (OPE)

- Matrix element of nonlocal gluonic operator:

$$M_{\mu i;\nu j}(P,z) = \langle N(P)|\mathrm{Tr}[F_{\mu i}(z)W(z,0)F_{\nu j}(0)W(0,z)]\,|N(P)\rangle$$

- A short-distance expansion can be applied to extract the Mellin moments with appropriate Wilson coefficients (perturbatively calculable)

$$M(P,z) = \sum_{n=0}^{\infty} \frac{(i\nu)^n}{n!} C_n(z^2,\mu^2)\, a^g_{n+2}(\mu)$$

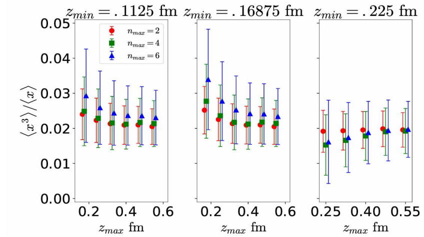
- Double ratio (DR) serves as renormalization and used for moment fit:

$$\mathfrak{M}_g(\nu,z^2) \equiv \left(\frac{M_g(\nu,z^2)}{M_g(\nu,0)|_{z=0}}\right) \Big/ \left(\frac{M_g(0,z^2)|_{p=0}}{M_g(0,0)|_{p=0,z=0}}\right)$$

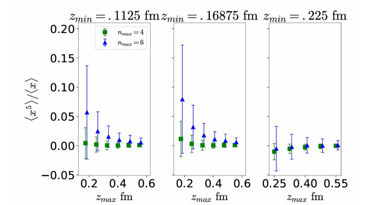
$$\mathfrak{M}_g(\nu,z^2)\langle x\rangle_g^{\mu^2} = \sum_{n=0}^{\infty}(-1)^n \frac{\nu^{2n}}{(2n)!}\langle x^{2n+1}\rangle_g^{\mu^2}\left[1-\frac{\alpha_s N_c}{2\pi}\left\{2\ln\left(\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right)\left(\frac{11}{12}+\frac{1}{1+n}-\frac{1}{2+n}+\frac{2}{3+n}-H_{4+n}\right)+\frac{n}{n+1}-H_n+\frac{H_n^2+H_n^{(2)}}{2}-\frac{3}{4}+\frac{3}{2+n}-\frac{3}{3+n}+\frac{1}{4+n}\right\}\right]$$

Mellin Moments

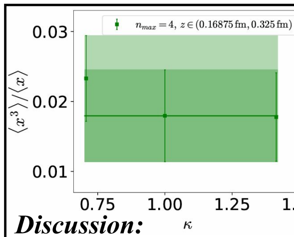
- <x^3> / <x> including all ensemble data



- <x^5> / <x> shows zero within error bars



Scale Evolution - Final Results



$$\kappa = \frac{\mu_0^2 z^2 e^{2\gamma_E}}{4} \qquad \kappa \in \left(\tfrac{1}{\sqrt{2}}, \sqrt{2}\right)$$

Final results at 2 GeV using scale evolution of Wilson coefficients:

<x^3 >/<x >= 0.018(6)

Discussion:

- <x^3 > accessible from nonlocal operators!
- Combining three ensembles has non-negligible effect compared to result from ensemble A ([1]).
- Further investigation needed of the continuum limit at different stages of the analysis.

References and Acknowledgements

[1]. Delmar et al., arXiv:2606.28102

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