A Software Package for Multiloop Feynman Diagram Calculations on the Lattice

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Abstract

- Automated Mathematica package for multiloop Feynman-diagram calculations.
- Supports lattice and dimensional regularization.
- Performs symbolic contractions, algebraic simplification, and numerical momentum integration.
- Handles general lattice actions, composite operators, and overlap fermions.
- Supports GIRS scheme, operator mixing, renormalization, and scheme conversions.
- Includes expression-compression tools for large intermediate results.
- Improves efficiency, reliability, and reproducibility of high-order perturbative work.

Scan to access the GitHub repository for source code and documentation.
github.com/Rinnoco/super_quanta

Some Basic Commands

- **Contraction among vertices** contract[…]: Performs Wick contractions and contracts pairs of fields.
- **Simplifications** simplifydelta[…]: Simplifies products of Kronecker delta functions by performing index contractions.

replace[…, {a_, b_}]: Replaces occurrences of a specific part of an expression (a) with another expression (b), with correct handling of implicit summations.
productDiracRulesDdim: A set of substitution rules which may be applied repeatedly in order to simplify products of Dirac matrices in D dimensions.
traceDiracRulesDdim: A set of substitution rules which may be applied repeatedly in order to simplify traces of Dirac matrices in D dimensions.
reducerho[...]: Reduces repeated Lorentz-index structures and simplifies momentum-dependent tensor expressions.
replacec2exact[…] / replacec2p[...]: All cosine functions that depend solely on the external momenta q_i can be replaced exactly with equivalent expressions involving sine functions. / A similar function as the above but is used for the internal momenta p_i.

- **Extraction of divergent/convergent terms** subtractionIRdiv[…]: Simplifies one-loop expressions by isolating IR divergent terms.

matchindices[…]: Takes an integrand with propagators independent of external momenta, and matches the directions of sines, so as to lead to an even integrand under $p[i] \rightarrow -p[i]$.
makeindependent[…]: Isolates the part to be integrated and makes it independent of the external momentum components.

- **Numerical integration over loop momenta:** integrator[...]: Generates optimized Fortran code for the numerical evaluation of lattice loop integrals.

extrapolate[...]: Performs infinite-volume extrapolation: $L \rightarrow \infty$ and estimates systematic uncertainties.

Computation of Feynman Diagrams on the lattice

- The first step in the evaluation of each diagram (e.g., in Fig. 1 or in Fig. 2) is the contraction of the vertices, which is performed automatically once the algebraic expressions of the vertices and the diagram topology (the "incidence matrix") are specified.
- Then we reduce the number of infrared-divergent integrals to a minimal set. To do this, we use subtractions among the propagators. IR convergent terms can be treated by Taylor expansion in the lattice spacing to the desired order. Alternatively, the extraction of the $(a\ q)$ dependence (a: lattice spacing, q: external momentum) may be performed using iteratively subtractions.
- The required numerical integrations of the algebraic expressions for the convergent loop integrands are performed by highly optimized Fortran programs; these are generated by our Mathematica 'integrator' routine. Momentum integration is replaced by finite sums over the Brillouin zone for finite hypercubic lattices L^4, with extrapolation to the infinite-volume limit: $L \rightarrow \infty$. Execution time is reduced through optimizations, including: precomputed trigonometric factors, symmetry reductions, restricted summation domains, inverse-tree organization of summands.
- Infinite-volume extrapolation uses multiple asymptotic fits with weighted averaging to estimate systematic uncertainties.

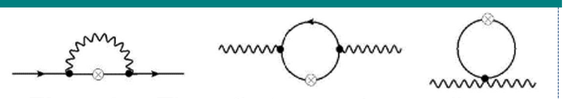

Fig. 1: The first one-loop diagram contributes to $\langle \psi(q_1)\ O_i(x)\ \bar{\psi}(q_2) \rangle$ and the remaining one-loop diagrams to $\langle A_\mu(q_1)\ O_i(x)\ A_\nu(q_2) \rangle$. Wavy (solid) lines → gluons, A_μ (quarks, ψ). Circled cross → quark bilinear operator, O_i.

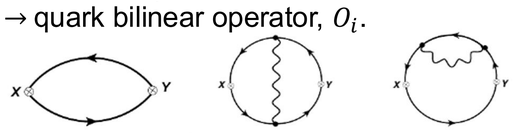

Fig. 2: The first diagram contributes to the tree-level and the remaining diagrams contribute to NLO of the Green's function $\langle O_i(x)\ O_j(y) \rangle$. Wavy (solid) lines → gluons, A_μ (quarks, ψ). Circled cross → quark bilinear operator, O_i.

Computation of Feynman Diagrams in the dimensional regularization

- Continuum loop integrals are evaluated in $D = 4 - 2\,\varepsilon$ using dimensional regularization.
- Massive propagators are treated through the 't Hooft–Feynman parameter procedure, combining denominators into a single quadratic form.
- Tensor integrals are reduced analytically, with explicit implementations up to rank-5 loop-momentum structures.
- Series expansion in ϵ extracts finite and divergent contributions, followed by renormalization-scale (μ) dependence and logarithmic simplifications.

Useful Commands for continuum integrals

- reducediamond[...]: Diamond-type two-loop topologies are reduced recursively into simpler integrals.
- ChetyrkinFormula[...]: Standard one-loop integrals are automated, performing symbolic integration over loop momenta.
- FourierTransformIntegrals[...]: Handles the symbolic evaluation of Fourier transform integrals, a crucial task in converting position-space quantities into momentum-space representations in QFT.

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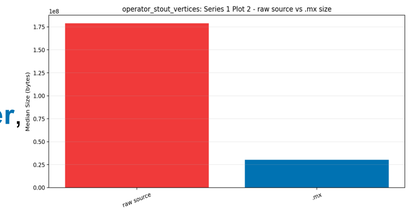
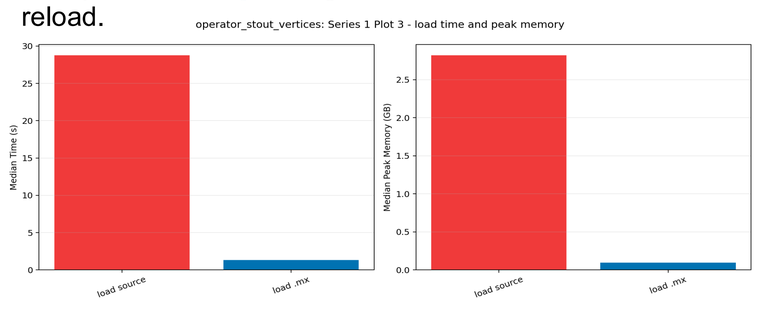
Shrinker Results

Compressed storage and caching tools (.mx, .gz, .zst, etc.) accelerate handling of multi-million-term symbolic expressions. Our experimental evaluation shows that native storage provides substantial practical gains compared with raw Mathematica source:

- the stored artifact is approximately **5.2 times smaller**,
- reloads about **25 times faster**, and
- requires about **16.2 times less** peak **memory** during reload.





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