Perturbative Renormalization of Noether Currents in SYM with Improved Discretization

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Abstract

- Eployment of Wilson (with single step stout smeared links) and clover term for gluinos (λ), and Symanzik-improved action for gluons (u_μ) in $\mathcal{N}=1$ supersymmetric Yang–Mills theory (SYM).
- Study of Noether currents, focusing on the chiral current and supercurrent operators (a tilde denotes stout smearing):

$$J_{5\mu} = \mathrm{Tr}(\bar{\lambda}\gamma_5\gamma_\mu\lambda), \qquad S_\mu = -\frac{1}{2}\mathrm{Tr}\left(\tilde{\tilde{F}}_{\rho\sigma}[\gamma_\rho,\gamma_\sigma]\gamma_\mu\lambda\right),$$

using improved discretization.
- Compute 2-pt and 3-pt Green's functions (GFs) to determine the renormalization factors of all fields and parameters of the theory including the critical gluino mass.
- Determine the renormalization of $J_{5\mu}$, and of the mixing matrix for S_μ upon renormalization. The mixing matrix includes both gauge-invariant and noninvariant operators.
- A gauge-fixing term, together with the compensating ghost-field (c) term, are also added to the action; these terms are the same as in the non-supersymmetric case; a standard "measure" term is added to the action account to the change of variables $u_\mu \to U_\mu$ (links).

Motivation

- Develop a lattice formulation of $\mathcal{N}=1$ SYM with explicitly broken SUSY and chiral symmetry at finite lattice spacing. Gluinos are defined on lattice sites, while gluons are represented by link variable: $U_\mu(x) = \exp\left(igaT^\alpha u_\mu^\alpha\left(x+\frac{a\hat{\mu}}{2}\right)\right)$, where T^α are the generators of the $SU(N_c)$ gauge group and a is the lattice spacing.
- Use of improved actions to reduce discretization effects and fine-tuning.
- Incorporate stout smearing improves stability in numerical simulations, at finite lattice spacing and reduces the size of required fine-tunings; stout-smeared links are defined as: $\tilde{U}_\mu(x) = \exp\left(i\,Q_\mu(x)\right)U_\mu(x)$, where $Q_\mu(x)$ is given by:

$$Q_\mu(x) = \frac{\omega}{2i}\left(V_\mu(x)U_\mu^\dagger(x) - U_\mu(x)V_\mu^\dagger(x) - \frac{1}{N_c}\mathrm{Tr}\left[V_\mu(x)U_\mu^\dagger(x) - U_\mu(x)V_\mu^\dagger(x)\right]\right),$$

the "stout coefficient", ω, is a tunable parameter, while $V_\mu(x)$ represents the sum over all staples associated with the link $U_\mu(x)$:

$$V_\mu(x) = \sum_{\nu\neq\mu}\left(U_\nu(x)\,U_\mu(x+\hat{\nu})\,U_\nu^\dagger(x+\hat{\mu}) + U_\nu^\dagger(x-\hat{\nu})\,U_\mu(x-\hat{\nu})\,U_\nu(x-\hat{\nu}+\hat{\mu})\right).$$

- Perform perturbative fine-tuning and provide calibration of lattice simulations.
- Renormalize Noether currents relevant to supersymmetric and chiral Ward identities, toward the restoration of SUSY and chiral symmetry in the continuum limit.

Computational Setup

- We employ the following lattice action in Euclidean space:

$$S_{SYM} = \sum_x\left\{\frac{2}{g^2}\left[c_0\sum_{\rm plaq}\mathrm{Re\,Tr}(1-U_{\rm plaq}) + c_1\sum_{\rm rect}\mathrm{Re\,Tr}(1-U_{\rm rect})\right] + a^4\left[\sum_\mu\mathrm{Tr}(\bar{\lambda}\gamma_\mu\tilde{D}_\mu\lambda) - \frac{a\,r}{2}\sum_\mu\mathrm{Tr}(\bar{\lambda}\tilde{D}^2\lambda) - \frac{a\,c_{\rm SW}}{4}\sum_{\mu\nu}\bar{\lambda}^\alpha\,\sigma_{\mu\nu}\,\hat{F}_{\mu\nu}^{\alpha\beta}\,\lambda^\beta\right]\right\}$$

- We begin by computing the renormalization factors of all fields and parameters of the theory. *Their expressions are presented here:*
- The operator smearing parameter, ω_O, is treated independently of the action smearing parameter, ω_A.
- We compute Green's functions in both dimensional regularization (DR) and lattice regularization (LR).
- The Feynman diagrams contributing to the 2-pt Green's functions are shown in Fig. 1.
- We determine the renormalization constants as well as operator mixing in the $\overline{MS}$ scheme on the lattice ($Z_O^{LR,\overline{MS}}$).
- For the operator S_μ, different choices of external momenta for the GFs are employed to extract the mixing pattern.
- The tree-level structures of the mixing operators naturally show up within the one-loop GFs of S_μ.
- The renormalized chiral current can be written in the following multiplicative form: $J_{5\mu}^R = Z_J\,J_{5\mu}^B$.
- The renormalized supercurrent operator is expressed as the following linear combination of bare operators:

$$S_\mu^R = Z_{SS}S_\mu^B + Z_{ST}T_\mu^B + Z_{SA1}\mathcal{O}_{A1}^B + \sum_{i=1}^{2}Z_{SBi}\mathcal{O}_{Bi}^B + \sum_{i=1}^{9}Z_{SCi}\mathcal{O}_{Ci}^B,$$

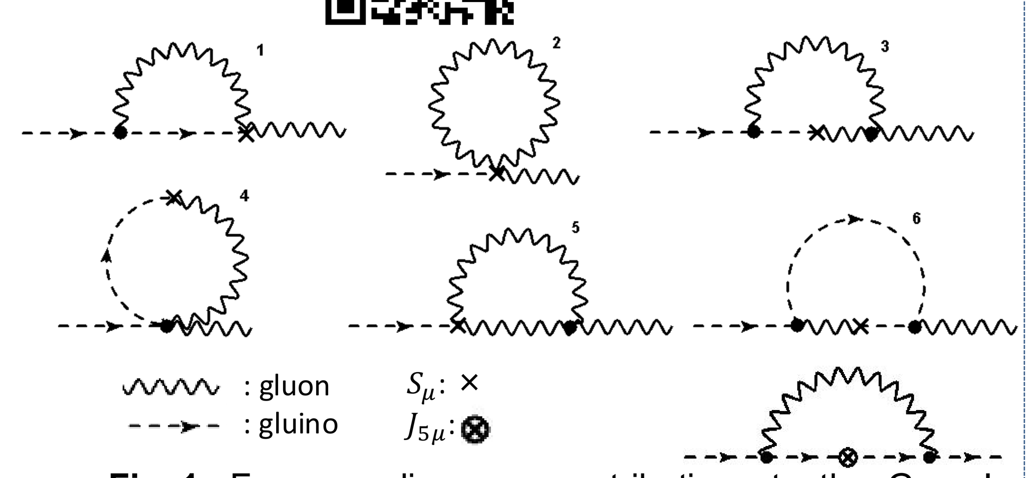

Fig 1: Feynman-diagrams contributions to the Green's function $\langle u_\nu\,S_\mu\,\bar{\lambda}\rangle$ are shown above. The diagram in the bottom right corresponds to the Green's function $\langle\lambda\,J_{5\mu}\,\bar{\lambda}\rangle$.

where

Class G: gauge invariant

$$T_\mu = 2\mathrm{Tr}(u_{\mu\nu}\gamma_\nu\lambda)$$

Class A: BRST invariant

$$\mathcal{O}_{A1} = \frac{1}{\alpha}\mathrm{Tr}\left((\partial_\nu u_\nu)\gamma_\mu\lambda\right) - ig\mathrm{Tr}\left([c,\bar{c}]\gamma_\mu\lambda\right)$$

Class B: vanishing by EOM

$$\mathcal{O}_{B1} = \mathrm{Tr}(u_\mu\not{D}\lambda) \qquad \mathcal{O}_{B2} = \mathrm{Tr}(\not{u}\gamma_\mu\not{D}\lambda)$$

Class C: all other operators with the same quantum numbers

$$\mathcal{O}_{C1} = \mathrm{Tr}(u_\mu\lambda) \quad \mathcal{O}_{C2} = \mathrm{Tr}(\not{u}\gamma_\mu\lambda) \quad \mathcal{O}_{C3} = \mathrm{Tr}(\not{u}\partial_\mu\lambda) \quad \mathcal{O}_{C4} = \mathrm{Tr}\left((\partial_\mu\not{u})\lambda\right)$$
$$\mathcal{O}_{C5} = \mathrm{Tr}\left((\partial_\nu u_\nu)\gamma_\mu\lambda\right) \quad \mathcal{O}_{C6} = \mathrm{Tr}(u_\nu\gamma_\mu\partial_\nu\lambda) \quad \mathcal{O}_{C7} = ig\mathrm{Tr}\left([u_\rho,u_\sigma][\gamma_\rho,\gamma_\sigma]\gamma_\mu\lambda\right)$$
$$\mathcal{O}_{C8} = ig\mathrm{Tr}\left([u_\mu,u_\nu]\gamma_\nu\lambda\right) \quad \mathcal{O}_{C9} = ig\mathrm{Tr}\left([c,\bar{c}]\gamma_\mu\lambda\right)$$

- The Z-factors are determined by the Green's functions $\langle\lambda\,J_{5\mu}\,\bar{\lambda}\rangle$ for the current, and $\langle u_\nu\,S_\mu\,\bar{\lambda}\rangle$, $\langle u_\nu\,u_\rho\,S_\mu\,\bar{\lambda}\rangle$, and $\langle c\,\bar{c}\,S_\mu\,\bar{\lambda}\rangle$ for the supercurrent.
- Only the 2-pt GFs of the supercurrent is considered, meaning that the Z-factors for $\mathcal{O}_{C7},\mathcal{O}_{C8},\mathcal{O}_{C9}$ are left undetermined.
- The results for the renormalizations of the Noether currents: $Z_J^{LR,\overline{MS}}$ and $Z_{SS}^{LR,\overline{MS}}, Z_{ST}^{LR,\overline{MS}}, Z_{SA1}^{LR,\overline{MS}}, Z_{SBi}^{LR,\overline{MS}}\,(i=1,2), Z_{SCi}^{LR,\overline{MS}}(i=1-6)$ *are presented here:*

Calculations, Results and Future Plans

- The Z-factors for the supercurrent are presented below in their most general form; for the numerical values of the coefficients c_{ijkl}^{SO}, see the second QR code.

$$Z_{SS}^{LR,\overline{MS}} = 1 + \frac{g^2}{16\pi^2}\Big[\frac{1}{N_c}\left(c_{0000}^{SS} + c_{0001}^{SS}\omega_O + c_{0002}^{SS}\omega_O^2\right) + N_c\big(c_{1000}^{SS} + c_{1001}^{SS}\omega_O + c_{1002}^{SS}\omega_O^2 + c_{1010}^{SS}\omega_A + c_{1011}^{SS}\omega_A\omega_O + c_{1020}^{SS}\omega_A^2 + c_{1100}^{SS}rc_{\rm SW} + c_{1101}^{SS}rc_{\rm SW}\omega_O + c_{1110}^{SS}rc_{\rm SW}\omega_A + c_{1111}^{SS}rc_{\rm SW}\omega_A\omega_O + c_{1200}^{SS}c_{\rm SW}^2 + c_{1201}^{SS}c_{SW}\omega_O\big)\Big],$$

$$Z_{ST}^{LR,\overline{MS}} = \frac{g^2}{16\pi^2}3N_c, \qquad Z_{SB1}^{LR,\overline{MS}} = \frac{g^2}{16\pi^2}N_c\left[c_0^{SB1} + c_1^{SB1}\omega_O - \log(a^2\overline{\mu}^2)\right],$$

$$Z_{SB2}^{LR,\overline{MS}} = \frac{g^2}{16\pi^2}N_c\left[c_0^{SB2} + c_1^{SB2}\omega_O - \frac{1}{2}\log(a^2\overline{\mu}^2)\right], \qquad Z_{A1}^{LR,\overline{MS}} = Z_{Ci}^{LR,\overline{MS}} = 0, \text{ for } i = 1,\dots,6 \;(i = 7,8,9 \text{ in forthcoming calculations}).$$

- The Z-factor for the chiral current is also presented below in its most general form; for the numerical values of the coefficients c_{ij}^J, see the second QR code.

$$Z_J^{LR,\overline{MS}} = 1 + \frac{g^2N_c}{16\pi^2}\left(c_{00}^J + c_{10}^Jc_{SW}r + c_{20}c_{SW}^2 + c_{01}^J\omega_A + c_{11}^Jc_{SW}r\omega_A + c_{02}^J\omega_A^2\right).$$

- Future work focuses on calculating the 3-pt Green's functions of S_μ to extend the renormalization program to a broader set of class-C operators, and exploring alternative lattice formulations, such as overlap fermions, to improve chiral symmetry and the realization of supersymmetry within this framework.

Acknowledgements: The projects EXCELLENCE/0524/0001 and EXCELLENCE/0524/0208 are implemented under the programme of social cohesion "THALIA 2021-2027" co-funded by the European Union, through Research and Innovation Foundation.

Cyprus University of Technology · RINNOCO Research · Innovation · Communication · University of Pittsburgh · University of Cyprus · Co-funded by the European Union · Republic of Cyprus · Research & Innovation Foundation