The fate of $U(1)_A$ symmetry in finite temperature QCD

Antonio Smecca

INFN - Sezione di Roma Tre

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University of Maryland

Istituto Nazionale di Fisica Nucleare
SEZIONE DI ROMA TRE

UNIVERSITY OF MARYLAND, COLLEGE PARK

Outline

- Motivations & why $U(1)_A$ is hard to probe
- Part I — probes of $U(1)_A$ restoration
  - $\pi \leftrightarrow \delta$ degeneracies
  - $m_l \to 0$ results
- Part II — Near-zero Dirac spectral density $\rho(\lambda)$
- Conclusions

Symmetries of the QCD lagrangian

The classical, massless, QCD Lagrangian is invariant under

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A.$$

In Nature, light quarks have small but non-zero masses, which makes the above symmetries approximate.

- $U(1)_V$ is realised at the quantum level and is associated to baryon number conservation
- $SU(N_f)_L \times SU(N_f)_R$ is spontaneously broken at $T = 0$ explaining the light meson spectrum
- Several LQCD studies place at $T \approx 154$ MeV the crossover transition into a chirally symmetric phase.

$\langle\overline{\psi}\psi\rangle$ is the order parameter of the phase transition, above T_c $(m_l = 0)$ $\langle\overline{\psi}\psi\rangle = 0$

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Is $U(1)_A$ **effectively** restored?

$U(1)_A$ is explicitly broken at the quantum level

Witten-Veneziano formula relates η' mass to the topological susceptibility χ_t

Pisarski and Wilczek [Pisarski, Wilczek, PRD 29 (1984) 338.] argued that the order of the phase transition in $N_f = 2$ QCD in $m_l = 0$ depends on if $U(1)_A$ gets *effectively* restored at T_c.

effectively $\Rightarrow$ $U(1)_A$ restoration is only approximate, so $U(1)_A$ breaking effects are strongly suppressed compared to those at $T = 0$

the anomaly removes $U(1)_A$ solely due to non-perturbative effects of topological nature $\Rightarrow T_{U(1)_A}$ should occur where topological excitations are much suppressed compared to $T = 0$

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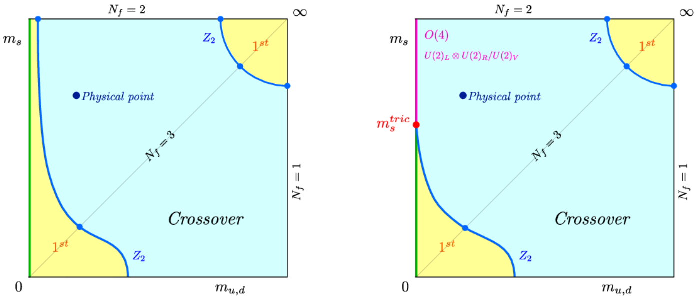
- the anomaly removes $U(1)_A$ solely due to non-perturbative effects of topological nature $\Rightarrow T_{U(1)_A}$ should occur where topological excitations are much suppressed compared to $T = 0$

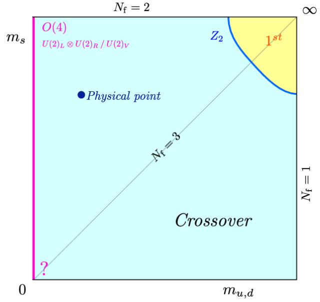
Open questions in the chiral limit

[Pisarski, Wilczek, PRD 29 (1984) 338.]



(a) First order scenario in the $m_s - m_{u,d}$ plane

(b) Second order scenario in the $m_s - m_{u,d}$ plane.



[Cuteri, Philipsen, Sciarra, JHEP 11 (2021) 141]

- If $T_{U(1)_A} \approx T_c$:

  either 1^{st} or 2^{nd} order phase transition

  $U(2)_L \otimes U(2)_R / U(2)_V$
- If $T_{U(1)_A} > T_c$:

  2^{nd} order phase transition $O(4)$

[Butti, Pelissetto, Vicari, JHEP 08 (2003) 029]
[Pelissetto, Vicari, PRD 88 (2013) 105018]
[Grahl, PRD 90 (2014) 117904]

Phenomenological implications

Heavy-ion collisions

enhancement in η'/π^0 and η/π^0

effective mass shift of η'

Shuryak, Comments Nucl. Part. Phys. 21, 235 (1994)

Kapusta, Kharzeev, McLerran, PRD 53, 5028 (1996)

Huang, Wang, PRD 53, 5034 (1996)

Bass, Moskal, Rev. Mod. Phys. 91, 015003 (2019)

Pisarski, Rennecke, PRL 132, 251903 (2024)

A. Y. Kotov, M. P. Lombardo and A. M. Trunin, Phys. Lett. B, 794, 83-88 (2019)

Axion cosmology

$U(1)_A$ breaking related to χ_t

$$m_a^2 = \chi_t/f_a^2$$

Bonati, D'Elia, Mariti *et al.* JHEP 03 (2016), 155

Petreczky, Schadler, Sharma, Phys. Lett. B 762, 498 (2016)

Frison, Kitano, Matsufuru *et al.* JHEP 09 (2016), 021,

Borsányi *et al.*, Nature 539, 69 (2016)

Lombardo, Trunin, Int. J. Mod. Phys. A 35, 2030010 (2020)

A. Y. Kotov, M. P. Lombardo and A. Trunin, JHEP, 09, 045 (2025)

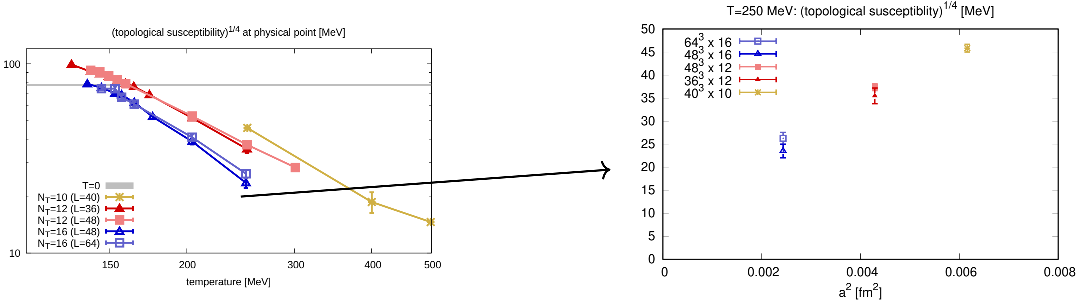
Athenodorou, Bonanno, Bonati *et al.* JHEP 10 (2022)

Bonanno, Bonati, D'Elia, arXiv:2604.28035

Bonanno, Bonati, D'Elia, arXiv:2510.03059

χ_t with MDWF [JLQCD, arXiv:2603.23022]

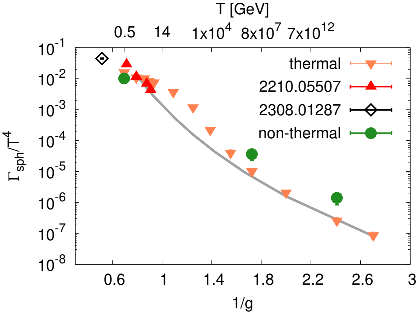
- $N_f = 2+1$ at physical point on $36^3 \times 12$, $48^3 \times 16$, $64^3 \times 16$
- finest $1/a \sim 4$ GeV, $m_{\text{res.}} \sim 0.01$ MeV at $T = 250$ MeV, but sizable discretisation error still observed
- χ_t and $U(1)_A$ anomaly in general is a difficult quantity to measure in LQCD



$N_t = 10, 12, 16$ show strong dependence on the lattice cutoff
Even on finest lattice with $1/a \sim 4$ GeV at $T = 250$ MeV, sizable discretisation error still observed

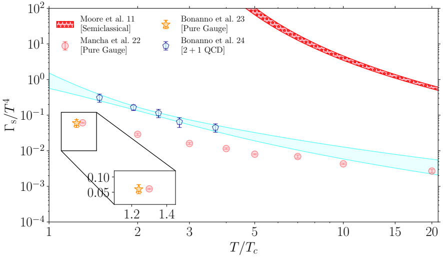
Sphaleron rate

The U(1)$_A$ anomaly ties chirality to gauge topology, $\partial_\mu j_5^\mu = 2N_f\, q(x)$ $(m_f = 0)$.
The real-time rate of topological transitions is the **sphaleron rate** $\Gamma_{\rm sph}$ which sets the *thermalization rate of axial charge* in the QGP (chiral magnetic effect)
The **topological rate** $\Gamma_{\rm top}(\omega, \mathbf{p})$ drives thermal axion production $(\Gamma_{\rm top}(0,0) = \Gamma_{\rm sph})$



[Guin, Sharma, arXiv:2604.07256 (accepted in PRD)]

effective 3D Hamiltonian for soft magnetic gluons SU(2) & SU(3); recovers $\Gamma_{\rm sph} \sim g^{10} T^4$ above the EW scale
Compute axion thermal production rate $R(T) = \int d^3\boldsymbol{k}\, \Gamma_{\rm top}(\boldsymbol{k}, T)$



[Bonanno *et al.* PRD 108 (2023) 074515 & PRL 132 (2024) 051903]

N_f=2+1 at the physical point;
light quarks *enhance* $\Gamma_{\rm sph}$; the semiclassical prediction (band) fails near the chiral crossover
new: momentum dependence of $\Gamma_{\rm top}(\omega = p)$

➜ **R. Dionisio** Mon 14:40

Probing $U(1)_A$ restoration

There is no order parameter for $U(1)_A$ to check for its effective restoration.

The $U(1)_A$ transformation mixes flavour non-singlet pseudoscalar (π) and scalar (δ/a_0) correlators. The same is true for the flavour singlet pseudoscalar (η) and scalar (σ/f_0):

$$\pi \overset{U(1)_A}{\longleftrightarrow} \delta \qquad \eta \overset{U(1)_A}{\longleftrightarrow} \sigma$$

Check (approximate) degeneracies between the channels using

- Screening $C_{\pi,\delta}(z)$ and temporal correlators $C_{\pi,\delta}(\tau)$
- screening masses $m_\pi - m_\delta$
- susceptibilities $\chi_\pi - \chi_\delta$ with $\chi_{\pi,\delta} = \int d^4x \, \langle O_{\pi,\delta}(x) O_{\pi,\delta}(0) \rangle$

Susceptibilities ratio [Chiu, Hsieh, arXiv:2602.14811]

$N_f = 2+1+1$ optimal DWF, $m_\pi^{\text{phys.}}$, $32^3 \times (16, 12, 10, 8, 4)$, $T_c \sim 150$ MeV
$a = (0.075, 0.069, 0.064)$ fm

dimensionless RG-invariant ratio

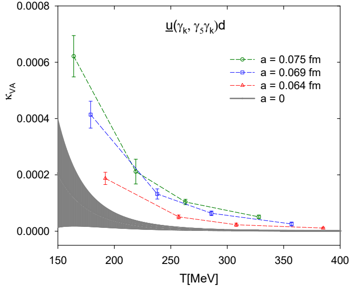
$$\kappa_{AB} = \frac{\chi_A^{\text{reg.}} - \chi_B^{\text{reg.}}}{\chi_A^{\text{reg.}} + \chi_B^{\text{reg.}}}$$

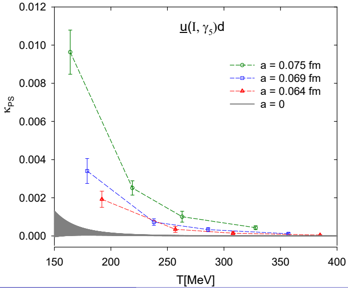
where $\chi_A^{\text{reg.}}(T, T_r) = \chi_A(T) - \chi_A(T_r)$ is the regularised susceptibility.

- $T_r \gg T_{\text{pc}}$ where symmetry **effectively** restored, $aT_r = 1/4$
- chiral and $U(1)_A$ **effectively** restored just above T_{pc}:

$\boxed{\kappa_{PS} = 0.81(5), \kappa_{VA} = 0.0078(9) \text{ @ } T \sim 41 \text{ MeV}}$

- $a \to 0$ upper bound on exact restoration





Wilson fermions

- [Brandt *et al.*, JHEP 12 (2016) 158] first result with Wilson fermions studying $U(1)_A$ restoration. $N_f = 2$, $m_\pi > m_\pi^{\text{phys.}}$, $m_l \to 0$ yields screening $m_\pi = m_\delta$ at T_{pc} within errors.
- FASTSUM [AS, Aarts, Allton *et al.*, arXiv:2604.11916]
  Anisotropic: $\xi = \frac{a_s}{a_\tau} > 1$, $N_f = 2+1$, $a_s \simeq 0.11$ fm, $L_s \in [3.08, 3.52]$ fm
  Generation 2 & 2L: $\xi \approx 3.5, m_\pi = 380, 240$ MeV, $T_{\text{pc}} \simeq 181, 167$ MeV, $a_\tau \simeq 0.035$ fm
  Generation 3: $\xi \approx 7, m_\pi = 380$ MeV, $T_{\text{pc}} \simeq 182$ MeV, $a_\tau = 0.015$ fm.

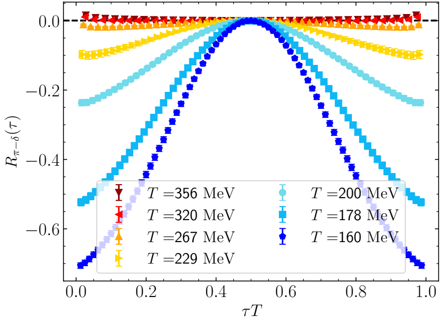
build dimensionless ratio

NOT the same as [Chiu, Hsieh, arXiv:2602.14811]

$$R_{\pi-\delta}(\tau, T) = \frac{\tilde{G}_\pi(\tau) - \tilde{G}_\delta(\tau)}{\tilde{G}_\pi(\tau) + \tilde{G}_\delta(\tau)},$$

$$\tilde{G}(\tau) = \frac{G(\tau)}{G(N_\tau/2)}$$

$R_{\pi-\delta}(N_\tau/2, T) = 0$ by construction



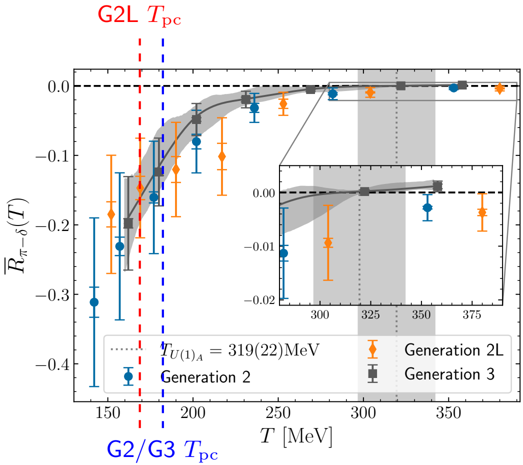
[AS, Aarts, Allton *et al.*, arXiv:2604.11916]

Wilson fermions

To check $U(1)_A$ breaking we define

$$\overline{R}_{\pi-\delta}(T) = \sum_{\tau=\tau_{\min}}^{N_\tau/2-1} R_{\pi-\delta}(\tau, T)$$

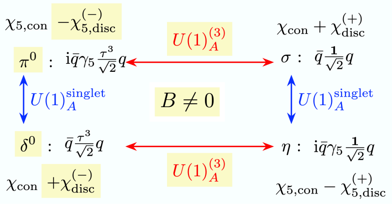
- $U(1)_A$ still broken at $T_{\rm pc}$
- $U(1)_A$ breaking is much suppressed for $T \approx 300$ MeV
- $m_l \to 0$ and $a_{s,\tau} \to 0$ needed to make conclusive claim



[AS, Aarts, Allton *et al.*, arXiv:2604.11916]

Susceptibilities in $B \neq 0$ [Ding, Hernández, Zhang, arXiv:2607.11625]

At $B \neq 0$, charged channels are no longer symmetry-equivalent to the neutral direction

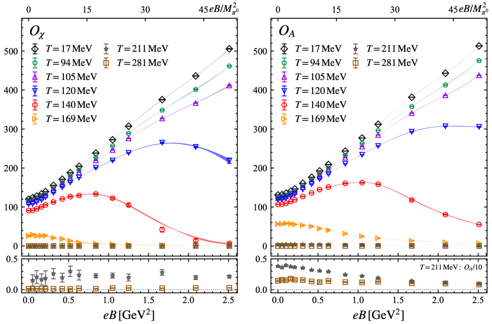


$U(1)_A^{(3)}$: $\pi^0 \leftrightarrow \sigma, \quad \delta^0 \leftrightarrow \eta$

$$O_\chi(B,T) \equiv \frac{m_s^2}{f_K^4}[\chi_{\pi^0}(B,T) - \chi_\sigma(B,T)]$$

$U(1)_A^{\mathrm{singlet}}$: $\pi^0 \leftrightarrow \delta^0, \quad \sigma \leftrightarrow \eta$

$$O_A(B,T) \equiv \frac{m_s^2}{f_K^4}[\chi_{\pi^0}(B,T) - \chi_{\delta^0}(B,T)]$$

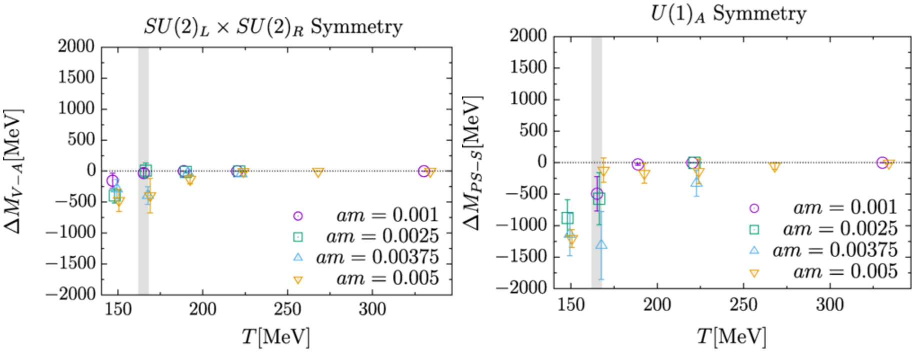


- low T: increase monotonically; counterpart of magnetic catalysis (MC) in O_χ; MC-like enhancement in O_A
- near T_{pc}: strong eB gives inverse MC in O_χ; inverse MC-like in O_A, reduction is milder and sets in at larger eB than O_χ

Towards $m_l \to 0$: Screening masses

JLQCD studied screening masses with MDWF with fermion masses below and above physical point [JLQCD, PRD 111 (2025) 11, 114506]

Lattice setup $N_f = 2$, $a \sim 0.075$ fm, $L \in [2.4, 3, 3.6]$ fm, $T \in [147, 330]$, $T_c \simeq 165$ MeV



[JLQCD, PRD 111 (2025) 11, 114506]

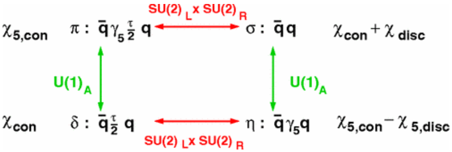
Towards $m_l \to 0$: Susceptibilities

[Ding *et al.*, PRL 126 (2021) 8, 082001]

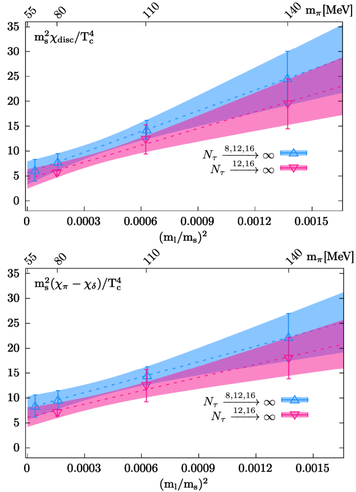
$N_f = 2+1$ HISQ
$a = (TN_\tau)^{-1} \in [0.12, 0.08, 0.06]$ fm
aspect ratio $N_s/N_\tau = [4-9]$
$T \approx 205$ MeV $\approx 1.6T_c$
results are compatible changing the order of limits $a \to 0$ and $m_l \to 0$

For $T \geq T_c$

$$\lim_{m_l \to 0} \chi_\pi - \chi_\delta = \lim_{m_l \to 0} \chi_{\rm disc}$$



[HotQCD, PRD 86 (2012) 094503]



[Ding *et al.*, PRL 126 (2021) 8, 082001]

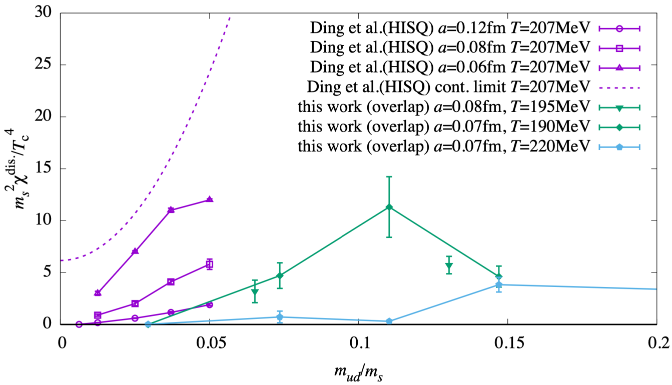
Towards $m_l \to 0$: Susceptibilities

[JLQCD, PTEP 2022 (2022) 2, 023B05]

reweighted Overlap on MDWF $N_f = 2$

$a = 0.074$ fm, $T \in [190 - 330]$ MeV, $L = 32$ (2.4 fm)

- A qualitative sharp drop is seen in both results
- mild discretisation effects in MDWF
- HISQ results deviate from zero as a decreases
- HISQ physical volumes larger than MDWF: HISQ $\approx [3.8 - 8.6]$ fm MDWF ≈ 2.4 fm



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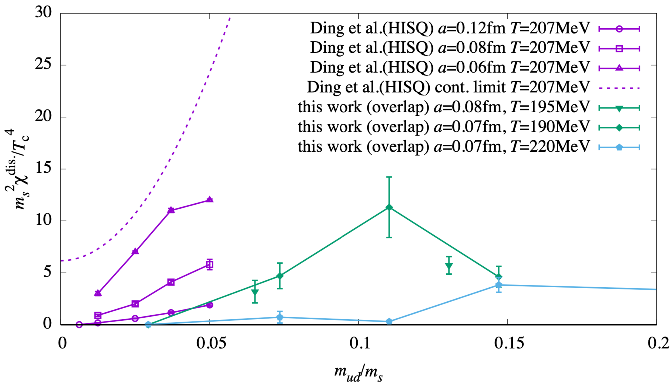

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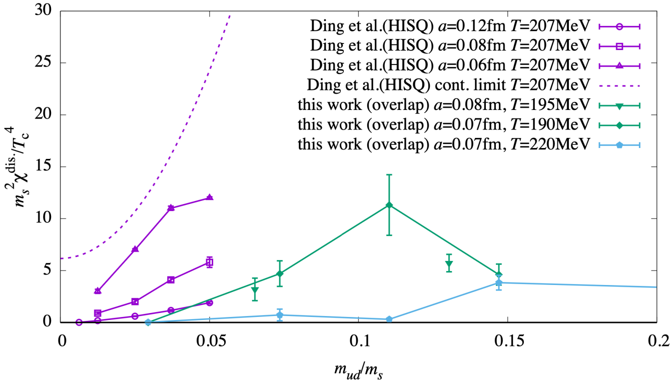

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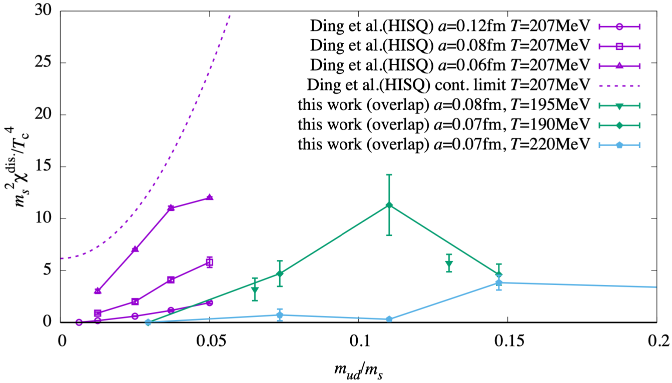

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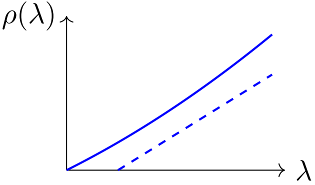
Dirac operator spectral density

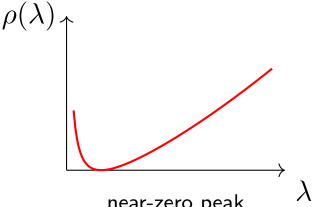
$$\chi_\pi - \chi_\delta = \int d\lambda \, \frac{2m_l^2 \rho(\lambda)}{(\lambda^2 + m_l^2)^2} \qquad \rho(\lambda) = \frac{1}{V} \sum_i \langle \delta(\lambda - \lambda_i) \rangle$$

- Here **effective** restoration takes a stronger meaning w.r.t Pisarski and Wilczek
  - ➜ anomaly invisible in $\chi_\pi - \chi_\delta$
- Gap in $\rho(\lambda)$ near zero $\Rightarrow \chi_\pi - \chi_\delta = 0$
  [Cohen, arXiv:nucl-th/9801061]
- $\rho(\lambda)$ analytic in m^2 *and* regular at $\lambda = 0$ implies anomaly invisible in a limited set of scalar/pseudoscalar multi-point functions above T_c
  [Aoki, Fukaya, Taniguchi, PRD86 (2012) 114512]
- Singular $\rho(\lambda)$ at $\lambda = 0$ (analyticity in m^2 kept) $\Rightarrow U(1)_A$ can stay broken [Giordano, arXiv:2510.24381]



regular at origin (or gapped)

$\chi_\pi - \chi_\delta = 0$



near-zero peak

$\chi_\pi - \chi_\delta \neq 0$

The fate of $U(1)_A$ depends on the near-zero behaviour of $\rho(\lambda)$

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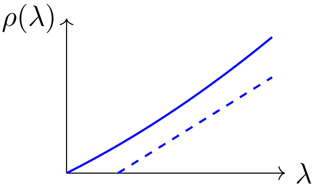

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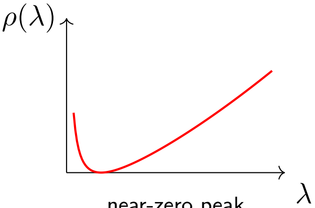

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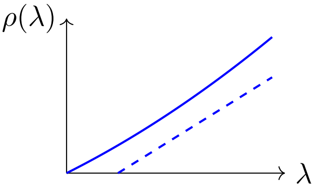
Dirac operator spectral density

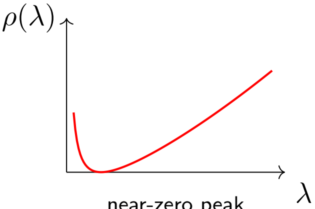
$$\chi_\pi - \chi_\delta = \int d\lambda \, \frac{2m_l^2 \rho(\lambda)}{(\lambda^2 + m_l^2)^2} \qquad \rho(\lambda) = \frac{1}{V} \sum_i \langle \delta(\lambda - \lambda_i) \rangle$$

- Here **effective** restoration takes a stronger meaning w.r.t Pisarski and Wilczek
  ➜ anomaly invisible in $\chi_\pi - \chi_\delta$
- Gap in $\rho(\lambda)$ near zero $\Rightarrow \chi_\pi - \chi_\delta = 0$
  [Cohen, arXiv:nucl-th/9801061]
- $\rho(\lambda)$ analytic in m^2 *and* regular at $\lambda = 0$ implies anomaly invisible in a limited set of scalar/pseudoscalar multi-point functions above T_c
  [Aoki, Fukaya, Taniguchi, PRD86 (2012) 114512]
- Singular $\rho(\lambda)$ at $\lambda = 0$ (analyticity in m^2 kept) $\Rightarrow U(1)_A$ can stay broken [Giordano, arXiv:2510.24381]



regular at origin (or gapped)

$\chi_\pi - \chi_\delta = 0$



near-zero peak

$\chi_\pi - \chi_\delta \neq 0$

The fate of $U(1)_A$ depends on the near-zero behaviour of $\rho(\lambda)$

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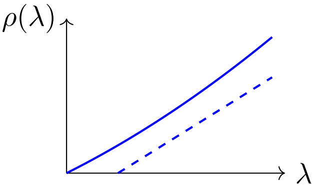

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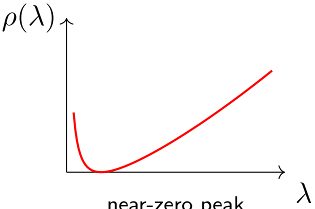

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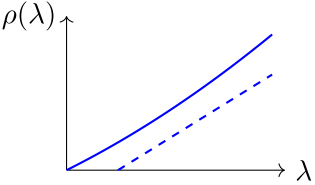

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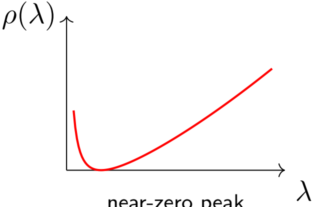

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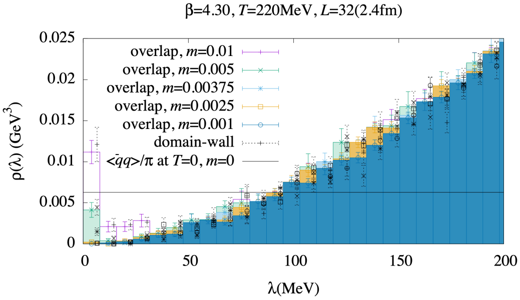
$\rho(\lambda)$ in the chiral limit

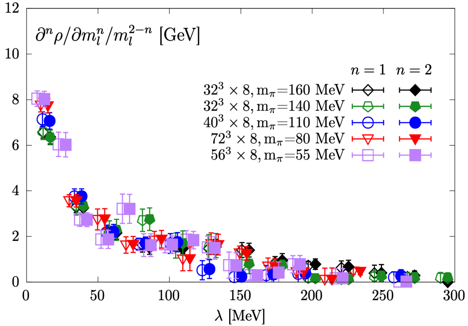
$N_f = 2$ reweighted overlap on MDWF

[JLQCD, PRD 103 (2021) 7, 074506]



$N_f = 2+1$ HISQ, $T \approx 205$ MeV

[Ding *et al.*, PRL 126 (2021) 8, 082001]



- peak disappears for $m_l \to 0$ with MDWF
- non-vanishing 1^{st} and 2^{nd} derivatives $\Rightarrow$ persistent peak with HISQ
- violation of GW equation largely affect near-zero λ

[Cossu, Fukaya, Hashimoto, Tomiya, Phys. Rev. D93, 034507 (2016), 1510.07395]

Possible IR transition

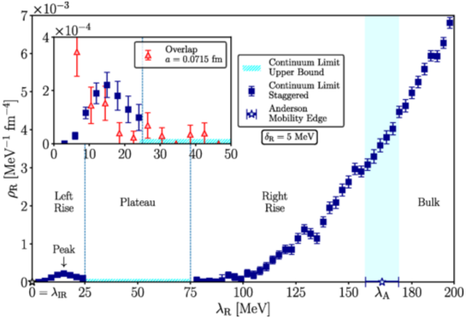
A peak in the near-zero region of $\rho(\lambda)$ has been related to the emergence of a new IR phase above $T_{\rm pc}$ [Alexandru, Horváth, PRD, 045038 (2015)]

- $N_f = 2+1$ stouted staggered fermions, $m_\pi^{\rm phys.}$, $a \to 0$ find $T_{IR} \approx 200 - 230$ MeV
- Infrared Dirac modes separate from the bulk
- Emergence of a distinct IR component
- Possible new thermal regime of QCD

- May provide a microscopic origin for the strongly coupled nature of the QGP
- Connects QCD infrared dynamics to Anderson-critical phenomena
- Raises the possibility of novel collective gluonic excitations in the medium

➜ **N. Dhindsa**, Fri 17:10

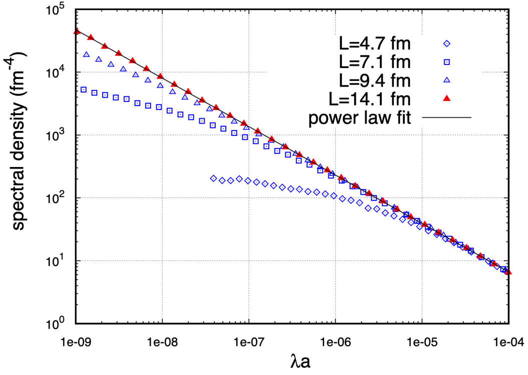
➜ **I. Horváth**, Fri 17:30



[Alexandru, Bonanno, D'Elia, Horváth, PRD 110 (2024) 7, 074515]

Why a peak in $\rho(\lambda)$?

- Explanation in terms of a instanton-anti-instanton model [Kovács PRL 132 (2024) 13, 131902] (see Kovacs@LATTICE '24)
- Near zero $\rho(\lambda) \propto \lambda^{\alpha}$ with $\alpha = -0.770(5)$ [Kovács PRL 132 (2024) 13, 131902]
- If the peak is due to topological fluctuations need at least $V_3 \approx \frac{T}{\Delta m^2} \approx \frac{T}{\chi_t}$ [Giordano arXiv:2510.24392]



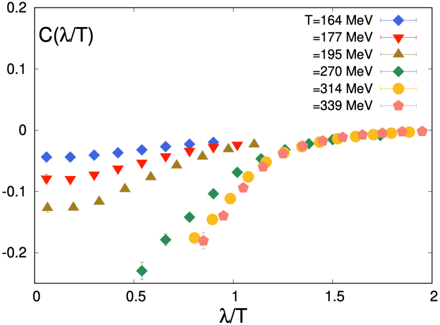
[from slides Kovacs@LATTICE '24]

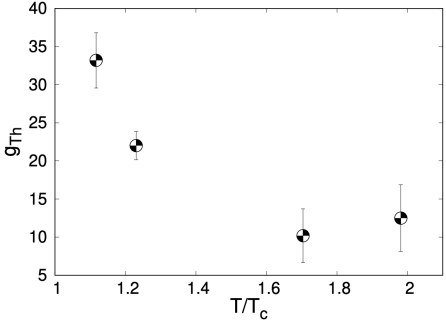
Delocalisation of near-zero λ

$U(1)_A$ breaking requires the delocalisation of near zero modes for $m \neq 0$
observed in [Meng *et al.*, JHEP 12 (2024) 101]

Delocalisation investigated in [Shanker, Pandey, Sharma, arXiv:2602.24227] with $N_f = 2+1$ MDWF
$L = 32,\ L \in [4 - 2.5]$ fm $m_\pi = 140$ MeV

Define $C(\lambda/T)$ as correlation between IR λ and fluctuation of $\mathrm{Re}.P(\boldsymbol{x})$ about its mean
strong correlation with (rare, random) $\mathrm{Re}.P$ wells $\Rightarrow$ localisation
$g_{\mathrm{Th}} = \frac{1}{\langle s \rangle} \langle \frac{\partial^2 \lambda_i}{\partial \eta^2} \rangle_{\eta \to 0}$, $\eta \to$ twist in spatial direction





Conclusions

- $U(1)_A$ restoration is a challenging problem to study in LQCD
- Strong suppression of $U(1)_A$ breaking found near $T_{\rm pc}$ with DWF and above $T_{\rm pc}$ with Wilson fermions ($m_\pi > m_\pi^{\rm phys.}$)
- $N_f = 2$ Results with chiral fermions point to a restoration of $U(1)_A$ near T_c in $m_l \to 0$ with several observables: screening masses, susceptibilities, near-zero $\rho(\lambda)$
- $N_f = 2+1$ continuum extrapolated HISQ results point to a broken $U(1)_A$ above T_c in $m_l \to 0$ and a persistent IR peak
- **If** $U(1)_A$ broken ($m_l = 0$), near-zero $\rho(\lambda)$ peak can be explained by instanton-anti-instanton model

I am grateful to T.W. Chiu, H. Fukaya, M.P. Lombardo, C. Bonanno, M. Giordano, S. Guin and D. Zhang, for sharing their material and for useful discussions

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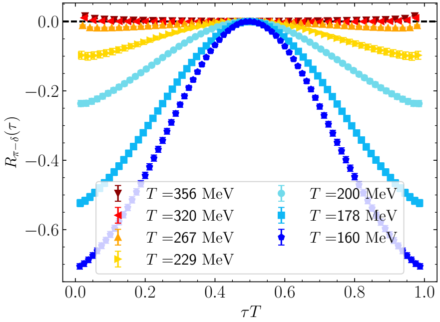
BACKUP SLIDES

Single pole ansatz

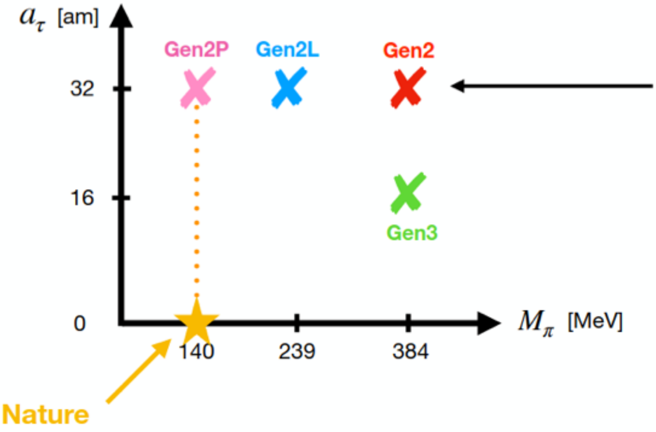
dimensionless masses $\tilde{m}_{\pi,\delta}/T$

$$R_{\pi-\delta}(\tau,T) = \frac{\cosh \tilde{m}_\pi x - \cosh \tilde{m}_\delta}{\cosh \tilde{m}_\pi x + \cosh \tilde{m}_\delta}$$
$$= \frac{1}{4}(\tilde{m}_\pi^2 - \tilde{m}_\delta^2)x^2 + \mathcal{O}(x^4)$$

where $x = \tau T - 1/2$

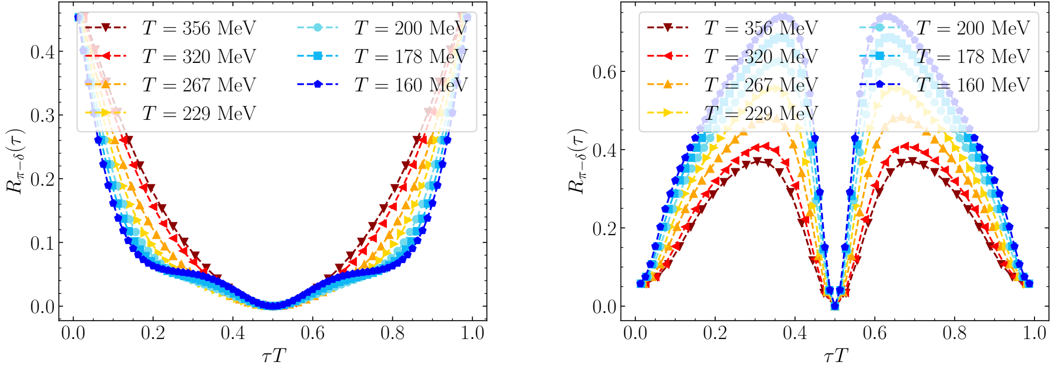


ensembles



a_τ [am]
Gen2P
Gen2L
Gen2
32
16
Gen3
0
M_π [MeV]
140
239
384
Nature

Free fermions



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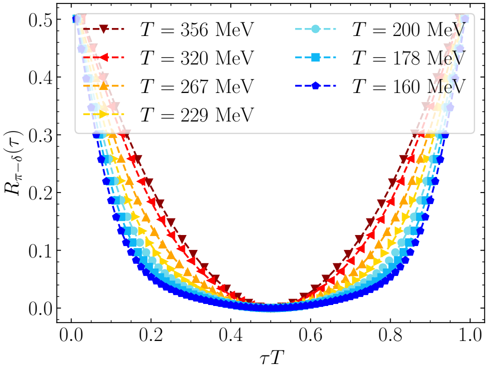
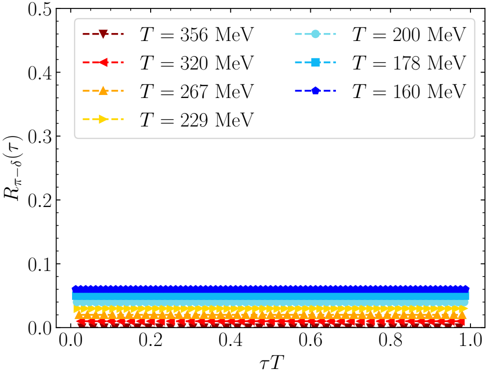
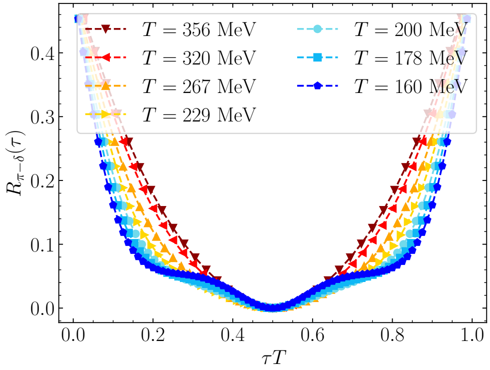
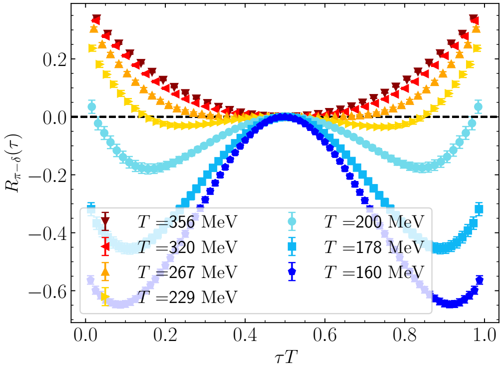




local fermions



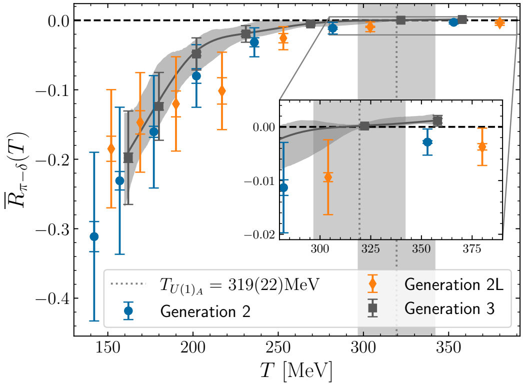


Wilson fermions

To check $U(1)_A$ breaking we define

$$\overline{R}_{\pi-\delta}(T) = \sum_{\tau=\tau_{\min}}^{N_\tau/2-1} R_{\pi-\delta}(\tau, T)$$

- $U(1)_A$ still broken at T_{pc}
- $U(1)_A$ breaking is much suppressed for $T \approx 300$ MeV
- $m_l \to 0$ and $a_{s,\tau} \to 0$ needed to make conclusive claim



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Define $C(\lambda/T)$ as C_n binned over λ/T with $C_n = \sum_{\boldsymbol{x}} |\Psi_n(\boldsymbol{x})|^2 [\mathrm{Re}.P(\boldsymbol{x}) - \langle \mathrm{Re}.P(\boldsymbol{x}) \rangle]$
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