Massachusetts Institute of Technology

# Nuclear and hypernuclear physics from lattice QCD

**Robert Perry**

**Lattice 2026, University of Maryland, College Park**

**30 July, 2026**

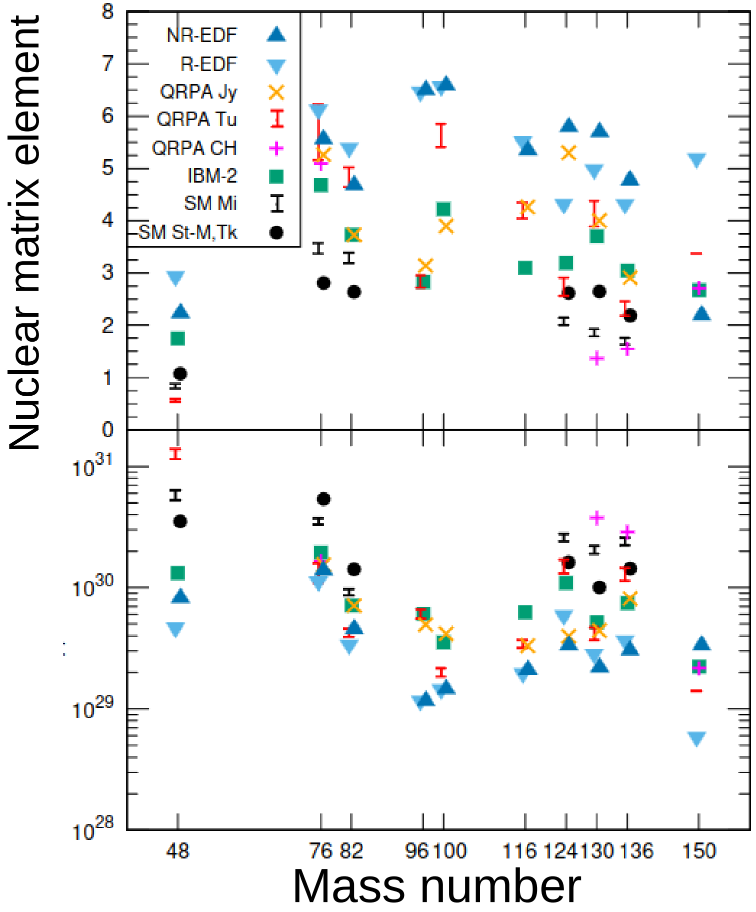
# Why nuclear physics?

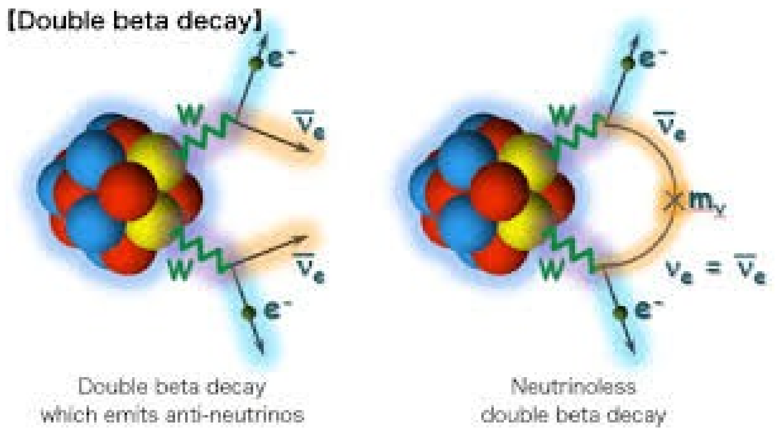
Rept.Prog.Phys. 80 (2017) 4, 046301

Interesting problem: observe the emergence of nuclear complexity from the Standard Model

Understanding nuclear physics is essential for BSM searches:

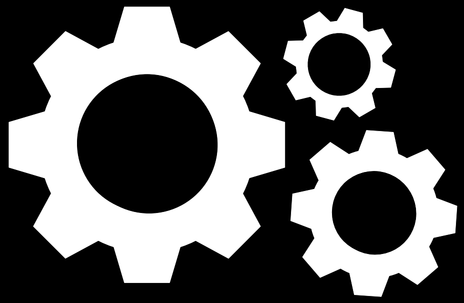
"Discovery potential" often limited by systematic uncertainties in nuclear targets




Order of magnitude discrepancy in the decay rate!

# “Lattice QCD is the only *ab-initio*, systematically improvable approach to the strong nuclear force”



# Outline

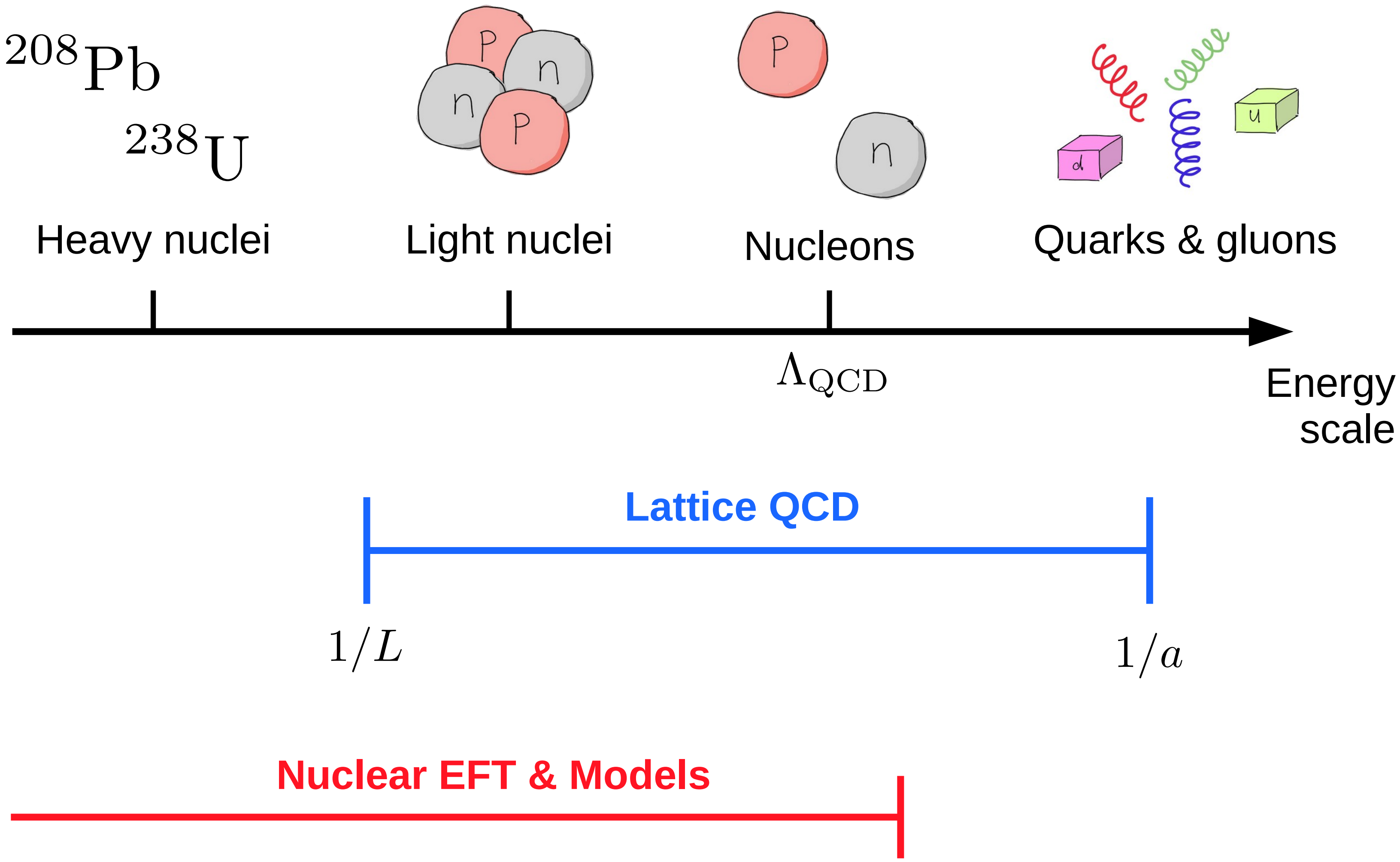
- Why nuclear physics?
- How do we study nuclear physics?
- Why are these calculations challenging?
- A historical perspective
- Towards systematically improvable nuclear physics
- Final thoughts & Conclusions

# Nuclear physics is a multi-disciplinary problem

Rosemary Zielinski
Wednesday, 9:40 AM
Margaret Brent B

Matthias Heinz
Wednesday, 10:00 AM
Margaret Brent B

Use information from small nuclear systems to constrain heavier systems



# How do we study nuclear physics?

**Commun.Math.Phys. 104 (1986) 177**
**Commun.Math.Phys. 105 (1986) 153-188**
**Nucl.Phys.B 339 (1990) 222-252**

**Few Body Syst. 29 (2000) 95-120**
**Phys.Rev.D 71 (2005) 094504**
**Phys.Rev.Lett. 99 (2007) 022001**

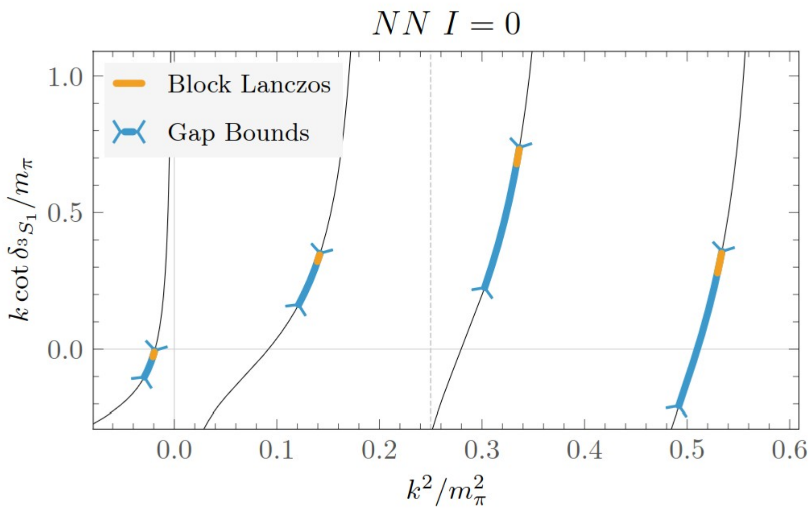
## Finite-volume quantization conditions

Compute finite-volume energies

$$\{E_0(L),\ E_1(L),\ E_2(L), \ldots\}$$

Quantization condition connects FV energies with infinite-volume phase-shift

$$k\cot\delta(k) = \frac{1}{\sqrt{\pi}L} Z_{00}(1, k^2L^2/(2\pi)^2)$$




## Nambu-Bethe-Salpeter wavefunction

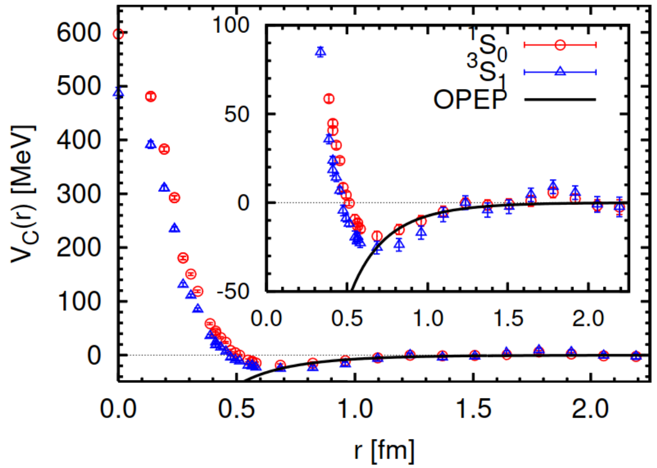
Compute NBS wave function

$$\phi_{k_n}(\vec{r})e^{-E_n t} = \langle 0|N(\vec{y},t)N(\vec{x},t)|n\rangle$$

Asymptotic spatial dependence determined by phase-shift

$$\phi_n(\vec{r}) \sim \frac{\sin(kr + \delta_0(k))}{kr}$$

Constrain effective potential



# How do we study nuclear physics?

Commun.Math.Phys. 104 (1986) 177
Commun.Math.Phys. 105 (1986) 153-188
Nucl.Phys.B 339 (1990) 222-252

Few Body Syst. 29 (2000) 95-120
Phys.Rev.D 71 (2005) 094504
Phys.Rev.Lett. 99 (2007) 022001

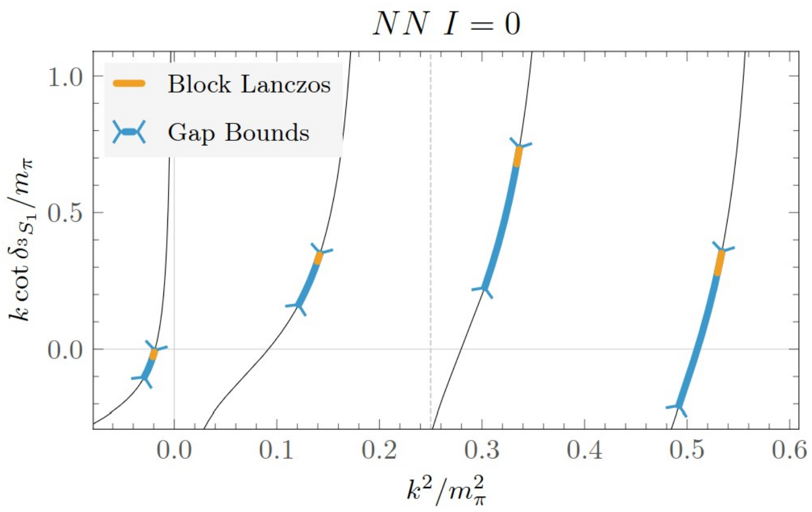
## Finite-volume quantization conditions

Compute finite-volume energies

$$\{E_0(L),\ E_1(L),\ E_2(L), \ldots\}$$

Quantization condition connects FV energies with infinite-volume phase-shift

$$k\cot\delta(k) = \frac{1}{\sqrt{\pi}L} Z_{00}(1, k^2L^2/(2\pi)^2)$$




## Nambu-Bethe-Salpeter wavefunction

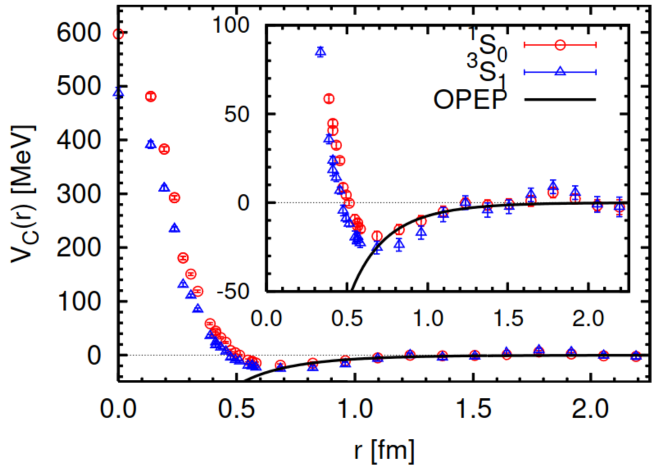
Compute NBS wave function

$$\phi_{k_n}(\vec{r})e^{-E_n t} = \langle 0|N(\vec{y},t)N(\vec{x},t)|n\rangle$$

Asymptotic spatial dependence determined by phase-shift

$$\phi_n(\vec{r}) \sim \frac{\sin(kr + \delta_0(k))}{kr}$$

Constrain effective potential



# Introduction to spectroscopy

Study two-point correlation functions

$$C_{ij}(t) = \langle \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) \rangle$$

Correlator exhibits a spectral decomposition

$$C_{ij}(t) = \sum_n \langle 0 | \mathcal{O}_i(0) | n \rangle \langle n | \mathcal{O}_j^\dagger(0) | 0 \rangle \, e^{-E_n t}, \qquad H |n\rangle = E_n |n\rangle$$

Define the effective mass

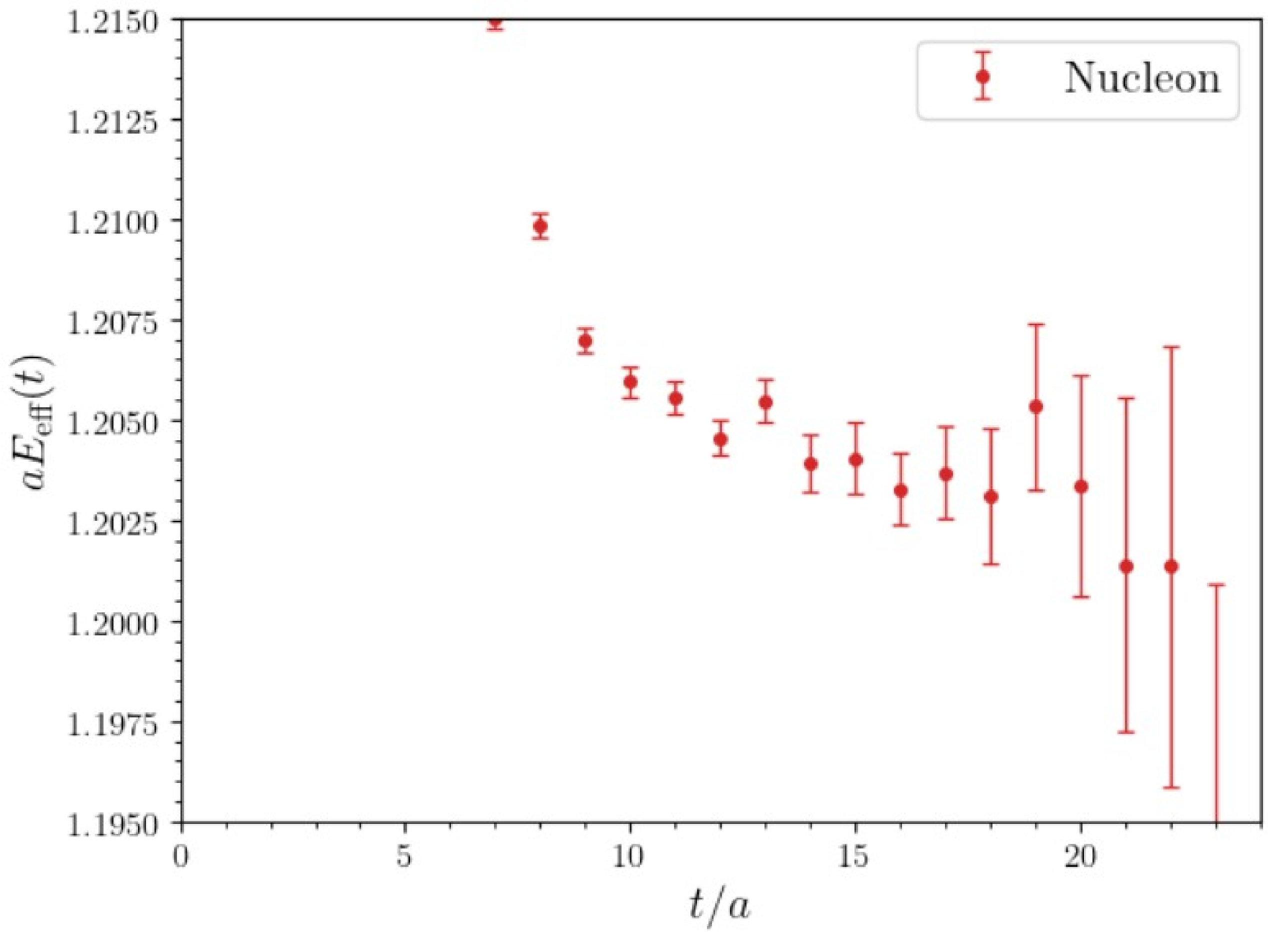
$$E_{\text{eff}}(t) = -\ln\left[\frac{C_{ij}(2t+1)}{C_{ij}(2t)}\right]$$

Large time limit

$$\lim_{t\to\infty} C_{ij}(t) = a_0 e^{-E_0 t} \quad \Longrightarrow \quad \lim_{t\to\infty} E_{\text{eff}}(t) = E_0$$

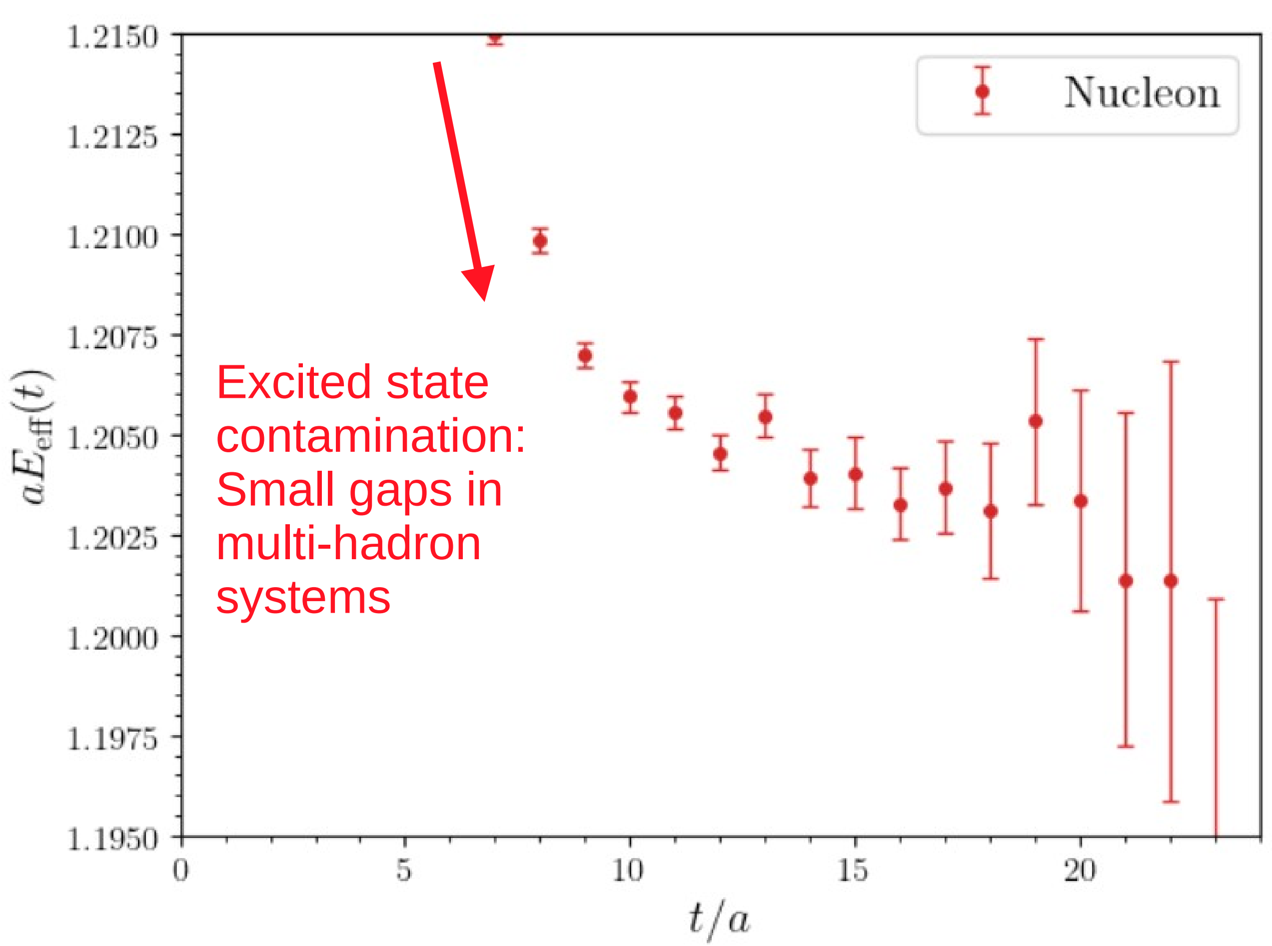
# What makes these calculations difficult?

**Phys.Rept. 103 (1984) 203-211**
**Theoretical Advanced Study Institute in Elementary Particle Physics (1989)**



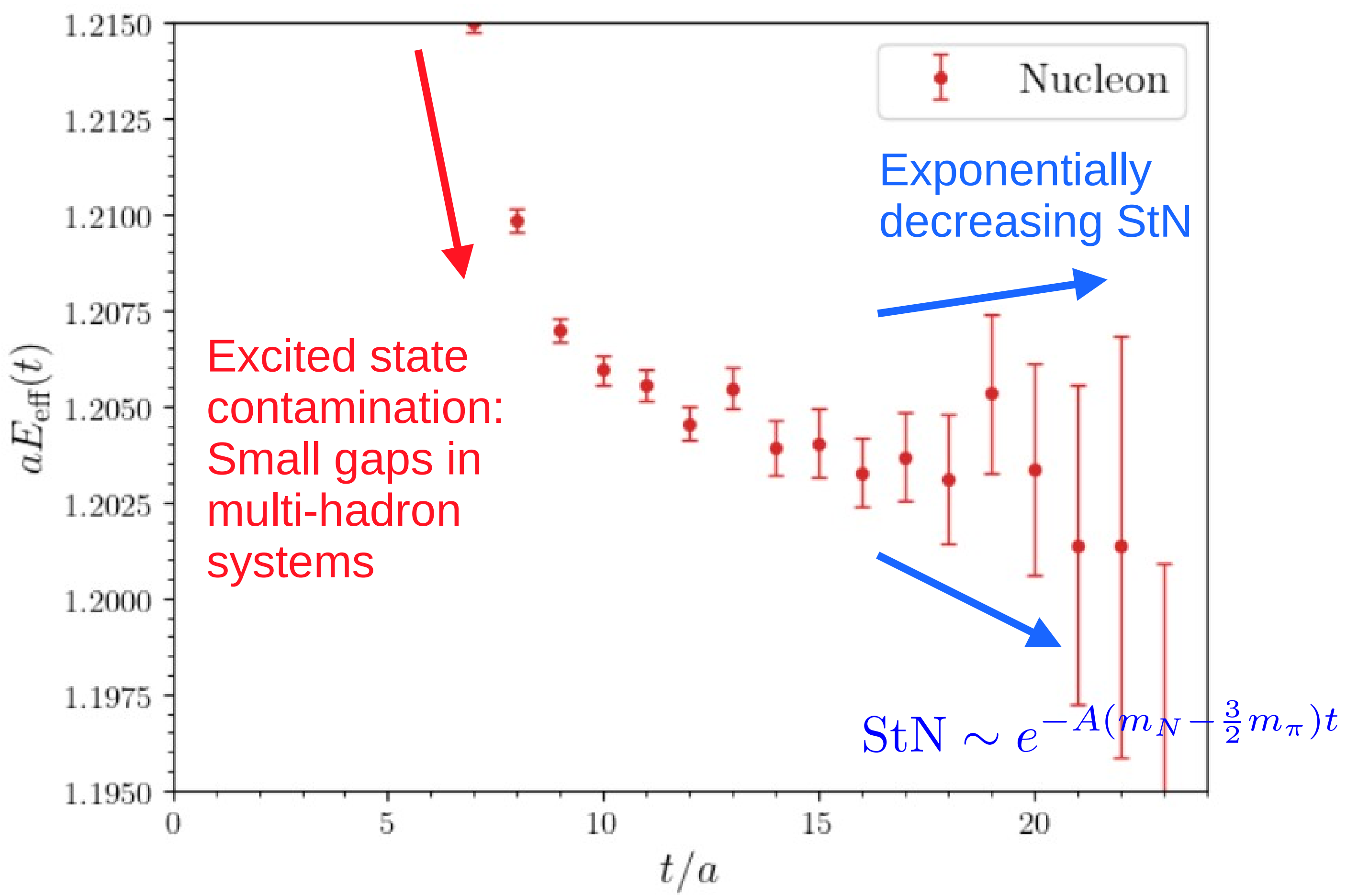
# What makes these calculations difficult?

**Phys.Rept. 103 (1984) 203-211**
**Theoretical Advanced Study Institute in Elementary Particle Physics (1989)**



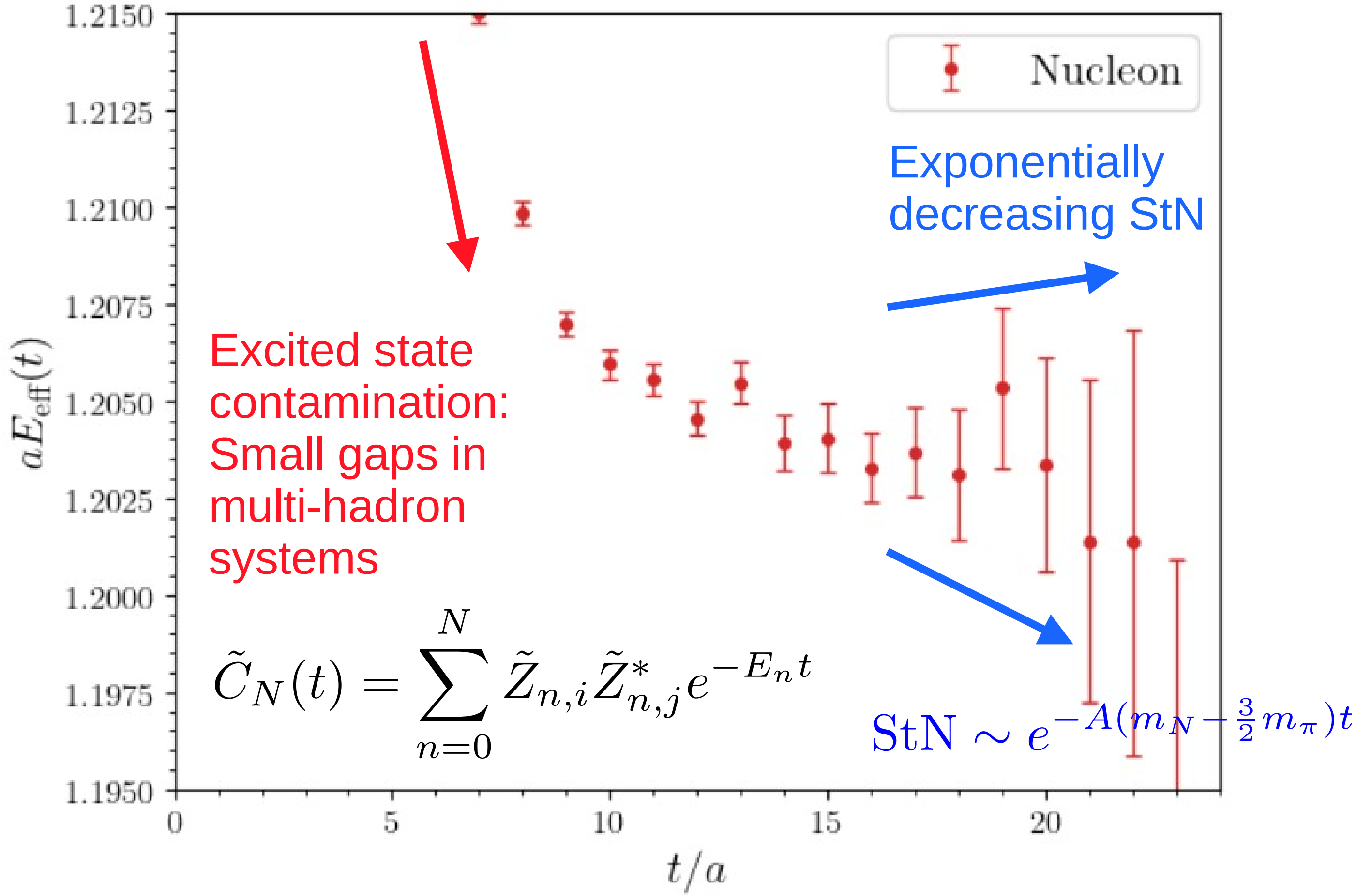
# What makes these calculations difficult?

**Phys.Rept. 103 (1984) 203-211**
**Theoretical Advanced Study Institute in Elementary Particle Physics (1989)**



# What makes these calculations difficult?

**Phys.Rept. 103 (1984) 203-211**
**Theoretical Advanced Study Institute in Elementary Particle Physics (1989)**

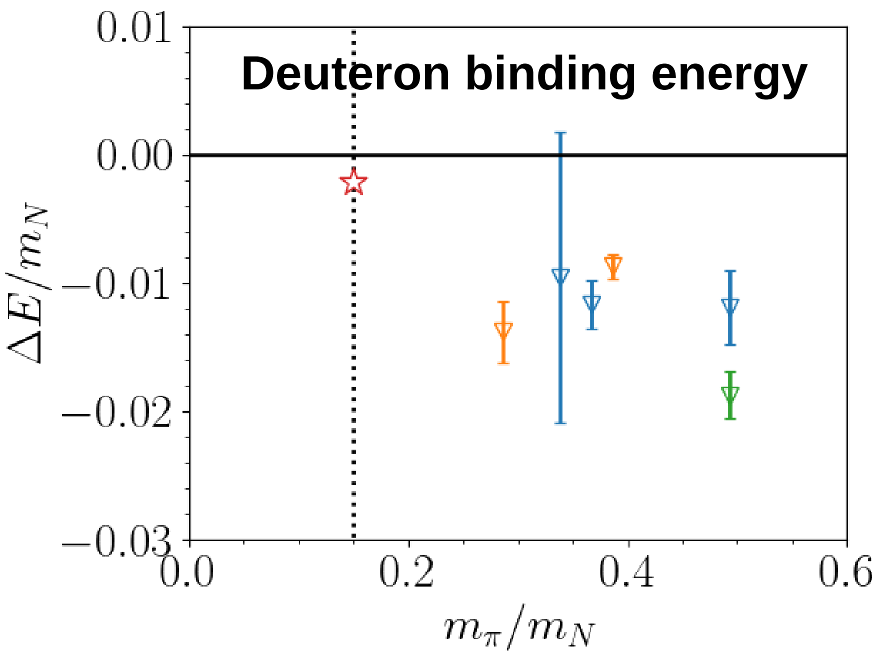



**Regularizing assumption: N-state-saturation**

There exists a region of Euclidean time in which the correlator is accurately described by a truncated spectral decomposition with M ≥ N states, where N states approximate genuine energy eigenstates up to statistical noise (e.g., the N lowest-energy states).

# Do two baryons bind at heavy quark masses? (circa 2018)

Phys.Lett.B 712 (2012) 437-441
Phys.Rev.D 85 (2012) 054511
Phys.Rev.D 86 (2012) 074514
Phys.Rev.D 87 (2013) 3, 034506
Phys.Rev.D 92 (2015) 1, 014501
Phys.Rev.D 92 (2015) 11, 114512
PoS CD15 (2016) 020
Phys.Lett.B 765 (2017) 285-292

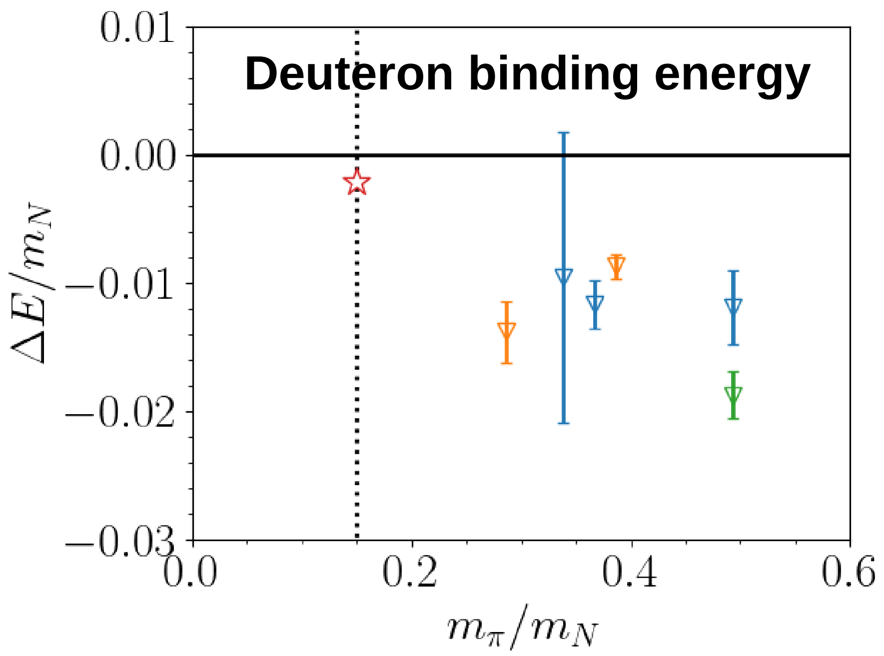



Spectroscopy: bound deuteron

HAL QCD: No bound state

## Do two baryons bind at heavy quark masses? (circa 2018)

**Phys.Lett.B 712 (2012) 437-441**
**Phys.Rev.D 85 (2012) 054511**
**Phys.Rev.D 86 (2012) 074514**
**Phys.Rev.D 87 (2013) 3, 034506**
**Phys.Rev.D 92 (2015) 1, 014501**
**Phys.Rev.D 92 (2015) 11, 114512**
**PoS CD15 (2016) 020**
**Phys.Lett.B 765 (2017) 285-292**




Spectroscopy: bound deuteron

HAL QCD: No bound state

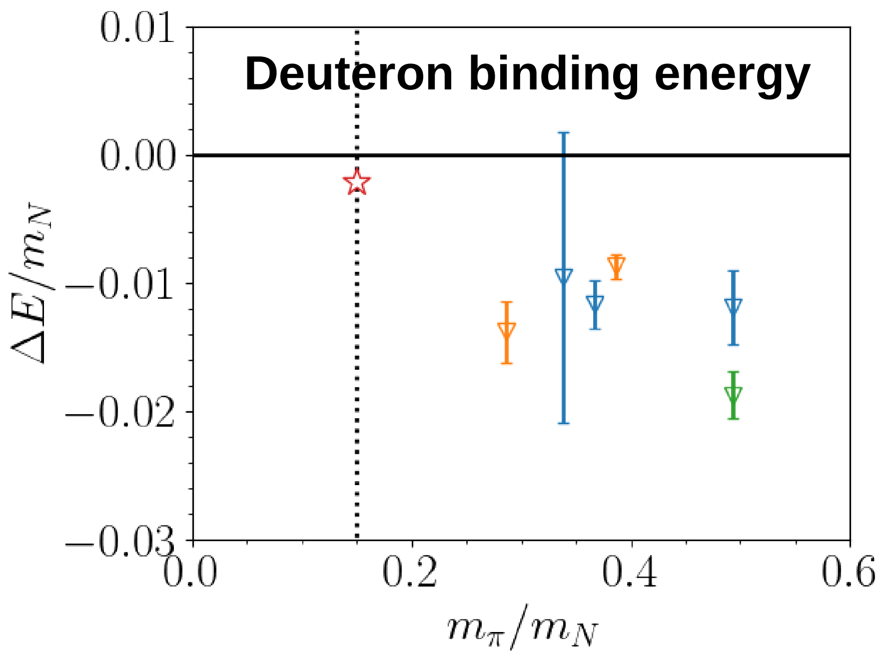
**Proof-of-principle** calculations, but some combination of:

- ✘ Limited operator set
- ✘ Single volume
- ✘ No continuum limit

under **N-state saturation assumption**

# Do two baryons bind at heavy quark masses? (circa 2018)

**Phys.Lett.B 712 (2012) 437-441**
**Phys.Rev.D 85 (2012) 054511**
**Phys.Rev.D 86 (2012) 074514**
**Phys.Rev.D 87 (2013) 3, 034506**
**Phys.Rev.D 92 (2015) 1, 014501**
**Phys.Rev.D 92 (2015) 11, 114512**
**PoS CD15 (2016) 020**
**Phys.Lett.B 765 (2017) 285-292**




Spectroscopy: bound deuteron

HAL QCD: No bound state

**Proof-of-principle** calculations, but some combination of:

- ✘ Limited operator set
- ✘ Single volume
- ✘ No continuum limit

under **N-state saturation assumption**

**Recent developments in all of these areas!**

# Variational methods

Nucl.Phys.B 215 (1983) 433-446
Nucl.Phys.B 339 (1990) 222-252

Phys.Rev.D 80 (2009) 054506

Phys.Rev.D 104 (2021) 3, 034502

Phys.Rev.Lett. 134 (2025) 24, 241901

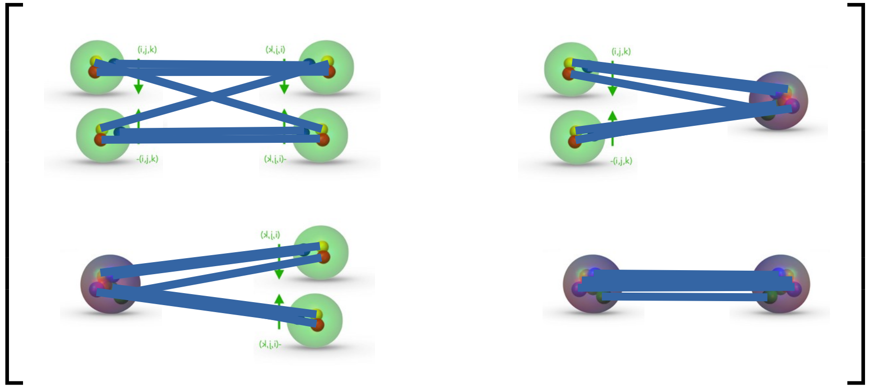
Juan Andres Urrea Nino
Monday, 2:00 PM
Juan Ramon Jimenez

Nilmani Mathur
Monday, 2:20 PM
Juan Ramon Jimenez

Sinya Aoki
Monday, 3:20 PM
Juan Ramon Jimenez

*Distillation* and *Sparsening* provide numerically tractable approaches to compute bilocal source operators: enables symmetric correlation functions & variational methods (GEVP, ...)

$$C_{ij}(t) =$$



**Symmetric** correlation functions satisfy variational bounds

$$E_{\text{eff}}(t) = -\log\left[\frac{C_{ii}(2t+1)}{C_{ii}(2t)}\right] \geq E_0$$

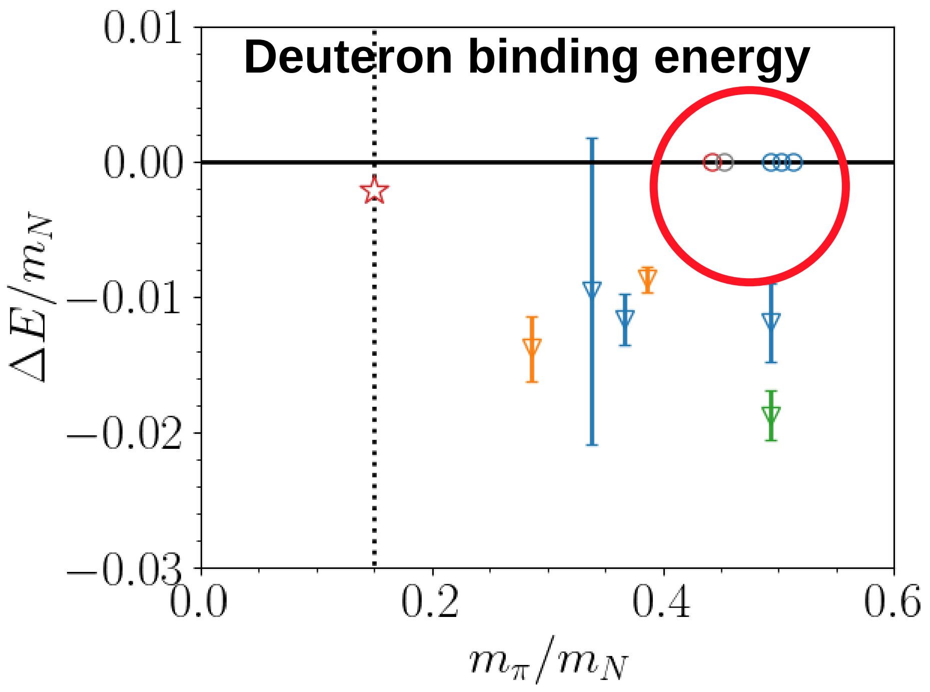
Protect you from overestimating binding: can still underestimate if interpreted as a point estimate rather than bound

**Formally, subtract out energy eigenstates: make correlator look more like single hadron (increase mass-gap)**

# Recent variational calculations

**Phys.Rev.D 99 (2019) 7, 074505**
**Phys.Rev.C 103 (2021) 1, 014003**
**Phys.Rev.D 107 (2023) 9, 094508**
**Phys.Rev.D 111 (2025) 11, 114501**
**Phys.Rev.C 113 (2026) 2, 024002**
**e-Print: 2601.22272 [hep-lat]**

Under an **N-state saturation** assumption, **no evidence** for bound deuteron or dineutron




Has been argued that previous deep bound states arise from use of asymmetric correlator: **not variational bound**

What about at lighter pion mass?

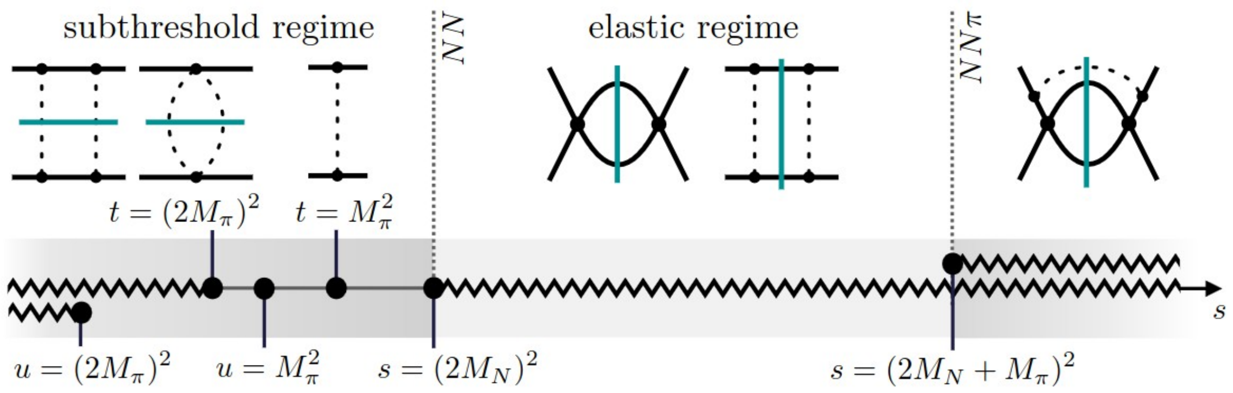
## Left-hand cuts become relevant at lighter pion mass

Phys.Rev.D 76 (2007) 034502
JHEP 10 (2021) 051
JHEP 06 (2024) 051
Phys.Rev.D 109 (2024) 7, L071506
JHEP 05 (2024) 168
JHEP 08 (2024) 075
Phys.Lett.B 864 (2025) 139442
JHEP 06 (2025) 186

Sebastian Dawid
Wednesday, 9:00 AM
Margaret Brent B

Lu Meng
Saturday, 11:00 AM
Colony Ballroom

Quantization conditions rely on analytic structure of FV amplitude




Luscher: right hand (unitarity) cut

Generalizations exist for 3-to-3 etc

For systems with light t-channel exchange, partial-wave projected amplitudes have additional branch cut singularities from crossed-channel exchanges.

Important for nuclear systems with **light** pions!

Now have several approaches to deal with this LHC

## Some studies at lighter pion mass suggest a virtual bound state...

**e-Print: 2605.16977 [hep-lat]**

Miguel Salg
Wednesday, 9:20 AM
Margaret Brent B

Malcolm Lazarow
Friday, 2:20 PM
Juan Ramon Jimenez

Joseph Moscoso
Friday, 3:00 PM
Juan Ramon Jimenez

Jorge Baeza-Ballesteros
Poster session

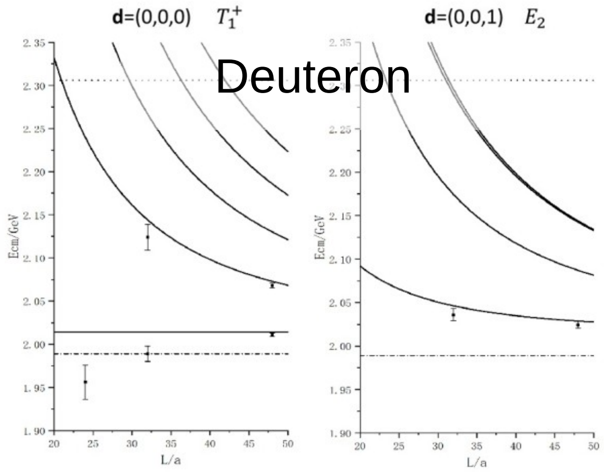
**Zhang et al: Two-nucleon systems at $m_\pi \approx$ 292 MeV from lattice QCD**

3 ensembles with different volumes

$a$ = 0.11 fm

Variational calculation: 2x2 correlation matrix

Hamiltonian framework to incorporate LHC



## Some studies at lighter pion mass suggest a virtual bound state...

e-Print: 2605.16977 [hep-lat]

Miguel Salg
Wednesday, 9:20 AM
Margaret Brent B

Malcolm Lazarow
Friday, 2:20 PM
Juan Ramon Jimenez

Joseph Moscoso
Friday, 3:00 PM
Juan Ramon Jimenez

Jorge Baeza-Ballesteros
Poster session

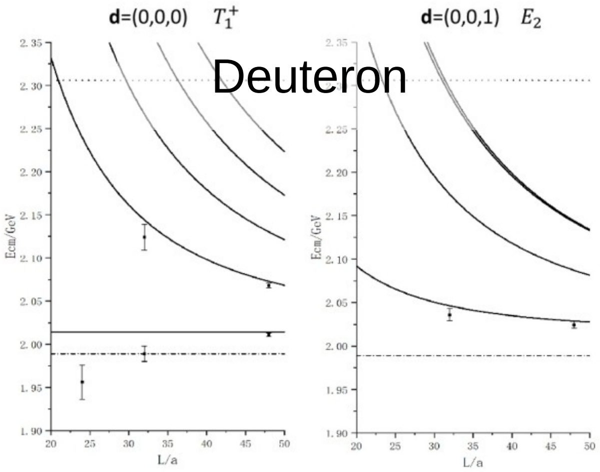
**Zhang et al: Two-nucleon systems at $m_\pi \approx$ 292 MeV from lattice QCD**

3 ensembles with different volumes

*a* = 0.11 fm

Variational calculation: 2x2 correlation matrix

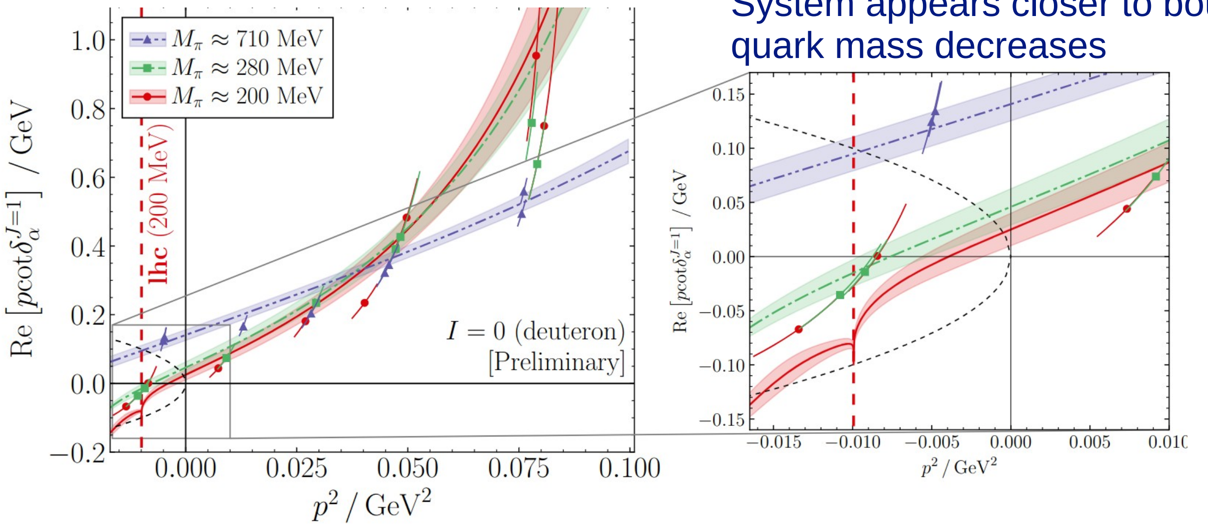
Hamiltonian framework to incorporate LHC




**BaSC: Quark mass dependence of two-nucleon systems**

CLS ensembles, lattice spacing *a* = 0.064 fm, $m_\pi \approx$ 280 and 200 MeV

Two different approaches for LHC; plane wave, relativistic Lüscher-like QC




System appears closer to bound as quark mass decreases

# Some studies at lighter pion mass suggest a virtual bound state...

e-Print: 2605.16977 [hep-lat]

Miguel Salg
Wednesday, 9:20 AM
Margaret Brent B

Malcolm Lazarow
Friday, 2:20 PM
Juan Ramon Jimenez

Joseph Moscoso
Friday, 3:00 PM
Juan Ramon Jimenez

Jorge Baeza-Ballesteros
Poster session

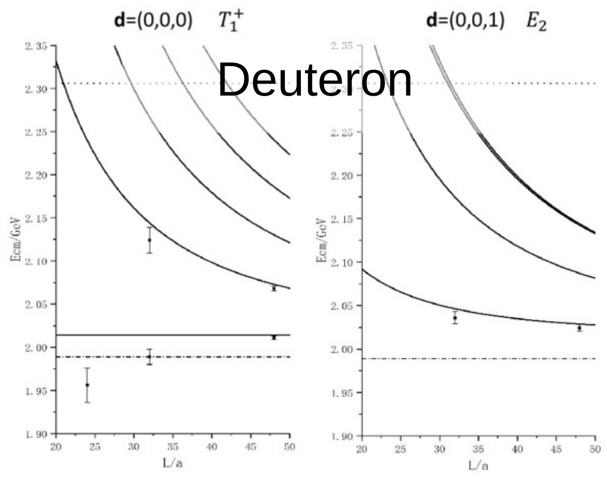
**Zhang et al: Two-nucleon systems at $m_\pi \approx$ 292 MeV from lattice QCD**

3 ensembles with different volumes

$a$ = 0.11 fm

Variational calculation: 2x2 correlation matrix

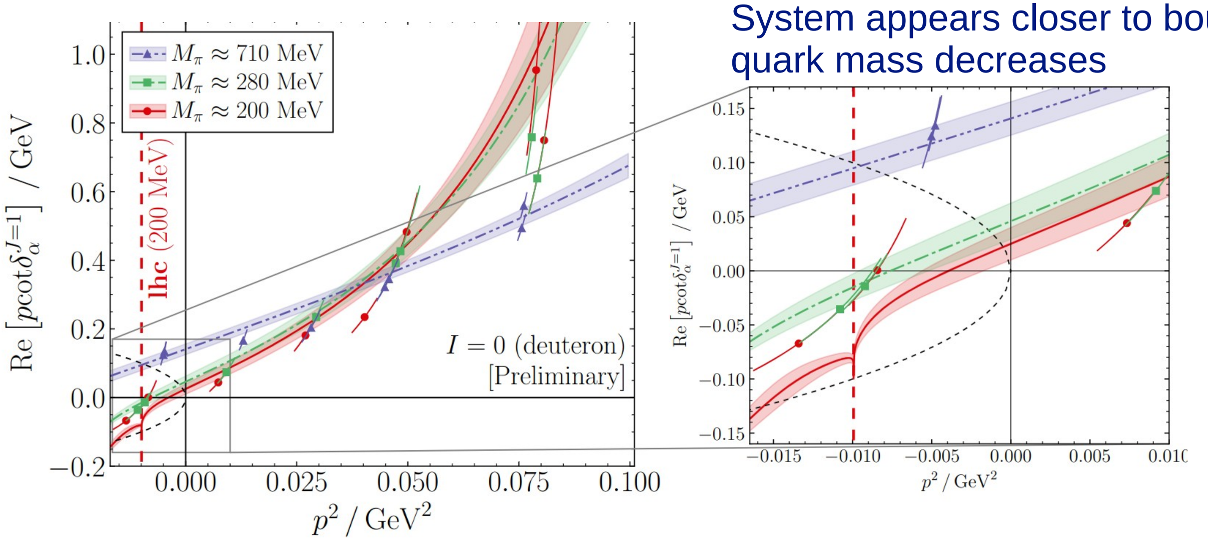
Hamiltonian framework to incorporate LHC




**BaSC: Quark mass dependence of two-nucleon systems**

CLS ensembles, lattice spacing $a$ = 0.064 fm, $m_\pi \approx$ 280 and 200 MeV

Two different approaches for LHC; plane wave, relativistic Lüscher-like QC




System appears closer to bound as quark mass decreases

Under **N-state saturation**, results predict a **virtual bound state**

# ...while others find evidence for a bound state

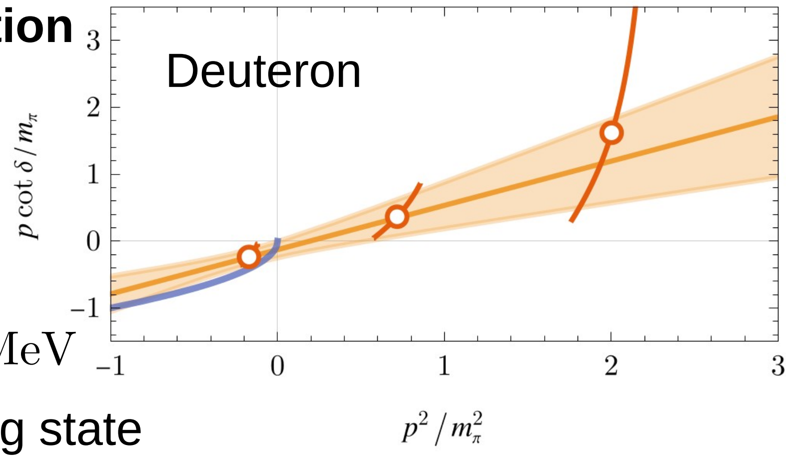
**e-Print: 2603.09554 [hep-lat]**
**e-Print: 2603.28872 [hep-lat]**

**Wang et al., First-Principles Determination of the Proton-Proton Fusion Matrix Element from Lattice QCD**

Variational calculation: 3x3 matrix

Single ensemble, *a* = 0.11 fm, $m_\pi \approx 432\ \mathrm{MeV}$

Favor a shallow bound state, but scattering state cannot be ruled out. ERE above t-channel cut



# ...while others find evidence for a bound state

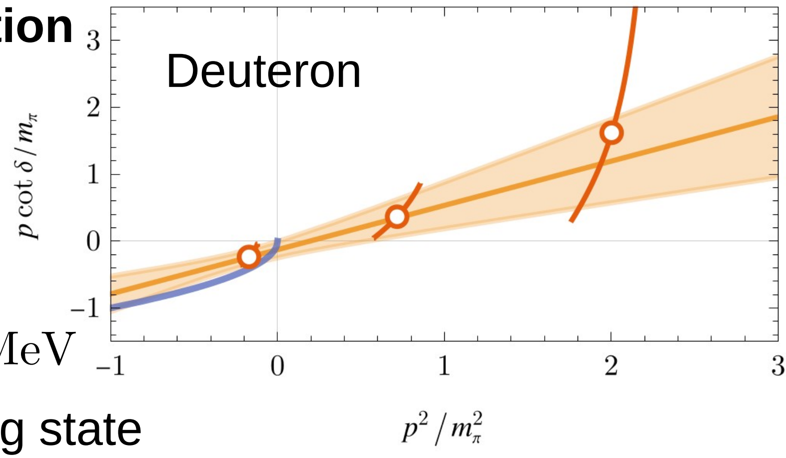
**e-Print: 2603.09554 [hep-lat]**
**e-Print: 2603.28872 [hep-lat]**

**Wang et al., First-Principles Determination of the Proton-Proton Fusion Matrix Element from Lattice QCD**

Variational calculation: 3x3 matrix

Single ensemble, *a* = 0.11 fm, $m_\pi \approx 432$ MeV

Favor a shallow bound state, but scattering state cannot be ruled out. ERE above t-channel cut



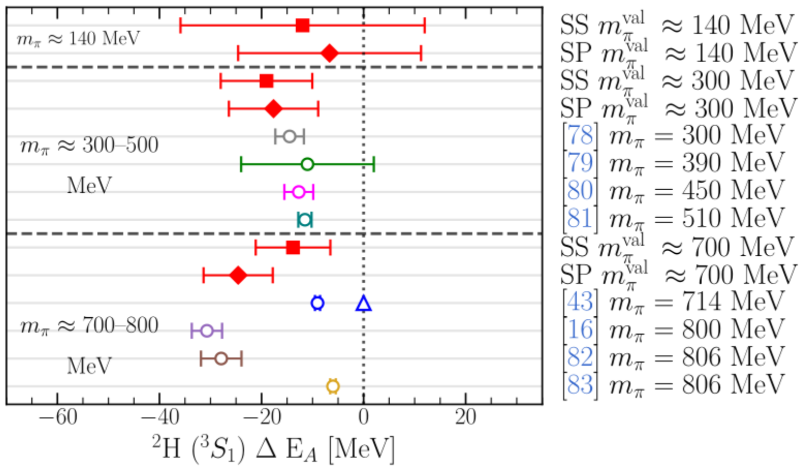

**Chakraborty et al., Quark-Mass Dependence of Light-Nuclei Masses from Lattice QCD and Trace-Anomaly Contributions to Nuclear Bindings**

Single ensemble, *a* = 0.076 fm, physical sea quarks

valence quarks $m_\pi \approx$ 700 to 140 MeV

Symmetric correlator analysis

Observe evidence for bound state



# ...while others find evidence for a bound state

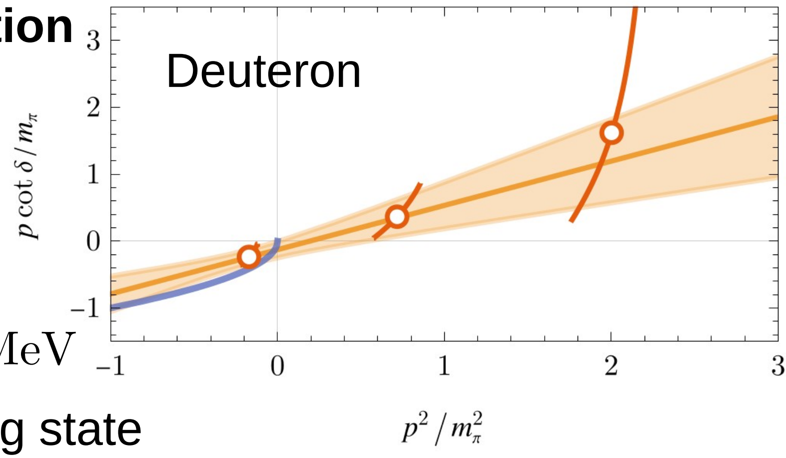
e-Print: 2603.09554 [hep-lat]
e-Print: 2603.28872 [hep-lat]

**Wang et al., First-Principles Determination of the Proton-Proton Fusion Matrix Element from Lattice QCD**

Variational calculation: 3x3 matrix

Single ensemble, *a* = 0.11 fm, $m_\pi \approx 432$ MeV

Favor a shallow bound state, but scattering state cannot be ruled out. ERE above t-channel cut



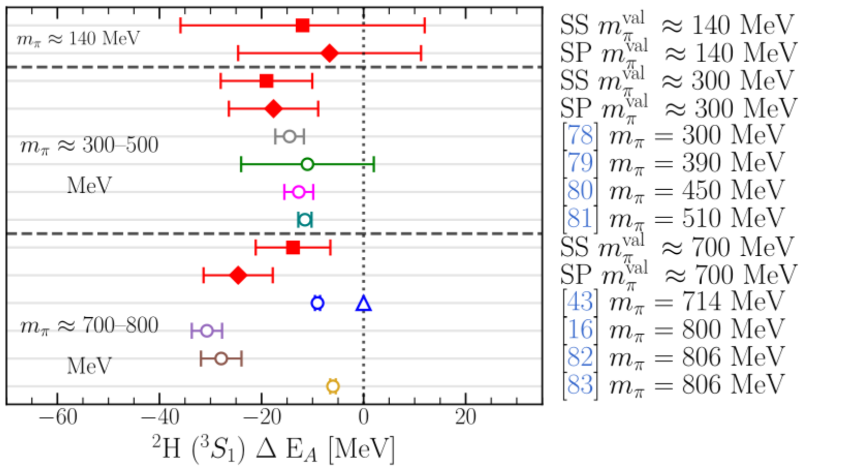

**Chakraborty et al., Quark-Mass Dependence of Light-Nuclei Masses from Lattice QCD and Trace-Anomaly Contributions to Nuclear Bindings**

Single ensemble, *a* = 0.076 fm, physical sea quarks

valence quarks $m_\pi \approx$ 700 to 140 MeV

Symmetric correlator analysis

Observe evidence for bound state




Under **N-state saturation**, results predict a **bound state**

# Lattice artifacts can be important

**Phys.Rev.Lett. 127 (2021) 24, 242003**
**PoS CD2024 (2026) 019**

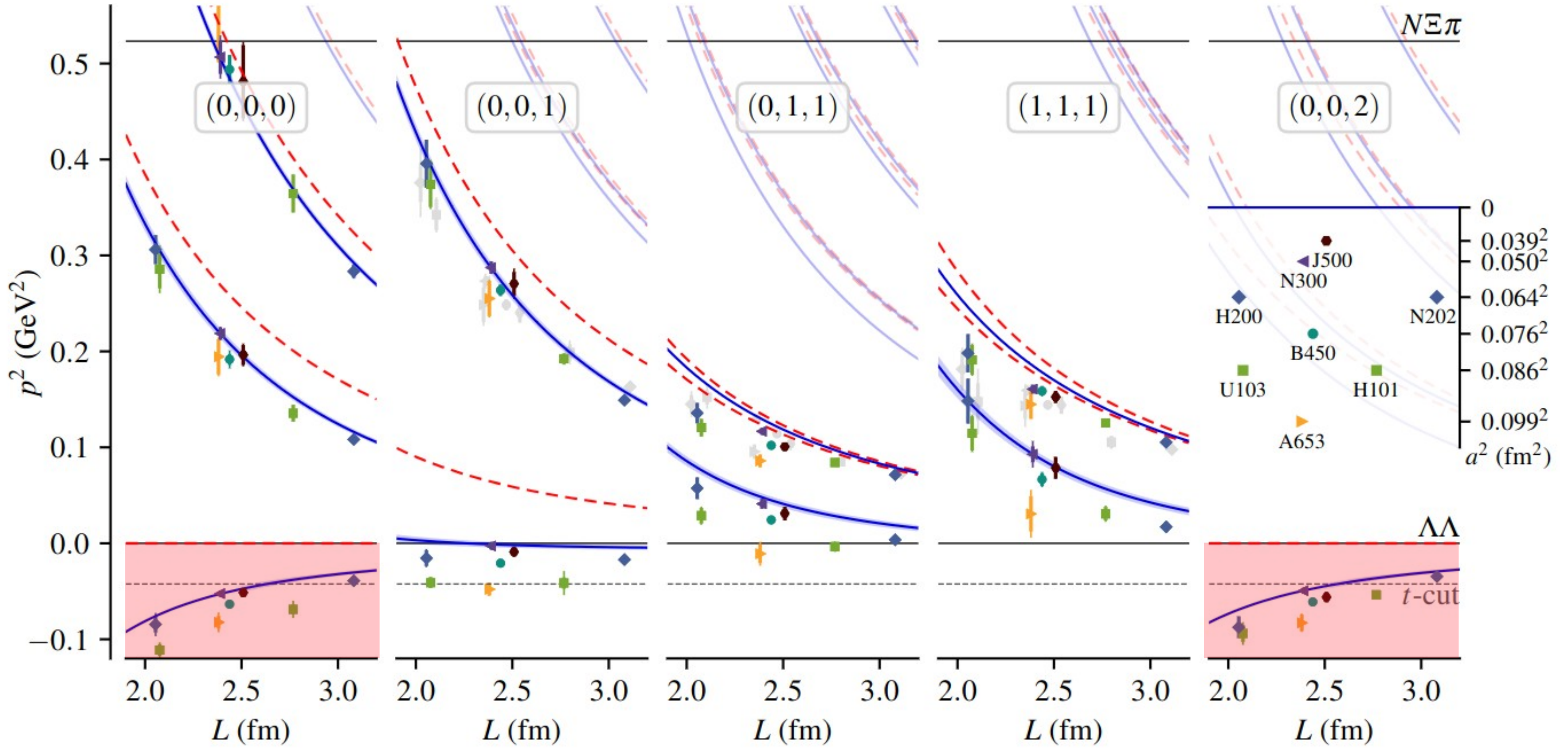
Juan Fernandez-de la Garza
Friday, 5:10 PM
Juan Ramon Jimenez

H-dibaryon (uuddss) possible bound state

2+1 Wilson-clover fermions, Lüscher-Weisz gauge action

Multi-ensemble with *a* = 0.1 to 0.04 fm, $m_\pi = m_K \approx 420\ \mathrm{MeV}$

At time did not have formalism for LHC: exclude energies at/below cut



# Lattice artifacts can be important

**Phys.Rev.Lett. 127 (2021) 24, 242003**
**PoS CD2024 (2026) 019**

Juan Fernandez-de la Garza
Friday, 5:10 PM
Juan Ramon Jimenez

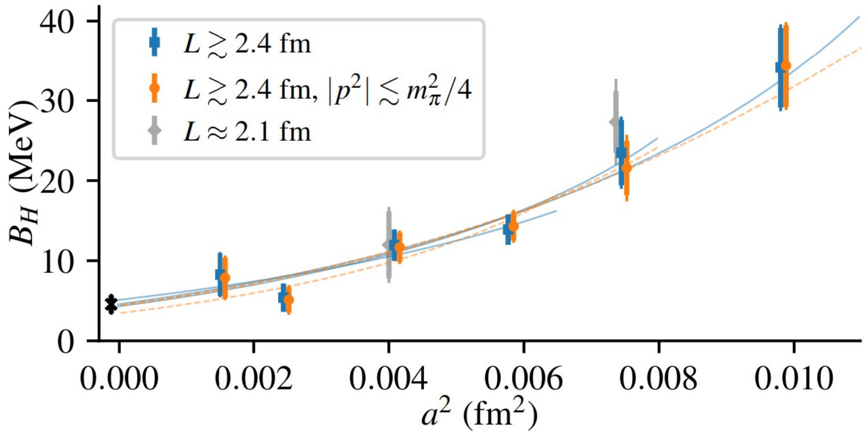
H-dibaryon (uuddss) possible bound state

2+1 Wilson-clover fermions, Lüscher-Weisz gauge action

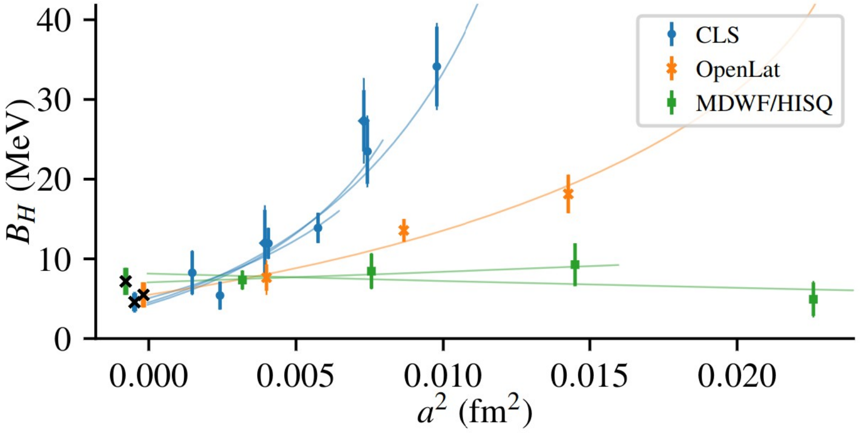
Multi-ensemble with *a* = 0.1 to 0.04 fm, $m_\pi = m_K \approx 420 \text{ MeV}$

At time did not have formalism for LHC: exclude energies at/below cut

$$B_H^{\mathrm{SU}(3)} = 4.56 \pm 1.13 \pm 0.63 \text{ MeV}$$




Different actions produce different lattice artifacts



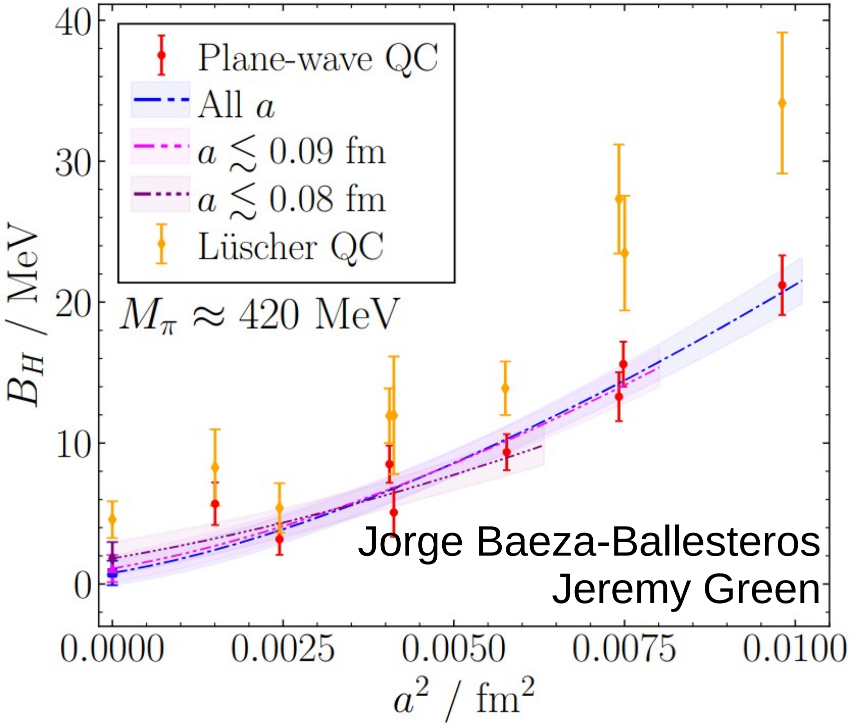
# Reanalysis of the H-dibaryon

**e-Print: 2605.22957 [hep-lat]**

Lin Qiu
Friday, 4:50 PM
Juan Ramon Jimenez

Two independent reanalyses employ QC which account for LHC and therefore allow for full dataset to be analyzed: updated binding energy

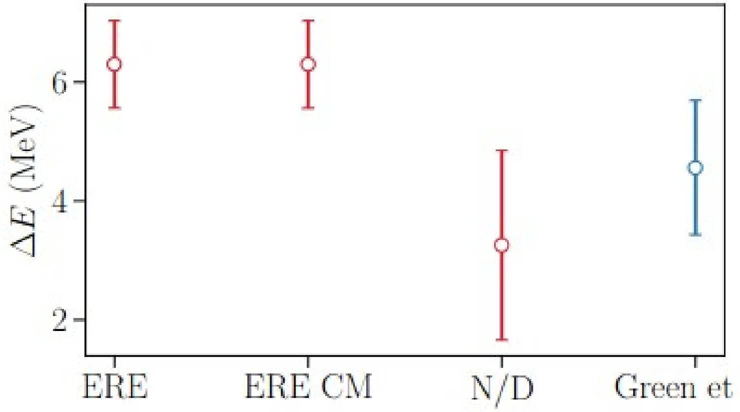
Plane wave QC




Jorge Baeza-Ballesteros
Jeremy Green

Roughly 2 sigma shift from previous analysis: system very close to unbound

N/D QC




Limited but non-negligible effect on the extraction of these observables

Result also prefers weaker bound state

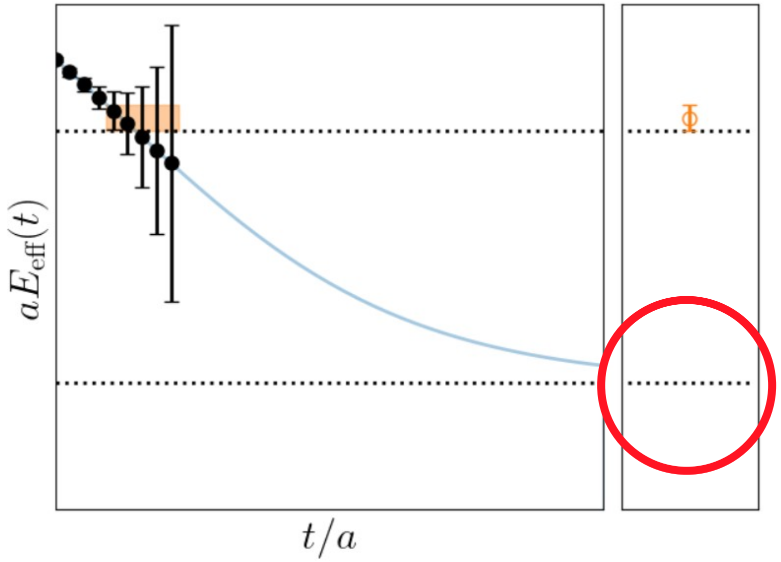
# Assumption of N-state saturation can fail

At least two failure modes to be worried about:

# Assumption of N-state saturation can fail

At least two failure modes to be worried about:

States **missing** from numerically determined spectrum*

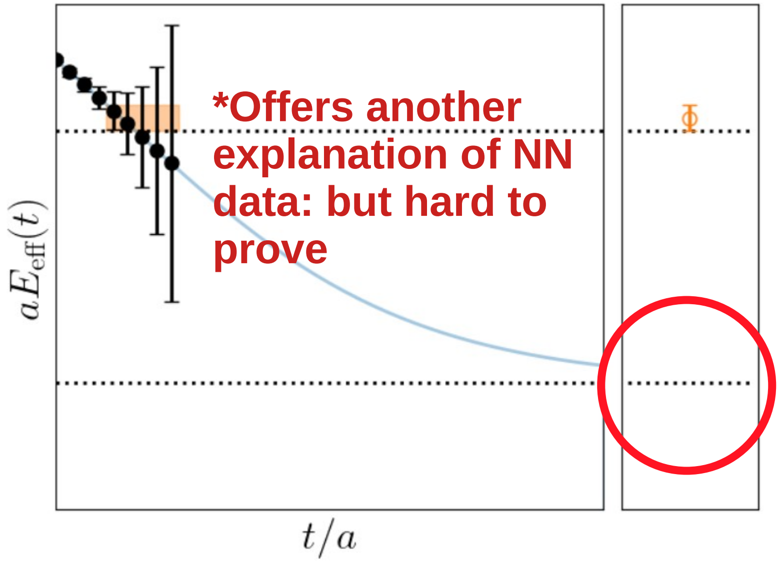



Nightmare scenario, but cannot exclude

## Assumption of N-state saturation can fail

At least two failure modes to be worried about:

States **missing** from numerically determined spectrum*

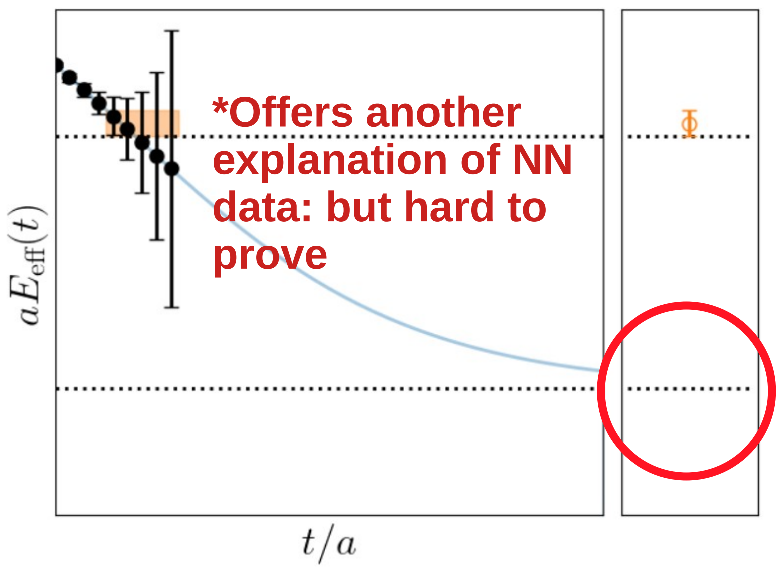



Nightmare scenario, but cannot exclude

## Assumption of N-state saturation can fail

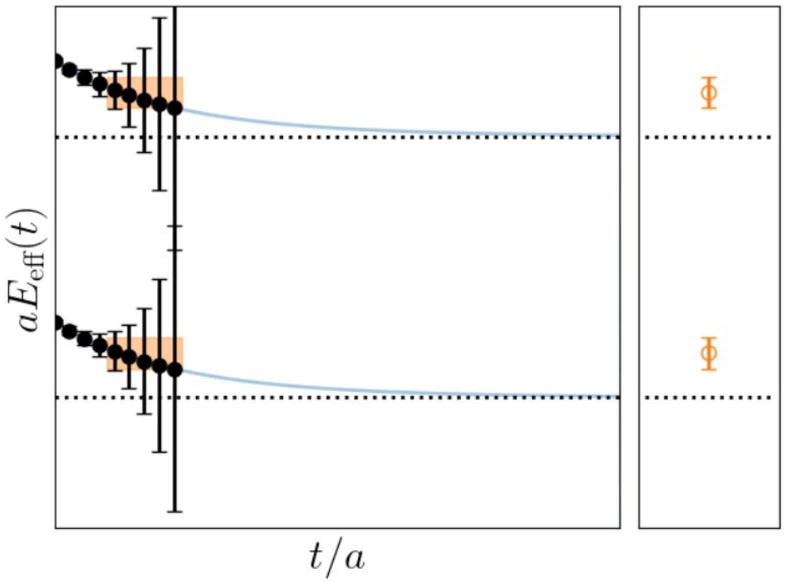
At least two failure modes to be worried about:

States **missing** from numerically determined spectrum*




Nightmare scenario, but cannot exclude

Spectrum is **known qualitatively**, but energy estimators are **biased by excited state effects**




Very plausible: can we do spectroscopy **without N-state saturation?**

# Two-sided bounds exist for eigenvalues

**Phys.Rev.Lett. 134 (2025) 24, 241901**
**e-Print: 2601.22272 [hep-lat]**

William Detmold
Friday, 4:10 PM
Juan Ramon Jimenez

Given an effective eigenvalue,

$$\tilde{\lambda} = \frac{C(2t+1)}{C(2t)} = \frac{\langle \mathcal{O}T^t|T|T^tO\rangle}{\langle \mathcal{O}T^t|T^tO\rangle}$$

Assuming **only** that T is Hermitian, **Residual bound:**

$$B = \frac{C_{ii}(2t+2)}{C_{ii}(2t)} - \left[\frac{C_{ii}(2t+1)}{C_{ii}(2t)}\right]^2$$

**defines a two-sided window** which **guarantees** the presence of **at least** one transfer-matrix eigenvalue.

$$\lambda_n \in [\tilde{\lambda} - \sqrt{B}, \tilde{\lambda} + \sqrt{B}]$$

Monotonicity of logarithm implies two-sided bounds on FV energies

$$E_n \in [-\ln(\tilde{\lambda} + \sqrt{B}), -\ln(\tilde{\lambda} - \sqrt{B})]$$

Assuming **no missing states** can use tighter **Gap bounds**:

$$E_n \in [-\ln(\tilde{\lambda} + B/G), -\ln(\tilde{\lambda} - B/G)]$$

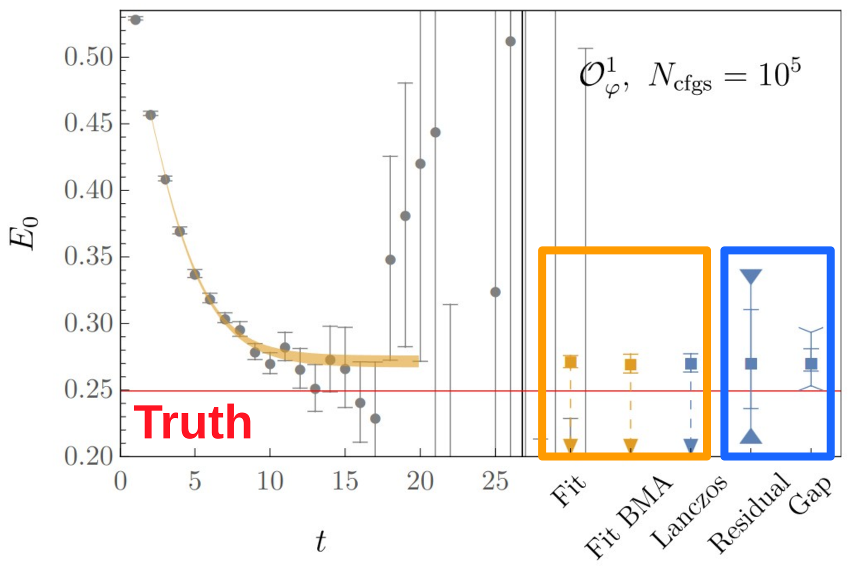
# Bounds produce better calibrated uncertainties

**Phys.Rev.Lett. 134 (2025) 24, 241901**
**e-Print: 2601.22272 [hep-lat]**
**e-Print: 2601.22273 [hep-lat]**

Validate in toy model where exact solution is known

Fits to correlator **wrong** at multiple sigma

Residual and gap bounds **cover true solution**



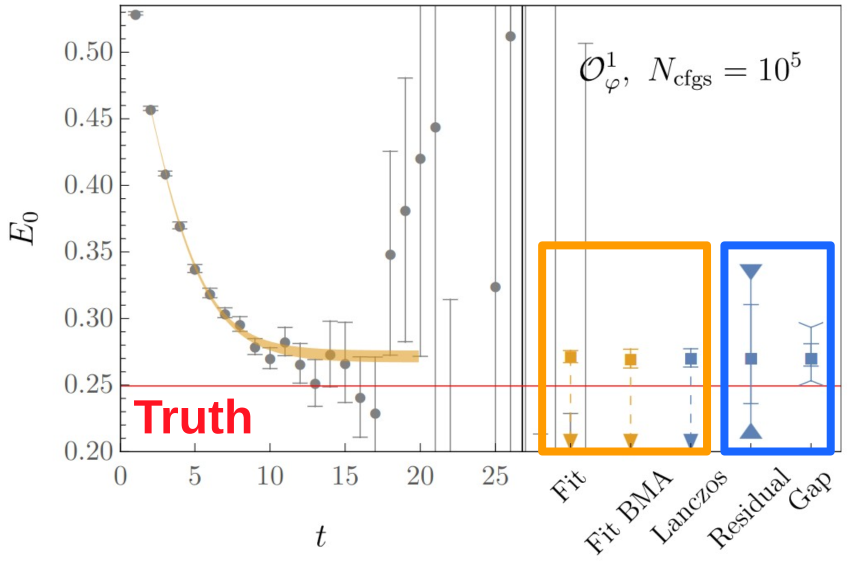
# Bounds produce better calibrated uncertainties

Phys.Rev.Lett. 134 (2025) 24, 241901
e-Print: 2601.22272 [hep-lat]
e-Print: 2601.22273 [hep-lat]

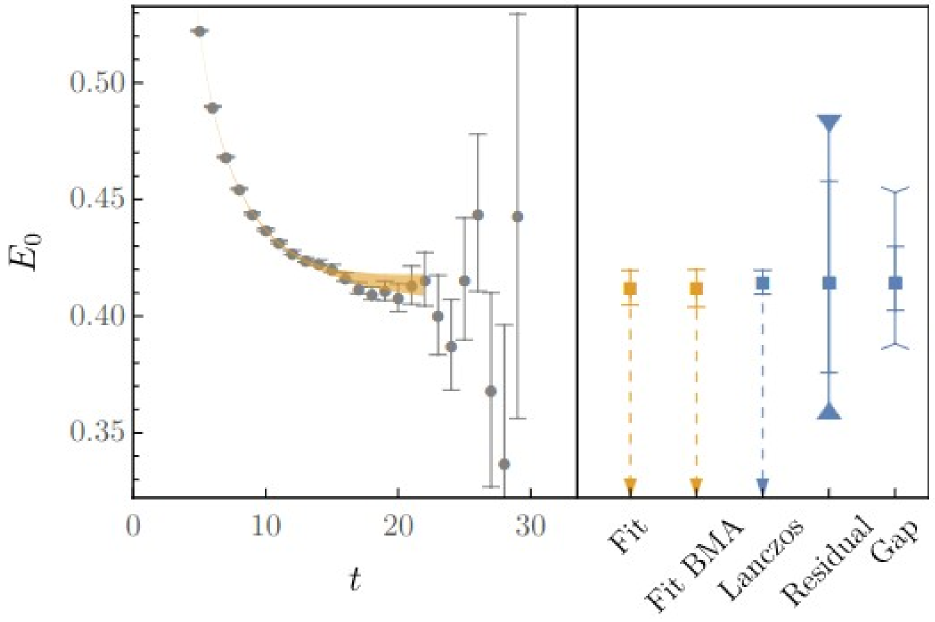
Validate in toy model where exact solution is known

Fits to correlator **wrong** at multiple sigma

Residual and gap bounds **cover true solution**




**Nucleon mass**




Constrain nucleon mass to 9-14% using residual bounds;

Constrain nucleon mass to 3-6% using gap bounds

**Systematically improvable** bounds offer path forward.

# NPLQCD: Multi-ensemble analysis of H-dibaryon

Michael Wagman
Friday, 4:30 PM
Juan Ramon Jimenez

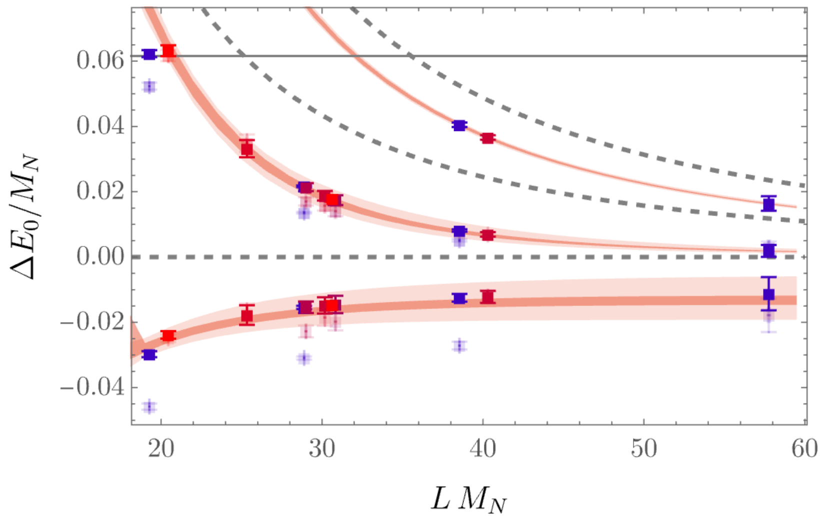
Gap bounds tight enough to be useful: H-dibaryon at $m_\pi \approx 800$ MeV

11 Wilson-clover ensembles spanning *a* = 0.077 fm to 0.15 fm

L = 2.5 fm to 7.2 fm

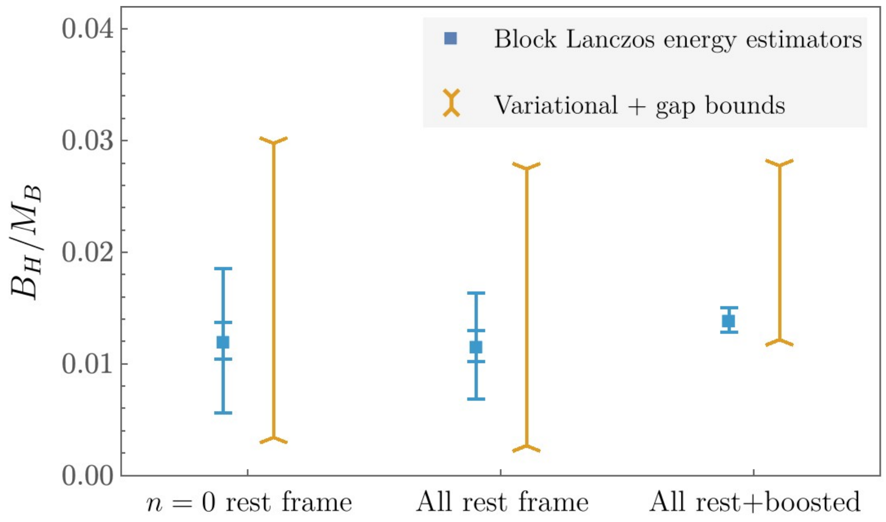
Spectroscopy using block Lanczos with **48 operators, 4 boosts**

Gap bounds computed under **no-missing-states assumption**




Expand K-matrix with effective range expansion

After accounting for lattice artifacts, mass mistuning, determine H-dibaryon binding energy $B_H/M_B = 0.0140^{+14}_{-2}$

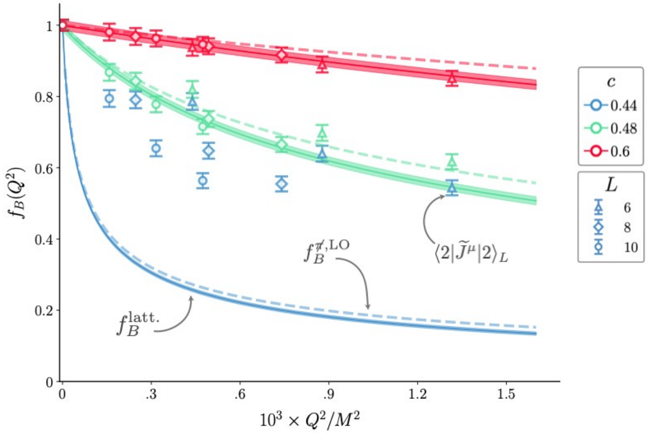


# Other progress in the field

**e-Print: 2602.20373 [hep-lat]**
**e-Print: 2603.28872 [hep-lat]**
**e-Print: 2603.09554 [hep-lat]**

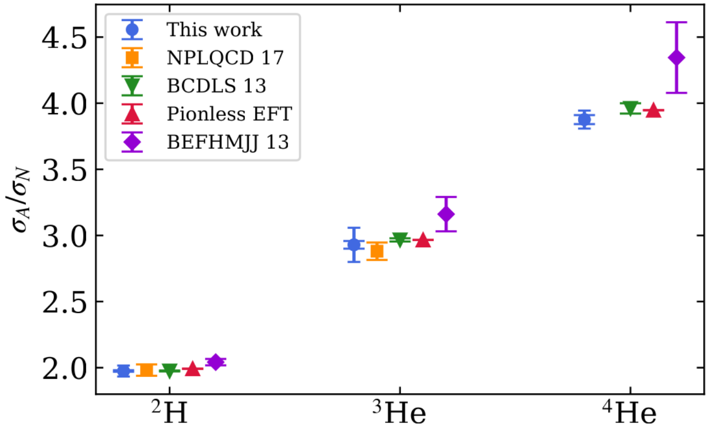
Takumi Doi
Jul 31, 2026, 2:00 PM
Juan Ramon Jimenez

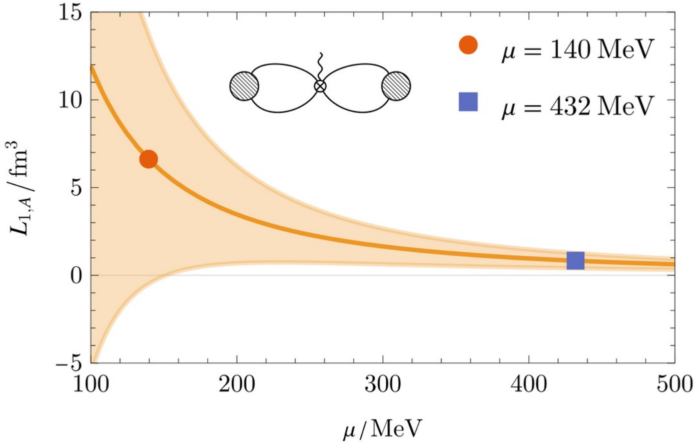
Koichi Murase
Jul 31, 2026, 2:40 PM
Juan Ramon Jimenez




**Moscoso et al:** Implemented $2 \to \mathcal{J} \to 2$ formalism in NN **lattice EFT**, Good agreement with theory

**Chakraborty et al:** masses of the deuteron, dineutron, $^3$He, $^4$He, nuclear sigma terms

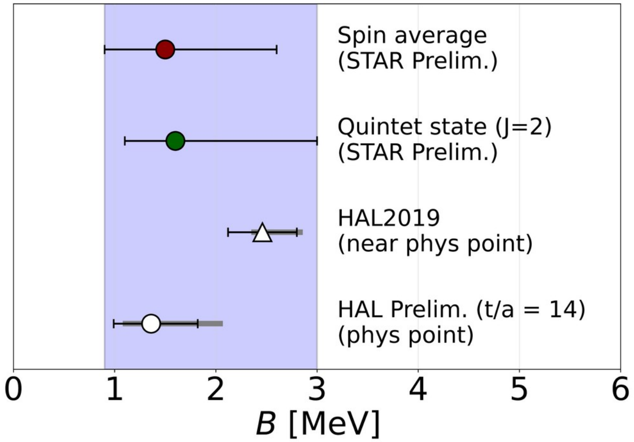



**Wang et al.:** Constrained two-body LEC relevant for pp-fusion

$$L_{1,A} = 6.0(7.1)\ \mathrm{fm}^3$$

**HAL QCD:** pΩ quasi-bound state at the physical point, agrees with recent STAR femtoscopy result



# Conclusions

Systematically improvable nuclear physics essential for ongoing experimental programs.

Previous LQCD calculations demonstrated that nuclear physics was possible, but did not control all sources of systematics

Modern calculations are starting to explore these systematics

- ✔ Large operator basis
- ✔ Multiple volumes
- ✔ Lattice spacing dependence
- ✔ Spectroscopy assumptions

Need more calculations which address all of these systematics

# Backup slides

# The variational method

Possible to make these statements clear.

$$C_{ii}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_i^\dagger(0)|0\rangle$$

Define wave function

$$|\psi_t\rangle = T^t\mathcal{O}_i^\dagger(0)\,|0\rangle \qquad C_{ii}(2t+1) = \langle\psi_t|T|\psi_t\rangle$$

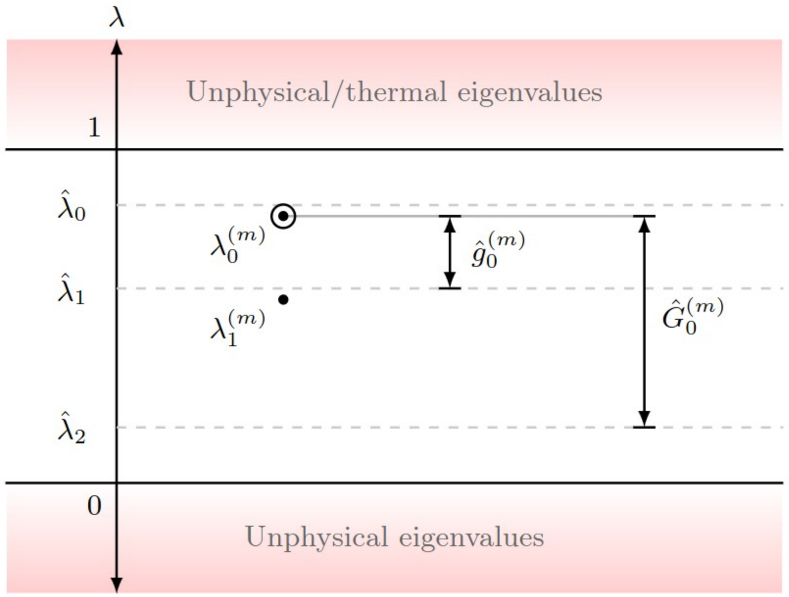
The variational principle from quantum mechanics:

$$\tilde{\lambda} = \frac{\langle\psi_t|T|\psi_t\rangle}{\langle\psi_t|\psi_t\rangle} = \frac{C(2t+1)}{C(2t)} \geq \lambda_0, \qquad T\,|n\rangle = \lambda_n\,|n\rangle$$

$$E_{\text{eff}}(t) = -\log\left[\frac{C(2t+1)}{C(2t)}\right] \geq E_0$$

# Bounds in the Lanczos framework

**Phys.Rev.Lett. 134 (2025) 24, 241901**
**Phys.Rev.D 112 (2025) 5, 054506**
**Phys.Rev.D 112 (2025) 1, 1**
**e-Print: 2503.17357 [hep-lat]**




**Residual bounds:** eigenvalue guaranteed to exist in two-sided window

$$\lambda_k^{(m)} \pm \sqrt{B_k^{L/R(m)}}$$

where

$$B_k^{R/L(m)} = \frac{\langle y_k^{R/L(m)} | [T - T^{(m)}]^2 | y_k^{R/L(m)} \rangle}{\langle y_k^{R/L(m)} | y_k^{R/L(m)} \rangle}$$

**Gap bounds:** When gaps between states are (approximately) known,

$$\lambda_k^{(m)} \pm \frac{B_k^{L/R(m)}}{G_k^{(m)}}$$

Where $G_k^{(m)}$ is gap to first "missing" state.

$$\hat{G}_k^{(m)} \equiv \left| e^{-E_{\text{miss}}} - \lambda_k^{(m)} \right|$$

$E_{\text{miss}}$: energy non in 1-1 correspondence with energy estimator

# Assumptions in spectroscopy

**Slide from M. Wagman**

Assume nothing

- **Lanczos residual bounds**

  MW, PRL 134 (2025)

$O(\sqrt{B})$ New directly calculable small parameter for quantifying systematic errors

Assume no missing levels in some energy range

- **Lanczos gap bounds**

$O(B)$ calculable error bounds

Detmold, MW, et al [NPLQCD], arXiv:2601.22272

Detmold, MW, et al [NPLQCD], arXiv:2601.22273

Assume energy estimators have saturated their asymptotic values

- **Statistical uncertainties on fits / energy estimators**

$O(e^{-\delta_1 t})$ systematics, no calculable error bounds

*Requires strong assumptions about energy spectrum and operator overlaps for multi-hadron systems where* $e^{-\delta_1 t}$ *is not a small parameter*

**More conservative**

**More precise**

37

# Assumptions in spectroscopy

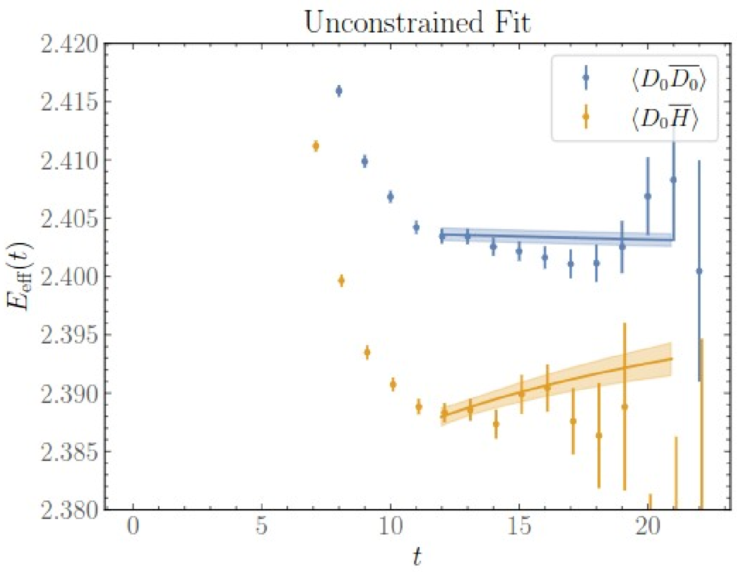
JHEP 10 (2016) 101
Phys.Rev.C 113 (2026) 2, 024002
e-Print: 2601.22272 [hep-lat]

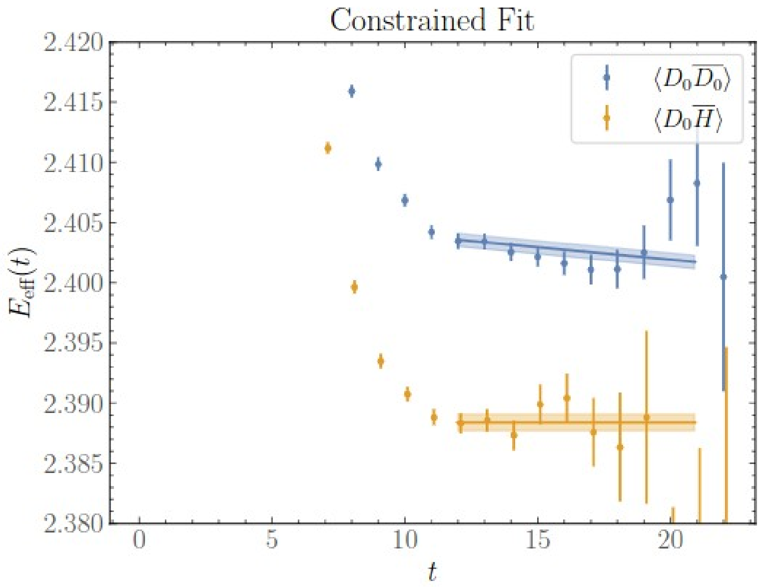
Compare symmetric and asymmetric correlators

$$C_{\rm sym}(t) = \sum_n |z_n|^2 e^{-E_n t} > 0 \qquad C_{\rm asym}(t) = \sum_n z_n z_n'^* e^{-E_n t} \not> 0$$

Excited states can interfere to cause false plateaus in asymmetric correlators

**20 Million propagators:**




Data precision **not sufficient** to make conclusion without additional assumptions

Asymmetric solution “unnatural”