Hadron structure from lattice QCD

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July 31, 2026

University of Maryland

Pre-emptive slide

>45 talks. 3 posters. Many papers since Lattice 2025.

I apologize to everyone whose work I do not mention.

There will be proceedings.

Please communicate omissions.

Highly subjective and partly random selection of highlighted results.

Plan

- Experiments
- Tools
- Pictures

Plan

- Experiments
  (Physics is **nature** written in math)
- Tools
  (Concepts, theory and algorithms: the fun part)
- Pictures
  (“Understanding” the structure)

What do we aim for?

Experiment = **Lattice** × Other Theory × Other Experiments

Standard model parameters and other input are determined in other experiments.

What do we aim for?

Isolate systematic effects in the extraction of data from experiments.

$$\frac{\boxed{\textbf{Experiment}}}{\boxed{\text{Other Theory}}} = \boxed{\textbf{Lattice}} \times \boxed{\text{Other Experiments}}$$

Example: exp. determination (using other theory) disagrees with precise Lattice $G_A(Q^2)$.

What do we aim for?

Predicting Lattice results.

$$\boxed{\textbf{Lattice}} = \frac{\boxed{\textbf{Experiment}}}{\boxed{\text{Other Theory}} \times \boxed{\text{Other Experiments}}}$$

If $\exists$ more reliable data: explore Lattice systematics. Examples: g_A & PDF $f_{u^+-d^+}(x, Q^2)$.

What do we aim for?

Constrain theory parameters and functions.

$$\boxed{\text{Other Theory}} = \frac{\boxed{\textbf{Experiment}}}{\boxed{\textbf{Lattice}} \times \boxed{\text{Other Experiments}}}$$

Example: GPDs and TMDPDFs from Lattice + Experiment.

What do we aim for?

Experiment $\neq$ Lattice $\times$ Other Theory $\times$ Other Experiments

NEW PHYSICS?

What happens if discrepancy?

What do we aim for?

Experiment $\neq$ **Lattice** $\times$ Other Theory $\times$ Other Experiments

NEW PHYSICS?

. . . or is Other Theory wrong or is Lattice wrong?

Must understand systematic uncertainties, before claiming a discovery!

NB: often Experiment ∩ Other Theory $\neq \{\}$ or Lattice ∩ Other Theory $\neq \{\}$.

What do we aim for?

Experiment $\neq$ Lattice $\times$ Other Theory $\times$ Other Experiments

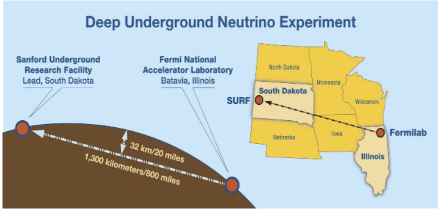
NEW PHYSICS?

. . . **or replace Lattice with Machine Learning?**

(suggestion from the protein folding community)

Hadron structure: the foundation and frontier of experiment

High-energy & intensity flagships



Large Hadron Collider (LHC)

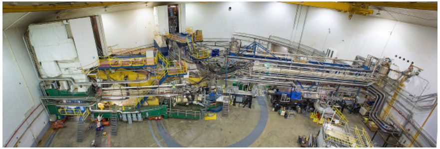
- Maps out gluons at large $0.1 < x < 0.5$.
- Flavour profiling in vector boson production: $u - \bar{u}$, $d - \bar{d}$ ($0.005 < x < 0.1$, LHCb: extreme forward/backward), $s \pm \bar{s}$ ($0.001 < x < 0.05$).
- Non-perturbative input needed for SM physics, BSM searches, heavy flavour physics.

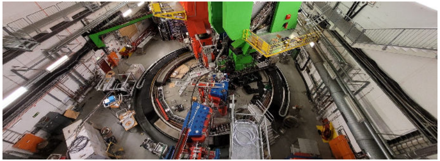
DUNE & Hyper-Kamiokande

- Neutrino-nucleon cross section needed.
- Form factors for ν scattering and resonance/pion production necessary.
- This will substantially improve the precision of neutrino masses and PMNS parameters.

Hadron structure: the foundation and frontier of experiment

The tomographic frontier





Electron-Ion Collider (EIC) & JLab & MAMI

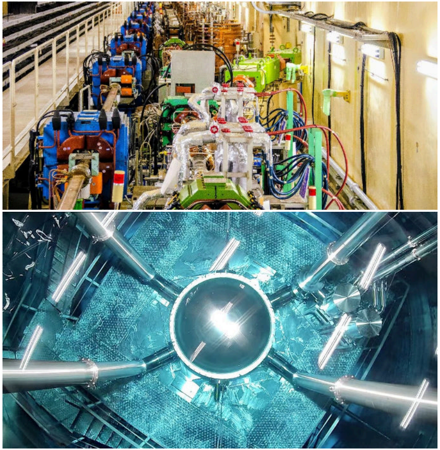
- Map 3D nucleon structure via GPDs and TMDs.
- Explore sea quarks and gluon imaging frontiers.
- Precision form factors, virtual Compton scattering.
- Lattice calculations are necessary and complementary.

AMBER@CERN

- Measure meson structure directly (Pion and Kaon PDFs via Drell-Yan).
- Validation of the proton charge radius in μ-p scattering. (At lower momentum also MUSE@PSI.)
- Produces data to compare with lattice.

Hadron structure: the foundation and frontier of experiment

Flavour factories & rare decays



Belle II & BESIII & LHCb

- Electroweak transition form factors needed.
- Light cone distribution amplitudes (LCDAs).
- Aim: CKM matrix elements and spectroscopy.

NA62@CERN

- Track ultra-rare kaon decay channels ($K \to \pi\nu\bar{\nu}$).
- SM expectation requires hadronic matrix elements.

XENONnT, PandaX-4T, LUX-ZEPLIN

- Search for weakly interacting dark matter.
- Nucleon matrix elements enter Monte-Carlo, in particular σ_s.

Hadron structure: the foundation and frontier of experiment

The precision electroweak frontier

JLab (MOLLER) & MESA (P2)

- Measure weak charges of e^- and p ($Q_W^e = 4\sin^2\theta_W - 1 \approx -Q_W^p$) via parity asymmetry in scattering.
- Test Standard Model running.
- Contributions from $Z\gamma$ box diagrams.

PSI & UCN@LANL & TUCAN@TRIUMF & FRM II

- Anomalies in neutron decay.
- Neutron electric dipole moments.
- Requires lattice QCD input, once discovered.
- $W\gamma$ and $\gamma\gamma$ box diagrams for β decay and atomic precision spectroscopy.

Why we are in the game: factorization of SM and QCD processes

Scale separation: $q \gtrsim \mu_F \gg \Lambda$

Short distance

Long distance

$$\underbrace{\sigma(q)}_{\text{cross section}} = \underbrace{C(q, \mu_F)}_{\text{coefficient function}} \otimes \underbrace{\langle O(\Lambda) \rangle (\mu_F)}_{\text{NP matrix element}} + O\left(\Lambda^2/q^2\right)$$

Coefficient function: **process-dependent**, perturbative.

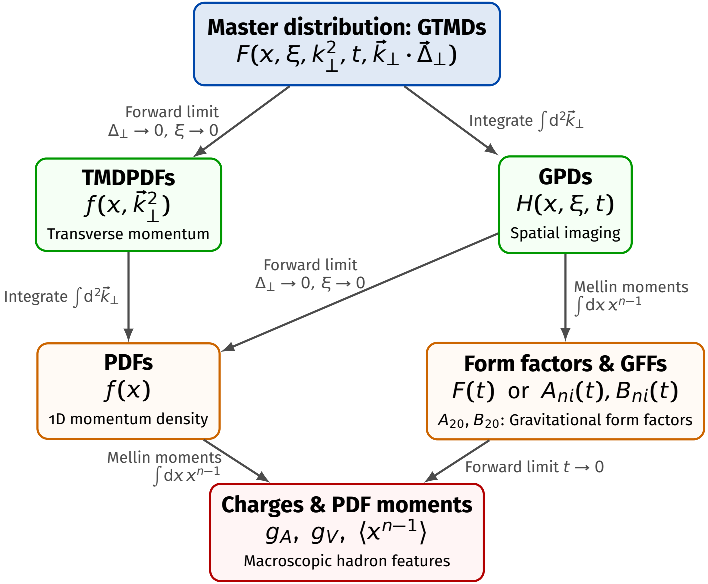
Matrix element: **universal**, non-perturbative. Example: PDFs.

Cross sections do not depend on the **factorization scale** μ_F.

- For simplicity: **renormalization scale** $\mu = \mu_F$.
- Often also: **hard kinematic scale** $q = \mu_F$.

! Factorization does not always hold at arbitrary perturbative orders.

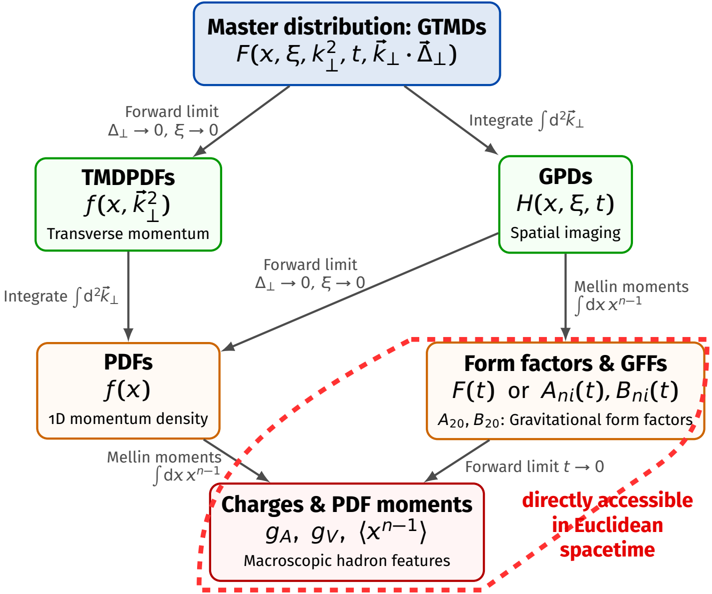
Hadron structure map



Kinematic definitions

- x: light cone "+" momentum fraction of parton. Forward limit: $x = x_B$.
- ξ: skewness: "+" momentum $P^+ \mapsto (1 - 2\xi)P^+ = P^+ + \Delta^+$.
- $\vec{k}_\perp$: Transverse parton momentum.
- $t = (\xi^2 M^2 - \vec{\Delta}_\perp^2)/(1 - \xi^2)$: squared hadron momentum transfer.
- Additional (logarithmic) dependence on the virtuality Q^2.

Hadron structure map



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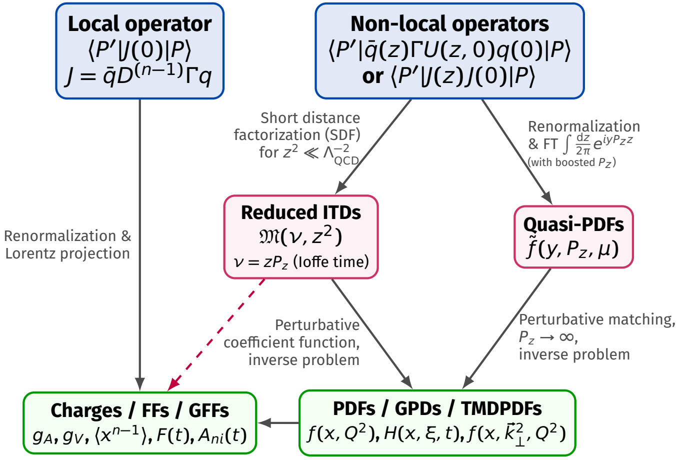
- Additional (logarithmic) dependence on the virtuality Q^2.

Other structure observables

- Transition form factors and GPDs. Andreas Jüttner, Mo 09:15
- What about resonances?
- Light cone distribution amplitudes.
- Higher twist contributions.
- Two-current observables, e.g., DPDs.
- Fragmentation functions (not amenable to lattice QCD).

From matrix elements to hadron structure



OPE of bilocal light-ray operator
$O_\Gamma(z) = \bar{q}(z/2)\Gamma U(z/2,-z/2)q(-z/2)$:

$$O_\Gamma = \sum_{n\geq 0} \frac{1}{2^n n!} z^{\mu_1}\ldots z^{\mu_n}\bar{q}(0)\Gamma \overleftrightarrow{D}_{\mu_1}\ldots\overleftrightarrow{D}_{\mu_n}q(0)$$

Twist $\tau = d - s$: "spin" with respect to the conformal light front group $SL(2,\mathbb{R})$.

d: Dimension of operator. s: Lorentz spin.

Leading twist: symmetrize and subtract traces.

Matrix elements $\langle P'|\bar{q}\cdots q|P\rangle$ then give **Mellin moments** of (fixed-twist) GPDs, i.e. GFFs.

Higher-twist contributions to scattering: usually suppressed by powers of Q.

Reduced ITDs (pseudo-PDFs) and quasi-PDFs
Inverse problem! Shoji Hashimoto, Mo 11:00
(same with PDFs from exp. cross-sections)

Higher-twist pollution:

$$\sim \Lambda^2 z^2 \text{ or } \frac{\Lambda^2}{x^2 P_z^2}, \frac{\Lambda^2}{(1-x)^2 P_z^2}.$$

Matching to continuum schemes

Matching (simplified picture)

For a given $\overline{\mathrm{MS}}$ scheme matrix element

$$\langle O\rangle_{\overline{\mathrm{MS}}}(\mu) = Z_O(\alpha(\mu), q)\cdot\langle O\rangle_{\mathrm{lat}}(a, q) + \mathcal{O}(a^2q^2)$$

Competing constraints

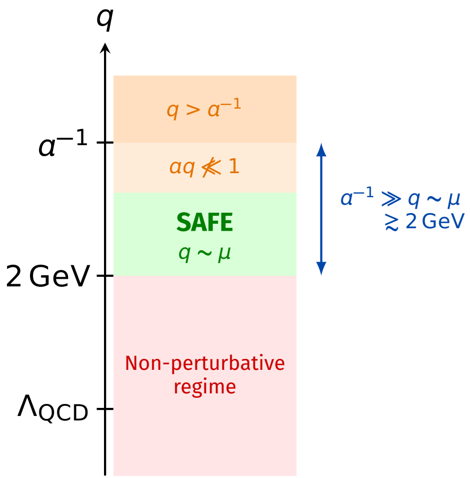
- **Continuum limit.** Only possible in the regime $q \ll a^{-1}$.
- **Perturbative matching.** Requires $\mu \gtrsim 2\,\mathrm{GeV}$.
- **No large logs.** Requires $q \sim \mu$ to ensure a feasible perturbative expansion of Z_O.

Alternative:

Step scaling with $q \propto \mu = L^{-1}$. Stefan Sint, Mo 10:00

Not always possible.

The window problem



What is the maximum a?

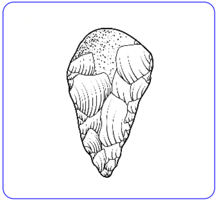
Is a small enough to allow for perturbative matching?

method	factorization scale	constraint ($\mu = 2\,\mathrm{GeV}$)	in real life
momentum factorization, e.g., RI'/SMOM	3-momentum $\lvert\vec{q}\rvert$	$a \ll \frac{\sqrt{3}\pi}{q} \approx 0.5\,\mathrm{fm}$	$a \lesssim q^{-1} \approx 0.1\,\mathrm{fm}$
gradient flow	flow time t	$a \ll \sqrt{8t} \sim \frac{4\sqrt{2}e^{-\gamma_E}}{\mu} \approx 0.3\,\mathrm{fm}$	$a \lesssim 0.3\,\mathrm{fm}$??
short distance factorization	distance $\lvert\vec{z}\rvert$	$a \ll z \sim \frac{2e^{-\gamma_E}}{\mu} \approx 0.1\,\mathrm{fm}$	$a \lesssim 0.1\,\mathrm{fm}$??
LaM"ET"	kinematics/ matching mix	$a \ll \lvert\vec{P}\rvert^{-1} \approx 0.07\,\mathrm{fm}$ ($P \gtrsim 3\,\mathrm{GeV}$)	$a \lesssim 0.07\,\mathrm{fm}$??

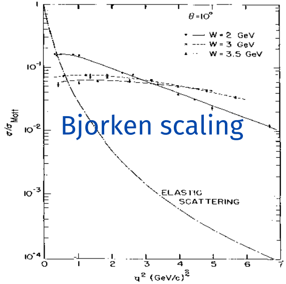
What is $\ll$? Depends on perturbative order, coefficients, and on power corrections.

For RI'/SMOM: $\ll\ \approx\ < \cdot \frac{1}{5}$. In other situations poorly known.

Evolution of lattice hadron physics tools and methods



The dawn of hadron structure



SLAC 1969 ($W = \sqrt{s}$)

Light cone PDF

Mellin moments

PHYSICAL REVIEW D VOLUME 6, NUMBER 12 15 DECEMBER 1972

Light-Cone Behavior of Perturbation Theory

N. Christ*†
Columbia University, New York, New York 10027
and
B. Hasslacher‡
University of Illinois, Urbana, Illinois 61801
and
A. H. Mueller†
Columbia University, New York, New York 10027

(Received 29 June 1972)

A technique introduced by Symanzik is used to derive a series of equations obeyed order by order in perturbation theory by the structure functions W_1 and νW_2 entering the cross section for inelastic electron scattering. These equations relate the q^2, ν, and coupling-constant dependence of W_1 and νW_2 in a manner reminiscent of the renormalization-group results of Gell-Mann and Low. The equations are used to compute the leading logarithmic contribution to νW_2 in a theory of fermions coupled to pseudoscalar particles and a theory of fermions coupled to vector particles.

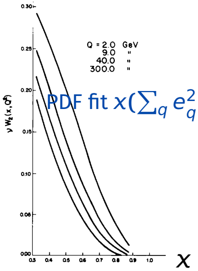
PHYSICAL REVIEW D VOLUME 9, NUMBER 4 15 FEBRUARY 1974

Asymptotically free gauge theories. II *

David J. Gross† and Frank Wilczek
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540

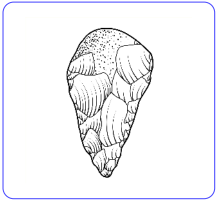
(Received 27 August 1973)

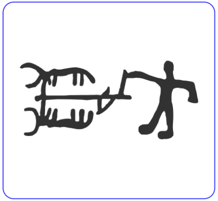
Deep-inelastic lepton-hadron scattering is analyzed in asymptotically free gauge theories of the strong interactions. The renormalization-group equations for the coefficients of the twist-two operators in the Wilson expansion are reviewed. A careful treatment of the mixing of operators with identical quantum numbers and dimensions is given. The relevant anomalous dimensions of the twist-two operators are calculated to second order in perturbation theory. These are used to calculate the asymptotic q^2 behavior of the moments of the structure functions. It is shown that the approach to the asymptotic limit is logarithmic,



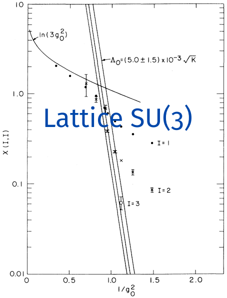
[A González-Arroyo et al, NPB159(79)512]

Evolution of lattice hadron physics tools and methods





Planting the seeds



[M Creutz, PRL(80)45]

Volume 116B, number 6 PHYSICS LETTERS 28 October 1982

background field 82

THE PROTON AND NEUTRON MAGNETIC MOMENTS IN LATTICE QCD

G. MARTINELLI
Laboratori Nazionali di Frascati, INFN, Frascati, Italy
G. PARISI
Università di Roma II, Tor Vergata, Rome, Italy
R. PETRONZIO
CERN, Geneva, Switzerland
and
F. RAPUANO
Istituto di Fisica G. Marconi, Università di Roma I, Rome, Italy

Received 17 June 1982

We have computed the proton and neutron magnetic moments on the lattice by a Monte Carlo simulation of QCD in the quenched approximation. The results are in remarkable agreement with the experimental values.

Nuclear Physics B (Proc. Suppl.) 30 (1993) 487–490
North-Holland
PROCEEDINGS SUPPLEMENTS

disconnected 93

Sea-Quark Effects in Nucleon Structure

Shao-Jing Dong and Keh-Fei Liu
Dept. of Physics and Astronomy, Univ. of Kentucky, Lexington, KY 40506, USA

We report results of our calculation on the πN σ term and the flavor-singlet axial charge of the nucleon in the framework of quenched lattice QCD. The disconnected insertions which involve contributions from the sea quarks are evaluated with the stochastic estimator with the Z_2 noise which yields minimum variances. We find the sea-quark contribution to the πN *sigma* term to be $\sim \frac{1}{2} - \frac{1}{4}$ of that of the dynamical fermion calculation. Whereas, the sea-quark contribution to the flavor-singlet g_A is negative and large enough to cancel the valence (connected insertion) contribution substantially to resolve the "proton spin crisis".

Nuclear Physics B306 (1988) 865–889
North-Holland, Amsterdam

sequential propagators 88

A LATTICE CALCULATION OF THE PION'S FORM FACTOR AND STRUCTURE FUNCTION

G. MARTINELLI
CERN, Geneva, Switzerland
C.T. SACHRAJDA
Physics Department, University of Southampton, Southampton S09 5NH, UK

Received 18 January 1988

In this paper we present the results of a lattice computation of the pion's form factor at several values of momentum transfer and the lowest two moments of the pion's structure function. The results are very satisfying, having small statistical errors and being consistent with experimental results. In addition, we present the details of the Monte Carlo simulation and an outline of the analysis of the results.

Hybrid/Hamiltonian MC

[S Duane et al, PLB(87)216]

Sea quark simulations 87

Volume 134B, number 5 PHYSICS LETTERS 19 January 1984

Feynman-Hellmann 83

THE ρ–$\pi\pi$ COUPLING CONSTANT IN LATTICE GAUGE THEORY

Steven GOTTLIEB
University of California, San Diego, La Jolla, CA 92093, USA
Paul B. MACKENZIE, H.B. THACKER
Fermi National Accelerator Laboratory, P.O Box 500, Batavia, IL 60510, USA
and
Don WEINGARTEN [1]
Brown University, Providence, RI 02912, USA

Received 9 September 1983

We present a method for studying hadronic transitions in lattice gauge theory which requires computer time comparable to that required by recent hadron spectrum calculations. This method is applied to a calculation of the decay $\rho \to \pi\pi$

Nuclear Physics B (Proc. Suppl.) 17 (1990) 361–364 361
North-Holland

covar. quark smearing 90

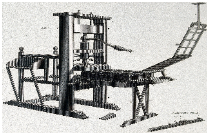
A STUDY OF SMEARING TECHNIQUES FOR HADRON CORRELATION FUNCTIONS *

Stephan GÜSKEN
Physics Department, University of Wuppertal, 5600 Wuppertal 1, Fed. Rep. of Germany
In collaboration with: U. Löw, K.-H. Mütter, R. Sommer, A. Patel and K. Schilling

We present a systematic study of smearing techniques, applied to increase the overlap of Hadron operators with their ground states. We find that the relevant parameters of the different schemes can be optimized such that the Hadron propagators are dominated by the ground state at time separation $t = 1$ already.

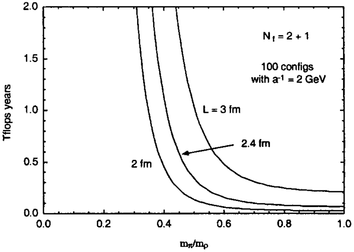
NP renormalization 93

Evolution of lattice hadron physics tools and methods



Exchanging ideas: new algorithms

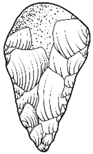
The Berlin wall 2001

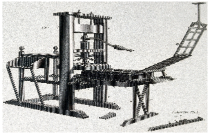
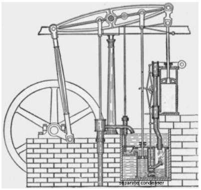


[A Ukawa, NPB(PS)(02)195]

- Multigrid revolution
- HMC evolution (Hasenbusch etc)
- Noise reduction methods (LMA, TSM, AMA)
- All-to-all propagators
- Improved analysis methods
- Using ChPT

Evolution of lattice hadron physics tools and methods



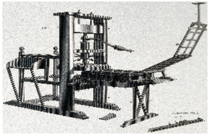
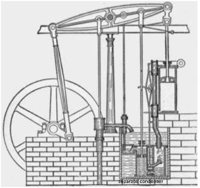
Industrial revolution

- $M_\pi \searrow M_{\pi,\text{phys}}$
- $L > M_\pi/4$
- Better statistics
- Better control of renormalization (Schrödinger functional, RI'SMOM matching at higher loop order)
- $a \to 0$
- Better control of excited states

- Distillation
- Momentum smearing
- Position space methods
- GPUs
- ...

Evolution of lattice hadron physics tools and methods

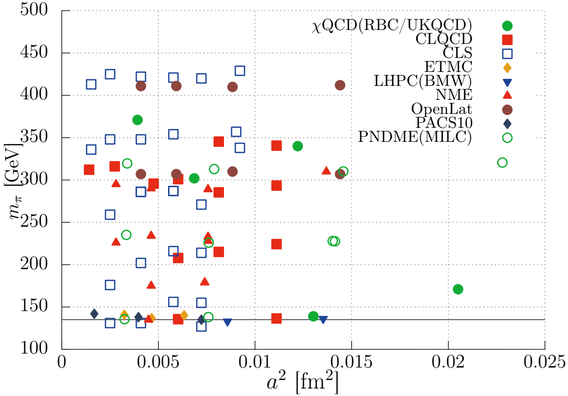


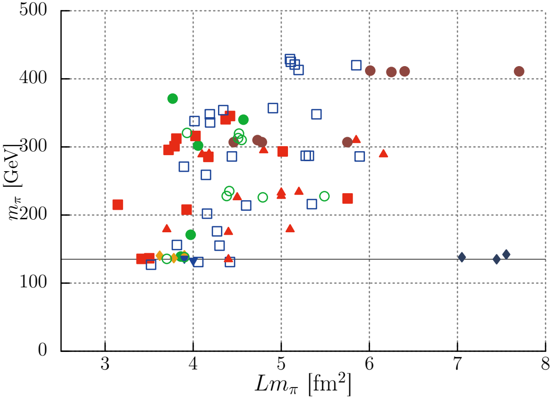




New horizons

- New methods and new observables. Nilmani Mathur, Mo 14:20 Luchang Jin, Mo 15:00 Sinya Aoki, Mo 15:20 Joshua Miller, Mo 16:10 Wayne Morris, Tu 14:00 Artur Avkhadiev, Tu 14:20 Yang Fu, Tu 14:40 William Good, Tu 16:30 Rui Zhang, We 09:40 Joseph Karpie, We 10:20 Raza Sufian, Fr 15:20
- Bi-local currents, four-point functions. Alex Chang, Mo 16:50 Christian Zimmermann, We 11:10 Benjamin Banneker, We 11:30 Sudip Shiwakoti, Fr 14:40 Geng Li, Fr 16:10
- Many applications of gradient flow regularization. Vaibhav Charar, Fr 14:00 Andrea Shindler, Sa 9:45
- Variance reduction. [T Harris, 2605.00643] Antonio Evangelista, Fr 15:00
- Multihadron interpolators to address excited states. Lorenzo Barca, Th 16:10
- Structure of resonances? Plans: Luka Jevsenak, Mo 16:50
- First steps of including isospin-breaking effects. Christian Schneider, Th 17:10

Landscape of ensembles in use for hadron structure calculations





Lattice-spacing and quark-mass dependence can thoroughly be investigated.
Least quantifiable systematics: Excited states and renormalization.
(+ **reconstruction and power corrections** for LaM“ET”, SDF).

Big problem: Signal to noise. New methods are needed/should be used!
E.g., kinematically enhanced interpolators: [R Zhang et al, 2501.00729], blending: [Z-C Hu et al, 2505.01719], Multigrid LMA and other methods, see [T Harris, 2605.00643].

Isovector pion PDF Mellin moments using the gradient flow

The gradient flow is a O(4)-invariant and gauge-covariant regulator.

Same symmetries as in the continuum theory.
No mixing with lower-dimensional lattice operators!

Continuum limit

$$\langle x^{n-1}\rangle^{\text{flow}}(t) = \langle x^{n-1}\rangle^{\text{flow}}(t,a) + \underbrace{c_2\frac{a^2}{t} + \text{other}\, a^2 + \ldots}_{\text{extrapolate } a\to 0}$$

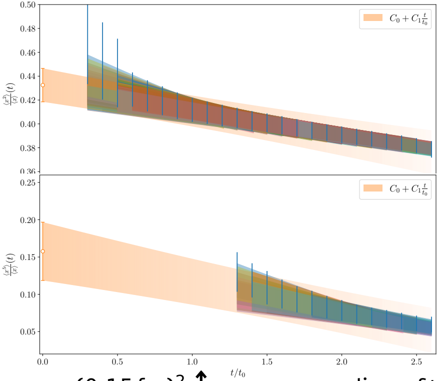
Small flow time expansion

$$\langle x^{n-1}\rangle^{\overline{\text{MS}}}(\mu) = C_0^{(n)}(\alpha_s(\mu))\langle x^{n-1}\rangle^{\text{flow}}(t) + \underbrace{\sum_i C_{2i}^{(n)}\langle O_{n,2i}^{\text{flow}}\rangle\, t + \ldots}_{\text{extrapolate } t\to 0}$$

$C_0^{(n)}$ available at two-loop order [R Harlander et al, 2511.17145].
Fermionic wavefunction renormalization cancels from ratios.

[OpenLAT: A Francis et al, 2510.26738]

Andrea Shindler, Sa 9:45 (will also present nucleon)



$\sim (0.15\,\text{fm})^2$ ↑

linear fit

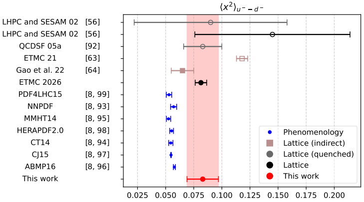
2nd nucleon gluon moment: [HadStruc: R Edwards et al, 2602.14260] Alex Sturzu, Tu 16:10

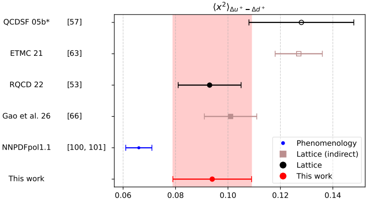
Perturbative one-loop matching to lattice regulators: Gregoris Spanoudes, Tu 17:10

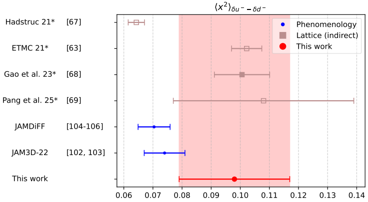
Nucleon σ terms: Rohith Karur, Th 16:50

Conventional isovector PDF moments for the nucleon

[E Taggi et al, 2605.02808] (LHPC) Figure: Marcel Rodekamp. $a \approx 0.116$ fm and $a \approx 0.093$ fm, $M_\pi \approx 135$ MeV.







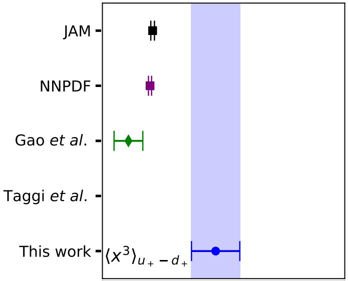
[ETMC, 2607.03458] First calculation of the fourth isovector moment.

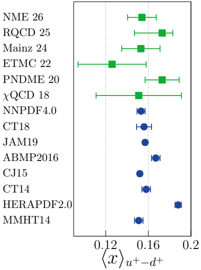
Christian Kummer, Mo 15:20

$a \approx 0.08$ fm, $M_\pi \approx 139$ MeV
(also A_{n0} GFFs at small Δ^2 up to $n = 4$).

$q^\pm = q \pm \bar{q}$

For $\langle x^4\rangle$ and higher, a^{-1} terms cannot be avoided (mixing with lower dimensional operators)!
Lower n for flavour singlets → gradient flow.





NME 26: R Gupta, Mo 14:20

Parton momentum fractions in the nucleon

[ETMC, 2607.21230] Constantia Alexandrou, Th 14:00, M_π^{phys}, $a = 0.05 - 0.08$ fm.

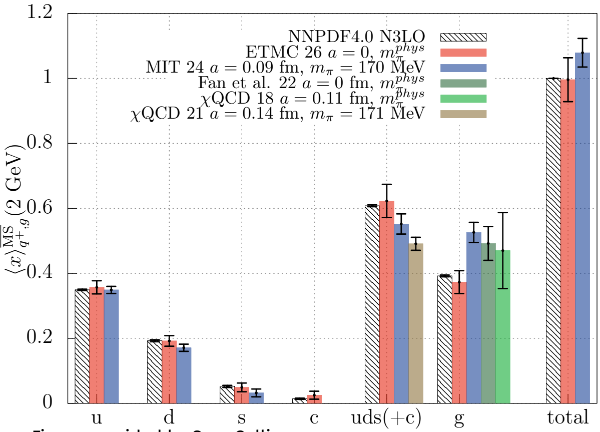


Figure provided by Sara Collins

A consistent picture emerges, once the systematics are under control.

- Continuum limit.
- Physical quark masses.
- Study of excited states.
- $N_f = 4$ non-perturbative renormalization to RI'MOM.
- Two-loop conversion to $\overline{\rm MS}$.
- Evanescent operator was considered for the first time, but an approximation was necessary.

[NNPDF4.0N3LO, 2402.18635] (moments computed using [LHAPDF, 1412.7420]).

[MIT 24, 2310.08484], [Fan et al. 22, 2208.00980], [χQCD 18, 1805.00531], [χQCD 21, 2111.09329].

The nucleon spin

Gauge invariant Ji spin decomposition

$$\frac{1}{2} = J_g + \sum_q J_{q^+} = J_g + \sum_q \left(\frac{1}{2} \Delta\Sigma_{q^+} + L_{q^+} \right)$$

[ETMC, 2607.21230] Constantia Alexandrou, Th 14:00, M_π^{phys}, $a = 0.05 - 0.08$ fm.

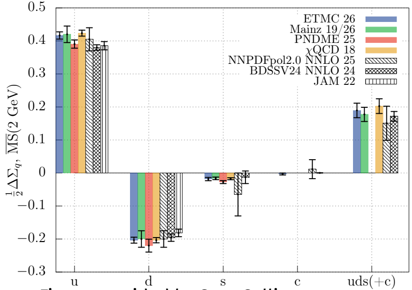
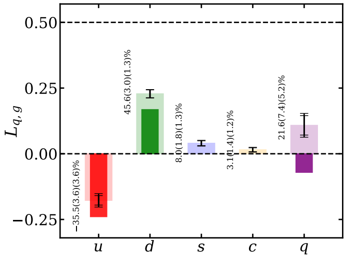
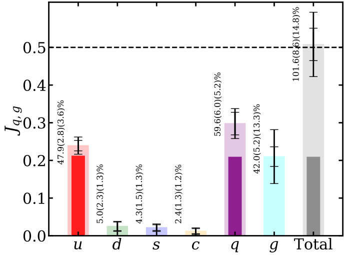


Figure provided by Sara Collins



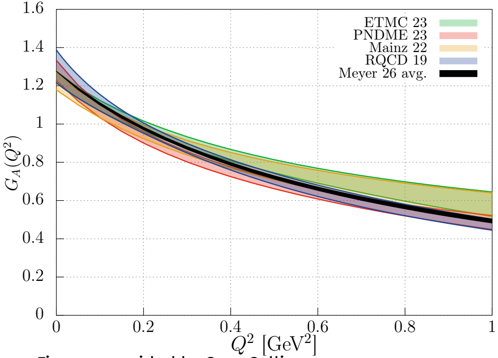


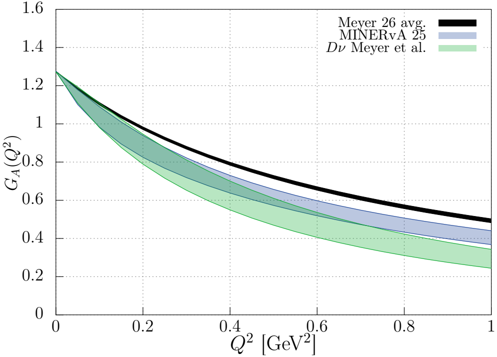
[Mainz 26, 2605.06559], [Mainz 19, 1911.01177], [PNDME 25, 2503.07100], [χQCD 18, 1806.08366], [NNPDFpol2.0NNLO, 2503.11814], [BDSSV24-NNLO, 2407.11635], [JAM 22, 2202.03372]. Big Lattice impact on Δs and $\Delta\Sigma$!

Nucleon isovector axial form factor: $G_A(Q^2)$

$$\langle N(p_f)|A^\mu|N(p_i)\rangle = \bar{u}(p_f)\left[\gamma^\mu G_A(Q^2) + q^\mu \frac{\tilde{G}_P(Q^2)}{2M_N}\right]\gamma_5 u(p_i)$$

Lattice average: [Meyer 26, 2601.02676] ($G_A(0) = g_A$ fixed to experiment).





Figures provided by Sara Collins

[ETMC 23, 2309.05774] [PNDME 23, 2305.11330] [Mainz 22, 2207.03440] [RQCD 19, 1911.13150] [MINERνA 25, 2512.14097]

[$D\nu$ A Meyer et al, 1603.03048]. See also [O Tomalak et al, 2601.21155]

PNDME update (not yet included) R Gupta, Mo 14:20

Nucleon isovector axial radius: $\langle r_A^2 \rangle$ [fm^2]

$$\langle r_A^2 \rangle = -6 \left. \frac{dG_A(Q^2)}{dQ^2} \right|_{Q^2=0}$$

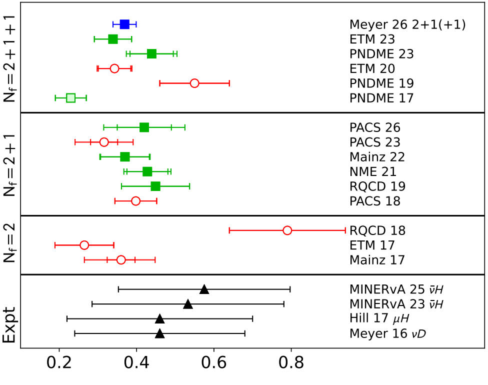
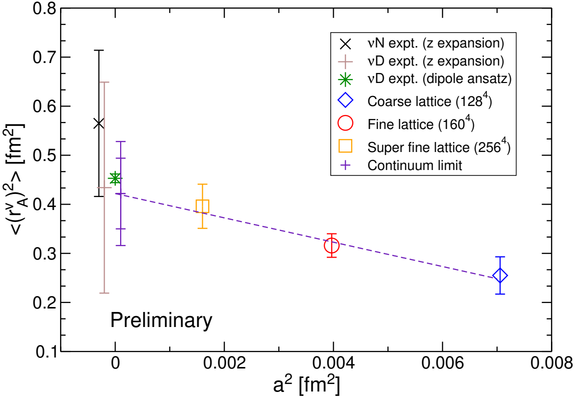


Figure provided by Sara Collins

Poster Tsuji Ryutaro (PACS)

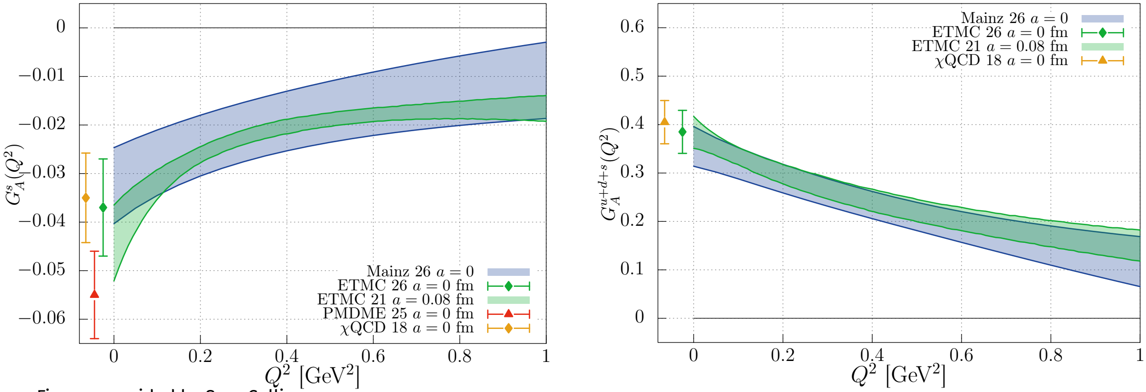
M_π^{phys}, $\boldsymbol{L \sim 10}$ **fm**, $a = 0.04 - 0.09$ fm.



See also [O Tomalak et al, 2601.21155].

Nucleon strange and singlet axial form factors

[Mainz 26, 2605.06559] $N_f = 2+1$ CLS, $M_\pi \geq 129$ MeV, $a = 0.05 - 0.09$ fm.



Figures provided by Sara Collins

ETMC 26 Constantia Alexandrou, Th 14:00, M_π^{phys}, $a = 0.05 - 0.08$ fm.

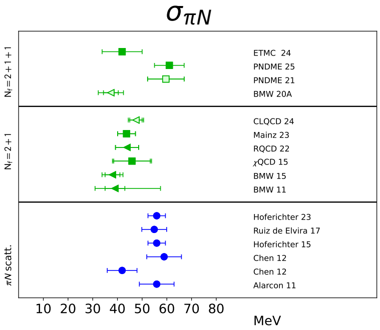
[ETMC 21, 2106.13468], [PNDME 25, 2503.07100], [χQCD 18, 1806.08366].

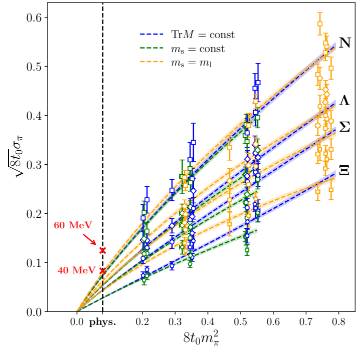
Octet baryon pion and strange σ terms

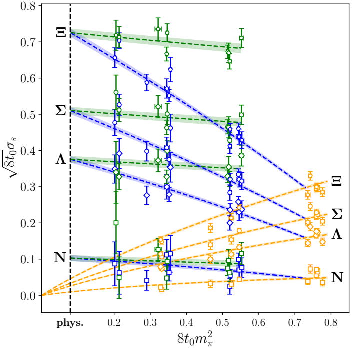
Direct determination: $\sigma_{\pi B} = m_{ud}\langle B|\bar{u}u + \bar{d}d|B\rangle$ and $\sigma_s = m_s\langle B|\bar{s}s|B\rangle$, $B \in \{N, \Lambda, \Sigma, \Xi\}$.

RQCD/Münster/JLab Pia Leonie Jones, Th 16:30 CLS, continuum limit.

orange: $m_s = m_u = m_d$, blue: $m_s + m_u + m_d =$ physical, green: $m_s =$ physical.







[FLAG 24, 2411.04268]: $N_f = 2+1$ average $\sigma_{sN} = 44.9(6.4)$ MeV.

Pion-nucleon sigma term

Direct determination: Lorenzo Barca, Th 16:10 $a = 0.09$ fm, $M_\pi = 222$ MeV.

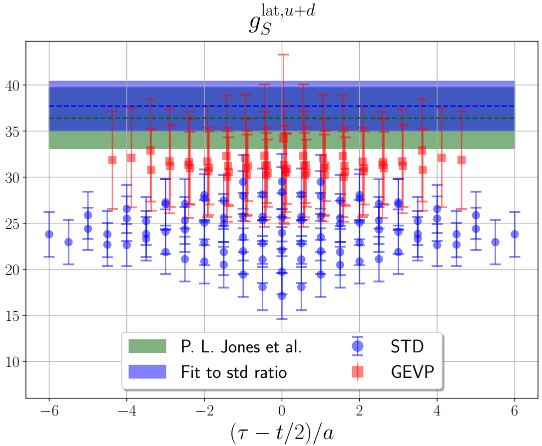
Standard analysis (STD):

- Single spatially extended nucleon interpolator.
- 13 source-sink separations ($t_{\text{sep}} = (0.3 - 1.4)$ fm).

Jones et al., 4 $t_{\text{sep}} = (0.7 - 1.2)$ fm.

Generalized eigenvalue problem analysis (GEVP):

- N and $N\sigma$-type interpolators.
- 11 $t_{\text{sep}} = (0.3 - 1.2)$ fm.
- σ is unstable. Lowest $\pi\pi$ level is seen.

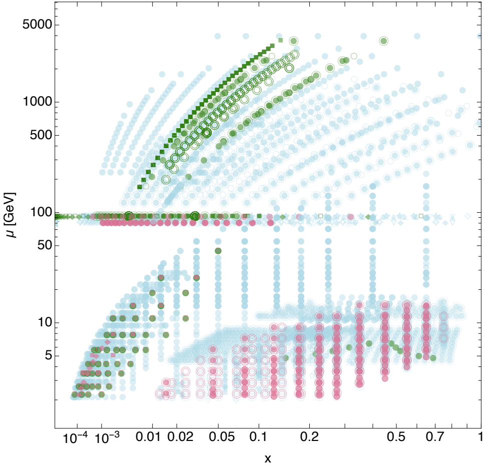


x-dependent ?PDFs from Ioffe-time distributions or quasi-?PDFs

- Lattice excels in flavour separation.
- New source of information.
- Very active field, many new methods and ideas.
- **Inverse problem with noisy data!**

[CT25, 2512.19779] x and μ of exp. data

• light blue: CT18 • green: new in CT25 • red: removed



PDFs Joshua Miller, Mo 16:10 William Good, Tu 16:30
Yamil Medrano, We 09:00 Qi Shi, We 09:20 Rui Zhang, We 9:40
Alex NieMiera, We 10:00 Yong Zhao, We 11:50 Raza Sufian, Fr 15:20 (photon)
Poster Nehal Khosla
Structure functions Chris Zimmermann, We 11:10 Jordan Mckee, We 11:30
LCDA Alex Chang, Mo 16:50
GPDs Manuel Colaço, Mo 14:40 Jack Holligan, Mo 15:00
Joseph Torsiello, Mo 16:30 Sicheng Liu, Tu 16:50 Joe Karpie, We 10:20
TMDPDFs Wayne Morris, Tu 14:00 Artur Avkhadiev, Tu 14:20
Yang Fu, Tu 14:40 Jinchen He, Tu 15:00 Jyn Peyton, Tu 15:20

x-dependent ?PDFs from Ioffe-time distributions or quasi-?PDFs

- Lattice excels in flavour separation.
- New source of information.
- Very active field, many new methods and ideas.
- **Inverse problem with noisy data!**
- At present: poor kinematic coverage.
- Large z/small P, xP, $\bar{x}P$: higher twist pollution.
- Use case: whenever little or no data exist. TMDPDFs, GPDs, transverse polarized PDFs.
- Quantifying the systematics: e.g. Joe Karpie, We 10:20 Chris Monahan, Sa 11:30

PDFs Joshua Miller, Mo 16:10 William Good, Tu 16:30
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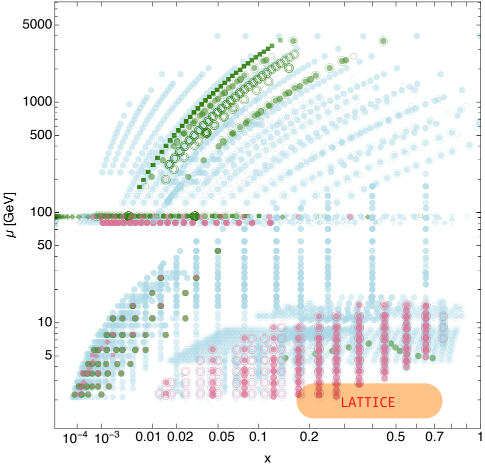

SDF, ITDs and pseudo/quasi-PDFs (references in write-up)

$\sqrt{s} \gg M, Q \gg M$ in DIS experiments but not on the light cone ($s < \infty$). Nevertheless can be related to PDFs: **collinear factorization**. Can we do the same on the lattice?

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$M^0(z,P) = \langle P|\bar{q}(z)\gamma^0 U(z,0)q(0)|P\rangle = 2P^0 M(\nu, z^2)$, where $z^\mu = (0, \vec{z})$, $z = |\vec{z}|$.

Ioffe time $\nu = -z_\mu P^\mu = \vec{z}\cdot\vec{P} \geq 0$ **Ioffe time distribution** $M(\nu, z^2)$.

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After taking the continuum limit: $$\mathfrak{M}(\nu, z^2) = \int_0^1 \mathrm{d}y\, C(y, z^2\mu^2) Q(\nu y, \mu^2) + \mathcal{O}(z^2)$$

Short-distance factorization (SDF). Q: ITD in $\overline{\mathrm{MS}}$ scheme.

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Relation to PDF: $Q(\nu, \mu^2) = \int_{-1}^1 \mathrm{d}x\, e^{i\nu x} q(x, \mu^2)$

$q(-x) = -\bar{q}(x) \rightarrow$ imaginary part: q^+, real part: q^-. **Inverse trafo:** difficult.

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Quasi-PDFs/LaMET: renormalization/matching/limits more involved. Harder for me to explain.

Pion quasi-PDF/TMPDF in Coulomb gauge

Rui Zhang, We 9:40 [A Grebe et al, 2606.16877] $P_z = 2.2$ GeV.

Coordinate $\vec{z}$, $z = |\vec{z}|$.

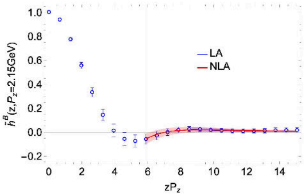
$$\tilde{q}(x, P_z) = P_z \int_{-\infty}^{\infty} \frac{dz}{2\pi} \tilde{h}(z, P_z) e^{ixP_z z} \tilde{h}(z, P_z).$$

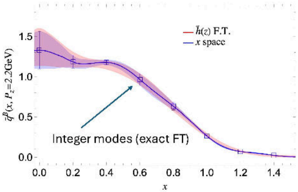
In Coulomb gauge: no z-dependent renormalization issue.

Fourier trafo can directly be taken, using data.

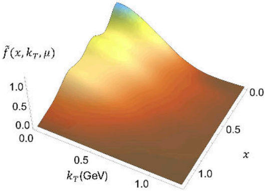
The problem at large z is transformed into a problem at small xP_z.

Consistent results.

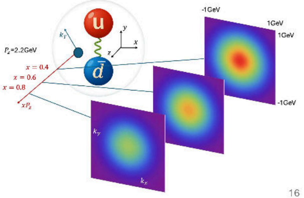




• 3D Momentum Distribution



• k_T distribution at different x



Collins-Soper rapidity anomalous dimension function

TMDPDF evolution factorizes in position space (related by 2D FT to momentum space).

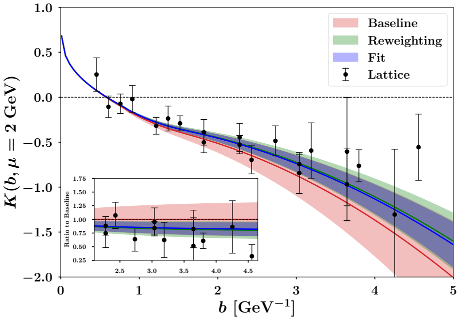
Rapidity evolution: $f_{q/g}(x, b_\perp; \mu, \zeta) = f_{q/g}(x, b_\perp; \mu, \zeta_0) \exp\left[\frac{1}{2}\gamma_\zeta(b_\perp, \mu) \ln \frac{\xi}{\xi_0}\right]$.

Rapidity anomalous dimension (Collins-Soper kernel) $\gamma_\zeta(b_\perp)$ computed from ratio of different $\zeta_n \propto P_n$.

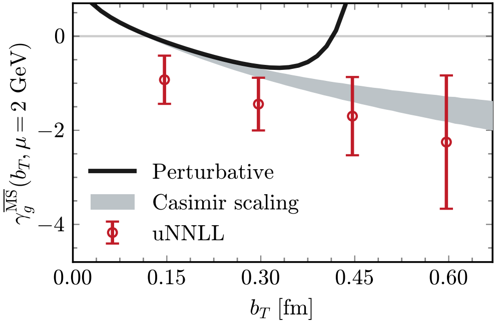
Independent of the hadron → use the pion! 4-loop result exists for $b_\perp \ll \mu^{-2}$ (most interesting for small $k_\perp \sim 1/b_\perp$).

Done in the continuum limit using LaMET in Coulomb gauge: Artur Avkhadiev, Tu 14:20 Yang Fu, Tu 14:40

Quark sector: Continuum limit, physical quark mass.
New: joint extraction also using experimental data.
[A. Avkhadiev et al., 2510.26489]



Gluon sector: First time!
$M_\pi = 172(3)$ MeV, $a = 0.15$ fm.
[A. Avkhadiev et al., 2607.24587]



Structure functions from current-current correlators

Hadronic tensor in Minkowski spacetime: χQCD Christian Zimmermann, We 11:10

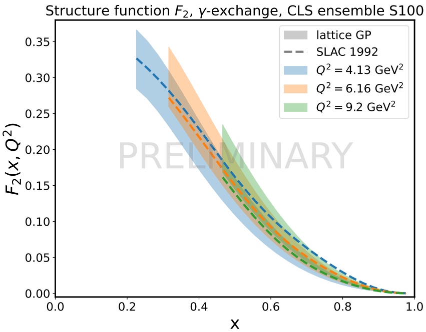
$W_{\mu\nu} = \frac{1}{4\pi}\int \mathrm{d}^4 z\, e^{iqz} \left\langle P \left| \left[J^\dagger_\mu(z), J_\nu(0) \right] \right| P \right\rangle$ $\rightarrow$ decomposes into structure functions.

$$W^E_{\mu\nu}(\vec{p},\vec{q},t) = \frac{1}{4E_{\vec{p}}}\int \mathrm{d}^3 z\, e^{-i\vec{q}\cdot\vec{z}} \left\langle P \left| T J^\dagger_\mu(\vec{z},t) J_\nu(0) \right| P \right\rangle = \int_{q_{\min}}^{\infty} \frac{\mathrm{d}q_0}{2E_{\vec{p}}} e^{-q_0 t} W_{\mu\nu}(p,q)$$

Problem: On-shell states with $(p+q)^2 = m^2$ propagate if $q_0 < q_{\min}$ $\rightarrow$ limits kinematic reconstruction range.

Strategy: Compute 4-point function stochastically; parametrize $W^E_{\mu\nu}$ via Euclidean structure functions.

Reconstruction of F_2 via Gaussian process regression (CLS ensemble S100, $M_\pi = 223\,\mathrm{MeV}$, $a = 0.085\,\mathrm{fm}$).



Structure functions from current-current correlators

Hadronic tensor in Minkowski spacetime: χQCD Christian Zimmermann, We 11:10

$$W_{\mu\nu} = \frac{1}{4\pi}\int \mathrm{d}^4 z\, e^{iqz} \left\langle P \left| \left[J^\dagger_\mu(z), J_\nu(0) \right] \right| P \right\rangle$$ → decomposes into structure functions.

$$W^E_{\mu\nu}(\vec{p}, \vec{q}, t) = \frac{1}{4E_{\vec{p}}}\int \mathrm{d}^3 z\, e^{-i\vec{q}\cdot\vec{z}} \left\langle P \left| T J^\dagger_\mu(\vec{z}, t) J_\nu(0) \right| P \right\rangle = \int_{q_{\min}}^{\infty} \frac{\mathrm{d}q_0}{2E_{\vec{p}}} e^{-q_0 t} W_{\mu\nu}(p, q)$$

Problem: On-shell states with $(p+q)^2 = m^2$ propagate if $q_0 < q_{\min}$ → limits kinematic reconstruction range.

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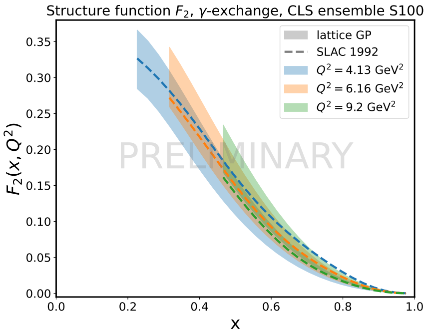
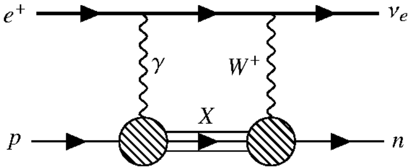


Compton forward amplitude: QCDSF Jordan Mckee, We 11:30

Same object for $t = 0$ via Feynman-Hellmann method for the vector-axialvector combination.

Nachtmann moment prediction:

First calculation of CP-violating Nachtmann moment, related via dispersion relation to the γW box diagram.



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Summary of lattice hadron structure calculations

Importance

- **Essential tool:** Enables diverse experiments to achieve their full physics potential.
- **Meson precision:** Has become indispensable for flavour physics (not covered here).
- **Baryon frontier:** Achieving similar precision goals remains a key challenge.
- **Complementary input:** Straightforward flavour separation directly enhances global fits. Moments vs. x-dependence. Clean separation of leading twist.

Impact (examples)

- Impact on **helicity** and **transversity** PDFs!
- Steady progress on lesser-known **GPDs, TMDs,** and **gluon structure**.

Precision milestones

- Nucleon **charges and form factors** are approaching the **1% precision** threshold.
- Now that systematics are under control, the first two isovector PDF **Mellin moments** show agreement with global experimental fits.

Field vitality

- Higher Mellin moments are now accessible via **gradient flow** techniques.
- Wealth of interesting new coordinate-space data for **bilocal matrix elements**.
- Highly active landscape driving novel methods to improve calculational efficiency and to quantify systematic errors.