Recent results in lattice BSM physics

Lattice 26

Simon Catterall

July 31, 2026

Topics

Review:

- Composite Higgs
- Dark matter
- Supersymmetric theories
- Lattice gravity

Focus area:

Recent developments: Lattice Anomalies, Kähler-Dirac Fermions and Symmetric Mass Generation

My personal take/bias ...

Things I won't have time to cover

- QCD contributions to axion-photon coupling Brandt et al. 2603.29153 (plenary Marko this meeting)
- Sphalerons - Bonano et al PRL 132 (2024) 051903 (Dionisio this meeting)
- Conformal window: Zeroes of partition function in 12 flavors. Saleh et al. 2606.14642. (Meurice this meeting)
- Tensor networks (Tanaka,Hayazaki,Aizawa,Sugimoto,Akiyama,...)
- Qubit regularizations of lattice gauge theories - Shailesh Chandrasekharan et al. 2603.01215 (Siew this meeting)
- Hamiltonian staggered fermions, disk chiral fermions (Kaplan and Sen) (Yamaoka, Dang (parallel)) ... Hersh Singh (plenary)

Apologies to anyone I missed ...

Composite Higgs

Light Higgs natural if composite Goldstone boson

Start with hypercolor gauge theory with group $G_{\rm HC}$
Add fermions with global symmetry G_F
Assume strong dynamics breaks $G_F \to H_F$ in I.R
Higgs in coset G_F/H_F

Minimal way to do this ?

Take $G_{\rm HC} = Sp(4)$
Couple to 2 Dirac (4 Weyl) fermions in fundamental (pseudoreal) rep [anomaly free]
Thus $G_F = SU(4)$ and $SU(4) \to Sp(4)$
5 Goldstones

Where is the Higgs ?

Identify an $SO(4)$ subgroup of $Sp(4)$ naturally as EW sector
custodial symmetry built in automatically!
Goldstones transform as $\mathbf{4} + \mathbf{1}$ under
$SO(4) = SU(2)_L \times SU(2)_R$
Higgs plus a singlet (dark matter ?)

Gauging $SU(2)_L$ automatically generates $SO(4)$ invariant
Higgs potential

Generating the Yukawas

Higgs has 2 roles in SM

- Break EW symmetry. Seen how to generate composite Higgs to do this by coupling to fermions Q in fundamental rep of $Sp(4)$.
- Give mass to SM fermions. $\leftarrow$ How to do this in composite Higgs models ?

Naively $QQ\psi\psi$. But need large Yukawa to get top $\rightarrow$ large FCNC

Solution: partial compositeness: SM fermion mixes with hypercolor composite baryon transforming as color triplet/EW doublet: $\delta\mathcal{L} = \lambda\psi\mathcal{O}_{\rm HC}$

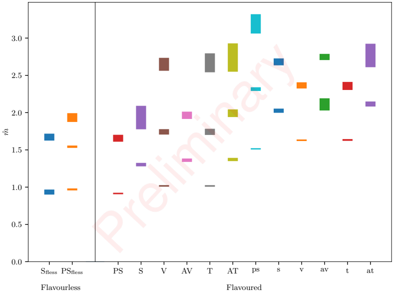
UV realization: 3 Dirac fermions Ψ in AS (real) rep $Sp(4)$

$\rightarrow$ chimera $\mathcal{O}_{\rm HC} = QQ\Psi$ $\leftarrow$ embed color in $SO(6)_F$

Composite Higgs - TELOS collaboration

Upper case denotes hadrons made with fund fermions. Lower case with AS fermions.

GBs live in 6 of $SU(4)_F$. Break to $5+1$ (1 is flavor singlet)



Left panel:S+PS states from flavor singlet mesons + glueballs

Right is flavored sector.

Chimera states - see Bennett et al. 2506.19804

See talk by Brito this meeting.

Dark matter

WIMP miracle: $\Lambda_{DM} \sim \Lambda_{EW}$ with direct SM coupling.
Growing tension with direct dark matter detection expts...

$\Downarrow$

Another scenario: dark matter might be a composite state from a new confining gauge theory.

- Dark matter as a stable baryon of $SU(N)$ or (Sp/SO) gauge theory.
- Dark matter as a Goldstone boson.
- Dark matter as a glueball.

Lattice can supply masses, LEC to EFT descriptions ..

Baryons as dark matter

Stealth dark matter (LSD collaboration)

- Dark matter is lightest baryon of $SU(4)$ hypercolor theory with 4 fund Dirac fermions carrying EW charges. Spin 0 neutral boson coupling only weakly (dim 7 ops) to SM (evades direct detection) $\Lambda_{DM} \sim 1 - 10\mathrm{TeV}$ (Appelquist et al. 1503.04203) But where are the dark pions ?
- Hyperstealth: $SU(4)$ with 1 (light) Dirac fermion (EW singlet) plus heavy (EW charged) sector. Integrate out heavy fermions $\rightarrow$ EFT that weakly couples dark baryon to SM. Allows $\Lambda_{DM} \sim 0.1 - 1\mathrm{GeV}$ (2412.14540) no light pion, good for small scale structure

Pions as dark matter

eg Strongly Interacting Massive Particle (SIMP) models:

SIMP requires $\chi\chi\chi \to \chi\chi$. Relic abundance leads to $\Lambda_{DM} \sim 0.1 - 10\mathrm{GeV}$ and no direct SM coupling

Ingredients: chirally broken gauge theory with non-zero $\pi_5(G/H)$ (5th homotopy group)
5pt vertex from WZW term:

$$\delta\mathcal{L} \sim \frac{N_c}{f_\pi^5}\epsilon^{\mu\nu\rho\lambda}\mathrm{tr}(\pi\partial_\mu\pi\partial_\nu\pi\partial_\rho\pi\partial_\lambda\pi)$$

Possible U.V completion (Maas et al. 2405.06506, TELOS 2603.19557, 2606.14546)

$Sp(4) + 2$ fund quarks $\to SU(4)/Sp(4)$ again !

Gravity waves from Dark Matter

First order phase transitions arising from deconfinement of strongly coupled dark sector in early Universe can produce stochastic background in gravitational waves

A broad peak in the power spectrum is expected whose location is

- EW scale dark matter $\rightarrow$ milliHertz (LISA)
- hyperstealth/SIMP dark matter $\rightarrow$ nanoHertz (binary pulsar arrays)

Pure SP/SU gauge theories have first order transitions.

Use lattice to provide theoretical prediction of spectrum ...

Early Universe Thermodynamics + Gravitational Wave from Hyperstealth Dark Matter (LSD)

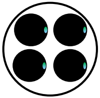
Nobu Matsumoto's talk
arXiv: 2602.23002 [hep-lat]

HSDM
SU(4-color) YM Gauge Theory
PRD 112, 075004 (2025)
arXiv:2412.14540

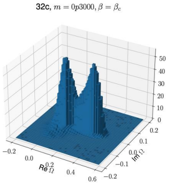
1 flavor of dark quarks

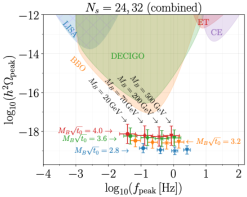
Feeble interactions with the Standard model today

DM = stable, 2^+ dark baryon



1st order confinement transition for heavy quark masses





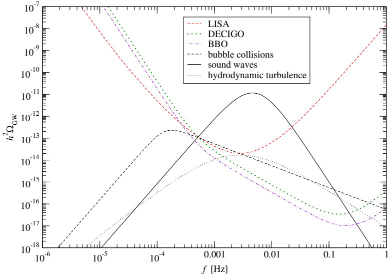
- **Polyakov loop effective potential from multi-histogram**
- **Sea-quark decreases the surface tension of the vacuum bubbles**
  → decreases the gravitational wave amplitude
- **Spectrum landscape from the quenched to the 2nd order endpoint**

Dynamical simulations reveal decreased surface tension vac bubbles
→ smaller amplitude for signal

Sp(4) Glueball Dark Matter - Panero et al.

Gravitational Waves from $\mathrm{Sp}(4)$ Dark Matter

- ★ Dark Matter as the glueball of an $\mathrm{Sp}(4)$ Yang–Mills theory
- ★ First-order confining phase transition at finite temperature
- ★ Production of primordial gravitational waves



M. Bruno, N. Forzano, M. Panero, and A. Smecca,
JCAP 01 (2026) 049, arXiv:2410.17122

Gravitational wave energy density vs frequency

Supersymmetric theories

Two main tracks for lattice:

- $\mathcal{N} = 1$ super Yang-Mills and Super QCD $\leftarrow$ simplest theory where expect confinement, χ-symmetry breaking, monopoles etc depending on (N_c, N_f). Great target for lattice simulation but discretization breaks SUSY - fine tuning problem to regain SUSY as $a \to 0$
- $\mathcal{N} = 4$ super Yang-Mills. Fascinating CFT with conjectured strong-weak coupling duality and holographic connections to quantum gravity in AdS_5. Planar limit understood but describes classical gravity. Finite $N \to$ quantum gravity. Lattice construction exists which preserves one supercharge.

Super Yang-Mills and super QCD

Use p theory as a guide to tuning needed

Tuning lattice $\mathcal{N} = 1$ SUSY theories towards the supersymmetric limit

E. Antoniou, M. Costa, E. Ioannou, H. Panagopoulos, G. Spanoudes

Main Goals / Ongoing Projects

Study one-loop renormalization in lattice $\mathcal{N} = 1$ supersymmetric gauge theories:

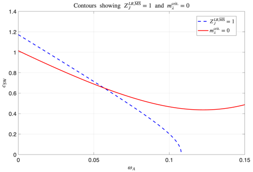
1. $\mathcal{N} = 1$ SYM with clover-improved, stout-smeared gluinos [poster by M. Costa]
2. $\mathcal{N} = 1$ SQCD using an overlap formulation [talk by M. Costa]

Strategy for part of the projects

Compute key quantities in $\mathcal{N} = 1$ SYM:

- the chiral current renormalization condition, $Z_J = 1$,
- the vanishing of the critical gluino mass, $m_\lambda^{\text{crit}} = 0$.

Their simultaneous solution identifies an improved region in the parameter space $(c_{\text{SW}}, \omega_A)$ (clover, stout).



Key result

The two improvement conditions intersect around:
$\omega_A \simeq 0.055$, $c_{\text{SW}} \simeq 0.67$.

Take-home message: The plot shows that perturbation theory can guide the tuning of c_{SW} and stout coefficient ω_A so that both chiral/supersymmetric restoration conditions are improved simultaneously.

see poster/talk by Costa at this meeting.

$\mathcal{N} = 4$ super Yang-Mills

Lattice construction $\mathcal{N} = 4$ Super Yang-Mills with exact SUSY:
Ingredients:

- Complex gauge links $\mathcal{U}_a = A_a + iB_a$ carries gauge field + scalars $\mathcal{U}_a^\dagger \mathcal{U}_a \neq 1$
- Fermions on sites, links and plaquettes $\eta(x)$, $\psi_a(x)$ and $\chi_{ab}(x)$ of A_4^* lattice.
- All fields expanded on algebra eg. $\mathcal{U}_b(x) = \sum_{A=1}^{N^2} \mathcal{U}_b^A(x) T^A$. Flat measure but does not break gauge invariance.
- $\delta S = 0$ for single scalar supersymmetry. $\delta \mathcal{U}_a(x) = \epsilon \psi_a(x) \ldots$
- Exact lattice symmetries (almost) completely eliminate need for fine tuning to get correct continuum limit

Recent improved action allows (RHMC) simulations at arbitrarily strong g (Catterall et al. 2302.16025)

Static potential and holography

$\mathcal{N} = 4$ dual to gravity in AdS_5 ← quark potential for $(g, N) \to \infty$ known from classical gravity calc.

Lattice for finite N and finite $\lambda = g^2 N$ ← probe quantum gravity

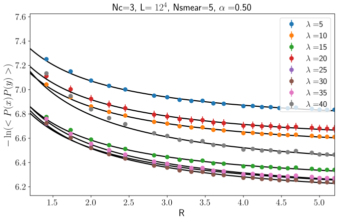


Figure: Correlator of Polyakov lines

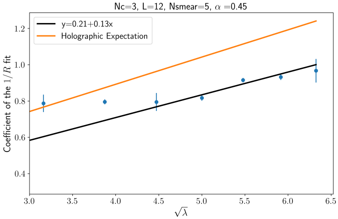


Figure: Coulomb coeff. vs 't Hooft coupling

Single non-abelian Coulomb phase with $\alpha \propto \sqrt{\lambda}$

To do list for $\mathcal{N} = 4$ ...

- Spectrum of conformal operators (Konishi ..) Compare to bootstrap/planar limit.
- Comparison of SYM thermodynamics with dual BHs ($D < 4$ well studied)
- Coulomb branch .. tests of strong/weak coupling duality
- ...

Quantum gravity

Quantizing Einstein gravity ...

$$Z = \int \mathcal{D}g \, e^{iS(g)} \quad \text{with } S = \frac{1}{4\pi G} \int d^4x \sqrt{g} \, (R - \Lambda)$$

Problem: non-renormalizable in perturbation theory

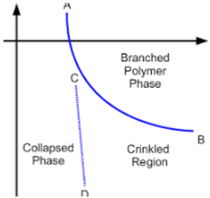
Weinberg: asymptotic safety: renormalizable about some non-perturbative U.V zero of $\beta(G)$

Need continuous phase transition in lattice theory

Lattice gravity

Replace $\int \mathcal{D}g$ by $\sum_T$ where T is a **triangulation**

$$Z = \sum_T e^{-S_{\rm EH}(T)}$$

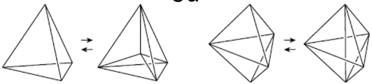


$$S_{\rm EH} = -\kappa_2 N_2 + \Lambda N_4 + \beta R^2$$

$$(\kappa_2 = \frac{1}{G})$$

Monte Carlo:
Random walk in space of 4d triangulations via local moves

3d



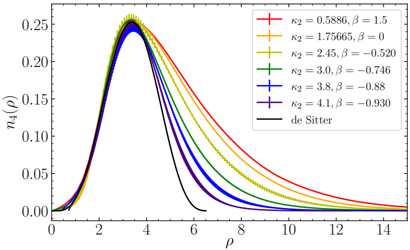
Two approaches:
Sum over all T's - Euclidean Dynamical Triangulations (EDT).
Sum over only those that admit a timeslicing - Causal Dynamical Triangulations (CDT)

Emergence of de Sitter space - EDT

de Sitter - space with uniform positive curvature $R = \Lambda > 0$

Average area of spherical region as function of radius

$$A(r) \sim \sin^3\left(r\sqrt{\Lambda}\right)$$

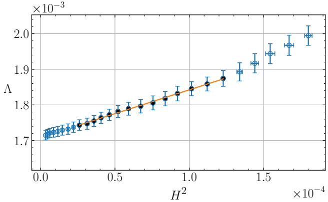


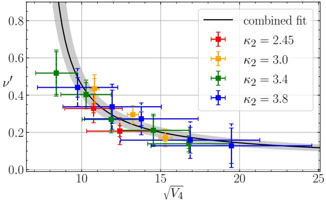
Lattice results suggest that long distance theory is close to classical de Sitter (Laiho et al. PRD111 (2025) 3 034514)

Running dark energy Λ

Hubble tension (maybe) suggests Λ is not constant

Compute $\frac{1}{3}\Lambda_{\mathrm{eff}} = (-\frac{\dot{a}}{a})^2 + \frac{1}{a^2}$ from lattice data with $a(t) = A^{\frac{1}{3}}(t)$ (Laiho et al.) Friedman eqn. $\rightarrow \Lambda_{\mathrm{eff}} = \Lambda_0 + \nu' H^2$



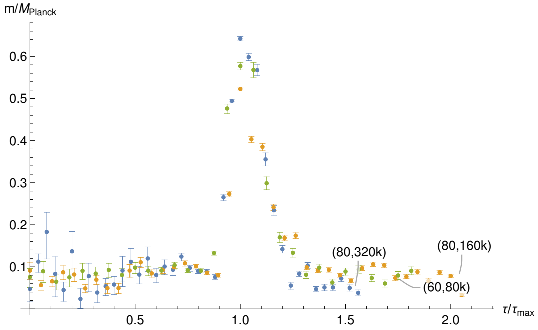


Dark energy larger in early Universe. Quantum effect !

Recent work → evidence for R^2 inflation (Laiho et al)

Recent CDT simulations

Extract mass of geon (gravitational glueball) as function of cosmological time from $< R(0)R(r) >$
Possible DM candidate or PBH.



Mass peaks near inflection point in de Sitter ...
A. Maas et al. 2504.11047

Anomalies, Kähler-Dirac fermions and Symmetric Mass Generation

Anomalies - textbook story (circa 1985)

3 types:

- Breaking of global symmetry under quantization. Acceptable and interesting ... eg ABJ.
- Anomaly in local (gauge) symmetry: ruins consistency: must cancel eg SM
- 't Hooft anomalies: Obstruction to gauging. Imagine coupling global symmetry to background gauge fields $\rightarrow$ anomaly $\mathcal{A}$
  BUT
  Since $\mathcal{A}$ RG invariant $\rightarrow$ I.R theory with global symmetry cannot be trivial - massless states eg SSB or TQFT

Historical understanding: Anomalies only arise with ∞ dof.
Cannot be seen on lattice ...
Anomalies only arise for continuous symmetries. One loop

Anomalies post 2015

- Anomalies for discrete symmetries eg time reversal
- Anomalies for higher form symmetries – act on extended objects eg center symmetry
- Non-invertible symmetries (not even a group !)

Understand in terms of new math (bordism groups etc).

Most interesting for us ...

Anomalies can be found in lattice models with finite dof !
Discrete symmetries - invisible in p. theory

Lattice anomalies

Modern take:

Bulk-boundary correspondence: onsite lattice symmetries cannot be anomalous unless theory forms boundary of some bulk TQFT

Ex: cannot capture chiral anomaly using lattice fermions (Nielsen Ninomiya) unless lattice forms boundary of 5D bulk where massive fermions generate a CS term $\leftarrow$ DWF setup

Bulk boundary generalizes this to any dimension (and other symmetries)...

Is this true ? Not necessarily in presence of gravity or non-trivial spacetime topology ... see also E Berkovitz's talk on bosonic lattice anomalies (this meeting)

Counter example - Kähler-Dirac fermions on curved space

Put fermions on p-simplices in a (oriented) triangulation
$\Phi = (\Phi_{\rm sites}, \Phi_{\rm links}, \cdots, \Phi_p, \cdots)$
Define a "derivative operator":

$$\delta\Phi(C_p) = \sum_{p-1} L(p,p-1)\Phi(C_{p-1}) \quad \text{with} \quad L(p,p-1) \in 0, \pm 1$$

$(\delta - \delta^\dagger)\Phi = 0$ - Kähler-Dirac eqn.

$U(1)$ symmetry $\Phi_p \to e^{i\alpha\Gamma}\Phi_p$, $\bar{\Phi}_p \to e^{i\alpha\Gamma}\bar{\Phi}_p$

where $\Gamma\Phi_p = (-1)^p \Phi_p$ and $\{\Gamma, (\delta - \delta^\dagger\}_+ = 0$

Onsite, exact $U(1)$ lattice symmetry

A gravitational anomaly

Take triangulation of sphere.

$$Z = \int \left(\prod_p d\bar{\Phi}_p \, d\Phi_p \right) e^{-S_{\mathrm{KD}}(\Phi,\bar{\Phi})}$$

Under $U(1)_\Gamma$

$$Z \to \int \left(e^{2i\alpha N_p \Gamma_p} \prod_p d\bar{\Phi}_p \, d\Phi_p \} \right) e^{-S_{\mathrm{KD}}(\Phi,\bar{\Phi})}$$

$$= e^{2i\alpha(N_0 - N_1 + N_2 - \ldots + N_d)} Z = e^{2\chi\alpha i} Z \leftarrow \textit{anomaly}!$$

On sphere $\chi = 2 \; U(1) \to Z_4$

Measure violates symmetry ! onsite lattice symmetry can be anomalous ... exact zero modes from spacetime topology

Discrete Kähler fermions = staggered fermions

Another important connection ...

Known since 1980's that (discrete) Kähler fermions are equivalent to staggered fermions in flat space.

Square lattice

$$\Phi_{\rm site} = \chi(x)$$
$$\Phi_{\text{link dir }\mu} = \chi(x+\mu)$$
$$\Phi_{\text{plaquette }\mu\nu} = \chi(x+\mu+\nu)\ldots$$

staggered phases arise from orientation and antisymmetry of forms

$$\Gamma \to \epsilon(x) \text{ and } U(1) \to U_\epsilon(1)$$

Kähler fermions are generalization of staggered to curved space

A 't Hooft anomaly for staggered fermions

$$S = \sum_{x,\mu} \bar{\chi}^a(x)\eta_\mu(x)\Delta_\mu\chi^a(x) - y^2 \sum_x (\bar{\chi}^a\chi^a)^2 \quad a = 1, N$$

Invariant under $SU(N)$ symmetry $\chi \to e^{i\beta.T}\chi \quad \bar{\chi} \to e^{-i\beta.T}\bar{\chi}$
Plus Z_4 symmetry:

$$\chi(x) \to \omega^k\chi(x) \quad \text{and} \quad \bar{\chi}(x) \to \omega^k\bar{\chi}(x) \quad \omega = i\epsilon(x), k = 0\ldots3$$

Imagine gauging the Z_4 symmetry. Insert Z_4 gauge links - action invariant. But measure transforms by factor

$$\prod_x \left[\omega^2(x)\right]^N$$

Anomaly ! Z invariant iff $N = 2k$

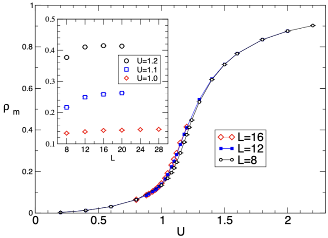
Symmetric Mass Generation

New paradigm for generating mass without breaking (chiral) symmetries ... well established in low dimensions but $D = 4$?
All fermion bilinears are zero and instead a symmetric four fermion condensate forms

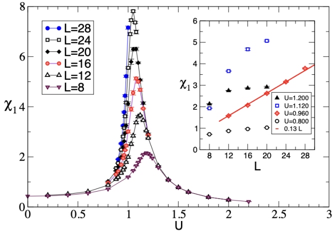
To gap states without breaking symmetries it is necessary to cancel all 't Hooft anomalies including this Z_4

Such SMG phases have been observed in $D \leq 3$

Symmetric Mass Generation in 3D



(a) $\langle \bar{\psi}_1 \psi_1 \bar{\psi}_2 \psi_2 \rangle$



(b) $< \sum_x \bar{\psi}^1(x)\psi^1(x)\bar{\psi}^2(0)\psi^2(0) >$

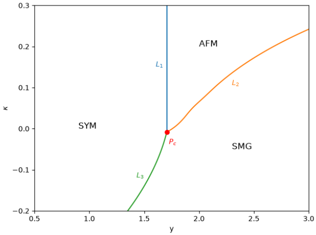
Figure: Ayyar & Chandrasekharan, PRD 91 (2015) 065035

No order parameter for phase transition !

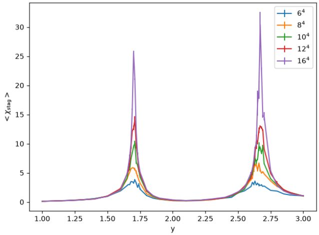
Also recently Banerjee, Chandrasekharan, Marinkovic, 2512.24836: SMG transition identified with multicritical point in larger parameter space (Marinkovic this meeting)

A 4D SMG phase ?

Lattice Higgs model: $S = S_{\rm stag} + iy\sum_x \bar{\chi}\sigma\chi - \kappa\sigma\Box\sigma + \sigma^2 + \ldots$
At P_c all bilinears vanish but $< \psi\psi\psi\psi > \neq 0$ and χ diverges. See G. Hartshaw's talk.



(a) Phase diagram



(b) $\chi_{\rm stag}$ at $\kappa = 0.2$

Figure: Anti-ferromagnetic bilinear and its susceptibility vs y at $\kappa = 0.2$

Also A Hasenfratz et al. for same model at $y = 0$ when $SU(2)$ symmetry is gauged (PRL134 (2025) 3 031602)

What is SMG (potentially) good for ?

- New mechanism for mass generation. May be useful in model building eg partial compositeness.
- Dark baryons without dark pions ?
- Mirror models for chiral lattice gauge theories .. gap all R handed mirrors without breaking chiral symmetries of target L handed fields. DWF constructions ...?
- Minimal model of (reduced) staggered fermions that is free of the anomaly in Z_4 is Pati-Salam GUT !

Summary

Lots of exciting lattice work on BSM theories !

- Minimal composite Higgs models based on Sp gauge theories well motivated and should yield useful input to model builders in the near future.
- Composite dark matter models while appealing are less constrained but current work helps to flesh out possibilities
- Progress in lattice SUSY. Holographic connections to quantum gravity being explored.
- Substantial progress on DT approaches to quantum gravity and recent efforts to make predictions for cosmology.
- Lattice theories can exhibit anomalies $\rightarrow$ theoretical understanding of the appearance of SMG phases in staggered fermions possibly also in 4D with appropriate dynamics Consequences for BSM model building and lattice chiral gauge theories ?

Backups

Reflection anomalies for 2d reduced staggered fermions

$$S = \sum_x \chi(x) \left[\eta_\mu(x)\Delta_\mu + iM(x)\right] \chi(x)$$

where $M(x) = m\xi_\mu(x)\xi_\nu(x+\mu) T_\nu T_\mu \leftarrow$ symmetric shift

Reflection symmetry: $\chi(x_0, \vec{x}) \stackrel{R_0}{\rightarrow} i\,(-1)^{x_0}\,\epsilon(x)\chi(L - x_0, \vec{x})$

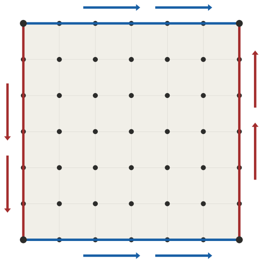
Integrate out fermions $\rightarrow Z = \mathrm{Pf}(\eta \cdot \Delta + iM)$

$$e^{i\frac{\pi}{2}\beta} = \frac{\mathrm{Pf}(-m)}{\mathrm{Pf}(m)}$$

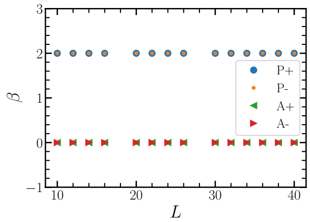
On orientable spaces reflection invariance $\rightarrow \beta = 0 \leftarrow \mathrm{Pf}$ indep of sign of mass.

Klein bottle

Instead use non-orientable lattice. Like switching on a non-trivial background gauge field for reflections



(a) Klein bottle



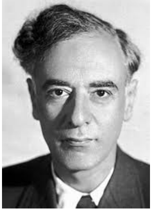
(b) β ($= 4 \times \eta$-invariant)

$\beta = 2$ implies $Z \to -Z$ as $m \to -m$ Anomaly!

Historical aside ... Landau vs Dirac

Finally, we would like to mention that Dirac [5] in a recently published work, also treats the problem from the same starting point. Both theories seem to be equivalent, apart from the complete difference in methods and equations. However, their precise connection is unclear to us. Dirac's theory, in contrast to ours, is hardly applicable to multi-electron problems.

On the theory of the magnetic electron, Landau and Iwanenko, Zeitschrift für Physik, March 1927.



$$(d - d^\dagger + m)\Phi = 0$$

Connection between Landau-Kähler and Dirac

From Kähler-Dirac field $\Phi = (\phi, \phi_\mu, \ldots)$ build matrix Ψ

$$\Psi = \phi I + \phi_\mu \gamma_\mu + \phi_{\mu\nu} \gamma_\mu \gamma_\nu + \ldots$$

Satisfies flat space Dirac equation

$$(\gamma_\mu \partial_\mu + m)\Psi = 0$$

$2^{d/2}$ columns of Ψ are (degenerate) Dirac spinors

But coupling to gravity is different ...

Reduced staggered fermions

A staggered field can be projected into 2 reduced fields

$$\chi(x) = \frac{1}{2}[1+\epsilon(x)]\chi(x) + \frac{1}{2}[1-\epsilon(x)]\chi(x)$$
$$\bar{\chi}(x) = \frac{1}{2}[1+\epsilon(x)]\bar{\chi}(x) + \frac{1}{2}[1-\epsilon(x)]\bar{\chi}(x)$$

$$S_K = \sum_x \bar{\chi}_+(x)\eta_\mu(x)\Delta_\mu\chi_-(x) + \bar{\chi}_-(x)\eta_\mu\Delta_\mu\chi_+(x) \leftarrow \text{discard}$$

Single reduced staggered/Kähler field carries 1/2 dof. Need 4 copies to cancel mod 2 anomaly.

Bulk-boundary correspondence

Partition function of massive fermions in d+1 dims is TQFT that encodes the 't Hooft anomaly for d massless fermions on boundary

Familiar in 3d. Integrate out massive fermions → Chern-Simons action i.e Z_{3d} picks up a phase
Add boundary → 2d massless edge states with chiral anomaly
Boundary anomaly cancels phase from 3d bulk - anomaly inflow.

2d reflection story similar - but anomaly cannot be seen in perturbation theory and bulk effective action cannot be written as a local Lagrangian. The massless boundary fermions experience a time reversal anomaly

Minimal anomaly free staggered model in 4d

Construct matrix representation for reduced field (in chiral basis):

$$\Psi^{4\times 4}_{\text{continuum}} = \begin{pmatrix} \psi_R & 0 \\ 0 & \psi_L \end{pmatrix} \leftarrow 1/2 \text{ dof}$$

Need 4 copies to cancel anomaly.
Spinor content:$(4, 1, 2) \oplus (\bar{4}, 2, 1)$

$$\text{Symmetry} : SU(4) \times SU_L(2) \times SU_R(2)$$

Pati-Salam GUT ! a chiral gauge theory containing the SM

if we gauge these global symmetries ...but $SU(2) \times SU(2)$ broken by lattice - only discrete subgroup - shift symmetries left ..

Shift symmetries

Staggered action invariant under shifts S_μ (offsite symmetries):

$$\chi(x) \stackrel{S_\mu}{\to} \xi_\mu(x)\chi(x+\mu), \quad \xi_\mu(x) = (-1)^{\sum_{\nu=\mu+1}^{d} x_\nu}$$

Shifts along orthogonal directions can be combined and (almost) form discrete group

$$G = (\pm I, \pm S_\mu, \pm i S_\mu S_\nu, \pm i S_\mu S_\nu S_\lambda, \ldots) \quad 2^{d+1} \text{ elements}$$

Almost means eg. $S_\mu S_{\mu\nu} = S_\nu T_\mu$ where T_μ is translation by two lattice spacings.

Shift symmetries correspond to a discrete subgroup of continuum flavor transformations. They ensure restoration of $SU(2^{d/2})$ flavor symmetry in continuum limit.

Mixed 't Hooft anomalies

However ..

$$[S_\mu, R_\mu] \neq 0$$

Thus shifts break when reflections are gauged $\leftarrow$ put theory on non-orientable manifold

Mixed 't Hooft anomaly $\leftarrow$ I.R non-trivial ... SSB ?

non-zero lattice anomalies server as precursor to continuum anomalies and predict SSB - here chiral symmetry