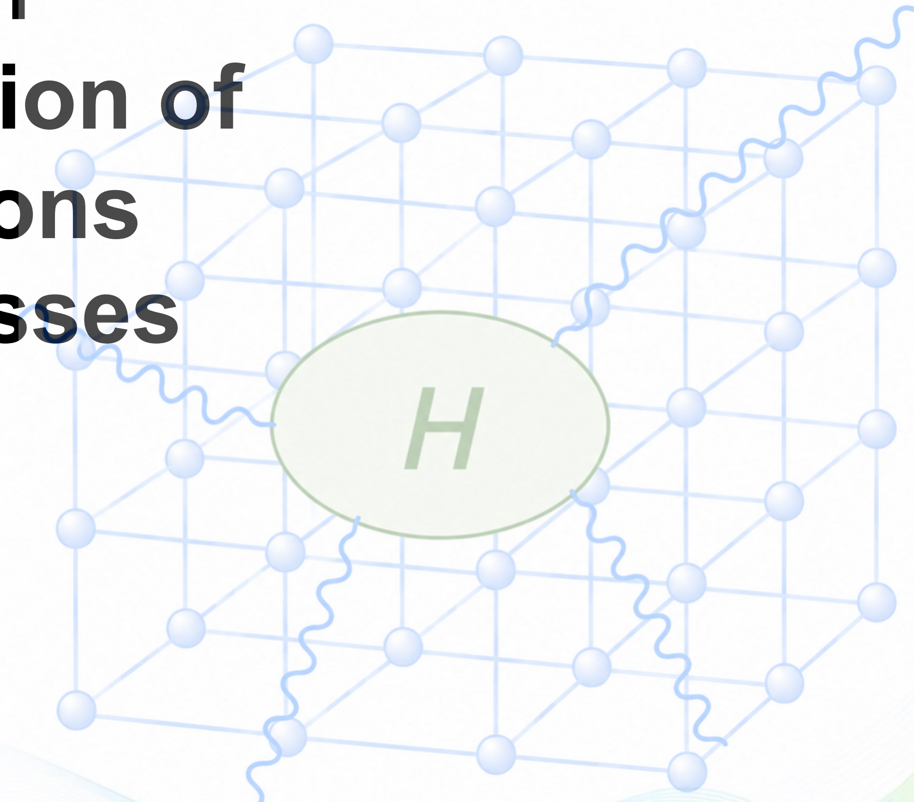
Recent Progress in Lattice Determination of Radiative Corrections in Hadronic Processes



Xin-Yu Tuo

Brookhaven National Laboratory

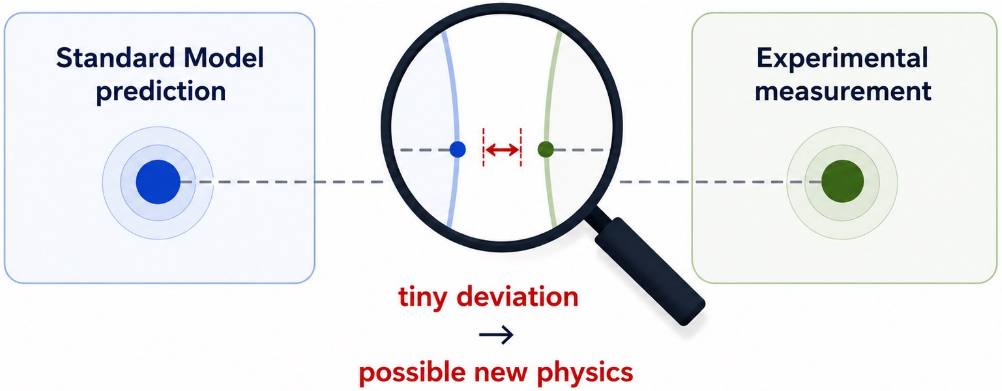
The 43rd International Symposium on Lattice Field Theory (Lattice 2026)

University of Maryland, College Park

July 26 – August 1, 2026

Why radiative corrections (RCs) matter

Precision frontier: indirect probe of new physics



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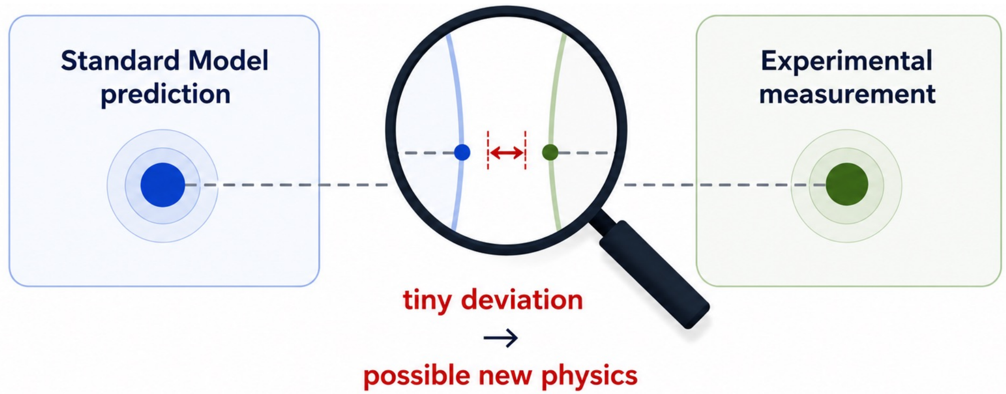

Key challenge: radiative corrections (RCs)

small

$$\alpha_{QED} \approx \frac{1}{137} \approx 0.7\%$$

but important

precision frontier enters the sub-percent era

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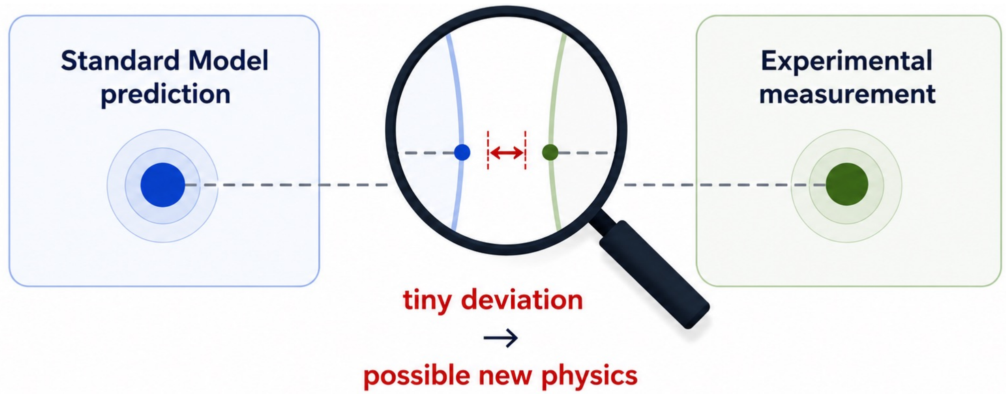
Precision frontier: indirect probe of new physics



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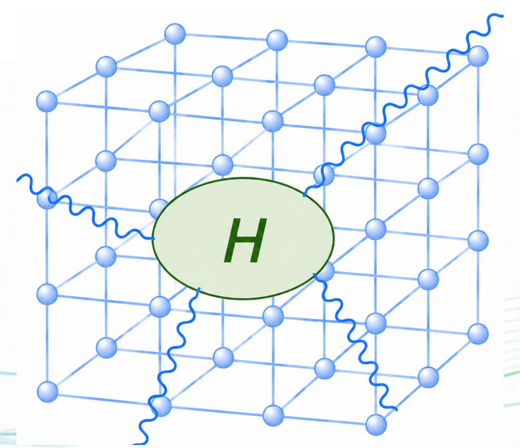
precision frontier enters the sub-percent era

QED coupled to nonperturbative QCD

→

major source of theory uncertainty



first-principles lattice QCD+QED

RCs: precision tests of the SM flavor structure

$$\frac{\text{precision } \sim \boldsymbol{O}(\mathbf{0.1\%})}{\text{RC size } \sim \boldsymbol{O}(\mathbf{1\%})}$$

first row's CKM unitarity: 2~3$\boldsymbol{\sigma}$ tension

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9983(6)_{V_{ud}}(4)_{V_{us}}$$

Particle Data Group, PRD 110 (2024) 030001

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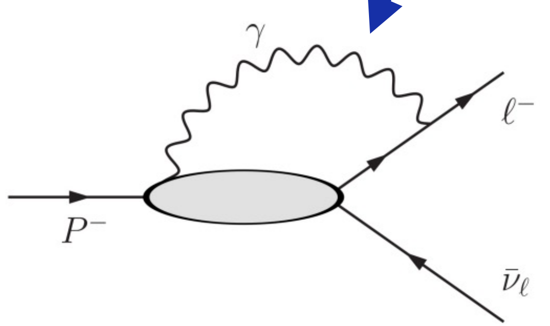

RC in $K_{\mu 2(\gamma)}/\pi_{\mu 2(\gamma)}$

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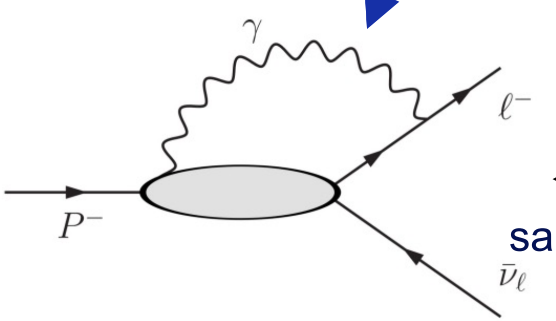
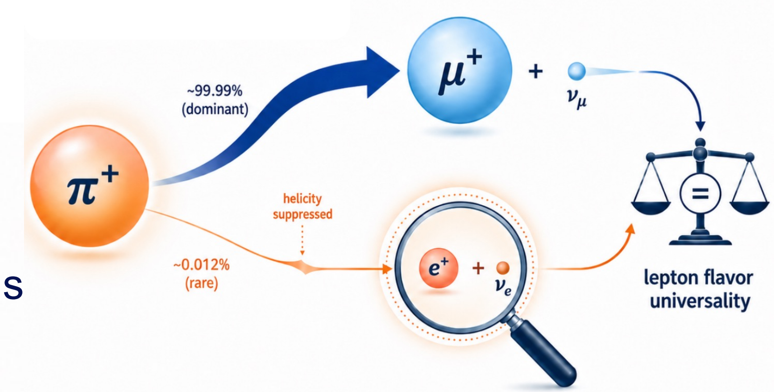
lepton-flavor universality



same RC diagrams



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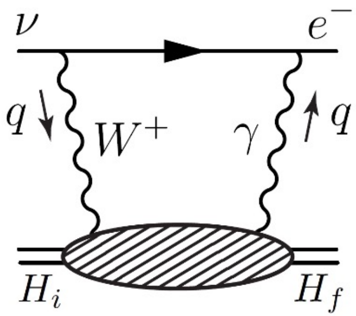

RC in $0^+ \to 0^+$ β decays

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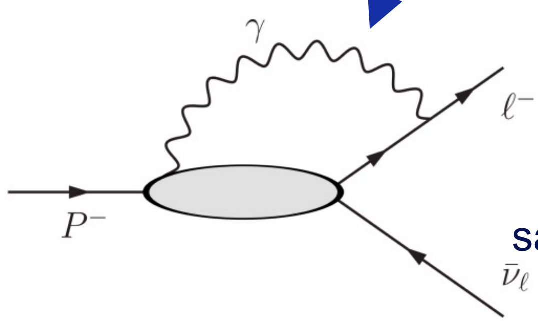

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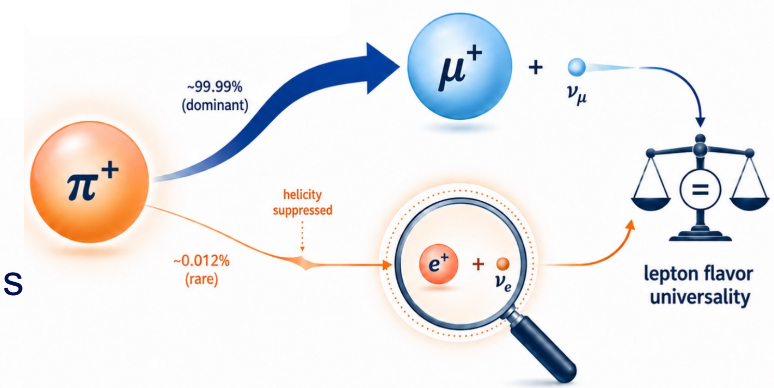

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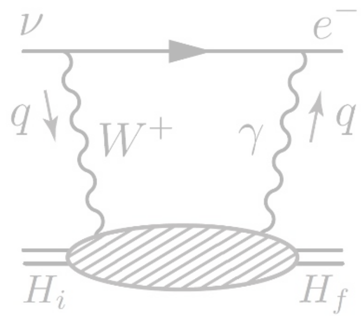
RC in $K_{\mu 2}/\pi_{\mu 2}$ and QED obstacle



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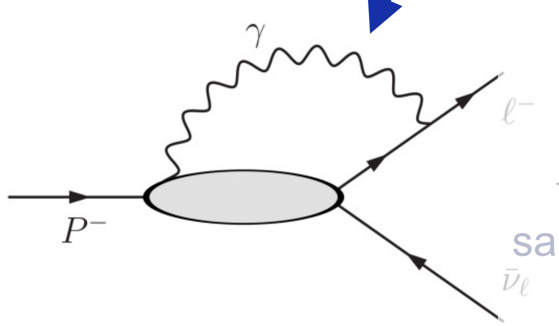

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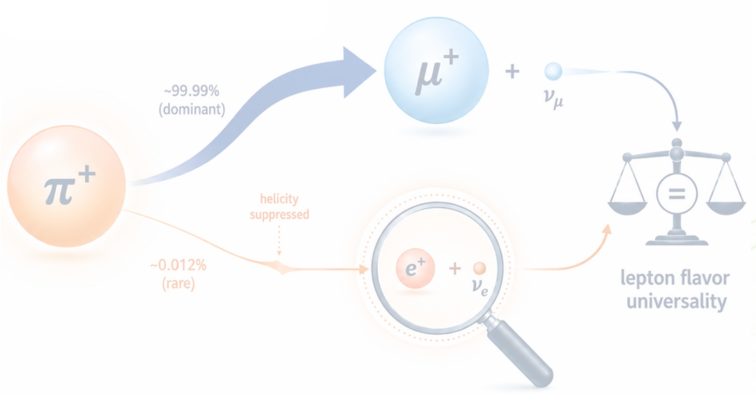

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Previous lattice results for isospin-breaking correction $\delta R_{K\pi}$

QED$_L$, ETMC/RM123

$$\delta R_{K\pi} = \delta R_K - \delta R_\pi = -0.0126(14)$$

D. Giusti *et al.*, PRL 120 (2018) 072001
M. Di Carlo *et al.*, PRD 100 (2019) 034514

QED$_L$, RBC & UKQCD

$$\delta R_{K\pi} = -0.0086(3)_{\text{stat}}\left(^{+11}_{-4}\right)_{\text{fit}}(5)_{\text{disc.}}(5)_{\text{quench.}}(39)_{\text{vol.}}$$

P. Boyle *et al.*, JHEP 02 (2023), 242

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	$O(1/L)$	$O(1/L^2)$	$O(1/L^3)$
$\delta_{\rm FV}^{{\rm QED}_L}/0.01$	-46.8%	$+26.2\%$	-39.3%

Obstacle: long-range QED
massless photon
→ large power-law finite-volume effects

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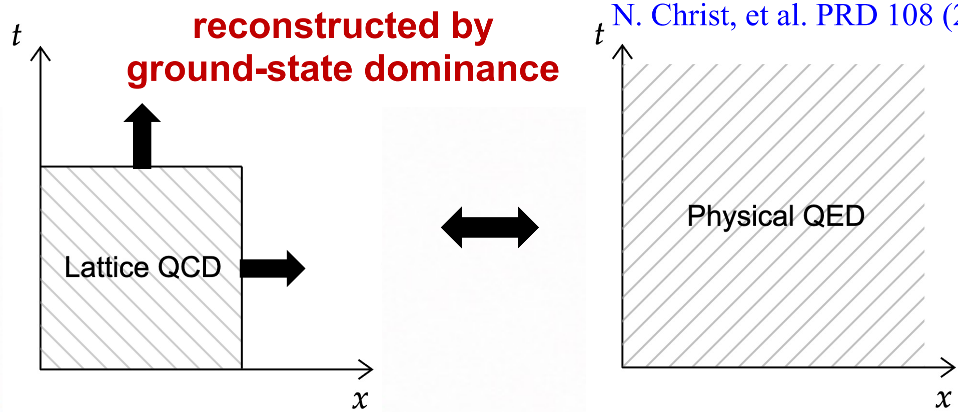
Infinite-volume reconstruction (IVR)

X. Feng, L. Jin. PRD 100 (2019) 094509
N. Christ, et al. PRD 108 (2023) 014501

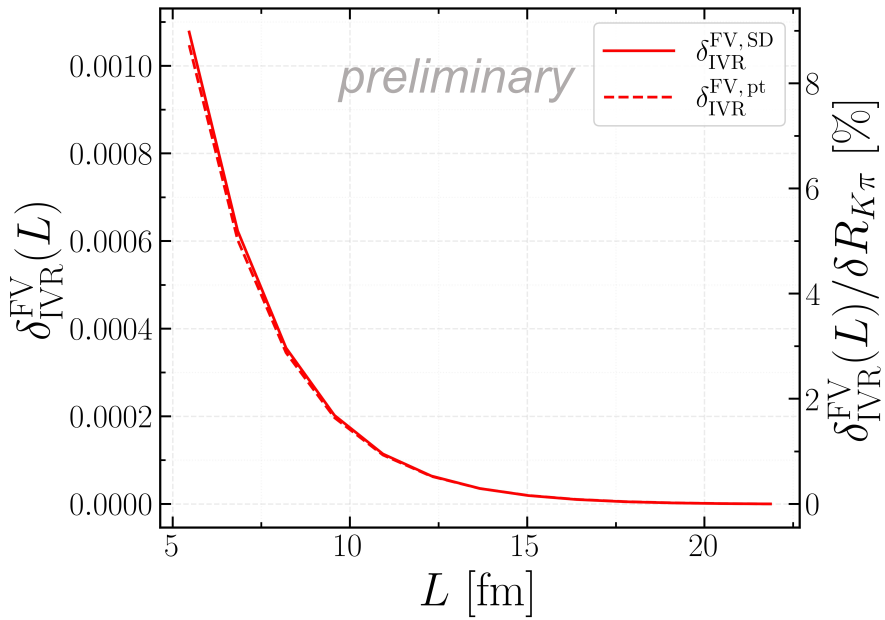


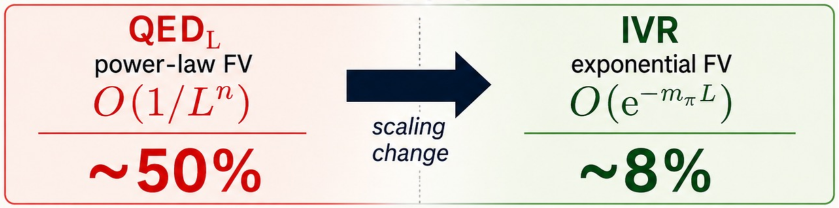
IVR: $O(1/L) \rightarrow O(e^{-mL})$

IVR result: FV-scaling changes

P. Boyle, N. Christ, X. Feng, T. Izubuchi, L. Jin, C. Sachrajda, and X. Tuo. manuscript in preparation (2026)

convergence of $O\left(e^{-mL}\right)$ FV effects





Estimation based on $L \sim 5.5$fm.
Residual $\sim$ 8% FV error is corrected by the point-particle approximation.

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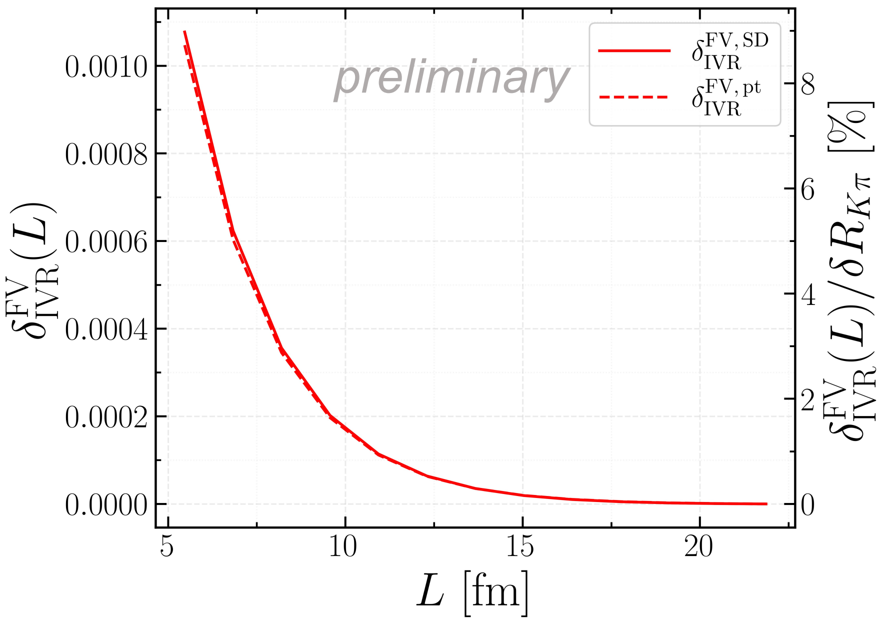
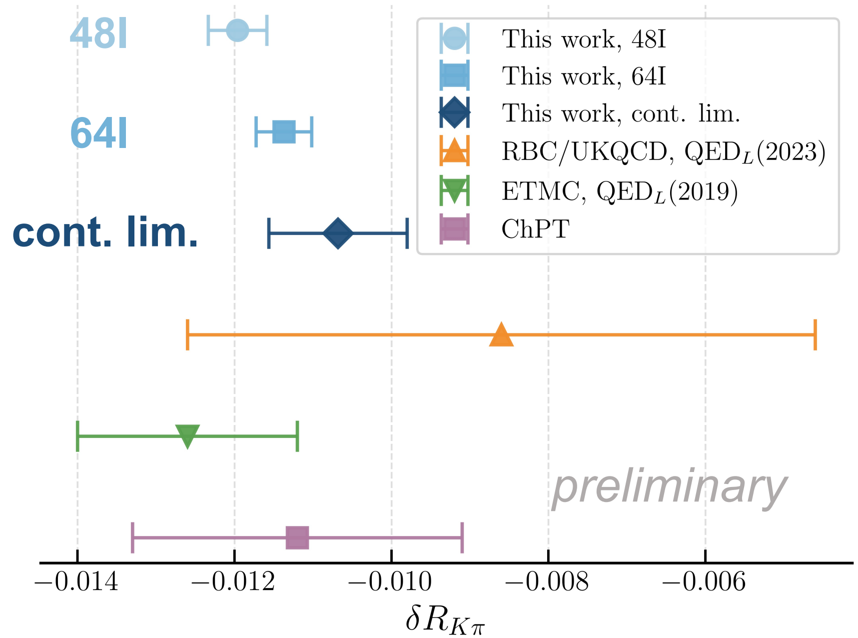
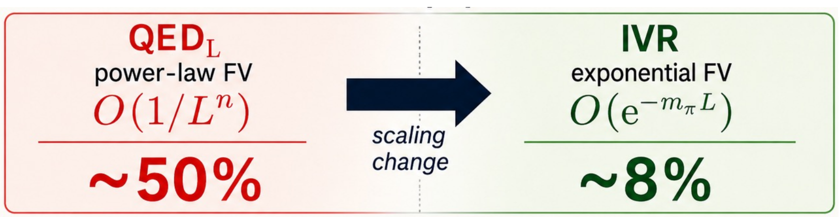


IVR result and comparison



M. Di Carlo, et al. PRD 100 (2019), 034514
P. Boyle, et al. JHEP 02 (2023), 242
V. Cirigliano, H. Neufeld, PLB 700 (2011), 7-10



$$\frac{|V_{us}|}{|V_{ud}|} = 0.23179(28)_{\mathrm{exp}}\boxed{(11)_{\delta R_P}}(65)_{f_P^{\mathrm{iso}}}$$

subdominant error

Estimation based on $L \sim 5.5$fm.
Residual ~ 8% FV error is corrected by the point-particle approximation.

Related processes: $\pi/K \to \ell^+\nu_\ell\gamma$

Tensions in lattice and experimental studies:

R. Frezzotti, et al. PRD 103 (2021) 053005
A. Kobayashi, et al. PLB 843 (2023) 138020

$\boldsymbol{\pi \to e\nu\gamma}$, **$5\sigma$ discrepancy**
lattice (ETMC) v.s. PIBETA exp.

$\boldsymbol{K \to e\nu\gamma}$, **$4\sigma$ discrepancy**
KLOE exp. v.s. E36 exp.

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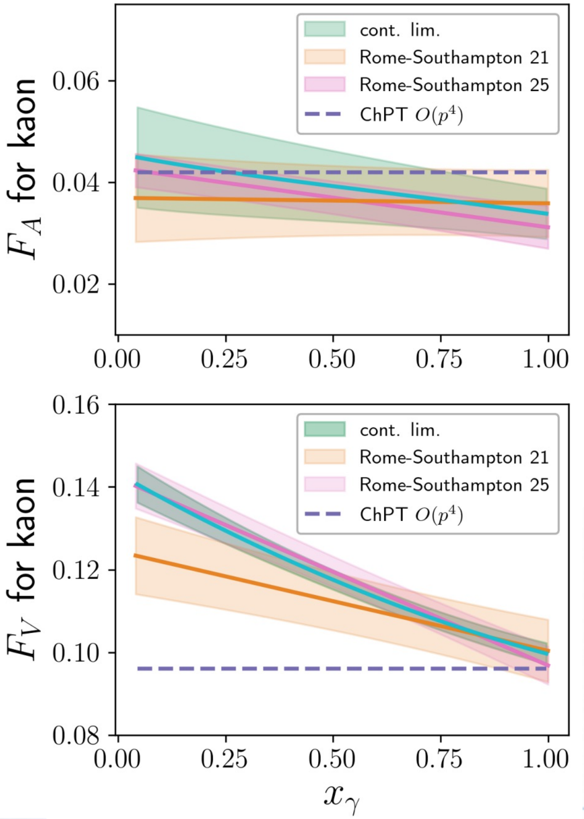
consistency: form factors



green
RBC & UKQCD

orange
ETMC 21

pink
ETMC 25

P. Boyle, N. Christ, X. Feng, T. Izubuchi, L. Jin,
C. Sachrajda, and X. Tuo. PRD 114 (2026) 014504

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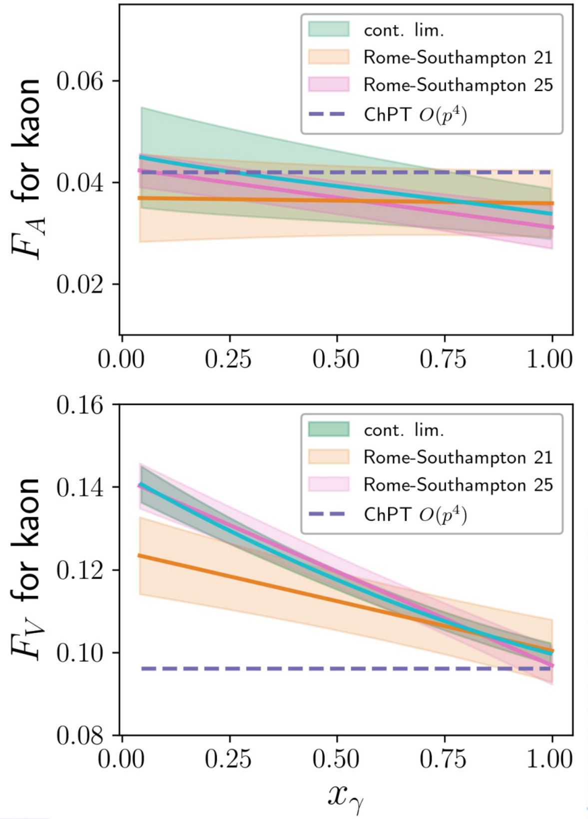

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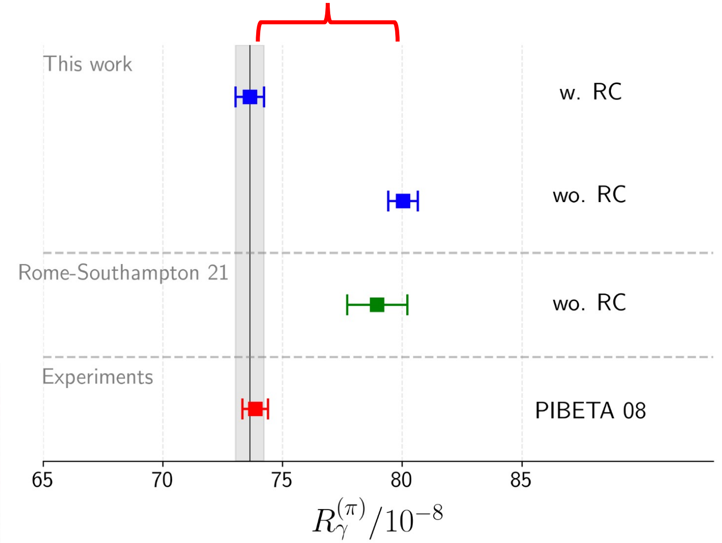
Importance of $O(\alpha^2)$ collinear RC

enhanced by $L_e = \log \frac{m^2_{\pi/K}}{m_e^2} \sim O(10)$

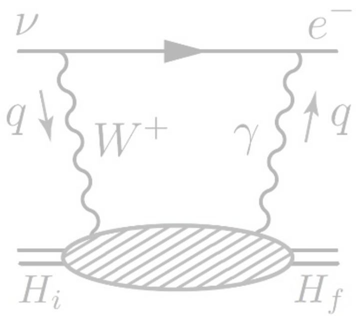
tensions in $\pi \to e\nu\gamma$: **solved**



tensions in $K \to e\nu\gamma$:
$O(\alpha^2)$ RC for KLOE and E36 are different.

NA62: new measurement with MC generator including $O(\alpha^2)$ effects.

RC in lepton-flavor-universality (LFU) ratio $R_{e/\mu}$



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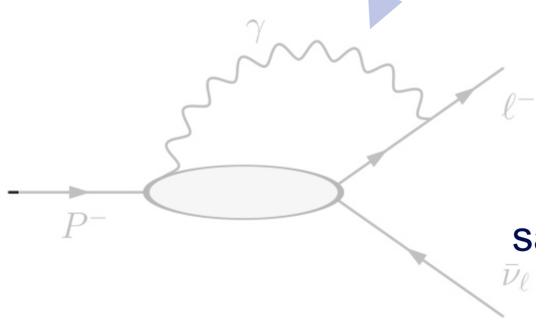

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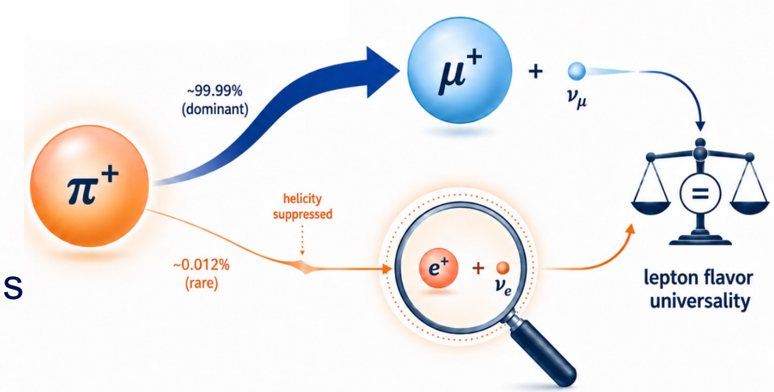

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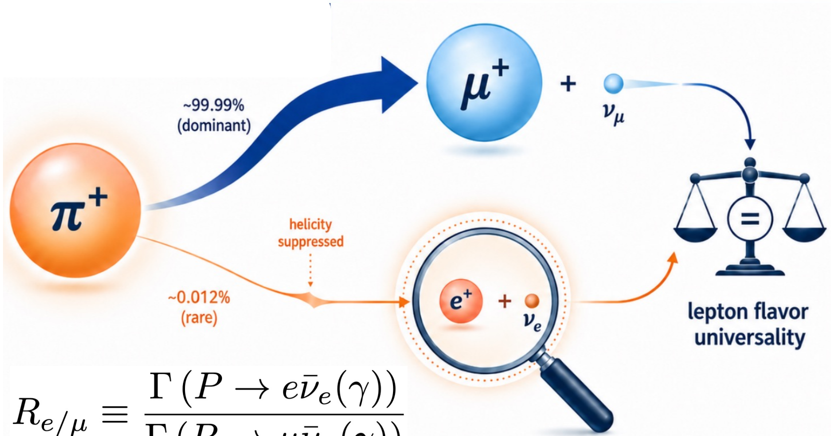

$$R_{e/\mu} \equiv \frac{\Gamma\left(P \to e\bar{\nu}_e(\gamma)\right)}{\Gamma\left(P \to \mu\bar{\nu}_\mu(\gamma)\right)}$$

ideal for precision LFU tests

helicity suppression

tiny SM rate → amplified BSM sensitivity

theoretically clean

cancellation of hadronic effects

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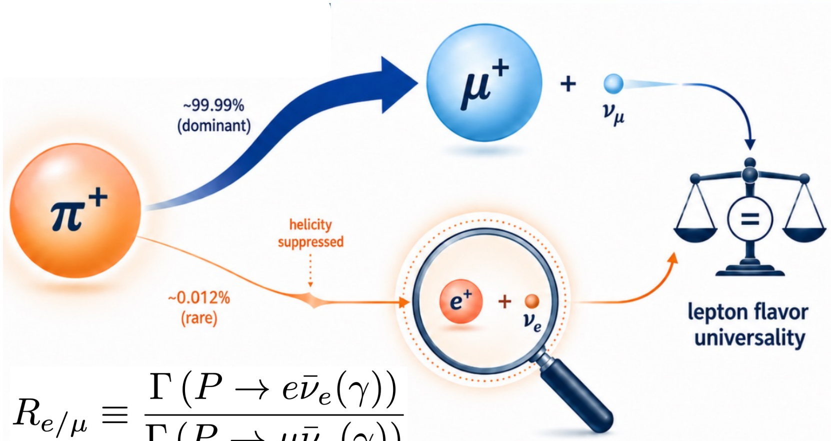

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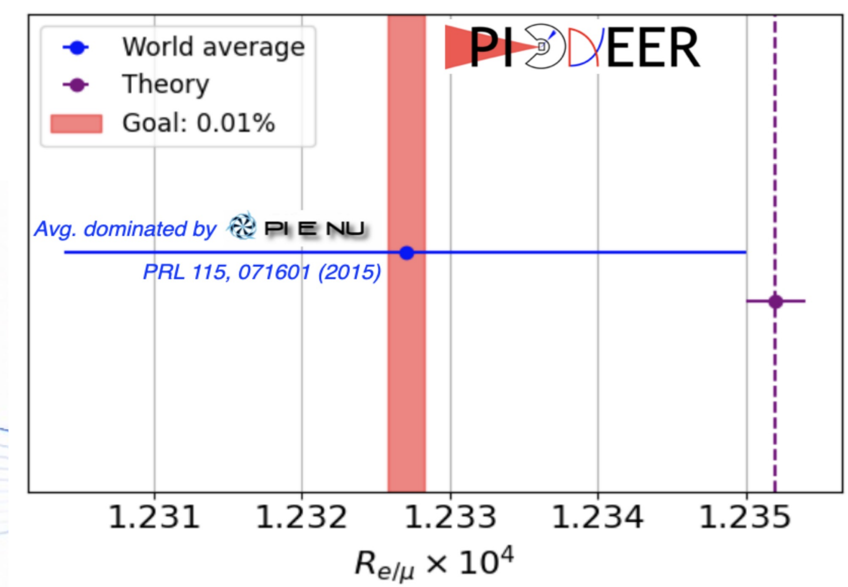
PIONEER: pushing toward 0.01% precision

A. Adelmann et al., 2025, [arXiv:2504.06375]

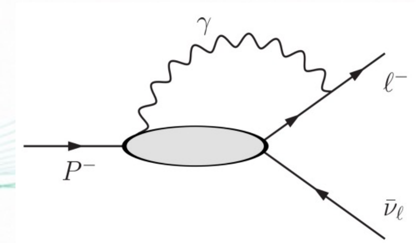


main theory bottleneck

V. Cirigliano, I. Rosell, PRL 99 (2007), 231801

Size of structure-dependent RC
(from ChPT, with error $\sim 0.01\%$)
$\sim \boldsymbol{O(0.1\%)}$

PIONEER projected precision
$\sim \boldsymbol{O(0.01\%)}$



Coulomb-Gauge IVR for a Tiny SD Signal

New challenge: much higher precision requirement

$K_{\mu 2}/\pi_{\mu 2}$: RC$\sim O(1\%)$ $\xrightarrow{10\times \text{ smaller signal}}$ $R_{e/\mu}$: SD RC$\sim O(0.1\%)$

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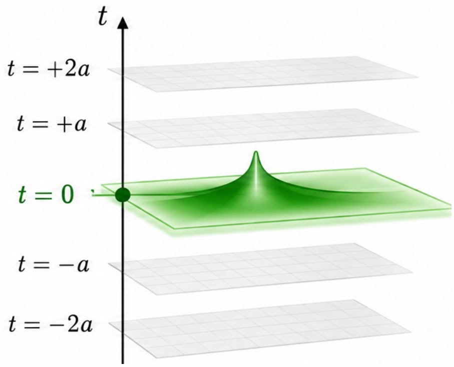
Novel method: IVR with Coulomb-gauge photons

suppresses soft-photon/long-distance region contribution → smaller FV effects

$$S^{\mathrm{Coul}}_{\sigma\rho}(k) = \frac{i}{\vec{k}^2} n_\sigma n_\rho + \frac{i}{k^2 + i\epsilon} \sum_{\lambda=1,2} \varepsilon_\sigma(k,\lambda)\varepsilon^*_\rho(k,\lambda)$$

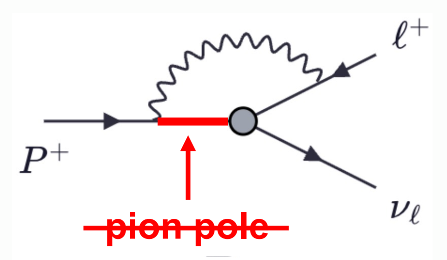
Coulomb potential instantaneous interaction



$$\frac{\delta(t)}{4\pi|\mathbf{x}|}$$

no on-shell soft photon

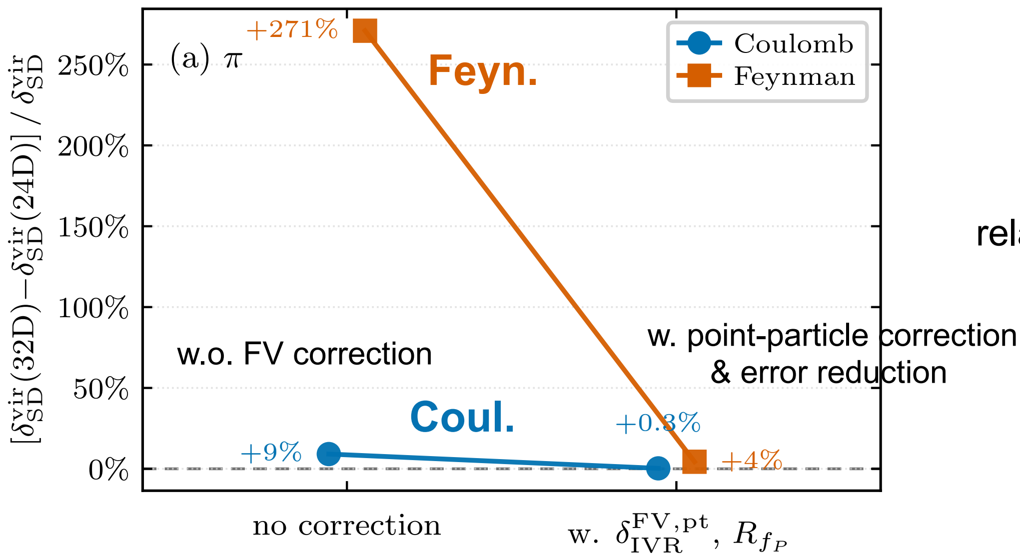
Transverse photons do not see the pion pole



no $1/(p \cdot k)$ soft enhancement

P. Boyle, N. Christ, X. Feng, T. Izubuchi, L. Jin, C. Sachrajda, and X. Tuo, 2026, [arxiv:2607.22358]

First lattice QCD+QED Determination of $R_{e/\mu}$



Comparison of FV effects

relative difference between two volume

(24D $m_\pi L = 3.3$, 32D $m_\pi L = 4.5$)

P. Boyle, N. Christ, X. Feng, T. Izubuchi, L. Jin,
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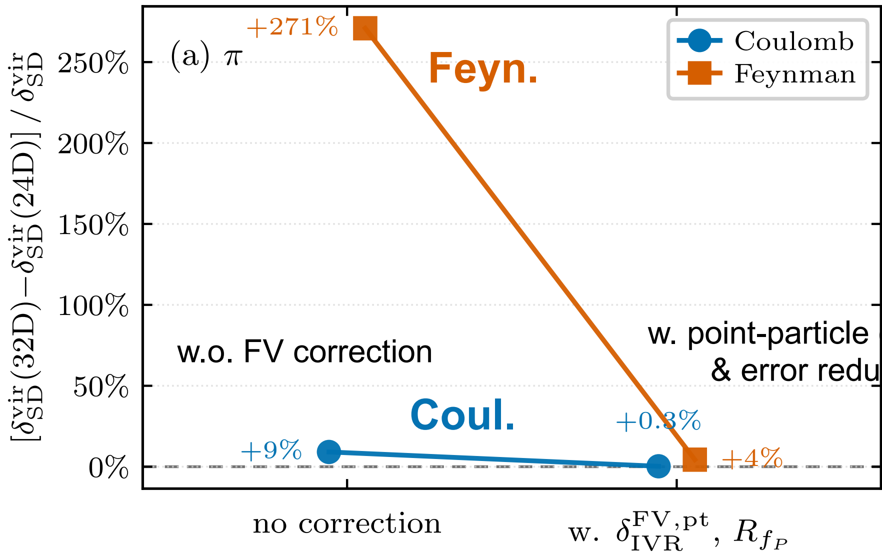

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Only $\sim \boldsymbol{O(1\%)}$ FV errors in Coulomb-gauge IVR!

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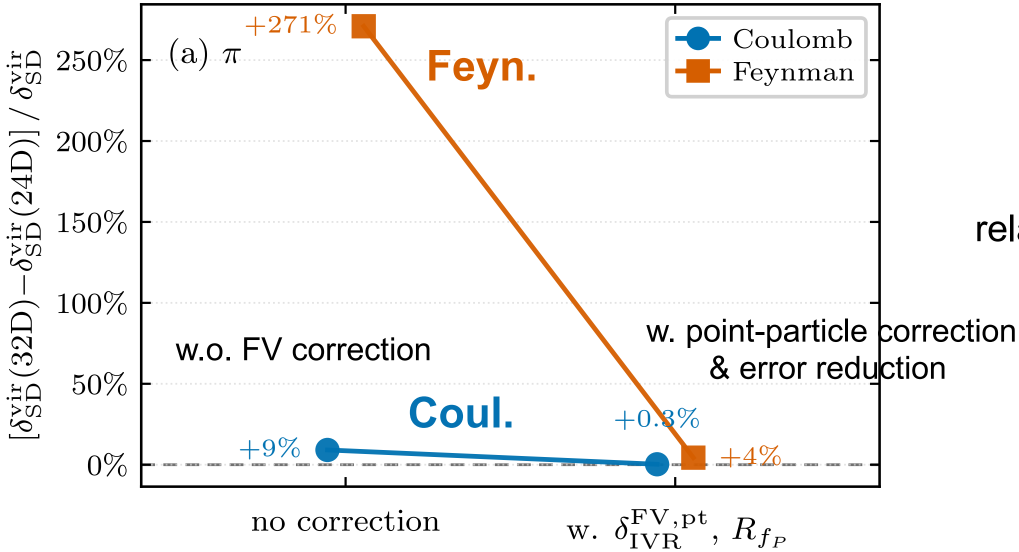

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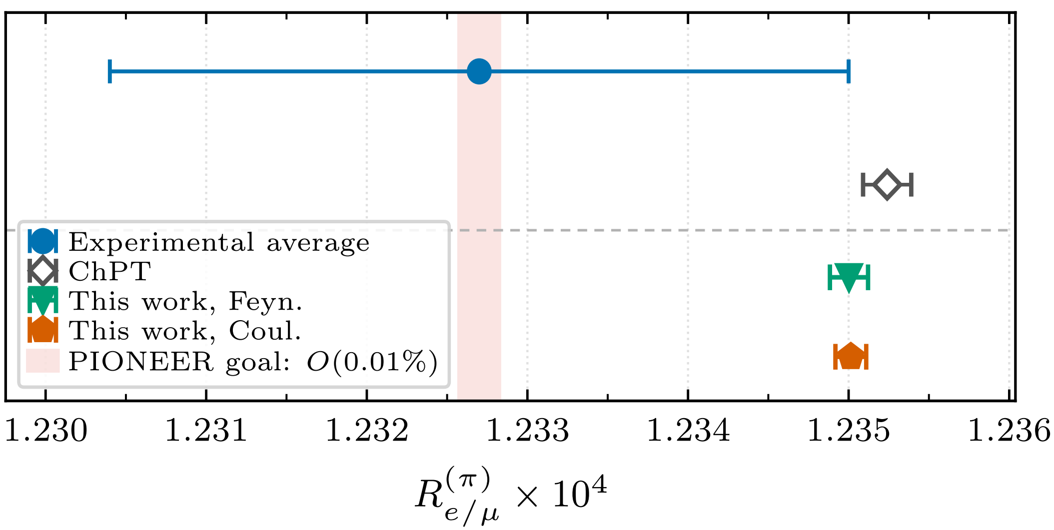
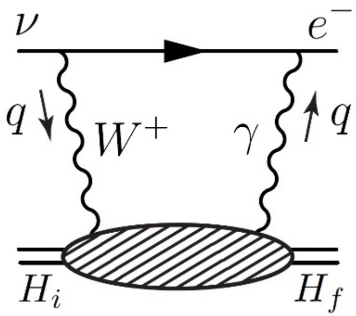


- most precise SM prediction to date
- independent check of ChPT: 1.3σ shift

P. Boyle, N. Christ, X. Feng, T. Izubuchi, L. Jin, C. Sachrajda, and X. Tuo, 2026, [arxiv:2607.22358]

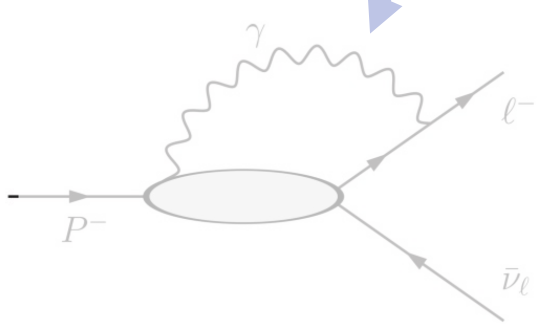
γW box diagram and impact on $|V_{ud}|$



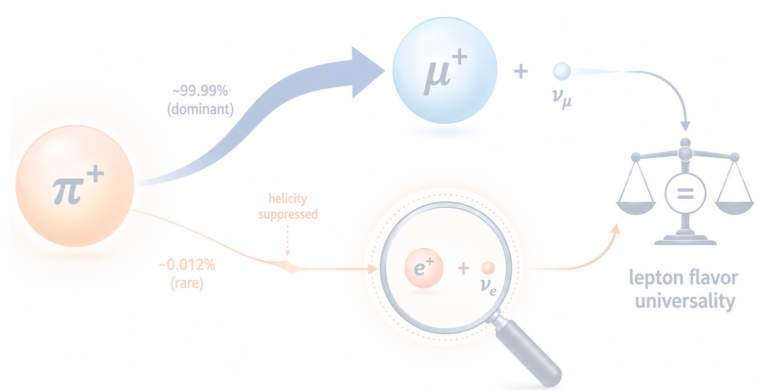
RC in $0^+ \to 0^+$ β decays

$$\boxed{|V_{ud}|^2} + \boxed{|V_{us}|^2} + |V_{ub}|^2 = 0.9983(6)_{V_{ud}}(4)_{V_{us}}$$

lepton-flavor universality



same RC



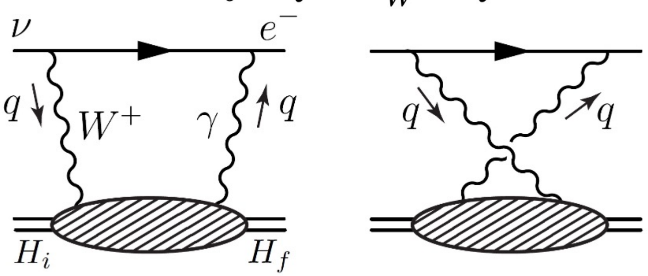
RC in $K_{\mu 2(\gamma)}/\pi_{\mu 2(\gamma)}$

RC in $\pi_{e2(\gamma)}/\pi_{\mu 2(\gamma)}$

γW box diagram and impact on $|V_{\mathrm{ud}}|$

Important uncertainty from γW box diagram

$$\Box_{\gamma W}^{VA} = \frac{3\alpha_e}{2\pi}\int \frac{dQ^2}{Q^2}\frac{m_W^2}{m_W^2+Q^2}M_n(Q^2)$$



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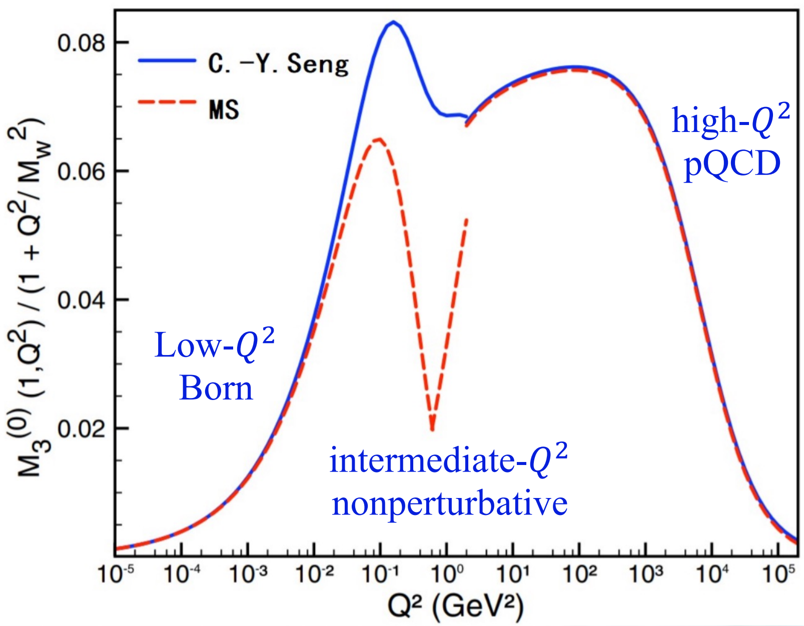


2006: VMD/large-N_C interpolation → **2018:dispersive analysis**

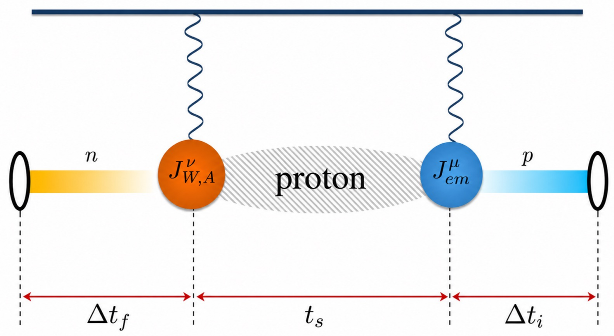
	PDG 2019		PDG 2020
V_{ud}	0.97420(21)	→	0.97370(14)
Δ_{CKM}	-0.00061(47)	→	-0.00149(45)

Marciano & Sirlin, PRL96, 032002 (2006)
Seng et.al. PRL 121, 241804 (2018)

IVR improves temporal convergence

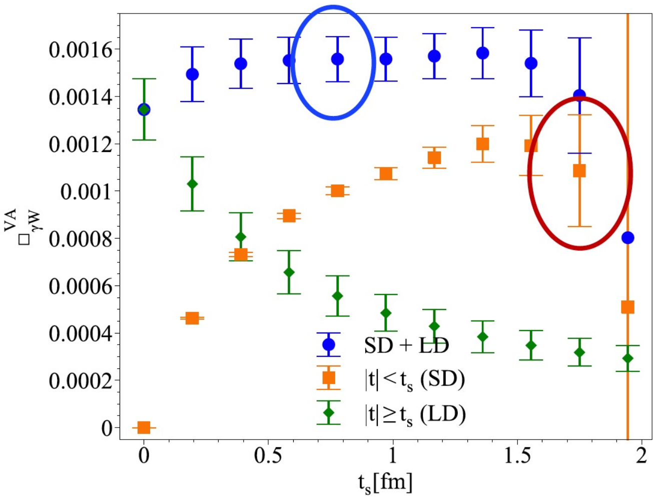
Difficulty

- slow convergence with t_s
- noise-to-signal ratio grows rapidly with t_s



IVR

P. Ma, et al. PRL 132 (2024) 191901



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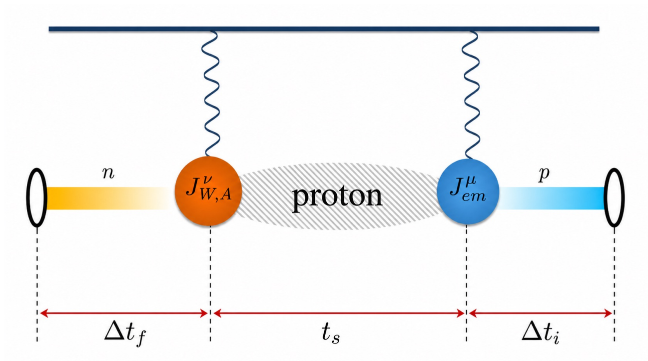

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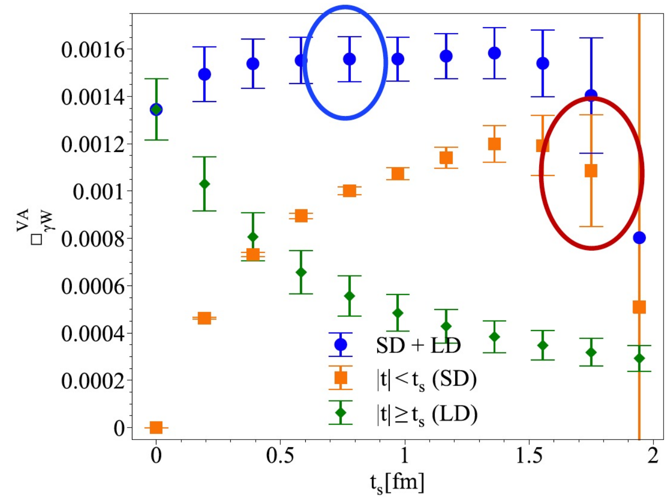
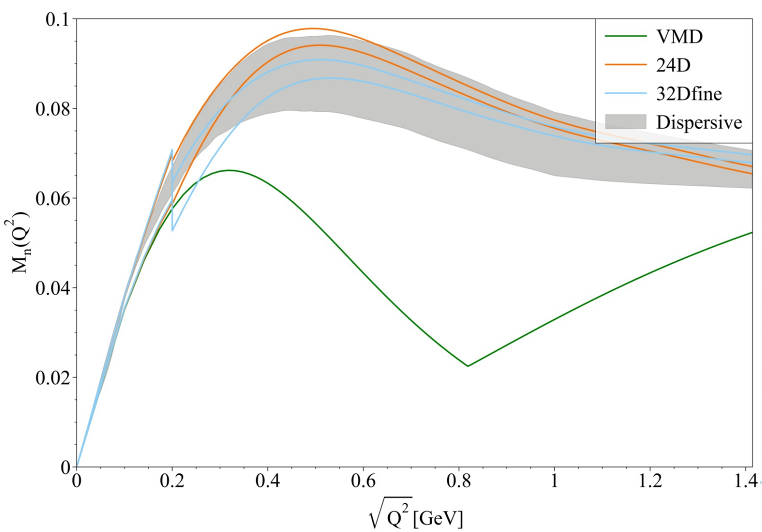


Lattice result and comparison



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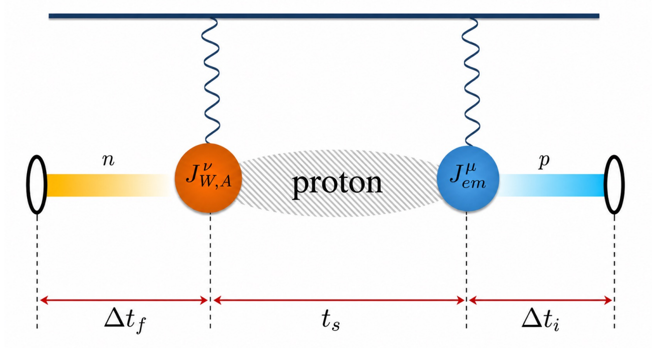

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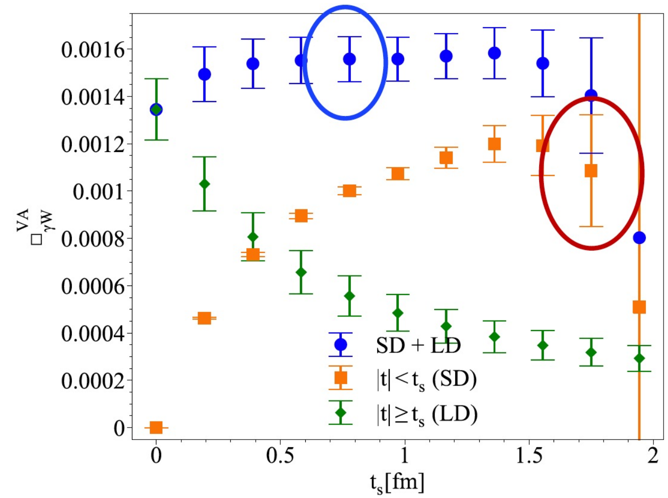

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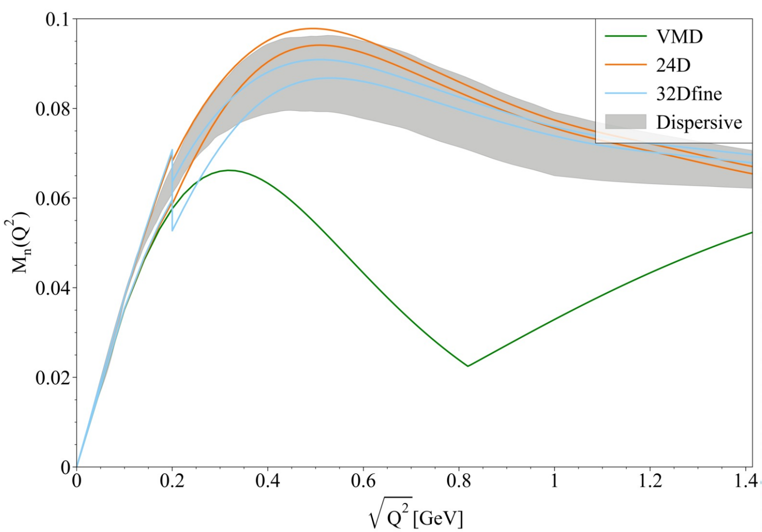
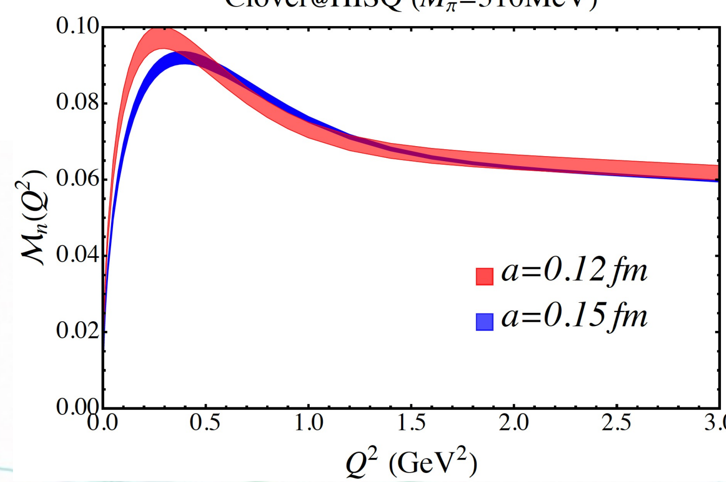


Independent calculation

by Fangcheng He, et al.

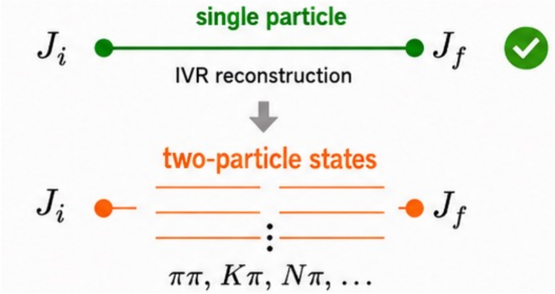
Clover@HISQ (M_π=310MeV)



Talk by Rajan Gupta (Monday 2:20pm)

Generalization: two-particle intermediate states

Obstacle: light intermediate states
Euclidean correlator ≠ physical amplitude



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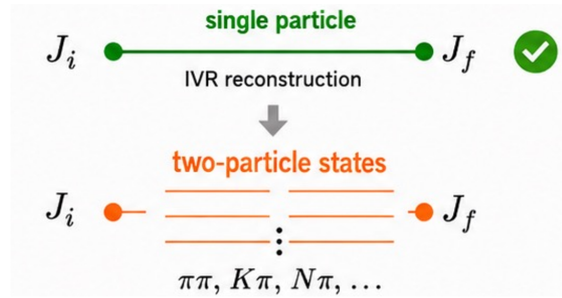
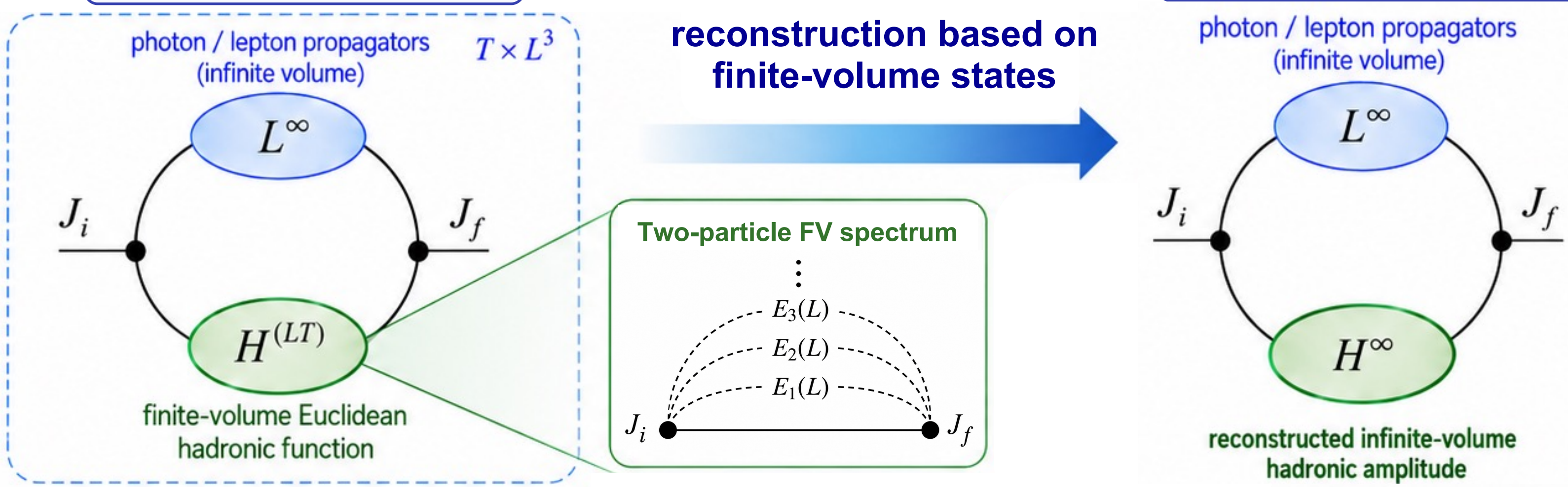


lattice calculation

reconstruction based on finite-volume states

physical amplitude



Infinite-volume EW kernel:
no power-law FV effect
from photon/lepton propagators

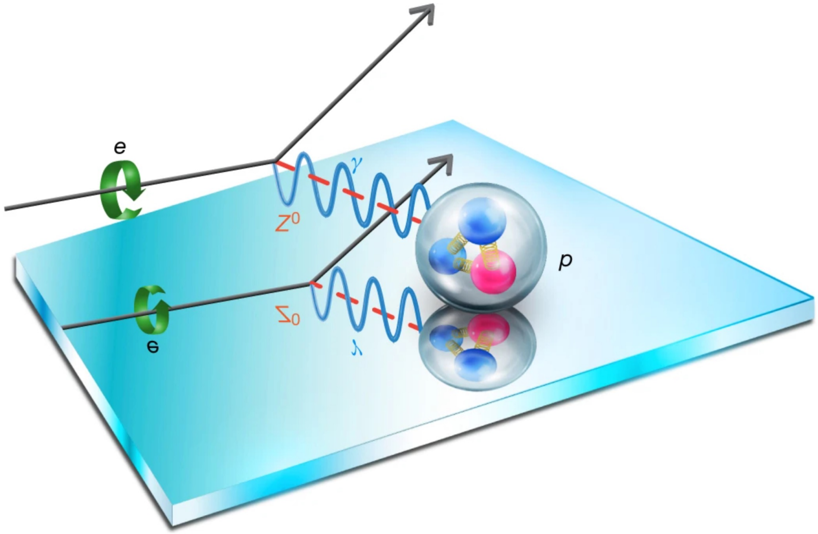
Light two-particle states:
power-law FV correction
+ exponentially growing terms

X. Tuo, X. Feng. PRD 112 (2025) 034512

The γZ box: precision meets the $N\pi$ threshold

Parity-violating ep scattering

D. Androić, et al. (Qweak Collaboration), Nature 557 (2018) 7704, 207



$e_L p$ vs. $e_R p$ → Q_W^p proton weak charge; $\sin^2\theta_W$ weak mixing angle

Tree level, $E = 0$:

$$Q_W^p = 1 - 4\sin^2\theta_W \approx 0.05$$

accidentally small

enhanced sensitivity to new physics

relative large electroweak radiative corrections (EWRC)

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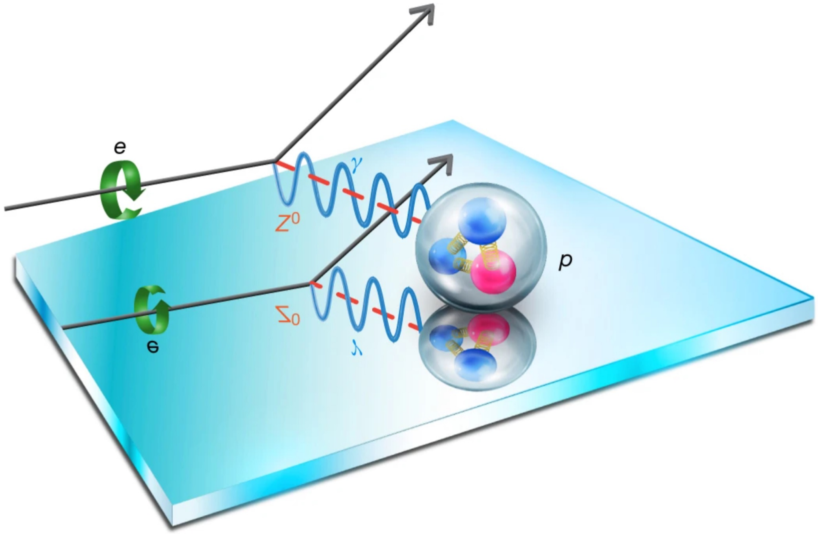

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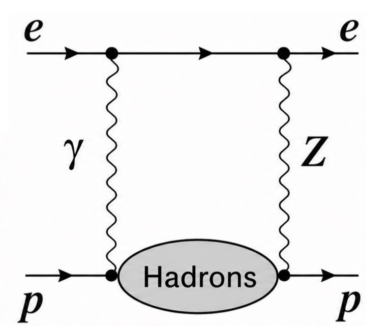
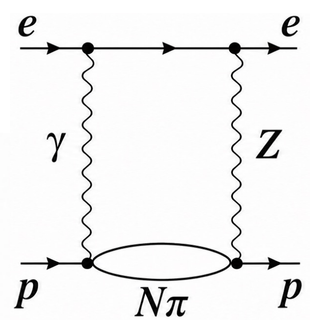
γZ box: nonperturbative EWRC

future P2 experiment at Mainz beam energy

$$E = 155\ MeV > E_{N\pi}^{th} = 150 MeV$$

D. Becker, et al. Eur.Phys.J.A 54 (2018) 11, 208





above $N\pi$ threshold

inelastic $N\pi e$ state

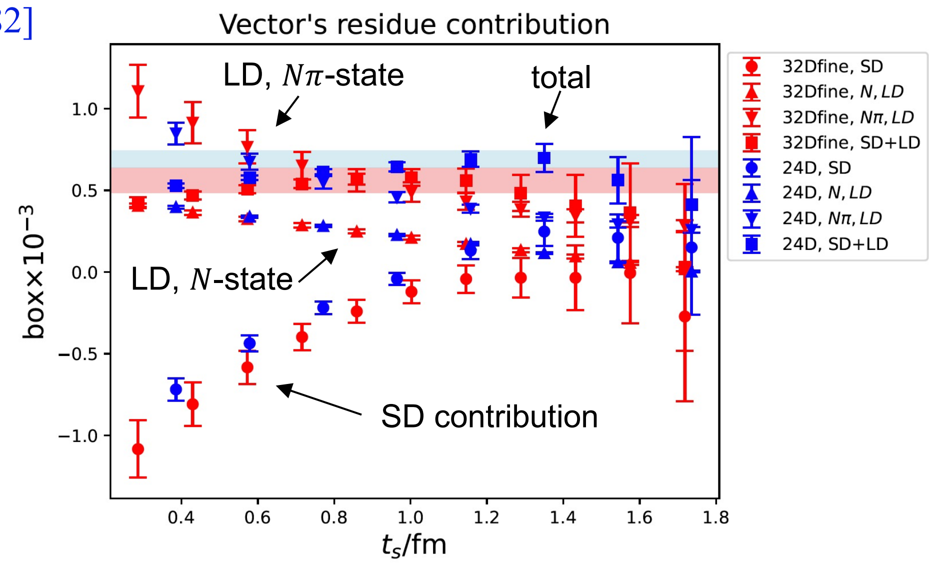
Reconstructing $N/N\pi$ contribution in the γZ box

Z. Zhang, et al., 2026, [arxiv: 2601.06582]

reconstructing N and $N\pi$ contribution

$N\pi$ inputs: finite-volume spectrum and matrix elements $\langle N\pi|J|N\rangle$ from

Y. Gao, Z. Zhang, et.al. PRL 134 (2025) 171904



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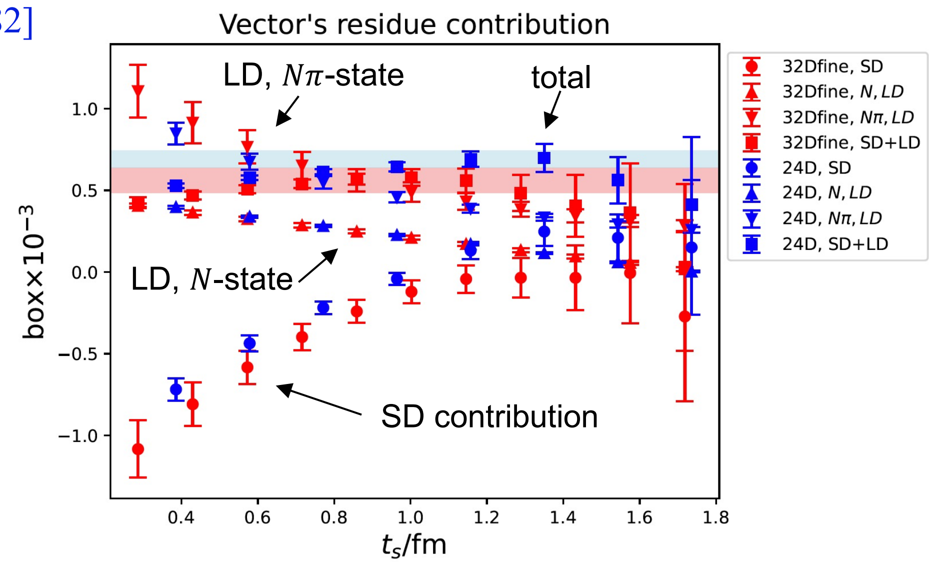
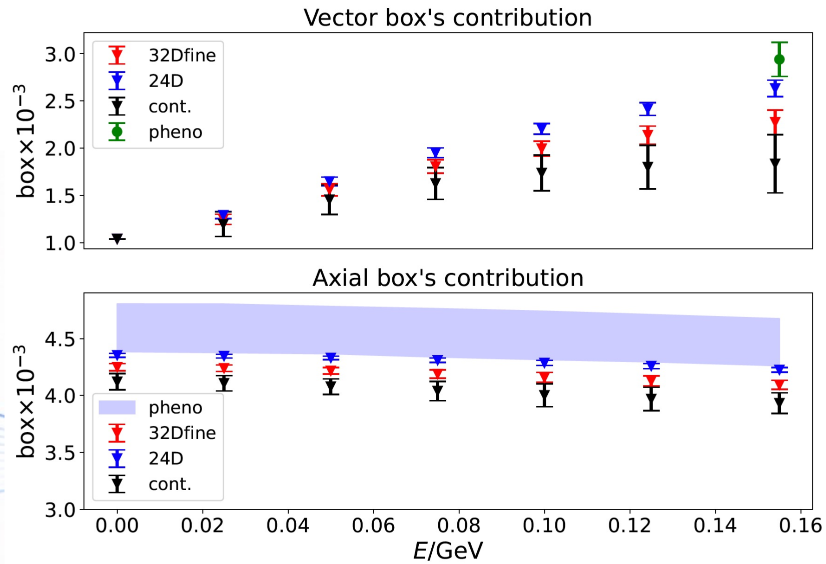


numerical results and comparison



Proton weak charge

$$Q_W^p = 0.06987(20)_{s_W^2}(10)_{WW}\boxed{(9)_{\gamma Z}}(44)_{\gamma\gamma}$$

subdominant error

"pheno": phenomenological determinations based on inclusive scattering data

M. Gorchtein, et.al. PLB 752 (2016) 135

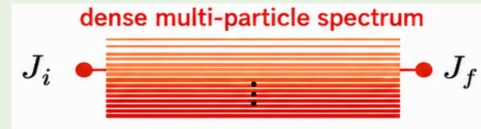
J. Erler, et. al. PRD 100 (2019) 05300

From State-by-State Reconstruction to Spectral Reconstruction

spectral function reconstruction

reconstruction from finite-volume states

Limitation: 1.expensive
2.hard to generalize to dense spectrum



D/B decays,
τ decays, $\cdots$

Talk by Shoji Hashimoto (Monday 11:00am)

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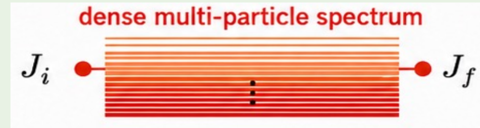
$K \to l\nu l' l'$
overlap benchmark



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light $\pi\pi$ states
$2m_\pi < \sqrt{k^2} < m_K$

Earlier studies were restricted to $2m_\pi > m_K$

X. Tuo, et.al. PRD 105 (2022) 054518

G. Gagliardi, et al. PRD 105 (2022) 114507

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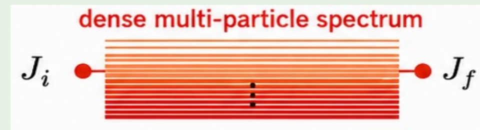

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$K \to l\nu l'l'$: successful example of SFR

R. Di Palma, et al. (2026) [arxiv: 2605.22742]
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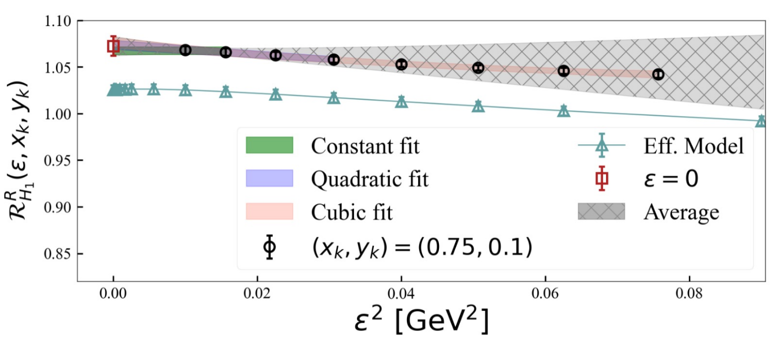

ε-smear photon propagator, extrapolate to $\varepsilon \to 0$

Talk by Francesco Sanfilippo (Tuesday 2:40pm)

From State-by-State Reconstruction to Spectral Reconstruction

spectral function reconstruction

reconstruction from finite-volume states

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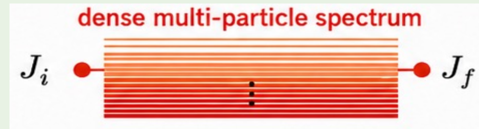

D/B decays, τ decays, ⋯

Talk by Shoji Hashimoto (Monday 11:00am)

$K \to l\nu l'l'$ overlap benchmark

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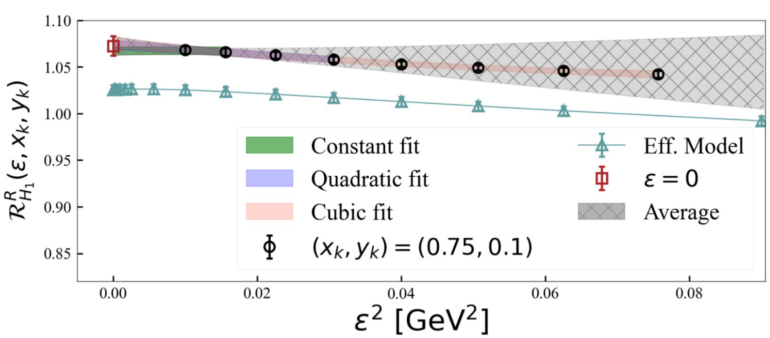
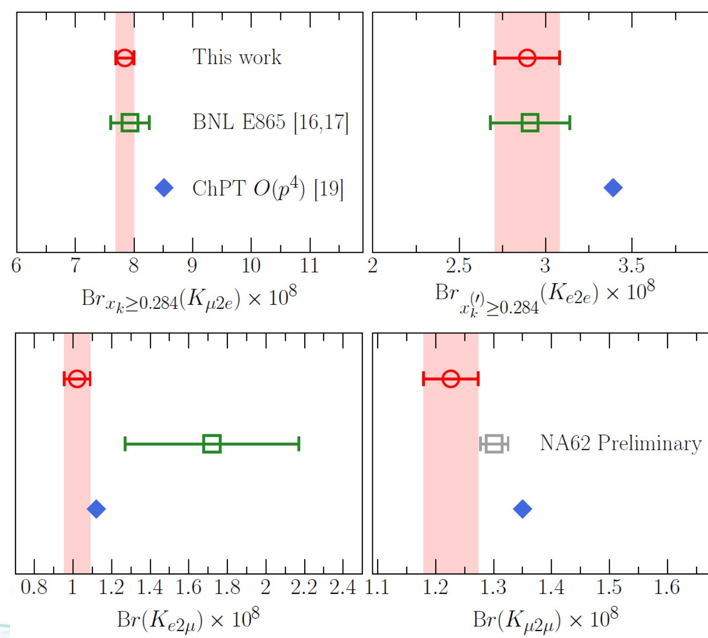


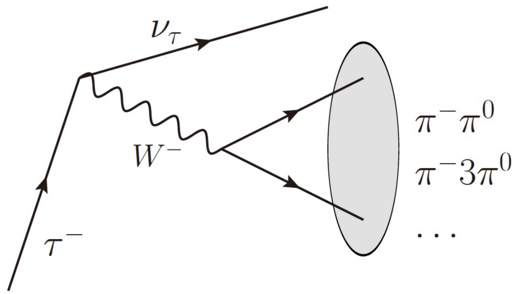
ε-smear photon propagator, extrapolate to $\varepsilon \to 0$

First complete lattice calculation



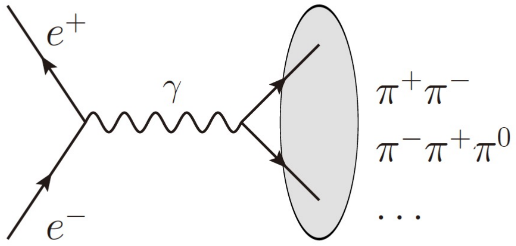
Talk by Francesco Sanfilippo (Tuesday 2:40pm)

τ decays for the muon $g-2$



isospin-breaking corrections

connect τ with HVP for muon $g-2$



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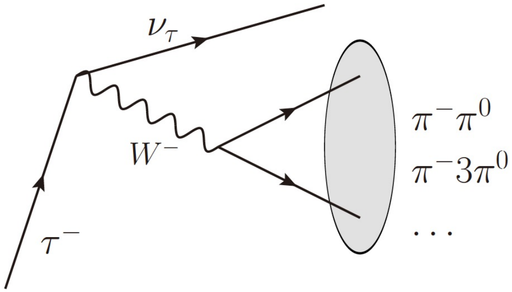

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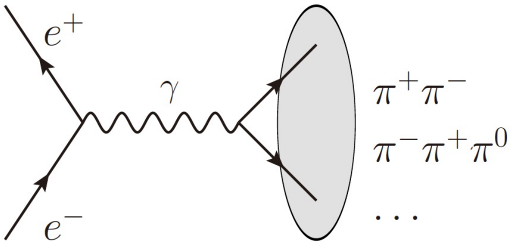
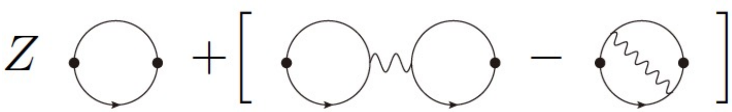


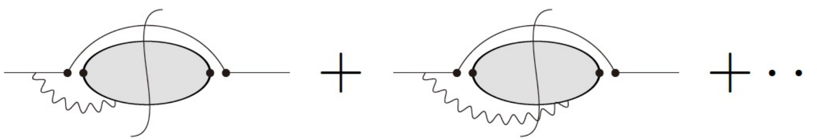
Strategy from Lattice QCD+QED
delicate short and long distance effects

M. Bruno, et al. (2026) [arxiv: 2607.00564]
P. Boyle, et al. (2026) [arxiv: 2606.04729]

Talk by Mattia Bruno (Friday 4:30pm)



factorizable corrections
charged vs neutral vector densities



non-factorizable corrections
severe inverse problems

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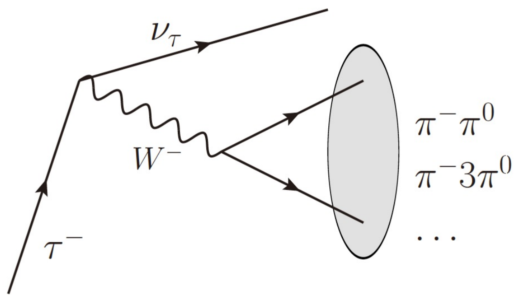

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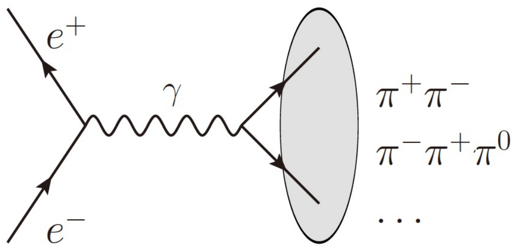

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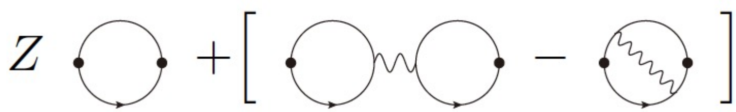

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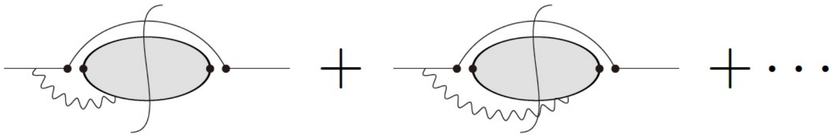

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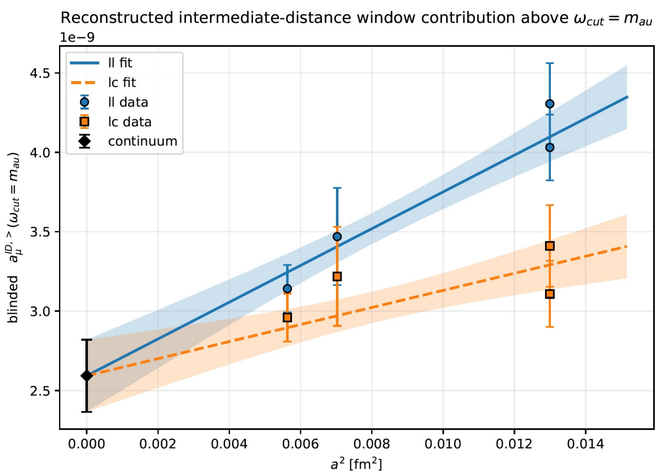


a fully inclusive strategy

→ factorizable effects
Talk by Mattia Bruno (Friday 4:30pm)

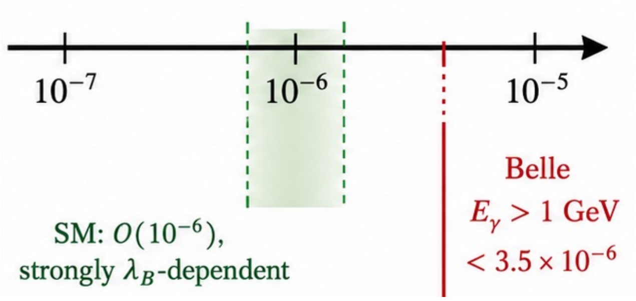
→ systematic effects of spectrum above the τ mass
Talk by Julian Parrino (Monday 5:10pm)

Ongoing Efforts and Future Opportunities

$B \to \ell\nu\gamma$ and λ_B

A direct probe of λ_B: key input to heavy-flavor QCD factorization

(first inverse momentum of light-cone distribution amplitude)



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Ongoing effort: Teseo San Jose, et al. lattice 2025, (UKQCD, JLQCD) [arxiv: 2603.19960]

Radiative decays of D/D_s
R. Frezzotti, et al. (ETMC) PRD 108 (2023) 074505
D. Giusti, et al. PRD 107 (2023) 074507
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RC in $D_s \to \ell\nu(\gamma)$

QED effects remove a CKM anomaly

$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 - 1$$

Before: 5.2σ $\xrightarrow{\text{complete 1-loop EW + point-like QED}}$ After: 1.1σ

point-like QED: T. Kitahara, et al. JHEP 05 (2026), 041

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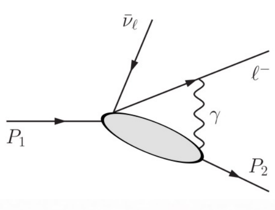
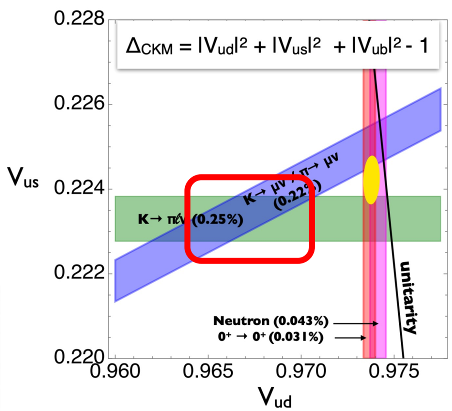
RC in $K_{\ell 3}$ decay





Related process: neutron β decay

QED vertex correction to g_A

Ongoing Efforts and Future Opportunities

$B \to \ell\nu\gamma$ and λ_B

A direct probe of λ_B: key input to heavy-flavor QCD factorization
(first inverse momentum of light-cone distribution amplitude)

Ongoing effort: Teseo San Jose, et al. lattice 2025, (UKQCD, JLQCD) [arxiv: 2603.19960]

Radiative decays of D/D_s
R. Frezzotti, et al. (ETMC) PRD 108 (2023) 074505
D. Giusti, et al. PRD 107 (2023) 074507
D. Giusti, et al. PRD 112 (2025) 054507

RC in $D_s \to \ell\nu(\gamma)$

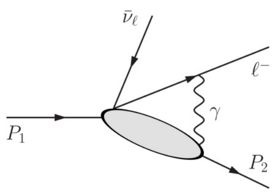
QED effects remove a CKM anomaly

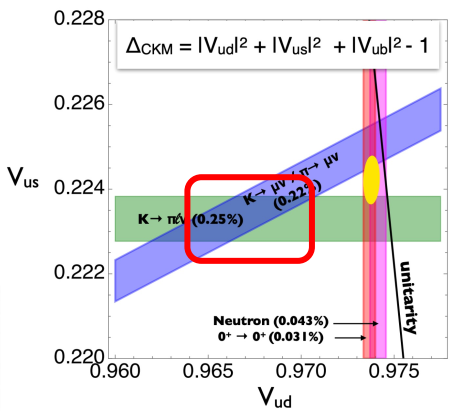
$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 - 1$$

Before	complete 1-loop EW + point-like QED	After
5.2σ	$\longrightarrow$	1.1σ

point-like QED: T. Kitahara, et al. JHEP 05 (2026), 041

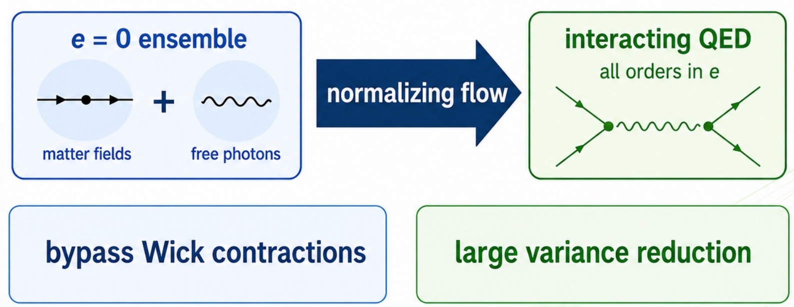
RC in $K_{\ell 3}$ decay





Related process: neutron β decay
QED vertex correction to g_A

All-orders QED from normalizing flow



Application to scalar QED
Nils Hermansson-Truedsson, Gurtej Kanwar, (2026), [arxiv:2605.22444]

Conclusion

1

Long-range QED is becoming controllable

IVR changes finite-volume effects from power-law to exponential scaling

2

First-principles RCs now impact precision tests

CKM unitarity, lepton-flavor universality, $\gamma W/\gamma Z$ boxes, inclusive τ decays, $\cdots$

3

The next frontier: multiparticle states and broader applications

Multiparticle reconstruction, spectral and inclusive approaches, physical applications in heavy-flavor decays, τ decays, $K_{\ell 3}$ decays, vertex correction to g_A, $\cdots$

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