

Flavour on the lattice

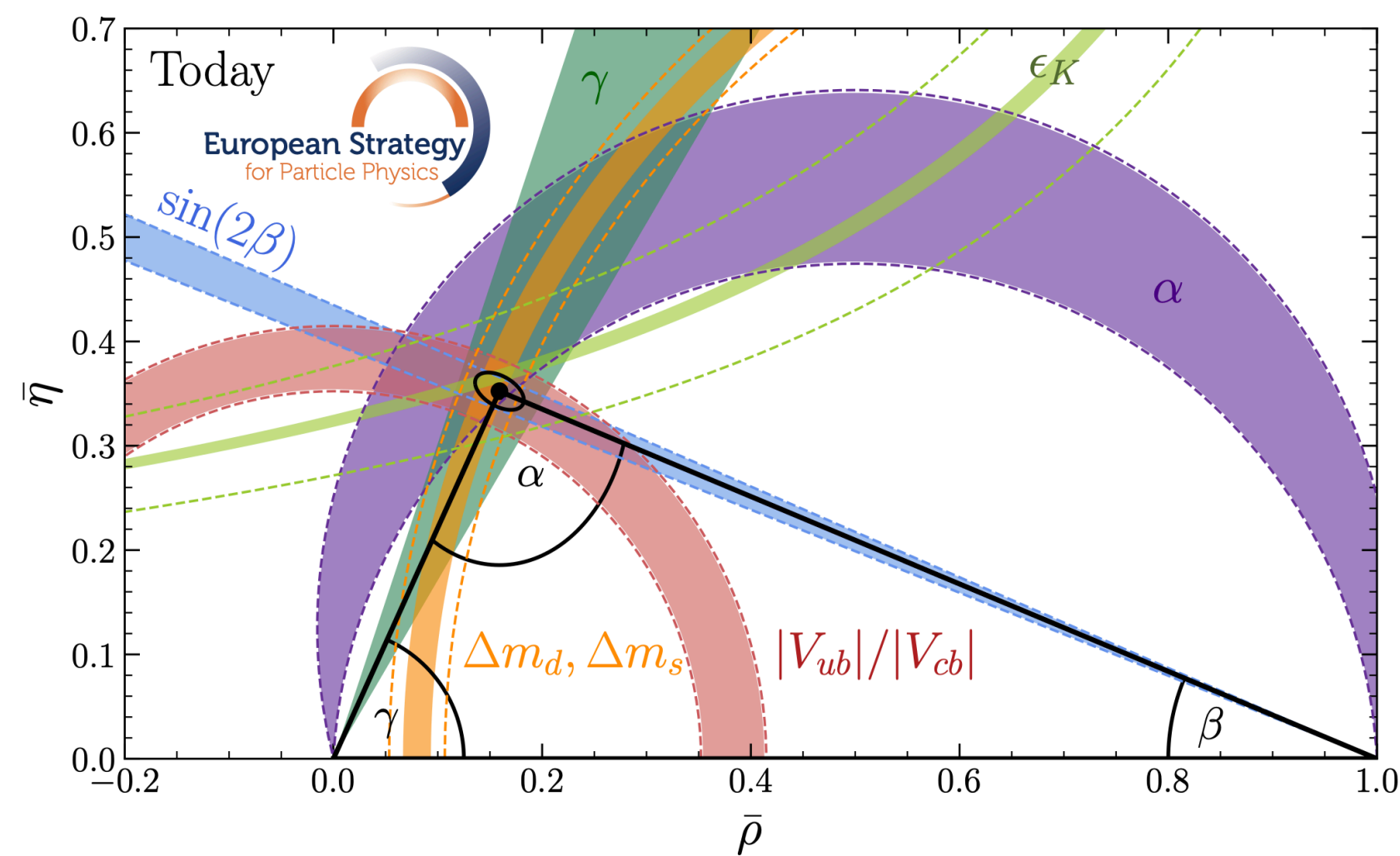
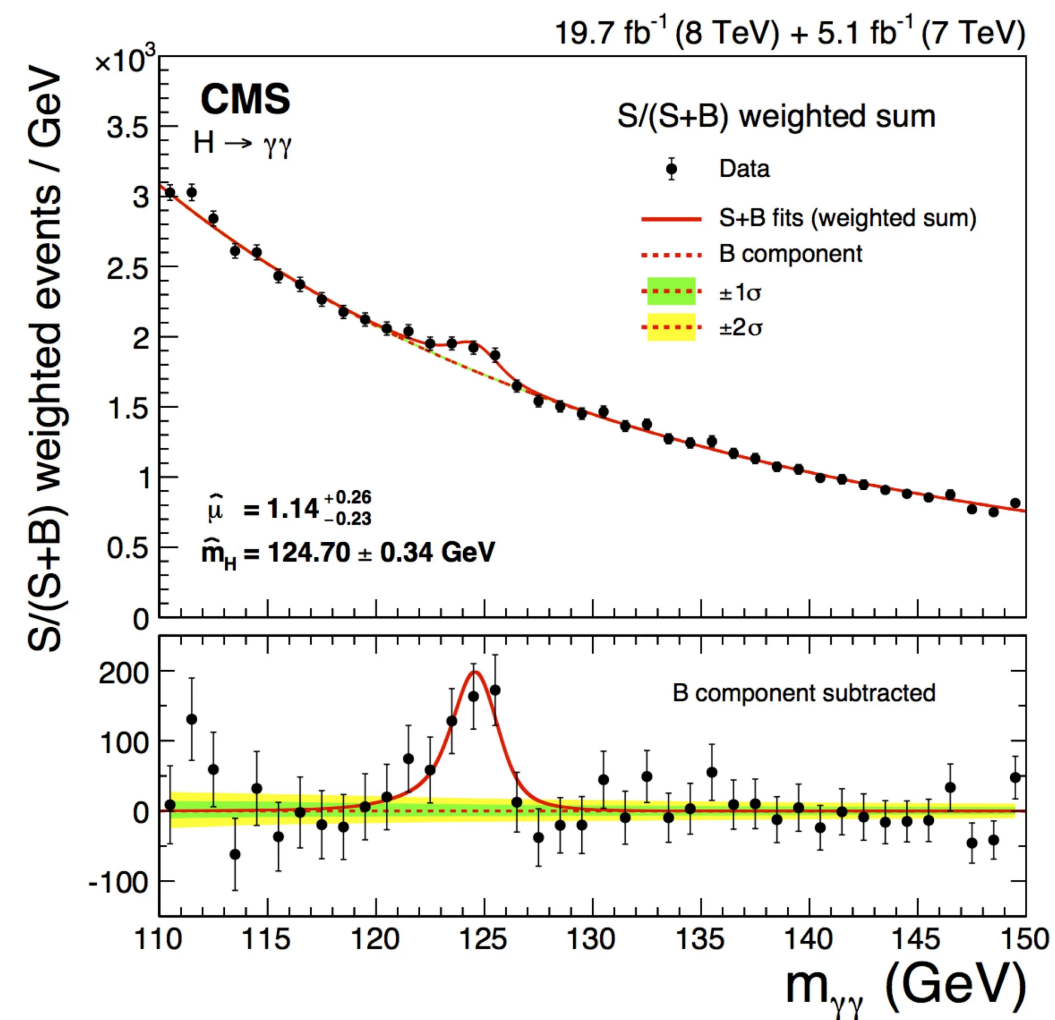


Andreas Jüttner



Testing the Standard Model

- Understand and predict and confirm *old* physics model-independently (QCD parameters, hadron spectrum, decay, scattering, ...)
- Search for new physics
 - *direct searches* — ‘bump in the spectrum’
 - *indirect searches* — overconstrain SM

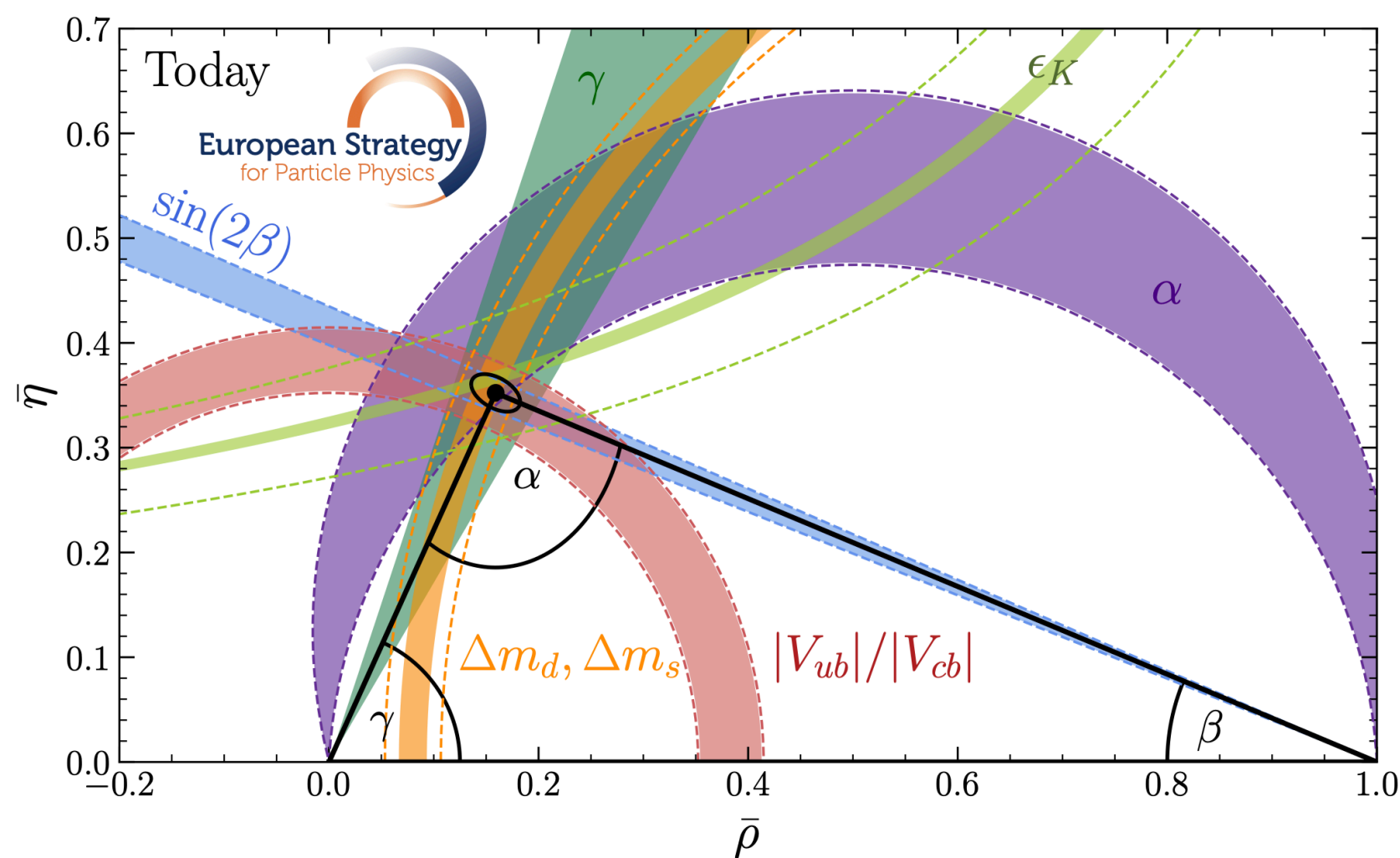
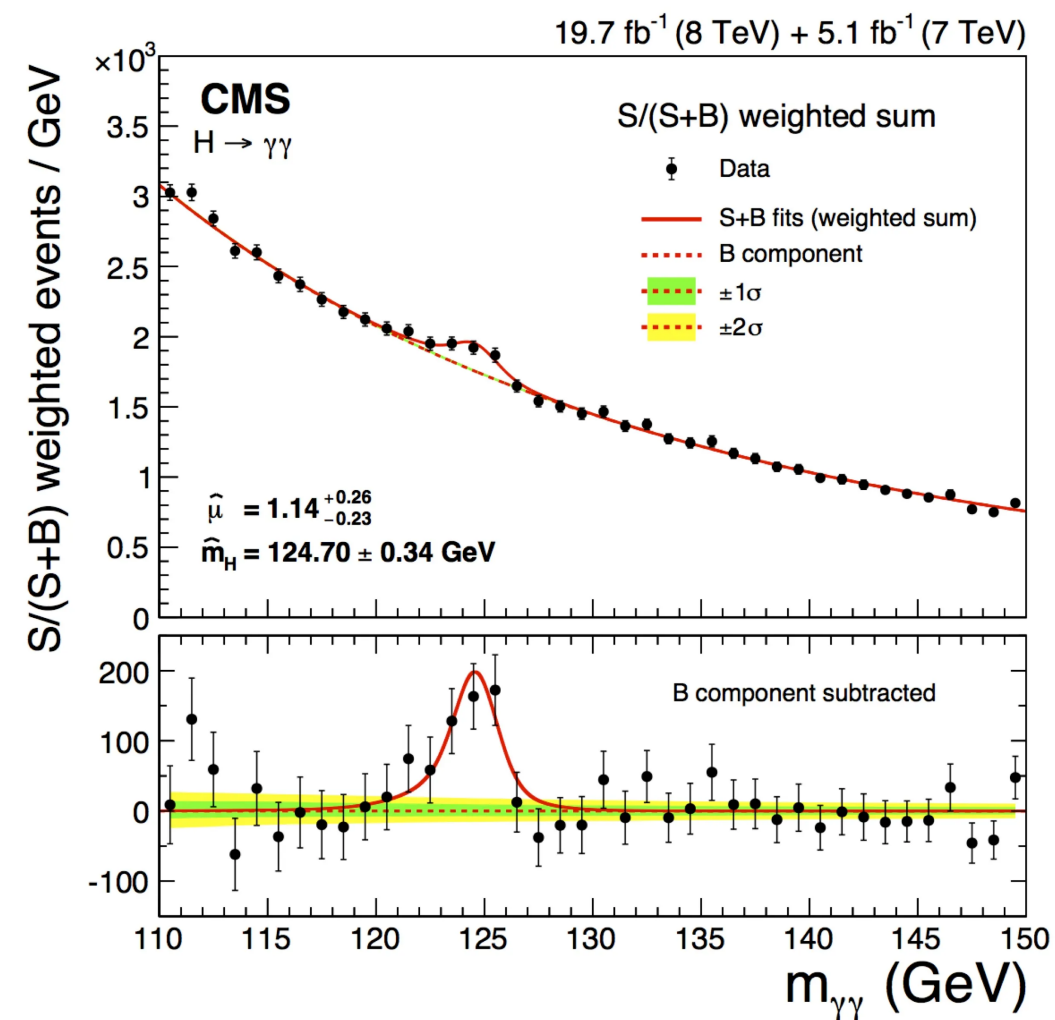


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Requires

- improving precision+accuracy
- discover new opportunities
- develop new techniques



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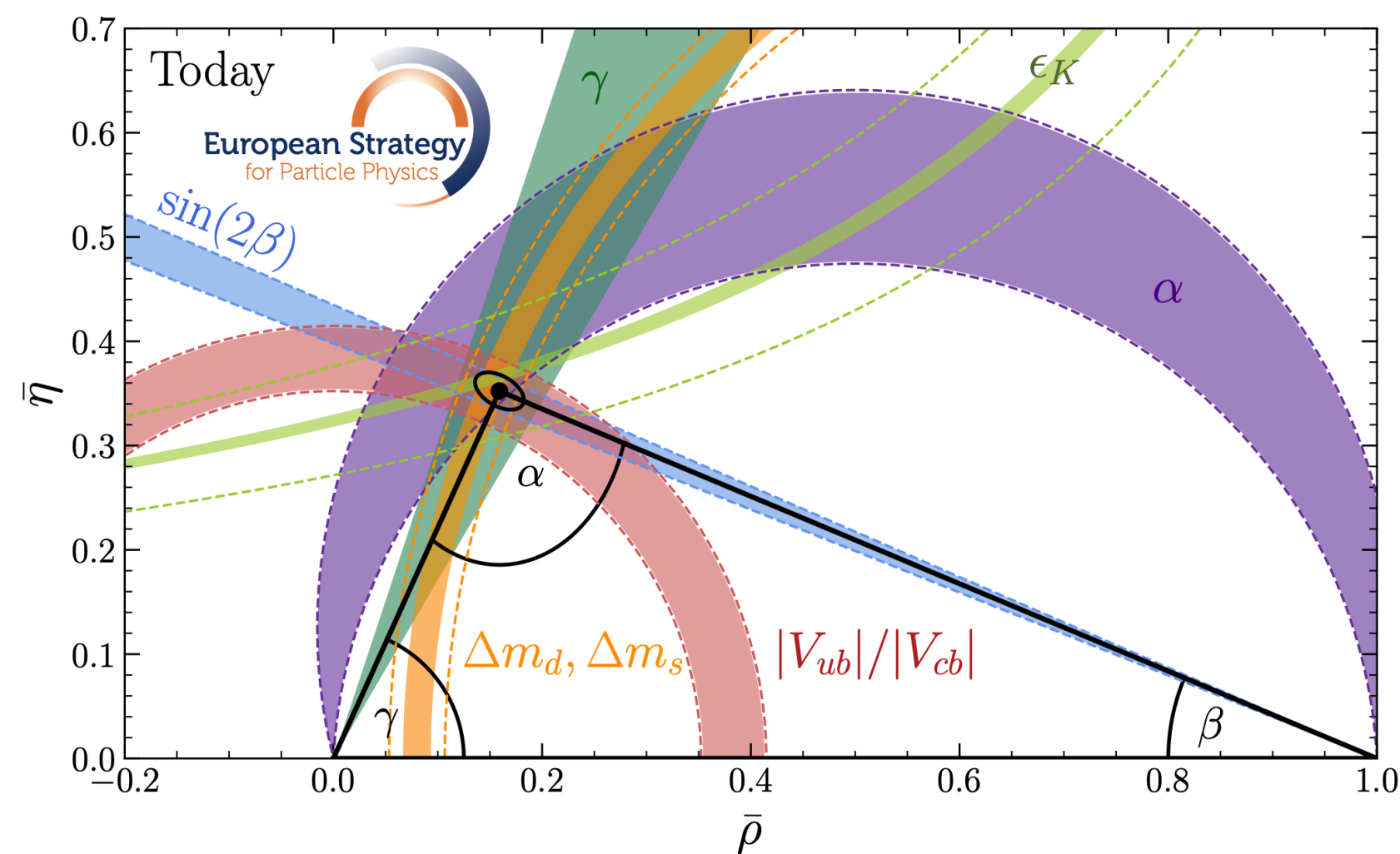
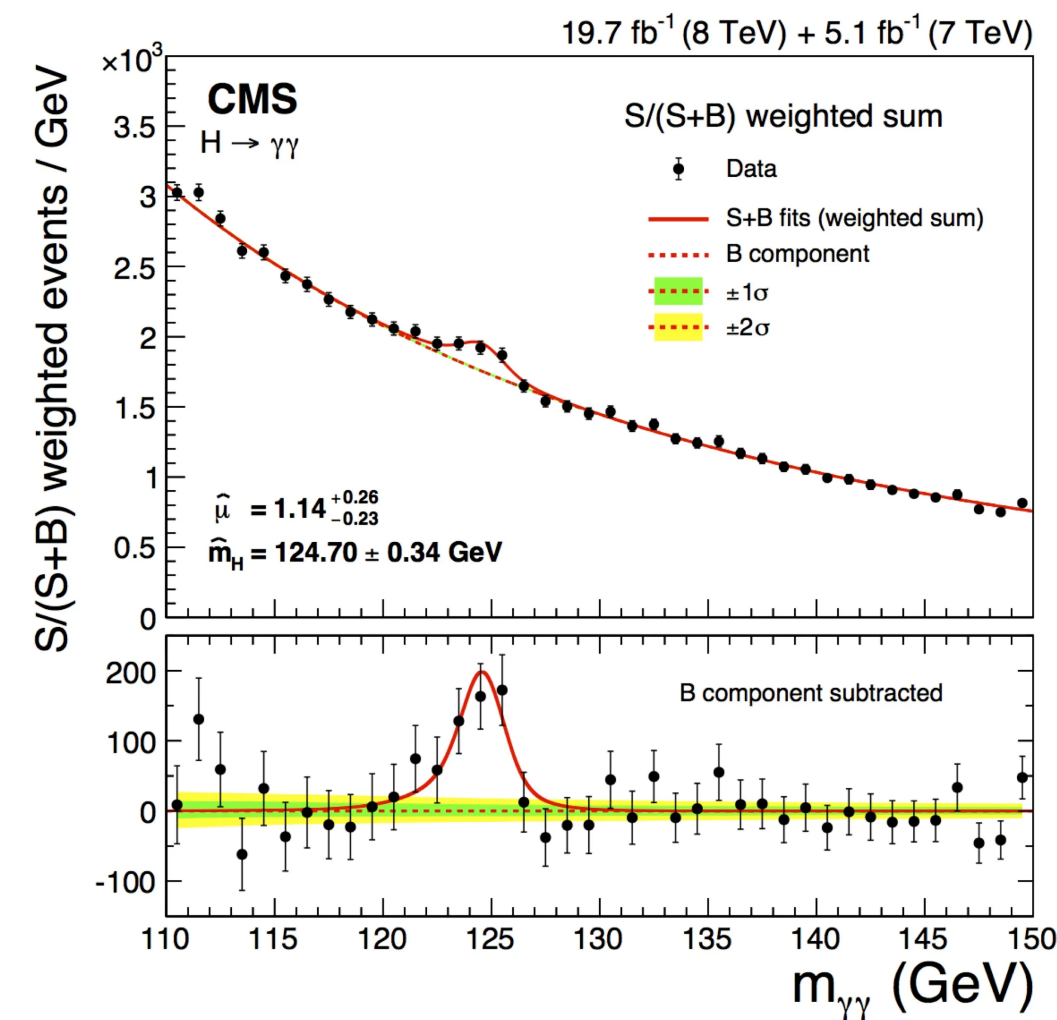
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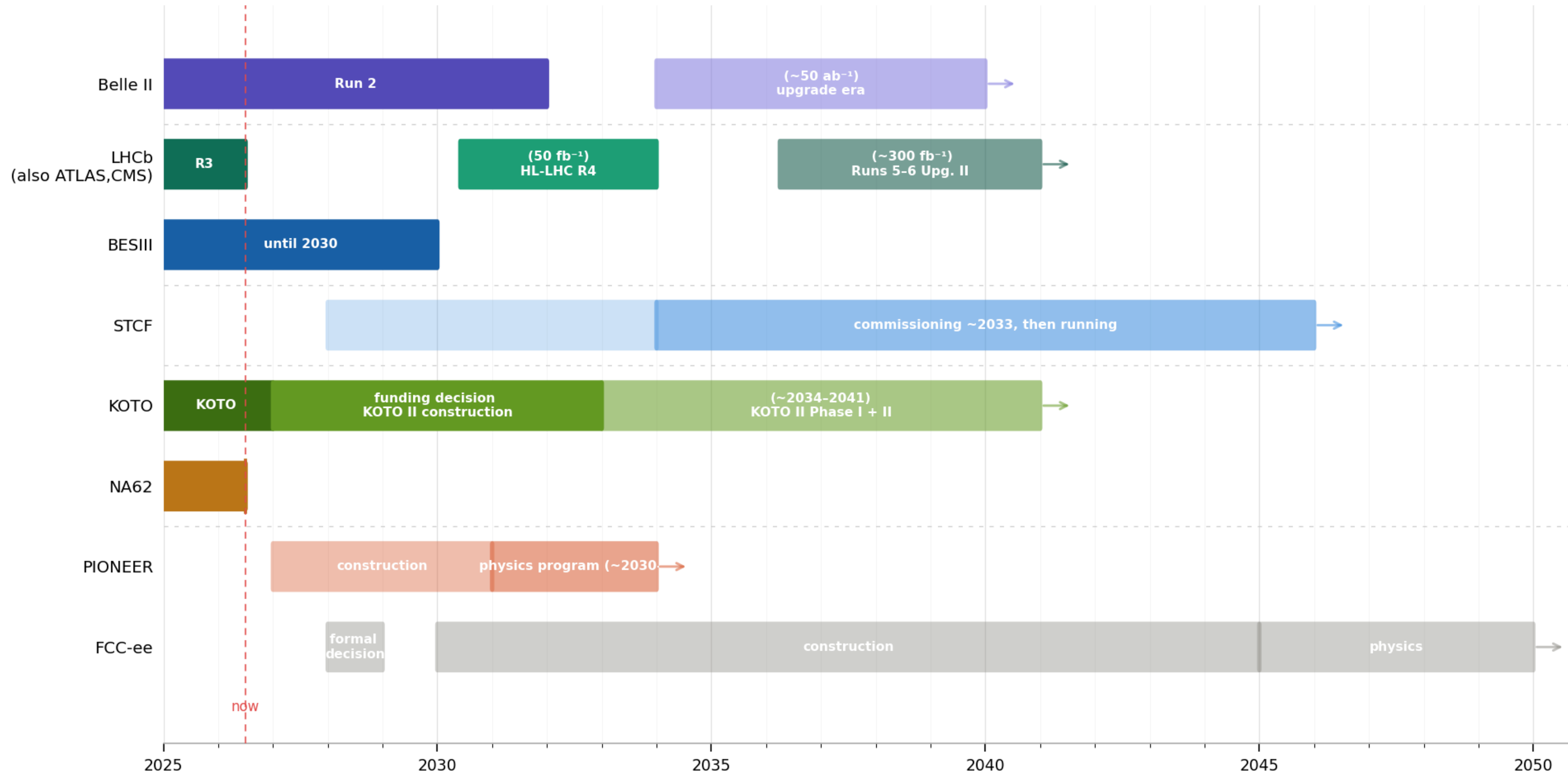
- improving precision+accuracy
- discover new opportunities
- develop new techniques

- **experiment + theory**

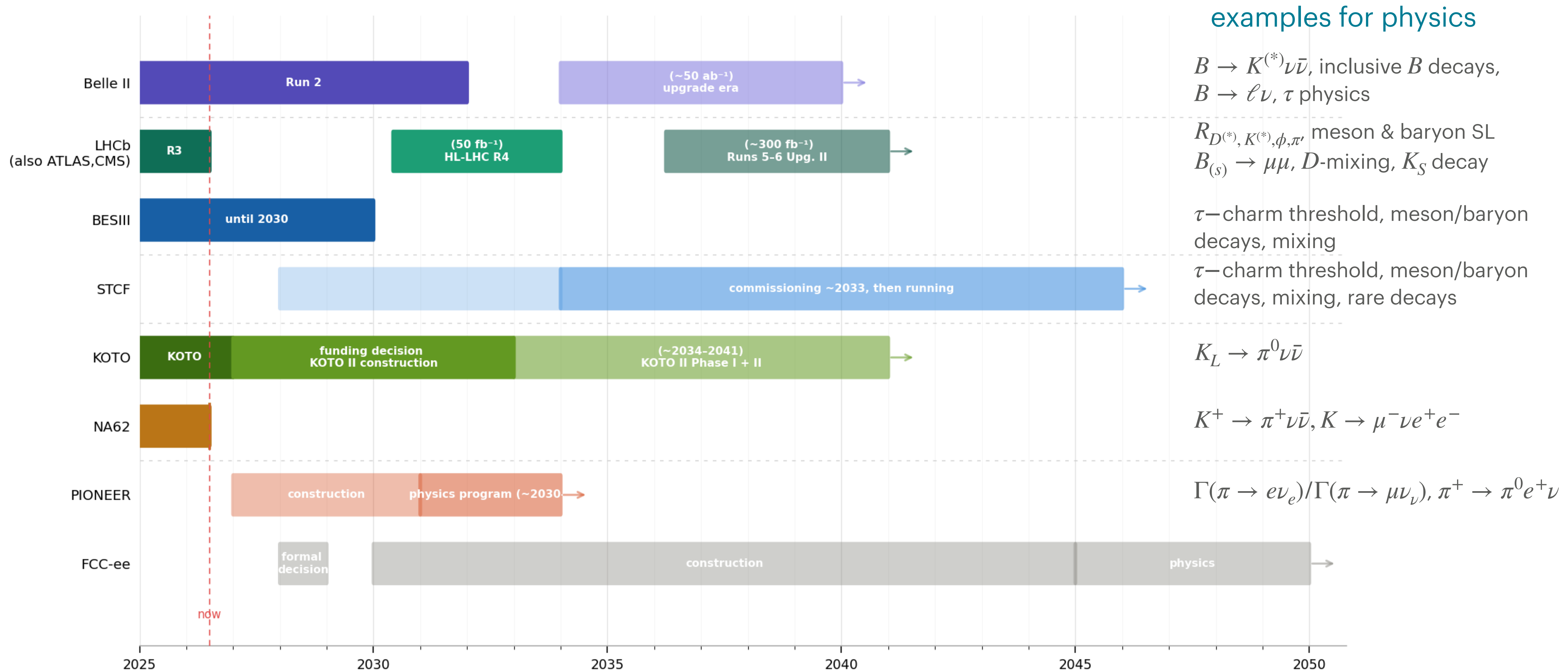
$$\Gamma = \underbrace{\text{Weak}}_{\text{PT}} + \underbrace{\text{EM}}_{\text{PT}} + \underbrace{\text{QCD}}_{\text{PT+nonPT}}$$



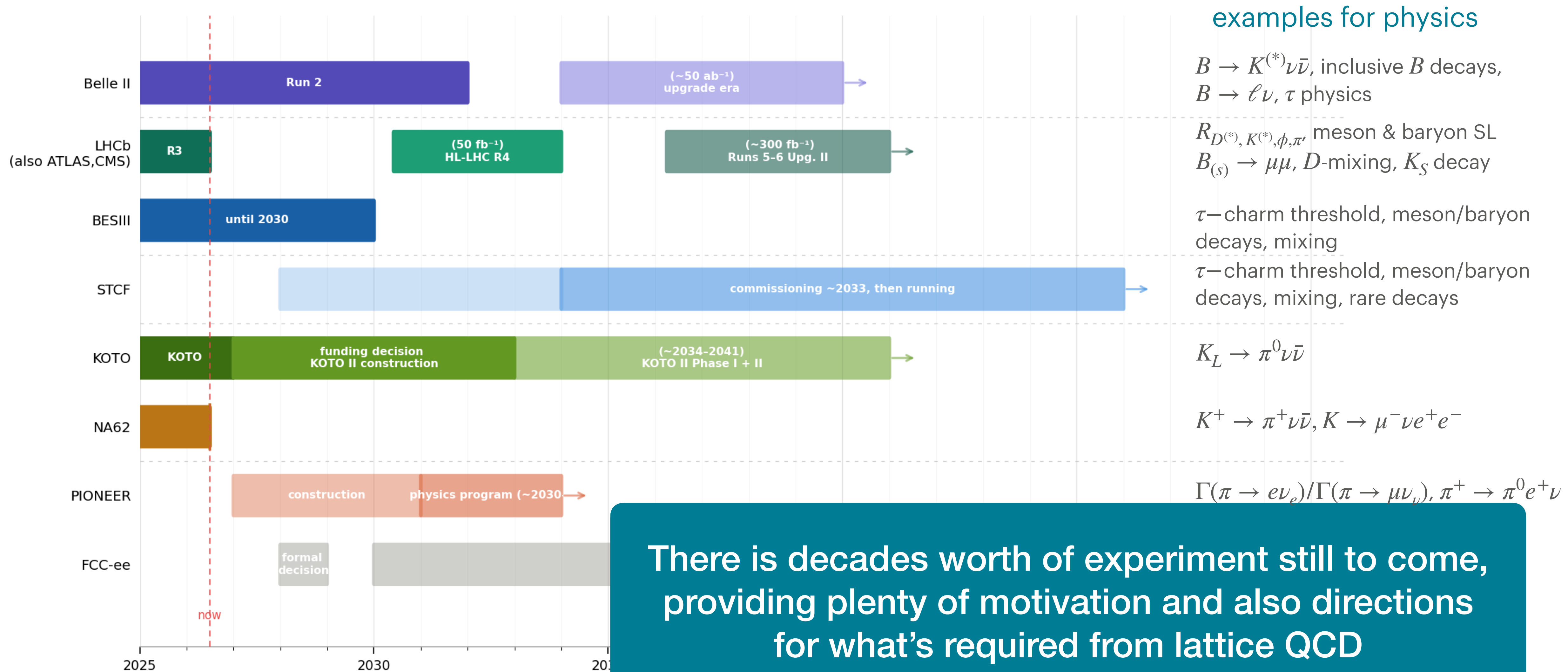
Flavour-Experimental landscape



Flavour-Experimental landscape



Flavour-Experimental landscape



Relevance for European Strategy

Submissions for flavour for the Update of the European Strategy for Particle Physics
(of 268 submissions)

pions:

- $\pi \rightarrow \ell \nu$
- $\pi^+ \rightarrow \pi^0 e^+ \nu$
- $|V_{ud}|$

[115] PIONEER

kaons:

- $K - \bar{K}$ mixing
- K (rare) L/SL decays
($K \rightarrow \pi \ell \ell (\nu \nu)$, $K \rightarrow \ell \ell$)
- rare hyperon decay
- $K_L - K_S$ interference,
- $|V_{us}|$

[55] Kaon community
[81] LHCb Upgrade II
[155] KOTO(-II)

charm:

- $D - \bar{D}$ mixing
- D (rare) decays
($D \rightarrow \pi \ell \ell$, $D \rightarrow hh \ell \ell$, ...)
- D hadronic decays
- charm baryonic decays
- $|V_{cd}|$, $|V_{cs}|$

[81] LHCb Upgrade II
[188] LEP3
[196] FCC
[205] Belle-II/Super KEK B
[223] ATLAS, Belle-II, CMS, LHCb
[231] STCF

bottom:

- $B_{u,d,s,c}$ (rare) L/SL decays
($\bar{B}_s^0 \rightarrow K \ell \nu$, $B \rightarrow D^{**} \ell \bar{\nu}$,
 $B_{(s)}^0 \rightarrow \mu^+ \mu^-$, $B^0 \rightarrow K^{*0} \mu^+ \mu^-$, ...)
- Λ_b SL/radiative decays
($\Lambda_b \rightarrow p \ell \nu$, $\Lambda_b^0 \rightarrow \Lambda \gamma$)
- $B_{(s)} - \bar{B}_{(s)}$ mixing
- hadronic decays
- inclusive/exclusive
- $|V_{ub}|$, $|V_{cb}|$

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[188] LEP3
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[223] ATLAS, Belle-II, CMS, LHCb

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CERN-ESU-2025-001
4 November 2025

Physics Briefing Book

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Electroweak Physics: Jorge de Blas¹, Monica Dunford² (Conveners), Emanuele Bagnaschi³ (Scientific Secretary), Ayres Freitas⁴, Pier Paolo Giardino⁵, Christian Grefe⁶, Michele Selvaggi⁷, Angela Taliercio⁸, Falk Bartels² (Contributors)

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Beyond the Standard Model Physics: Fabio Maltoni^{8,31}, Rebeca Gonzalez Suarez³² (Conveners), Benedikt Maier³³ (Scientific Secretary), Timothy Cohen^{28,78,*}, Annapaola de Cosa^{34,*}, Nathaniel Craig³⁵, Roberto Franceschini³⁶, Loukas Gouskos³⁷, Aurelio Juste³⁸, Sophie Renner¹⁴, Lesya Shchutska²⁸ (Contributors)

Dark Matter and Dark Sector: Jocelyn Monroe^{34,46}, Matthew McCullough⁷ (Conveners), Yohei Ema^{7,1} (Scientific Secretary), Paolo Agnes⁴⁷, Francesca Calore⁴⁸, Emanuele Castorina⁴², Aaron Chou⁴⁹, Monica D'Onofrio⁵⁰, Maksym Ovchinnikov^{7,1}, Tina Pollmann¹⁹, Josef Pradler^{39,86}, Yotam Soreq⁵², Julia Katharina Vogel⁵³ (Contributors)

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Editors: Roger Forty⁷, Karl Jakobs⁷⁴, Hugh Montgomery⁷⁵, Mike Seidel^{28,76}, Paris Spicas^{7,77}

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e) L/SL decays

$\nu, B \rightarrow D^{**} \ell \bar{\nu},$
 $u^-, B^0 \rightarrow K^{*0} \mu^+ \mu^-, \dots)$

relative decays

$\nu, \Lambda_b^0 \rightarrow \Lambda \gamma)$

mixing

decays

exclusive

$b |$

ade II

[196] FCC

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“Theoretical calculations, such as those in **lattice QCD**, must be supported on a global level to guarantee the full exploitation of the increased experimental precision.”

THE EUROPEAN STRATEGY FOR PARTICLE PHYSICS:
2026 UPDATE

Deliberation Document
by the European Strategy Group



[196] FCC
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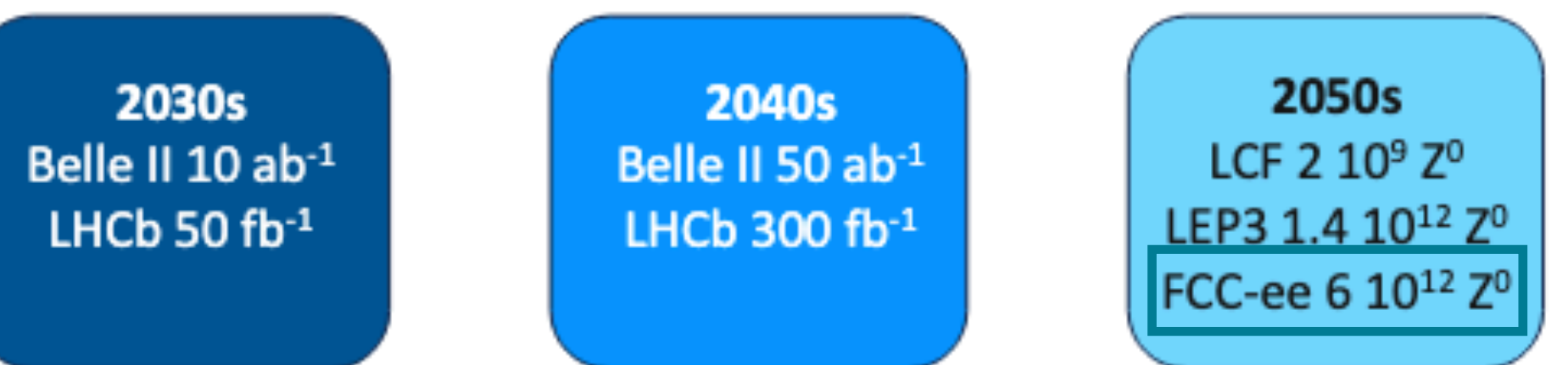
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With 6×10^{12} Z-bosons FCC-ee will be a formidable flavour factory. Fingers crossed for a positive decision in 2028!

Matrix-element classification

Weak gauge bosons so heavy that we can replace them by point-interaction described by an Effective Hamiltonian H_{eff} $\rightarrow$ compute their expectation values in QCD(+QED)

Minkowski $\leftrightarrow$ Euclidean

1st order EW:

$$\langle P' | H_{\text{eff}} | P \rangle$$

e.g.

- tree-level leptonic and semileptonic decay
- short-distance contributions to 2nd-order processes (rare decays, mixing, ...)

$$\langle 0 | A_\mu | K \rangle$$

$$\langle \pi | V_\mu | B \rangle$$

$$\langle B | O^{\Delta B=2} | \bar{B} \rangle$$

"i"s and signs

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1st order EW: $\langle P' H_{\text{eff}} P \rangle$	e.g. • tree-level leptonic and semileptonic decay • short-distance contributions to 2nd-order processes (rare decays, mixing, ...)	$\langle 0 A_\mu K \rangle$ $\langle \pi V_\mu B \rangle$ $\langle B O^{\Delta B=2} \bar{B} \rangle$	"i"s and signs
1st order EW: $\langle P' P'' H_{\text{eff}} P \rangle$ 2nd order EW: $\langle P' T[H_{\text{eff},1} H_{\text{eff},2}] P \rangle$	e.g. • hadronic decay • long-distance effects in rare decays • meson mixing	$\langle K T[j_{\text{em}} O_{1,2}] B \rangle$ $\langle B T[O^{\Delta B=1} O^{\Delta B=1}] \bar{B} \rangle$	"i"s and signs AND depending on kinematics: Lüscher or spectral reconstruction

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e.g.
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 • short-distance contributions
 processes (rare decays, mixing)

Part I

$$\langle 0 | A_\mu | K \rangle$$

$$\langle \pi | V_\mu | B \rangle$$

$$\langle B | O^{\Delta B=2} | \bar{B} \rangle$$

“ i ”s and signs

1st order EW:
 $\langle P' P'' | H_{\text{eff}} | P \rangle$
 2nd order EW:
 $\langle P' | T[H_{\text{eff},1} H_{\text{eff},2}] | P \rangle$

e.g.
 • hadronic decay
 • long-distance effects in rare decays
 • meson mixing

Part II

$$\langle K | T[j_{\text{em}} O_{1,2}] | B \rangle$$

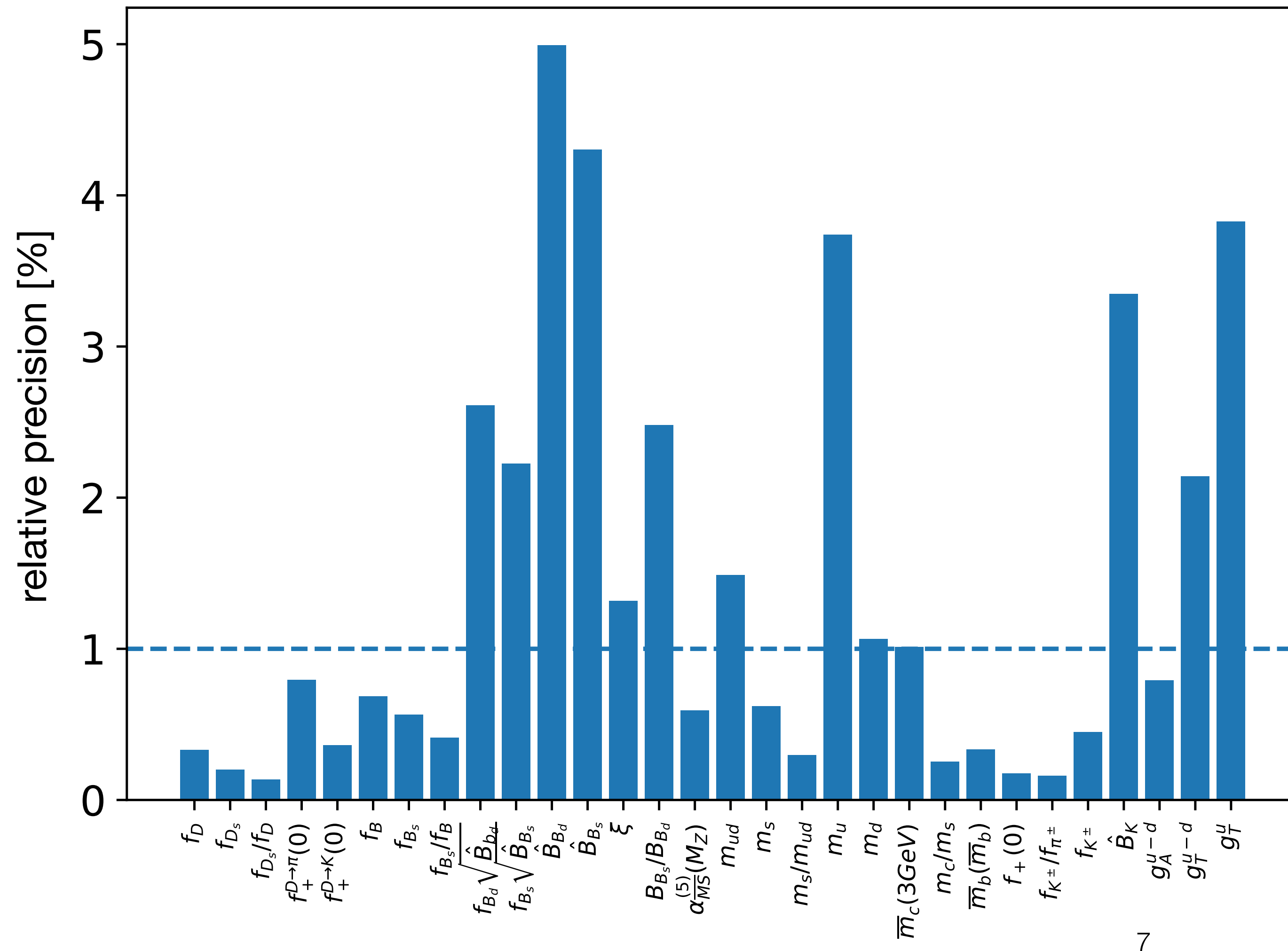
$$\langle B | T[O^{\Delta B=1} O^{\Delta B=1}] | \bar{B} \rangle$$

“ i ”s and signs
AND
 depending on kinematics:
 Lüscher or spectral
 reconstruction

Part I — 1st order EW

(decay constants, form factors, LECs, ...)

Precision quantities discussed in FLAG24



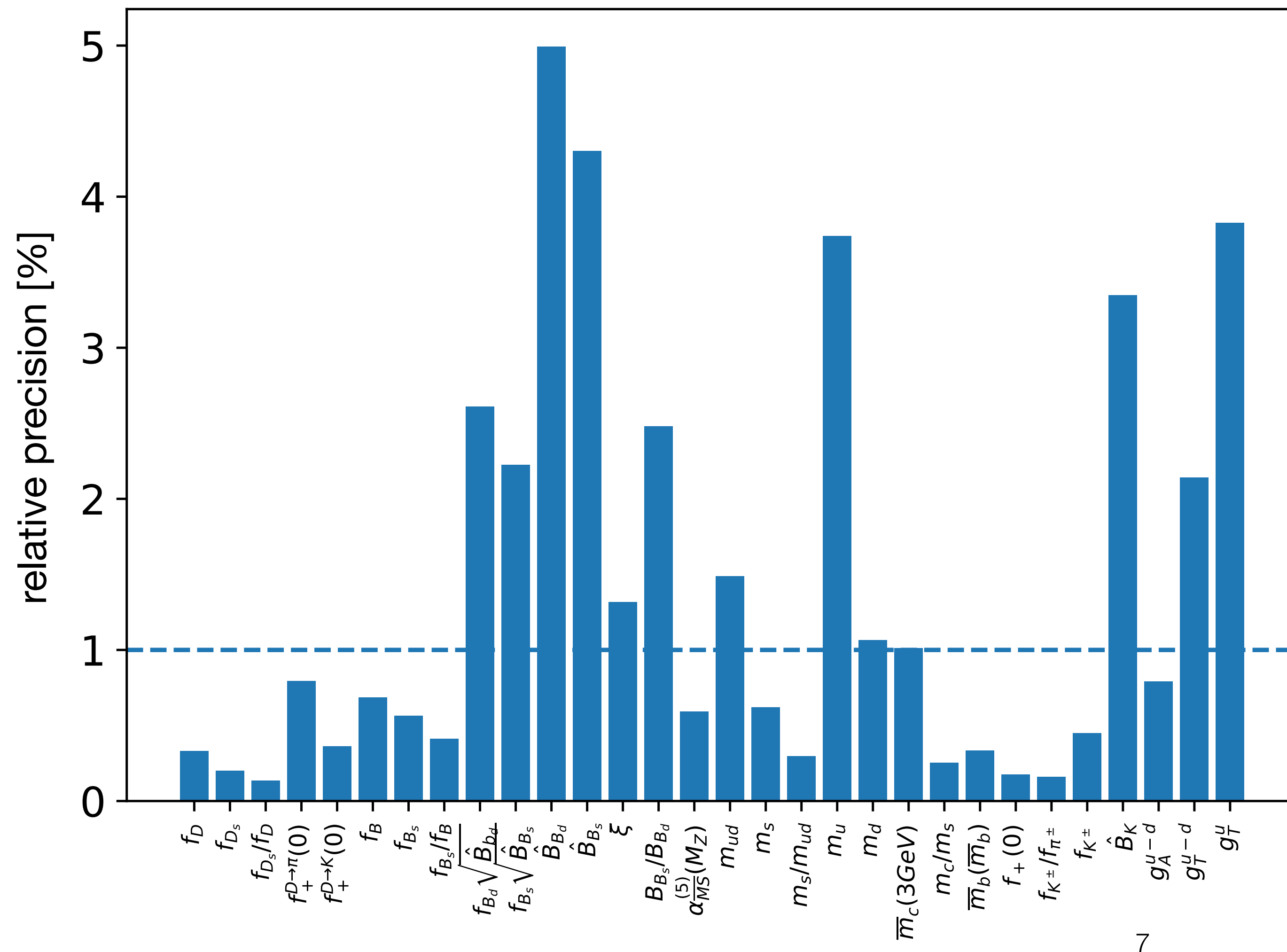
FLAG
Flavour Lattice Averaging Group

<http://flag.unibe.ch>

Phys.Rev.D 113 (2026) 1, 014508

Lattice-QCD results increasingly
hitting 1%-error barrier.

Precision quantities discussed in FLAG24



<http://flag.unibe.ch>

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Lattice-QCD results increasingly
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“Edinburgh consensus”

	QCD	isoQCD
M_{π^+}	135.0 MeV	135.0 MeV
M_{K^+}	491.6 MeV	494.6 MeV
M_{K^0}	497.6 MeV	494.6 MeV
$M_{D_s^+}$	1967 MeV	1967 MeV
$M_{B_s^0}$	5367 MeV	5367 MeV
f_{π^+}	130.5 MeV	130.5 MeV

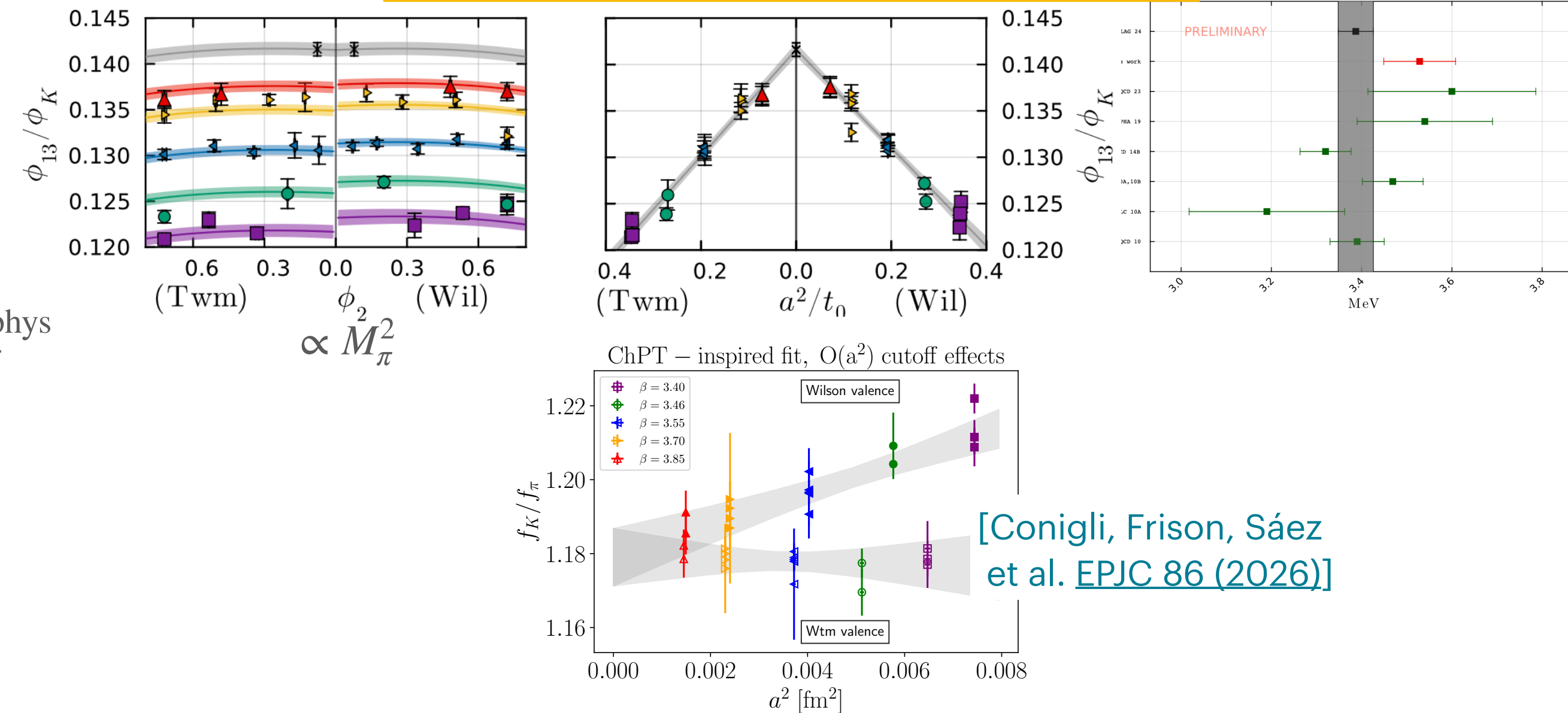
Quark Masses and decay constants

Talk by **Fernando Perez Panadero**

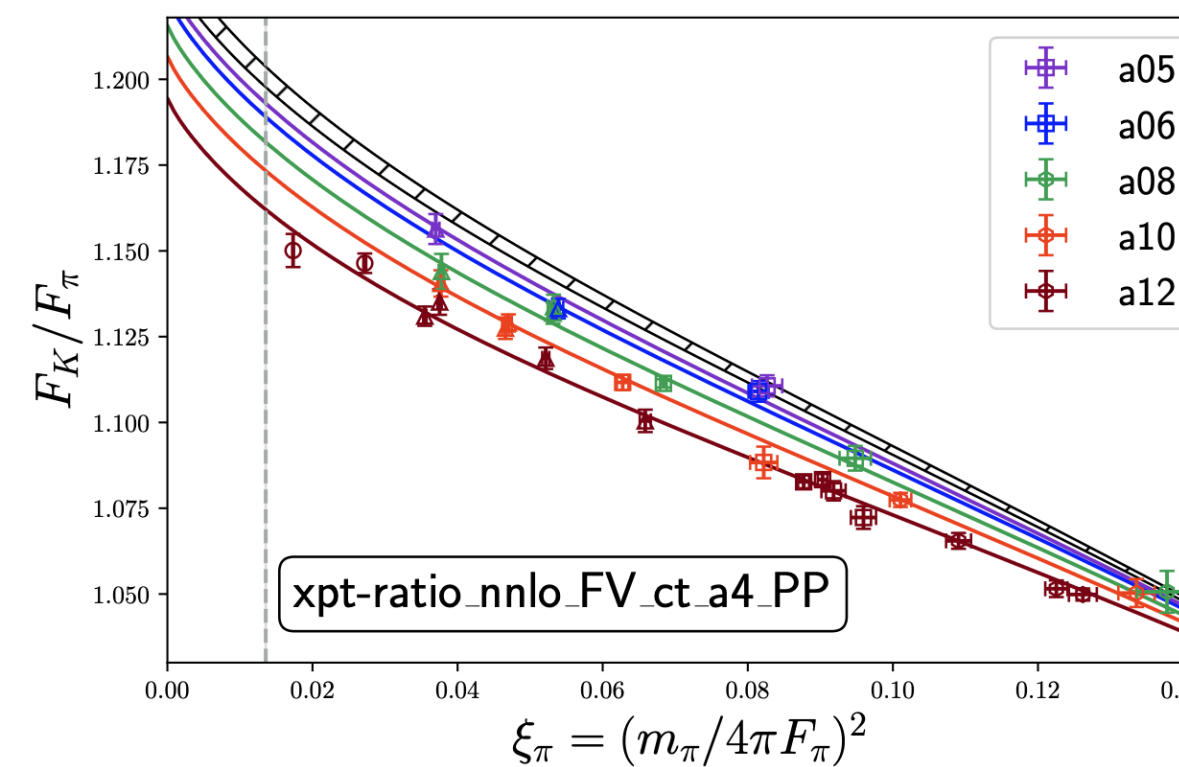
- light- and strange masses following
[ALPHA EPJC 80 (2020), EPJC 86 (2026)]
and decay constants [Conigli, Frison, Sáez et al. EPJC 86 (2026)]
- $N_f = 2 + 1$ CLS, Wilson and TM mixed valence,
five values of $a \gtrsim 0.38$ fm, $\text{tr}(M_q) = 2m_l + m_s \approx \text{const.}$, $M_\pi \gtrsim M_\pi^{\text{phys}}$
- Edinburgh consensus
- quark masses SF-renorm. and running
- decay constant SU(3) χ PT extrapolation and NLO FVE
 $f_K/f_\pi = 1.1872(59)_{\text{stat}}(84)_{\chi\text{-cont}} (\sim 1\%)$

[Hudspith et al. arXiv:2605.06560]

- $N_f = 2 + 1 + 1$ MDWF, $a \sim 0.05 - 0.12$ fm
 $M_\pi \sim 0.135 - 460$ MeV
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- NNLO SU(3) χ PT extrapolation with $O(a^4)$
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- $f_K/f_\pi = 1.1962(34) (\sim 0.3\%)$



Talks by **Andre Walker-Loud**
and
Zack Hall



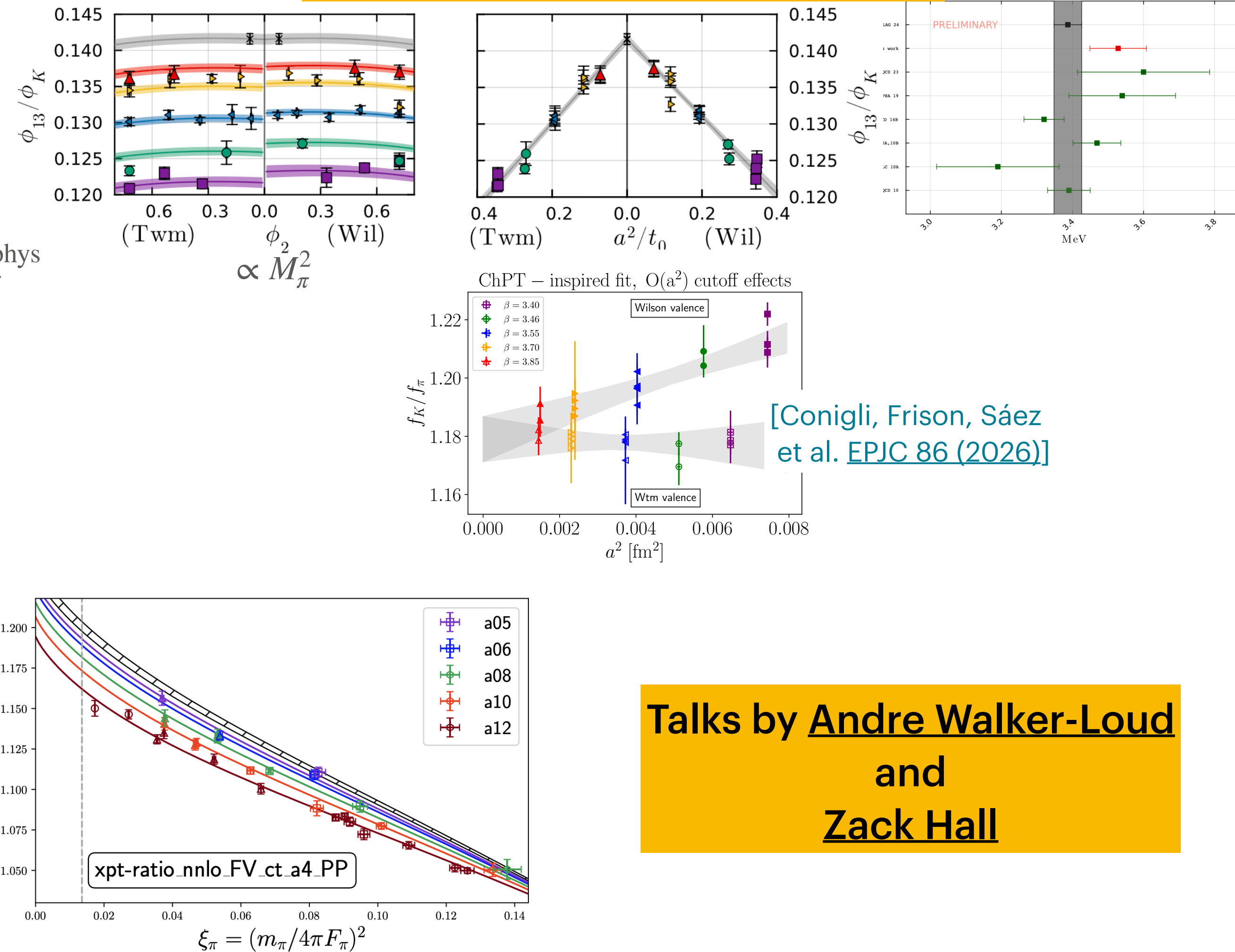
Quark Masses and decay constants

Talk by **Fernando Perez Panadero**

- light- and strange masses following
[ALPHA EPJC 80 (2020), EPJC 86 (2026)]
and decay constants [Conigli, Frison, Sáez et al. EPJC 86 (2026)]
- $N_f = 2 + 1$ CLS, Wilson and TM mixed valence,
five values of $a \gtrsim 0.38$ fm, $\text{tr}(M_q) = 2m_l + m_s \approx \text{const.}$, $M_\pi \gtrsim M_\pi^{\text{phys}}$
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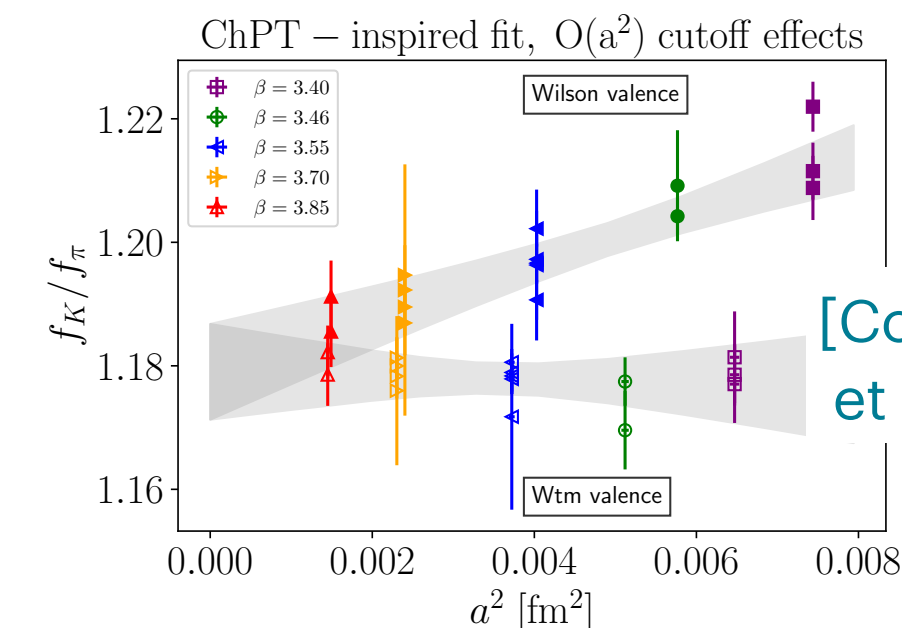
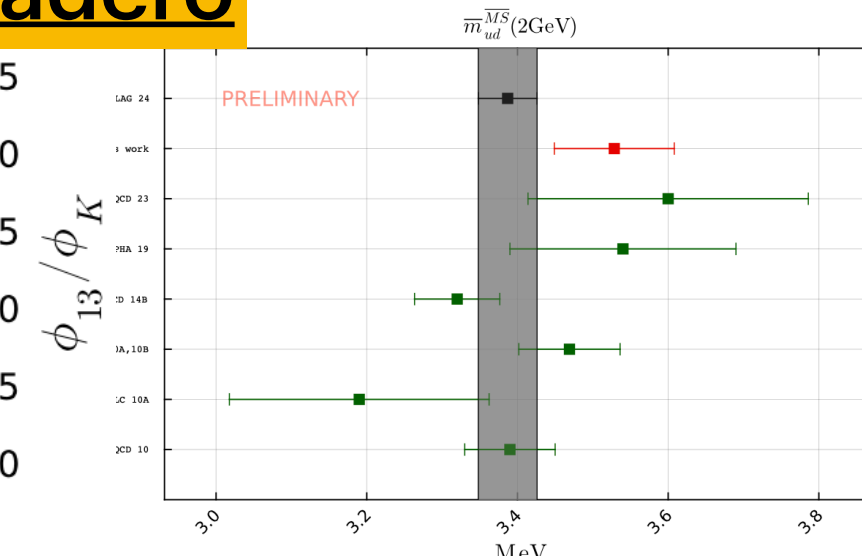
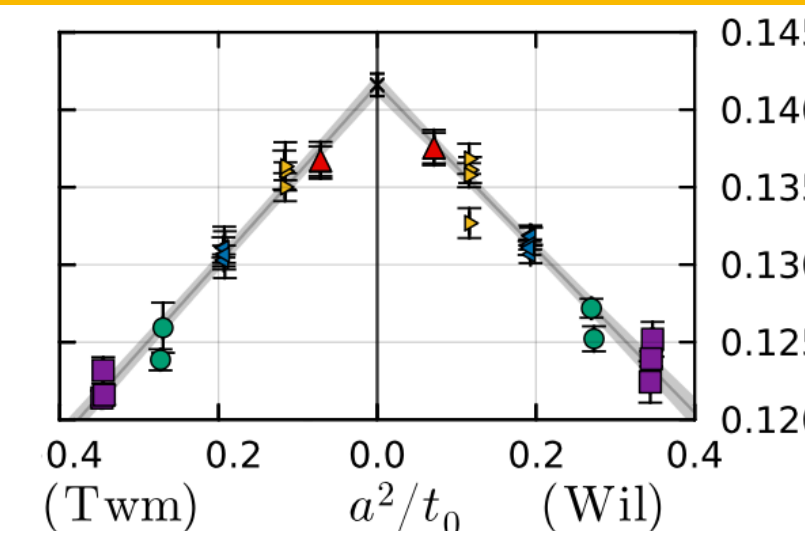
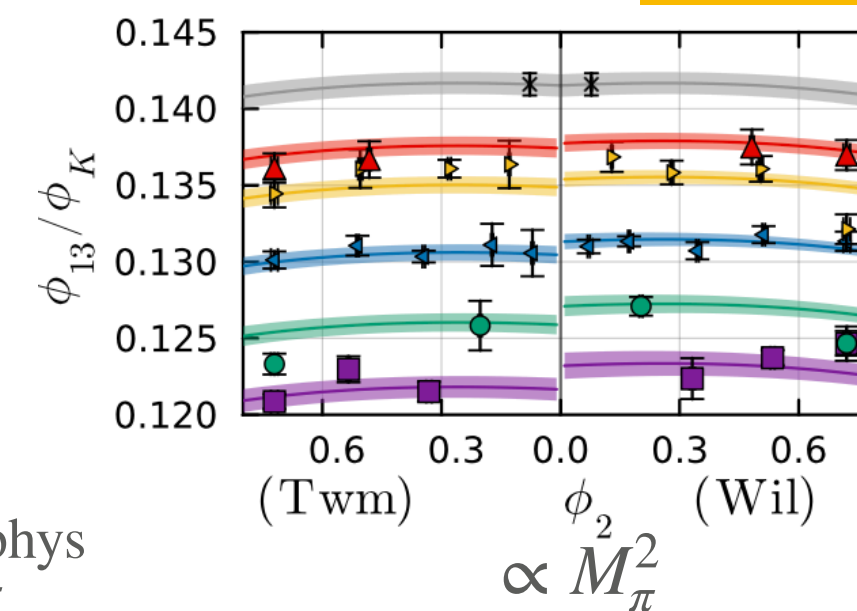
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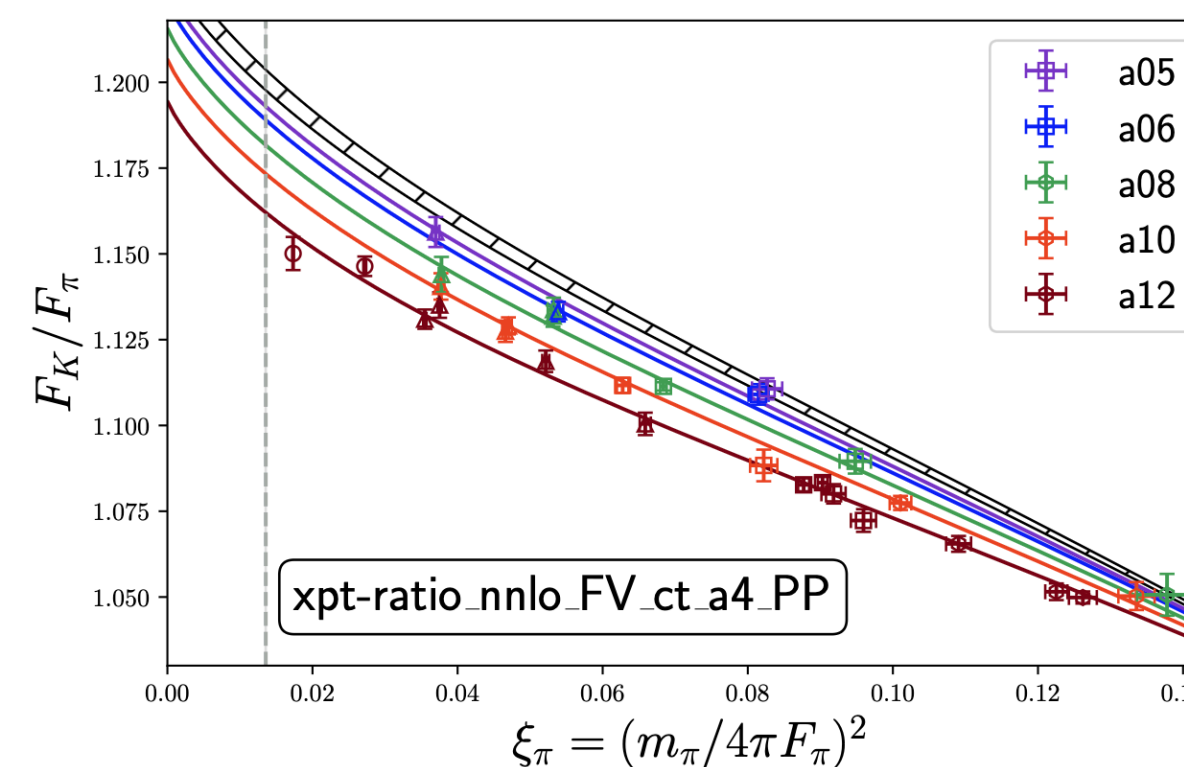
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[Conigli, Frison, Sáez et al. EPJC 86 (2026)]



Talks by **Andre Walker-Loud**
and
Zack Hall

Separate talks on quark masses with gradient flow by **Mingle Dai**
and on $B_c^{(*)}$ decay constants by **Navdeep Singh Dhindsa**

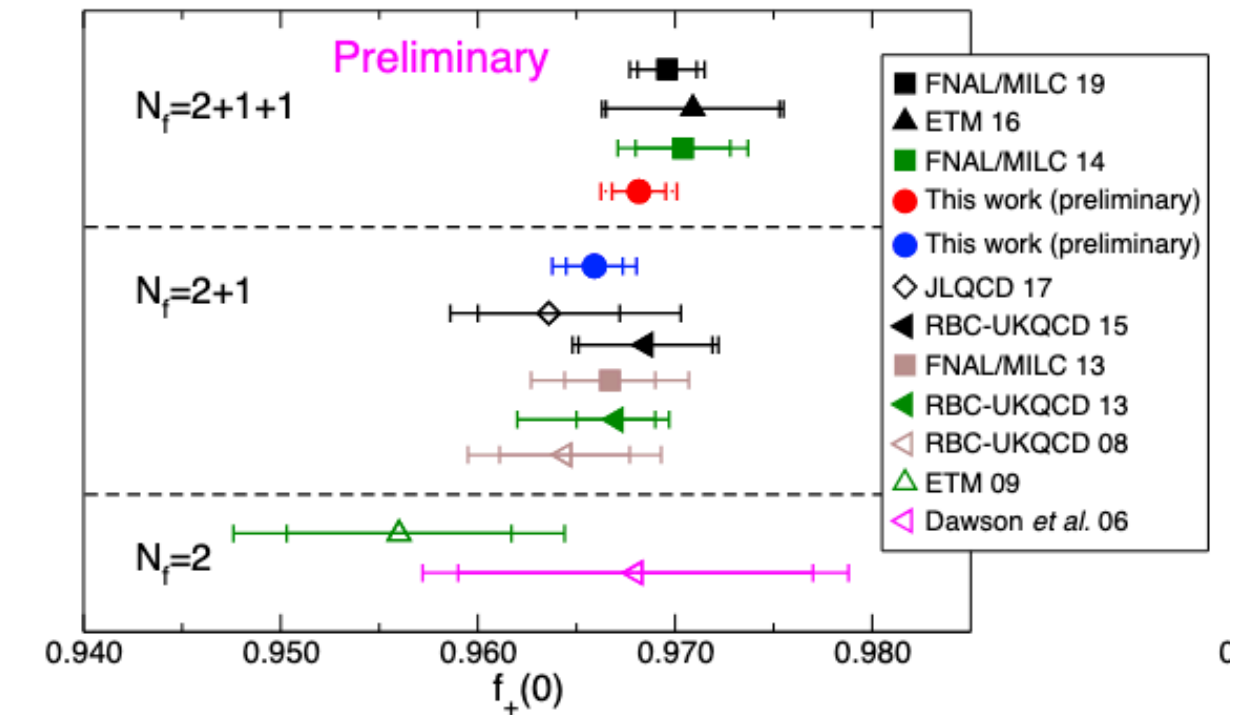
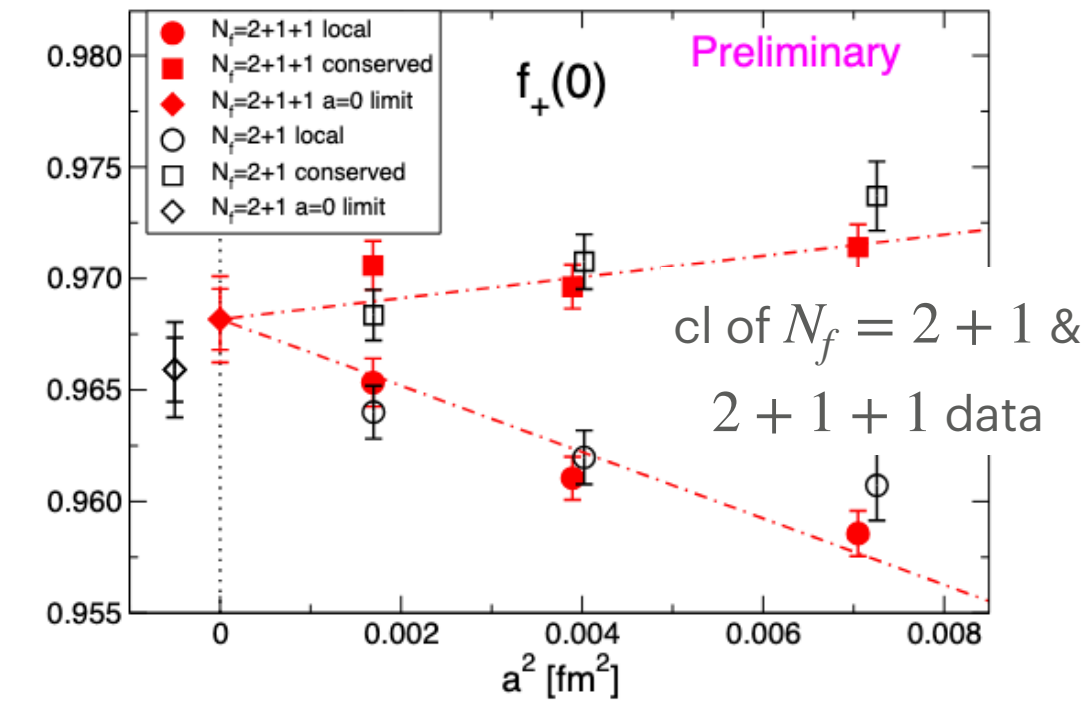
Kaon semileptonic decay $K \rightarrow \pi \ell \nu_\ell$

determination of LECs and $|V_{us}|$

PACS collaboration based on [\[PRD 101 \(2020\), PRD 106 \(2022\), PoS \(LATTICE2024\)\]](#)

- $N_f = 2 + 1$ & $2 + 1 + 1$, Wilson M_π^{phys} , $a \sim 0.04, 0.06, 0.08$, $L \sim 10$ fm
- PDG input: π^- , K^0 , Ξ^0
- NLO χ PT fits
- IB corrections NLO ChPT smaller than stat. error
- sea-charm effects smaller than stat. error

Talk by Takeshi Yamazaki

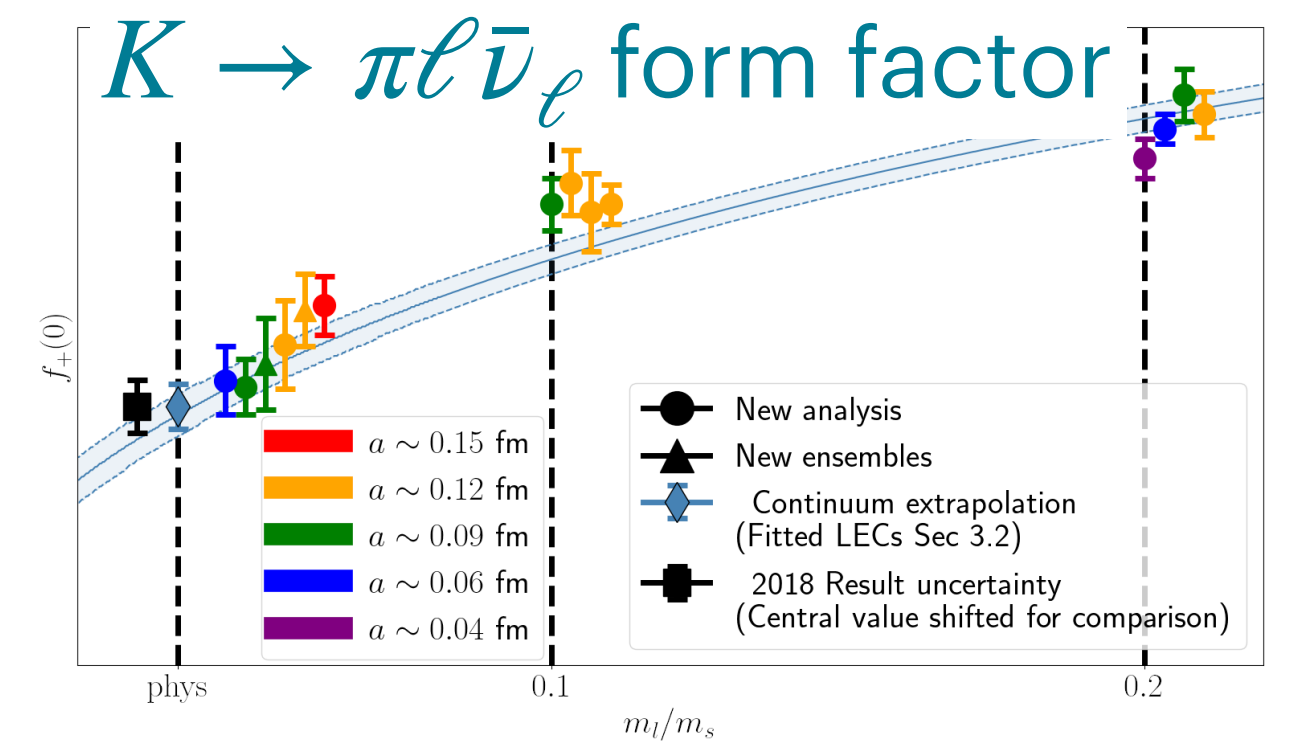
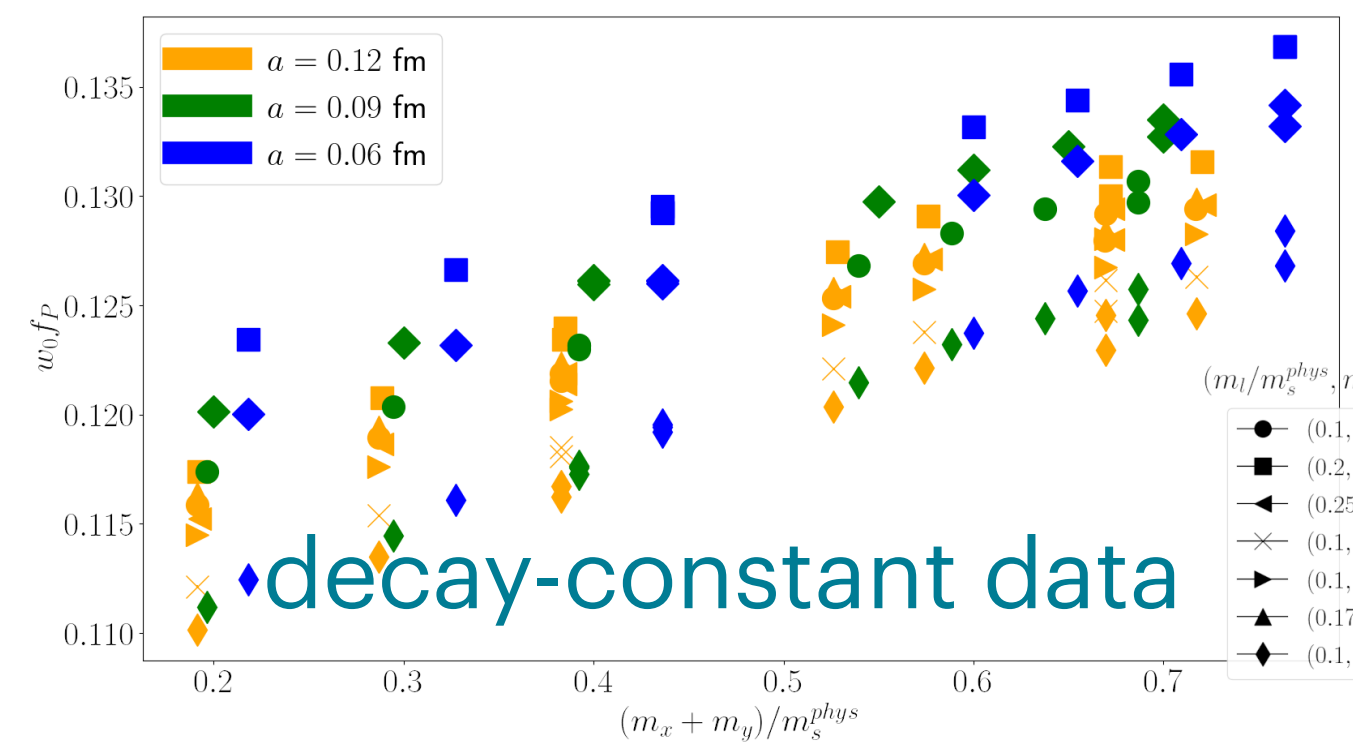


FNAL/MILC Collaboration

(Material from Ramón Merino)

Talk by Ramón Merino

- FNAL/MILC HISQ $N_f = 2 + 1 + 1$ ensembles
- multiple PQ light and strange masses:
 $m_l = (1/20, \dots, 1/5)m_s^{\text{phys}}$, $m_s = (1/10, \dots, 3/5)m_s^{\text{phys}}$
- fit ansatz with terms NLO PQSChPT, NNLO PQChPT, analytic terms $O(\alpha_s a^2, \alpha_s a^2 m, m^2)$
- $\rightarrow F_0, B_0, L_4, L_5, 2L_6 - L_4, 2L_8 - L_5 + \text{PQChPT-const.}$
- also updated $f_+(0)$ analysis correlated with LECs



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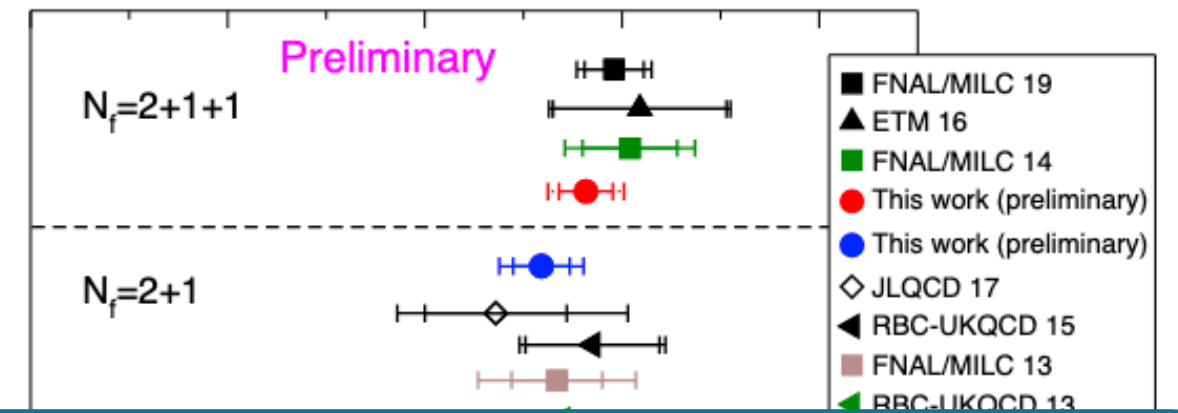
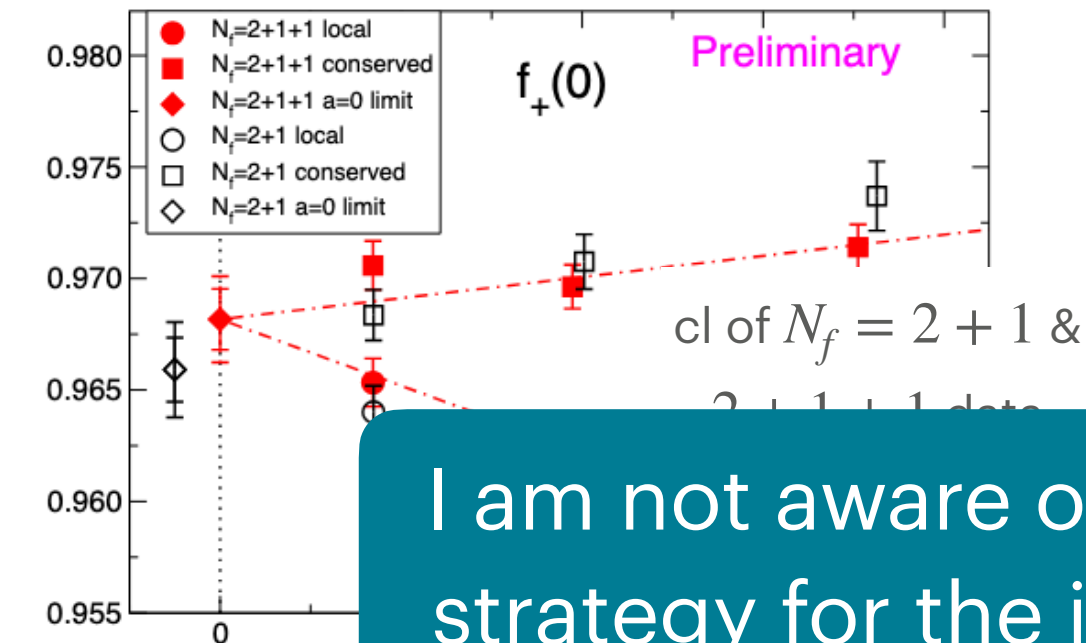
FNAL/MILC Collaboration

(Material from Ramón Merino)

Talk by Ramón Merino

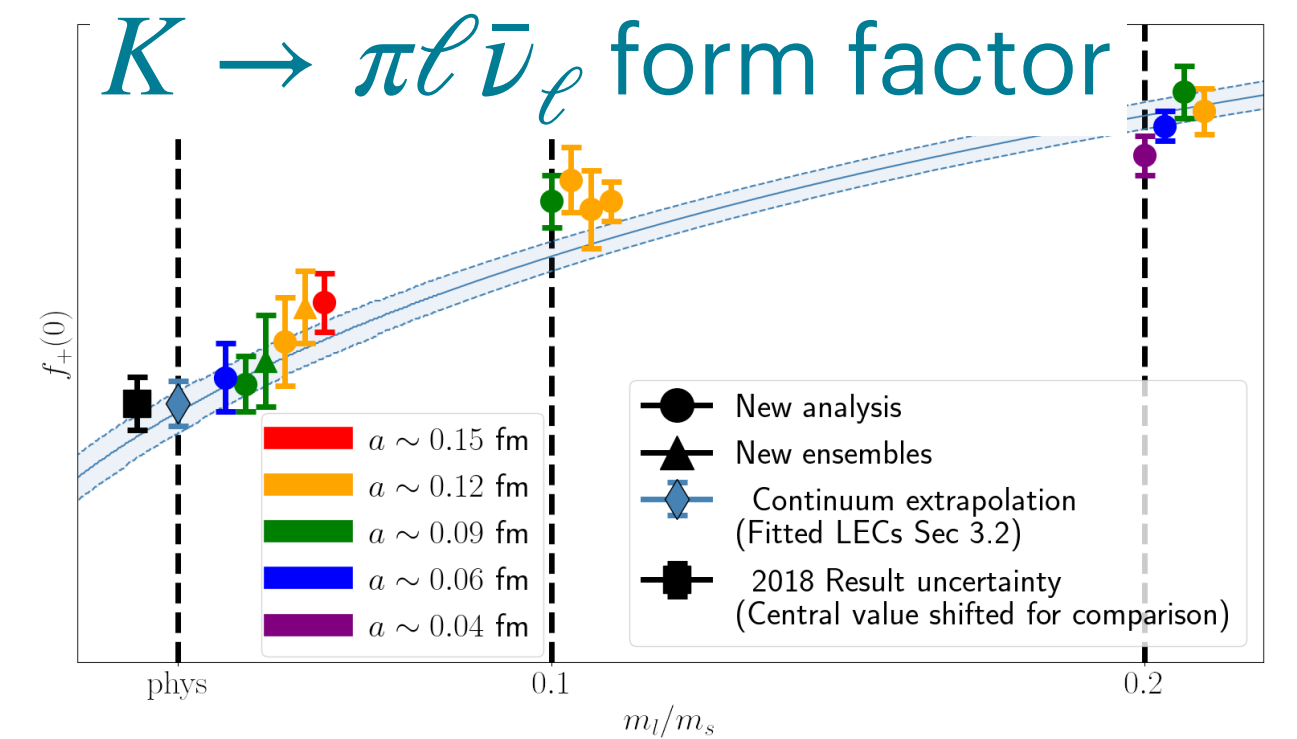
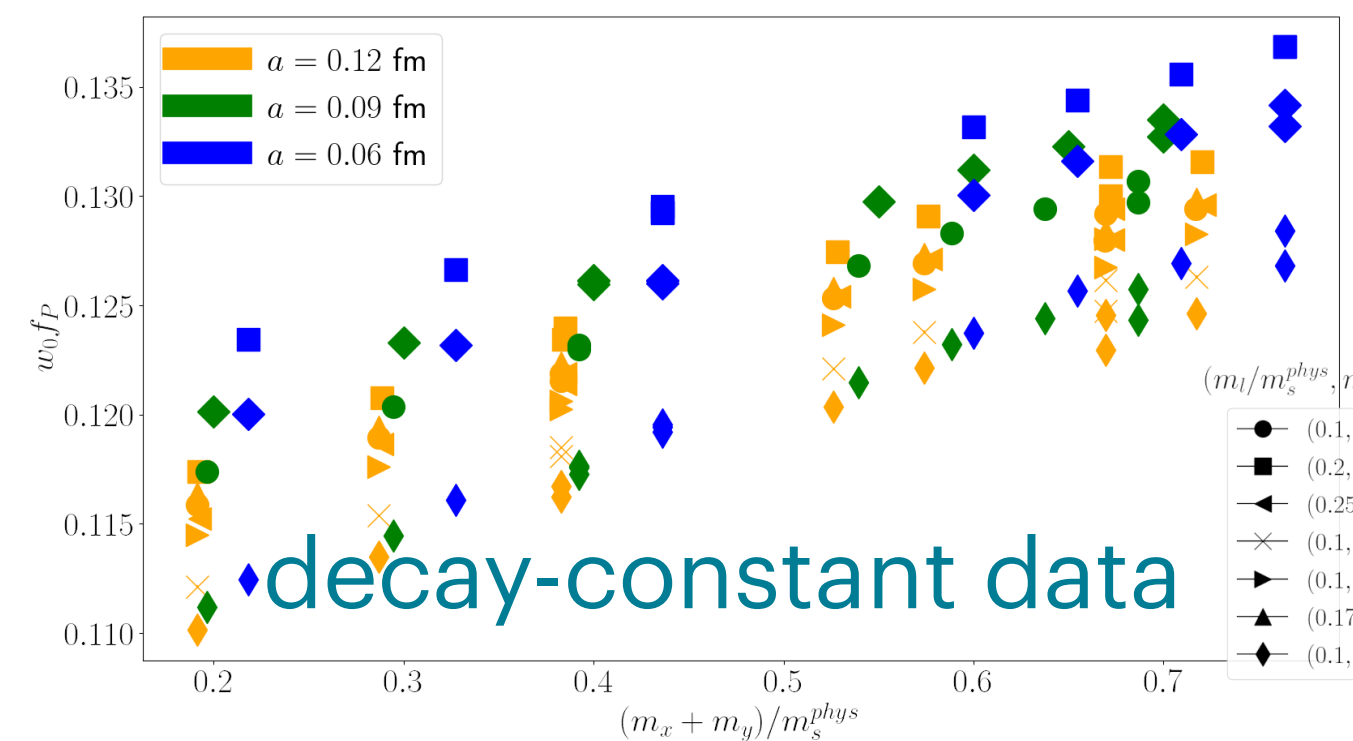
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- also updated $f_+(0)$ analysis correlated with LECs

Talk by Takeshi Yamazaki



I am not aware of a fully developed or executed strategy for the inclusion of IB/QED corrections for this quantity (and any semileptonic decay with charged leptons final states).

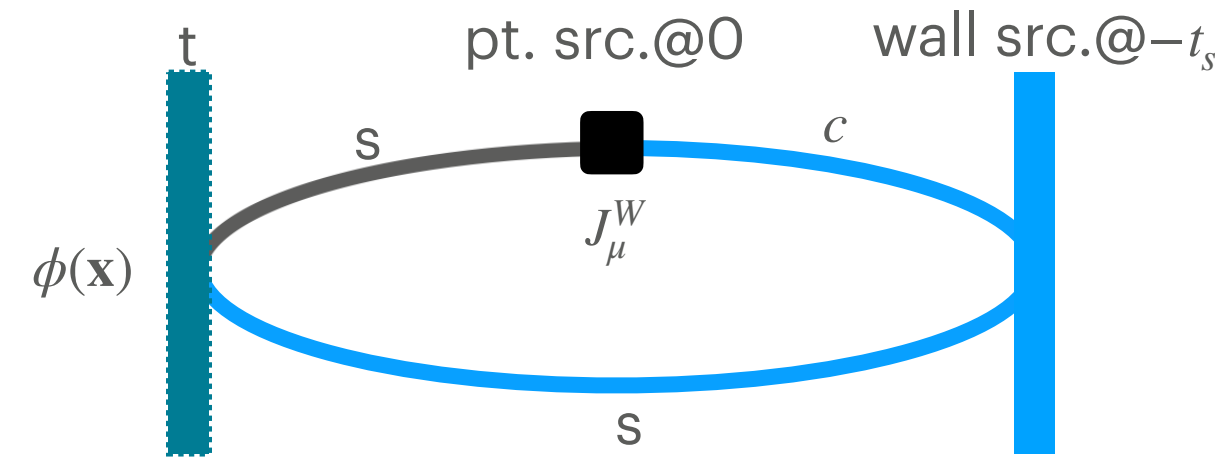
See [Christ et al., [arXiv:2402.08915](#)]



D_s semileptonic determination of $|V_{cs}|$

$$D_s \rightarrow \phi \ell \nu_\ell \text{ [Fan et al. arXiv: 2510.14478]}$$

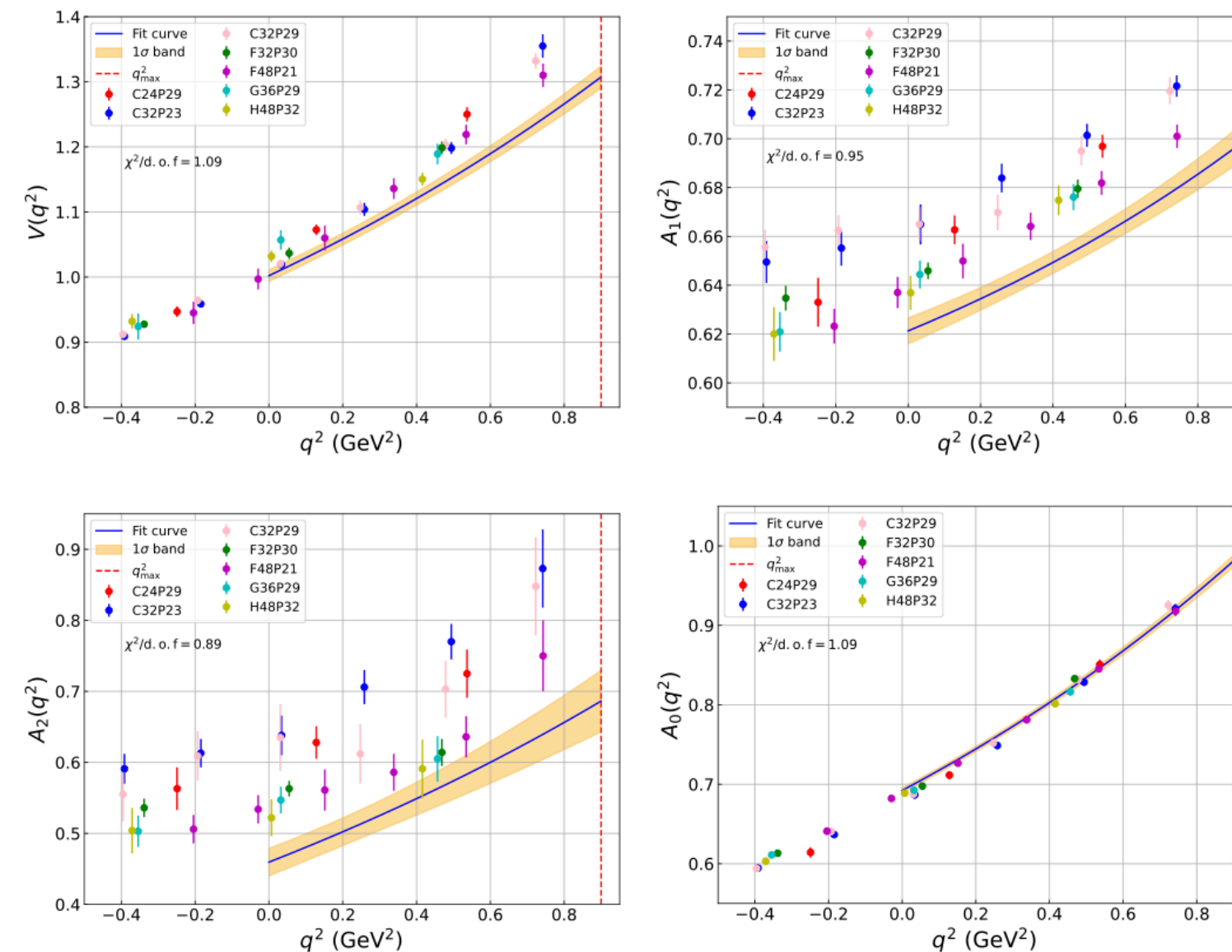
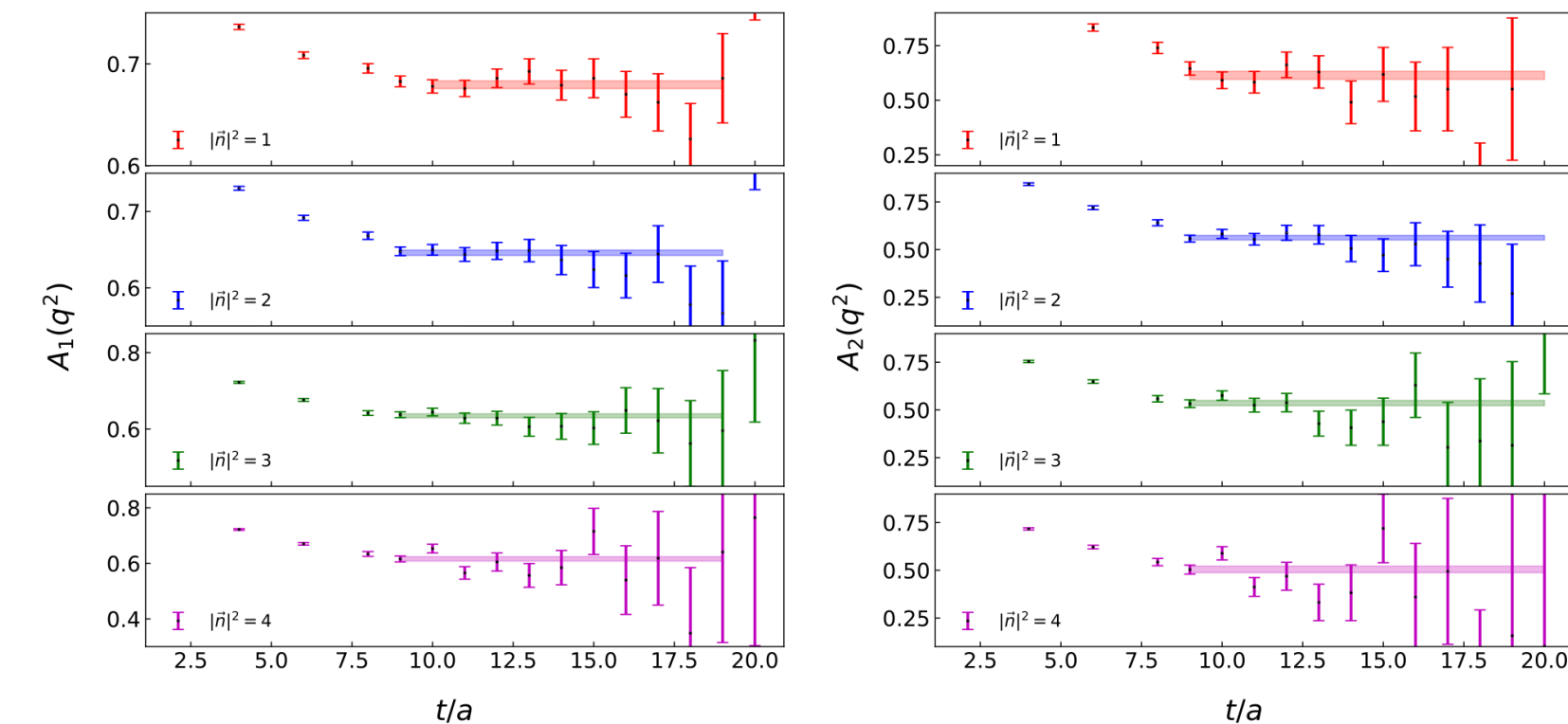
- seven $N_F = 2 + 1$ CLQCD Wilson Clover ensembles
 $a = 0.052 - 0.105 \text{ fm}$, $M_\pi = 210 - 320 \text{ MeV}$
- Measured by BaBar, CLEO, BESIII
- scalar-function analysis [Tuo et al. PRD 105 (2022)]
only connected contributions
- ground-state fits (various src-sink seps. up to $1.6 - 2.0 \text{ fm}$)
(long time-range, but maybe an ES analysis would still be helpful to ensure GS correctly identified)



$$a \approx 0.078, M_\pi \approx 300 \text{ MeV}$$

(i) $V(q^2)$ at F32P30.

(ii) $A_0(q^2)$ at F32P30.



$|V_{cs}|$ determination

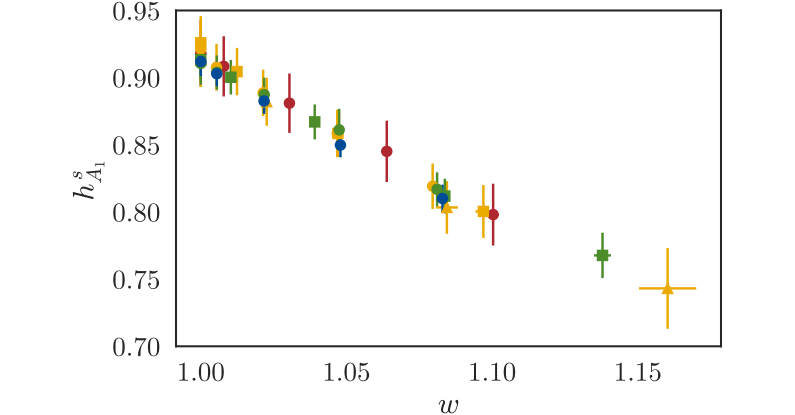
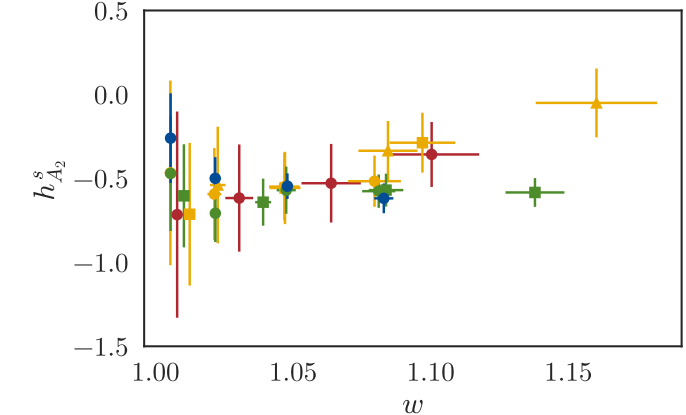
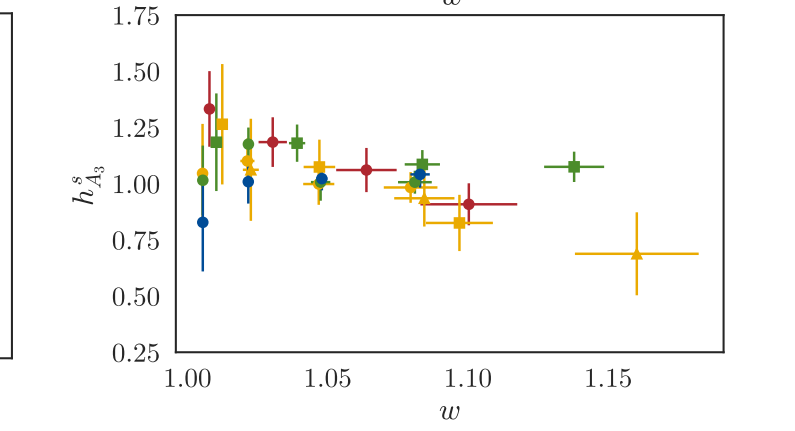
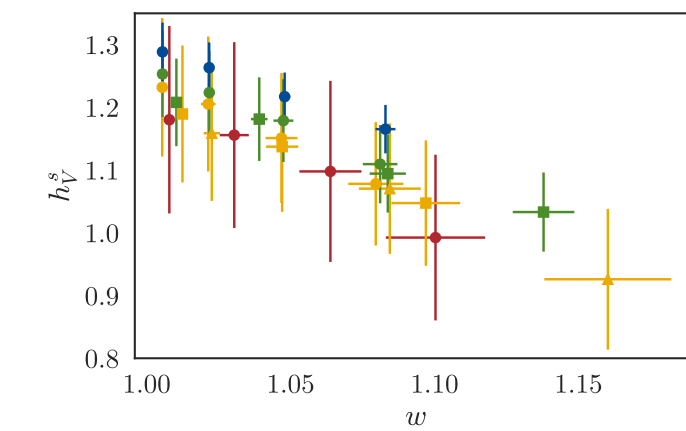
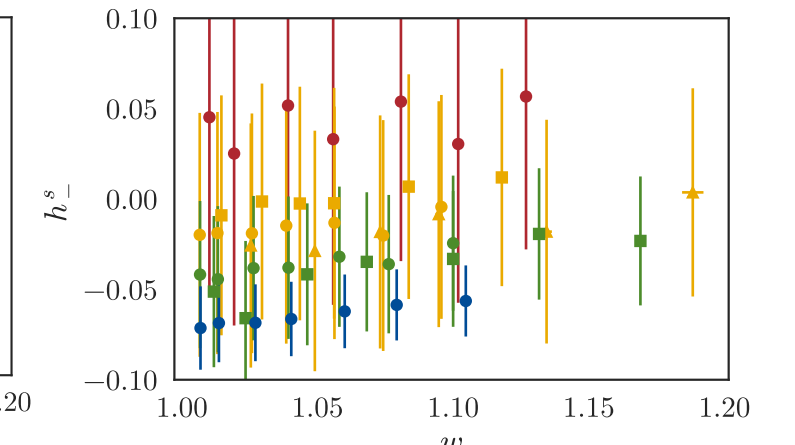
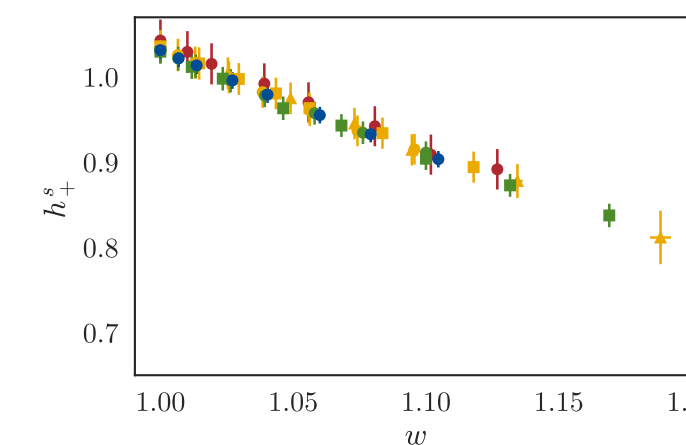
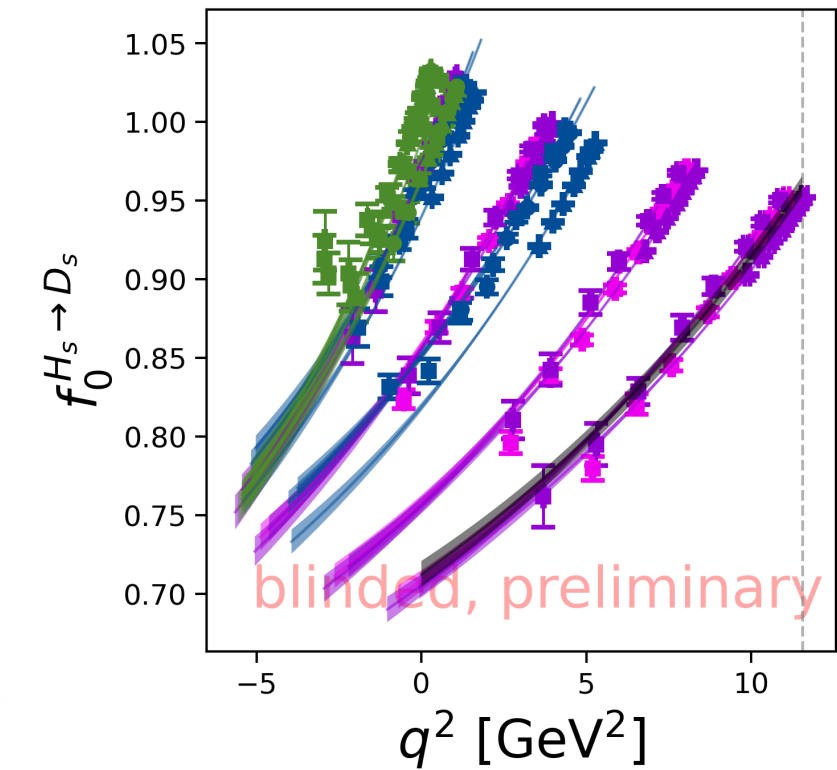
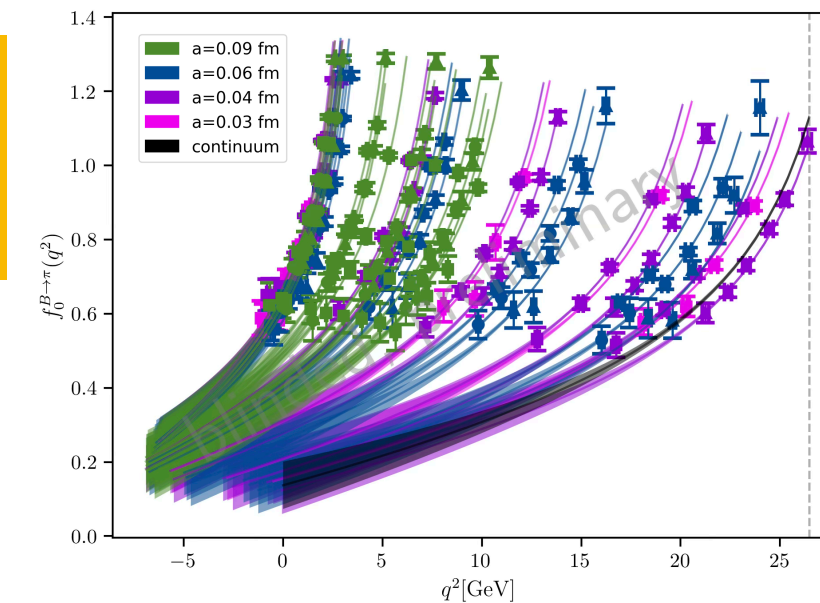
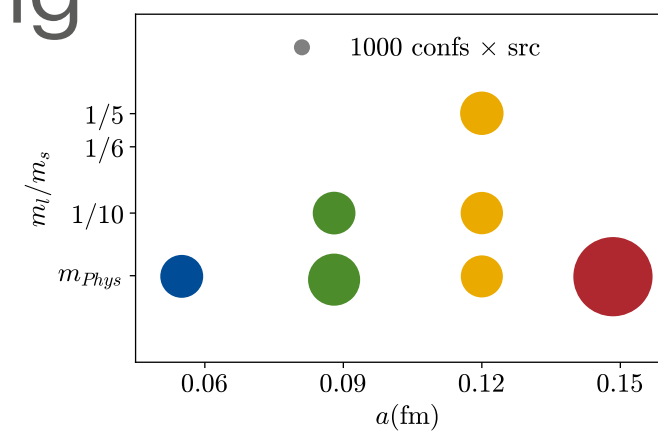
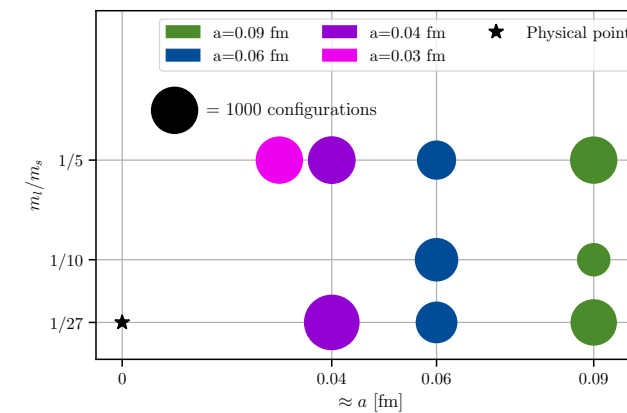
Unitarity		0.97349 ± 0.00016	Phys. Rev. D 110, 030001 (2024)
HFLAV		0.9701 ± 0.0081	arXiv:2411.18639
PDG		0.975 ± 0.006	Phys. Rev. D 110, 030001 (2024)
BESIII		0.9623 ± 0.0046	Phys. Rev. D 110, 112006 (2024)
HPQCD		1.017 ± 0.064	Phys. Rev. D 90, 074506 (2014)
This work		0.945 ± 0.027	This work, e channel
This work		0.952 ± 0.026	This work, μ channel

$B_{(s)}$ semileptonics

determination of $|V_{ub}|$, $|V_{cb}|$, rare decay

- $N_f = 2 + 1 + 1$ FNAL/MILC HISQ, $a = 0.03 - 0.09\text{fm}$
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Talks by Nicholas Cassar
and Akhil Chauhan

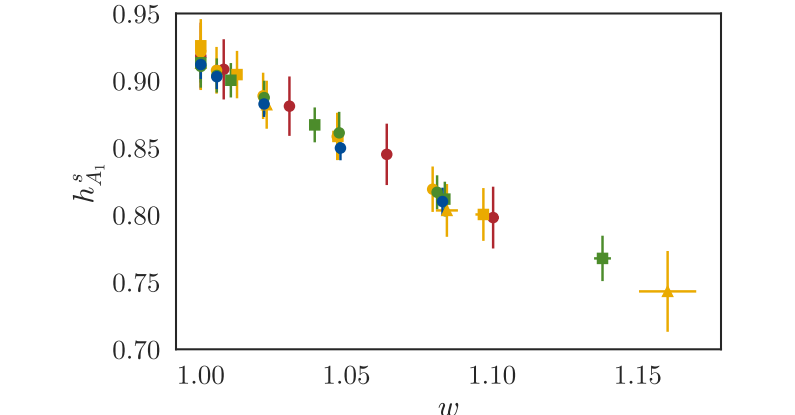
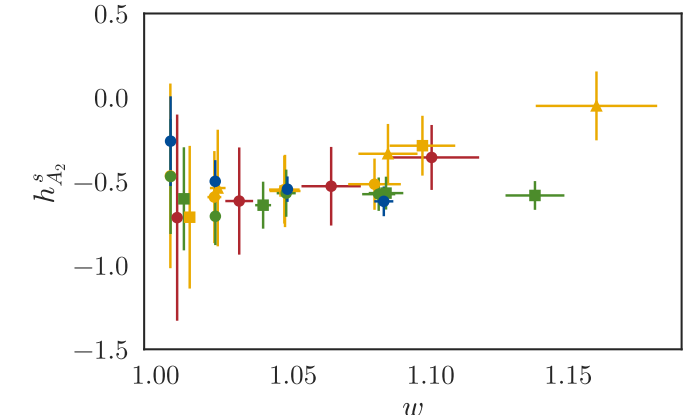
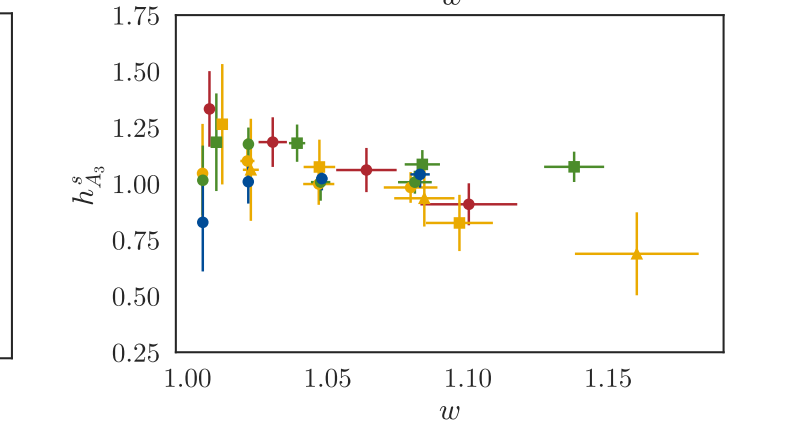
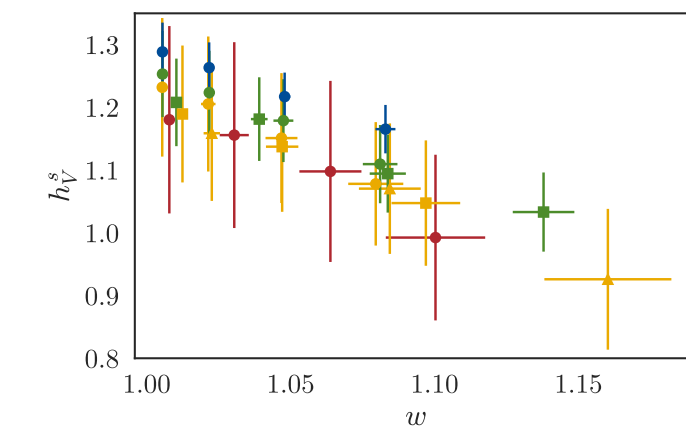
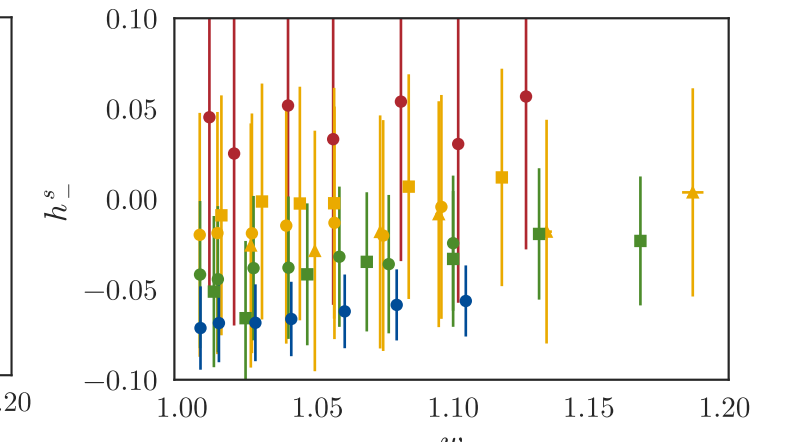
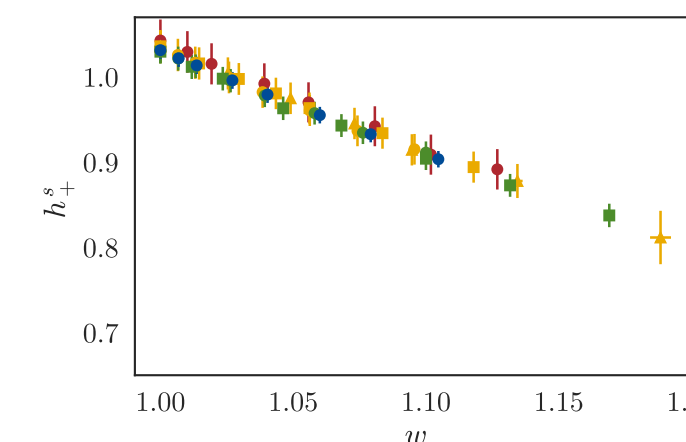
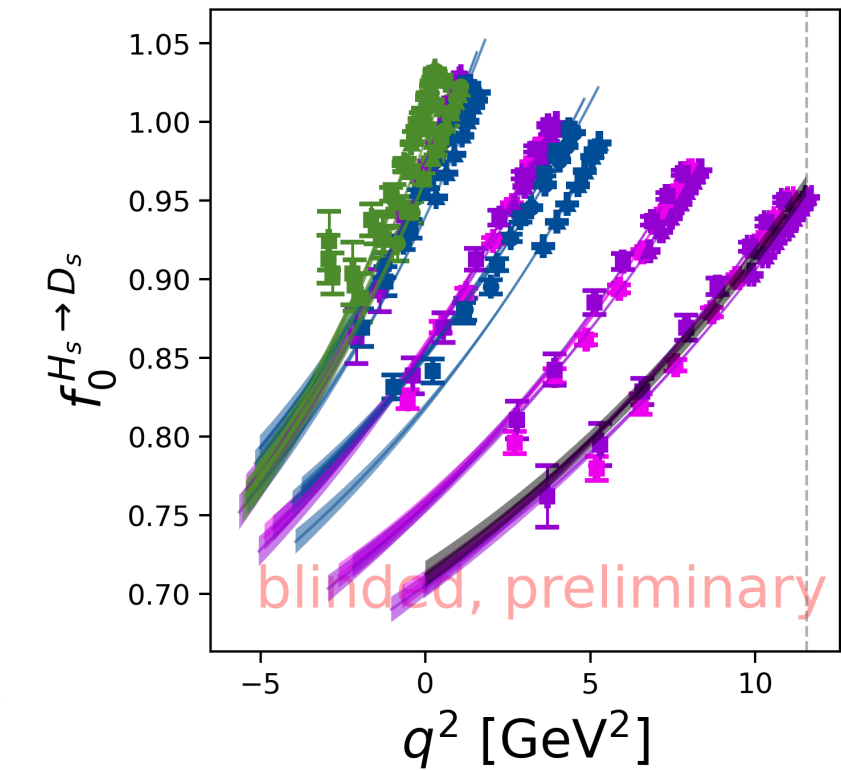
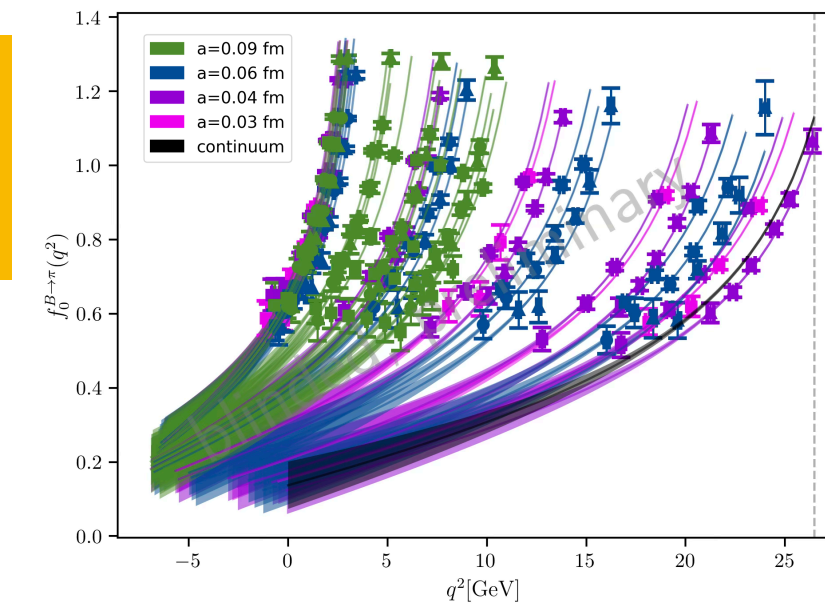
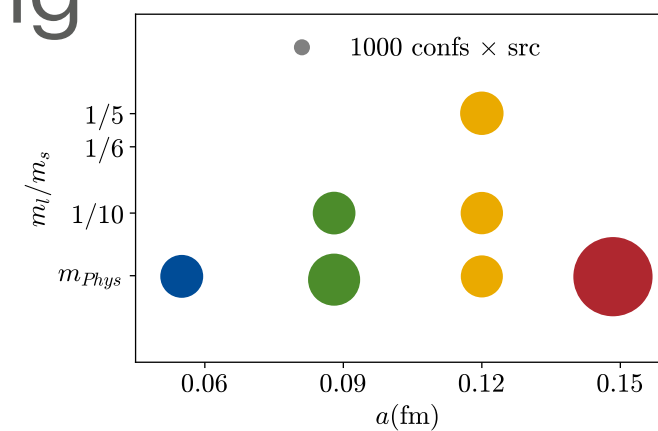
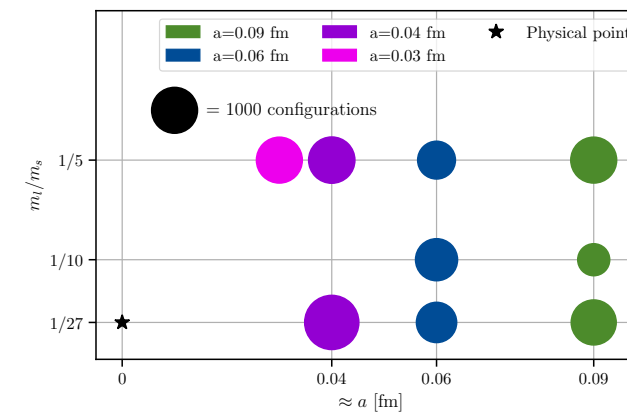


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Talks by Nicholas Cassar
and Akhil Chauhan



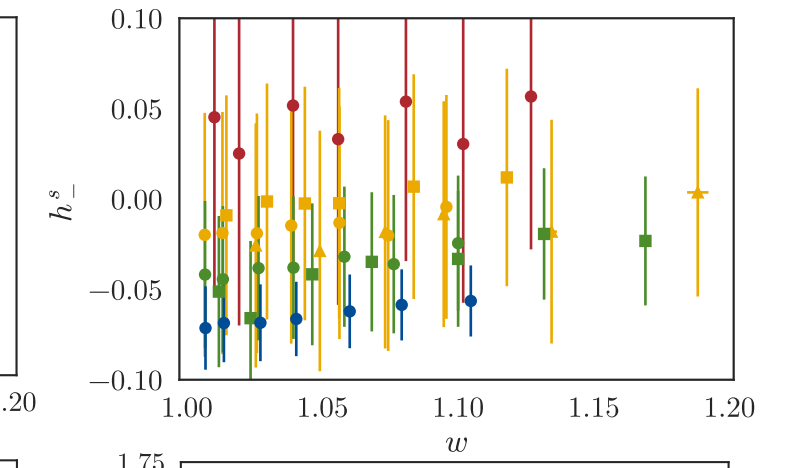
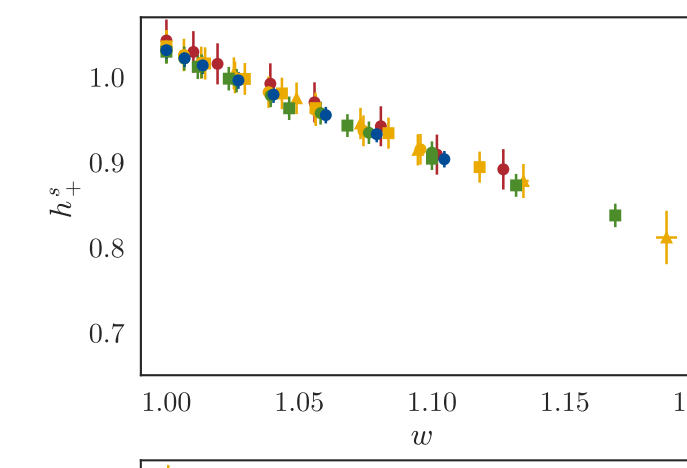
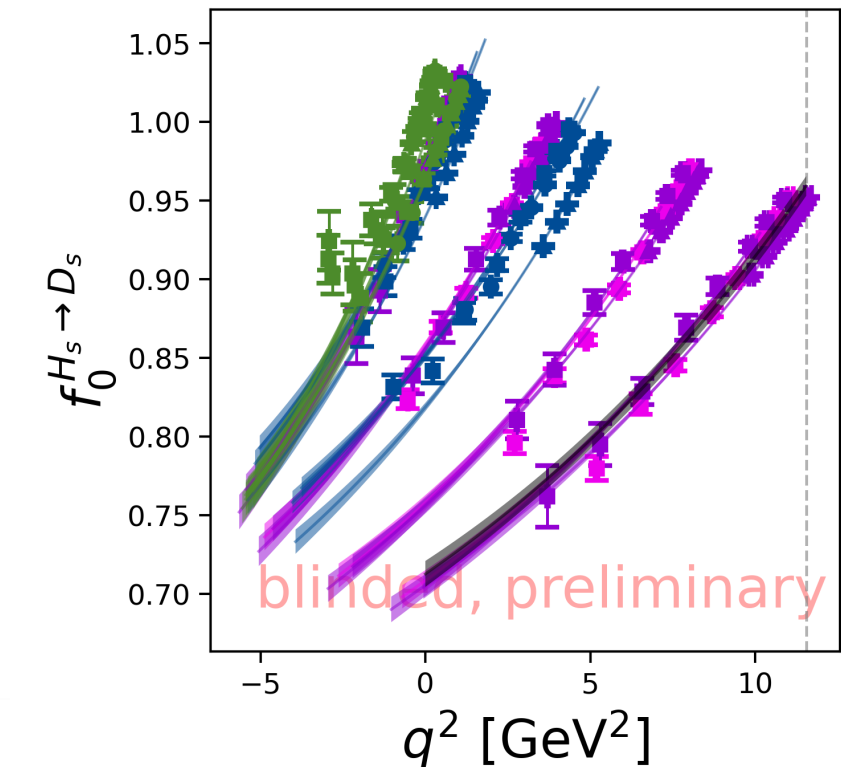
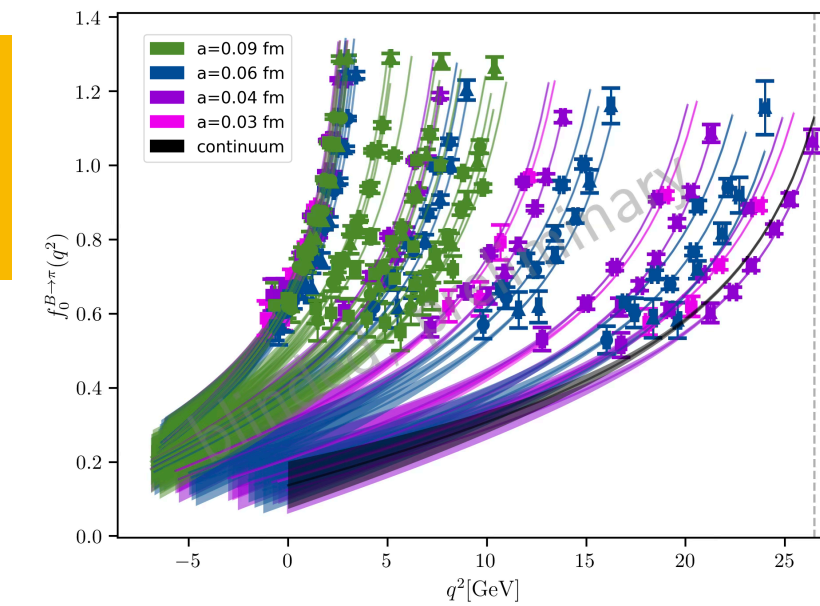
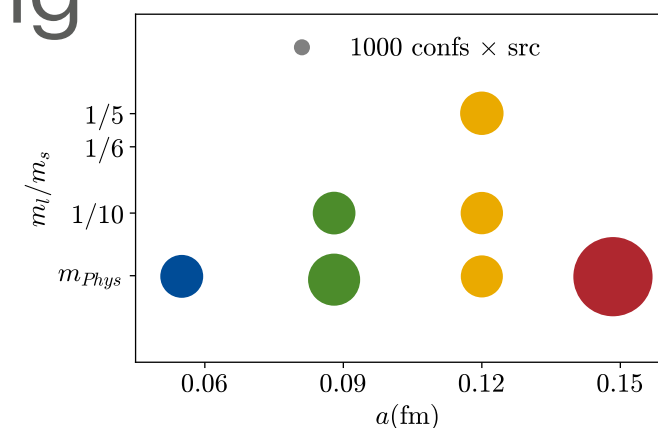
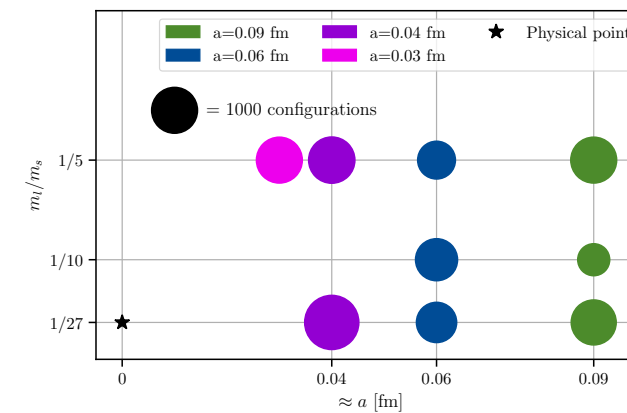
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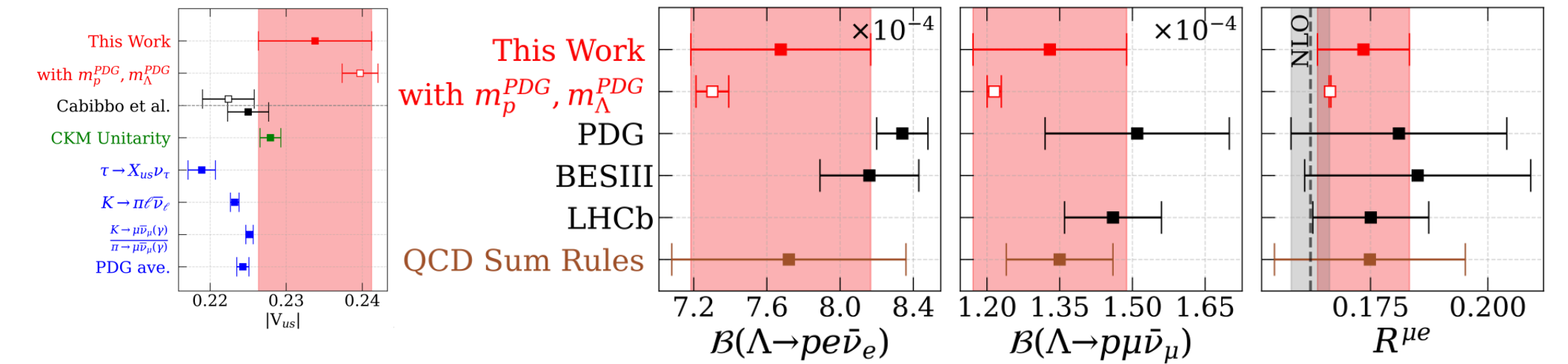
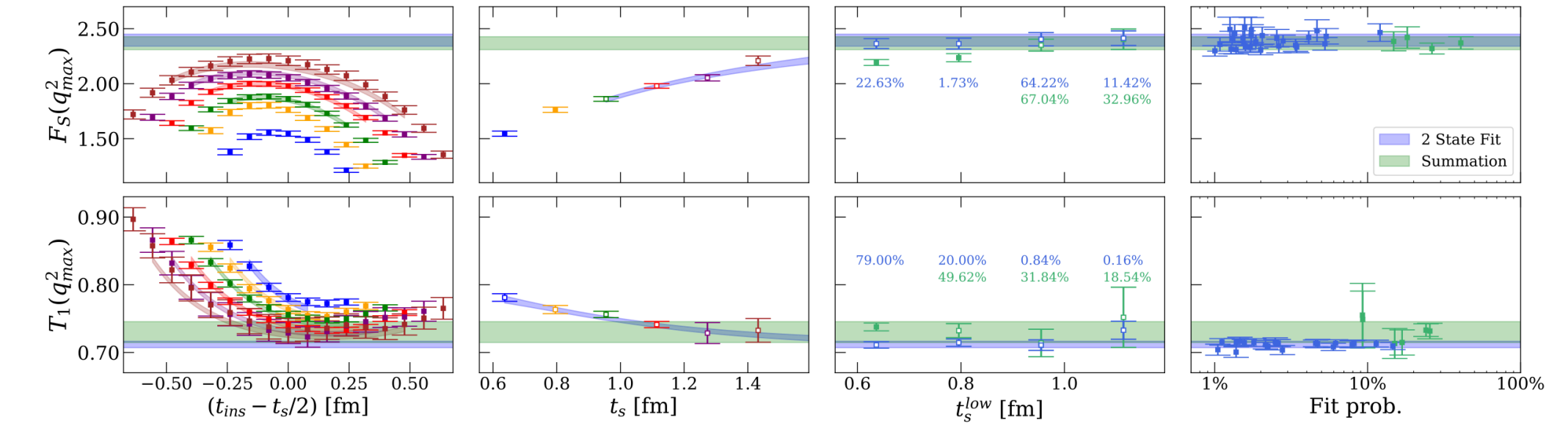
I am eagerly looking forward to these new results coming out.
Tensions amongst form factor computations for SL D and $B_{(s)}$ decay discussed already in Tobi Tsang's Liverpool talk still persist and need to be resolved urgently!

- KEK/Osaka talk on $B_s \rightarrow K\ell\nu$ by Junya Tokoro
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Baryon transitions

$\Lambda \rightarrow p \ell \nu$ [Bacchio, Konstantinou [PRL 135 \(2025\)](#), Alexandrou et al. [PRD 114 \(2026\)](#)]

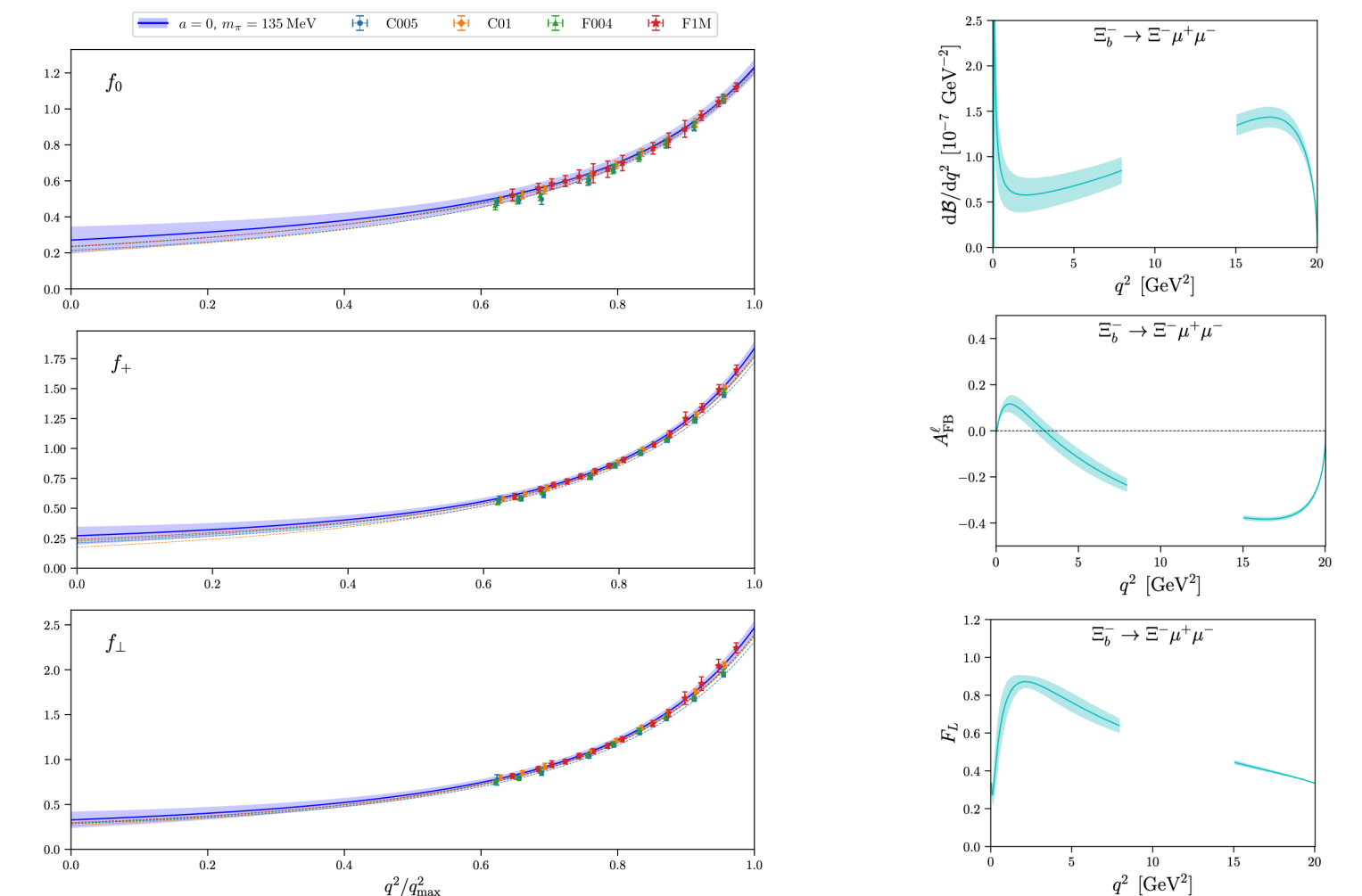
- $s \rightarrow u$ transition $\rightarrow |V_{us}|$ (not yet competitive)
- experimentally measured at BESIII and LHCb
- **single ensemble** $N_f = 2 + 1 + 1$, $a = 0.08\text{fm}$, $\mathbf{M}_\pi^{\text{phys}}$
- ES: summation method and ES-fits
- Good agreement with experiment, with relatively large errors



$\Xi_b^+ \rightarrow \Xi \mu \mu$, $\Xi_b \rightarrow \Xi \gamma$ [Farrell, Meinel [PRD 111 \(2025\)](#)], [Farrell, Meinel [PRD 113 \(2026\)](#)]

- $N_f = 2 + 1(\text{DWF})$, $a = 0.002 - 0.01$, $M_\pi > 230\text{MeV}$, RHQ b dark
- ‘chiral-continuum-kinematic’ fit based on BGL-type z -expansion
- rare $b \rightarrow s$ transition **complementary to**
 $B \rightarrow K^{(*)} \mu \mu$, $B_s \rightarrow \phi \mu \mu$, where tensions seen
- initial state polarised, plenty of Dirac structure, angular observables
- being analysed by LHCb, thus LQCD results are proper *prediction*

Talk by Stefan Meinel



Some remarks on Part I

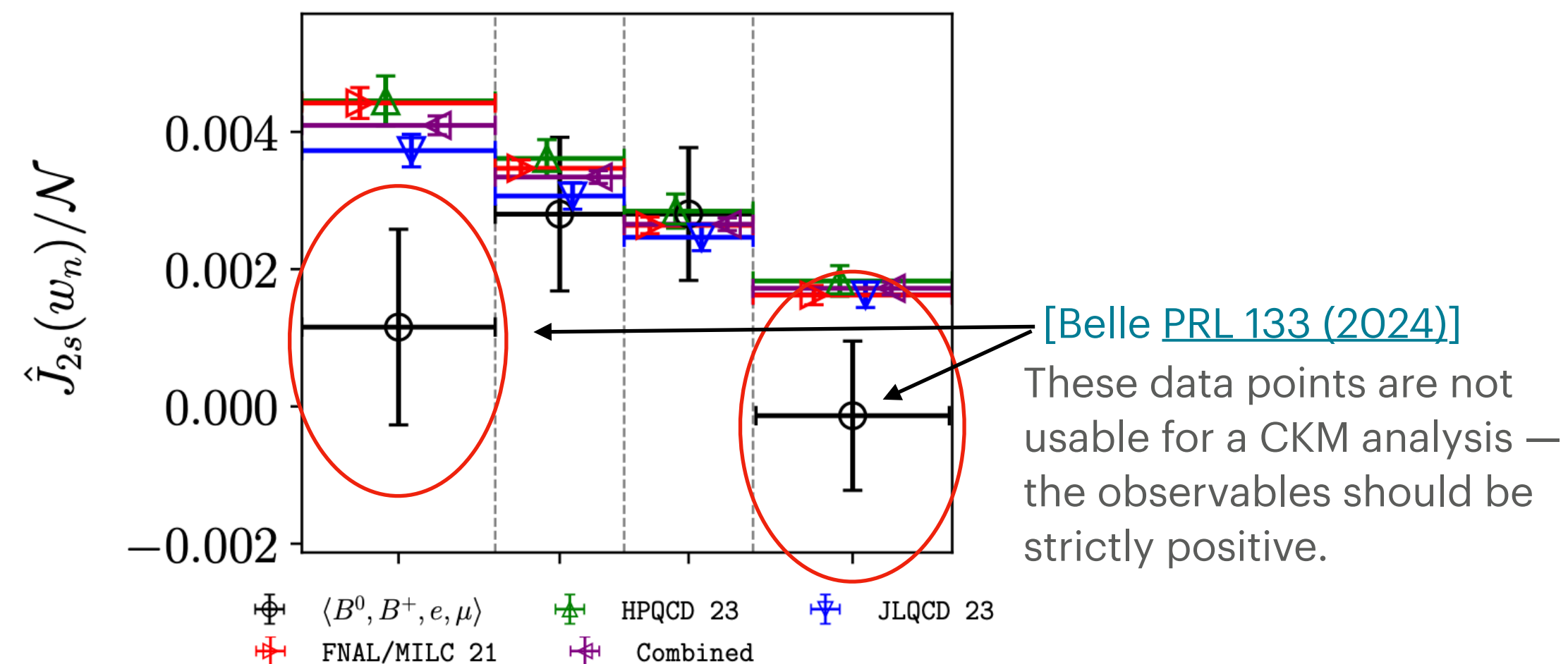
Data representation:

We need better access to th. & exp. data for reproducibility and data comprehension. Experiment and theory should

- provide data in digital format (e.g. [Zenodo](#), [HEPdata](#)) together with code (e.g. fit functions with parameters)
- central values + covariance matrix is not always enough — we need histograms or resampling data

$B \rightarrow D^* \ell \nu_\ell$ angular analysis

[Bordone, AJ, Ollie Heald et al. [arXiv:2606.23410](#)]



Some remarks on Part I

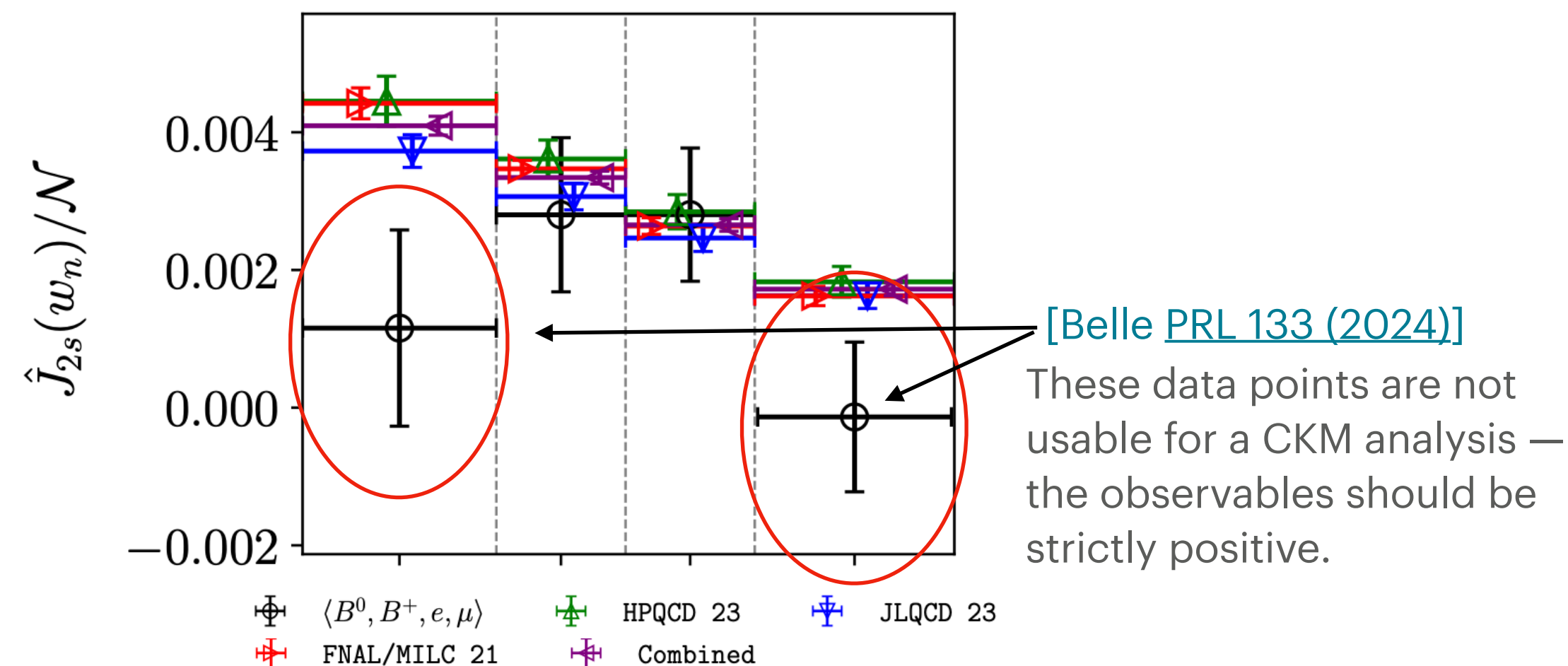
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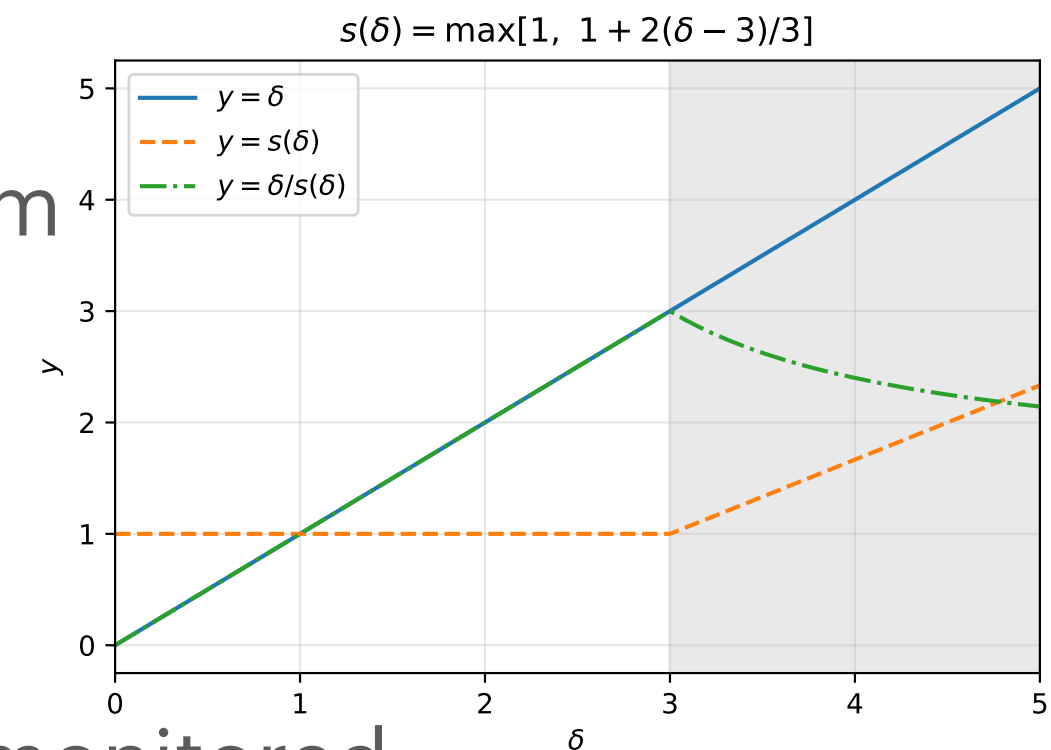
[Bordone, AJ, Ollie Heald et al. [arXiv:2606.23410](#)]



Continuum limit:

FLAG considering data-driven criterium

$$\delta(a) \equiv \frac{|Q(a) - Q(0)|}{\sigma_Q}$$



- not currently used as criterium but monitored where possible
 - error inflation $s(\delta) = \max[1, 1 + 2(\delta - 3)/3]$ in FLAG averages?
 - required information hard/possible to extract (complicated global fits, a , m_q , kinematics, ...)
 - Related idea: provide less complex analysis also for simple benchmark quantities (e.g. $f(q_{\max}^2)$, $f(q_{\text{ref}}^2)$, ...)
- see [Della Morte, Tsang [EPJST 233 \(2024\)](#)]

Some remarks on Part I

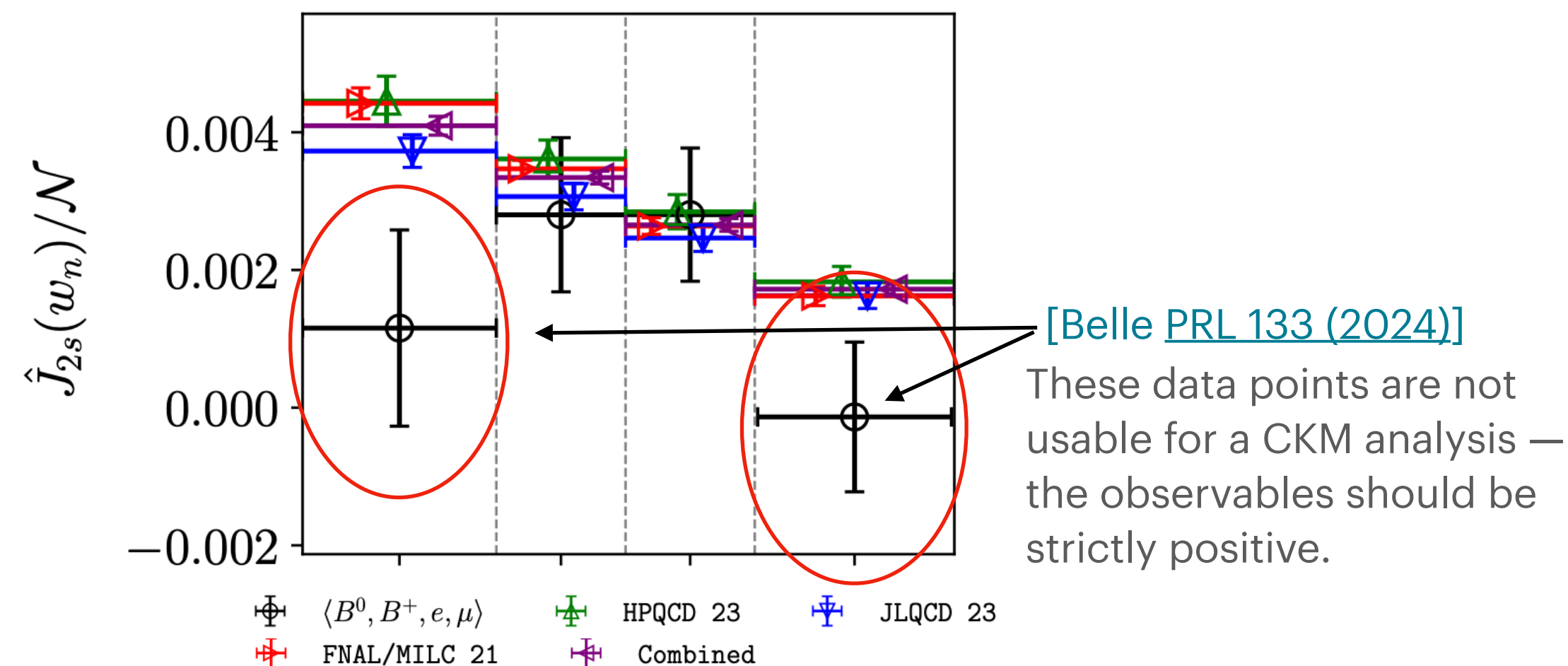
Data representation:

We need better access to th. & exp. data for reproducibility and data comprehension. Experiment and theory should

- provide data in digital format (e.g. [Zenodo](#), [HEPdata](#)) together with code (e.g. fit functions with parameters)
- central values + covariance matrix is not always enough — we need histograms or resampling data

$B \rightarrow D^* \ell \nu_\ell$ angular analysis

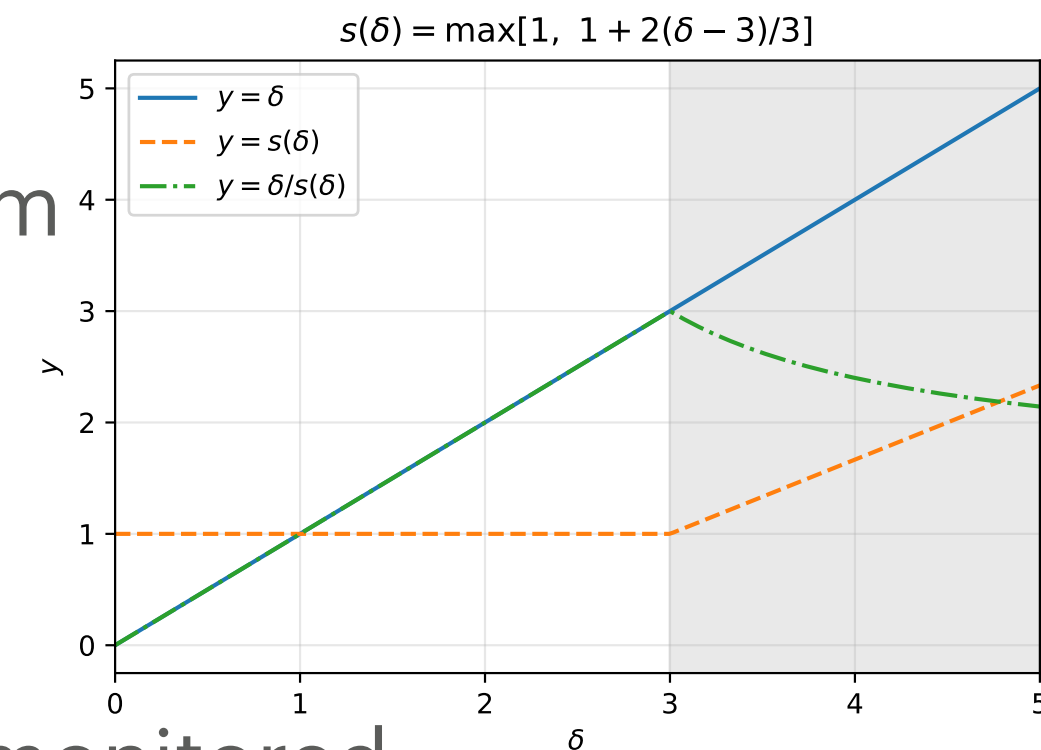
[Bordone, AJ, Ollie Heald et al. [arXiv:2606.23410](#)]



Continuum limit:

FLAG considering data-driven criterium

$$\delta(a) \equiv \frac{|Q(a) - Q(0)|}{\sigma_Q}$$



- not currently used as criterium but monitored where possible
 - error inflation $s(\delta) = \max[1, 1 + 2(\delta - 3)/3]$ in FLAG averages?
 - required information hard/possible to extract (complicated global fits, a , m_q , kinematics, ...)
 - Related idea: provide less complex analysis also for simple benchmark quantities (e.g. $f(q_{\max}^2)$, $f(q_{\text{ref}}^2)$, ...)
- see [Della Morte, Tsang [EPJST 233 \(2024\)](#)]

Some remarks on Part I

**We need to extend set of observables
for which we control QED/IB corrections**

inclusive τ decay **Talk by Erik Bäske**

radiative decays **Talk by Xin-Yu Tuo**

systematics **Talks by Davide Giusti,
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ϵ'/ϵ [[Christ, Lundstrum arxiv:2606.26336](#)]

a lot of activity in context of $(g - 2)_\mu$

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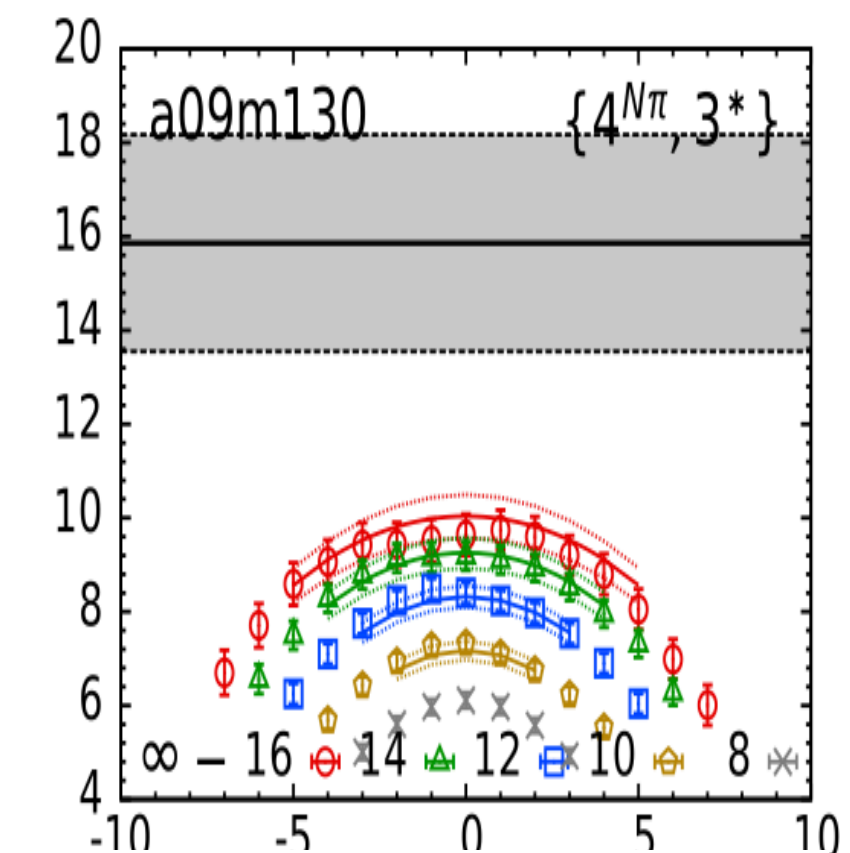
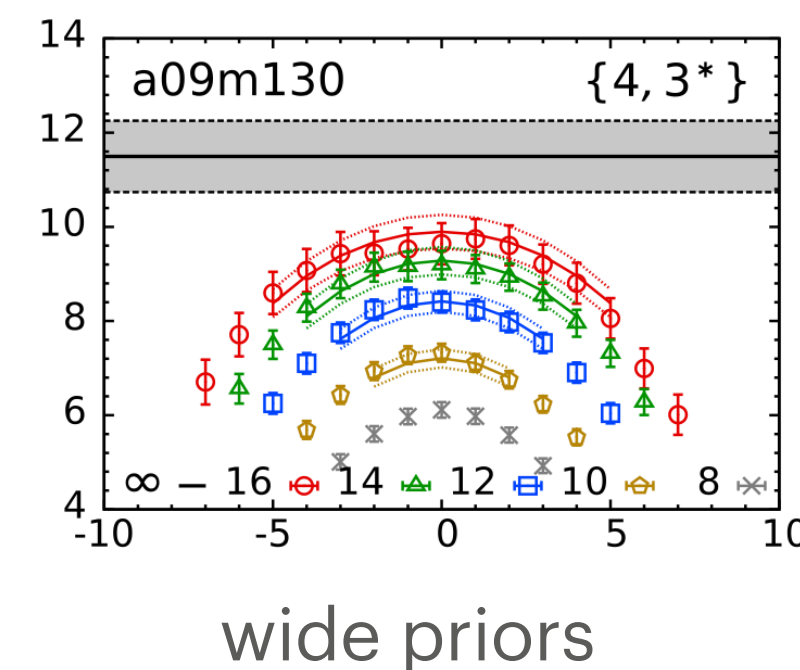
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Excited-state effects in form-factor analysis

[Tiburzi [PRD 91 \(2015\)](#)], [Bär et al. [EPJC 83 \(2023\)](#), [PRD 103 \(2021\)](#),
[PRD 101 \(2020\)](#), [PRD 100 \(2019\)](#), [PRD 99 \(2019\)](#)]

- NME community has long recognised ES effects as significant and challenging systematic effect
- EFT can provide excellent guidance
- loads of work in NME community
see talk by [[Lorenzo Barca at CERN 2026](#), also [PRD 112 \(2025\)](#)]

Two good fits for $\sigma_{\pi N}$,
two different results



[Gupta et al. [PRL127 \(2021\)](#)]

narrow-width priors
including free $N\pi$, $N\pi\pi$

Excited states in $B \rightarrow \pi \ell \nu_\ell$

[Bär et al. [EPJC 83 \(2023\)](#)]. [D'Anna et al. [POS 2026 246](#)]

preliminary results by Oliver Bär, Antoine Gérardin, Simon Kuberski, Rainer Sommer

Two analysis methods:

1. ratio with overlaps from 2pt-fit

$$\mathcal{M}_\mu(t_\pi, t_h; \mathbf{p}) = \frac{4E_\pi(\mathbf{p})C_\mu^{(3)}(t_\pi, t_h; \mathbf{p})}{\langle 0 | \hat{\mathcal{O}}_\pi | \pi \rangle \langle B | \hat{\mathcal{O}}_B | 0 \rangle} e^{E_B t_h} e^{E_\pi(\mathbf{p}) t_\pi}$$

HM χ PT predicts large ES state contamination

$$\mathcal{M}_k(t_\pi, t_h; \mathbf{p}) = p_k h_\perp(\mathbf{p}) (1 + \delta h_\perp(t_\pi, t_h; \mathbf{p}) + \dots)$$

$$\delta h_\perp(t_\pi, t_h; \mathbf{p}) = -e^{-E_\pi(\mathbf{p}) t_h} + \text{NLO} + \dots$$

2. GEVP for 2pt determines ground-state EV $v_{0,j}(t, t_0)$

$$\mathcal{M}_\mu(t_\pi, t_h; \mathbf{p}) = \frac{e^{E_B t_h/2}}{\sqrt{\lambda_0(t_h, t_0)}} \frac{R_1(t_h)}{\langle 0 | \mathcal{O}_\pi | \pi \rangle} C_{\mu,j}^{(3)}(t_\pi, t_h; \mathbf{p}) v_{0,j}(t_h, t_0)$$

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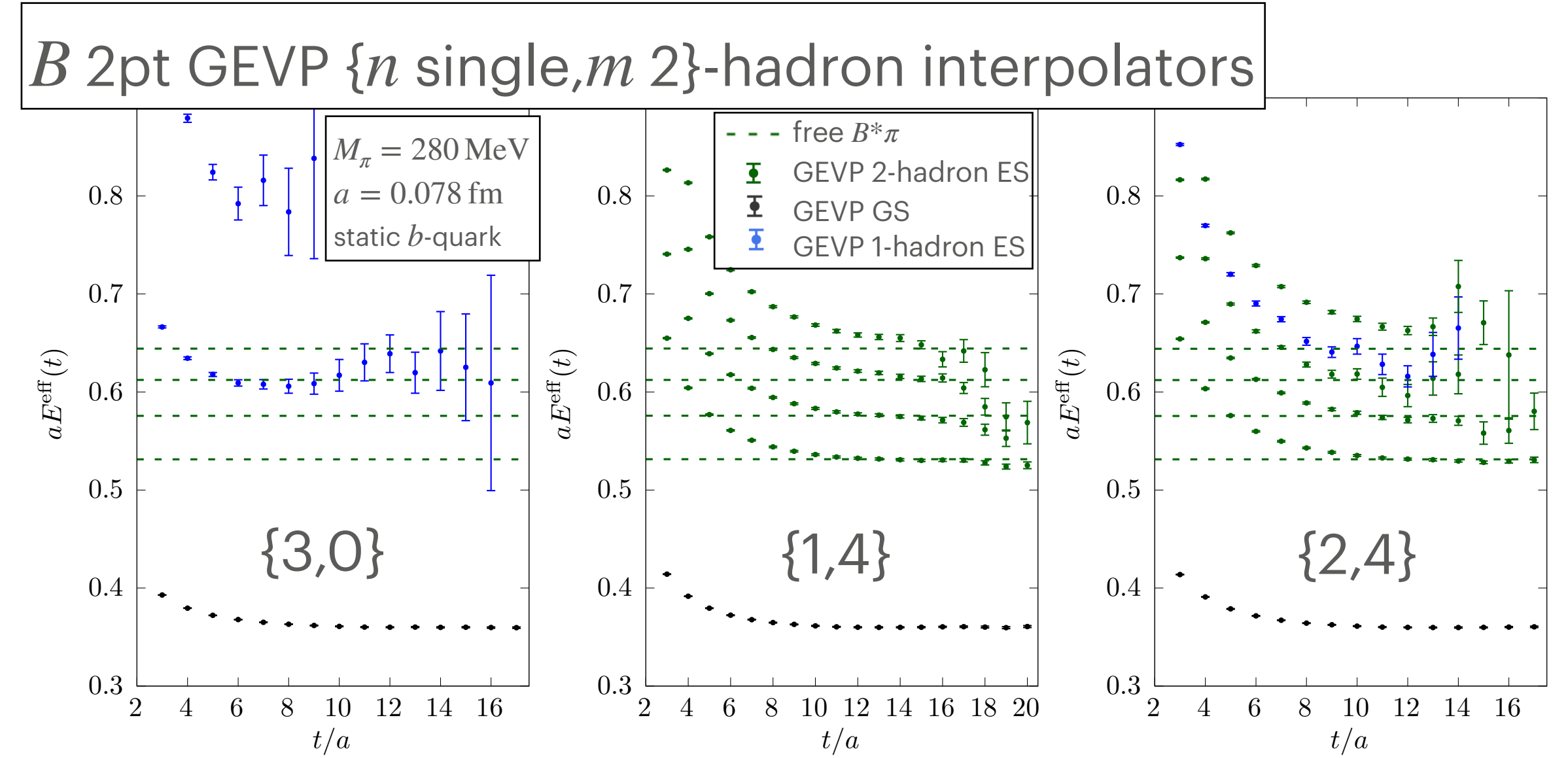
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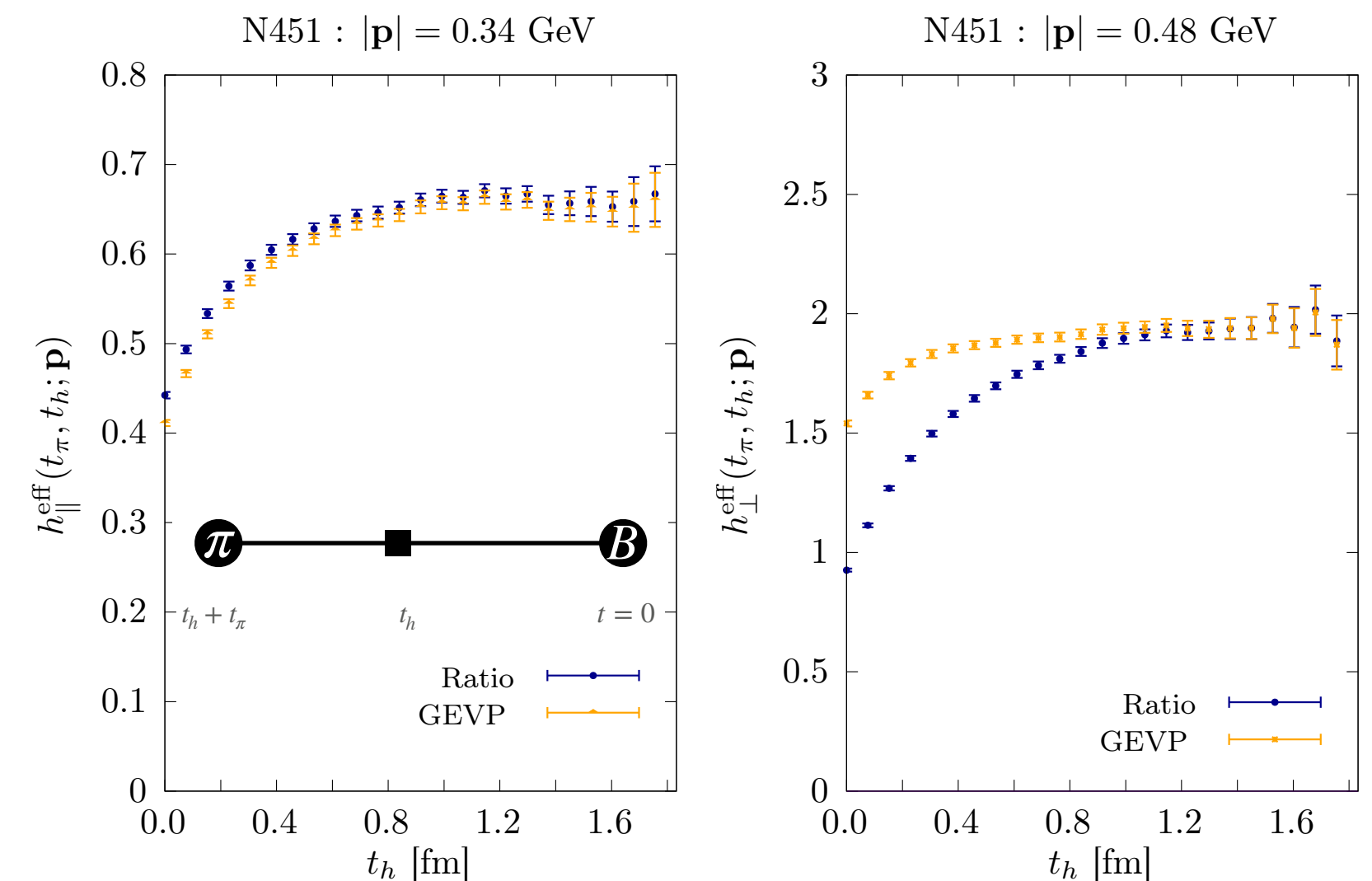
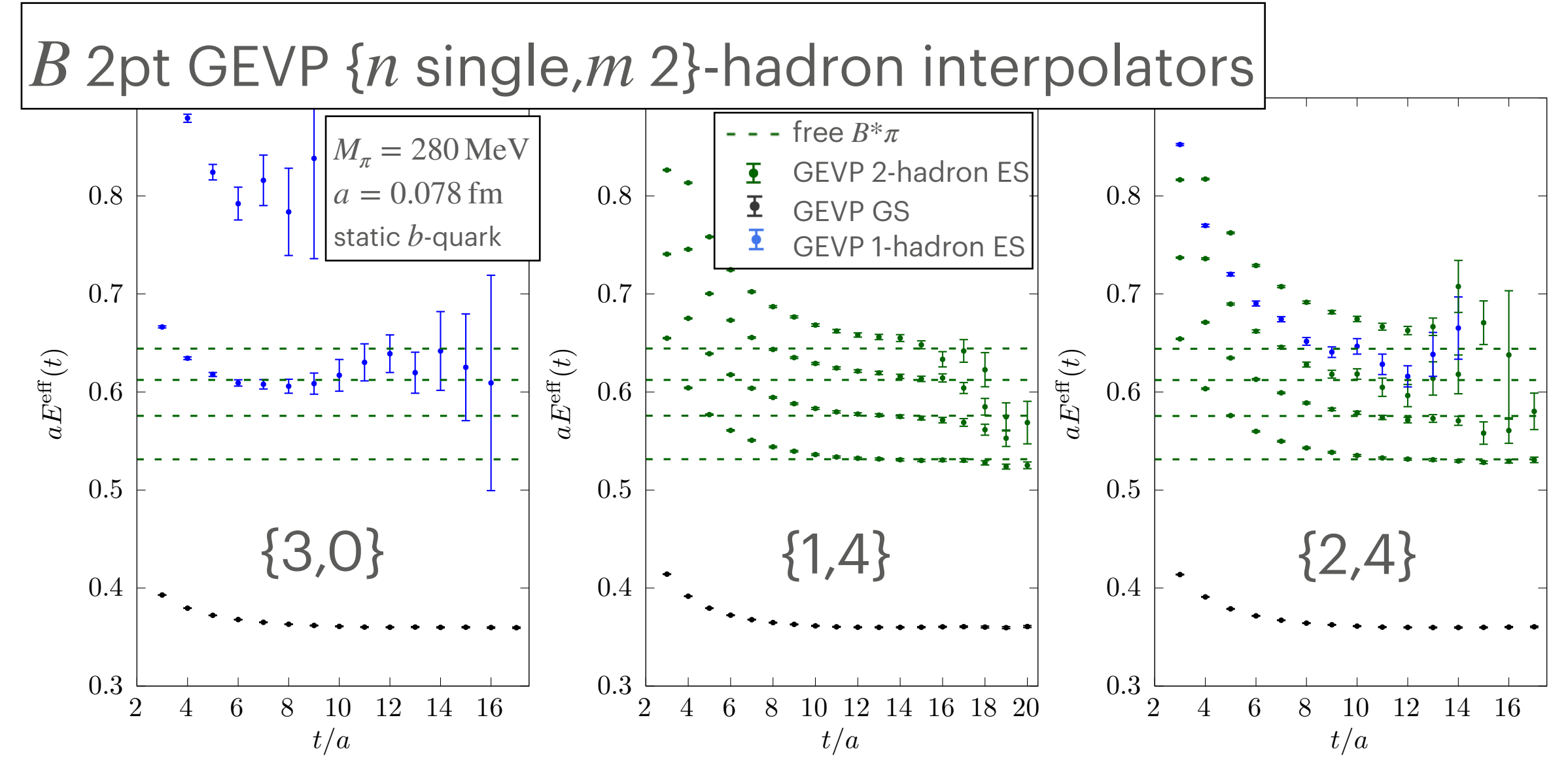
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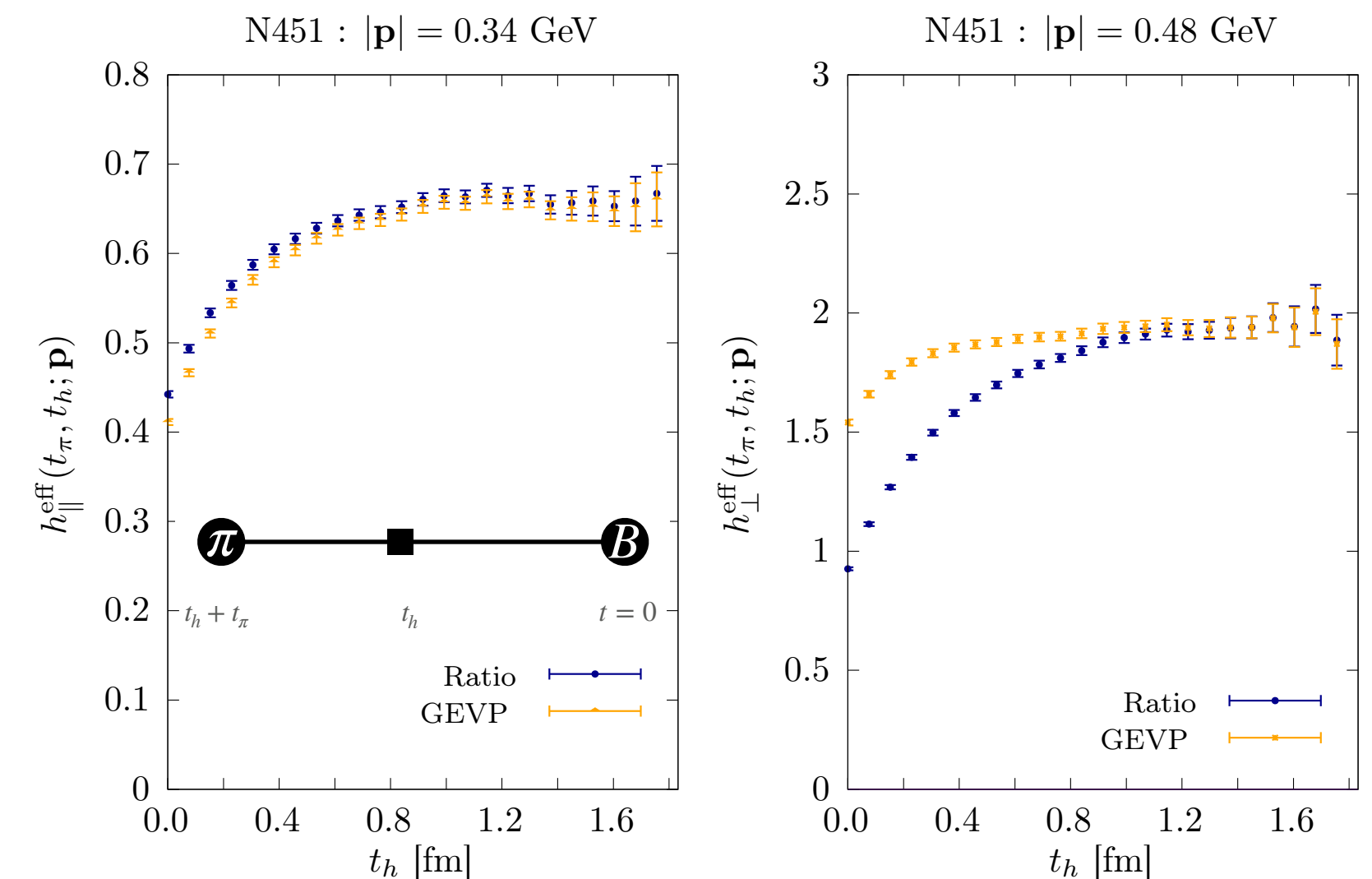
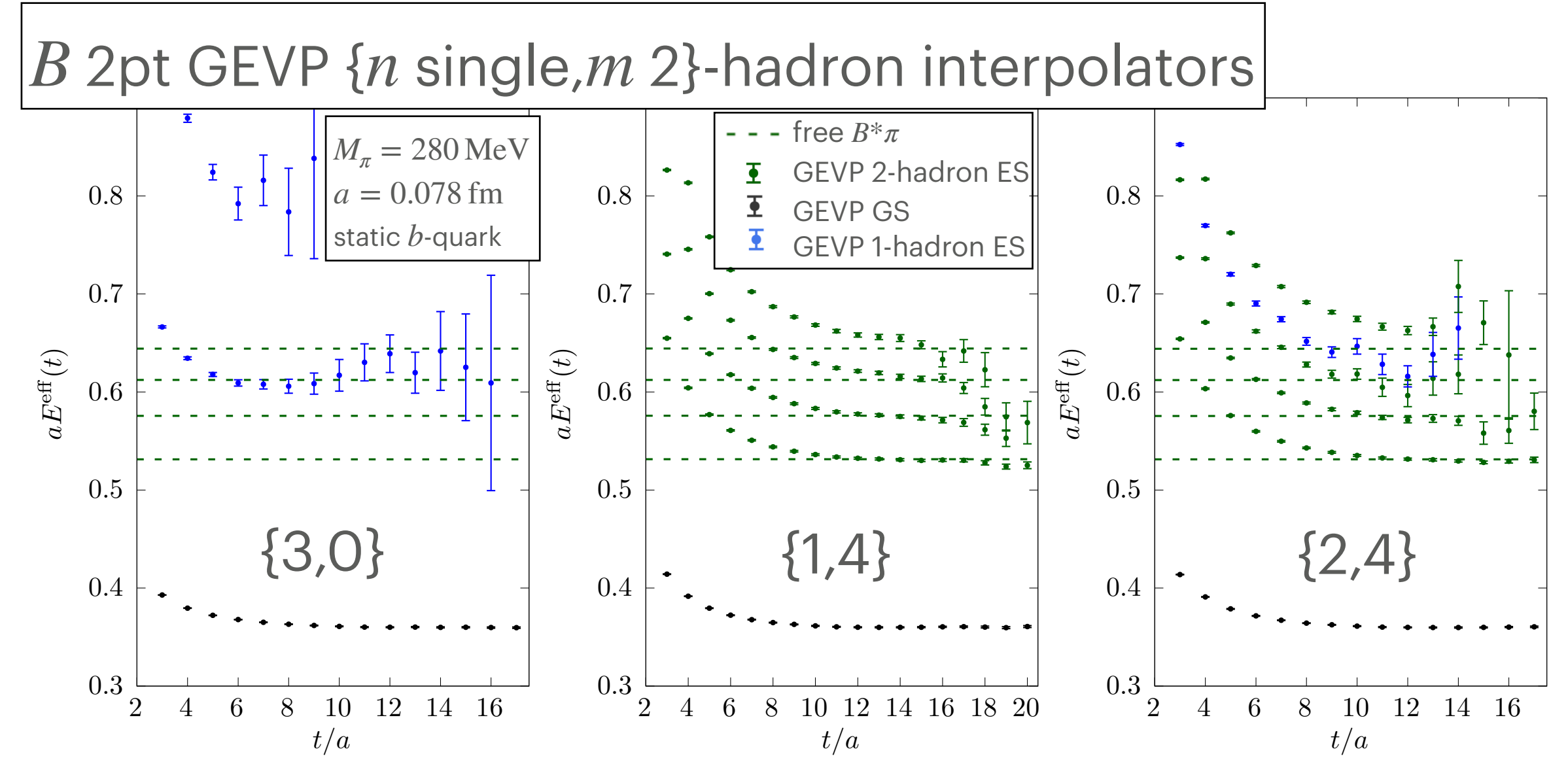
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- multi-hadron interpolators required to see $B^* \pi$ states
- large ES in $h_\perp$ as predicted in HM χ PT
- removing the $B^* \pi$ state reduces excited state contamination substantially
- enforcing naive 1st ES from 2pt in 3pt induces bias



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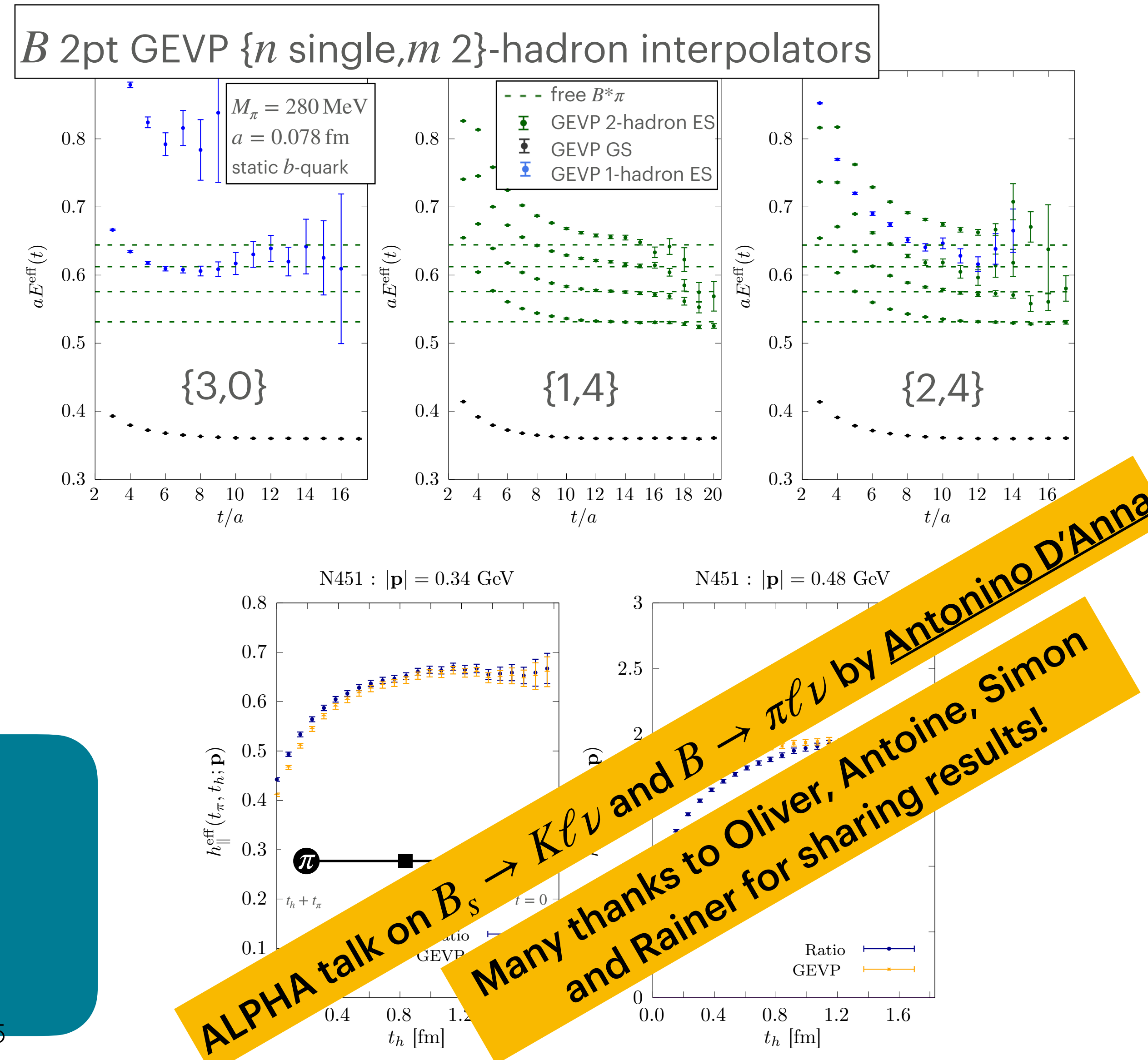
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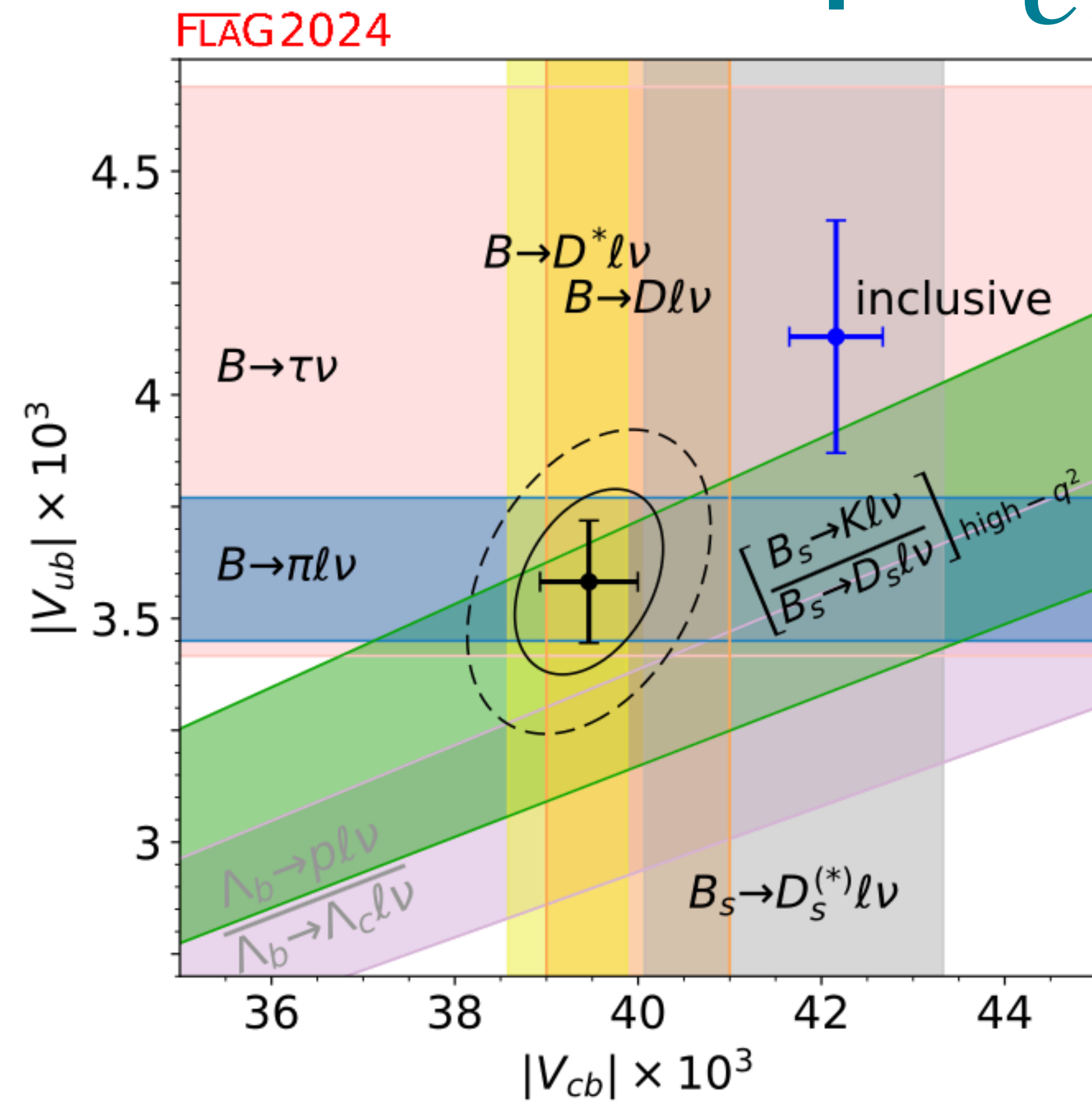
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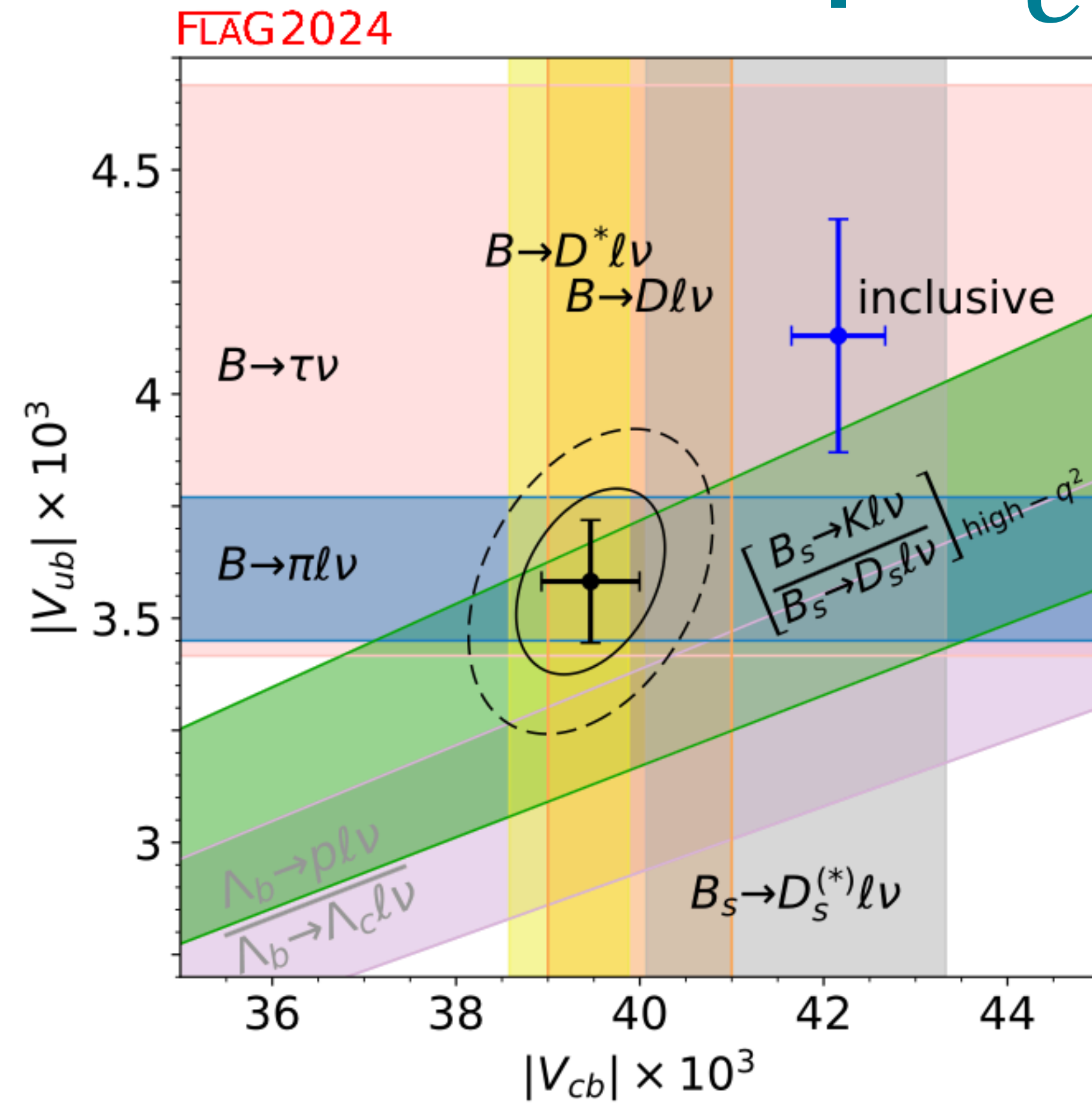
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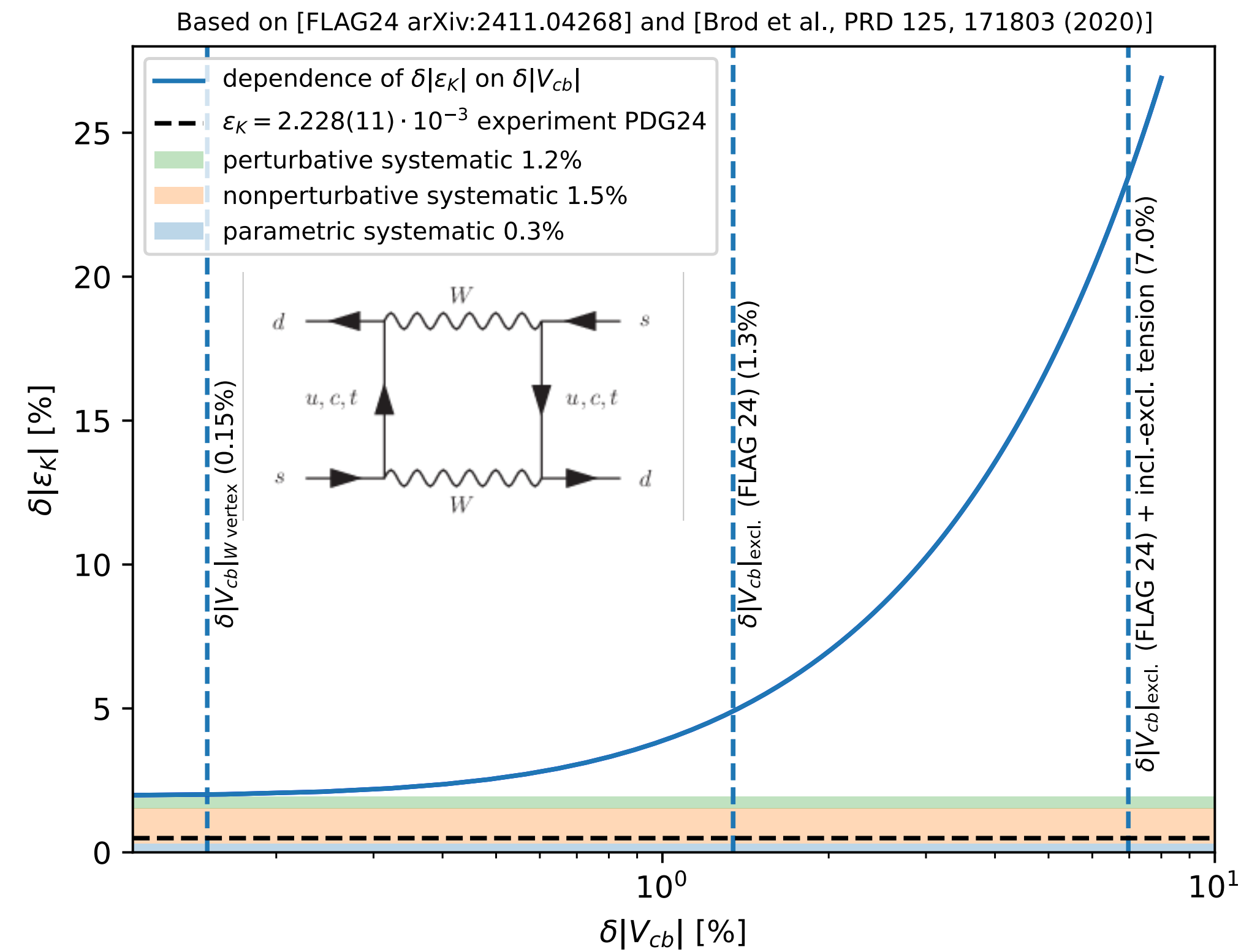
$|V_{cb}|$ puzzle



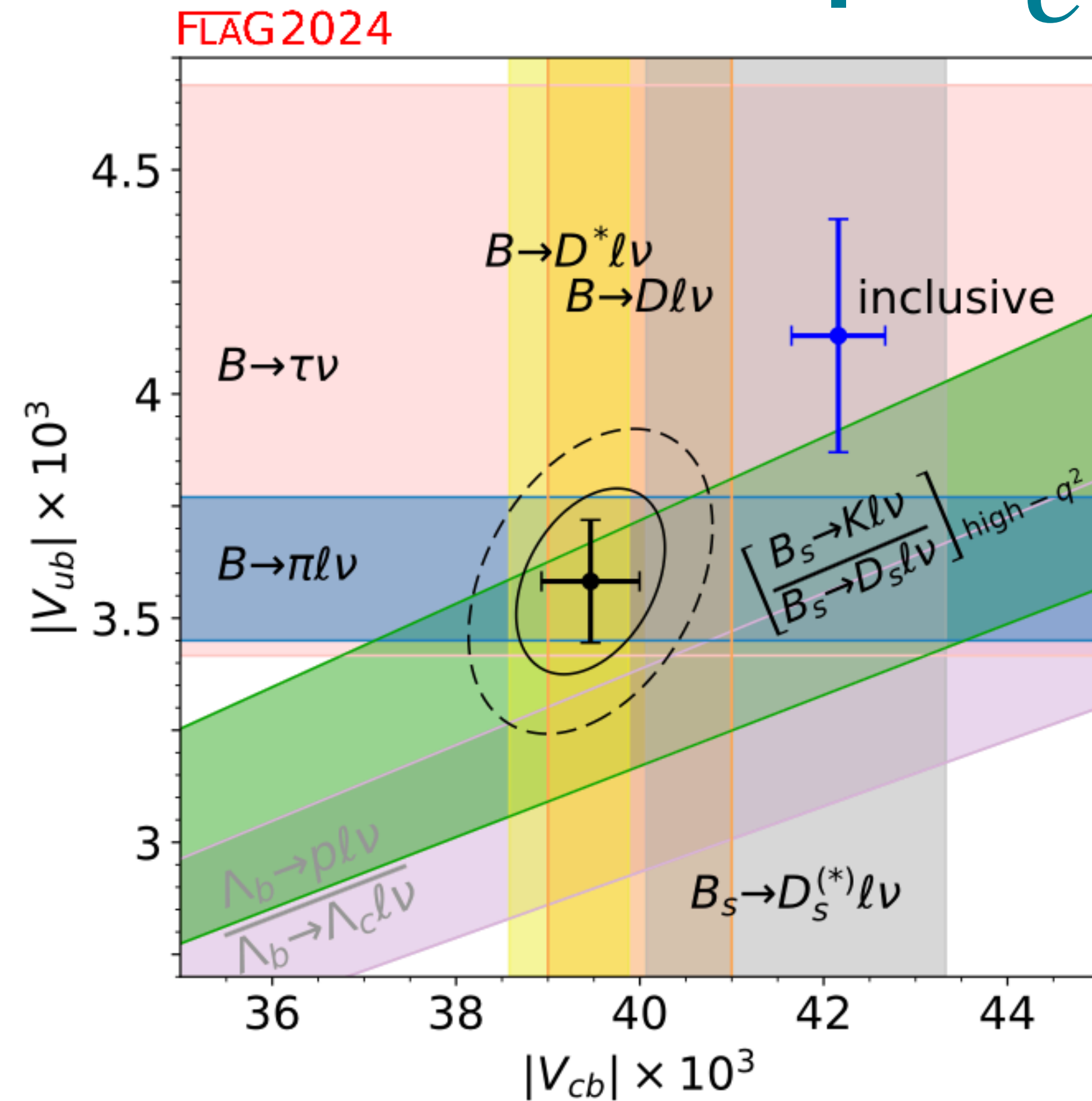
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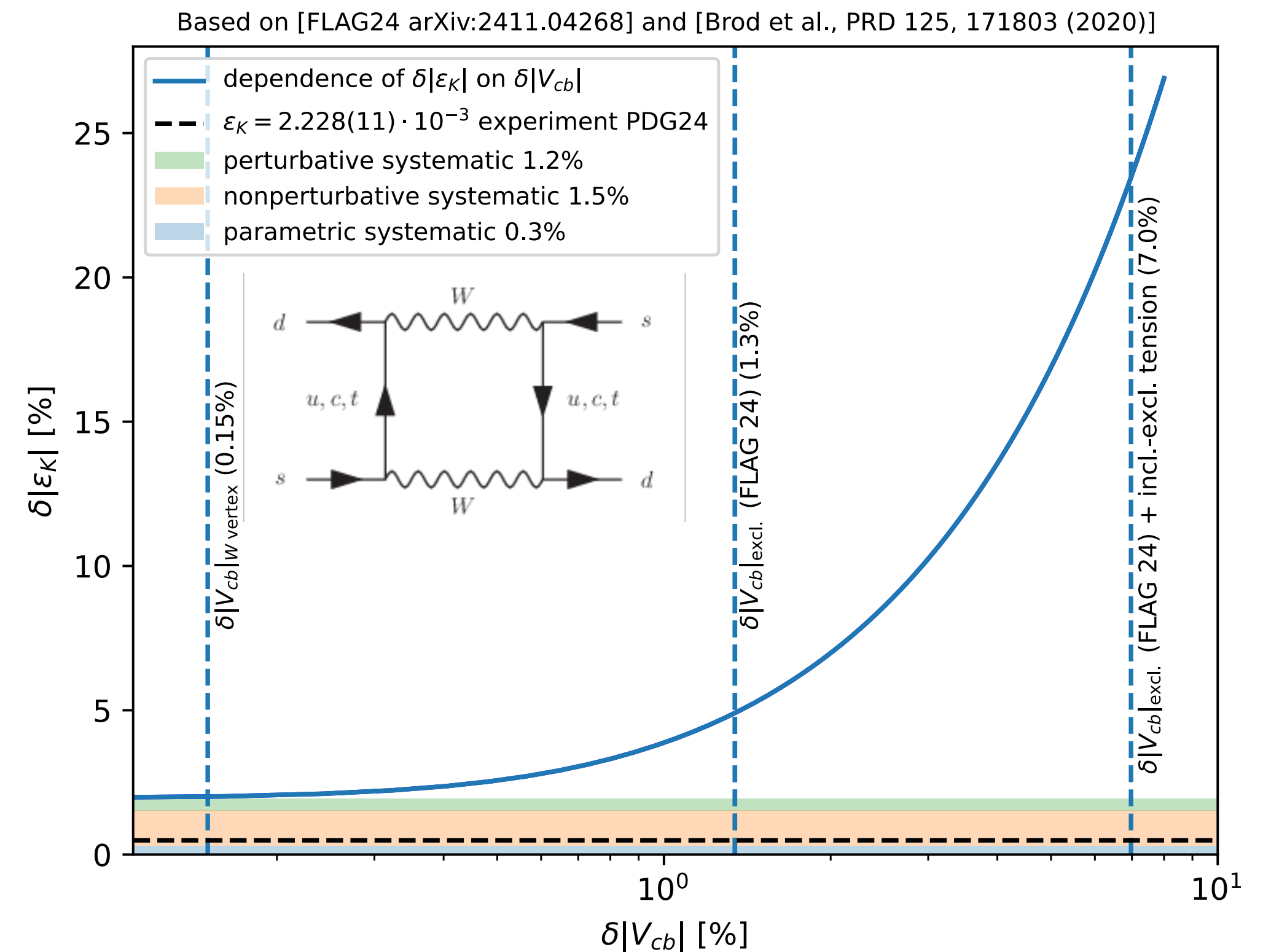
ϵ_K of kaon mixing depends on $|V_{cb}|$



$|V_{cb}|$ puzzle

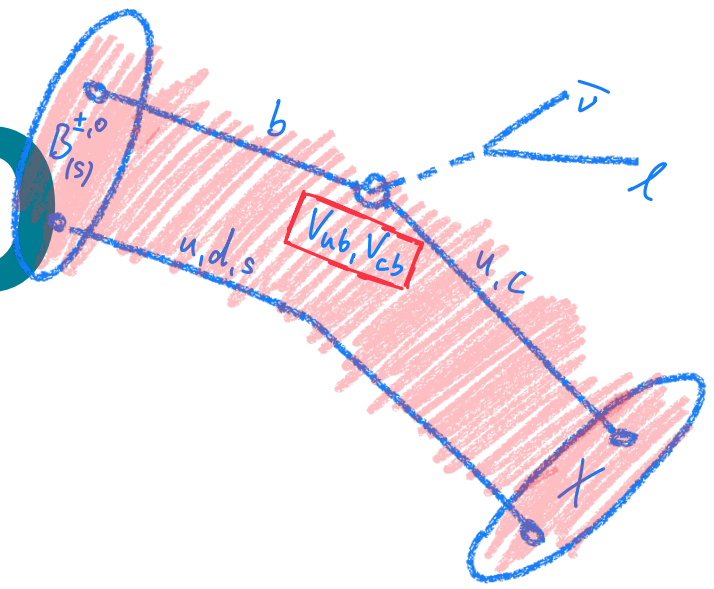


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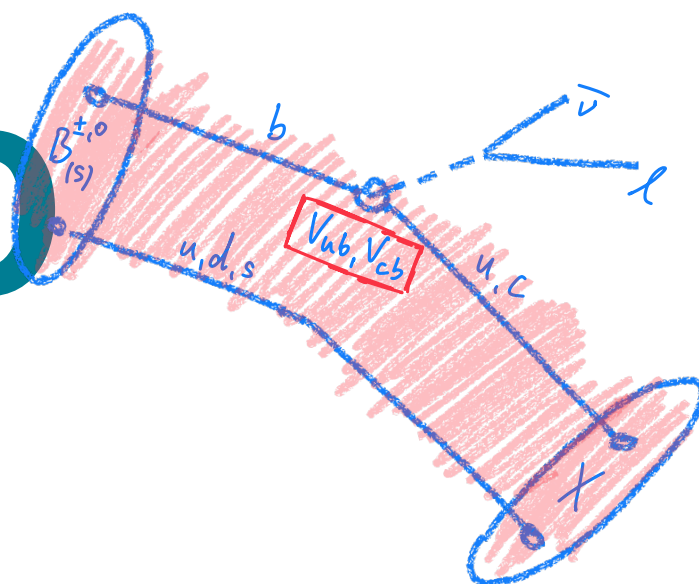
- V_{cb} crucial input to highly sensitive SM probes
- relevant question even on time scale of FCC- ee
- also relevant for $B \rightarrow \mu\mu$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$, $\Delta M_{d,s}$, ...

NEW: Inclusive decay in Lattice QCD



- inclusive $|V_{cb}|$ results to date based on perturbation theory/OPE
- new ideas leading to first pheno-relevant lattice computations

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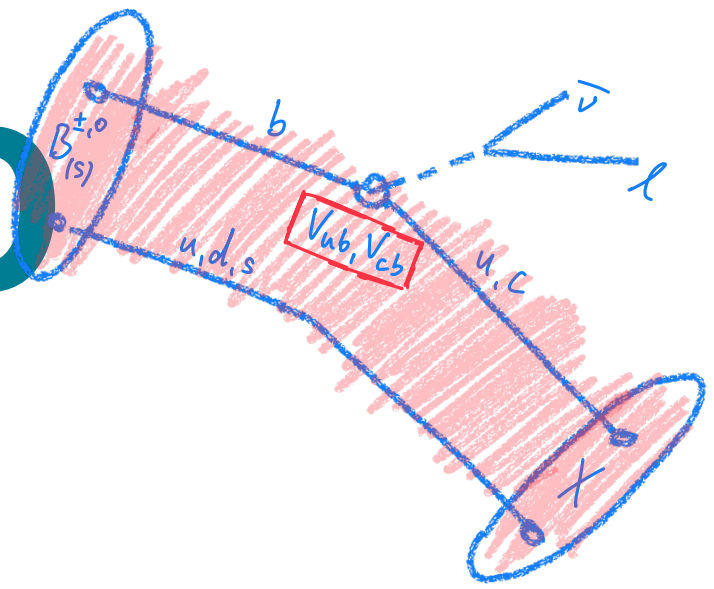
$$1) \quad \frac{d\Gamma}{dq^2 d\omega} \sim \frac{G_F^2 |V_{cb}|^2}{8\pi} \tilde{L}^{\mu\nu}(\omega, \mathbf{q}) W_{\mu\nu}(\omega, \mathbf{q})$$

↙ leptonic tensor
↖ hadronic tensor

Inclusive differential decay rate

Leptonic tensor $\tilde{L}$, hadronic tensor W

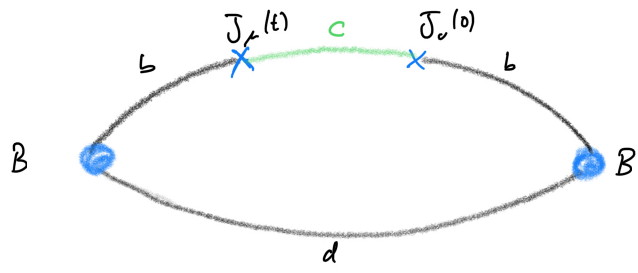
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Inclusive differential decay rate
Leptonic tensor $\tilde{L}$, hadronic tensor W

2) 
$$C_{\mu\nu}(t, \mathbf{q}) = \int_{\omega_{\min}}^{\infty} d\omega W_{\mu\nu}(\omega, \mathbf{q}) e^{-\omega t}$$

Euclidean 4pt function is Laplace transform of hadronic tensor

$$W^{\mu\nu}(p_B, q) = \frac{1}{2E_B} \sum_{X_c} (2\pi)^3 \delta^{(4)}(p_B - q - p_{X_c}) \langle B(\mathbf{0}) | (\tilde{J}^\mu(q^2))^\dagger | X_c(p_{X_c}) \rangle \langle X_c(p_{X_c}) | \tilde{J}^\nu(q^2) | B(\mathbf{0}) \rangle$$

-

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Expand leptonic tensor in $e^{-a\omega}$

-

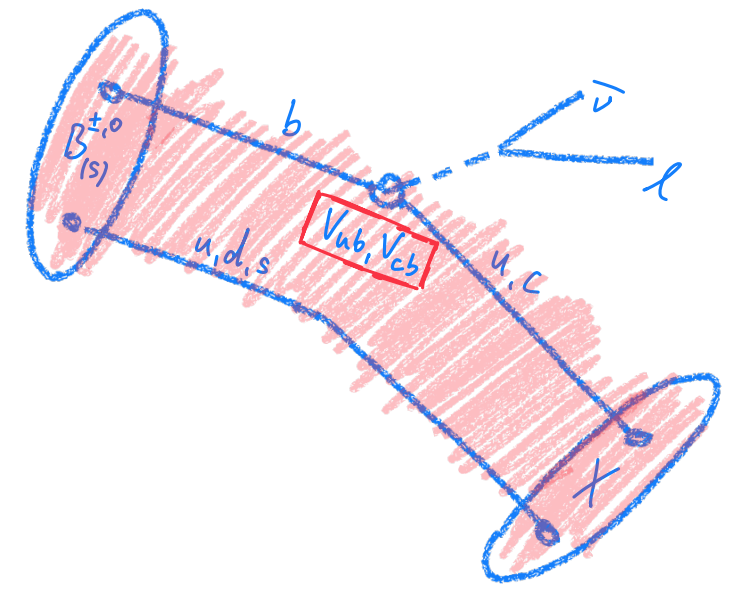
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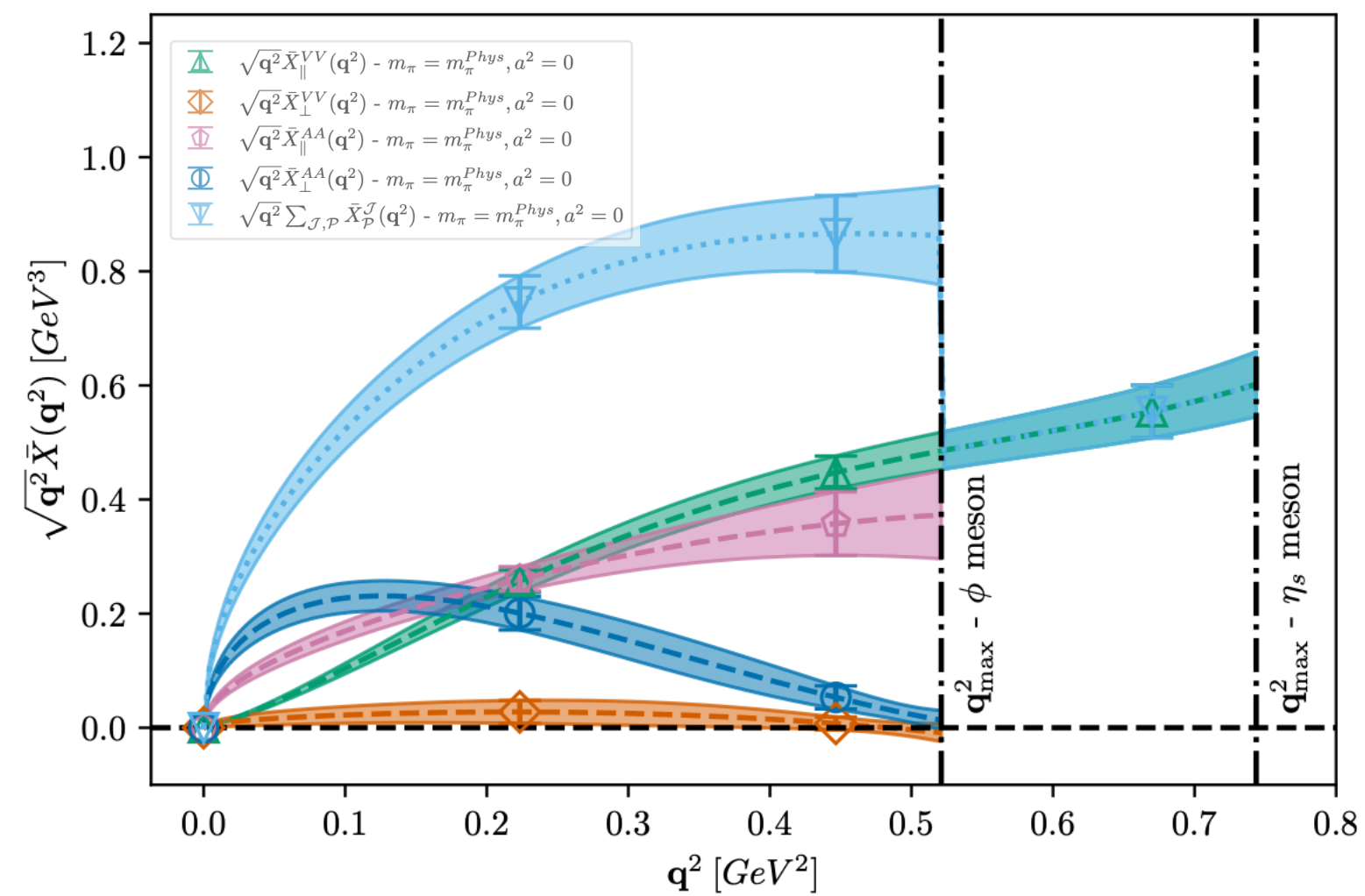
(Backus-Gilbert/HLT, Chebyshev, other???)

Inclusive $D_s \rightarrow X \ell \bar{\nu}_\ell$



[Kellermann et al. [PRD 112 \(2025\)](#), [arXiv:2604.22201](#)]

- $N_f = 2 + 1$ DWF (JLQCD),
 $m_{ud} > m_{ud}^{\text{phys}}$, no disconnected
- Chebyshev
- $c \rightarrow s$ transition
- full systematic-error budget

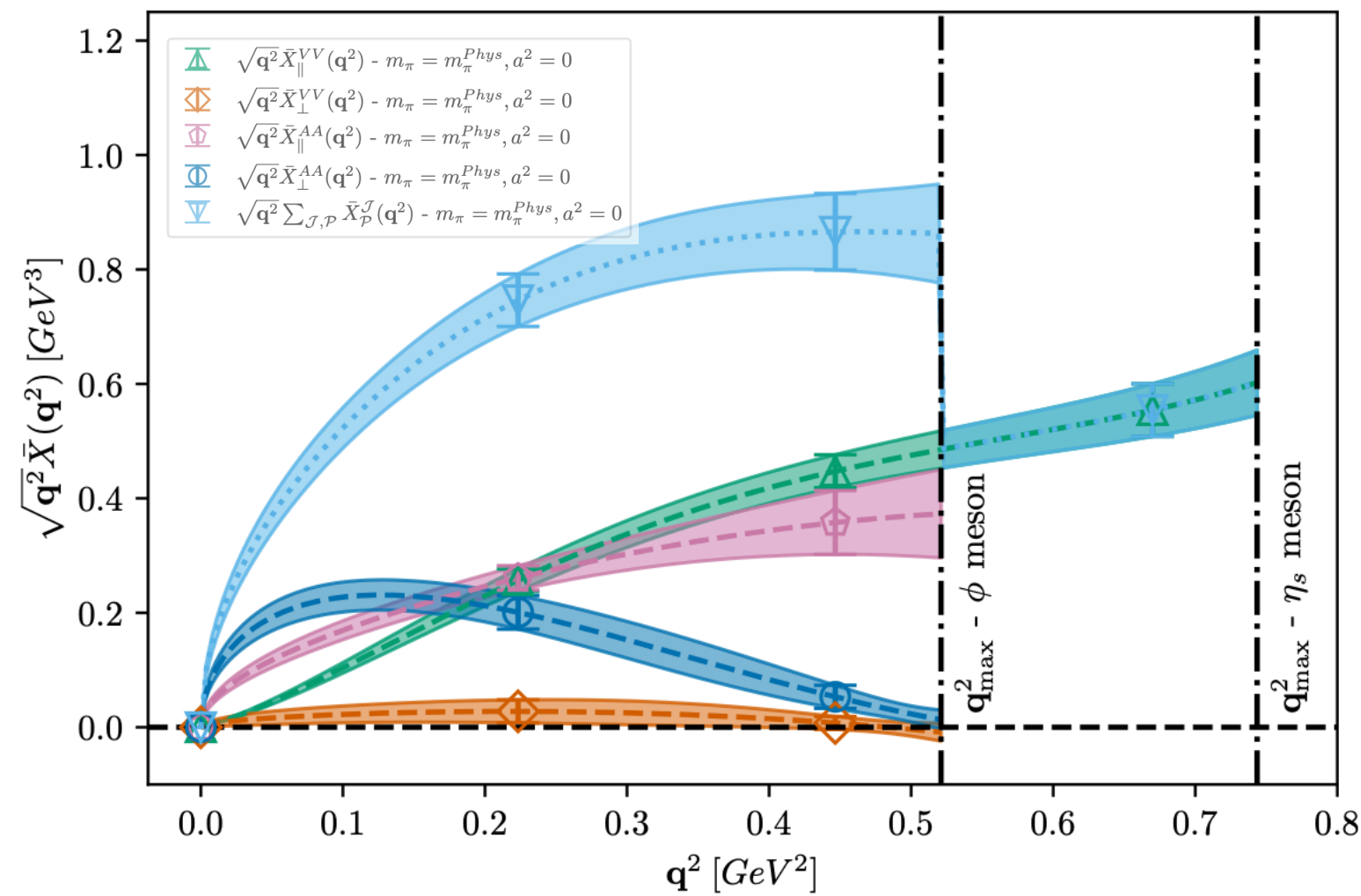


talk by Ryan Kellermann

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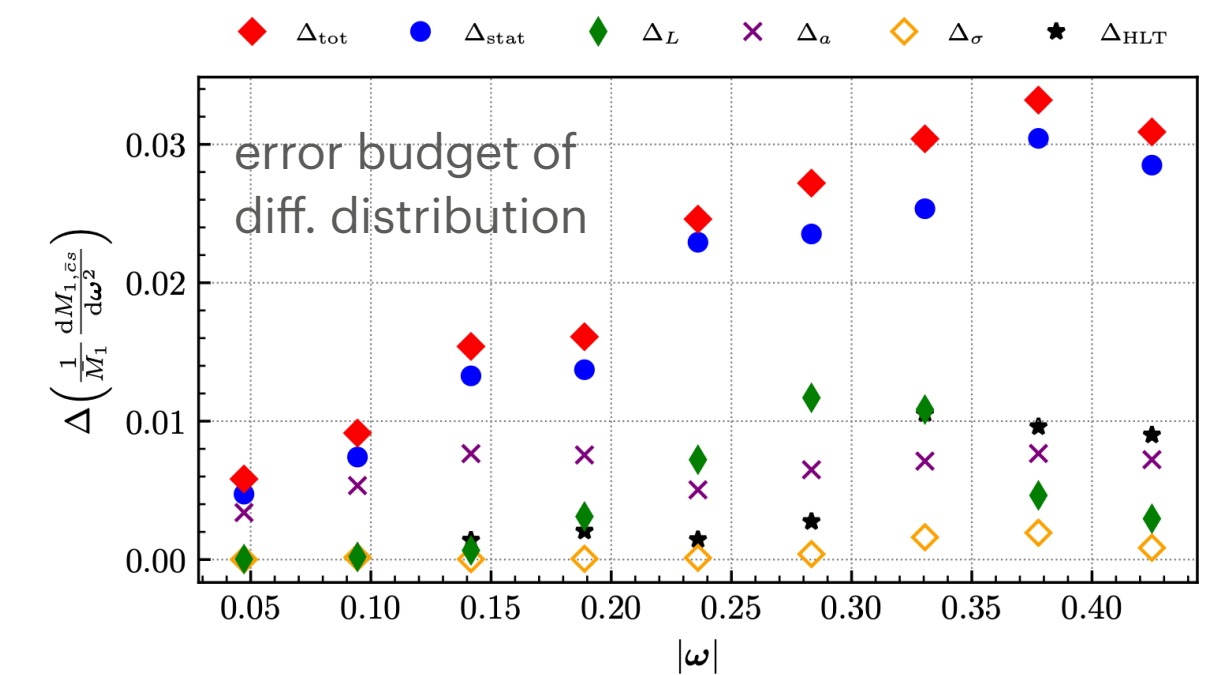
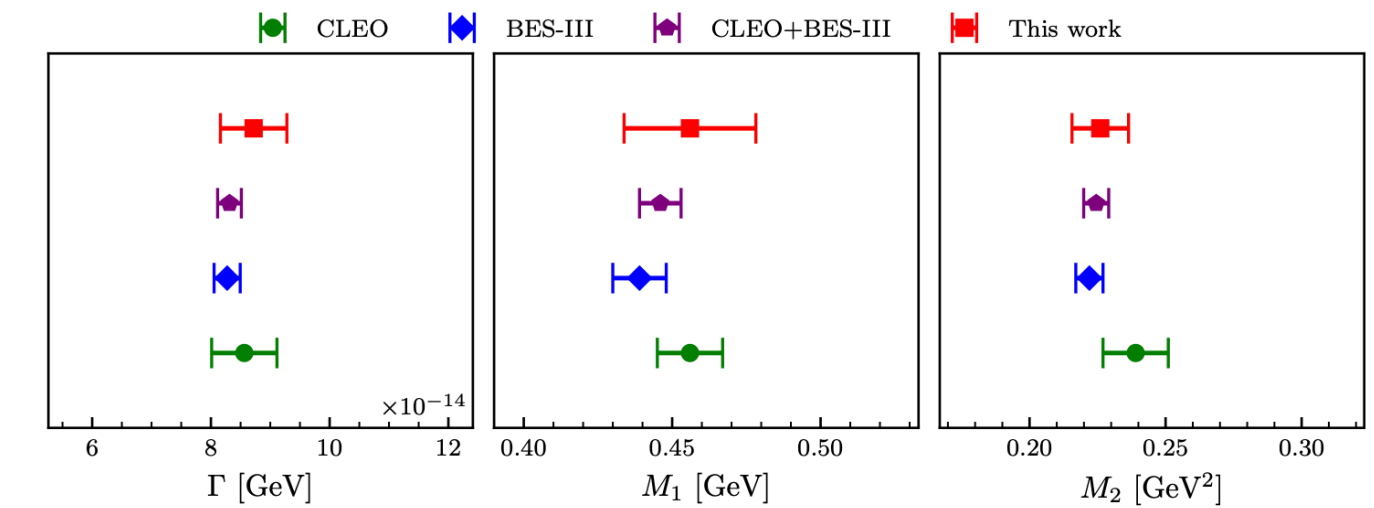
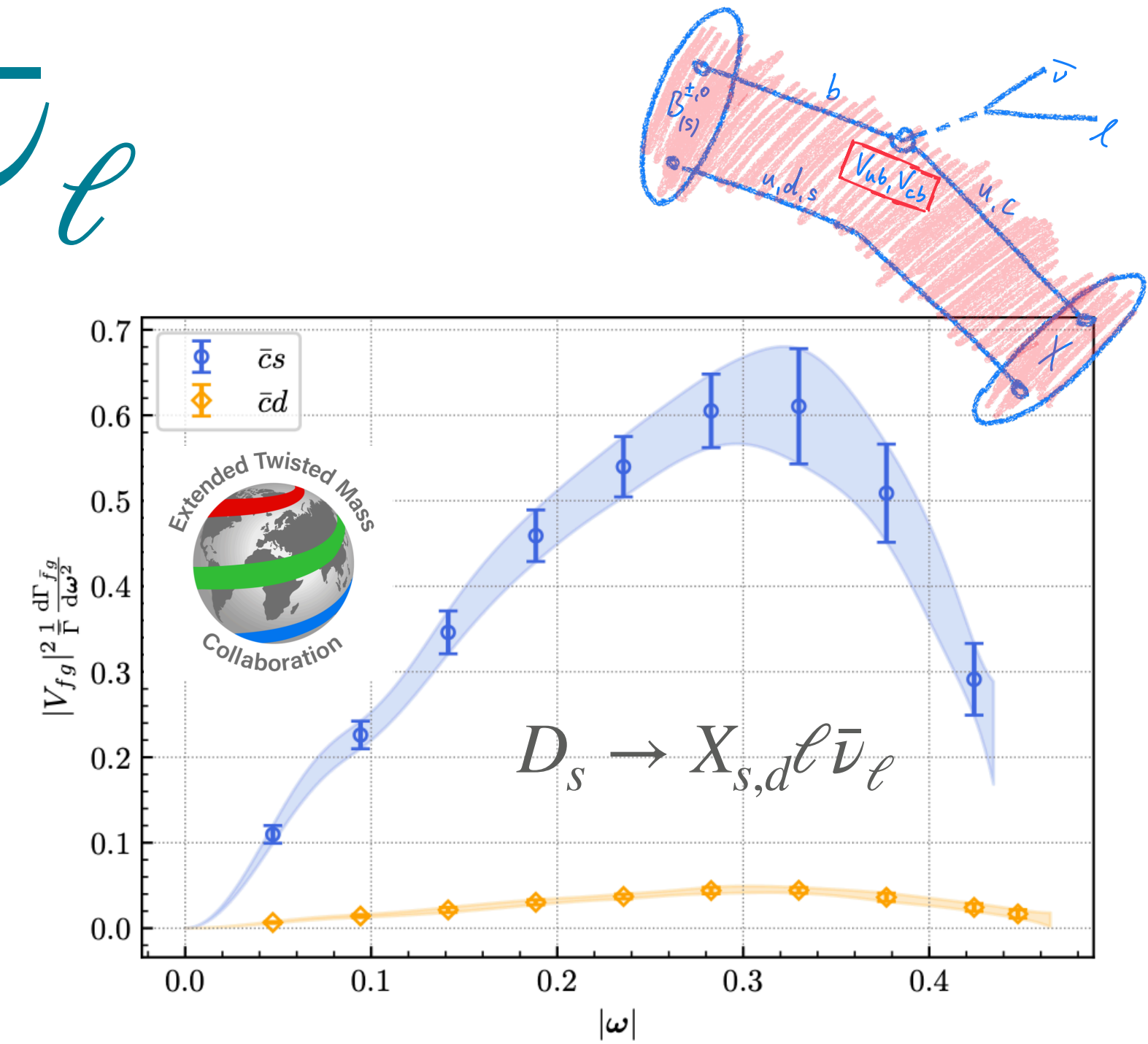
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[De Santis et al. PRL135 (2025), PRD 112 (2025)]

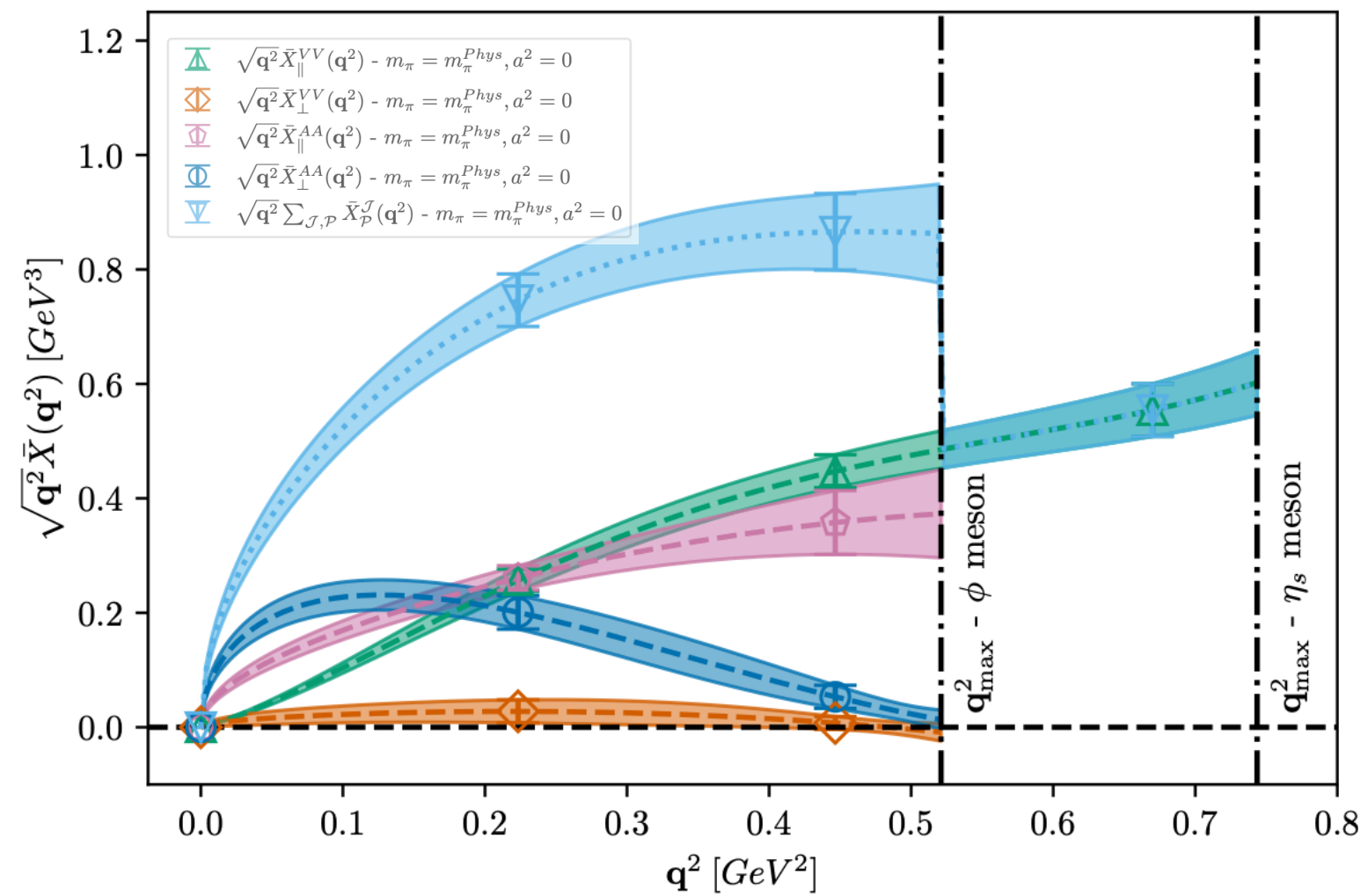
- $N_f = 2 + 1 + 1$ TM, $a \sim 0.05 - 0.08$ fm,
 m_q^{phys} , multiple volumes, disconnected
- Backus-Gilbert — Hansen-Lupo-Tantalo
- both $c \rightarrow s$ and $c \rightarrow d$ transitions
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[De Santis et al. PRL135 (2025), PRD 112 (2025)]

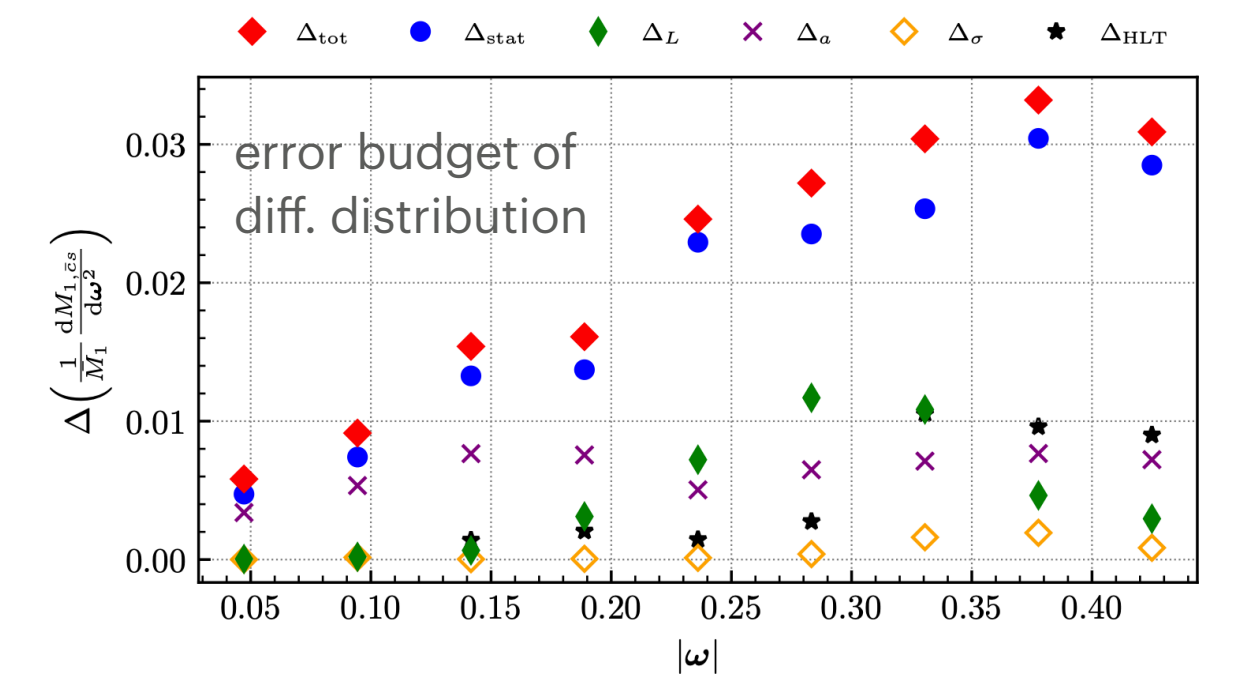
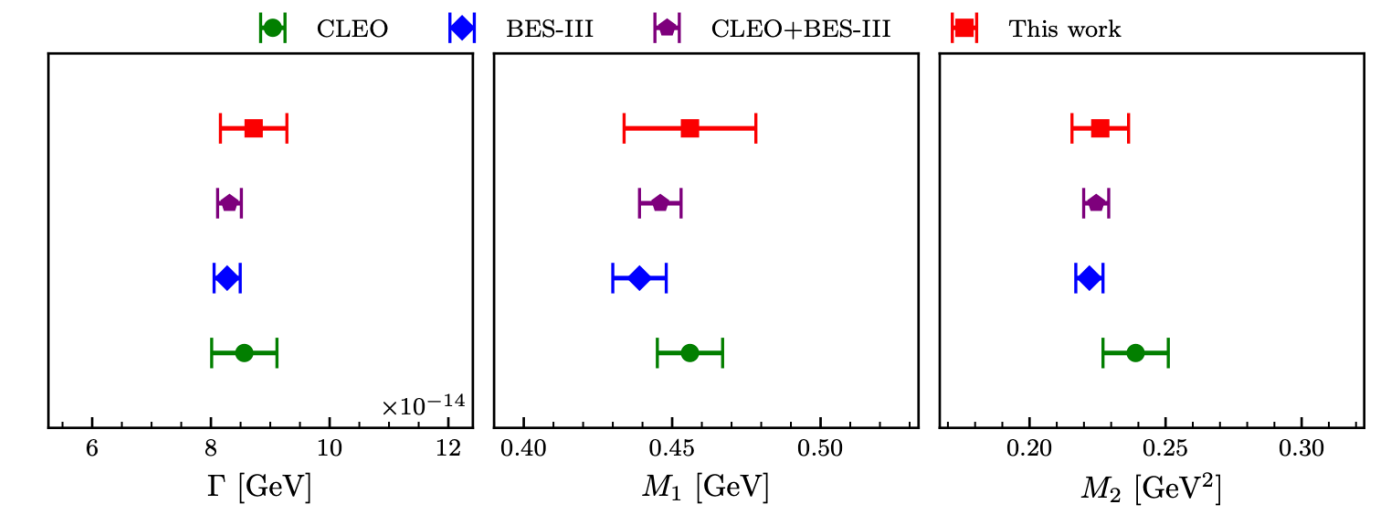
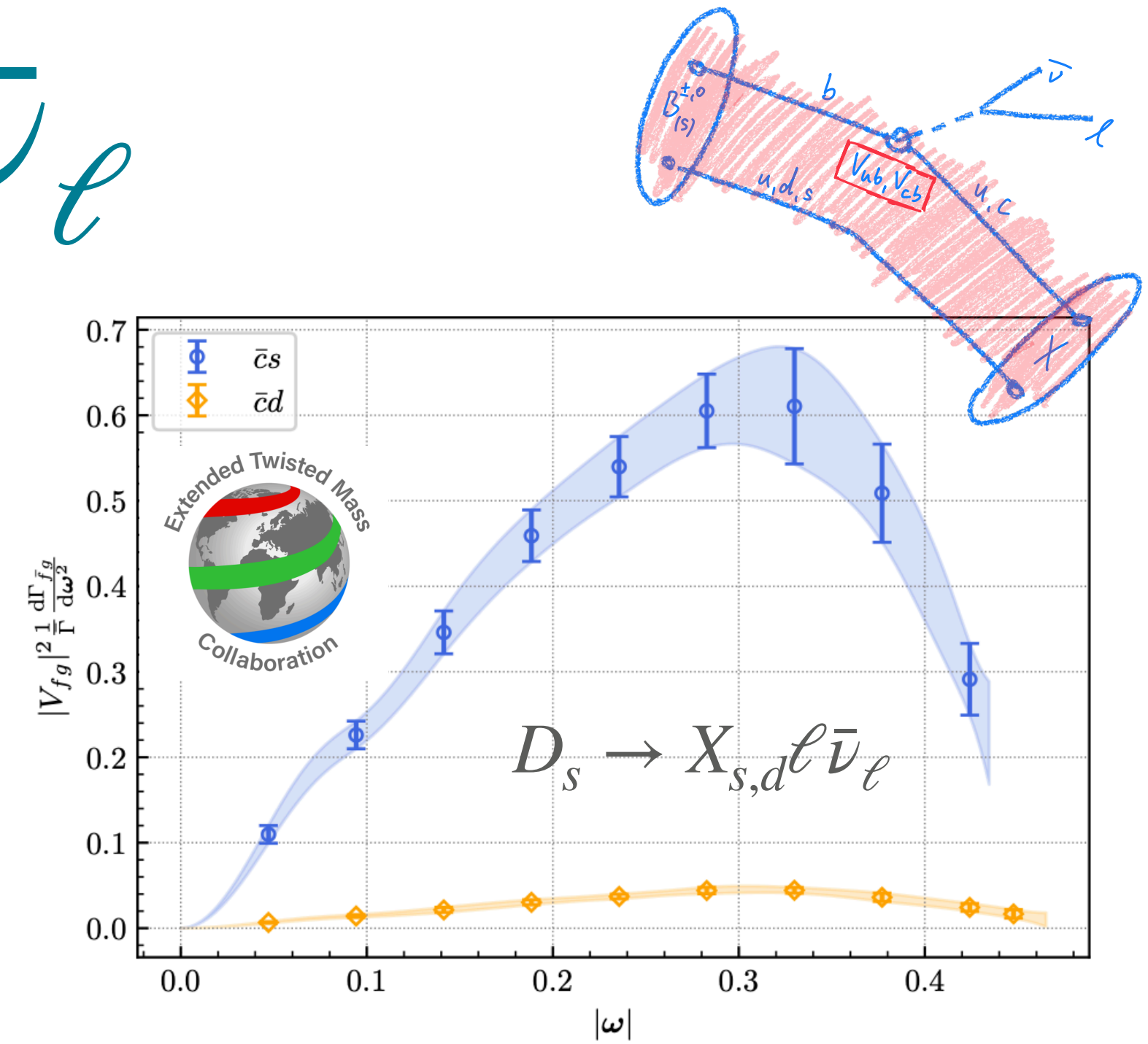
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$$\frac{\Gamma}{|V_{cs}|^2} = 0.856(46) \left({}^{+0.050}_{-0.048} \right) \times 10^{-13} \text{ GeV}$$

[Kellermann et al. arXiv:2604.22201]

$$\frac{\Gamma}{|V_{cs}|^2} = 0.853(46)(30) \times 10^{-13} \text{ GeV}$$

[De Santis et al. PRD 112 (2025), PRL 135 (2025)]

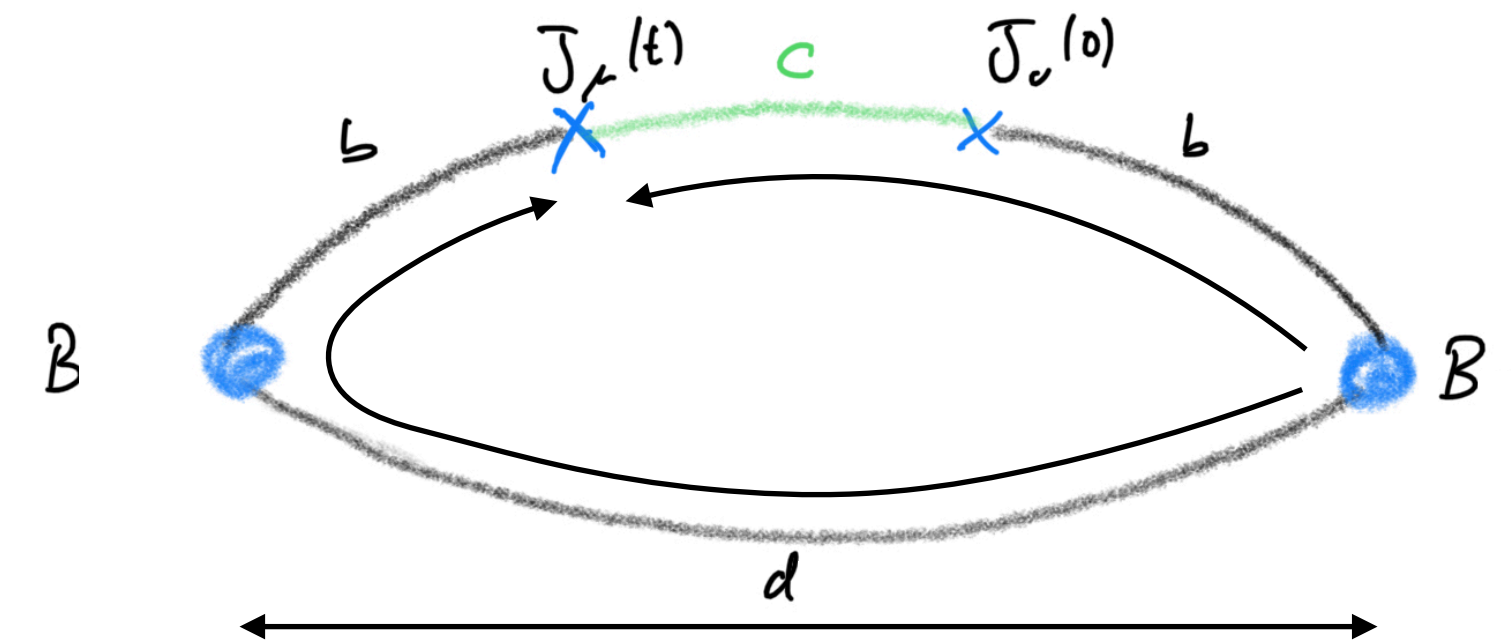
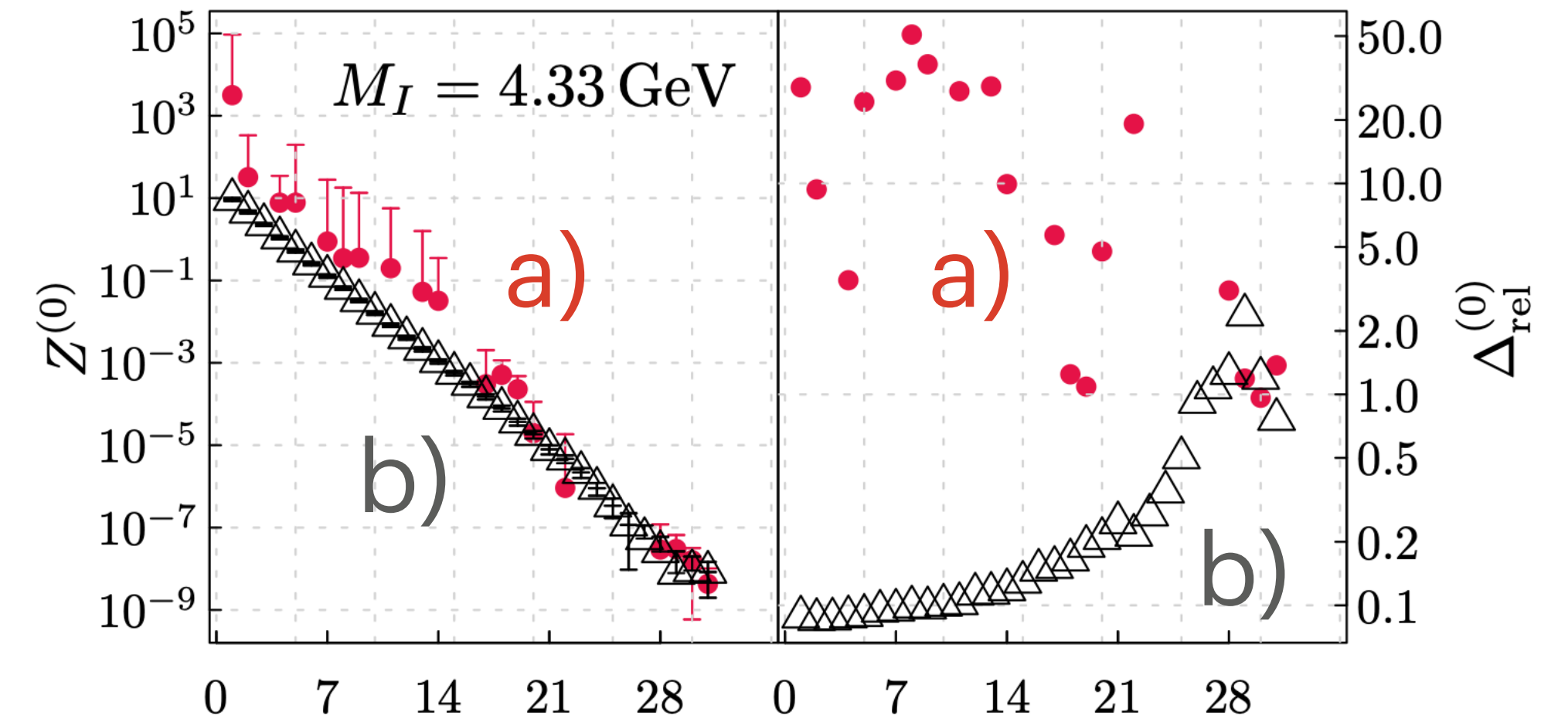


Inclusive $B_s \rightarrow X_c \ell \bar{\nu}_\ell$

ETMC

[De Santis et al. [arXiv:2607.01116](https://arxiv.org/abs/2607.01116)]

- $N_f = 2 + 1 + 1$ ETM, $a \sim 0.05 - 0.08$ fm, multiple L
- simulations for $\Gamma(M_i, M_f)$ with $M_i = 2 - 4.33$ GeV, $M_f = 0.72 - 1.6$ GeV with constant M_f/M_i (constant phase space) followed by interpolation to exact static result (OPE)
- substantial variance reduction (see r.h.s.)
- Hansen-Lupo-Tantalo

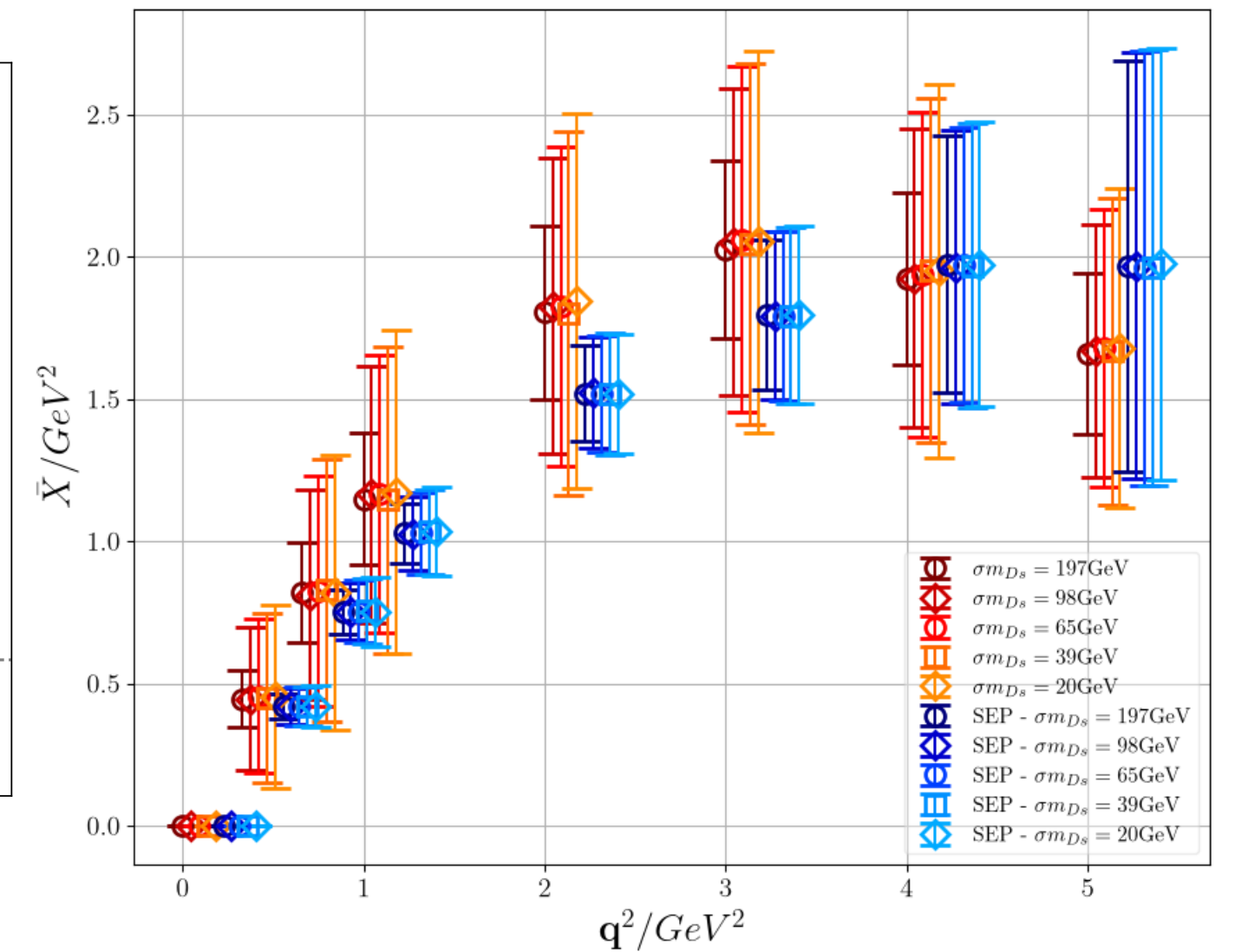
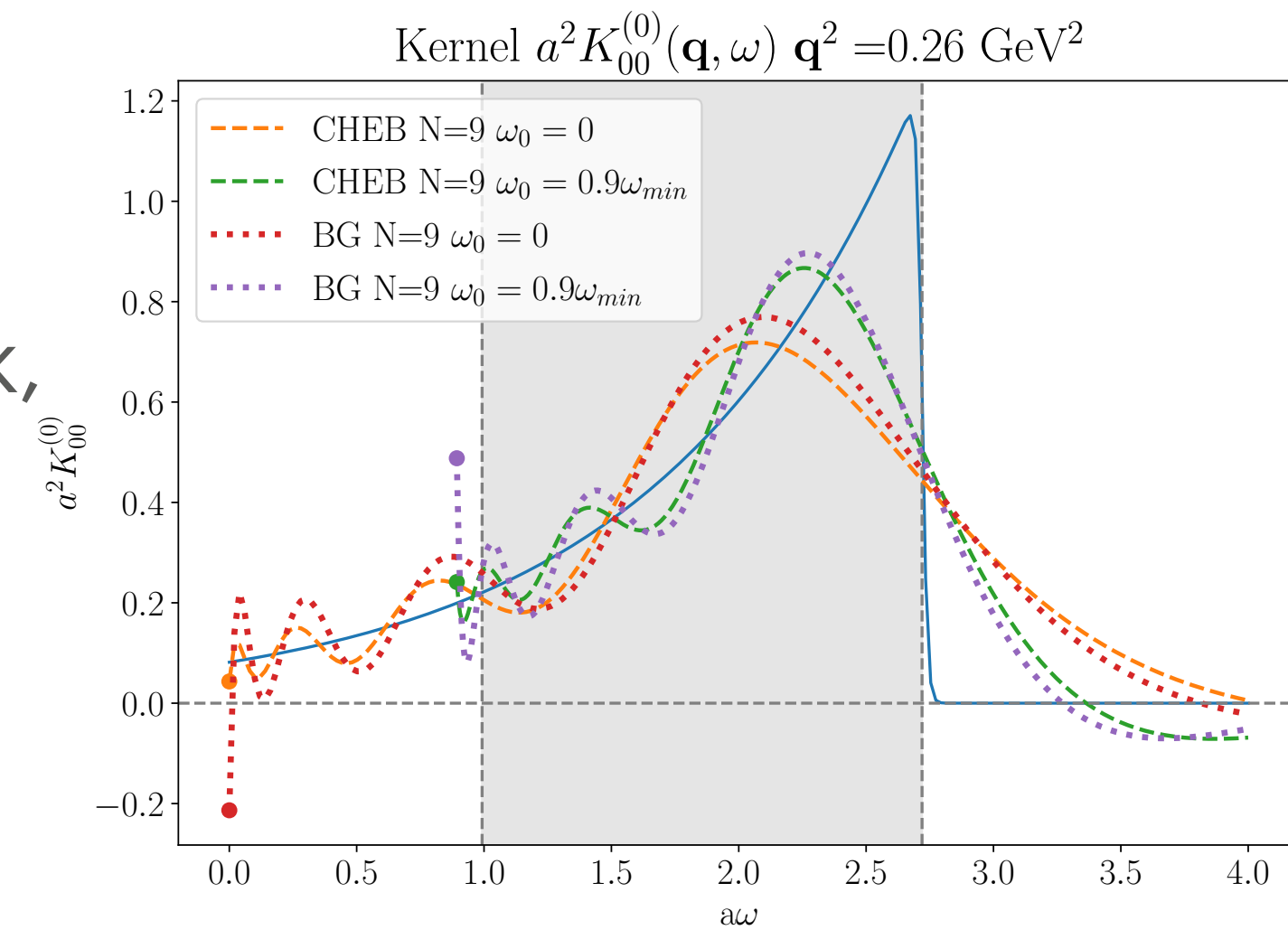


- a) keep ΔT fixed and vary t
- b) vary ΔT and keep $B - j$ distances fixed

Inclusive $B_s \rightarrow X_c \ell \bar{\nu}_\ell$

CERN-KEK-Mainz-Soton-[in preparation]

- $N_f = 2 + 1$ DWF with *physical* RHQ b quark, $a \sim 0.07 - 0.11$ fm, multiple L
- adopted variance reduction
- Chebyshev reconstruction



One idea they use:

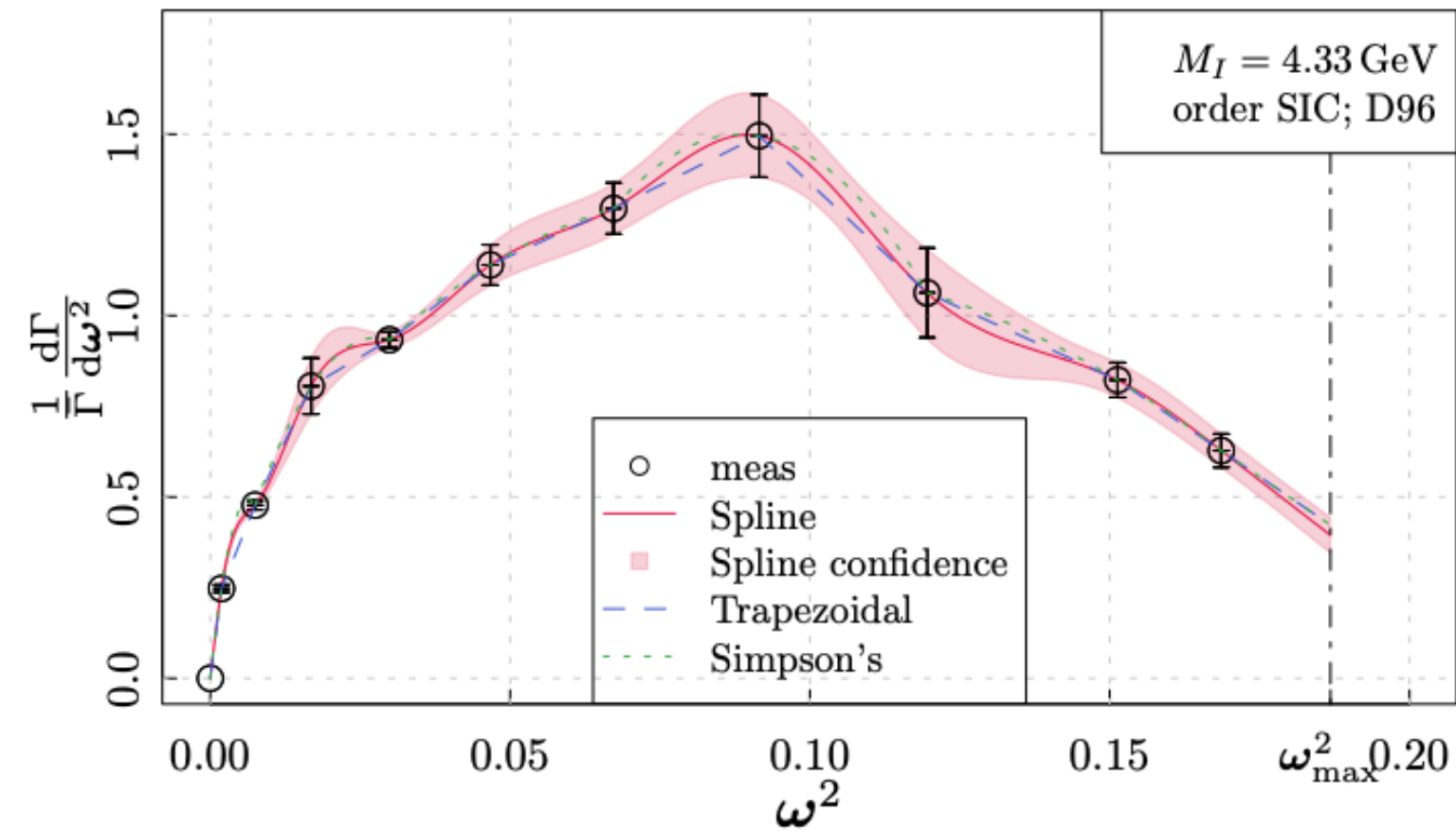
- Leptonic kernel sometimes hard to reconstruct with low-order polynomial
 $\rightarrow$ reconstruct only ES: $X = \underbrace{(X - X_{\text{GS}})}_{\text{Chebyshev}} + \underbrace{X_{\text{GS}}}_{3\text{pt}} \rightarrow$ reduced statistical noise and systematics

talk by **Ryan Kellermann**

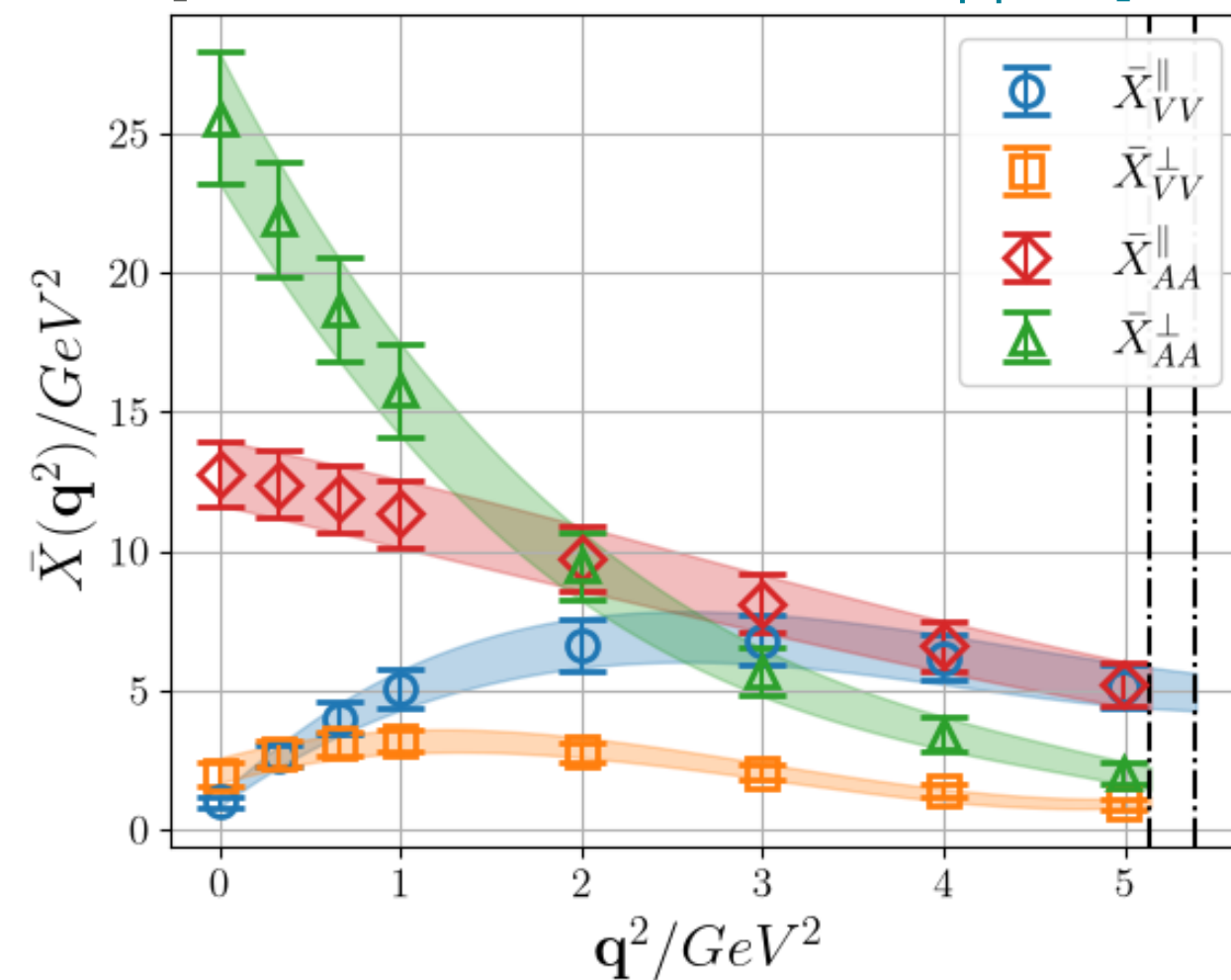
Summary $B_s \rightarrow X_c \ell \bar{\nu}_\ell$

Lattice predictions for
differential decay rates

[De Santis et al. [arXiv:2607.01116](https://arxiv.org/abs/2607.01116)]



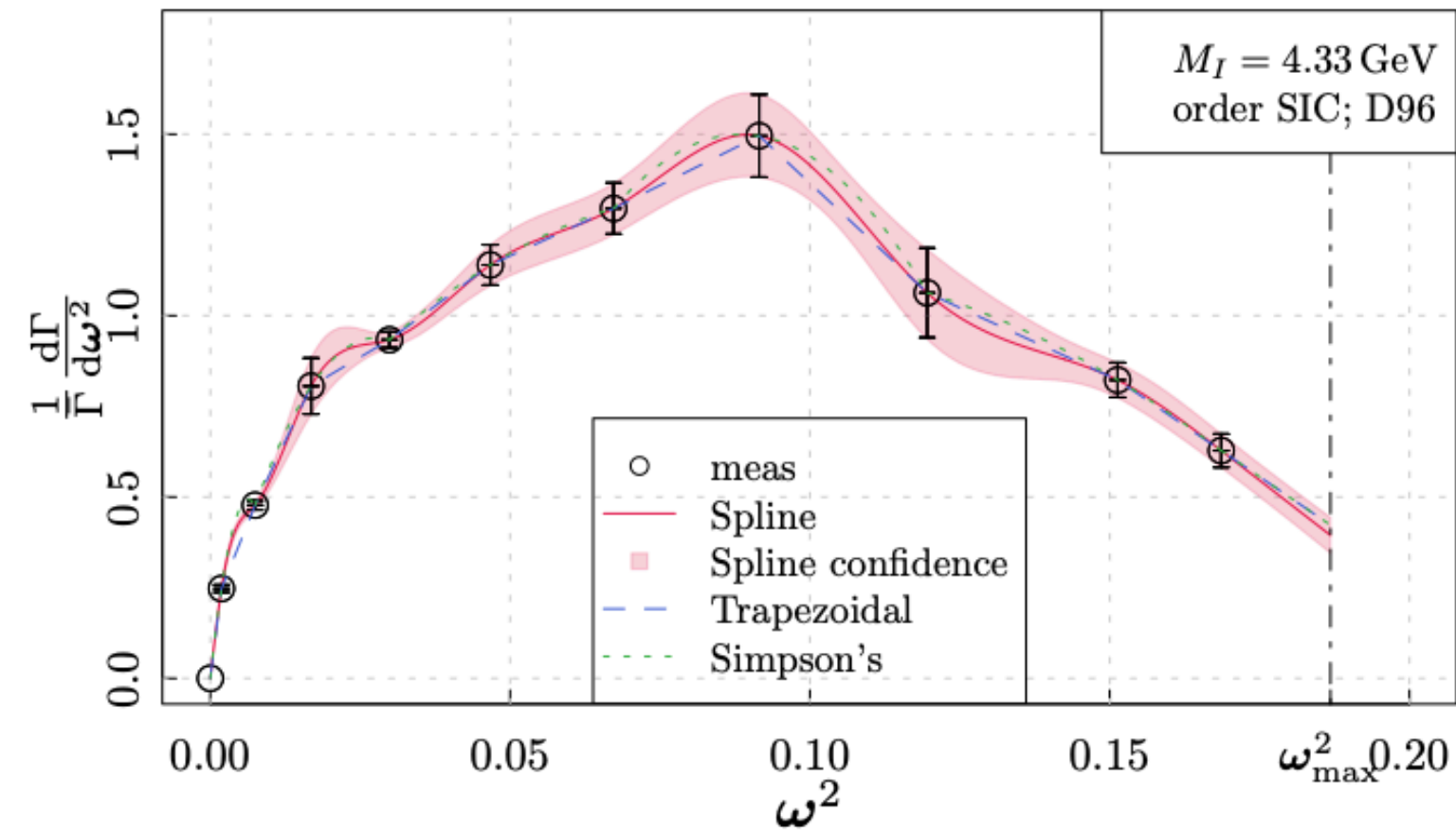
[CERN-KEK-Mainz-Soton, to appear]



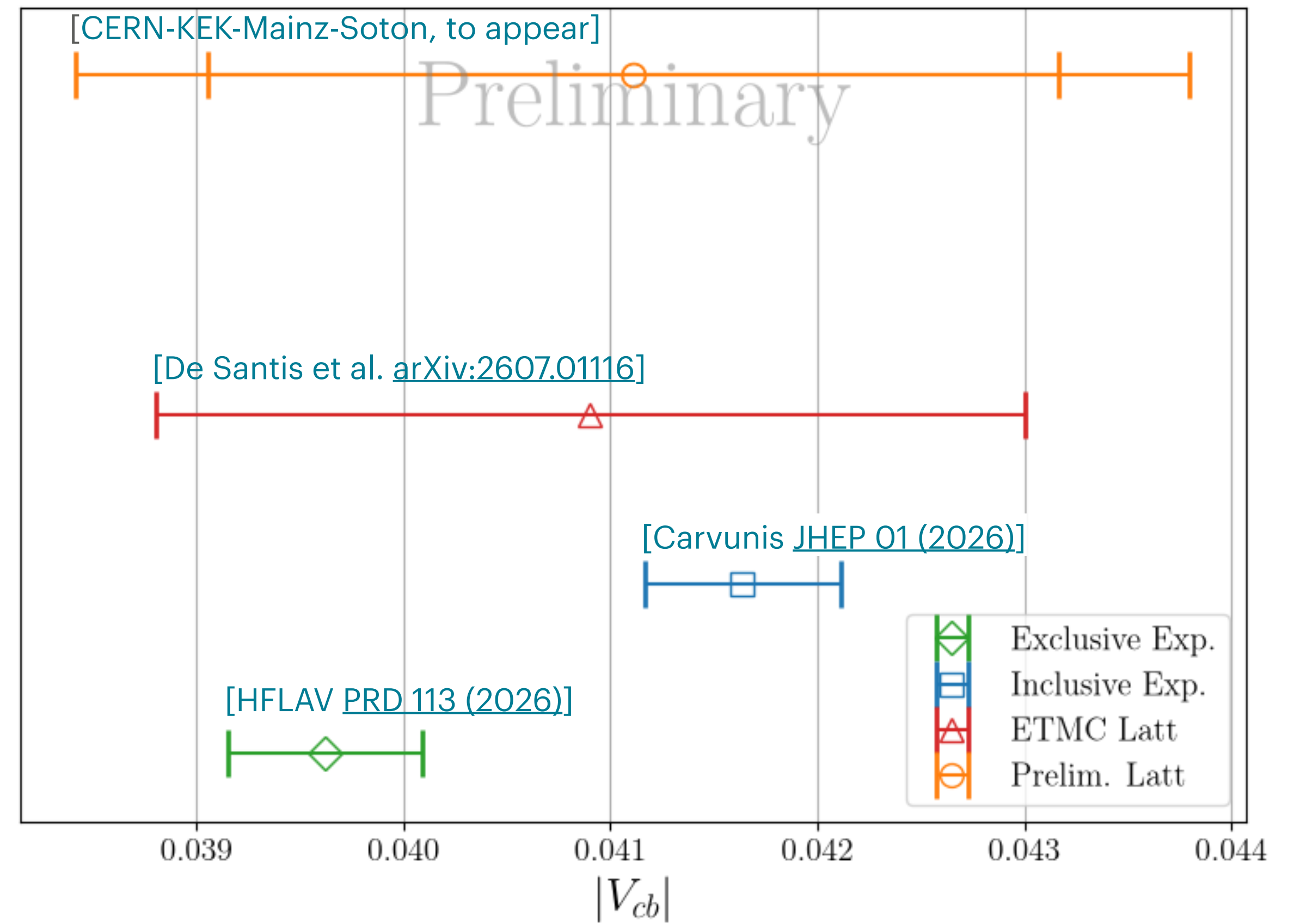
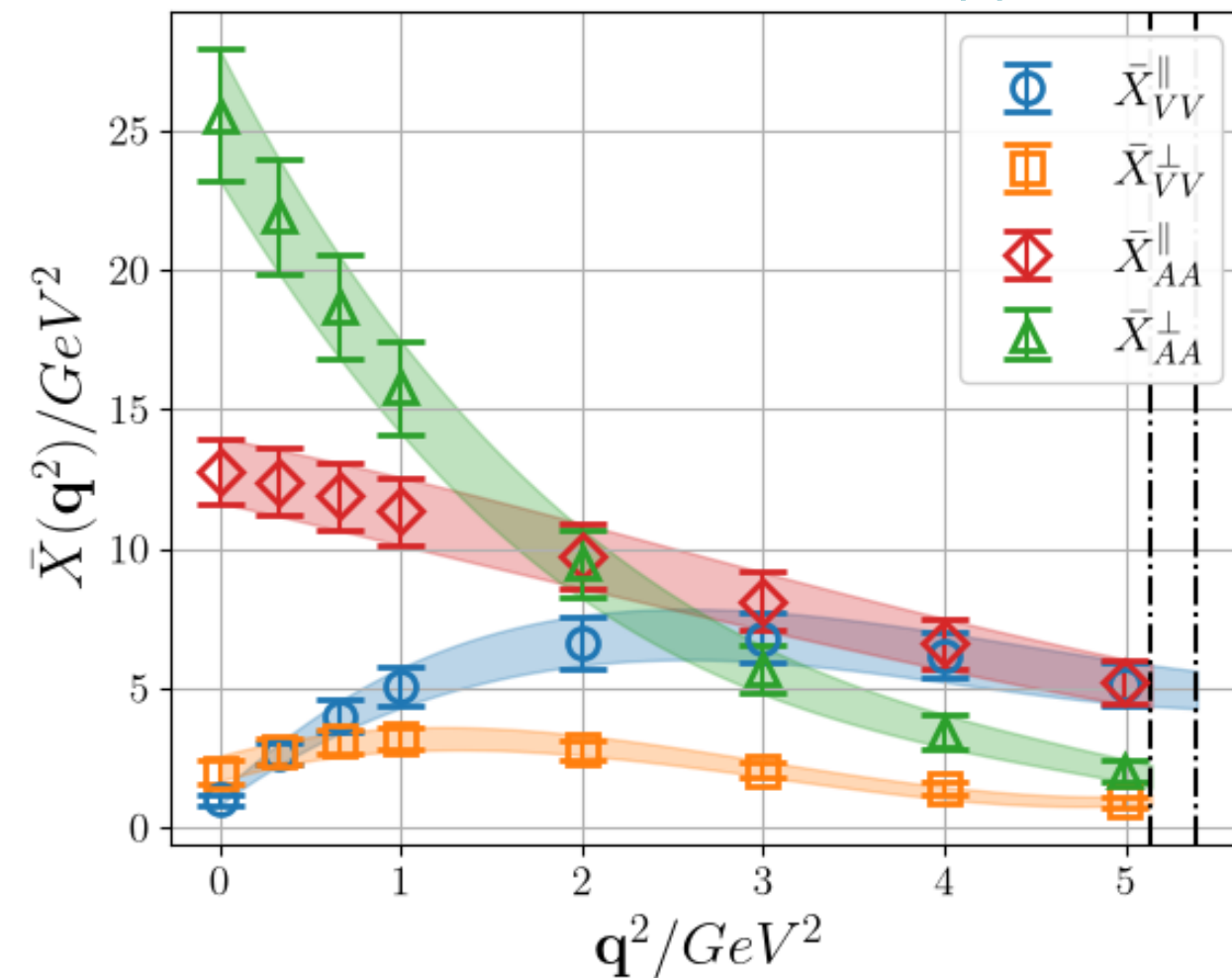
Summary $B_s \rightarrow X_c \ell \bar{\nu}_\ell$

Lattice predictions for
differential decay rates

[De Santis et al. [arXiv:2607.01116](#)]



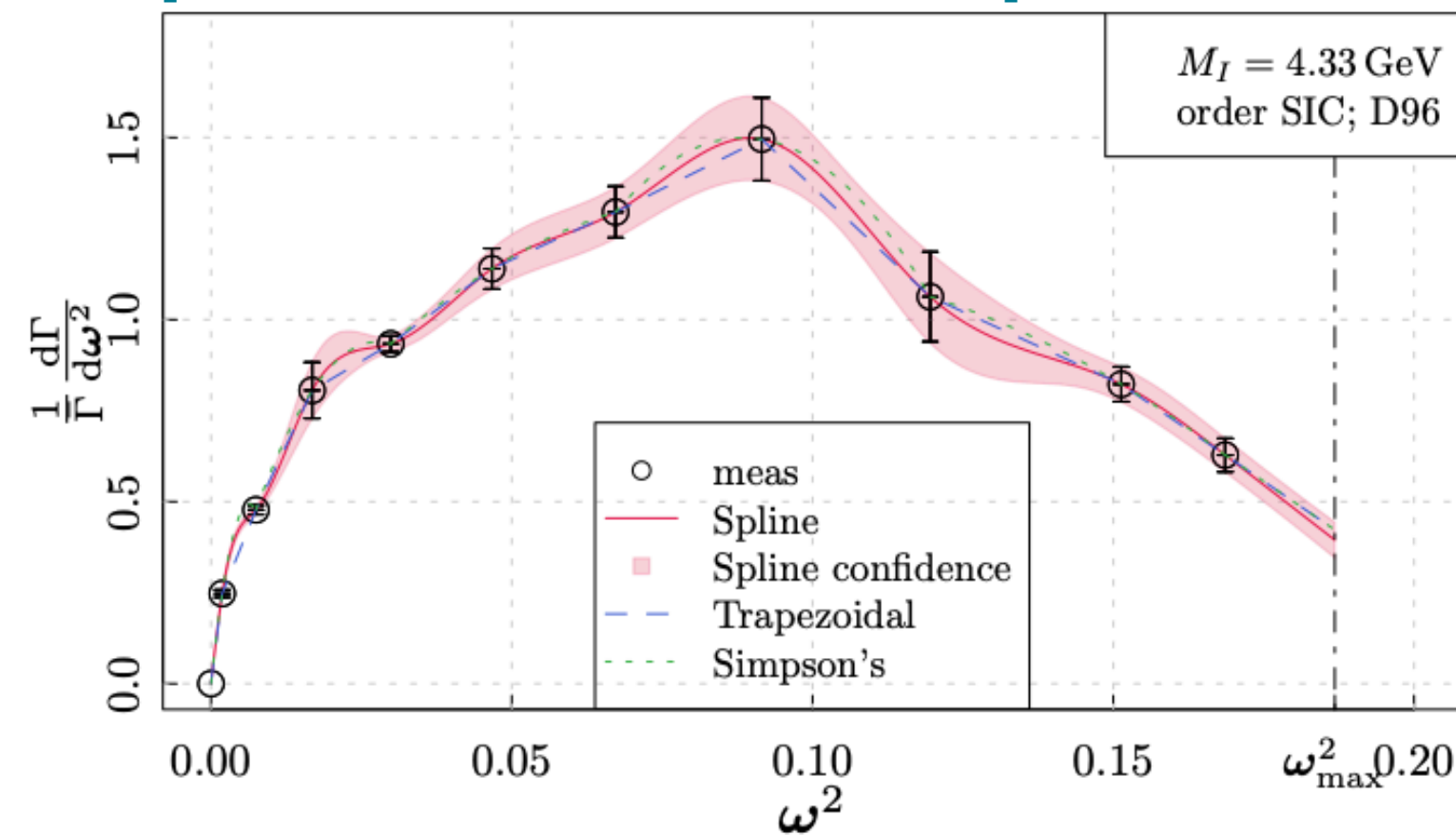
[CERN-KEK-Mainz-Soton, to appear]



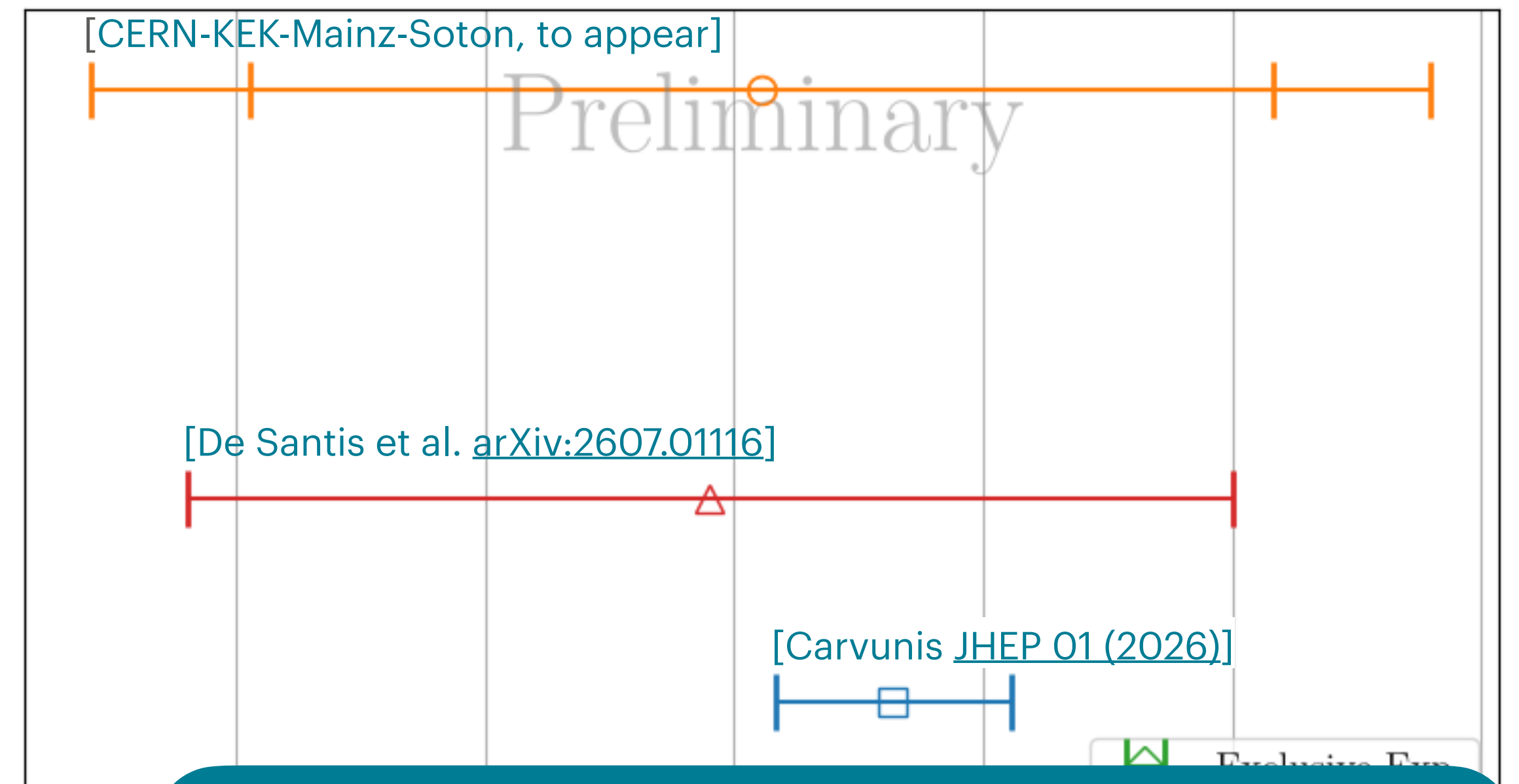
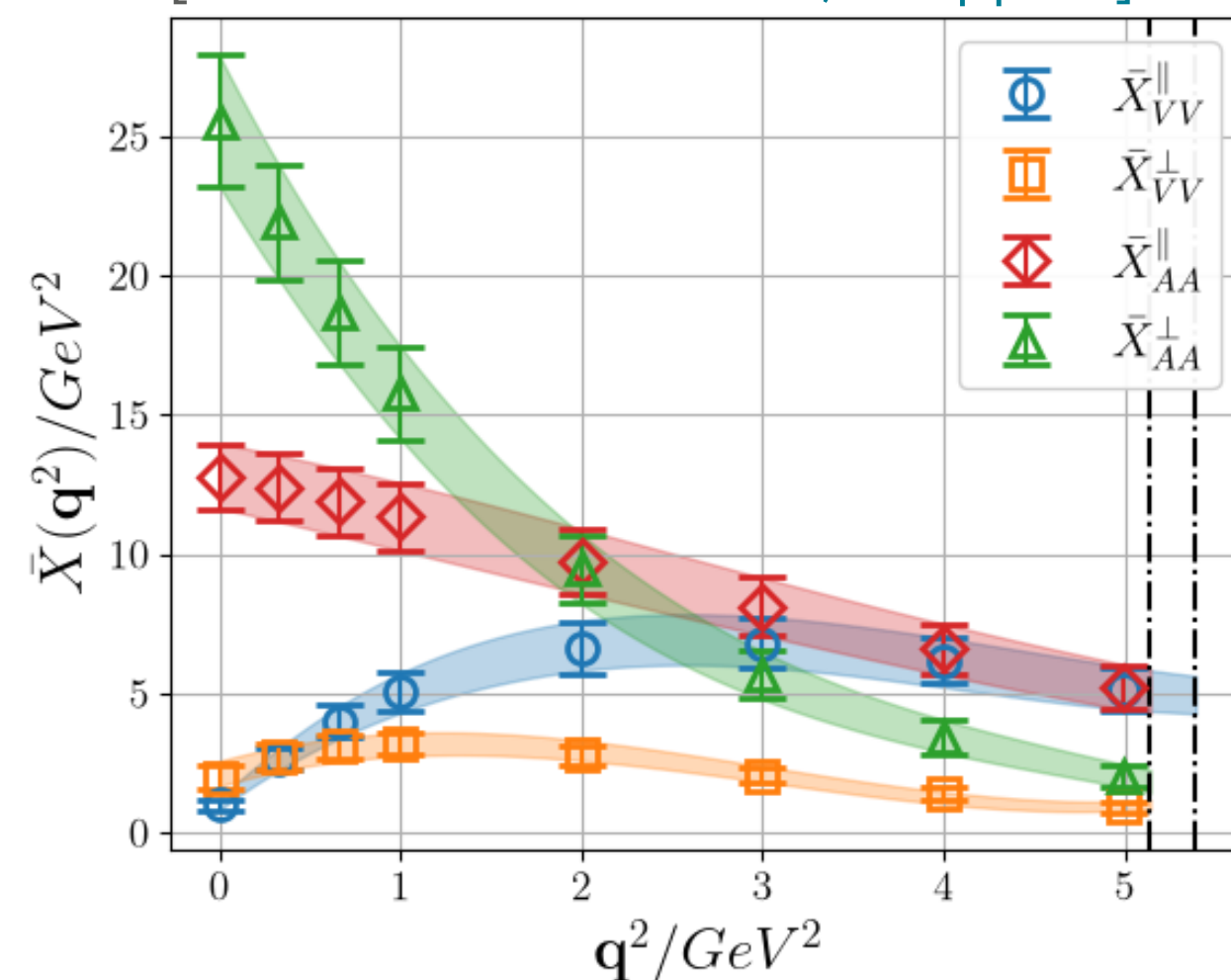
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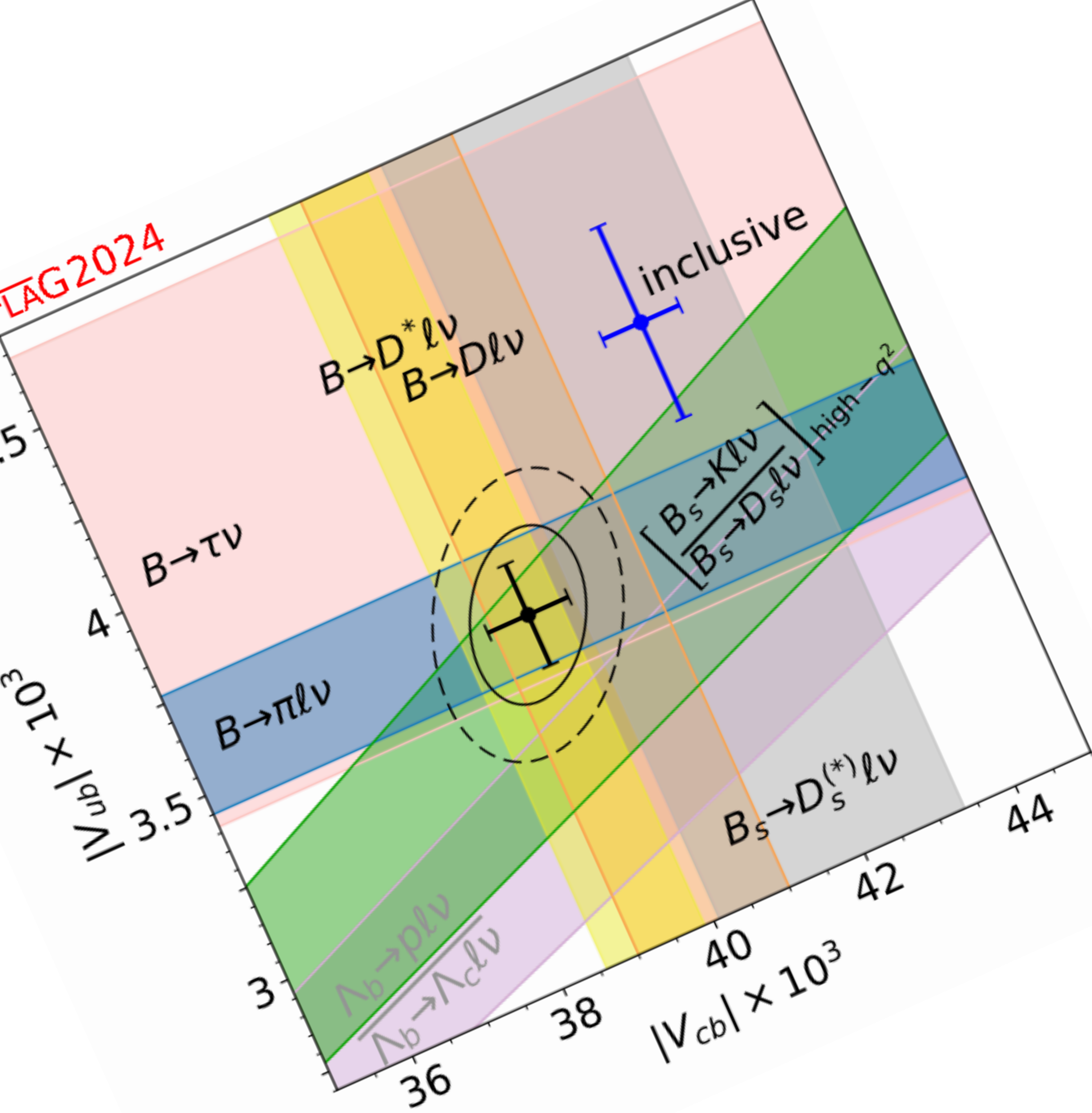
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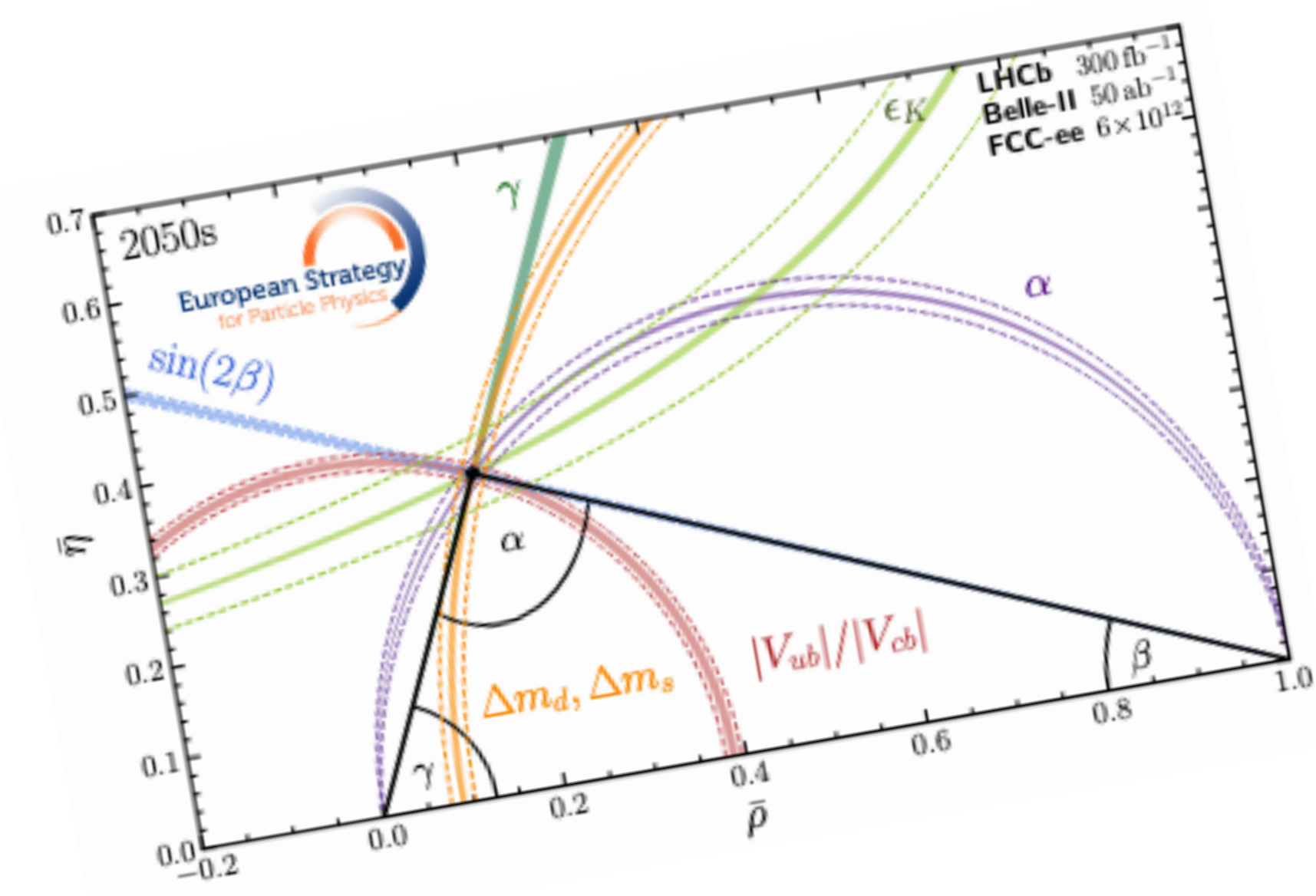
- first-ever lattice results for inclusive semileptonic D_s and B_s decay
- two independent analyses, largely different approaches in agreement
- plenty of ideas for how to improve calculation



$$\Delta M_{d,s}$$

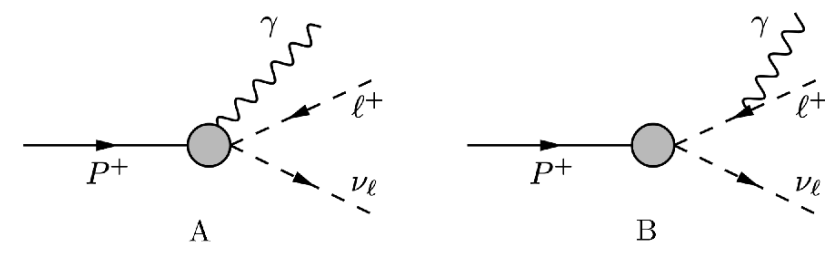
$$B \rightarrow \mu \mu$$

$$K_L \rightarrow \pi^0 \nu \bar{\nu}$$



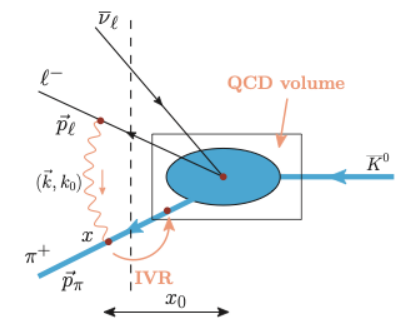
Part II — 2nd order EW

(hadronic decay, long-distance effects, ...)



Radiative meson decay

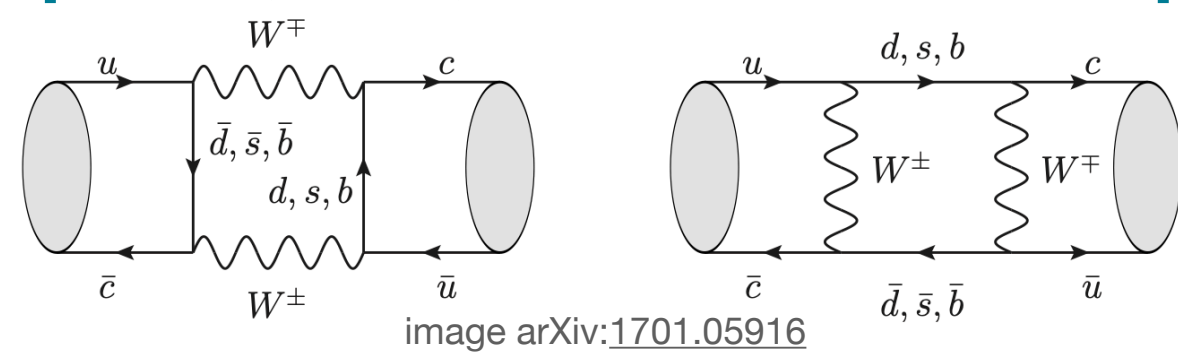
[Frezzotti et al. [PRD 103 \(2021\)](#)], [Giusti et al. [PRD 107 \(2023\)](#),
[PRD 112 \(2025\)](#)], Di Palma et al [PRD 111 \(2025\)](#)],
[JLQCD and UKQCD [2603.19960](#)]



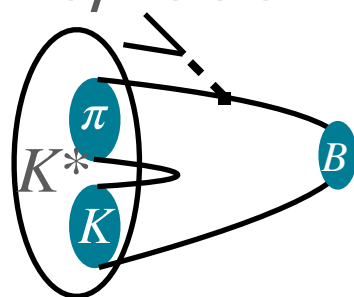
QCD+QED for semileptonic decays
[Christ et al. [arXiv:2402.08915](#)]

long-distance contributions to charm mixing

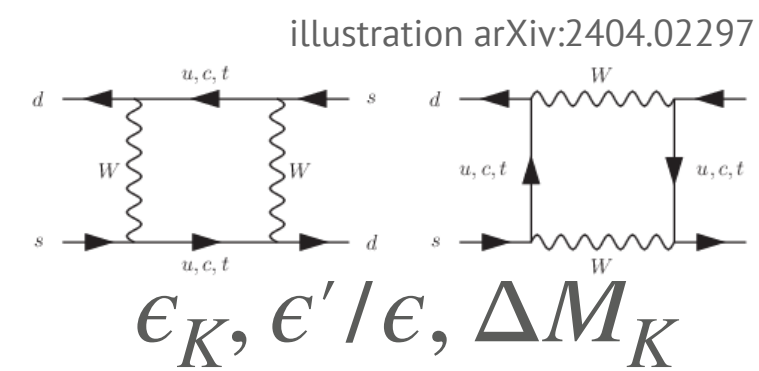
[Di Carlo, Erben, Hansen [arXiv:2504.16189](#)]



SL decay into unstable mesons/resonances



[Erben et al. [arXiv:2603.17900](#)],
[Leskovec et al. [PRL 134 \(2025\)](#)]



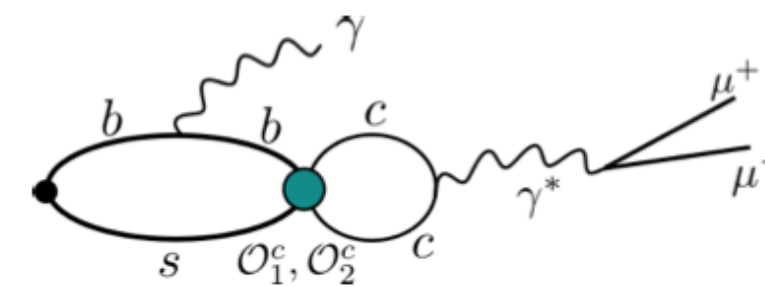
[Bai et al. [PRD 2024](#)]
[Christ et al. [arXiv:2606.26336](#)]

**Talks by Jonas Hildebrand
Masaaki Tomii**

2nd order EW / finite-volume techniques

	γ	quarks	μ	e	τ	ν
$3_q \rightarrow 2_q$	$b \rightarrow s \gamma$	$B_s \leftrightarrow \bar{B}_s$	$b \rightarrow s \mu \mu$ $b \rightarrow c \mu \nu$	$R_{bs}(e/\mu)$ $R_{bc}(e/\mu)$	$b \rightarrow s \tau \tau$ $b \rightarrow c \tau \nu$	$b \rightarrow s \nu \nu$
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$2_q \rightarrow 2_q$ $2_q \rightarrow 1_q$	$c \rightarrow u \gamma$	$D \leftrightarrow \bar{D}$	$c \rightarrow u \mu \mu$ $c \rightarrow s \mu \nu$ $c \rightarrow d \mu \nu$	$R_{cu}(e/\mu)$ $R_{cs}(e/\mu)$ $R_{cd}(e/\mu)$		$c \rightarrow u \nu \nu$
$2_q \rightarrow 1_q$ 1_q	$s \rightarrow d \gamma$	$K \leftrightarrow \bar{K}$	$s \rightarrow d \mu \mu$ $s \rightarrow u \mu \nu$ $d \rightarrow u \mu \nu$	$R_{sd}(e/\mu)$ $R_{su}(e/\mu)$ $R_{du}(e/\mu)$		$s \rightarrow d \nu \nu$

G. Isidori ESPP



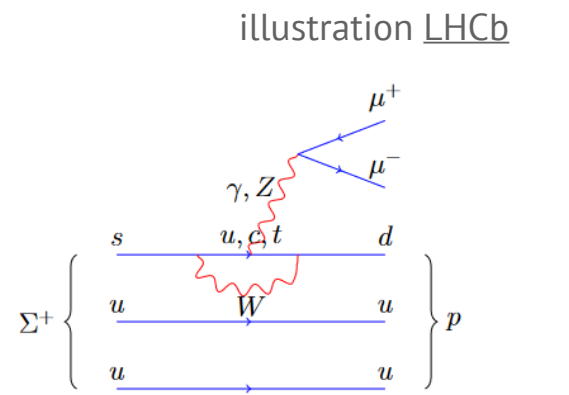
long-distance contributions to radiative decays, e.g. $B_s \rightarrow \mu \mu \gamma$

[Frezzotti et al., [PRD 2024](#)]

long-distance effects in rare leptonic decays

e.g. $K \rightarrow \ell \ell$

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rare hyperon decay
[Erben et al. [JHEP 07 \(2025\)](#)]

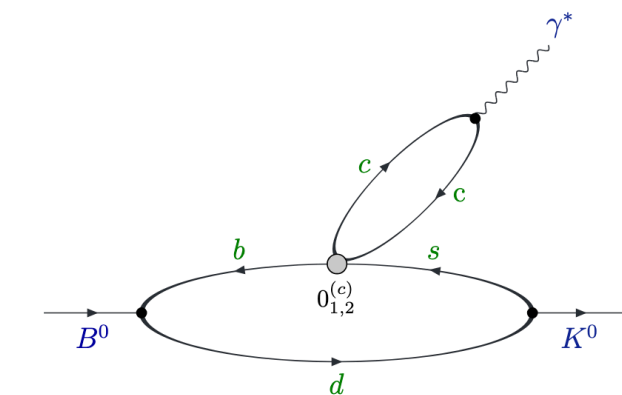


long-distance contributions to rare kaon decay

[Boyle et al. [PRD 2023](#)]

[Hodgson et al., [arXiv:2501.18358](#)]

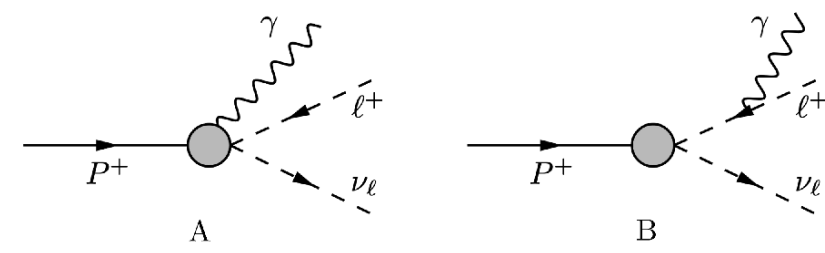
[Di Palma et al. [arXiv:2605.22742](#), [arXiv:2605.22727](#)]
[Tuo et al, [PRD 105 \(2022\)](#)], [Gagliardi et al. [PRD 105 \(2022\)](#)]



charming Penguin contributions to rare semileptonic decay

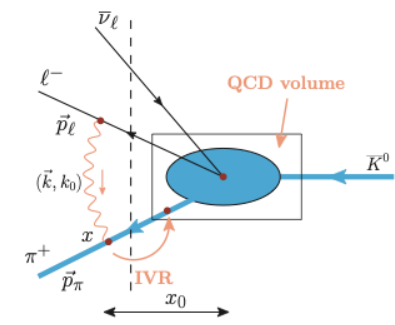
e.g. $B \rightarrow K^* \ell \ell$, $D \rightarrow \pi \ell \ell$

[Frezzotti et al. [PRD 108 \(2023\)](#), [PRD 113 \(2026\)](#)]
[AJ [2603.15487](#)]



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[Frezzotti et al. [PRD 103 \(2021\)](#)], [Giusti et al. [PRD 107 \(2023\)](#),
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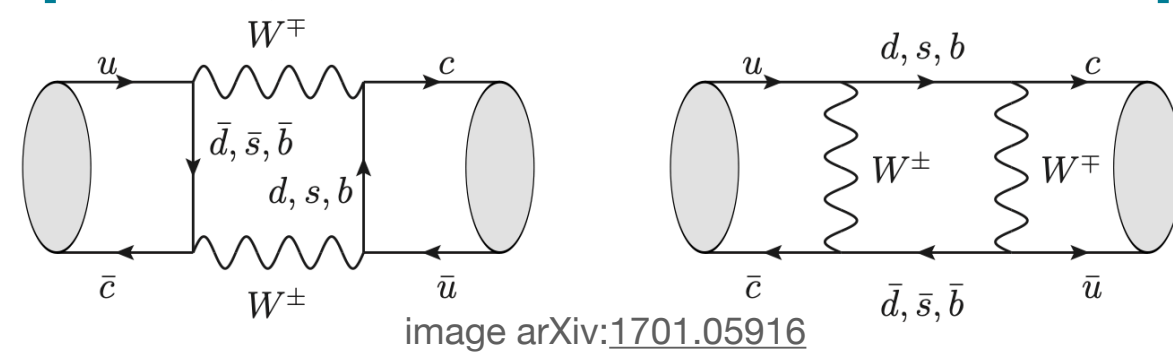
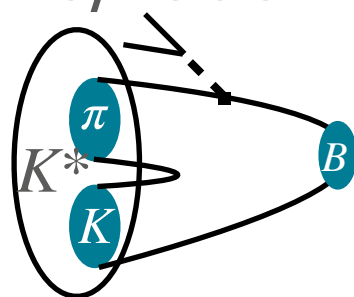
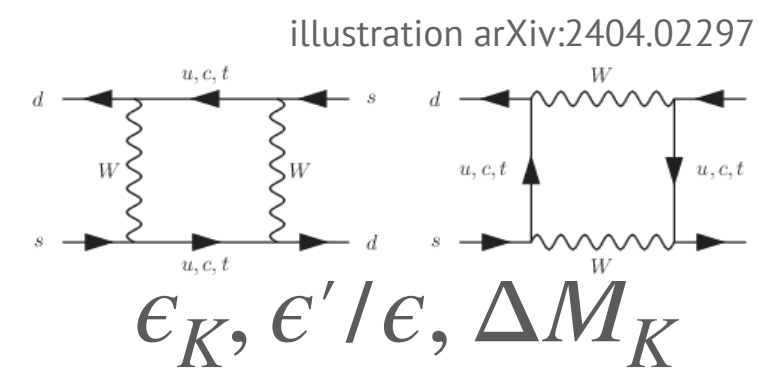


image [arXiv:1701.05916](#)

SL decay into unstable mesons/resonances



[Erben et al. [arXiv:2603.17900](#)],
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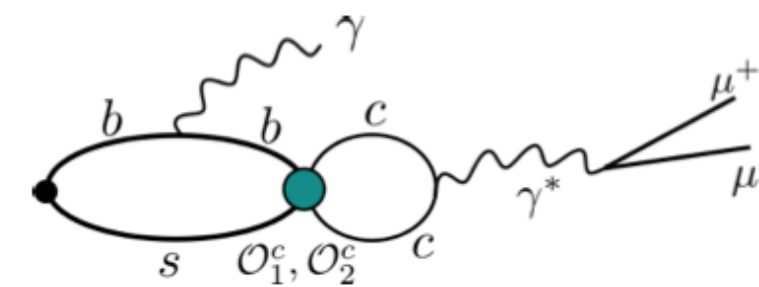
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Talks by Jonas Hildebrand
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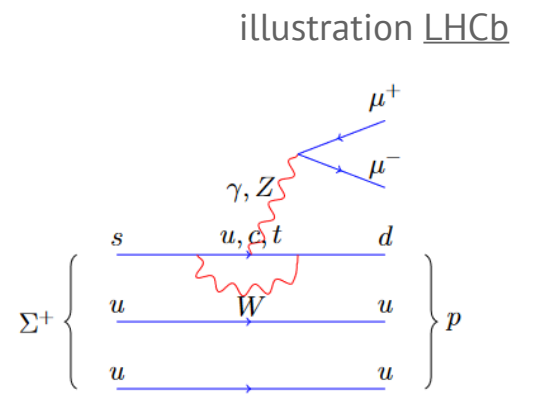
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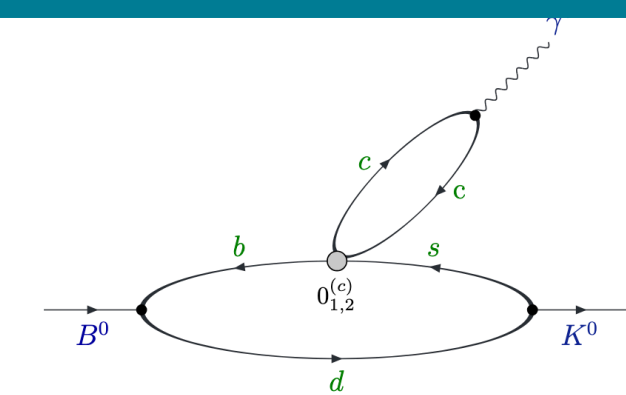
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[Frezzotti et al. [PRD 108 \(2023\)](#), [PRD 113 \(2026\)](#)]

[AJ [2603.15487](#)]

Reminder

Matrix-element classification

There is increasing activity on 2nd-order EW processes, often requiring development of exciting and new techniques.

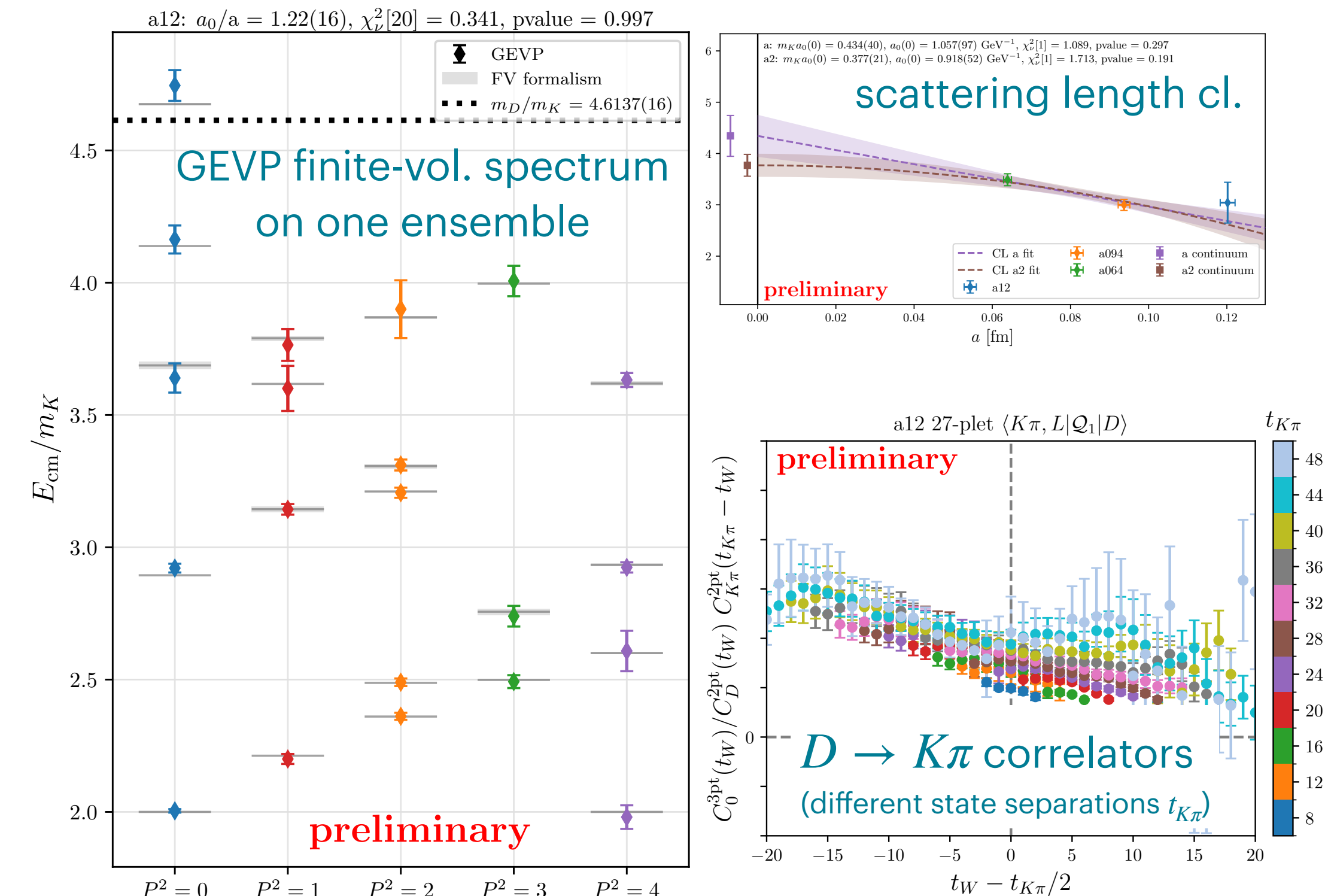
Minkowski $\leftrightarrow$ Euclidean

1st order EW: $\langle P' H_{\text{eff}} P \rangle$	e.g. • tree-level leptonic and semileptonic decay • short-distance contributions to 2nd-order processes (rare decays, mixing, ...)	$\langle 0 A_\mu K \rangle$ $\langle \pi V_\mu B \rangle$ $\langle B O^{\Delta B=2} \bar{B} \rangle$	"i"s and signs
1st order EW: $\langle P' P'' H_{\text{eff}} P \rangle$ 2nd order EW: $\langle P' T[H_{\text{eff},1} H_{\text{eff},2}] P \rangle$	e.g. • hadronic decay • long-distance effects in rare decays • meson mixing	$\langle K T[j_{\text{em}} O_{1,2}] B \rangle$ $\langle B T[O^{\Delta B=1} O^{\Delta B=1}] \bar{B} \rangle$	"i"s and signs AND depending on kinematics: Lüscher or spectral reconstruction

Hadronic D decay

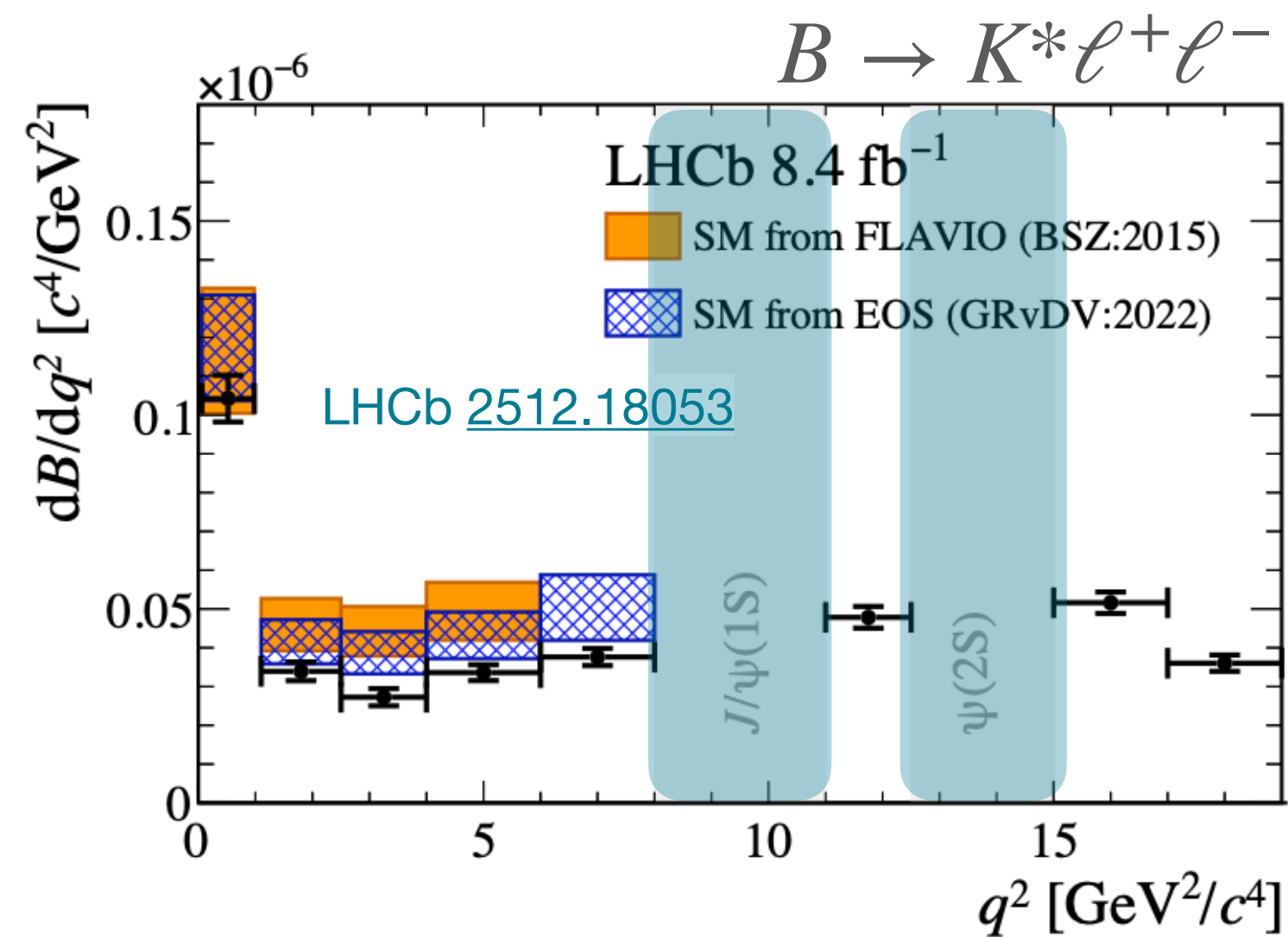
Material sent to me by Matthew Black

- $\Delta A_{CP} = A_{CP}(D \rightarrow K^+ K^-) - A_{CP}(D \rightarrow \pi^+ \pi^-)$ — LHCb
“Observation of CP violation in charm decays”
[LHCb PRL 122 (2019)]
- difficult due to contaminations by higher-multiplicity final states
- simplify $SU(3)_F$ symmetric point $M_\pi = M_K \sim 410\text{MeV}$
wt. stabilised Wilson fermions (OpenLat)
 - D just above 4π threshold $\rightarrow$ suppressed
 - concentrate on Cabibbo-favoured $D^+ \rightarrow \bar{K}\pi^+$
(no penguin & flavour-singlet contributions)
- Lüscher finite-volume technique
- despite simplifications still useful for pheno!



Talk by Max Hansen

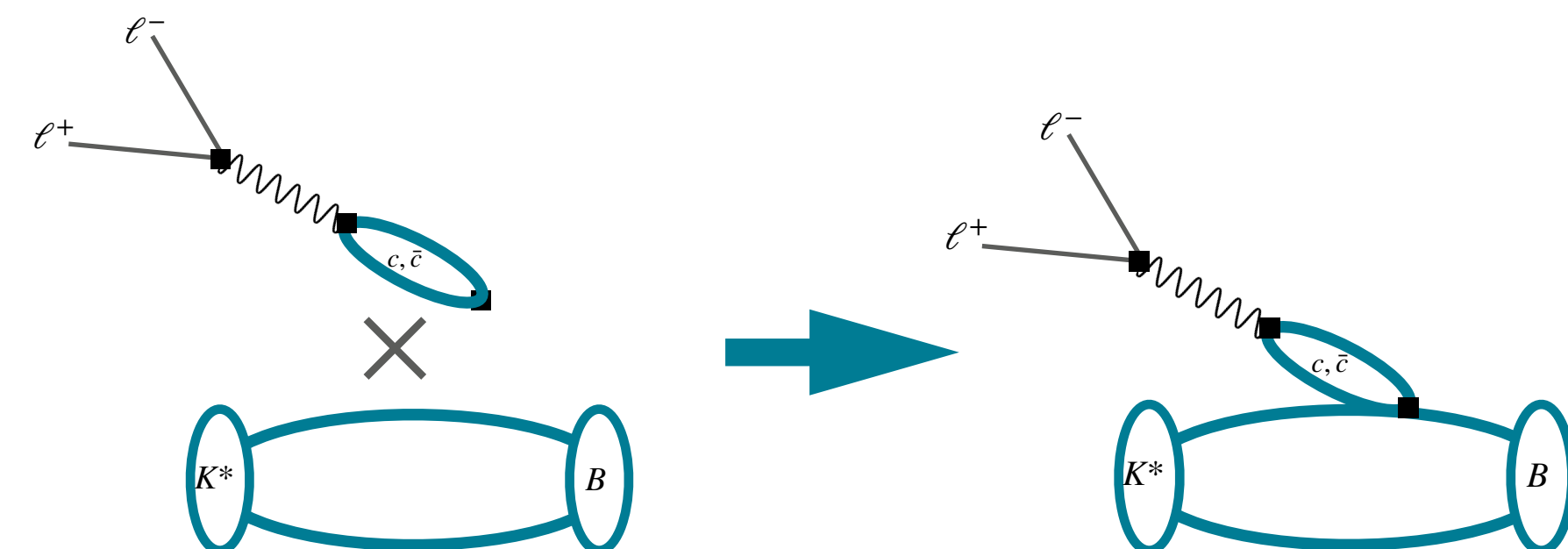
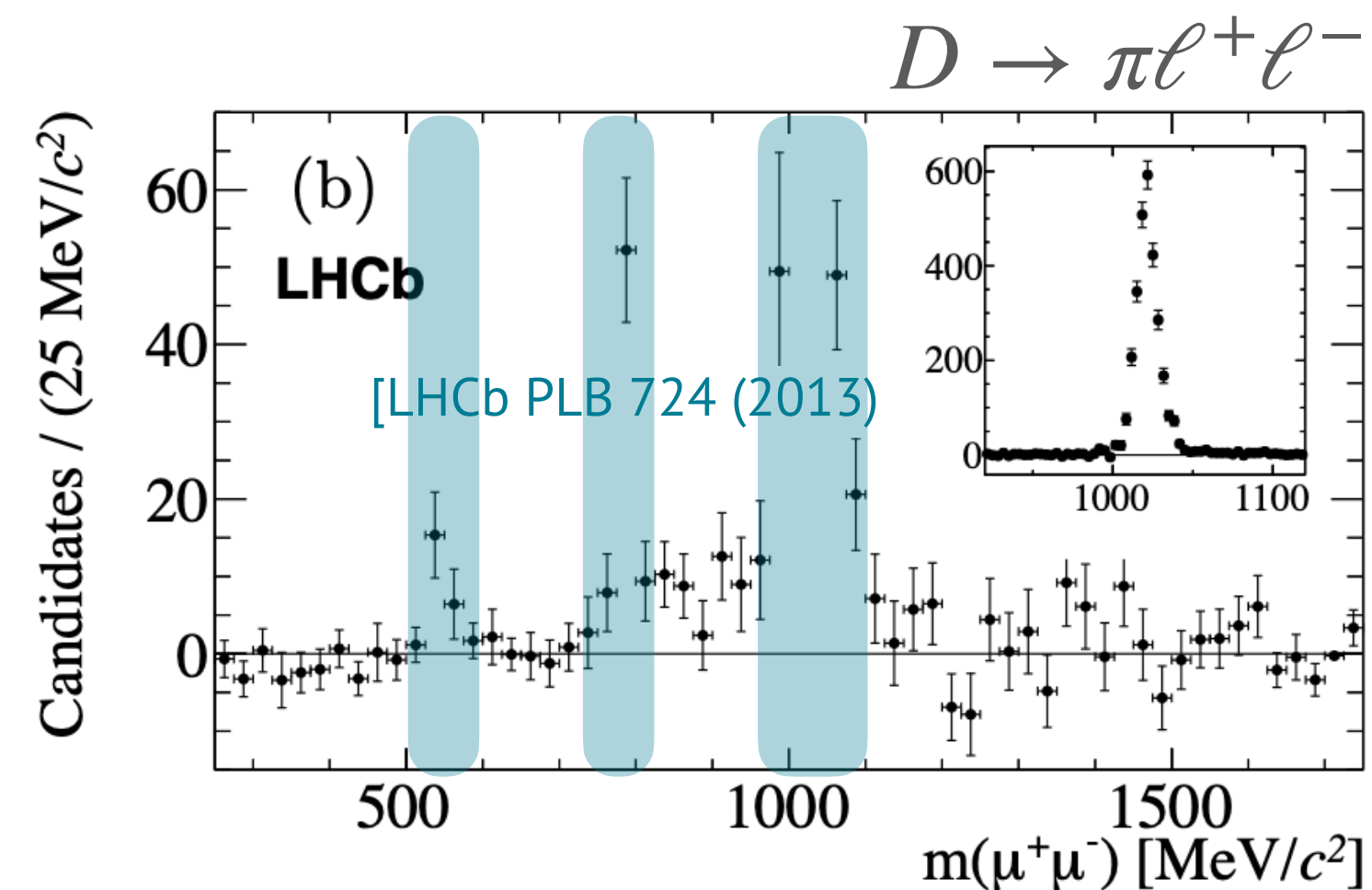
Rare semileptonic B and D decays



Challenge 1: $B \rightarrow K^* \ell^+ \ell^-$ final state unstable to date treated in narrow-width approximation

Challenge 2: long-distance contributions

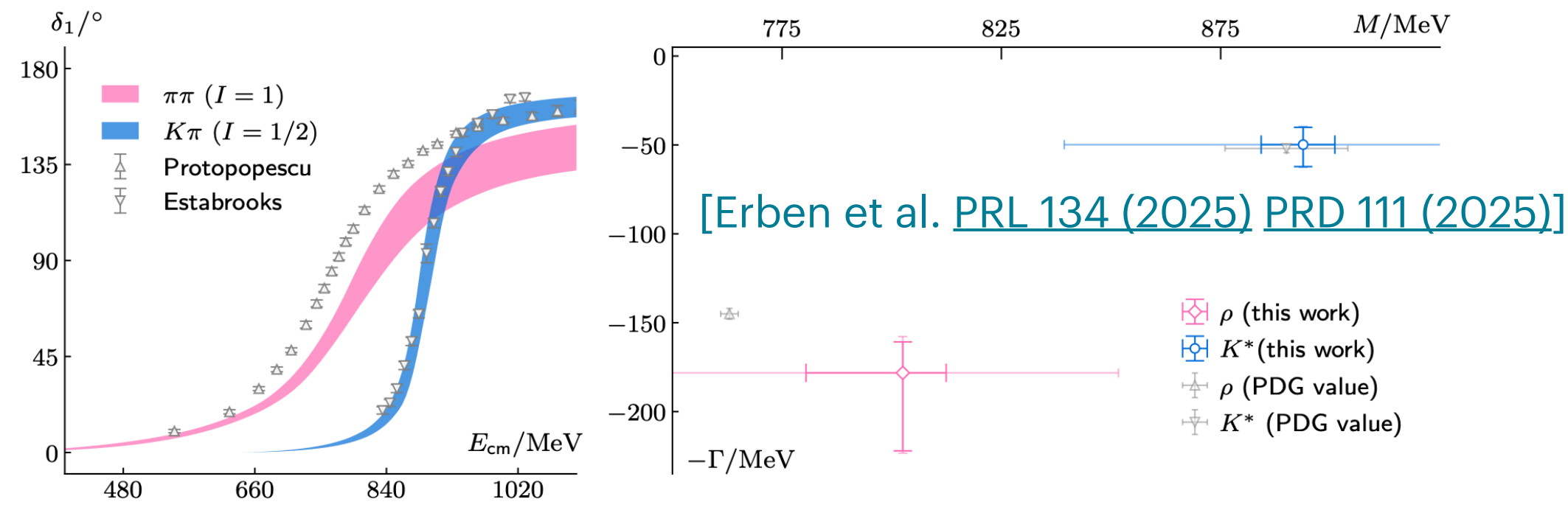
quark-loops ($c\bar{c}$ in $B \rightarrow K^{(*)} \ell^+ \ell^-$ or $d\bar{d}, s\bar{s}$ in $D \rightarrow \pi \ell^+ \ell^-$) so far only computed in factorisation, disp.-rel., LCSR, ... no 1st-principles prediction of strong phase



Challenge 1

$B \rightarrow K^*(K\pi)\mu\mu, B \rightarrow \rho\ell\nu_\ell$

first step: lattice calculation of resonances $\rho(770)$ and $K^*(892)$



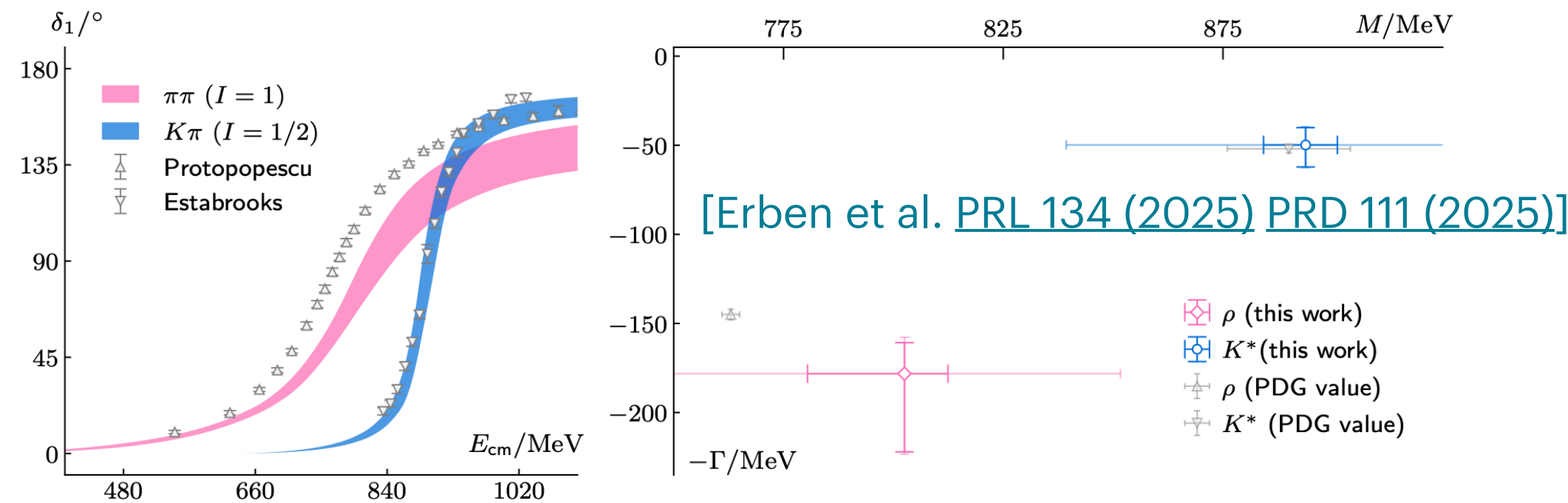
- single lattice-spacing/volume
- results in agreement with experiment
- can now be systematically improved



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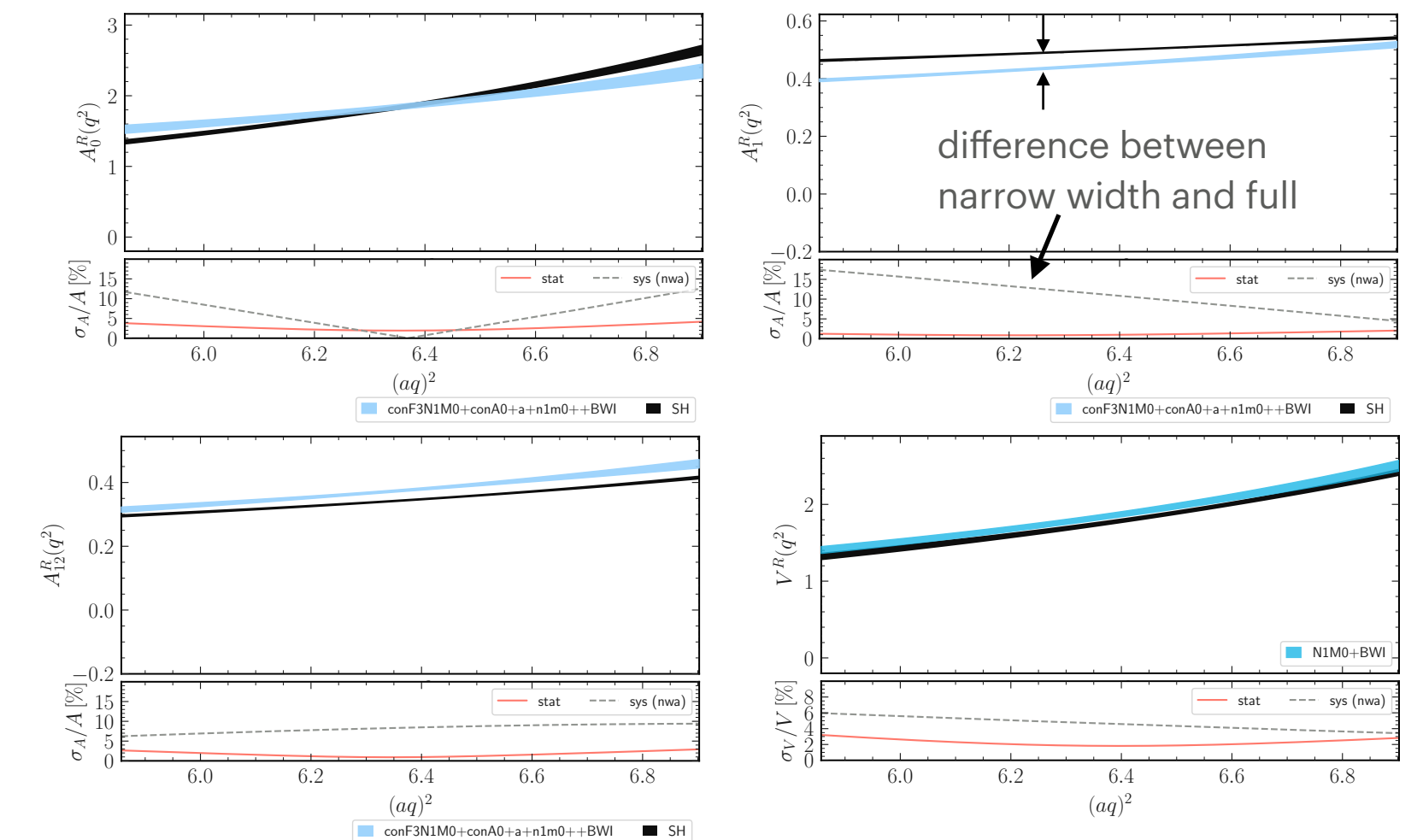
- single lattice-spacing/volume
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- **second step:** transition matrix element with unstable final state

(work on $B \rightarrow \rho\ell\bar{\nu}$ and $B \rightarrow K^*\ell^+\ell^-$ under way) [Agadjanova et al., [NPB 910 \(2016\)](#)], [Leskovec et al. [PRL 134 \(2025\)](#)], [Erben et al. [arXiv:2603.17900](#)]

- how bad is narrow-width approximation?
 - for narrow resonance expect 5% shift between narrow-width approximation and full treatment [Briceño, Hansen [PRD 92 \(2015\)](#)]
 - Plots: Leskovec et al. find significant differences of the expected size
 - They now also extend their existing work including ensembles with $M_\pi = 174, 320 \text{ MeV}$ and $a = 0.07, 0.11 \text{ fm}$

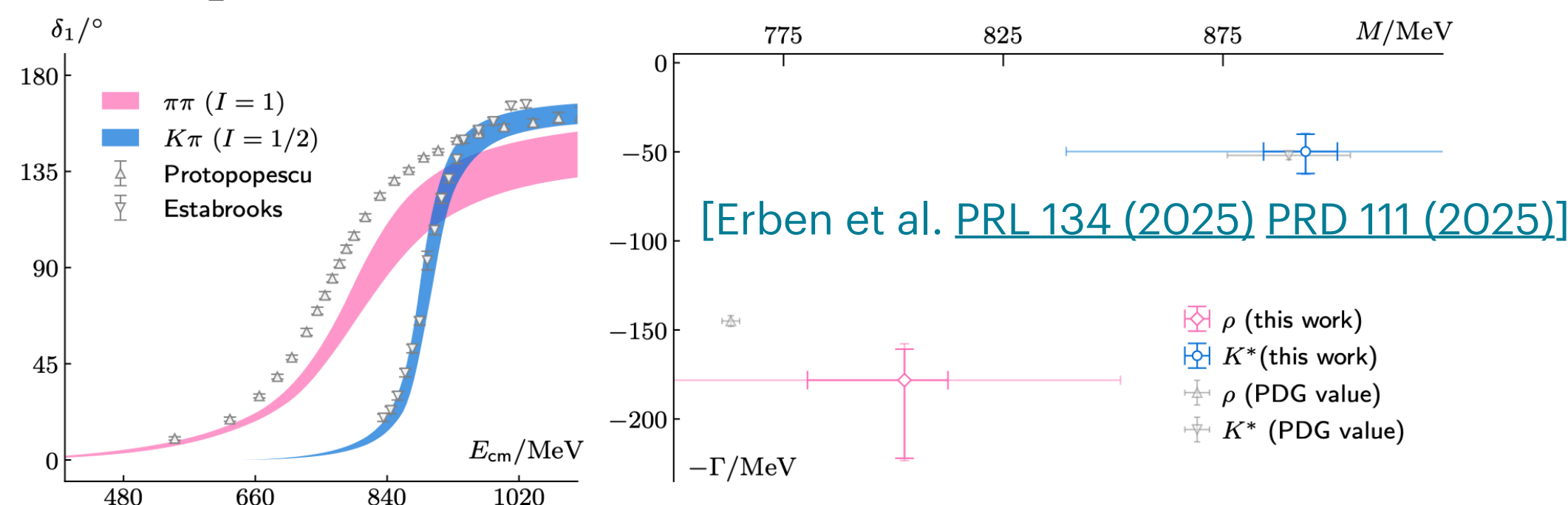
talk by **Luka Leskovec**



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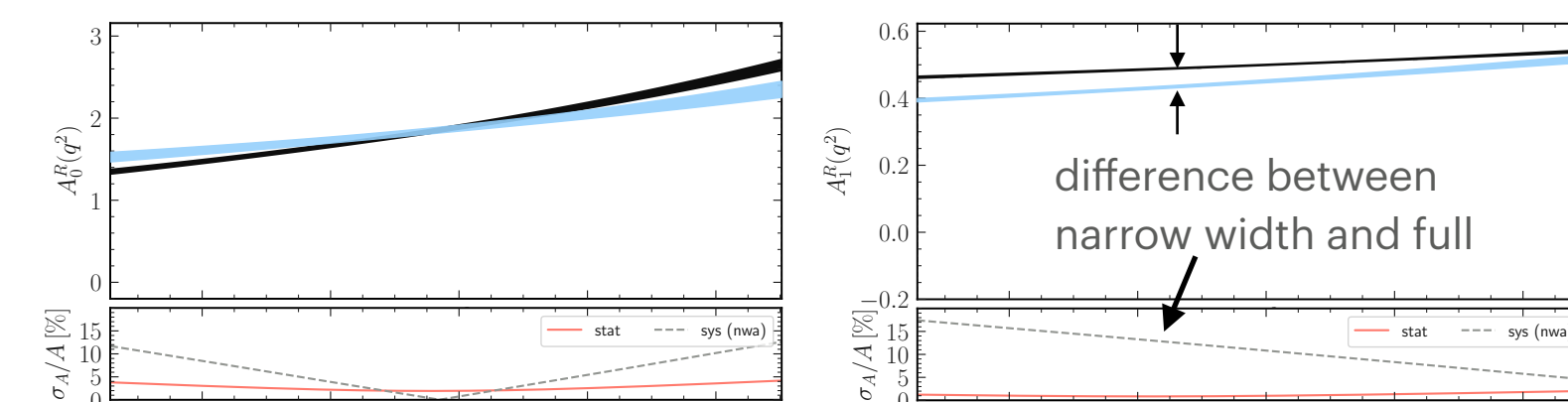


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talk by **Luka Leskovec**

- long-standing uncertainty due to narrow-width approximation in hadronic form factors now being addressed
- relevant for central NP searches at e.g. LHC

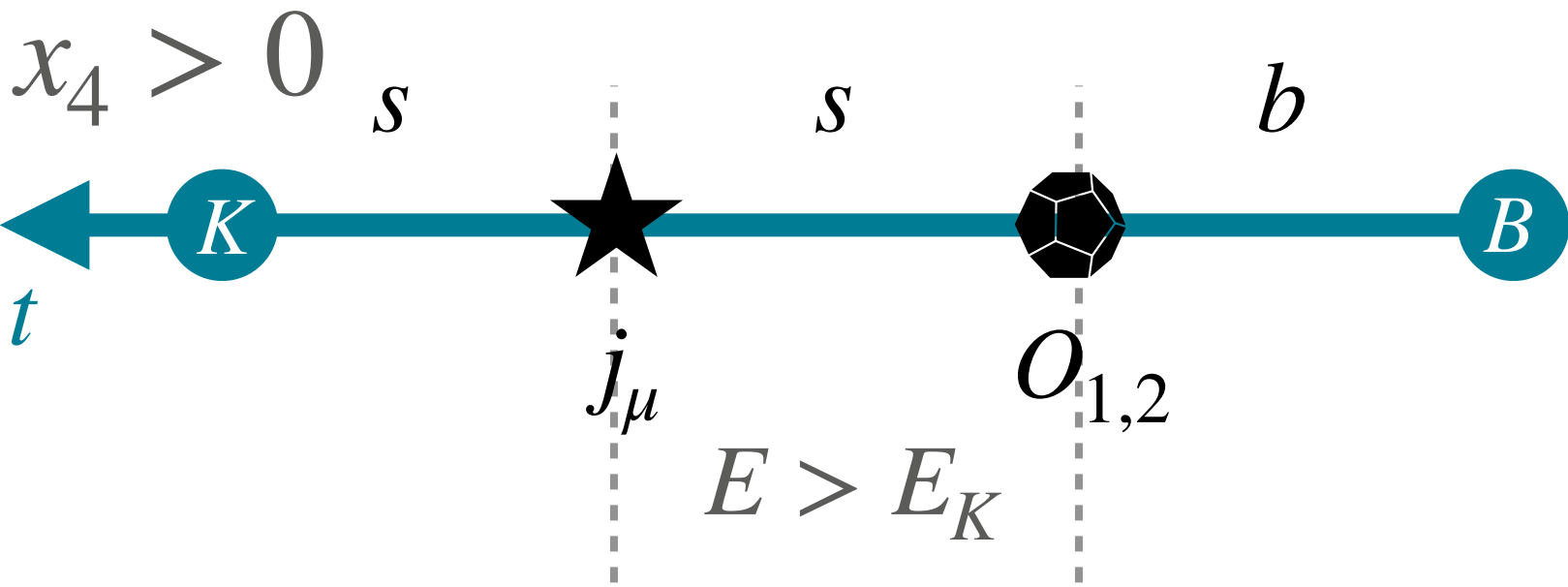
Minkowski $\leftrightarrow$ Euclidean

example 2nd order amplitude: $H_\mu(q) = i \int d^4x e^{iq \cdot x} \langle K(\mathbf{k}) | T\{j_\mu^{\text{elm}}(x), O_{1,2}(0)\} | B(\mathbf{0}) \rangle$

challenge 2

Minkowski $\leftrightarrow$ Euclidean

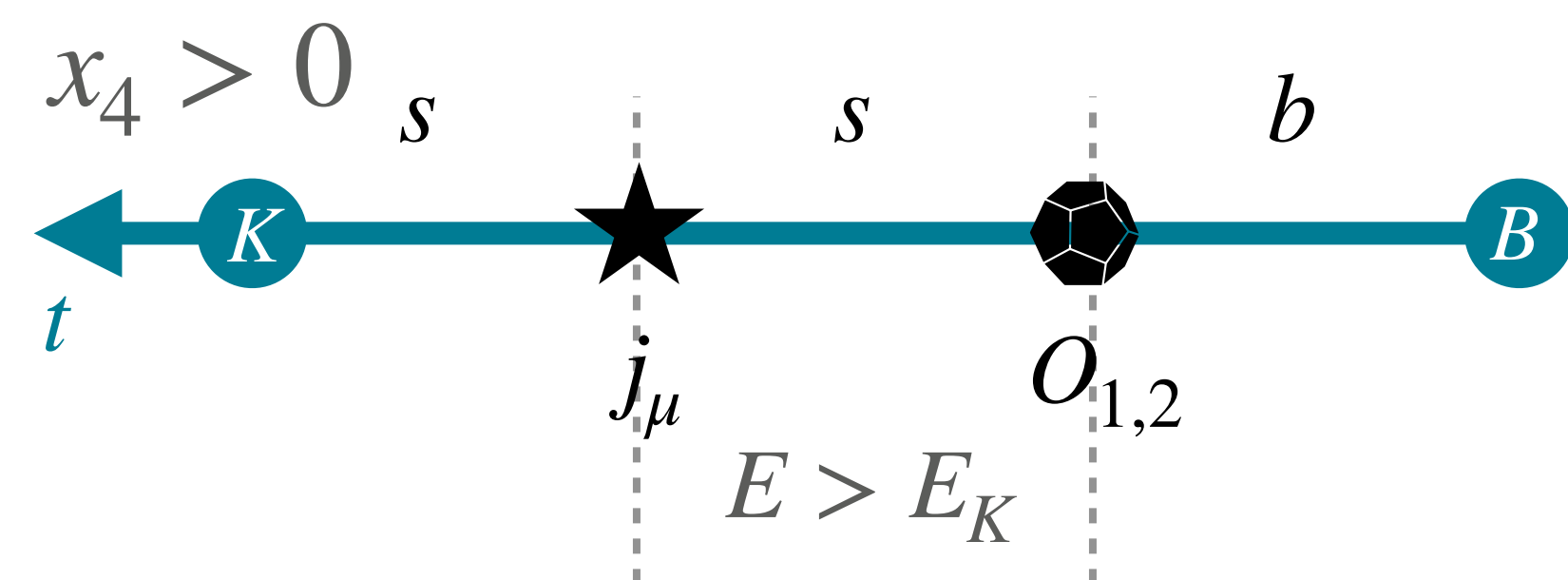
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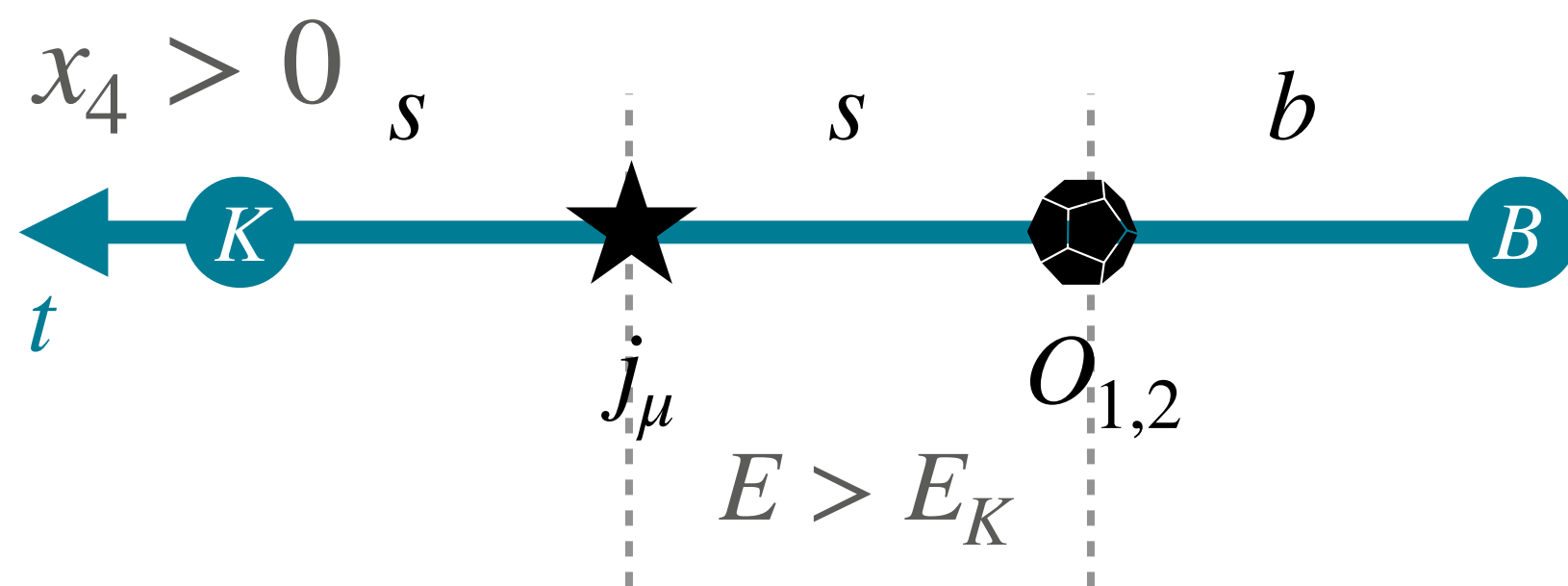
$x_4 > 0$: on-shell intermediate states possible

$$C_\mu^+(\mathbf{k}, q_0) = \int_0^\infty dx_4 \sum_n \langle K(\mathbf{k}) | j_\mu^{\text{elm}}(\mathbf{k}, 0) e^{-(\hat{H} - M_B)x_4} \frac{|n\rangle\langle n|}{2E_n} O_{1,2}(0) | B(\mathbf{0}) \rangle$$

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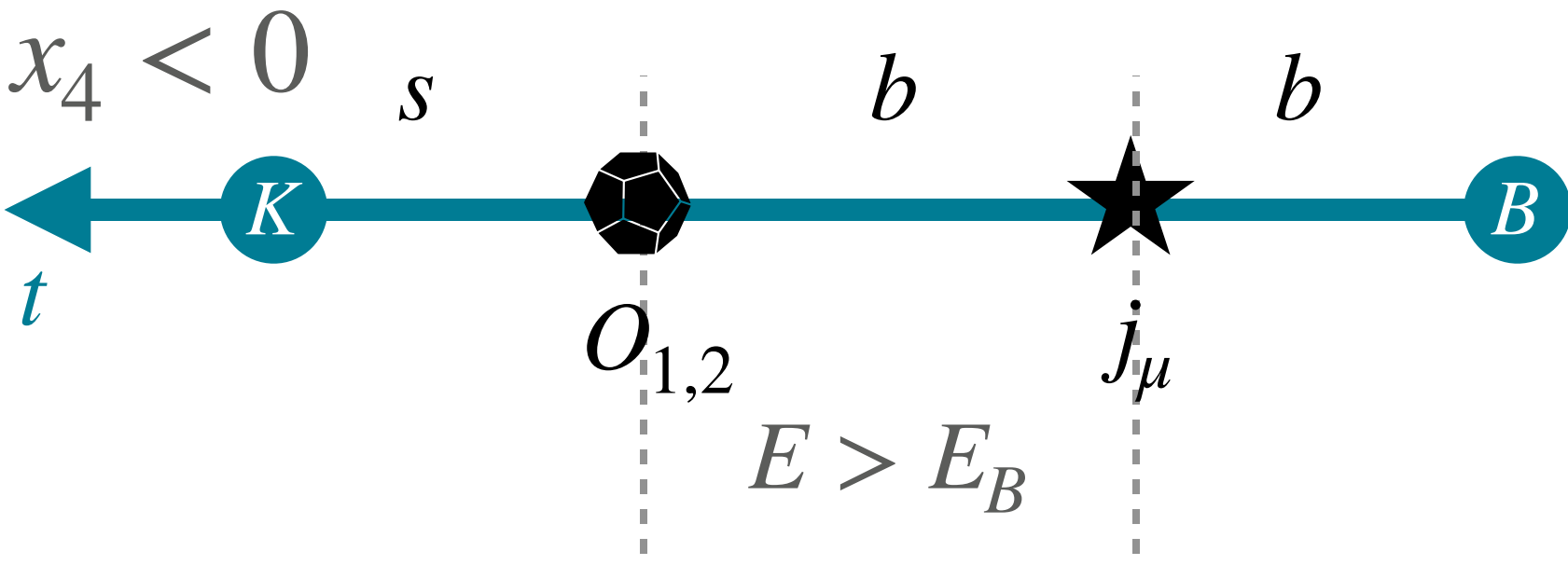
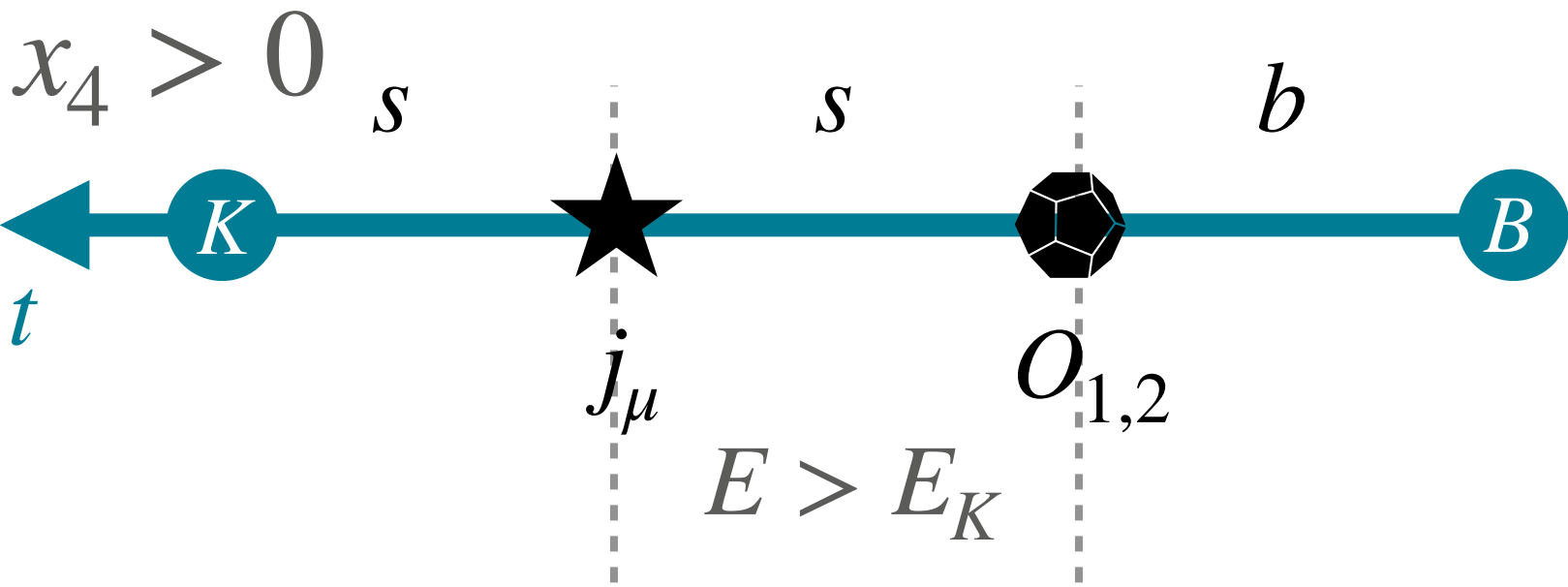
Then integrate Euclidean 4pt correlator over time:

- In Euclidean signature resonance states with $E_n - M_B < 0$ lead to divergent contributions under time integration
- naive lattice calculation not feasible

Challenge 2

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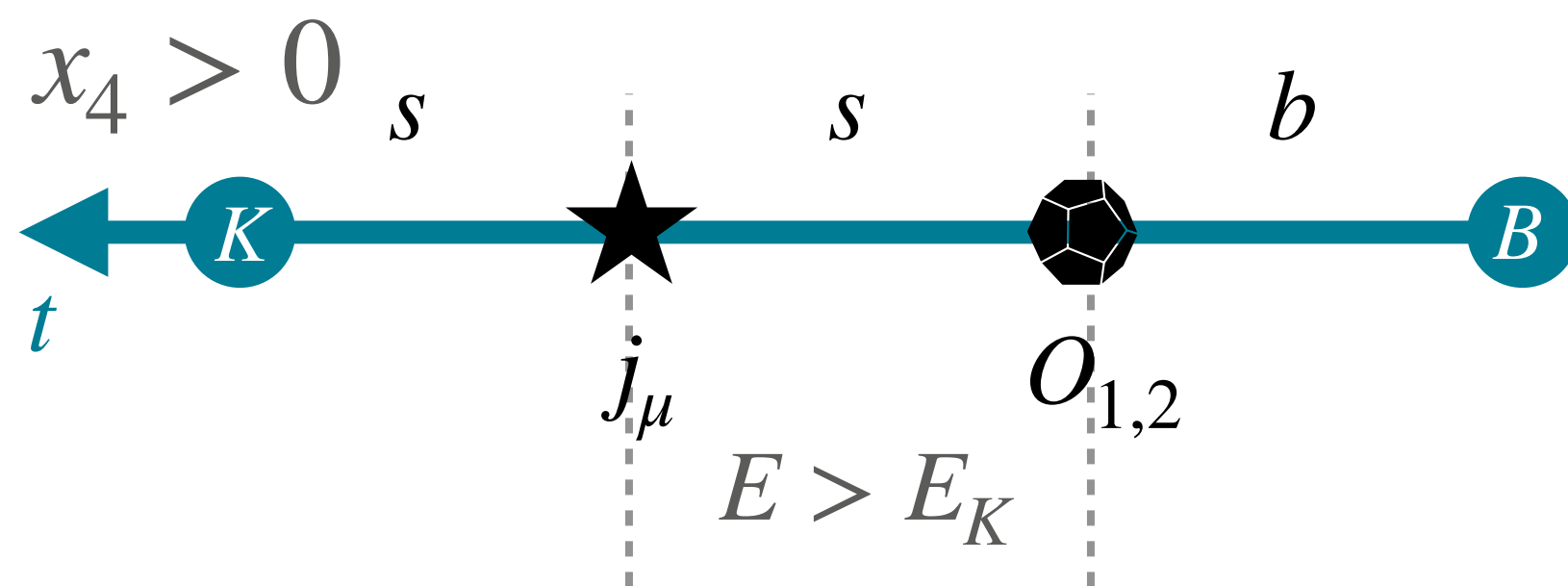
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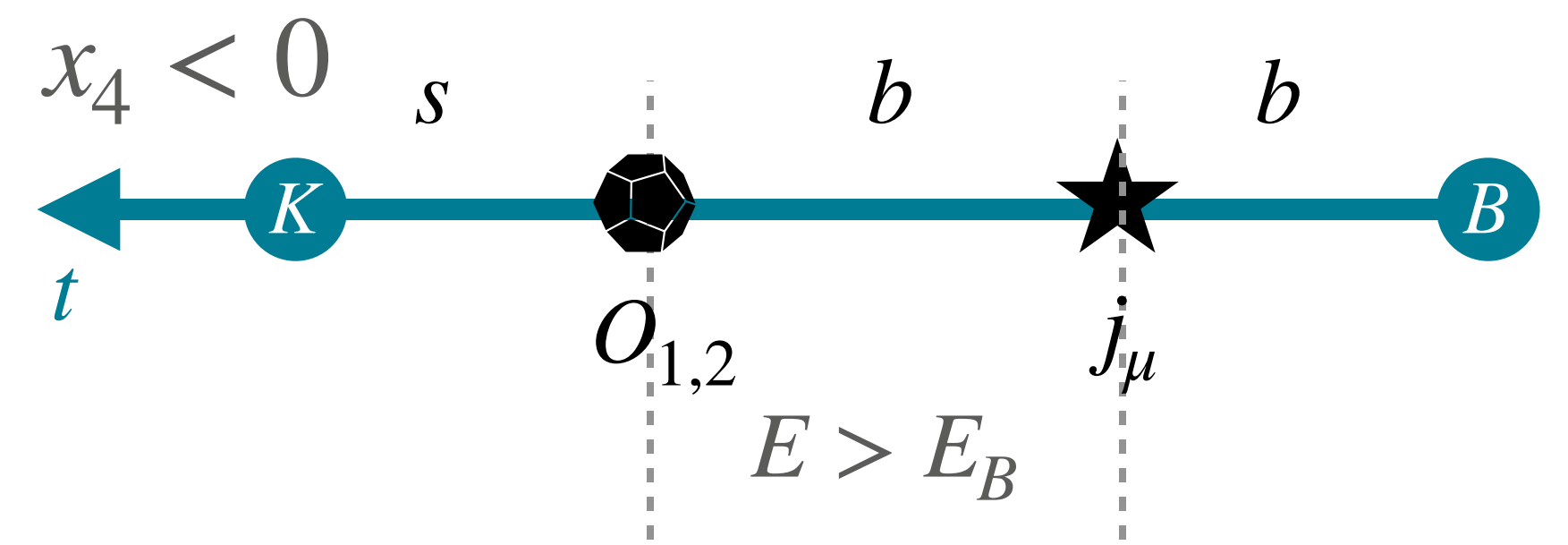


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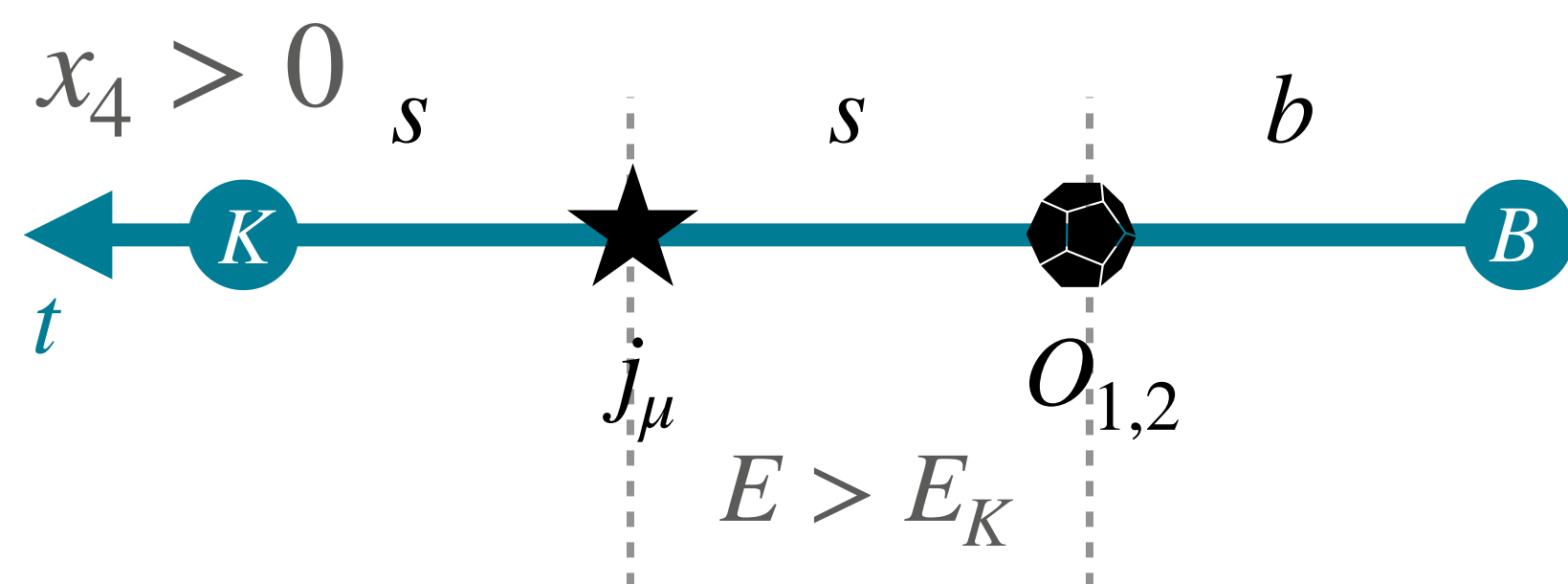
$x_4 < 0$: no on-shell intermediate states

$$C_\mu^-(\mathbf{k}, q_0) = \int_0^\infty dx_4 \sum_n \langle K(\mathbf{k}) | O_{1,2}(0) \frac{|n\rangle\langle n|}{2E_n} e^{-(\hat{H} - M_K)x_4} j_\mu^{\text{elm}}(\mathbf{k}, 0) | B(\mathbf{0}) \rangle$$

Challenge 2

Minkowski $\leftrightarrow$ Euclidean

example 2nd order amplitude: $H_\mu(q) = i \int d^4x e^{iq \cdot x} \langle K(\mathbf{k}) | T\{j_\mu^{\text{elm}}(x), O_{1,2}(0)\} | B(\mathbf{0}) \rangle$

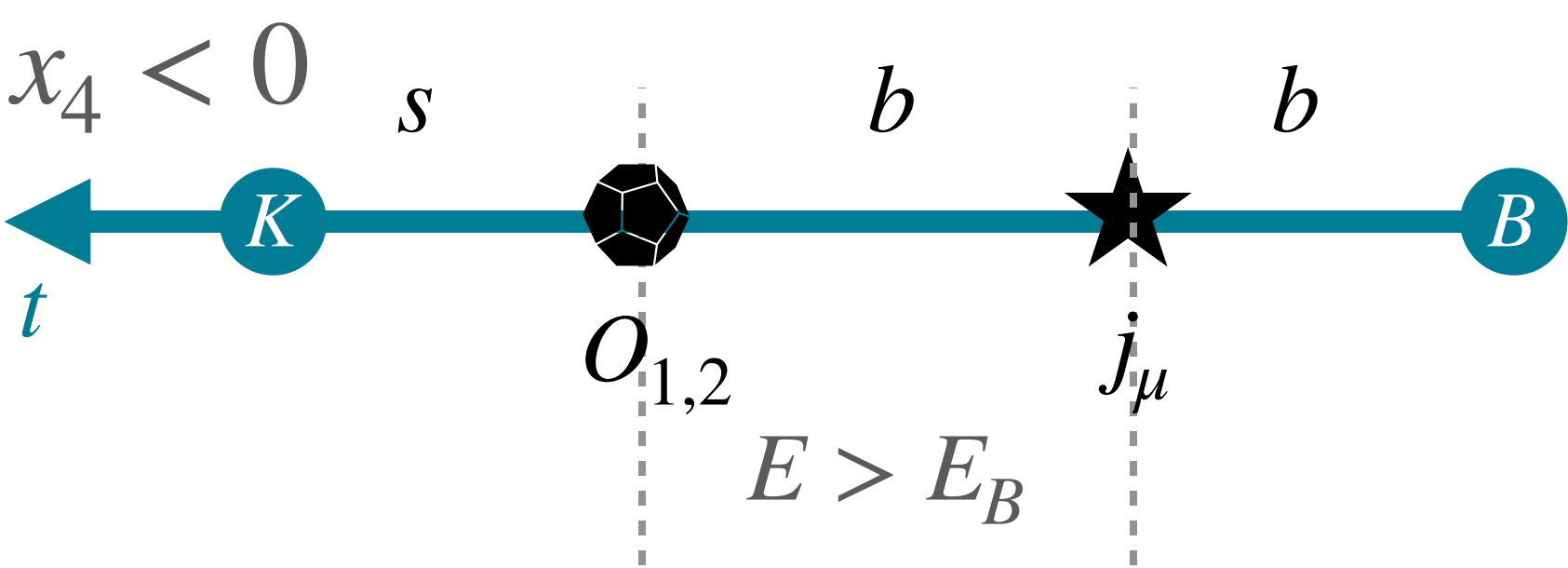


$x_4 > 0$: on-shell intermediate states possible

$$C_\mu^+(\mathbf{k}, q_0) = \int_0^\infty dx_4 \sum_n \langle K(\mathbf{k}) | j_\mu^{\text{elm}}(\mathbf{k}, 0) e^{-(\hat{H} - M_B)x_4} \frac{|n\rangle\langle n|}{2E_n} O_{1,2}(0) | B(\mathbf{0}) \rangle$$

Then integrate Euclidean 4pt correlator over time:

- In Euclidean signature resonance states with $E_n - M_B < 0$ lead to divergent contributions under time integration
- naive lattice calculation not feasible



$x_4 < 0$: no on-shell intermediate states

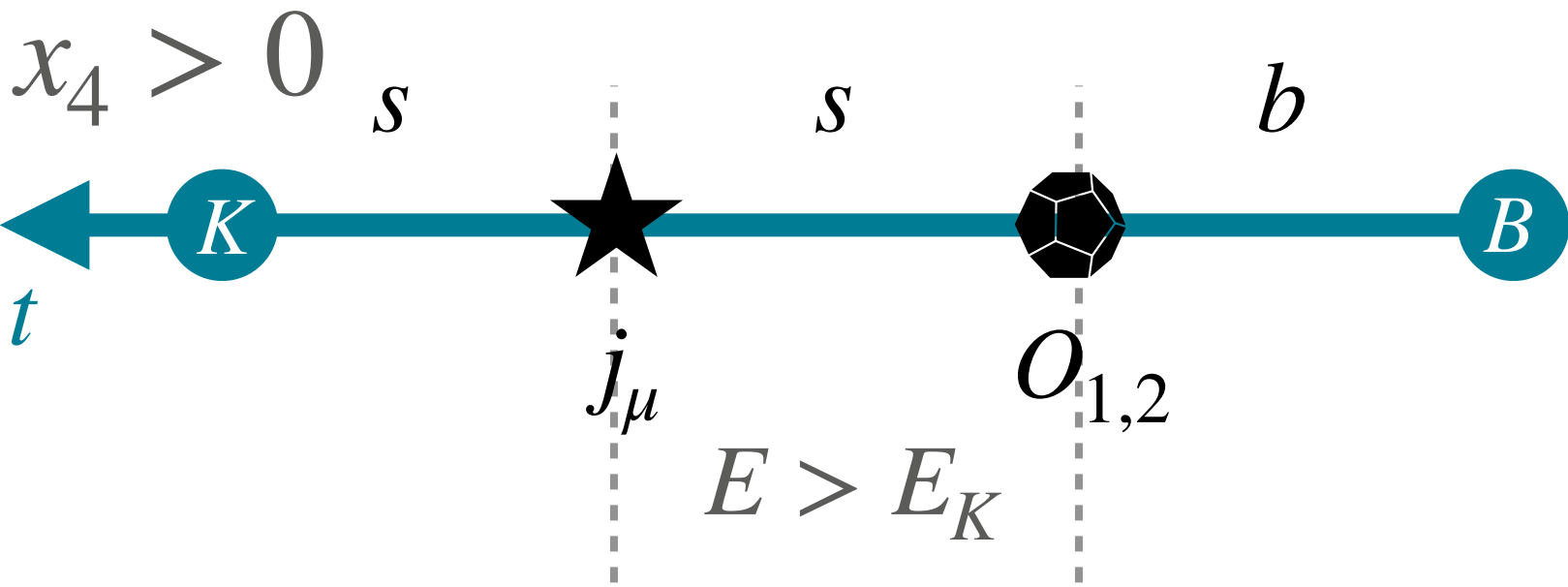
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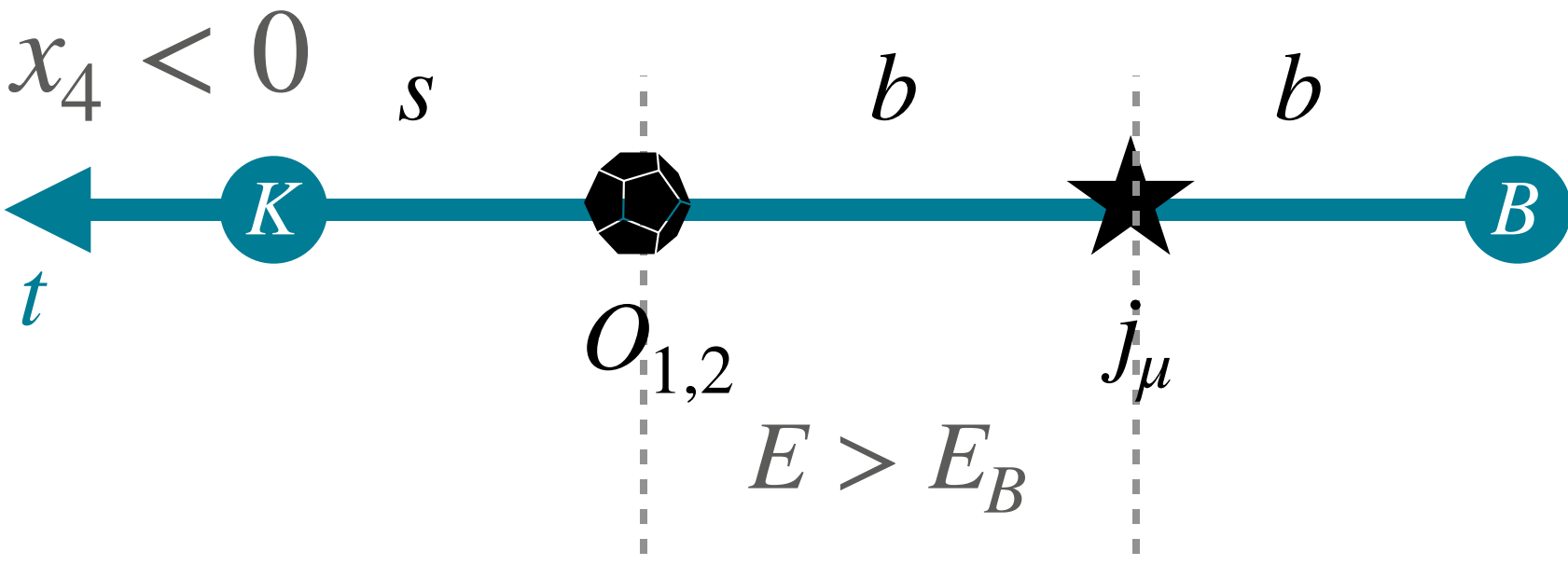
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spectral-reconstruction methods required

Talk by Shoji Hashimoto



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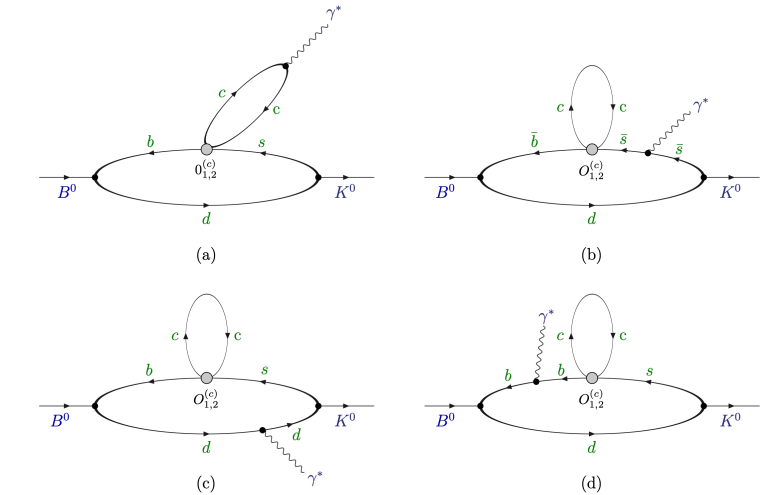
conventional methods sufficient

Challenge 2

Charming penguins in $B \rightarrow K\ell^+\ell^-$

[Frezzotti et al. [PRD 113 \(2026\)](#)]

- paper contains comprehensive computational strategy for reconstruction of amplitude H
- $N_f = 2 + 1 + 1$ ETMC, OS valence with $m_l^{\text{phys}}, m_s^{\text{phys}}, m_c^{\text{phys}} = m_b/2$, ensemble with $a \sim 0.08$ fm



amplitude

$$H_{1,2}^\nu(\mathbf{q}) = i \int d^4x e^{iq \cdot x} \langle K(\mathbf{p}') | T[j_{\text{EM}}^\nu(x) O_{1,2}(0)] | B(\mathbf{0}) \rangle$$

reconstruction

[Bruno, Hansen [JHEP06\(2021\)](#)]

[Frezzotti et al. [PRD 108 \(2023\)](#)]

$$H_{1,2}^{\nu+}(\mathbf{q}) = \int_{E_+^*}^{\infty} \frac{dE}{2\pi} \frac{\rho_{1,2}^{\nu+}(E, \mathbf{q})}{E - M_B - i\epsilon}$$

$$= \sum_{n=1}^{n_{\text{max}}} g_n(M_B, \epsilon) e^{-aE_K n} C_{1,2}^{\nu+}(an, \mathbf{q})$$

- onset of resonances visible in studied channels
- gray band (bottom right) is resonance model
- smearing limited by volume and resonance widths:
 $1/L \lesssim \epsilon \ll \Delta$ (Δ is scale over which H_μ varies a lot)

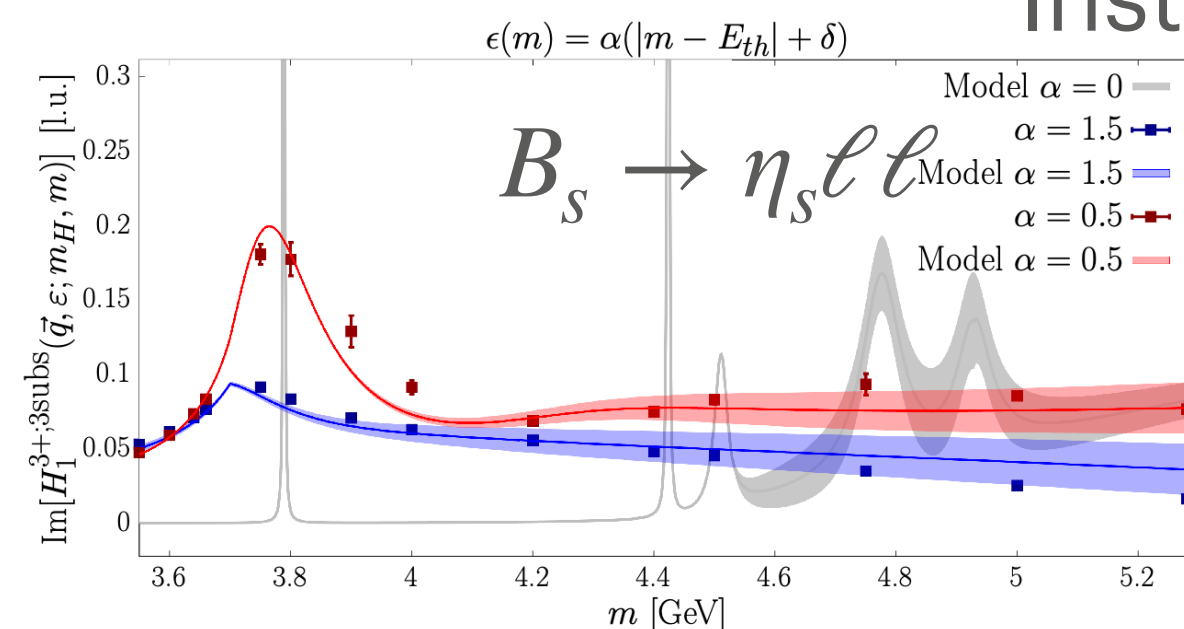
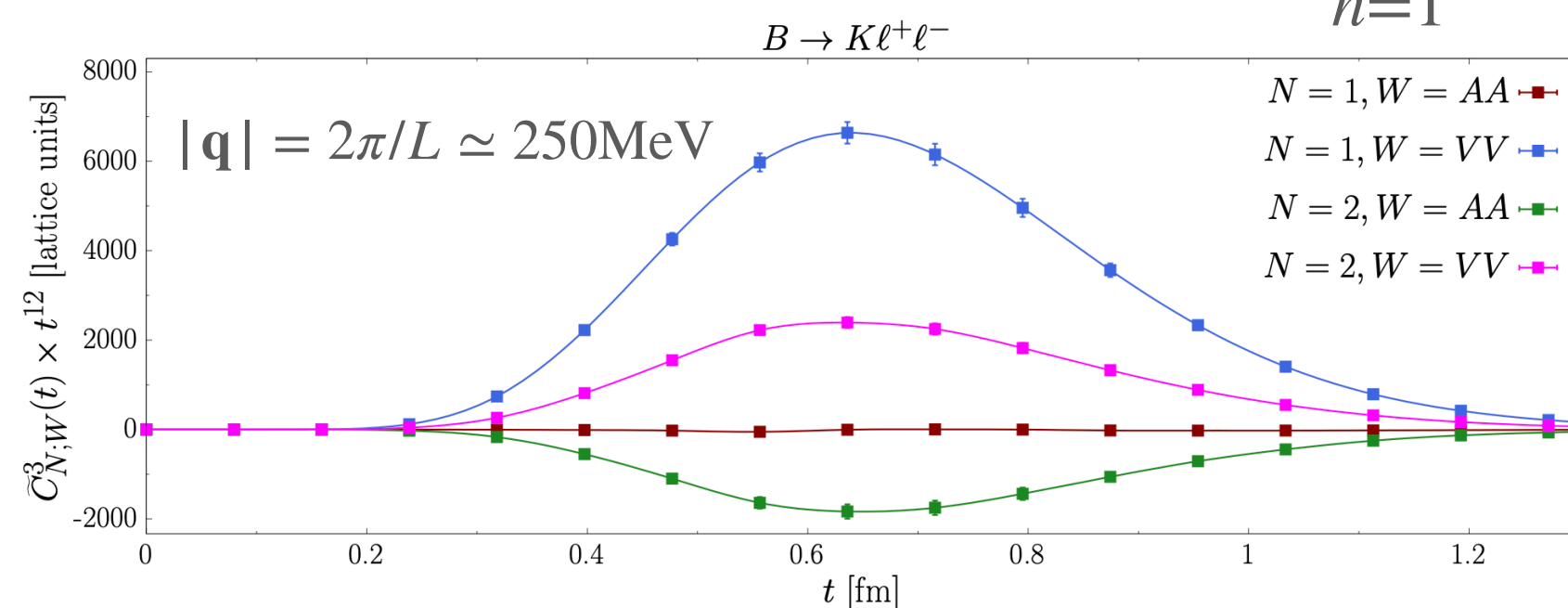
taking $\epsilon \rightarrow 0$ limit not currently possible.

Instead: A. reconstruct in tail region

B. fit model to lattice data and deduce physical resonance structure

Also: Renormalisation poses challenges:

Talk by [Gregoris Spanoudes](#)

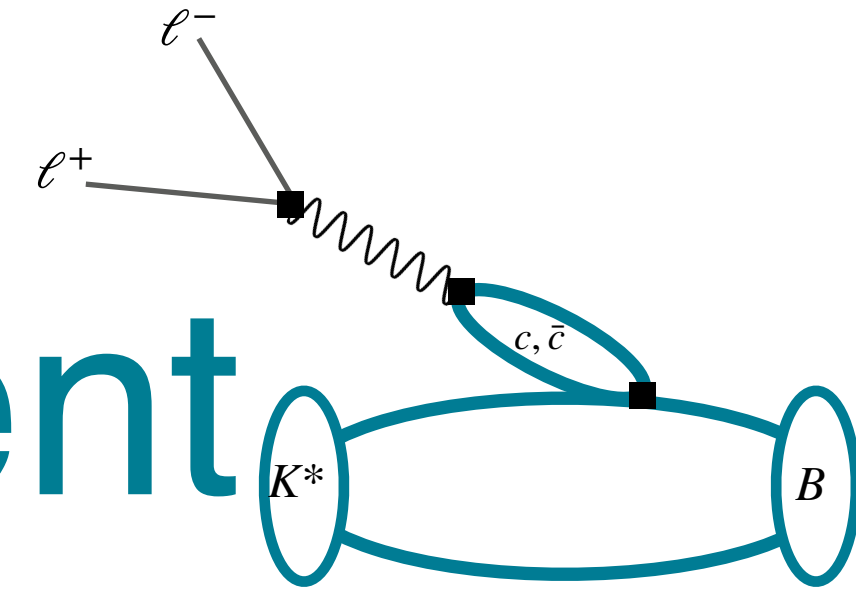


Challenge 2

Smeared theory and experiment

[AJ 2603.15487]

Idea: Smear such that $\langle O(q^2) \rangle_\epsilon|_{\text{experiment}} = \langle O(q^2) \rangle_\epsilon|_{\text{theory}}$



Challenge 2

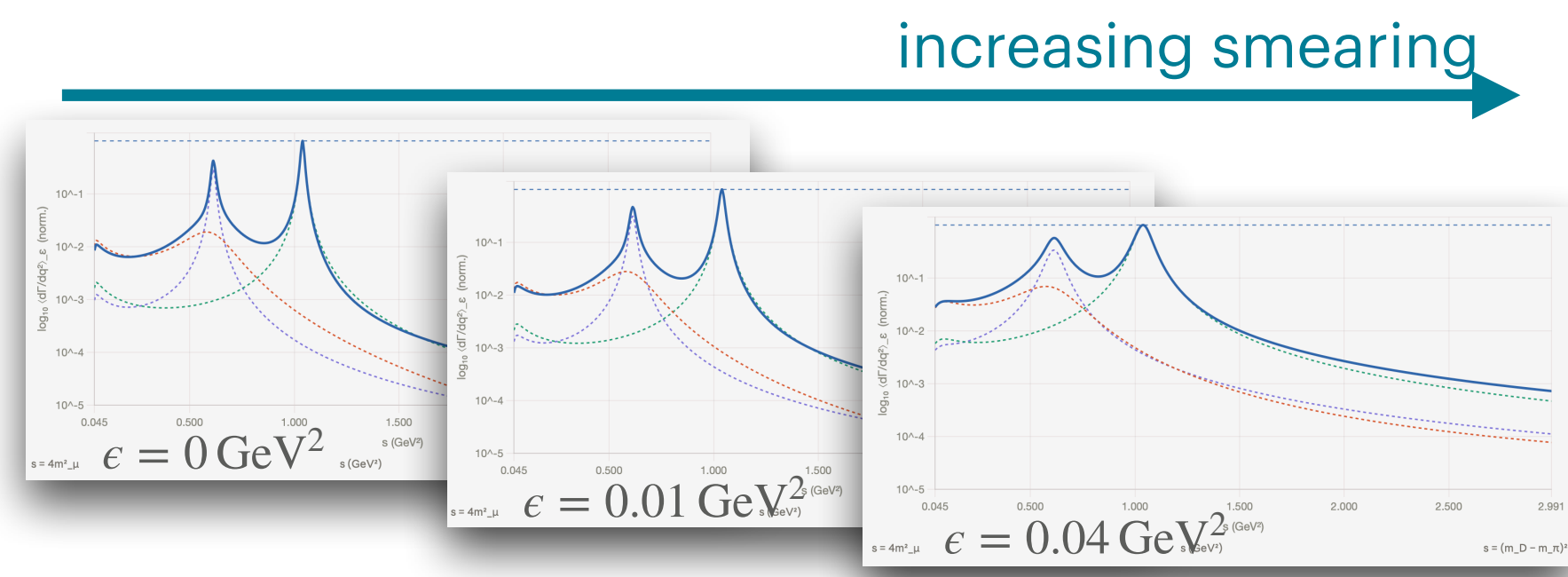
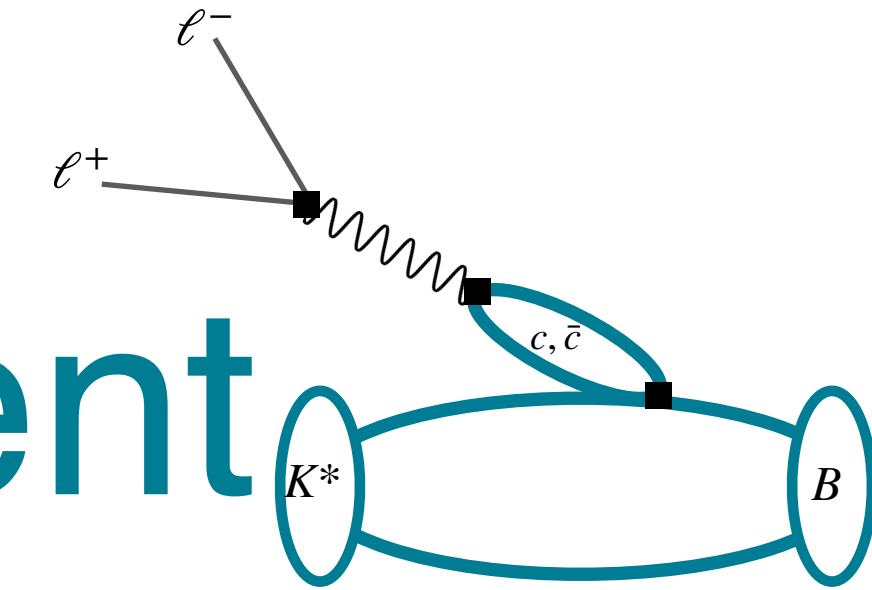
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Cauchy/Poisson
kernel



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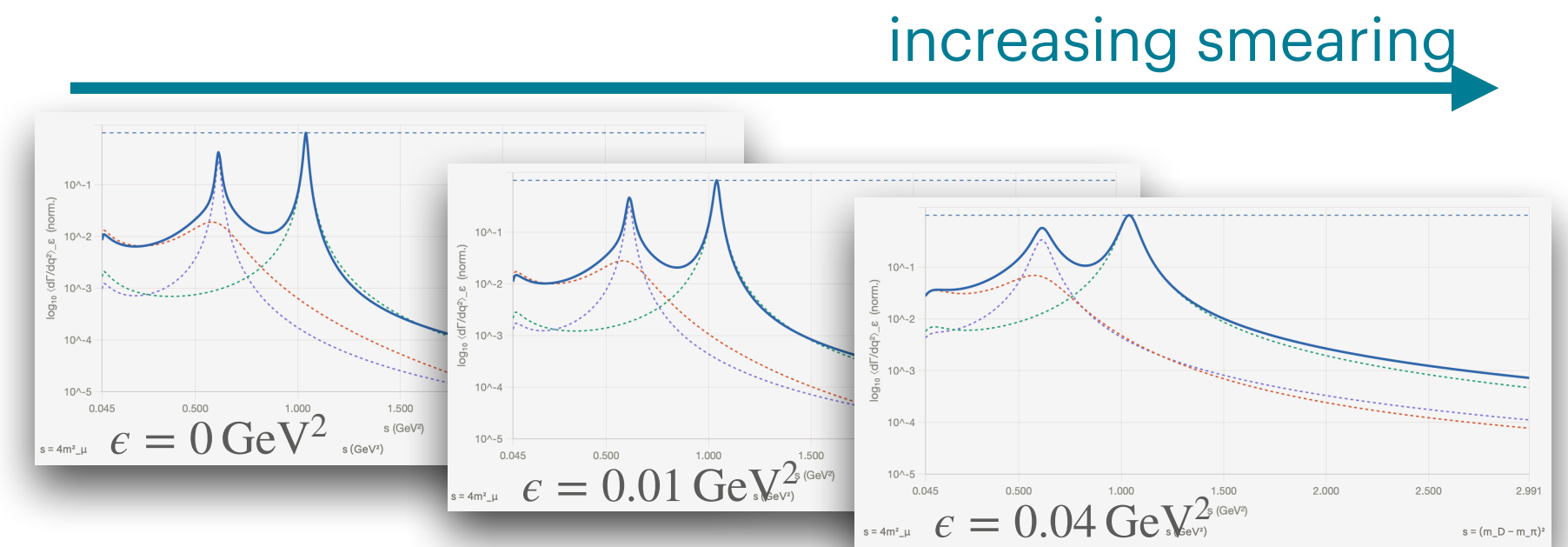
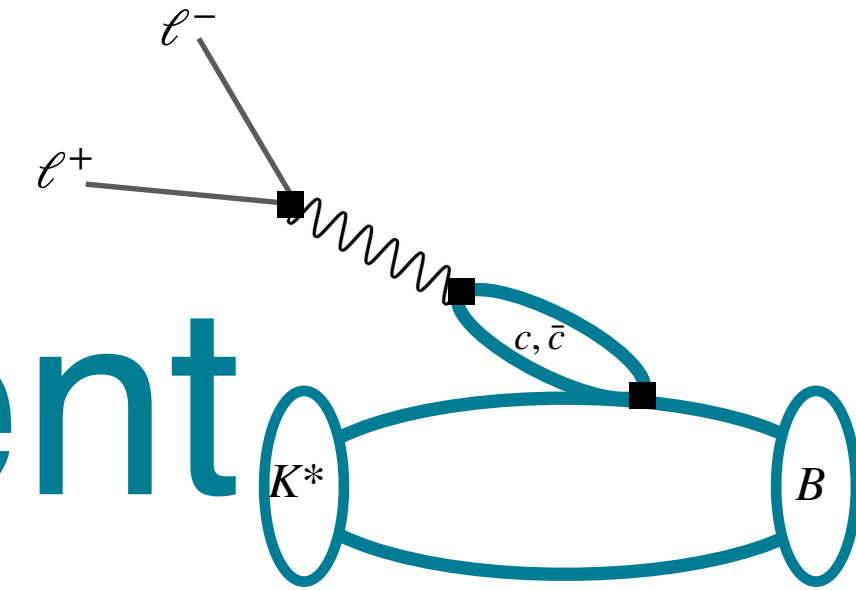
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- Shown to work for processes
 - linear in amplitude:** e.g. rare SL decay interference $\sim \text{Im}[H_{\text{SD}}^*(q^2 + i\epsilon)H_{\text{LD}}(q^2 + i\epsilon)]$ (e.g. $B \rightarrow K\ell\ell$ or $D \rightarrow \pi\ell\ell$)

$$2M_i \int_{\omega_{\text{th}}}^{\infty} d\omega \frac{\rho_T(\omega^2 - p_\omega^2)}{\omega^2 - p_\omega^2 - q^2 - i\epsilon}$$

reconstructed from 4pt function in terms of combined energy-momentum smearing



Challenge 2

Smearred theory and experiment

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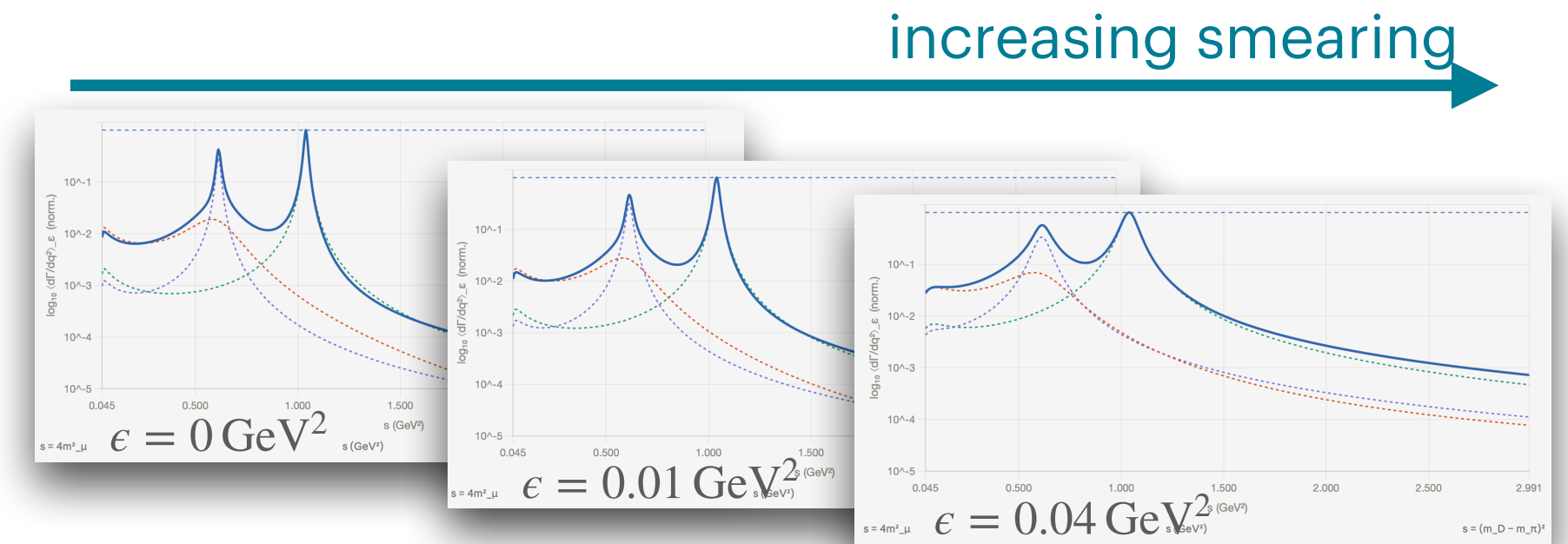
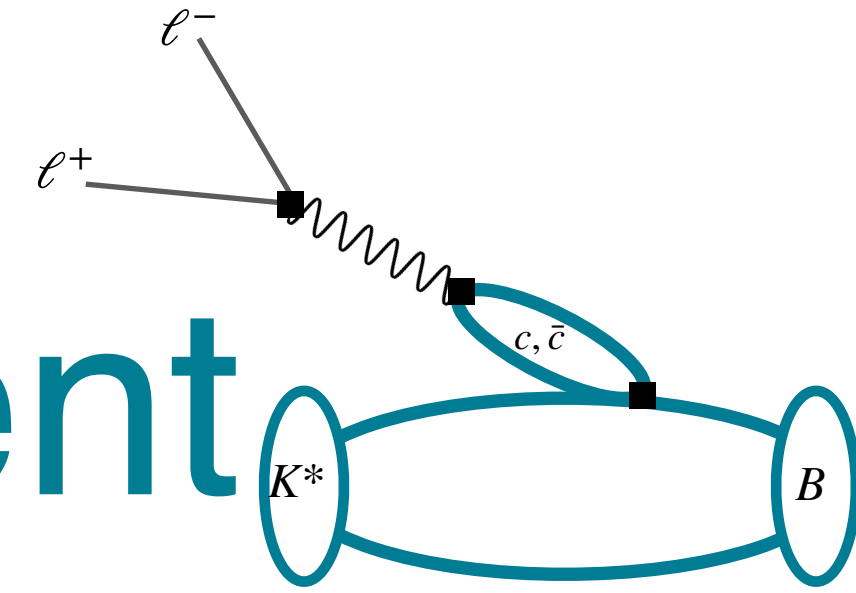
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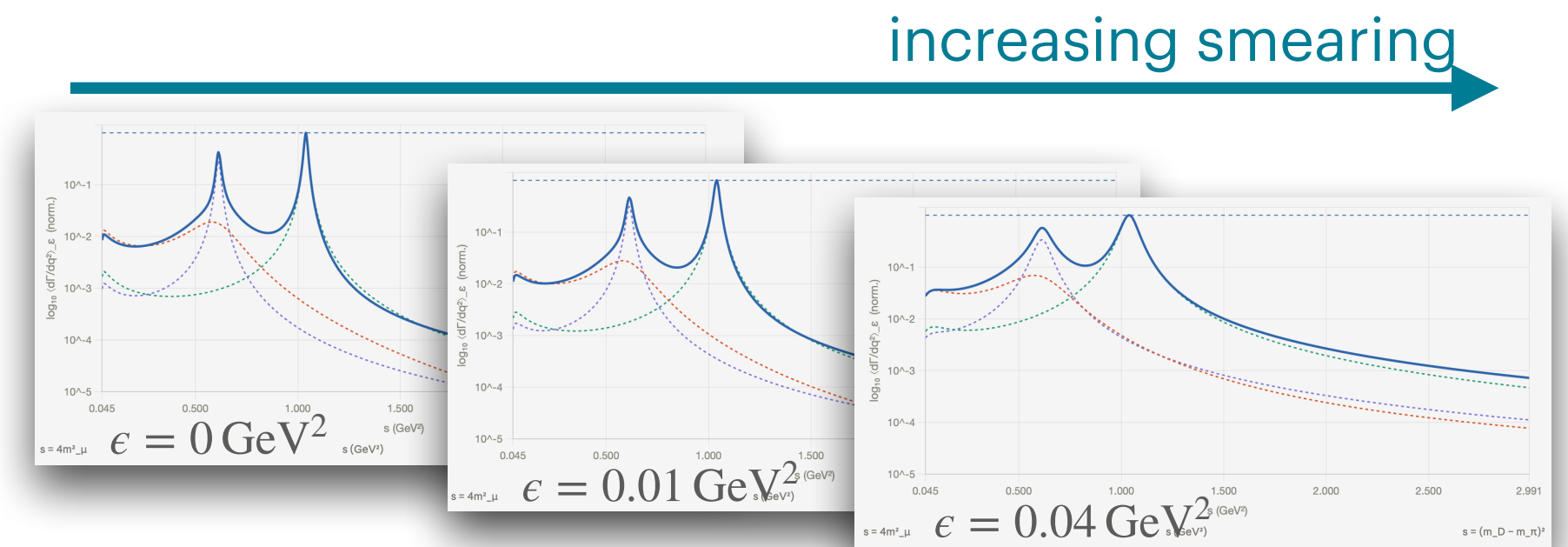
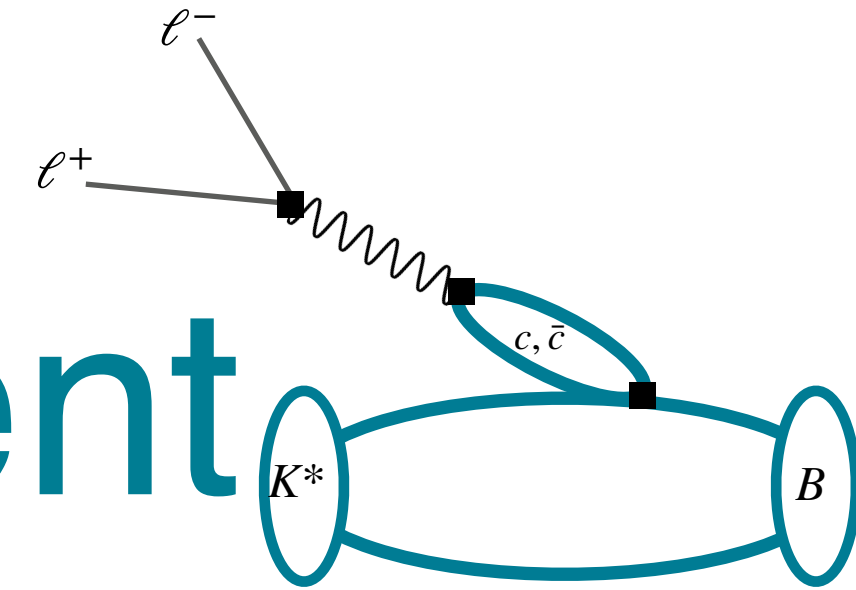
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 - lattice computation therefore feasible in narrow-resonance region
 - requires modified experimental analysis



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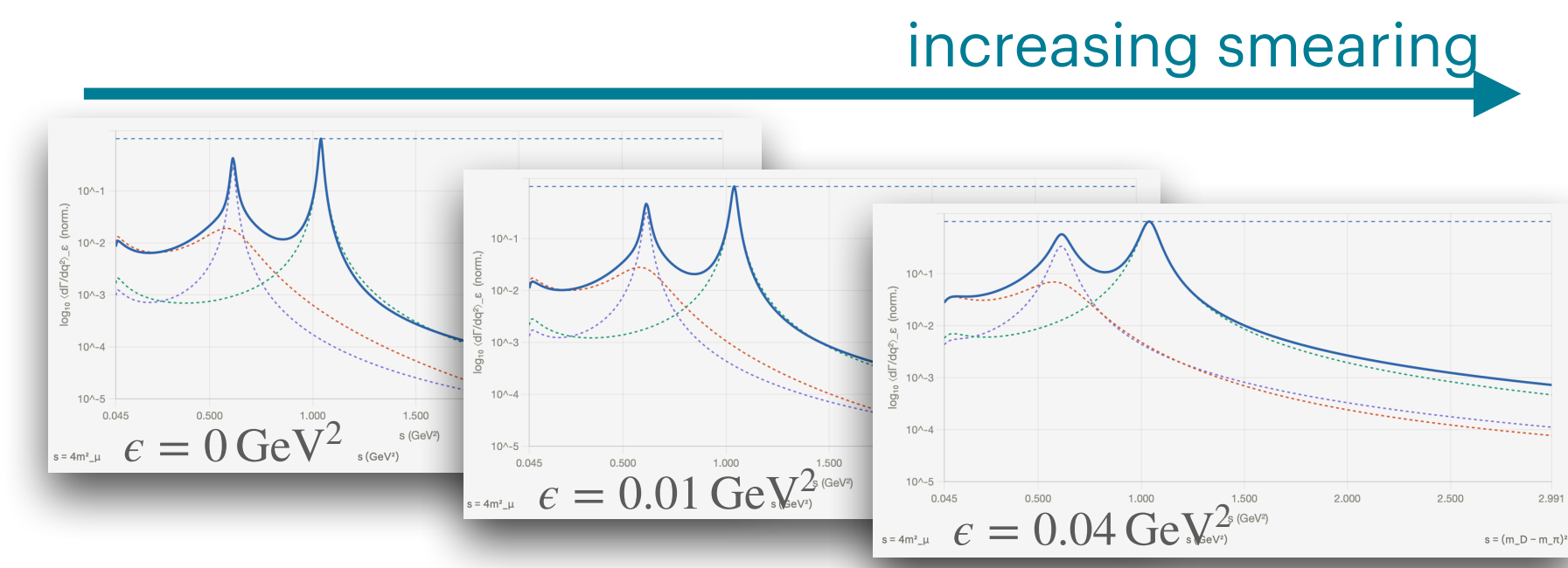
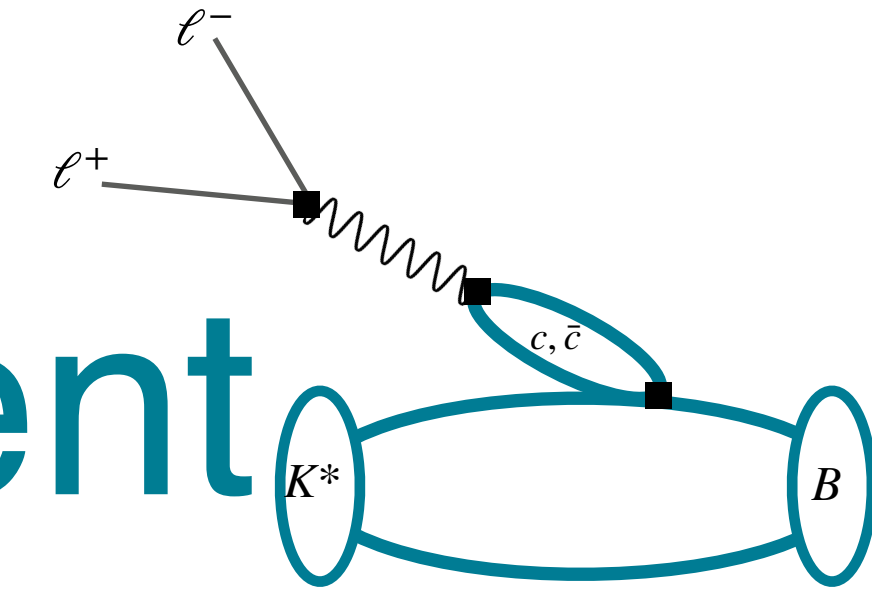
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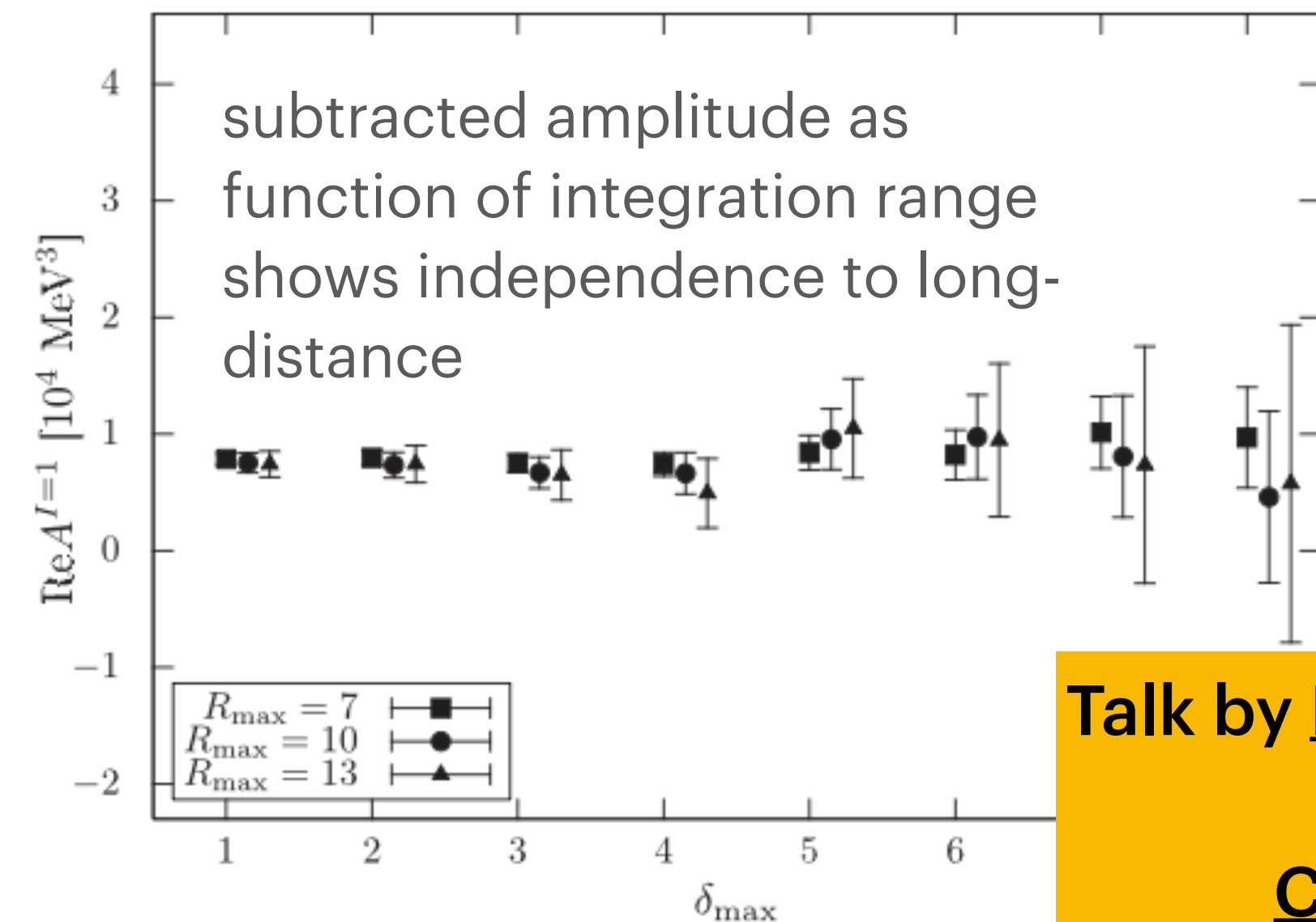
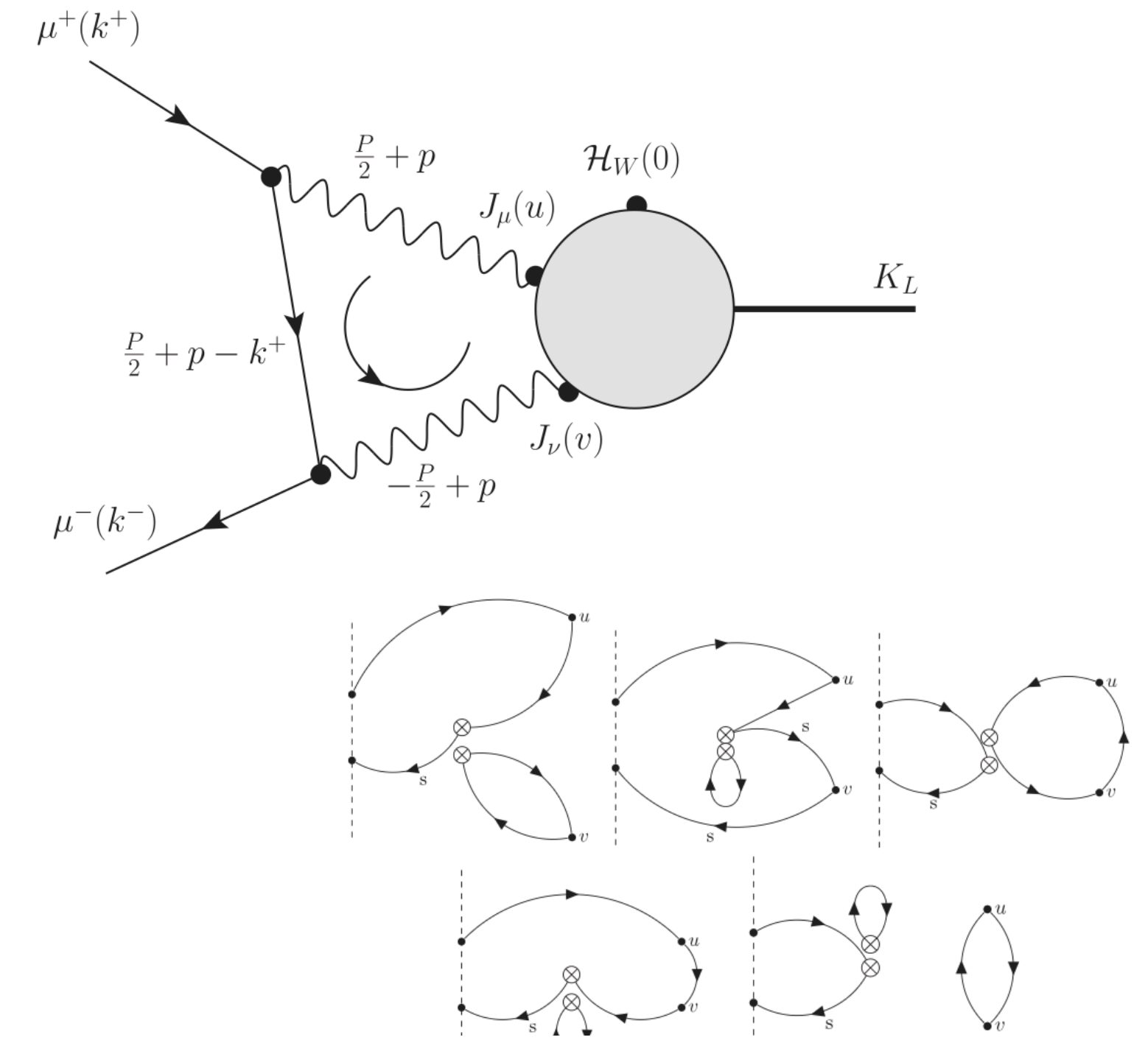
More general: Devise *convenient* observables that can be computed more easily, are accessible to experiment, but still allow for determining SM parameters and searches for NP!

$$K_L \rightarrow \mu\mu$$

[Chao, Christ, PRD 110 (2024)]

[Boyle et al. PRD 112 2026]

- measured at BNL E871 $\mathcal{B} = 6.84(11) \times 10^{-9}$ [E871 PRL 84 (2000)]
- at 2nd order mediated by either $2W$ or $2W + 1Z$
SD contributions known at NNLO $\mathcal{B}_{\text{SD}} = 0.79(12) \times 10^{-9}$ [Gorbahn & Haisch PRL 97 (2006)]
- $\rightarrow$ 3rd order $G_F \alpha^2 (W + 2\gamma)$ -exchange crucial but nonperturbative
- unresolved sign between SD and LD2 γ part $\rightarrow$ only lattice can resolve
- exploratory calculation: $N_f = 2 + 1$ DWF, $24^3 \times 64$, $a^{-1} \approx 1$ GeV, M_π^{phys}
(no GIM $\rightarrow$ renormalisation [Chao, Chist, Hu arXiv: arXiv:2607.14077])
- amplitude $\sim L^{\mu\nu}(u - v) \langle T \{ J_\mu(u) J_\nu(v) \mathcal{H}_W(0) K(-t_{\text{sep}}) \} \rangle$
with exponentially growing (in time) terms
 $\rightarrow$ explicit subtraction of π^0 intermediate state
($\pi\pi\gamma$ state kinematically not allowed in simulation (10% effect))
- section titled “Using lattice QCD to compute analytically continued Minkowski-space Green’s functions” is entertaining to read!



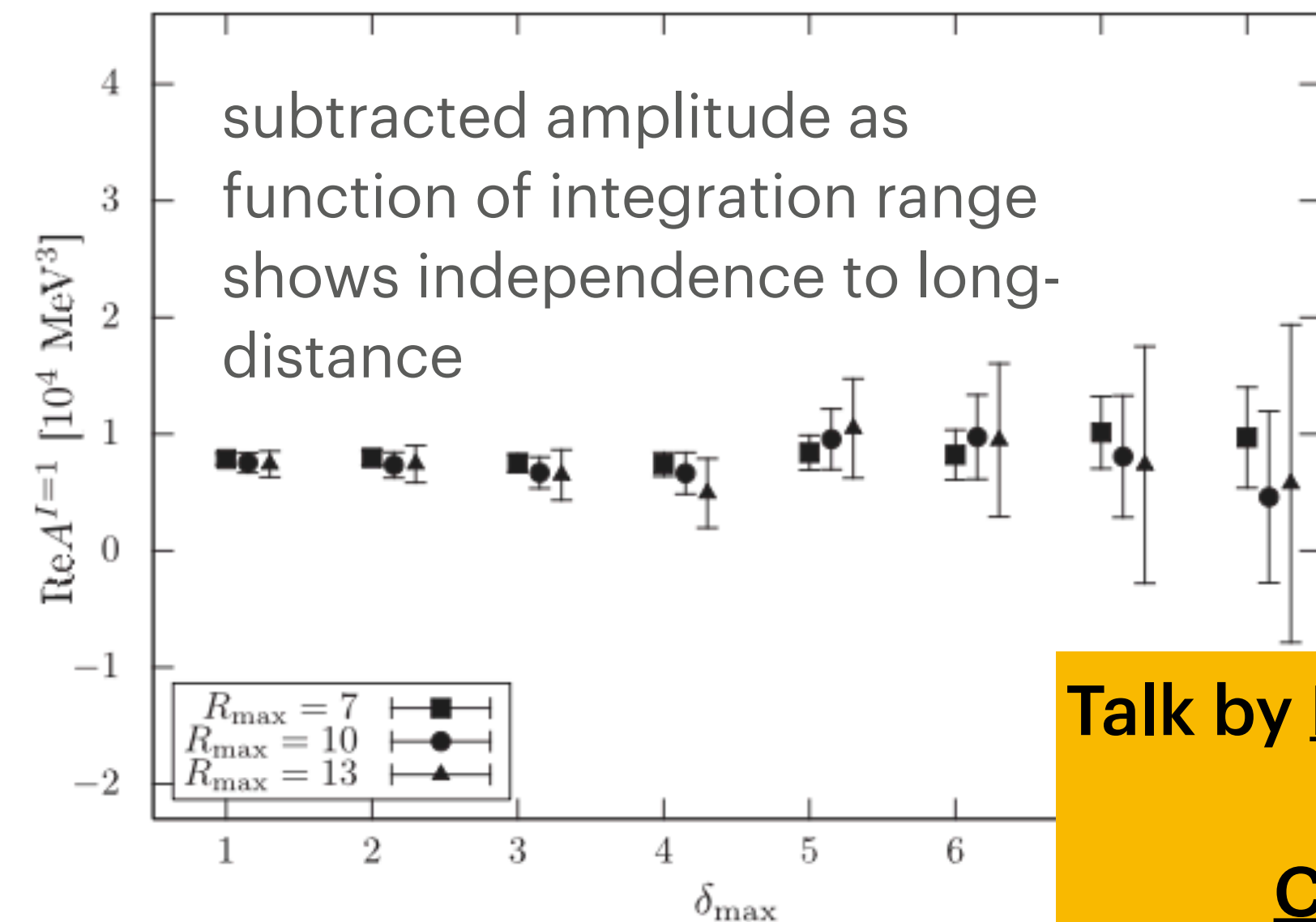
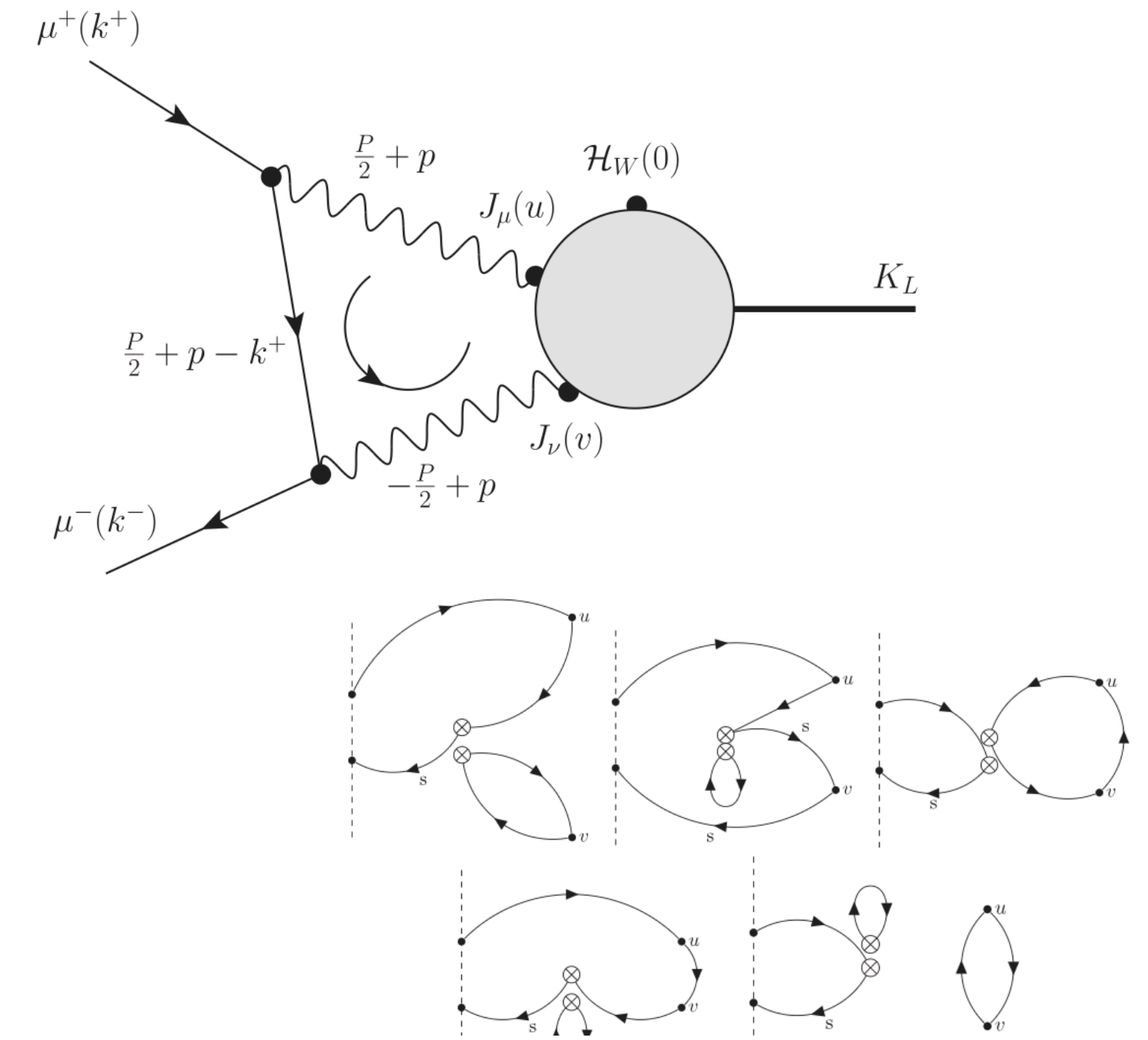
Talk by Norman Christ
and
Ceran Hu

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**Talk by Norman Christ
and
Ceran Hu**

$$K_L \rightarrow \ell^- \bar{\nu}_\ell \ell'^+ \ell'^- \quad (\ell, \ell' \in \{e^-, \mu\})$$

[Di Palma et al. [arXiv:2605.22742](#), [arXiv:2605.22727](#)]

earlier work [Tuo et al, [PRD 105 \(2022\)](#)], [Gagliardi et al. [PRD 105 \(2022\)](#)]

- $N_f = 2 + 1 + 1$ TM, physical u, d, s $a \sim 0.05 - 0.08$ fm,

multiple L , including quark-disconnected

- rare in SM $\sim O(G_F^2 \alpha_{\text{em}}^2)$
- alternative CKM determination, test of LFU, NP experimental constraint for $\ell \neq \ell'$ (BNL E865, NA62)

- $$H_W^{\mu\nu}(k^2, \mathbf{k}, \mathbf{p}) = \int dt_\gamma e^{iE_\gamma t_\gamma} \int d\mathbf{y} e^{i\mathbf{k} \cdot \mathbf{y}_\gamma} \langle 0 | T[j_W^\nu(0) j_{\text{em}}^\mu(y_\gamma)] | K(\mathbf{p}) \rangle$$

$$= H_{\text{pt}}^{\mu\nu}(E_\gamma, \mathbf{k}, \mathbf{p}) + H_{\text{SD}}^{\mu\nu}(E_\gamma, \mathbf{k}, \mathbf{p})$$

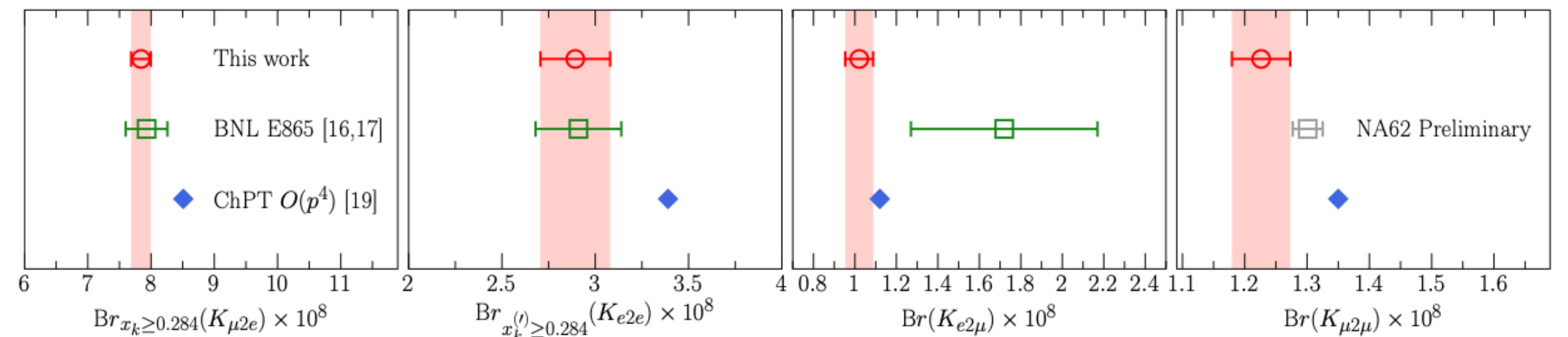
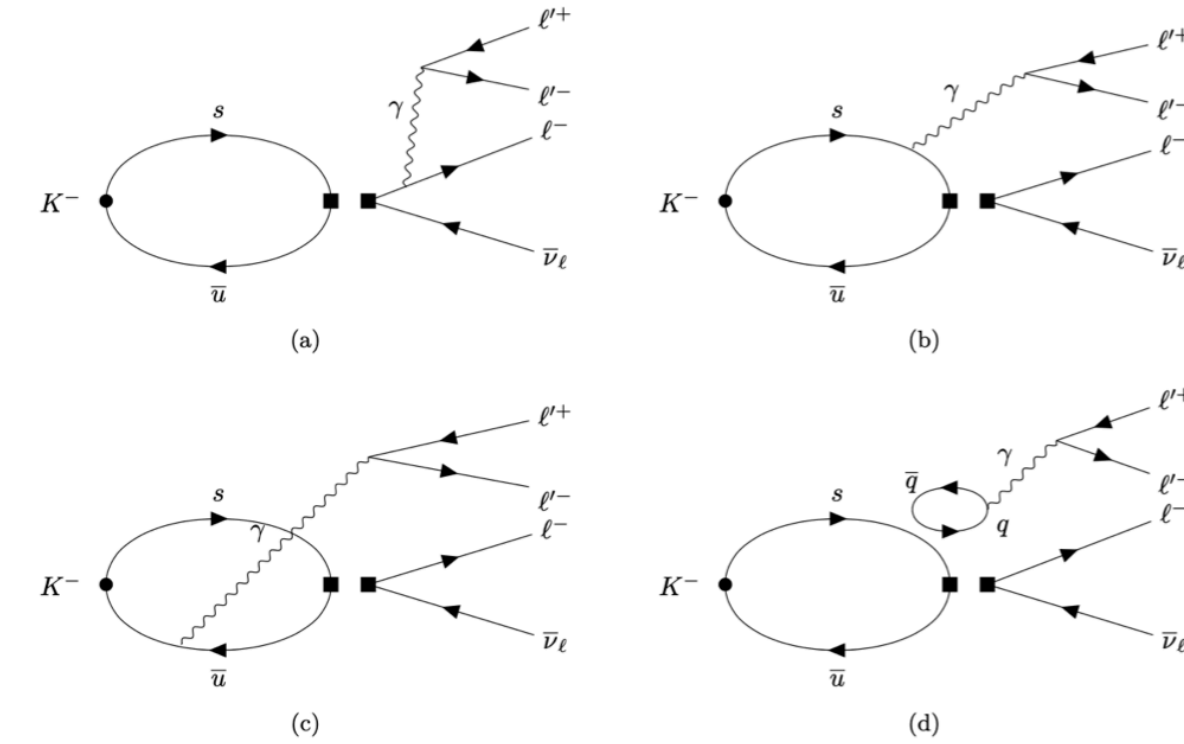
diagram b): $t_\gamma < 0$: $H_W^{\mu\nu}(E_\gamma, \mathbf{k}, t_W) = \int_{-\infty}^{\infty} dt_\gamma C_W^{\mu\nu}(t_\gamma, E_\gamma, \mathbf{k}, t_W)$

→ standard methods

diagram c): $t_\gamma < 0$: $\pi\pi$ intermediate states →

spectral reconstruction required

feasible off peak ($1/L \ll \epsilon$)



Talk by **Francesco Sanfilippo**

Summary

Rich experimental landscape for at least 20 years, if not longer

- provides for plenty of motivation for lattice flavour physics
(in fact, experimentalists and phenomenologists are counting on us!)
- now is a good time for getting involved and help determining future experimental directions



Workshop series to support flavour case
at FCC before 2028 decision making
Get in touch if interested!!!!

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- QED wall — further formal development required
- Edinburgh consensus being adopted
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PART II:

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There is more material that I couldn't cover in the backup slides.
meson life times — D-meson mixing — LECs — radiative meson decays



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I am grateful for contributions from and discussion with:

Matthew Black
Claudio Bonanno
Akhil Chauhan
Alessandro De Santis
Antoine Gérardin
Steve Gottlieb
Ryan Kellermann
Luka Leskovec
Ramón Merino
Fernando P. Panadero
Alejandro Saez
Abhishek Samlodia
Teseo San Jose Perez
Rainer Sommer
Gregoris Spanoudes
Nazario Tantalo
Xin-Yu Tuo
Alejandro Vaquero
Takeshi Yamazaki

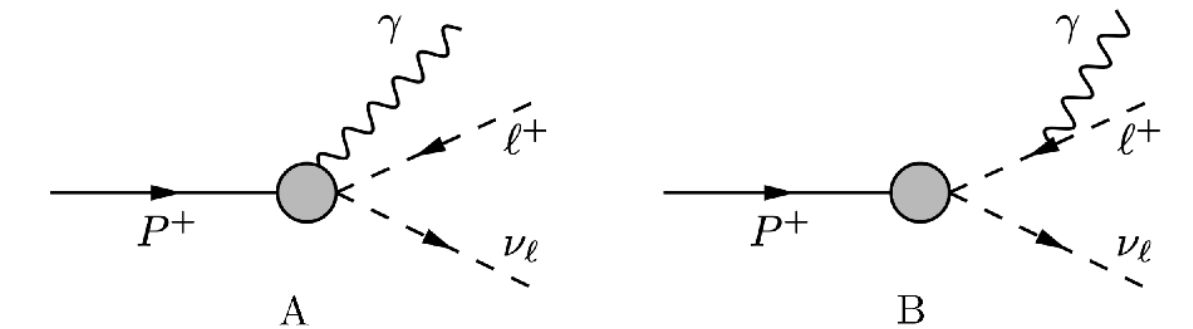
Further material

Radiative decays $P^+ \rightarrow \ell^+ \nu_\ell \gamma$

Recent work by [Frezzotti et al. [PRD 103 \(2021\)](#)], [Giusti et al. [PRD 107 \(2023\)](#), [PRD 112 \(2025\)](#)], [Di Palma et al [PRD 111 \(2025\)](#)], [JLQCD and UKQCD [2603.19960](#)]

Here: [Boyle [PRD 114 \(2026\)](#)]

- CKM determination and sensitive probe for potential BSM contributions via non-SM currents
- structure dependent contrib. relevant for real-photon emission and LQCD for QED corrections to $P \rightarrow \ell \nu_\ell$
- $P = \pi, K$, measured at PIBETA, KLOE, E36, ISTRA, OKA, E787 — tensions!
- **NEW:** at $O(\alpha^2)$ collinear radiative corrections can be sizeable,
leading to enhancements $\ln(M_P^2/m_e^2)$ and $\ln(2E_{\gamma_2, \max}^*/M_P)$ [Boyle [PRD 114 \(2026\)](#)]
- IR div. cancels between virtual-loop+real photon emission and emission of two real photons



diff. BR $\frac{d^2 R_\gamma^{\text{RC}}(x_\gamma, y_\ell)}{dx_\gamma dy_\ell} \sim \underbrace{A(x_\gamma, y_\ell)}_{f_P, F_V, F_A}^{\text{lattice}} + \frac{\alpha}{2\pi} (L_e - 1) \underbrace{A^{\text{RC}}(x_\gamma, y_\ell)}_{\text{analytical}}$ [Kuraev et al. [PRD 69 \(2004\)](#)]

$x_\gamma = \frac{2p \cdot k}{M_P^2} \rightarrow \text{photon energy}$
 $x_\ell = \frac{2p \cdot p_\ell}{M_P^2} \rightarrow \text{lepton energy}$

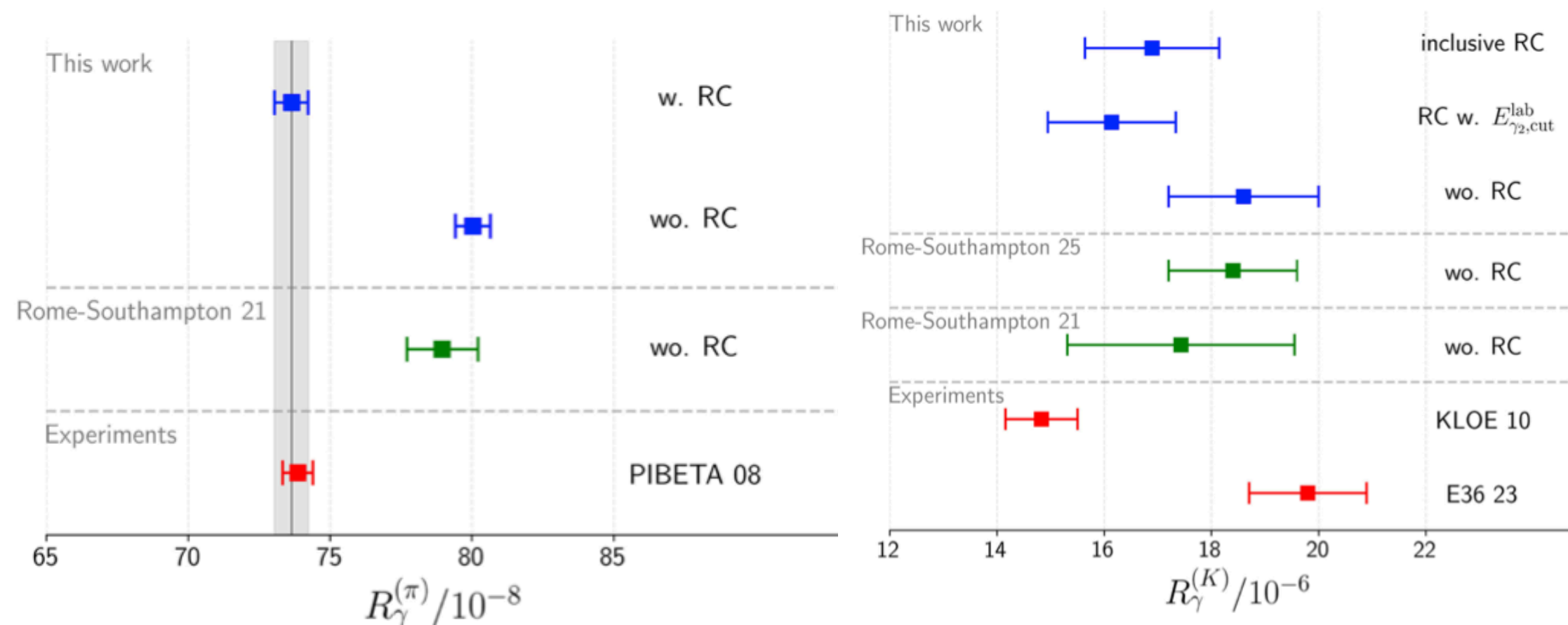
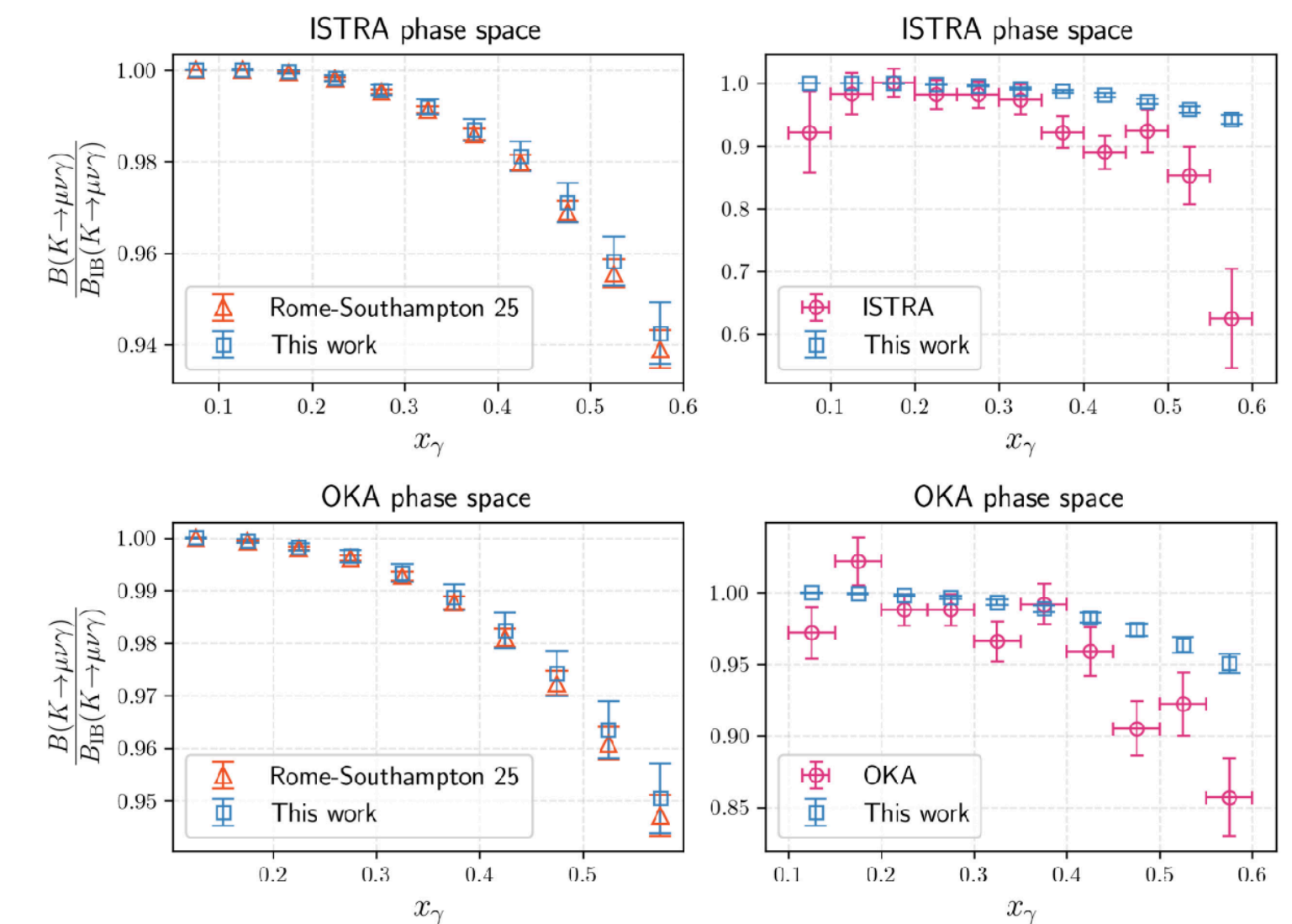
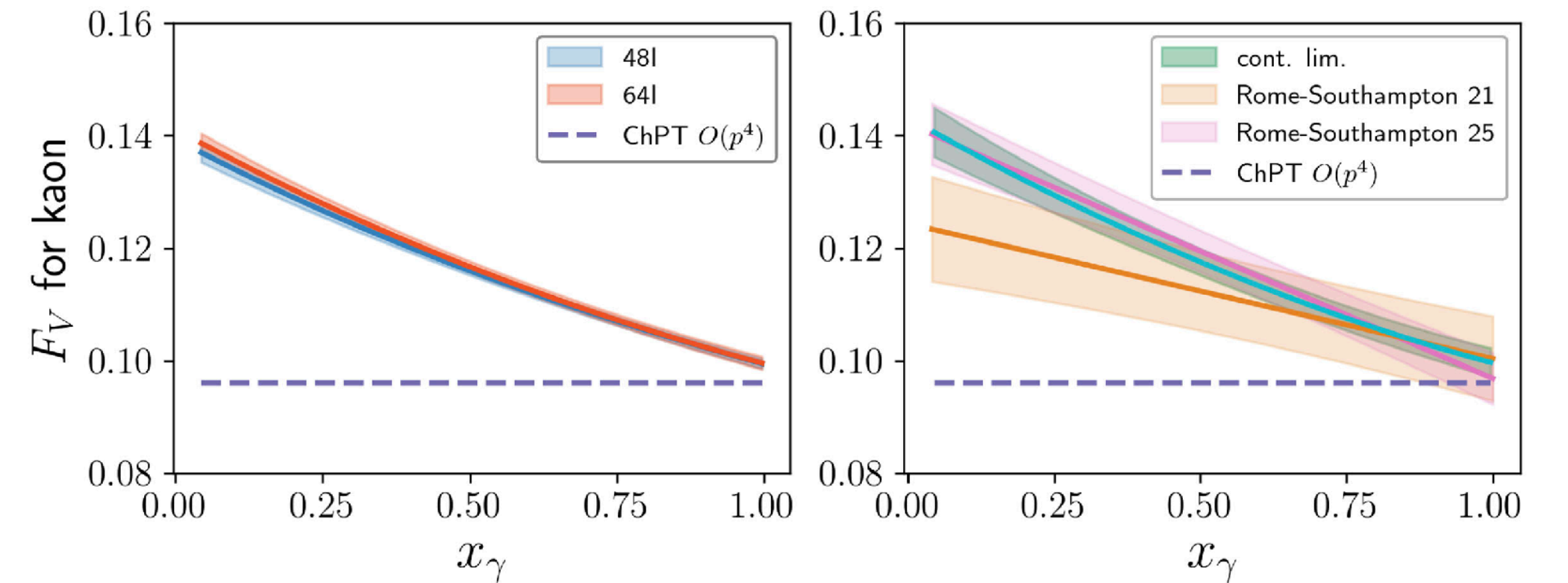
$$H_E^{\mu\nu}(k, p) = -i \int_{-\infty}^{\infty} dt \int d^3 \vec{x} e^{k^0 t - i \mathbf{k} \cdot \mathbf{x}} \langle 0 | T \{ J_{\text{em}}^\mu(\mathbf{x}, t) J_W^\nu(0) \} | P(p) \rangle \sim R_1, R_2, F_V, F_A, f_P$$

(conventional methods sufficient)

Radiative decays $P^+ \rightarrow \ell^+ \nu_\ell \gamma$

[Boyle PRD 114 (2026)]

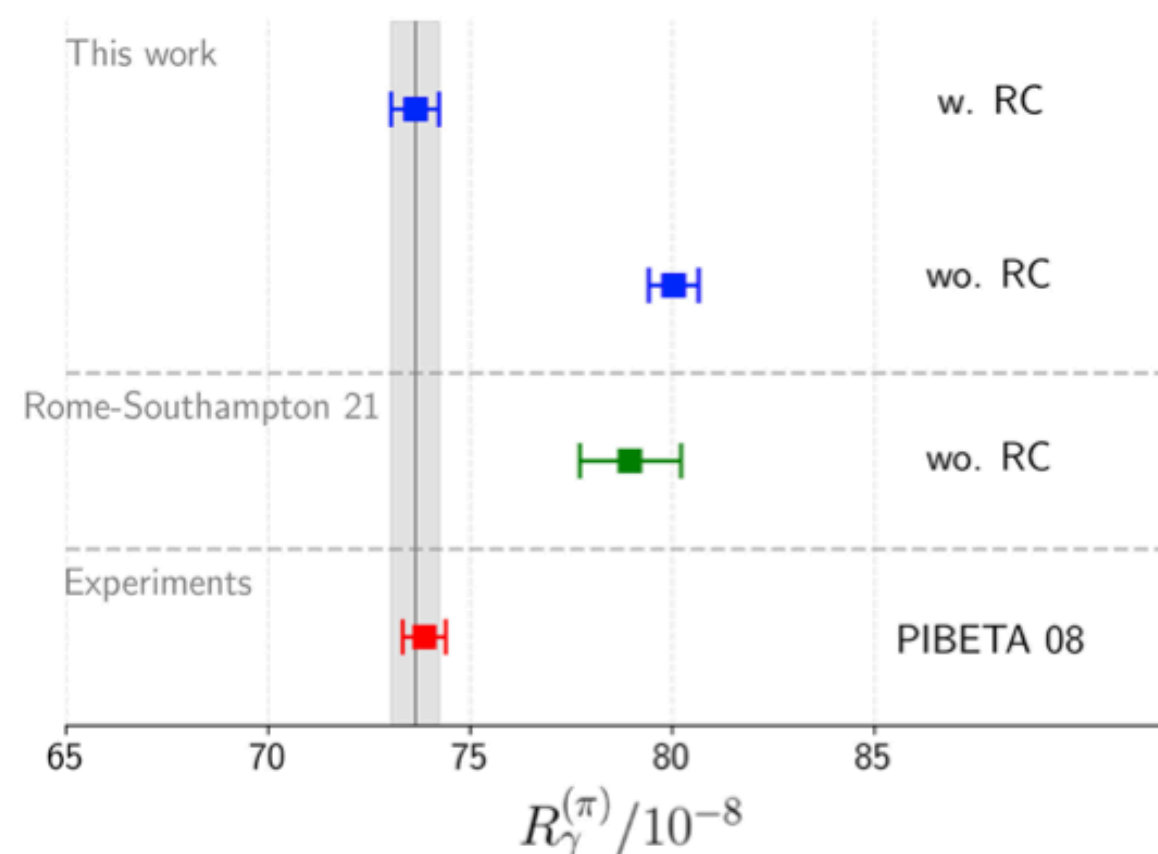
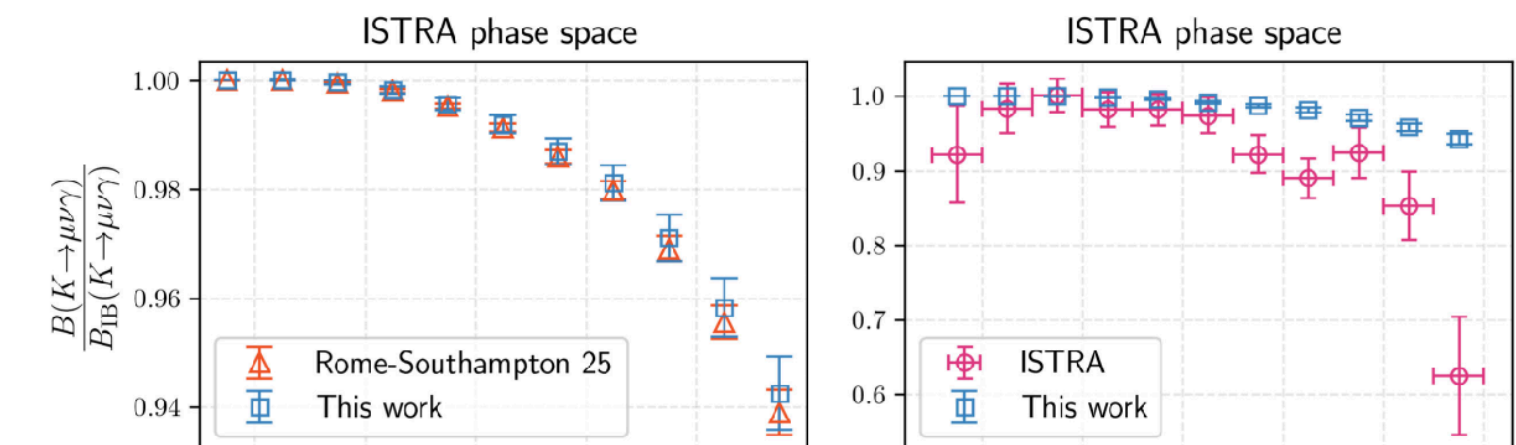
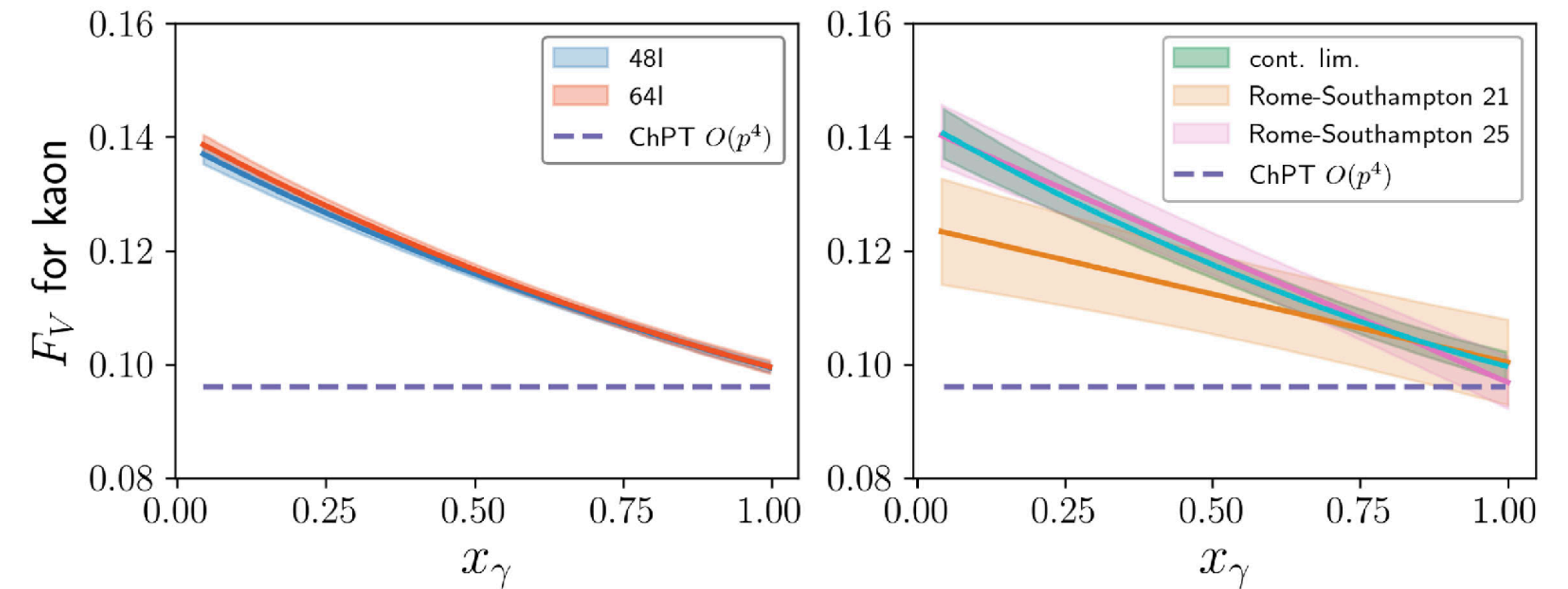
- new results for F_A, F_V
- $N_f = 2 + 1$ DWF (RBC/UKQCD), $a^{-1} \sim 1.7, 2.4 \text{ GeV}$, M_π^{phys}
- **specifics of experimental treatment of collinear 2nd photons (inclusive or energy cut) relevant!**
- treatment of inner-bremsstrahlung photons in $K \rightarrow e \nu_e \gamma$ differs between KLOE and E36 (KLOE one electron with 20MeV cut in lab frame, E36 inclusive)
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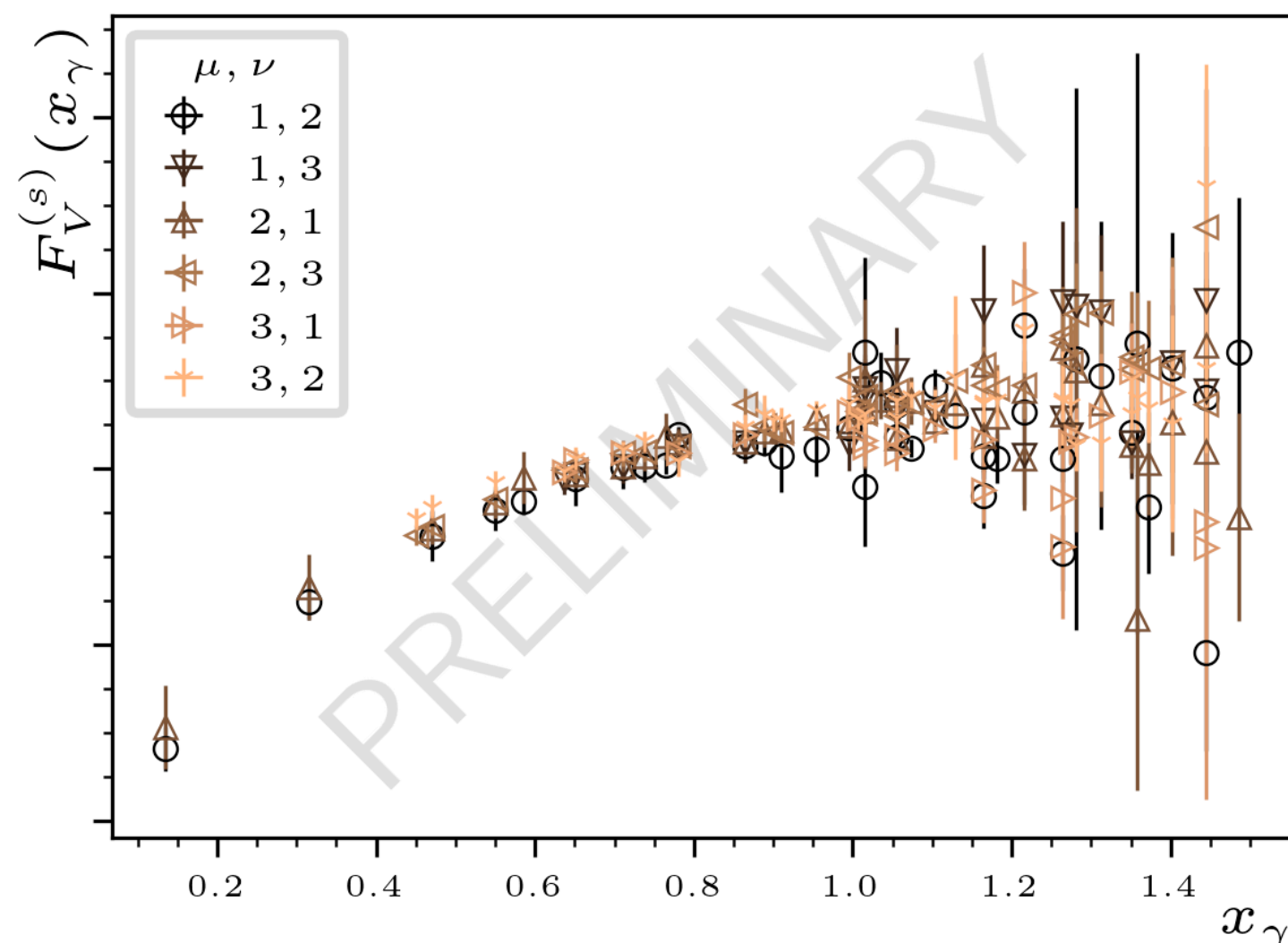
Including collinear photons removes tension encountered before.
→ Radiative corrections need to take into account experimental details!

This is something we have to engage with. With increasing precision, details of experimental will be much more relevant for what precisely theory needs to predict.

Other radiative decays/transitions

[JLQCD and UKQCD 2603.19960]

- $D_{(s)}, B_{(c)} \rightarrow \ell \nu_\ell \gamma$ real radiative decays
- access to internal meson structure, part of QED corrections in $D_{(s)}, B_{(c)} \rightarrow \ell \nu_\ell$, alternative route to CKM
- $N_f = 2 + 1$ DWF, $a > 0.04\text{fm}$, $M_\pi > 280\text{MeV}$
- computational strategy similar to $\pi/K \rightarrow \ell \nu_\ell \gamma$



[CLQCD PRD 113 (2026)]

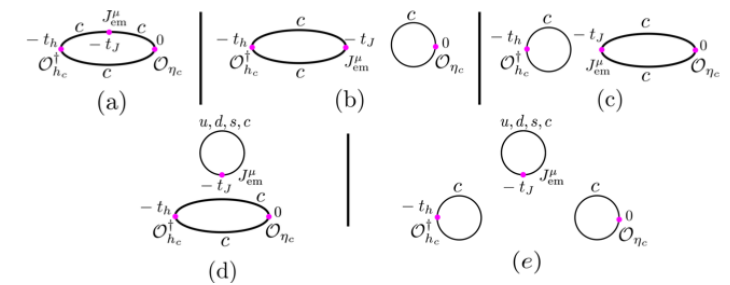
$$D^{*+} \rightarrow D^+ \gamma \rightarrow g_{D^{*+}D^+ \gamma}, D^{*0} \rightarrow D^0 \gamma \rightarrow g_{D^{*0}D^0 \gamma}, D_s^{*+} \rightarrow D_s^+ \gamma \rightarrow g_{D_s^{*+}D_s^+ \gamma}$$

- $N_f = 2 + 1$ CLQCD Wilson Clover, $a \geq 0.05\text{fm}$
- strong couplings relevant in HMChPT
- experimentally measured
- tension in branching fraction?

$$\mathcal{B}(D^{*+} \rightarrow D^+ \gamma) = 0.30(7)_{\text{this work}} \% \leftrightarrow 1.68(5)_{\text{CLEO1998}} \%$$

[Bečirević et al. PRD 112 (2025), PLB 868 (2025), arxiv:2604.15256]

$$h_{c,b} \rightarrow \eta_{c,b} \ell^+ \ell^-, \chi_{c1} \rightarrow J/\psi \ell^+ \ell^-$$



- study of internal structure of mesons and their dynamics as heavy-heavy bound states
- $N_f = 2 + 1 + 1$ ETMC, $a \geq 0.06\text{fm}$, no disconnected
- Total decay rates agree with experiment for $\ell = e, \mu$ for χ_{c1} decay, but 3σ tension found for $h_c \rightarrow \eta_c e^+ e^-$

Meson life times

[Suzuki et al. PRD 102 (2020)]

[Black et al. arXiv:2603.28516 arxiv:2603.28517]

- experimentally measured $\rightarrow$ lattice computation
- here life-time $\tau = 1/\Gamma$ within HQET expansion
- total rate

$$\Gamma_H = \Gamma_3 + \dots + 16\pi^2 \left[\Gamma_6 \frac{\langle \overline{\mathcal{O}_6} \rangle}{m_Q^3} + \Gamma_7 \frac{\langle \overline{\mathcal{O}_7} \rangle}{m_Q^4} + \dots \right]$$

where $\langle \overline{\mathcal{O}_d} \rangle = \langle H | \mathcal{O}_d | H \rangle / 2M_H$ with $\Delta Q = 0$ ops.

- concentrate on dim-6 matrix elements:

$$\mathcal{O}_1 = (\bar{Q}\gamma_\mu(1-\gamma_5)q)(\bar{q}\gamma^\mu(1-\gamma_5)Q)$$

$$T_1 = (\bar{Q}\gamma_\mu(1-\gamma_5)T^A q)(\bar{q}\gamma^\mu(1-\gamma_5)T^A Q)$$

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- operators mix under renormalisation
 - work in the limit where no mixing with lower-dim. ops
 - consider only life-time ratios, where no eye-diagrams contribute
- six $N_f = 2 + 1$ RBC/UKQCD DWF ensembles, $a^{-1} \sim 1.8, 2.4, 2.8$ GeV

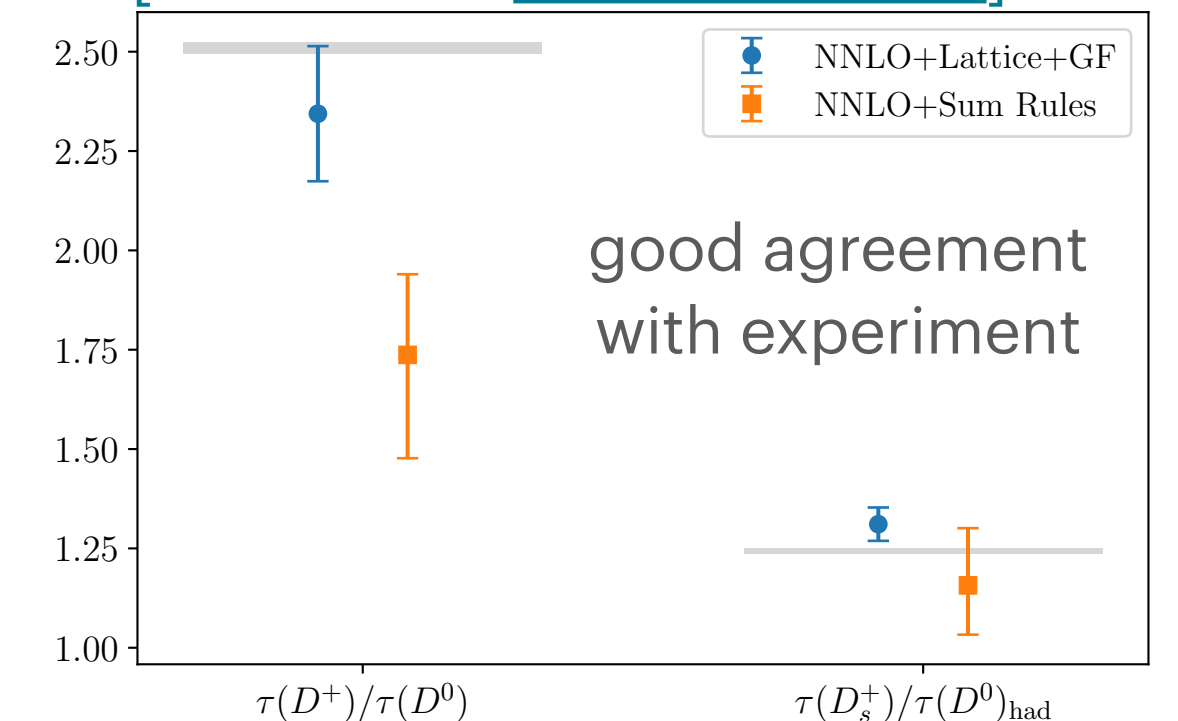
- aiming at full error budget for D_s
- procedure:
 - cont. limit for bag parameters at fixed flow-time τ
 - apply inverse matching matrix $\zeta(\tau, \mu)$

$$\langle \tilde{O}_n(\tau) \rangle = \sum_m \zeta_{n,m}(\tau, \mu) \langle \mathcal{O}_m^{\overline{\text{MS}}} \rangle(\mu) + O(\tau)$$

(done at various μ and then RG-run to ref. scale μ_0)
 - extrapolate $\tau \rightarrow 0$

**Talks by Andrea Shindler
Oliver Witzel**

[Moretti et al. arXiv:2604.24841]



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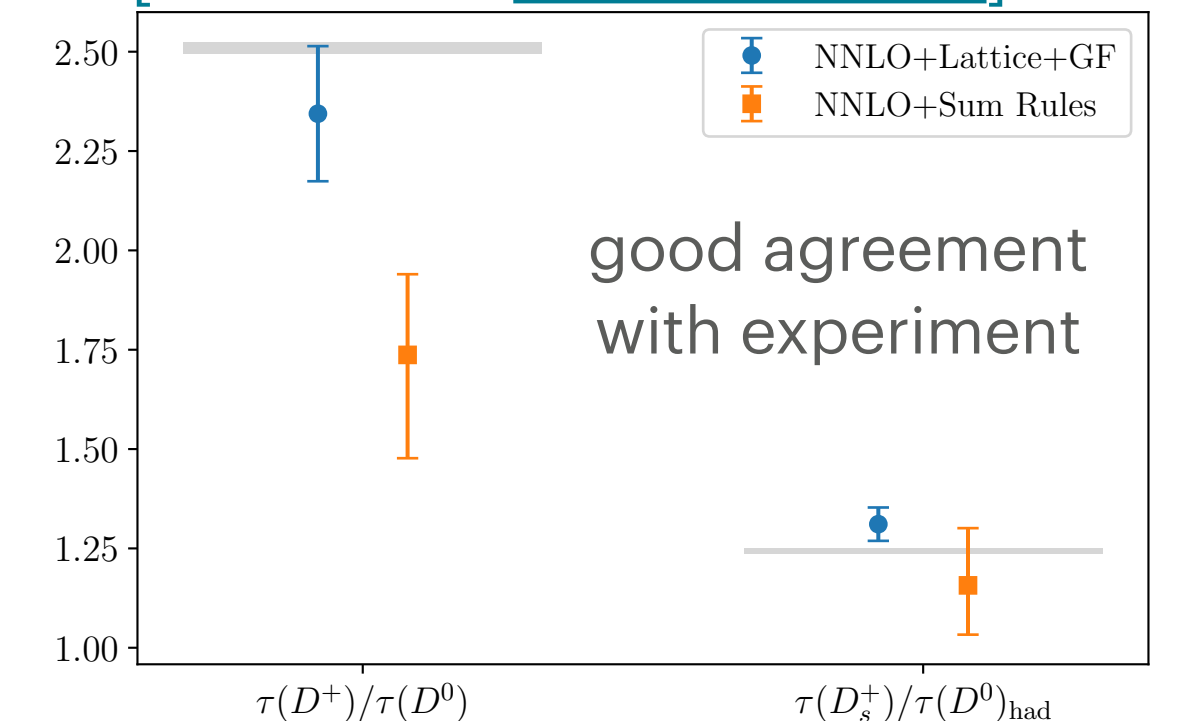
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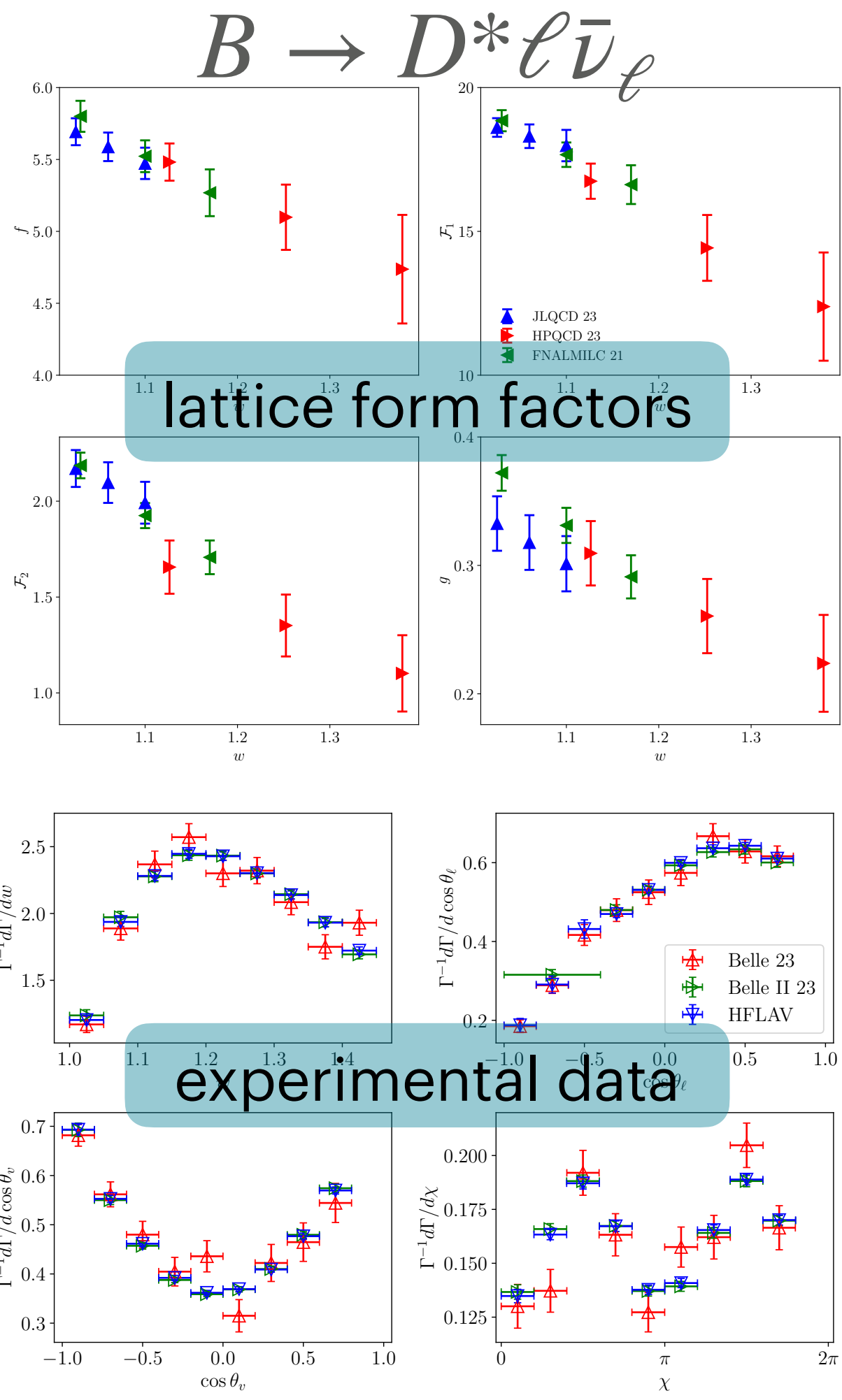


would be nice to also have K and B mixing bag parameters where there are tensions (see FLAG) (could it be renormalisation?)

Dissecting exclusive $|V_{cb}|$

[JLQCD 23 PRD 109 (2024)]
[HPQCD 23 PRD 109 (2024)]
[FNAL/MILC 21 EPJC 82 (2022)]

Belle (II) Phys.Rev.D 108 (2023) 9, 9
Phys.Rev.D 108 (2023) 1, 012002

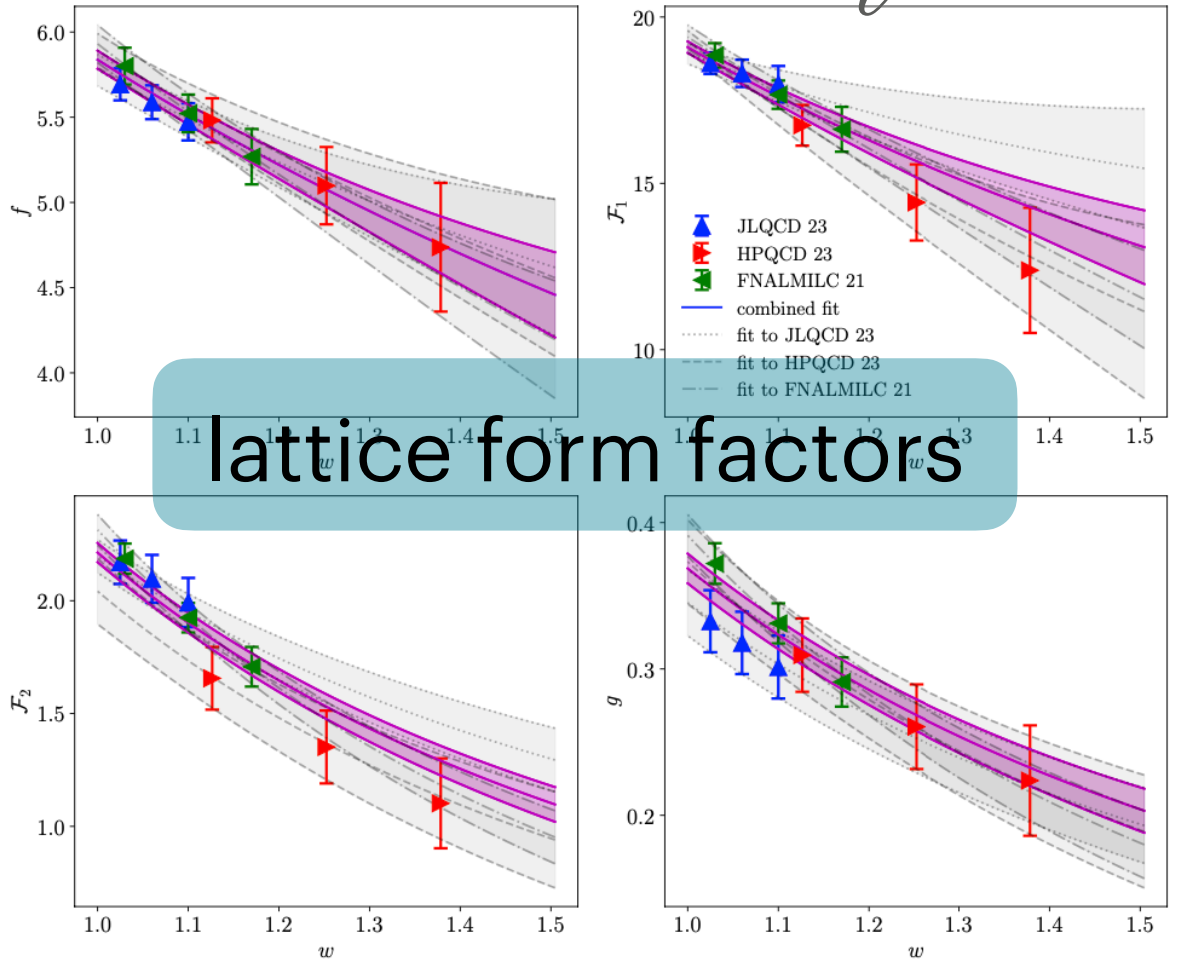


Related work: Fedele et al. PRD 108 (2023) 5, 5, Flynn, AJ, Tsang JHEP 12 (2023), Martinelli et al. EPJC 85 (2025) 3, 242, PRD 111 (2025) 1, 013005, EPJC 84 (2024) 4, 400, PRD 106 (2022) 9, 093002, , EPJC 82 (2022) 12, , PRD 105 (2022) 3, 034503, PRD 104 (2021) 9, 094512, Di Carlo et al. PRD 104 (2021) 5, 054502 Ray, Nandi JHEP 01 (2024) 022, Gambino PLB 795 (2019) 386-390, Bigi PLB 769 (2017) 441-445, JHEP 11 (2017) 061, Bernlochner et al. PRD 100 (2019) 1, 013005, [Bordone, AJ, Ollie Heald et al. arXiv:2606.23410]

Dissecting exclusive $|V_{cb}|$

$$B \rightarrow D^* \ell \bar{\nu}_\ell$$

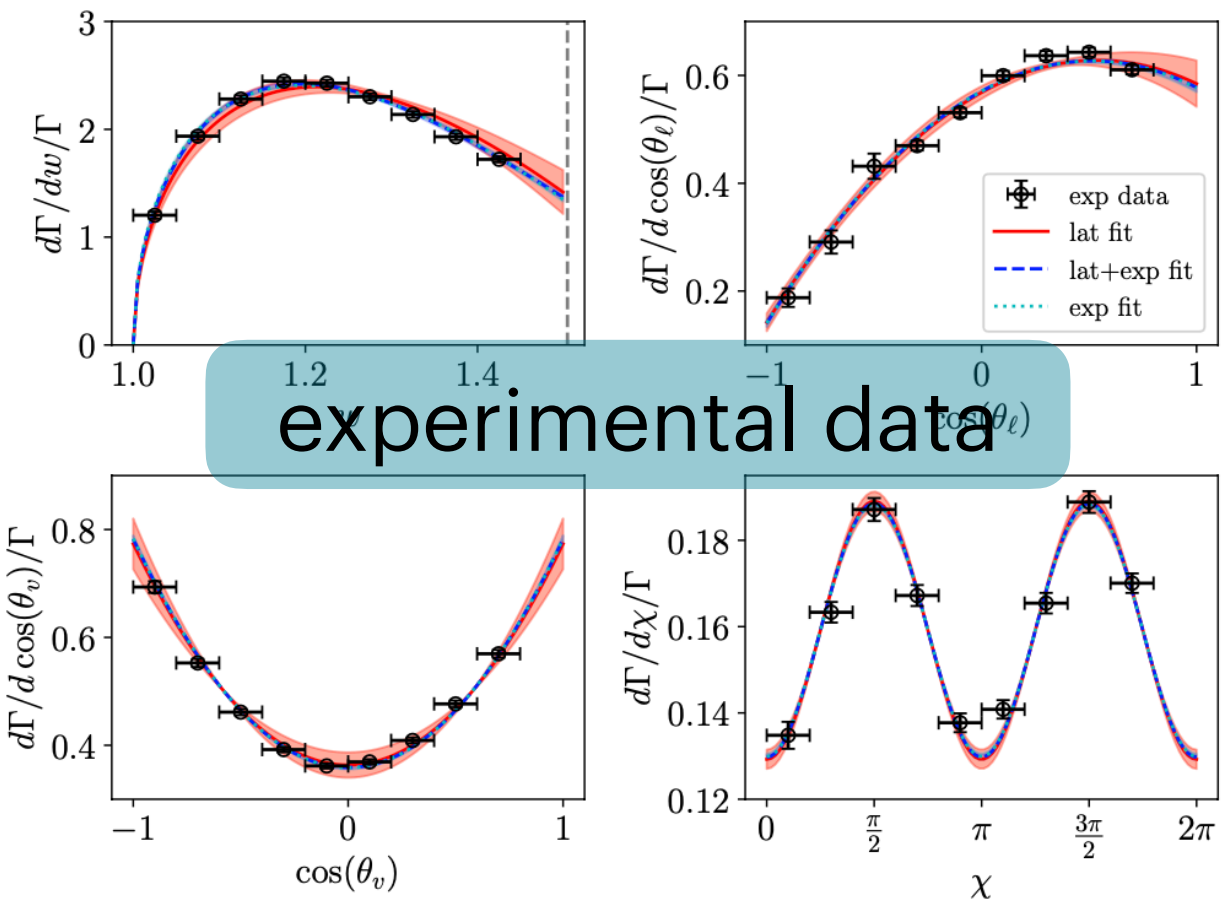
[JLQCD 23 PRD 109 (2024)]
[HPQCD 23 PRD 109 (2024)]
[FNAL/MILC 21 EPJC 82 (2022)]



lattice form factors

[Bordone, AJ, EPJC 85 (2025)]
also [Bordone, AJ, Ollie Heald, arXiv:2606.23410]

Belle (II) Phys.Rev.D 108 (2023) 9, 9
Phys.Rev.D 108 (2023) 1, 012002



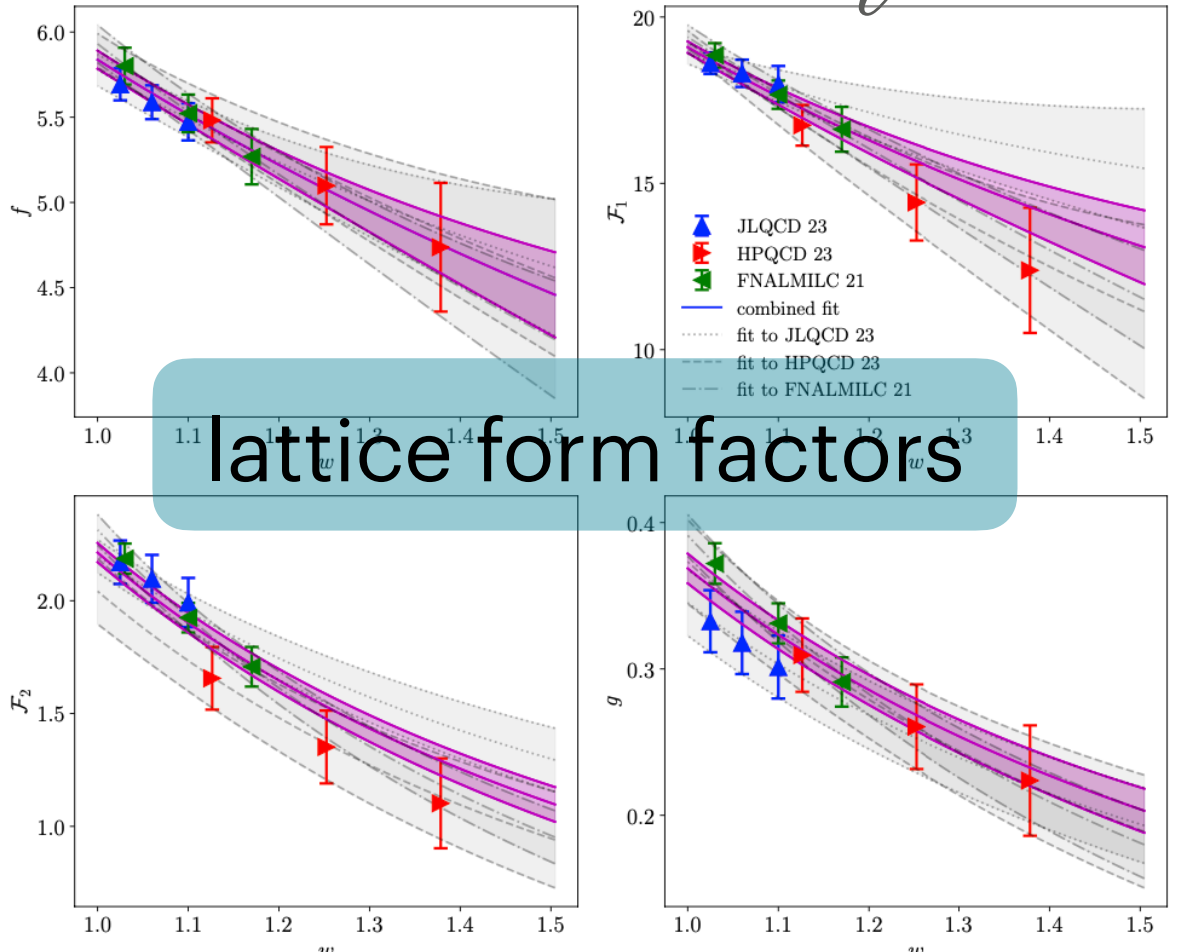
experimental data

Related work: Fedele et al. PRD 108 (2023) 5, 5, Flynn, AJ, Tsang JHEP 12 (2023), Martinelli et al. EPJC 85 (2025) 3, 242, PRD 111 (2025) 1, 013005, EPJC 84 (2024) 4, 400, PRD 106 (2022) 9, 093002, , EPJC 82 (2022) 12, , PRD 105 (2022) 3, 034503, PRD 104 (2021) 9, 094512, Di Carlo et al. PRD 104 (2021) 5, 054502 Ray, Nandi JHEP 01 (2024) 022, Gambino PLB 795 (2019) 386-390, Bigi PLB 769 (2017) 441-445, JHEP 11 (2017) 061, Bernlochner et al. PRD 100 (2019) 1, 013005, [Bordone, AJ, Ollie Heald et al. arXiv:2606.23410]

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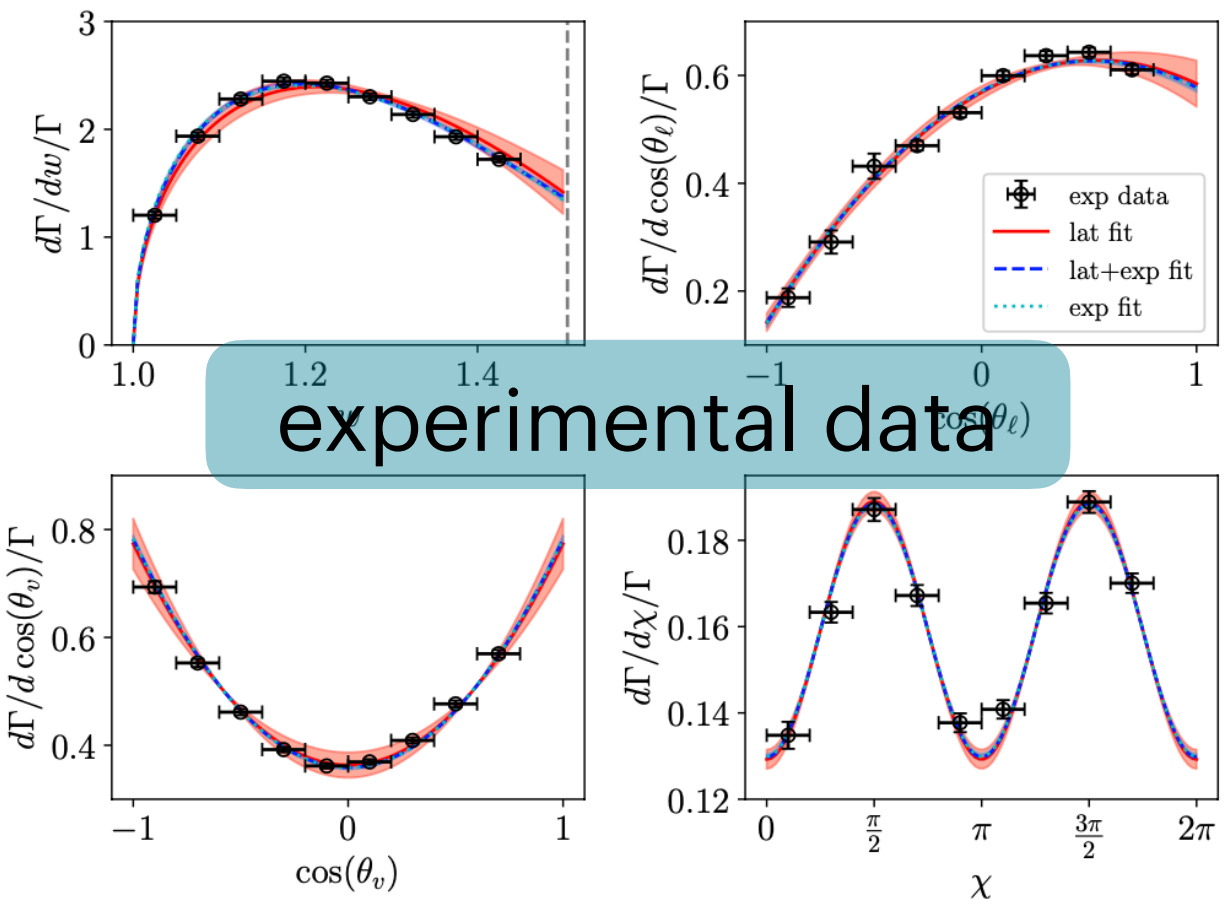
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[JLQCD 23 PRD 109 (2024)]
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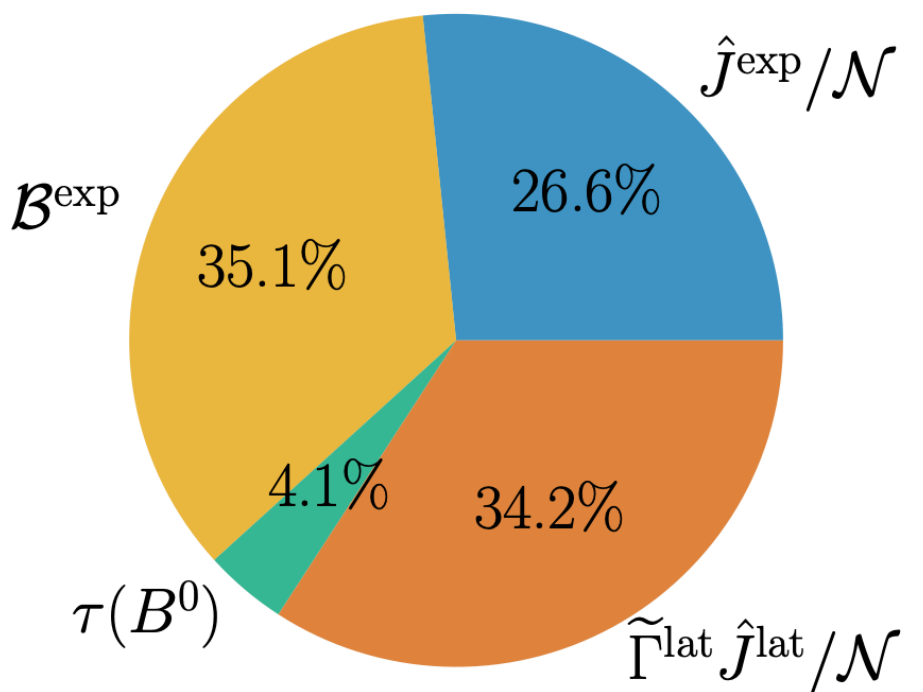
Belle (II) Phys.Rev.D 108 (2023) 9, 9
Phys.Rev.D 108 (2023) 1, 012002



Combined experimental and lattice analysis:

- $|V_{cb}|$ computed from
 - lattice form factors
 - normalised differential decay rate
 - B life time
 - total branching fraction
- $|V_{cb}|$ puzzle persists with new lattice and experimental data
- all components in the error budget can and will be improved (e.g. above discussion of $b \rightarrow c$ transitions)

$|V_{cb}|$ relative-error budget:

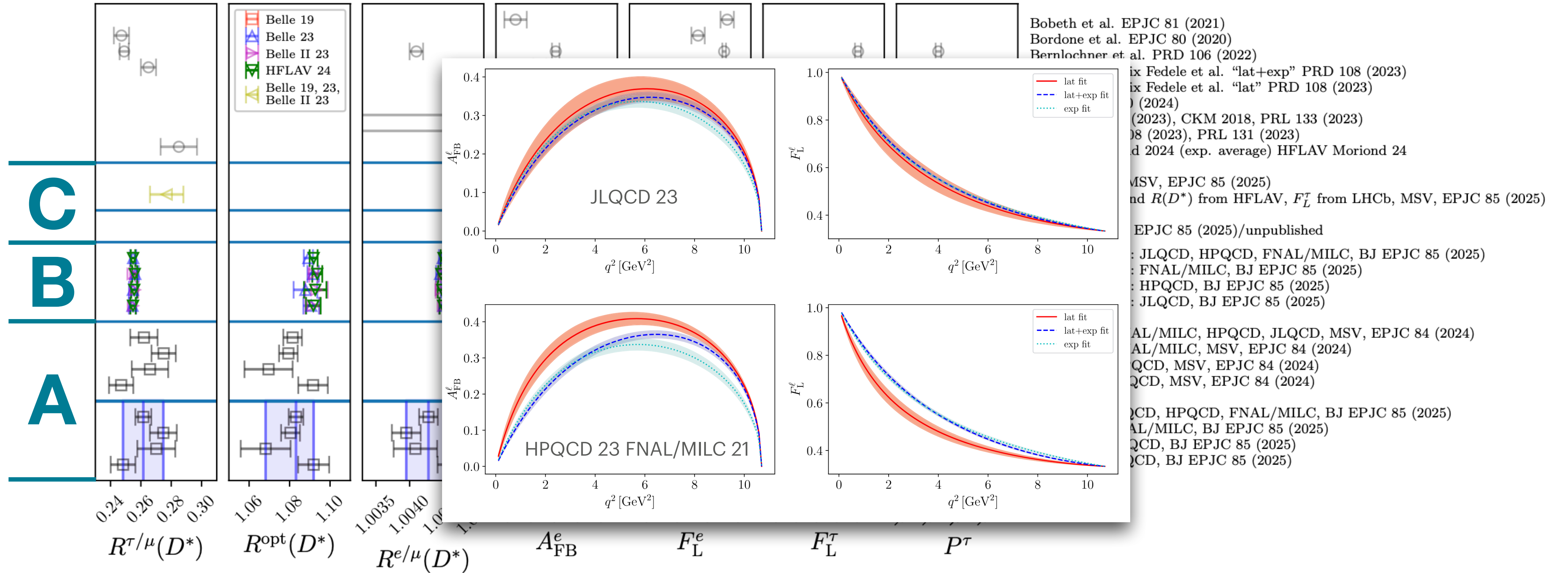


Error on branching fraction and lattice dominate error budget
[Bordone, AJ, Ollie Heald, arXiv:2606.23410]

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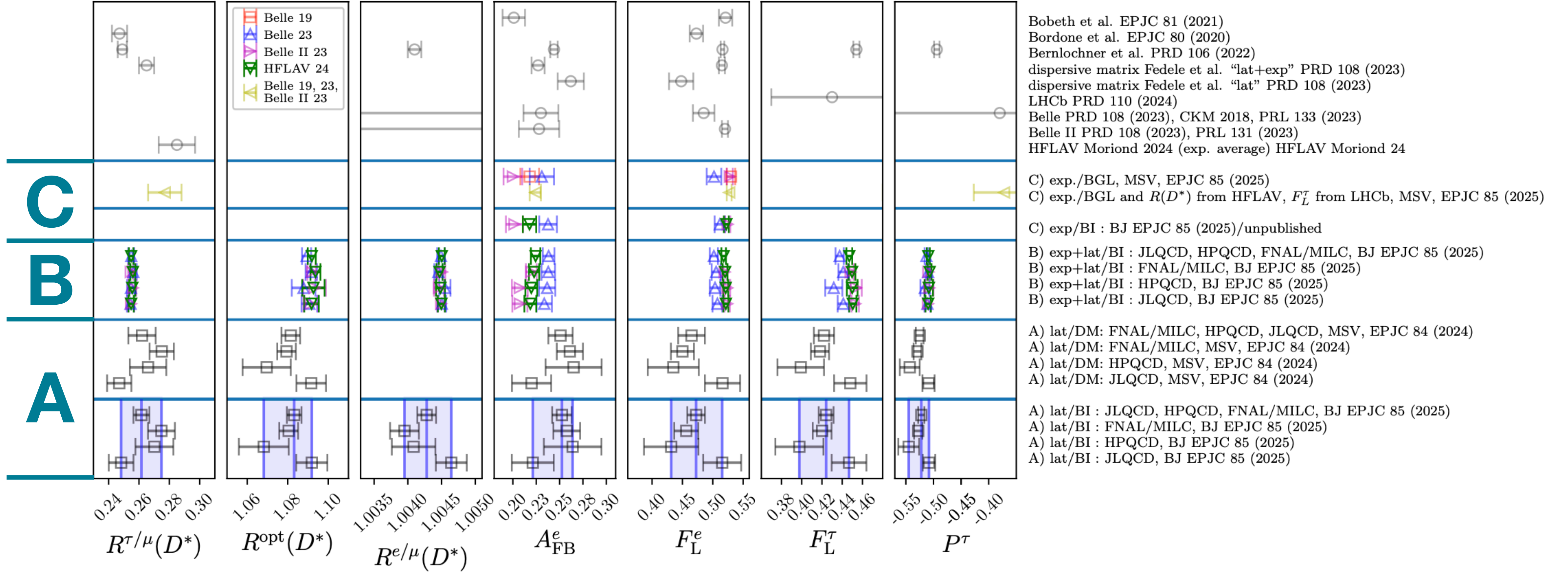
Other $B \rightarrow D^* \ell \bar{\nu}_\ell$ observables

[Bordone, AJ, EPJC 85 (2025)]



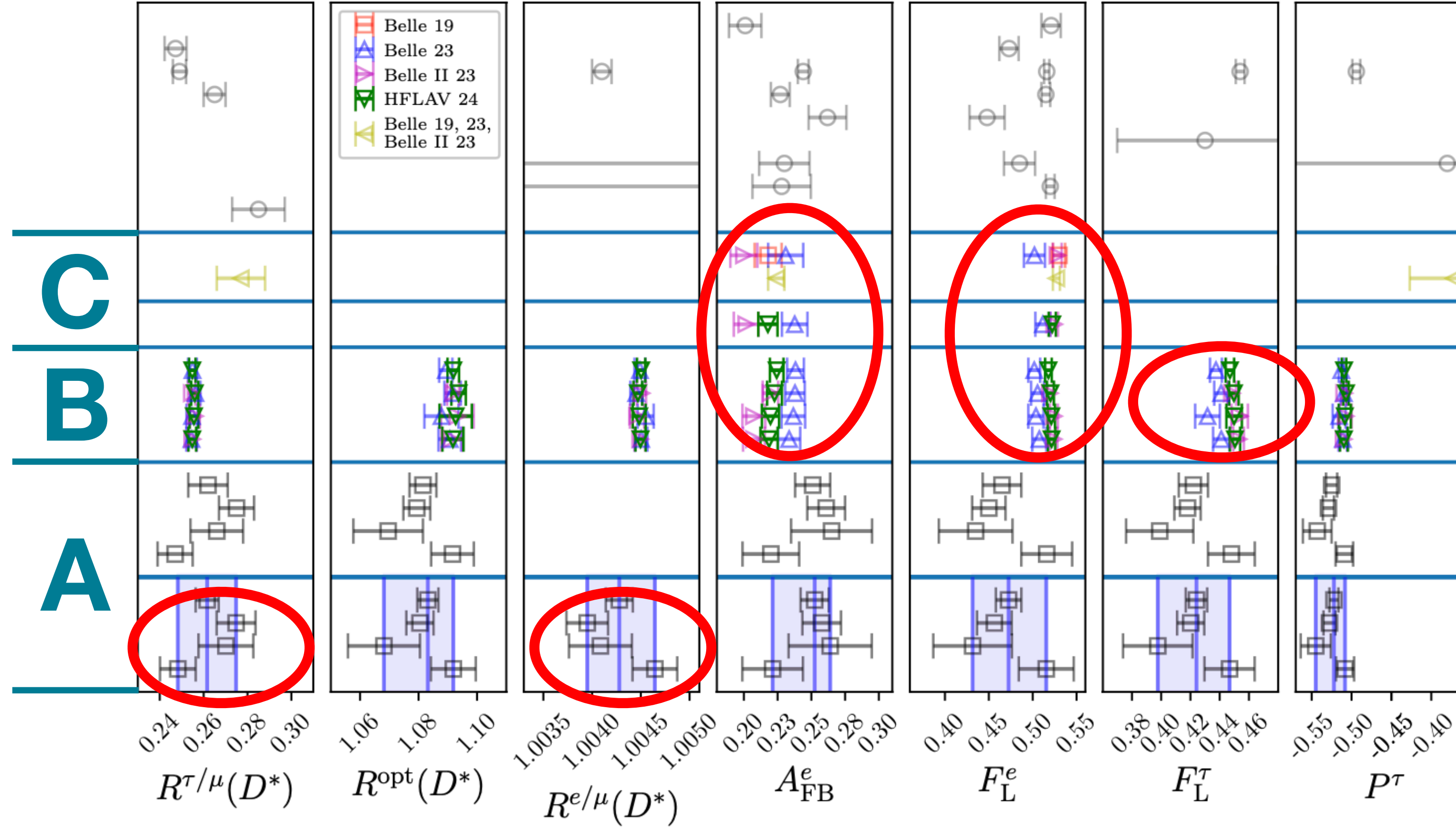
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Bobeth et al. EPJC 81 (2021)
 Bordone et al. EPJC 80 (2020)
 Bernlochner et al. PRD 106 (2022)
 dispersive matrix Fedele et al. “lat+exp” PRD 108 (2023)
 dispersive matrix Fedele et al. “lat” PRD 108 (2023)
 LHCb PRD 110 (2024)
 Belle PRD 108 (2023), CKM 2018, PRL 133 (2023)
 Belle II PRD 108 (2023), PRL 131 (2023)
 HFLAV Moriond 2024 (exp. average) HFLAV Moriond 24

C) exp./BGL, MSV, EPJC 85 (2025)
 C) exp./BGL and $R(D^*)$ from HFLAV, F_L^τ from LHCb, MSV, EPJC 85 (2025)

C) exp/BI : BJ EPJC 85 (2025)/unpublished

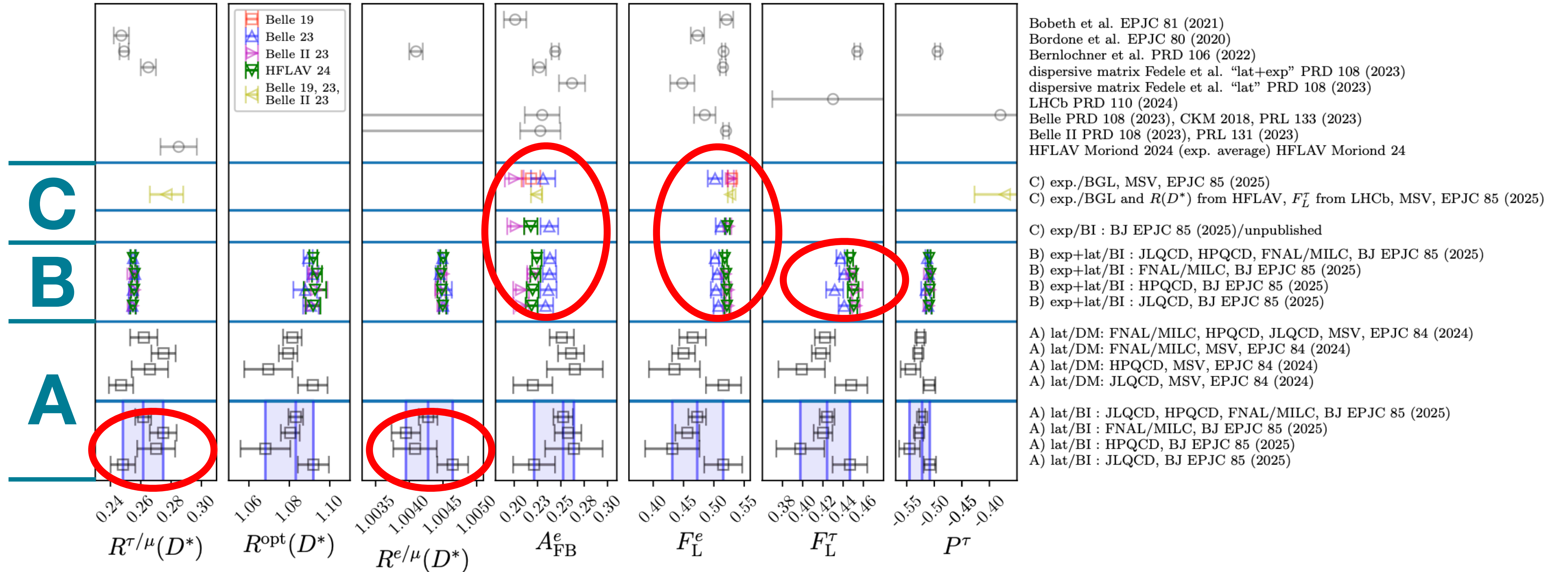
B) exp+lat/BI : JLQCD, HPQCD, FNAL/MILC, BJ EPJC 85 (2025)
 B) exp+lat/BI : FNAL/MILC, BJ EPJC 85 (2025)
 B) exp+lat/BI : HPQCD, BJ EPJC 85 (2025)
 B) exp+lat/BI : JLQCD, BJ EPJC 85 (2025)

A) lat/DM: FNAL/MILC, HPQCD, JLQCD, MSV, EPJC 84 (2024)
 A) lat/DM: FNAL/MILC, MSV, EPJC 84 (2024)
 A) lat/DM: HPQCD, MSV, EPJC 84 (2024)
 A) lat/DM: JLQCD, MSV, EPJC 84 (2024)

A) lat/BI : JLQCD, HPQCD, FNAL/MILC, BJ EPJC 85 (2025)
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Other $B \rightarrow D^* \ell \bar{\nu}_\ell$ observables

[Bordone, AJ, EPJC 85 (2025)]



analysis reveals tensions amongst
lattice as well as amongst experimental
data sets, but $|V_{cb}|$ analysis not sensitive to tensions

D -meson mixing — long-distance

Material sent to me by M. Di Carlo, F. Erben, M. T. Hansen and R. Hodgson

[Di Carlo et al. [JHEP 07 \(2025\)](#)]

First exploratory and preliminary results on $a^{-1} = 1.73 \text{ GeV}$

$M_\pi \approx 275 \text{ MeV}$, three volumes $\{20, 24, 32\}^3 \times 64$

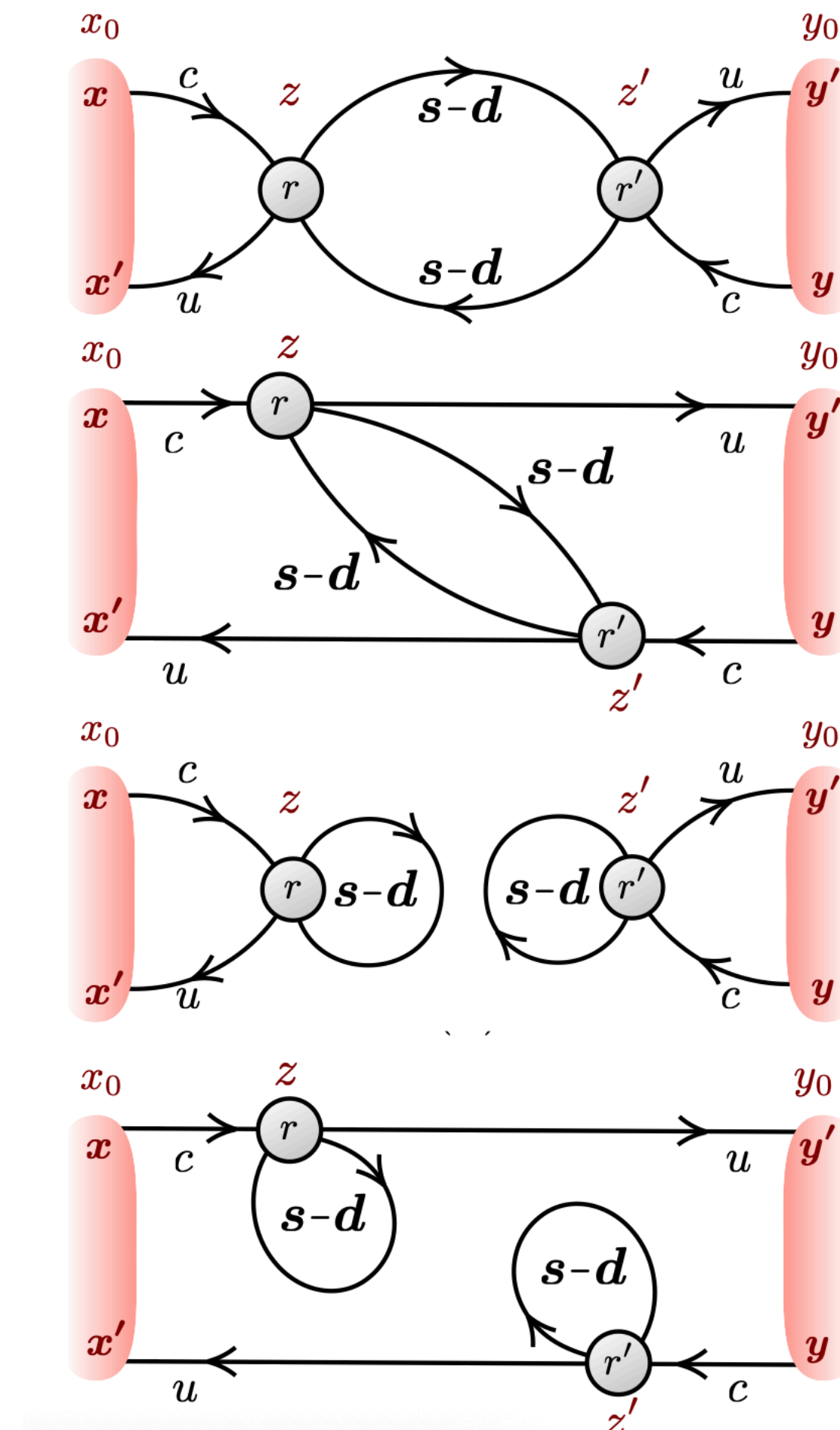
$$\mathcal{M}_{D^0 \rightarrow \bar{D}^0} = \underbrace{-\langle \bar{D}^0, \mathbf{p}_D | \mathcal{H}_w(0) | D^0, \mathbf{p}_D \rangle}_{\text{short-distance } (\Delta C = 2)} + \underbrace{\frac{i}{2} \int d^4x \langle \bar{D}^0, \mathbf{p}_D | T\{\mathcal{H}_w(x)\mathcal{H}_w(0)\} | D^0, \mathbf{p}_D \rangle}_{\text{long-distance } (\Delta C = 1)}$$

spectral reconstruction

$$\mathcal{M}_{D^0 \rightarrow \bar{D}^0}^{\text{LD}} = \frac{i}{2} \int d^4x \langle \bar{D}^0, \mathbf{p}_D | T\{\mathcal{H}_w(x)\mathcal{H}_w(0)\} | D^0, \mathbf{p}_D \rangle = \lim_{\epsilon \rightarrow 0} \int \frac{d\omega}{2\pi} \frac{\rho(\omega)}{\omega - E_D - i\epsilon}$$

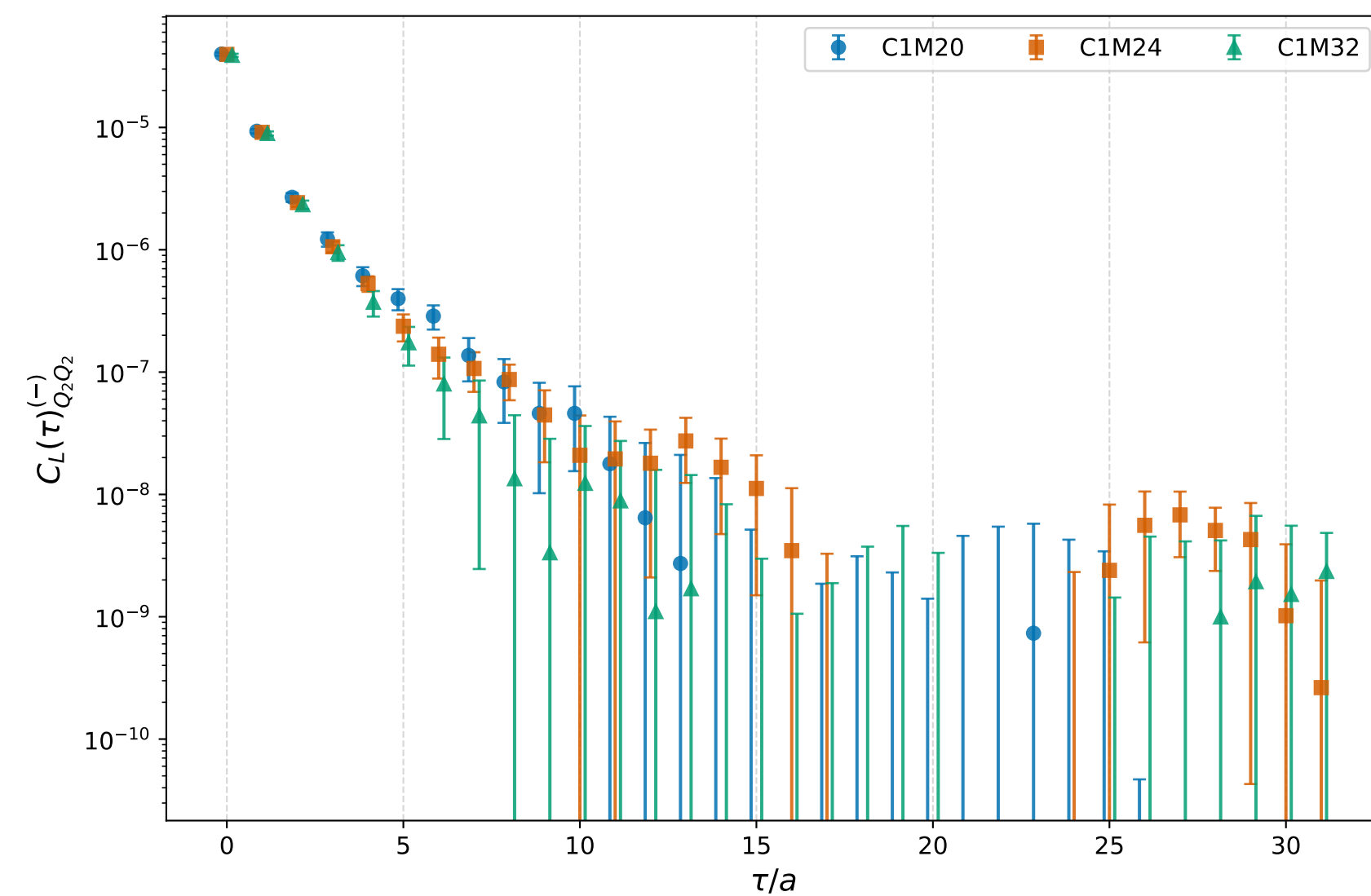
$$\rho(\omega) = \langle \bar{D}^0, \mathbf{p}_D | \mathcal{H}_w(0) (2\pi)^4 \delta(\hat{H} - \omega) \delta^3(\hat{\mathbf{P}} - \mathbf{p}_D) \mathcal{H}_w(0) | D^0, \mathbf{p}_D \rangle$$

6



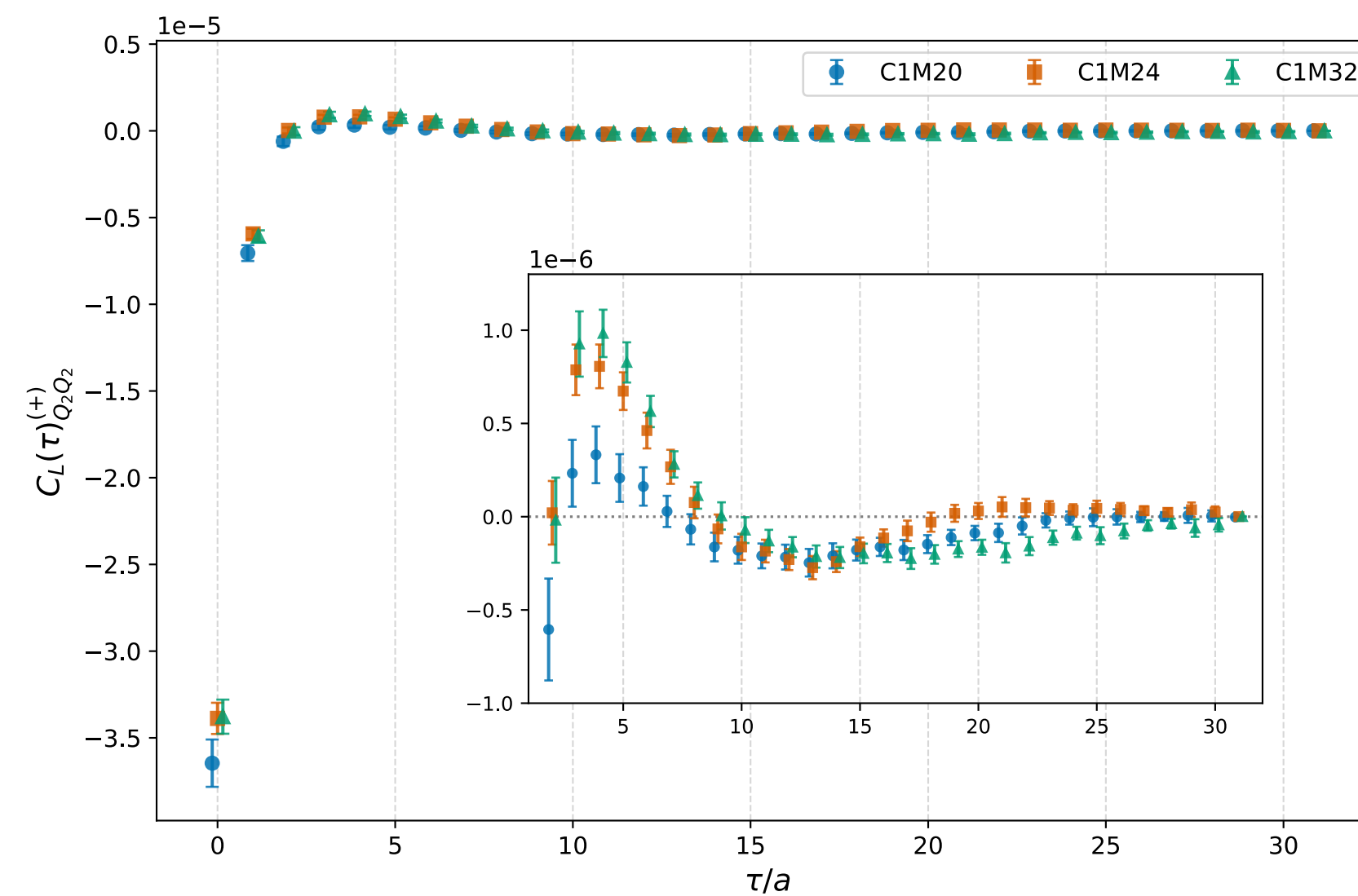
D-meson mixing — long-distance

negative-parity

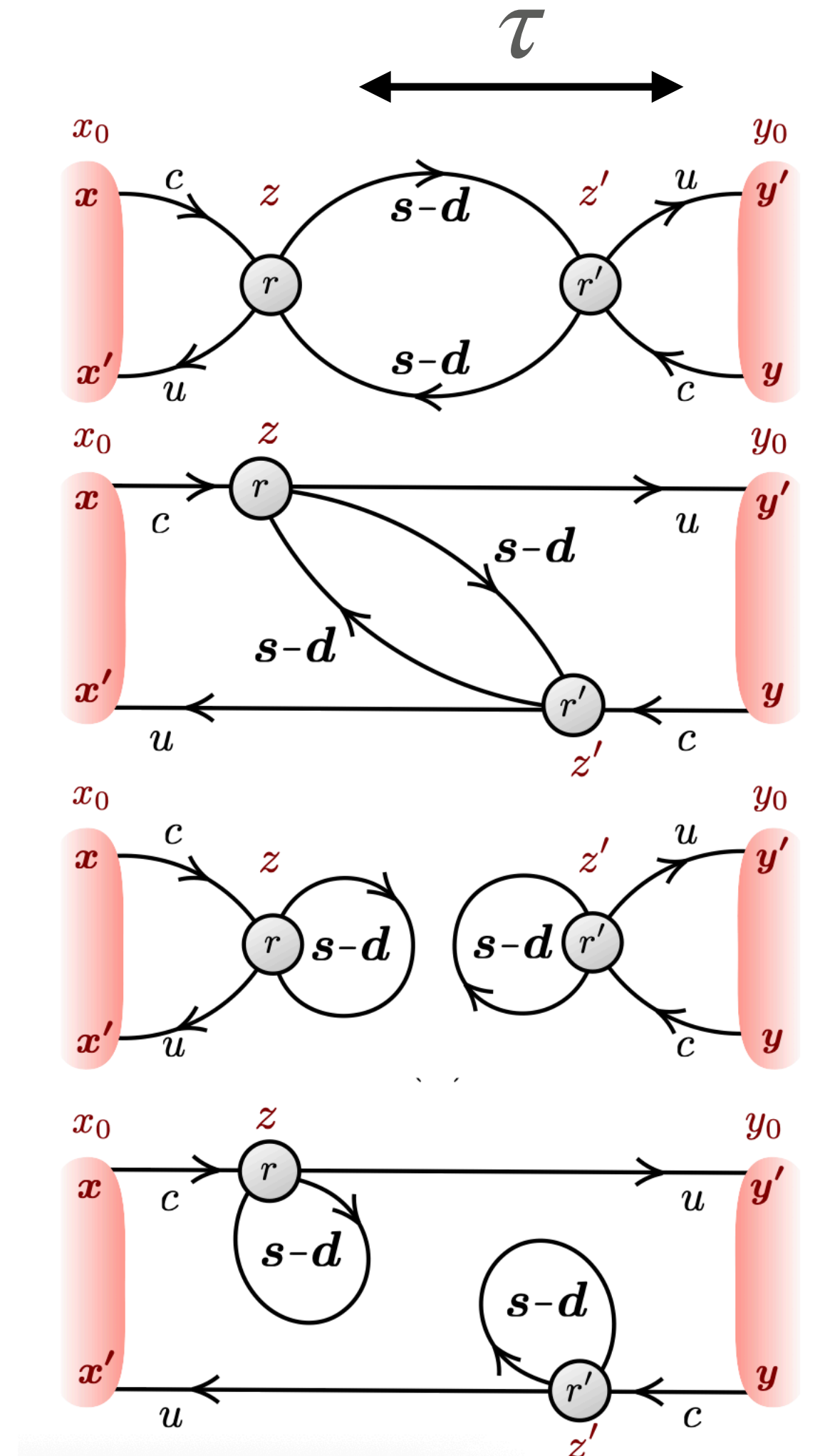


signal-to-noise deteriorating → will spectral reconstruction produce a signal?

positive parity



sign-change indicating non-positive-definite spectral density (as expected)



LECs

[Bonanno et al, [PRD 113 \(2026\)](#)]

- $N_f = 2 + 1$ staggered, $a \sim 0.08 - 0.15$ fm, varying $M_\pi > 260$ MeV, fixed m_s^{phys}
- l_7 scheme and scale-independent — strong-IB effects
e.g. $M_{\pi^+}^2 - M_{\pi^0}^2 = \frac{1}{4}(m_d - m_u)^2 \frac{8B^2}{F^2} l_7 [1 + \mathcal{O}(m_l)]$
- currently least-well determine $SU(2)$ LEC
- axion mass, self-coupling and $a\pi$ scattering $\rightarrow l_7$
- Here: compute it from pion mass splitting
[de Divitiis et al. [JHEP 04 \(2012\)](#), Giusti et al. [PRD 95 \(2017\)](#), Frezzotti et al. [PRD 104 \(2021\)](#)]
- Extrapolations: SYMEFT-based cl, NNLO χ PT (but no $S\chi$ PT ansatz)

