The recent precision determination of $\alpha_s(m_Z)$ by the ALPHA collaboration

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talk based on
Dalla Brida, Höllwieser, Knechtli, Korzec, Ramos, S., Sommer
"*High-precision calculation of the quark–gluon coupling from lattice QCD*".
Nature 652, 328–334 (2026).

ALPHA Collaboration

Lattice '26, University of Maryland, 27 July 2026

Outline

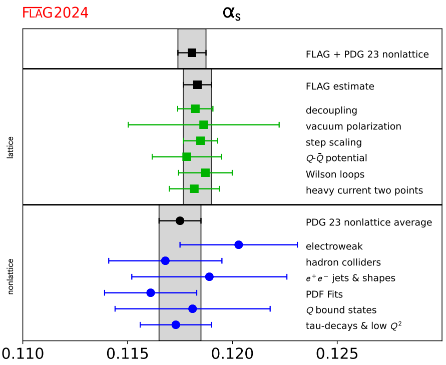
- Acknowledgments
- Current status of $\alpha_s(m_Z)$
- The ALPHA collaboration project
- Reformulation in terms of the Λ-parameter
- Large scale differences and step-scaling solution
- Decoupling of charm and bottom quarks
- Scale setting and result for $\Lambda_{\overline{\mathrm{MS}}}^{(3)}$
- Decoupling of heavy quarks as a renormalization tool
- Numerical set-up, continuum and decoupling limits
- Final results, conclusions and remarks

Acknowledgments

- My co-authors and I are particularly indebted to **Martin Lüscher** who founded the ALPHA collaboration in 1994 and who invented and developed many of the essential ingredients and tools that made this work possible.
- Special thanks also to **Peter Weisz** and **Ulli Wolff** whose contributions, expertise and advice were crucial in many ways.
- This work has a long history with origins going back to the 1990's. Many current and former members of the ALPHA collaboration have contributed to it. We are grateful for this common journey, and look forward to the future.

Disclaimer: I will peruse material and results from closely related previous ALPHA publications without always referencing it explicitly. Additional co-authors are P. Fritzsch ($N_\mathrm{f} = 3$ steps scaling), M. Bruno, S. Schaefer and H. Simma (scale setting in ALPHA 17) and A. Nada (decoupling).

Current status of $\alpha_s(m_Z)$



- PDG 2025 (non-lattice):

  $$\alpha_s(m_Z) = 0.1178(10) \quad [0.9\%]$$

- vs. FLAG 24 (lattice)

  $$\alpha_s(m_Z) = 0.1183(7) \quad [0.6\%]$$

  total error = step-scaling & decoupling error, statistics dominated!

- Most other results (on and off the lattice) systematics dominated (hadronization models, perturbative truncation at low energy scales,...)

ALPHA '26: improves and combines step-scaling & decoupling methods:

$$\alpha_s(m_Z) = 0.11876(58) \quad [0.5\%] \quad \text{(statistics dominated)}$$

ALPHA collaboration project

Build on CLS effort [Bruno et al, JHEP 1502 (2015) 043]:

- $N_\mathrm{f} = 2 + 1$ state of the art lattice QCD simulations
- nonperturbatively O(a) improved Wilson quarks & Lüscher-Weisz gauge action;
- open boundary conditions (avoids topology freezing)

Use 3 input parameters from experiment, e.g.

$$m_\pi, m_K, m_p \qquad \Rightarrow \qquad m_u = m_d, m_s, g_0$$

Everything else becomes a prediction, for instance

$$\alpha_\mathrm{x}^{(N_\mathrm{f}=3)}(100 \times m_p) \quad \text{(in any mass-independent renormalization scheme x)}$$

Final goal: $\alpha_s^{(N_\mathrm{f}=5)}(m_Z)$ in the $\overline{\mathrm{MS}}$-scheme

- Requires matching $N_\mathrm{f} = 3 \to 4 \to 5$ across the charm and bottom thresholds;

$\Rightarrow$ perturbation theory to 4-loop order satisfactory [Athenodorou et al. '18, ALPHA'26]

- ALPHA '26: scale setting via $\sqrt{t_0}$ (averaged, s.later).
- N.B. Error of 0.5% in $\alpha_s(m_Z)$ corresponds to 2.6% scale uncertainty

$\Rightarrow$ isospin breaking effects subdominant.

The QCD Λ-parameter vs. $\alpha_s(\mu) = \bar{g}^2(\mu)/4\pi$

The coupling $\alpha_{\mathrm{x}}(\mu)$ can be traded for its associated Λ-parameter:

$$\Lambda = \mu\varphi(\bar{g}(\mu)) = \mu\left[b_0\bar{g}^2(\mu)\right]^{-\frac{b_1}{2b_0^2}} \mathrm{e}^{-\frac{1}{2b_0\bar{g}^2(\mu)}} \exp\left\{-\int_0^{\bar{g}(\mu)} dg \left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g}\right]\right\}$$

- <u>exact</u> solution of Callan-Symanzik equation: $\left(\mu\frac{\partial}{\partial\mu} + \beta(\bar{g})\frac{\partial}{\partial\bar{g}}\right)\Lambda = 0$
- Number N_{f} of massless quarks is fixed.
- If the coupling $\bar{g}(\mu)$ non-perturbatively defined so is its β-function!
- $\beta(g)$ has asymptotic expansion $\beta(g) = -b_0 g^3 - b_1 g^5 - b_2 g^7 ..$

  $$b_0 = (11 - \tfrac{2}{3}N_{\mathrm{f}})/(4\pi)^2, \qquad b_1 = (102 - \tfrac{38}{3}N_{\mathrm{f}})/(4\pi)^4, \quad \ldots$$

  $b_{0,1}$ are universal, scheme-dependence starts with 3-loop coefficient b_2.
- Scheme dependence of Λ <u>almost</u> trivial:

  $$g_{\mathrm{X}}^2(\mu) = g_{\mathrm{Y}}^2(\mu) + c_{\mathrm{XY}} g_{\mathrm{Y}}^4(\mu) + \ldots \quad \Rightarrow \quad \frac{\Lambda_{\mathrm{X}}}{\Lambda_{\mathrm{Y}}} = \mathrm{e}^{c_{\mathrm{XY}}/2b_0}$$

$\Rightarrow$ can use $\Lambda_{\overline{\mathrm{MS}}}$ as reference (even though the $\overline{\mathrm{MS}}$-scheme is purely perturbative!)

The QCD Λ-parameter and $\alpha_s(\mu) = \bar{g}^2(\mu)/4\pi$

$$\Lambda = \mu\varphi(\bar{g}(\mu)) = \mu\left[b_0\bar{g}^2(\mu)\right]^{-\frac{b_1}{2b_0^2}} \mathrm{e}^{-\frac{1}{2b_0\bar{g}^2(\mu)}} \exp\left\{-\int_0^{\bar{g}(\mu)} dg\left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g}\right]\right\}$$

- Continuum relation, exact at any scale μ:
  - require large μ to evaluate integral perturbatively
  - require small μ to match hadronic scale

$\Rightarrow$ problem of large scale differences:

- The scale μ must reach the perturbative regime: $\mu \gg \Lambda$
- lattice cutoff must still be larger: $\mu \ll a^{-1}$
- spatial volume must be large enough to contain pions: $L \gg 1/m_\pi$
- Taken together a naive estimate gives

$$L/a \gg \mu L \gg m_\pi L \gg 1 \quad \Rightarrow L/a \simeq \mathrm{O}(10^3)$$

$\Rightarrow$ widely different scales cannot be resolved simultaneously on a single lattice!

Step-scaling

35 years ago: Strategy with application to the 2-dimensional $O(3)$ model:

Nuclear Physics B359 (1991) 221–243
North-Holland

A NUMERICAL METHOD TO COMPUTE THE RUNNING COUPLING IN ASYMPTOTICALLY FREE THEORIES

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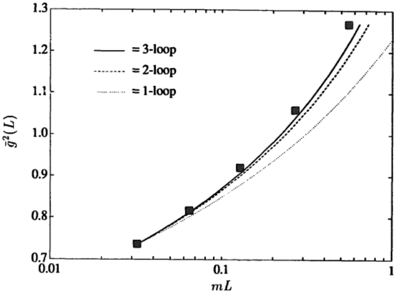
Received 18 February 1991

We propose to compute the running coupling $\bar{g}^2$ in lattice gauge theories and non-linear σ-models through a finite-size scaling analysis. The idea is shown to work well in the O(3) σ-model, where we succeed to determine $\bar{g}^2$ (in, say, a momentum subtraction scheme) at energies up to about 30 times the mass of the particles in the theory.



- Widely different scales cannot be resolved simultaneously on a *single* lattice

$\Rightarrow$ break calculation up in steps [Lüscher, Weisz, Wolff '91; Jansen et al. '95]:

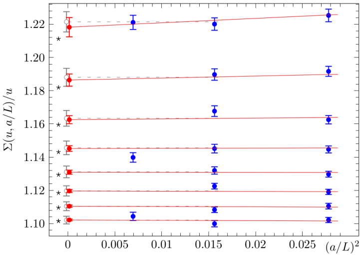
1. define $\bar{g}^2(L)$ that runs with the space-time volume, i.e. $\mu = 1/L$
2. construct the continuum step-scaling function for $u \in [u_{\text{min}}, u_{\text{max}}]$

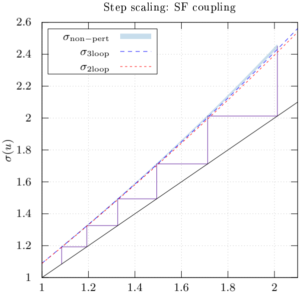
$$\sigma(u) = \lim_{a/L \to 0} \Sigma(u, a/L), \qquad \Sigma(u, a/L) = \bar{g}^2(2L)\Big|_{u=\bar{g}^2(L)}$$

$\Sigma(u, a/L)$ requires a pair of lattices, $(L/a,\ 2L/a)$, no large scale differences!

3. Recursively step up/down in scale by factors of 2:

$$\bar{g}^2(L_{\text{max}}) = u_{\text{max}} \equiv u_0, \quad u_k = \sigma(u_{k+1}) = \bar{g}^2(2^{-k} L_{\text{max}}), \quad k = 0, 1, \ldots$$





Finite volume couplings

If $\langle O_g \rangle$ can be perturbatively expanded we define

$$\bar{g}^2(L) \stackrel{\text{def}}{=} \langle O_g \rangle = g^2 + \mathrm{O}(g^4)$$

where

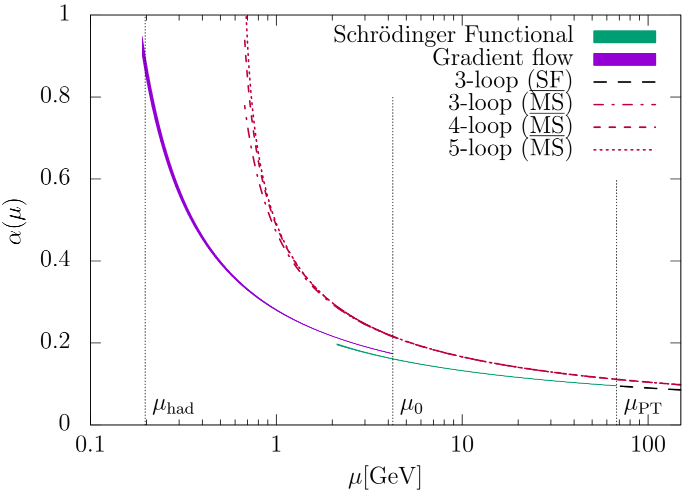
- all quark masses are set to zero;
- all dimensionful parameters are taken in fixed proportion to L, making $\mu = 1/L$ the only scale.

Consider 2 (classes of) finite volume schemes:

- $\bar{g}_{\mathrm{SF}}(L)$: from Schrödinger functional (SF) with Abelian background field [Lüscher et al. '92]
- $\bar{g}_{\mathrm{GF}}(L)$: from gradient flow observable $\langle \mathrm{tr}\{G_{kl}(t,x)G_{kl}(t,x)\}\rangle$ in finite volume with SF boundary conditions [Fritzsch & Ramos '13]

Use GF coupling at low energies, SF coupling at high energies and match non-perturbatively at $\mu_0 \approx 4\,\mathrm{GeV}$.

Non-perturbative running of $\alpha_{\rm x}$ in $N_{\rm f} = 3$ QCD for x = GF, SF

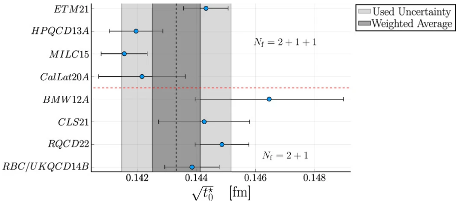


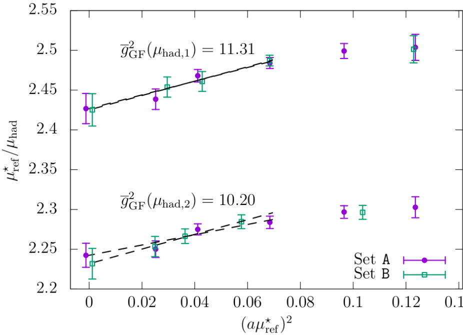
Scale definitions:

$$\bar{g}^2_{\mathsf{GF}}(1/\mu_{\mathsf{had}}) = 11.31, \qquad \bar{g}^2_{\mathsf{SF}}(1/\mu_0) = 2.012, \qquad \mu_{\mathsf{PT}} = 2^4\mu_0$$

Scale Setting

- Define $\sqrt{t_0^*}$ and $\mu_{\text{ref}}^* = 1/\sqrt{8t_0^*}$ at the symmetric mass point $(m_\pi = m_K)$
- Relation to the physical point: $\sqrt{t_0/t_0^*} = 1.0003(31)$ [RQCD 22, G.S. Bali et al.]
- Obtained from a variety of hadron masses/decay constants and both $N_\text{f} = 2+1$ and $N_\text{f} = 2+1+1$





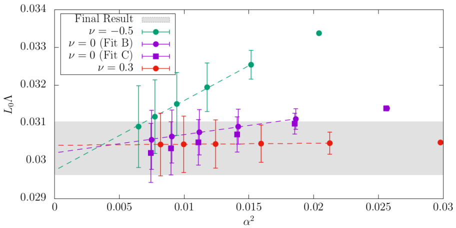
$$\sqrt{t_0^*} = 0.1433(7)_{\text{stat}}(4)_{\text{sys}}(17)_{\text{robust}}(19)_{\text{tot}}\ \text{fm}$$

N.B. The error is deliberately enlarged ("robust error") to cover the central values of all precise results!

$$\Rightarrow \quad \mu_{\text{had}} = 200.6(3.0)\text{MeV}, \qquad \mu_0 = 4.385(94)\text{GeV}, \qquad \mu_{\text{PT}} = 70.1(1.3)\text{GeV}$$

Result for $\Lambda_{\overline{\mathrm{MS}}}^{(3)}$ [ALPHA '17, ALPHA '26]

- The Λ-parameter can now be evaluated at scales $2^n\mu_0$ $(L_0 = 1/\mu_0)$, $n = 0, \dots, 5$ including μ_{PT} $(n = 4)$.
- The remaining uncertainty is parametrically $\propto \alpha^2(\mu_{\mathrm{PT}})$ if the β-function is known to 3-loop order

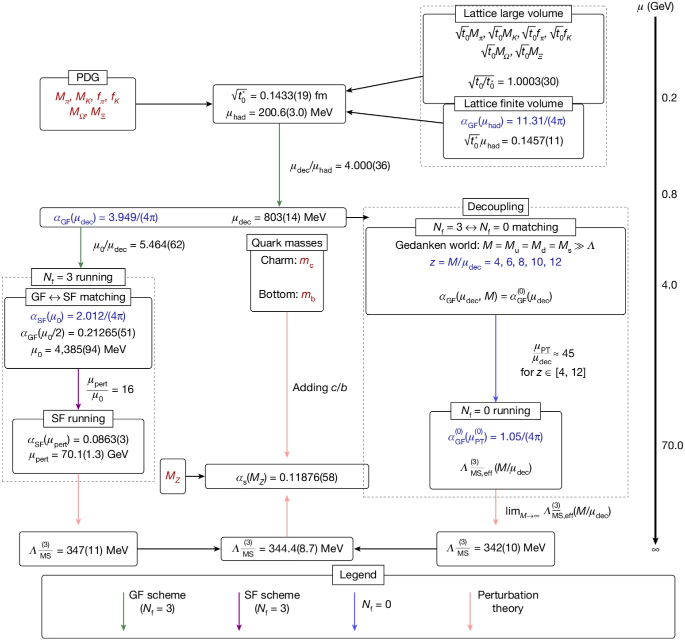


- Note: observation of this dependence requires data over a large range of scales, so that α^2 varies significantly!
- Result:

$$\Lambda_{\overline{\mathrm{MS}}}^{(3)} = 347(9)_{\mathsf{stat}}(6)_{\mathsf{sys}}(4)_{\mathsf{robust}}(11)_{\mathsf{tot}}\ \mathrm{MeV} = 347(11)\ \mathrm{MeV} \quad [3.2\%]$$

- Previously $341(12)\mathrm{MeV}$; Changes due to improved continuum extrapolations, new scale determination.

Flow chart of strategy:



Decoupling of charm and bottom quarks

- Mass-independent schemes: N_f is fixed!

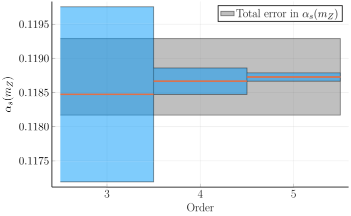
$\Rightarrow$ match: $N_\mathrm{f} = 5 \to 4$ (bottom) and $N_\mathrm{f} = 4 \to 3$ (charm)

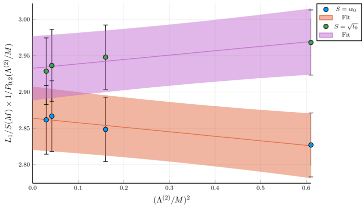
- Standard perturbative procedure: up to 5/4-loop running/decoupling in $\overline{\mathrm{MS}}$ scheme;

  [Bernreuther and Wetzel '82; Grozin et al. '11; Chetyrkin et al.'05; Schröder and Steinhauser '05; Kniehl et al. '06; Gerlach, Herren and Steinhauser '18]

- compare successive orders: perturbative error $\ll \Delta\alpha_s(m_Z)$

- PT ignores observable-dependent power corrections; Test in model $N_\mathrm{f} = 2 \to 0$ using $\sqrt{t_0}$ and w_0 [Athenodorou et al '18, ALPHA '26]





$$\underbrace{\frac{\sqrt{t_0(M)}}{L_1}}_{\text{non-pert.}} \times \underbrace{\frac{1}{P_{0,2}\left(\frac{M}{\Lambda^{(2)}}\right)}}_{\text{PT}} = \frac{\Lambda^{(2)} L_1}{\Lambda^{(0)}\sqrt{t_0}} + \underbrace{\mathcal{O}\left[\left(\frac{\Lambda^{(2)}}{M}\right)^2\right]}_{\text{very small!}}$$

Corrections are compatible with zero even at the charm threshold!

Decoupling of heavy quarks as an alternative strategy to compute $\Lambda_{\overline{\mathrm{MS}}}^{(3)}$

- Consider QCD with $N_\mathrm{f} = 3$ heavy quarks of Renormalization Group Invariant (RGI) mass M

$$M = \lim_{\mu\to\infty} \overline{m}_\mathrm{x}(\mu) \left[2b_0 \bar{g}_\mathrm{x}^2(\mu)\right]^{-\frac{d_0}{2b_0}}, \qquad d_0 = 1/(2\pi^2)$$

  with $\overline{m}_\mathrm{x}(\mu)$ the running mass in scheme x.

- At scales $\mu \ll M$, QCD($N_\mathrm{f} = 3$) is described by an effective theory, QCD($N_\mathrm{f} = 0$) ie.pure Yang-Mills theory:

$$\bar{g}_\mathrm{x}^{(3)}(\mu/\Lambda_\mathrm{x}^{(3)}, M) = \bar{g}_\mathrm{x}^{(0)}(\mu/\Lambda_\mathrm{x}^{(0)}) + \mathrm{O}(\mu^2/M^2)$$

- In PT and the $\mathrm{x} = \overline{\mathrm{MS}}$ scheme this leads to

$$[\bar{g}_{\overline{\mathrm{MS}}}^{(0)}(\mu = m_\star)]^2 = C\left(\bar{g}_{\overline{\mathrm{MS}}}^{(3)}(m_\star)\right)[\bar{g}_{\overline{\mathrm{MS}}}^{(3)}(m_\star)]^2, \qquad m_\star = \overline{m}_{\overline{\mathrm{MS}}}(m_\star),$$

  and for $\mu = m_\star$ one has

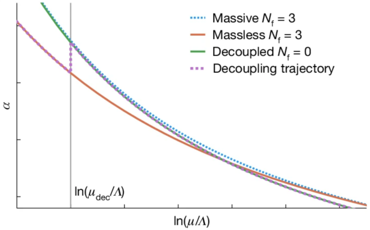
$$C(x) = 1 + c_2 x^4 + c_3 x^6 + c_4 x^8 + \ldots$$

  with c_n, $n \leq 4$ known (4-loop order).

Main idea:

- Relate couplings for $N_\mathrm{f} = 3$ and $N_\mathrm{f} = 0$ non-perturbatively at the scale μ_dec and trace coupling in $N_\mathrm{f} = 0$ theory (decoupling trajectory):

$\Rightarrow$ step scaling for $N_\mathrm{f} = 0$ instead of $N_\mathrm{f} = 3$ (much easier)!



Reformulate decoupling in terms of the respective Λ-parameters:

$$\frac{\Lambda_{\overline{\rm MS}}^{(3)}}{\mu_\mathrm{dec}} = \frac{\Lambda_{\overline{\rm MS}}^{(0)}}{\Lambda_\mathrm{x}^{(0)}} \times \lim_{M/\mu_\mathrm{dec}\to\infty} \left[\frac{\varphi_\mathrm{x}^{(0)}\left(\bar{g}_\mathrm{x}^{(3)}(\mu_\mathrm{dec}, M)\right)}{P\left(\frac{M}{\mu_\mathrm{dec}} \Big/ \frac{\Lambda_{\overline{\rm MS}}^{(3)}}{\mu_\mathrm{dec}}\right)} \right]$$

where P is the perturbative function:

$$P(M/\Lambda_{\overline{\rm MS}}^{(3)}) = \frac{\varphi_{\overline{\rm MS}}^{(0)}\left(g_\star\sqrt{C(g_\star)}\right)}{\varphi_{\overline{\rm MS}}^{(3)}(g_\star)} \qquad g_\star = g_{\overline{\rm MS}}^{(3)}(m_\star)$$

Coupling definitions

Set-up such that it benefits from previous work: running quark mass [Campos et al '18], $N_\mathrm{f} = 3$ coupling [ALPHA'17]

- Definition of massless renormalized couplings: use gradient flow GF scheme in finite volume

$$\bar{g}^2_{\mathsf{GF}}(\mu) = \mathcal{N}^{-1} \sum_{k,l=1}^{3} \frac{t^2 \langle \mathrm{tr}\,\{G_{kl}(t,x) G_{kl}(t,x)\}\, \delta_{Q,0} \rangle}{\langle \delta_{Q,0} \rangle} \Bigg|^{x_0 = T/2,\; c=\sqrt{8t}/L}_{\mu=1/L, T=L,\; M=0}$$

- use both $T = L$ (GF scheme) and $T = 2L$ (GFT scheme) with projection to topological charge $Q = 0$ sector (part of scheme definition)
- 1-parameter families of schemes, parameter $c = \sqrt{8t}/L$
- $T = 2L$ chosen to suppress both cutoff effects linear in a and large mass effects linear in $1/M$ from Euclidean time boundaries.

Numerical set-up

Lines of constant physics:

- $\bar{g}^2_{\mathrm{GF}}(\mu_{\mathrm{dec}}) = 3.949 \quad \Rightarrow \mu_{\mathrm{dec}} = 803(14)\mathrm{MeV}$
  Varying $L/a = 1/(a\mu_{\mathrm{dec}})$ between $L/a = 12 - 48$ defines a sequence of values $\beta = 6/g_0^2 \in [4.3, 5.2]$
- Define range of values $z = M/\mu_{\mathrm{dec}} \in \{4, 6, 8, 10, 12\}$ up to $\mathrm{O}(a^2)$ effects (non-trivial!) and find corresponding bare mass values.

$\Rightarrow$ This requires fully non-perturbative renormalization [Campos et al '18] and $\mathrm{O}(a)$ improvement of bare quark masses and coupling (including b_g [ALPHA '24])

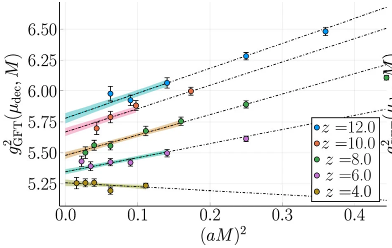
- At these values of the bare parameters choose $T = 2L$ and compute the couplings in a massive scheme

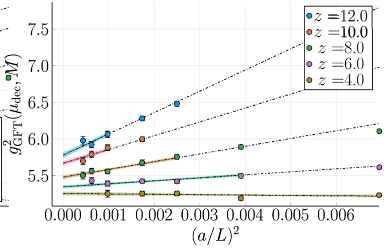
$$\bar{g}^{(3)}_{\mathrm{GFT}}(\mu_{\mathrm{dec}}, M)$$

- require aM to be small and $z = ML = M/\mu_{\mathrm{dec}} \gg 1$

$\Rightarrow$ potentially a difficult multiscale problem; using $\mu = 1/L$ alleviates this.

Continuum extrapolations





Data for $z = M/\mu_{\rm dec} \in \{4, 6, 8, 10, 12\}$ (here with $c = 0.36$), extrapolated to $a = 0$ using

- global fits (bands in plot);

$$\bar{g}^2(z_i, a) = C_i + p_1[\alpha_{\overline{\rm MS}}(a^{-1})]^{\hat{\Gamma}}(a\mu_{\rm dec})^2 + p_2[\alpha_{\overline{\rm MS}}(a^{-1})]^{\hat{\Gamma}'}(aM_i)^2 \,.$$

- fit form motivated by Symanzik and large mass expansions.
- cut in $(aM)^2 < 0.16$, with fixed $\hat{\Gamma} \in [-1, 1]$ and $\hat{\Gamma}' \in [-1/9, 1]$

Combining with pure gauge theory results [Dalla Brida & Ramos '19]

- The continuum values of the massive couplings $g^{(3)}_{\mathsf{GFT}}(\mu_{\rm dec}, M)$ can now be matched with the corresponding $g^{(0)}_{\mathsf{GFT}}(\mu_{\rm dec})$
- The step-scaling procedure in pure gauge theory gives us the function $\varphi^{(0)}_{\rm GF}(g)$

$$\frac{\Lambda^{(0)}_{\overline{\rm MS}}}{\mu_{\rm dec}} = \frac{\Lambda^{(0)}_{\overline{\rm MS}}}{\Lambda^{(0)}_{\mathsf{GF}}} \times \varphi^{(0)}_{\rm GF}\left(\bar{g}^{(0)}_{\mathsf{GF}}(\mu_{\mathsf{dec}})\right) .$$

$\Rightarrow$ requires matching GFT$(c = 0.36)$ and GF$(c = 0.3)$ schemes in QCD$(N_{\rm f} = 0)$:

$$\bar{g}^{(0)}_{\rm GF}(\mu) = \chi_{\rm c}\left(\bar{g}^{(0)}_{\rm GFT}(\mu)\right)$$

$\Rightarrow$ define $g = \chi_{\rm c}\left(\bar{g}^{(3)}_{\rm GFT}(\mu, M)\right)$ as input to $\varphi^{(0)}_{\rm GF}(g)$

- Combining all this at $\mu = \mu_{\mathsf{dec}}$, solve equation for the target quantity, ρ,

$$\rho \times \underbrace{P\left(z/\rho\right)}_{\mathsf{PT} + \mathsf{O}\left(\alpha^4_{\overline{\rm MS}}(m_\star)\right)} = \frac{\Lambda^{(0)}_{\overline{\rm MS}}}{\mu_{\rm dec}}, \qquad \rho = \frac{\Lambda^{(3)}_{\overline{\rm MS},{\rm eff}}}{\mu_{\rm dec}} = \frac{\Lambda^{(3)}_{\overline{\rm MS}}}{\mu_{\rm dec}} + {\rm O}(1/z^2)$$

- Corrections $\mathsf{O}(1/z)$ from boundaries strongly suppressed due to $T = 2L$ (was monitored)

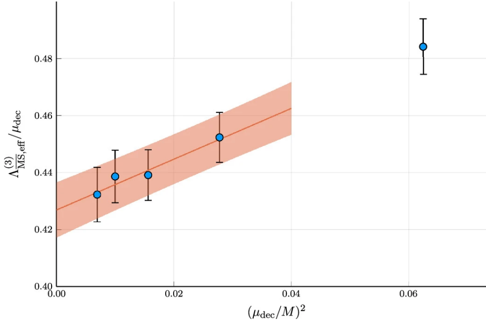
Decoupling limit extrapolation

Extrapolate continuum results for $z \to \infty$:

$$\Lambda^{(3)}_{\overline{\mathrm{MS}},\,\mathrm{eff}} = \Lambda^{(3)}_{\overline{\mathrm{MS}}} + \frac{B}{z^2}\left[\alpha_{\overline{\mathrm{MS}}}(m_\star)\right]^{\hat{\Gamma}_m}$$

Extrapolation at

- fixed $c \in [0.3, 0.42]$ (here $c = 0.36$)
- fixed $\hat{\Gamma}_m \in [0, 1]$ (here $\hat{\Gamma}_m = 0$, variation with $\hat{\Gamma}_m$ is used as error estimate)



Using $\mu_{\text{dec}} = 803(14)\,\mathrm{MeV}$ obtain the decoupling result:

$$\Lambda^{(3)}_{\overline{\mathrm{MS}}} = 342(9)_{\text{stat}}(4)_{\text{sys}}(4)_{\text{robust}}(10)_{\text{tot}}\,\mathrm{MeV} = 342(10)\,\mathrm{MeV} \quad [2.9\%]$$

Recall and compare to direct step-scaling result:

$$\Lambda^{(3)}_{\overline{\mathrm{MS}}} = 347(9)_{\text{stat}}(6)_{\text{sys}}(4)_{\text{robust}}(11)_{\text{tot}}\,\mathrm{MeV} = 347(11)\,\mathrm{MeV} \quad [3.2\%]$$

Final result and Conclusions

- Decoupling and direct $N_\mathrm{f} = 3$ step-scaling strategies are affected by very different systematics; Pearson correlation coefficient $= 0.33$

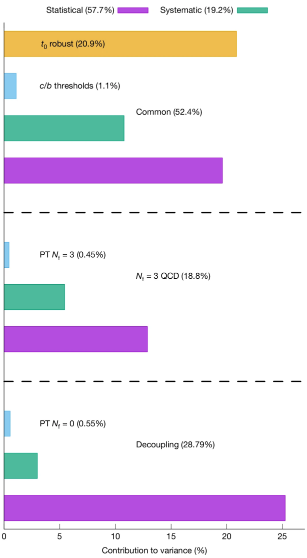
$\Rightarrow$ Take average

$$\Lambda^{(3)}_{\overline{\mathrm{MS}}} = 344.4(6.8)_\mathsf{stat}(3.7)_\mathsf{sys}(4.0)_\mathsf{robust}(8.7)_\mathsf{tot}\ \mathrm{MeV}$$

- Use $m_c^* = 1275(5)\ \mathrm{MeV}$, $m_b^* = 4171(20)\ \mathrm{MeV}$ [FLAG 2021], $m_Z = 91.188(2)\ \mathrm{GeV}$ [PDG 2024] and determine

$$\alpha_s(m_Z) = 0.11876(45)_\mathsf{stat}(25)_\mathsf{sys}(27)_\mathsf{robust}(58)_\mathsf{tot}$$

- Total uncertainty is 0.5%, statistical error dominates!
- The "robust" error in the scale $\sqrt{t_0}$ is to be eliminated by the lattice community.
- Benchmark result for $\alpha_s(m_Z)$; use in hadron collider phenomenology and Higgs physics!
- Prediction of the Standard Model, only using low energy hadronic input.



Final Remark

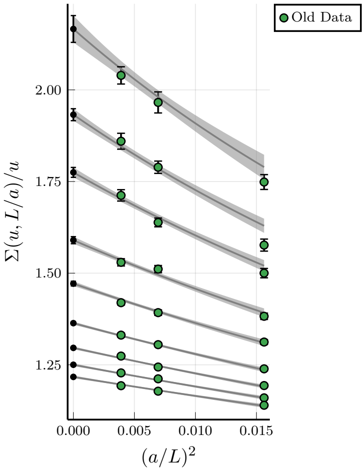
I would like to emphasize that this result heavily relies on a major community effort; Besides the works I have described, it crucially depends on

- Knowledge of the perturbative 5-loop β-function and 4-loop decoupling relations in the $\overline{\mathrm{MS}}$-scheme (cf. references in the paper)
- 2-loop (lattice) perturbation theory, both in finite and infinite volume
- Algorithmic improvements in lattice QCD

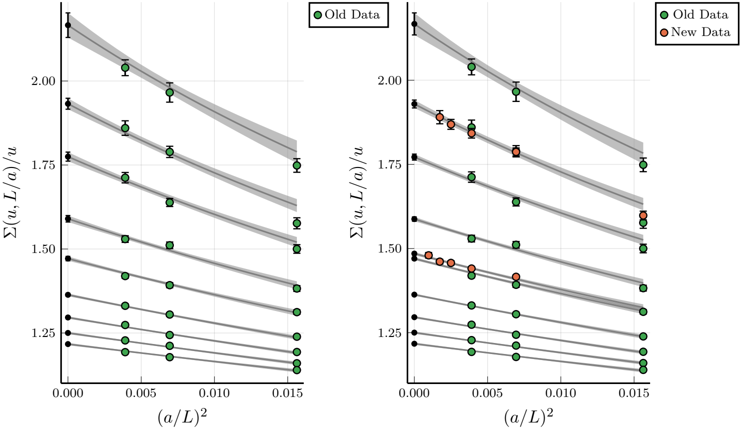
Thank you!

Improvement 1: ALPHA 17 step-scaling function for GF coupling

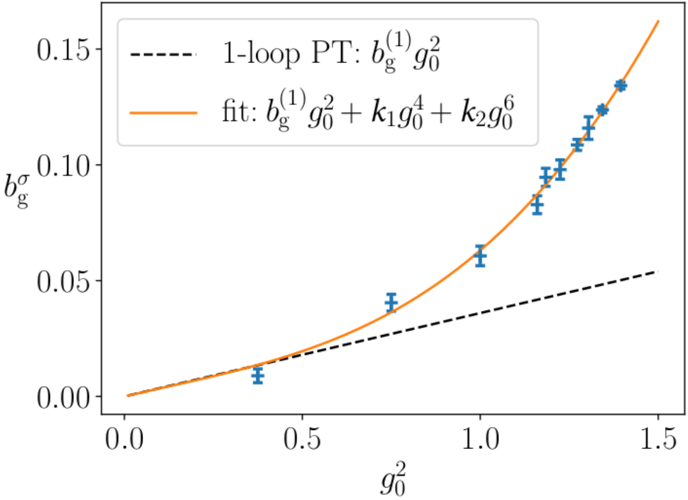
old data: significant cutoff effects, small lattices ($L/a = 8$) given smaller weight:



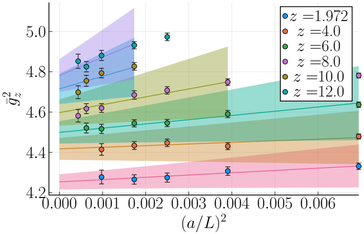
with new data from ALPHA coll. HQET project (courtesy Fritzsch, Kuberski, Heitger): very nice confirmation & improvement of old continuum extrapolations!



Nonperturbative result for b_g, [ALPHA '24]

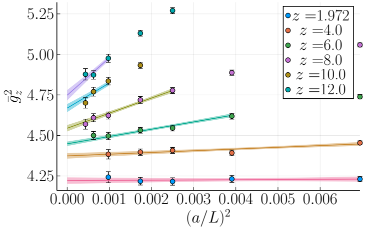


Improvement 2. The continuum extrapolation of massive couplings



Previous determination [ALPHA '22]

- Most of error from estimate of $b_g - b_g^{1-\text{loop}}$
- This is a systematic!
- But error in $\bar{g}^2(\mu, M)$ subdominant (assuming 100 percent error on 1-loop b_g)



NP determination of b_g [ALPHA '24]

- Much more precise continuum values
- Completely removes largest systematic effect in α_s