Lattice 2026

# (3+1)D U(1) with a $\theta$-Term

## A Path to Quantum Simulations with Qudits

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30.07.26 - College Park, MD

# (3+1)D U(1) with a $\theta$-Term

## Showing a sign problem - a case for quantum computing

### The Obstacle

- Classical Monte-Carlo shows a sign problem
- Interesting physics at $\theta \neq 0$

# (3+1)D U(1) with a $\theta$-Term

## Showing a sign problem - a case for quantum computing

### The Obstacle

- Classical Monte-Carlo shows a sign problem
- Interesting physics at $\theta \neq 0$

### The Opportunity

- Hamiltonian formalism has no sign problem!
- Simulation on QC possible even at high entanglement

# The Hamiltonian

## Based on Kan et al., Phys. Rev. D 104, 034504 (2021)

$$\hat{H} = \hat{H}_E + \hat{H}_B + \tilde{\theta}\hat{Q} + \mathcal{O}\left(\theta^2\right)$$

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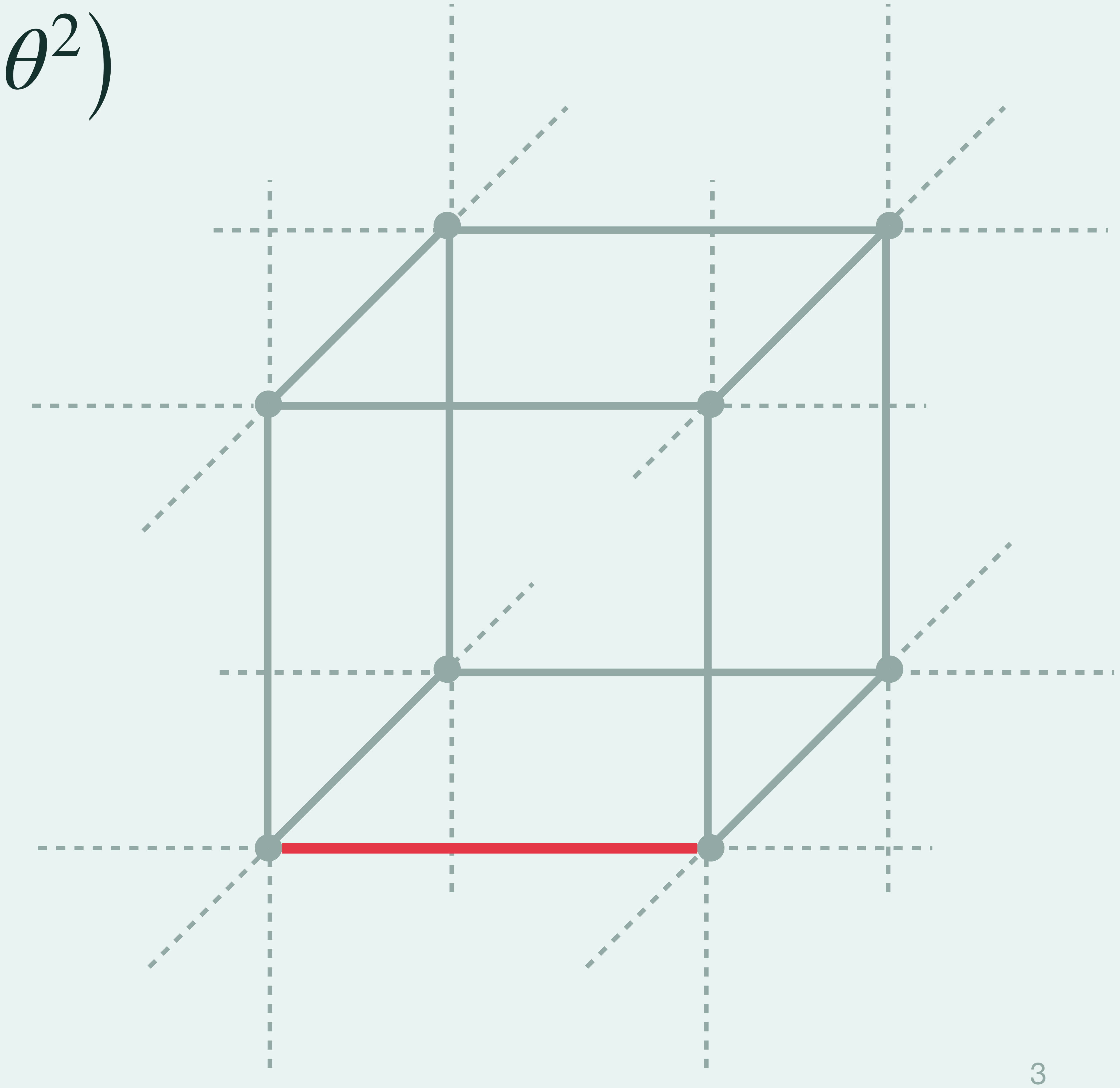
$$\hat{H} = \hat{H}_E + \hat{H}_B + \tilde{\theta}\hat{Q} + \mathcal{O}\left(\theta^2\right)$$

$$\hat{H}_E = \frac{1}{2\beta}\sum_{\vec{n},j}\hat{E}^2_{\vec{n},j}$$



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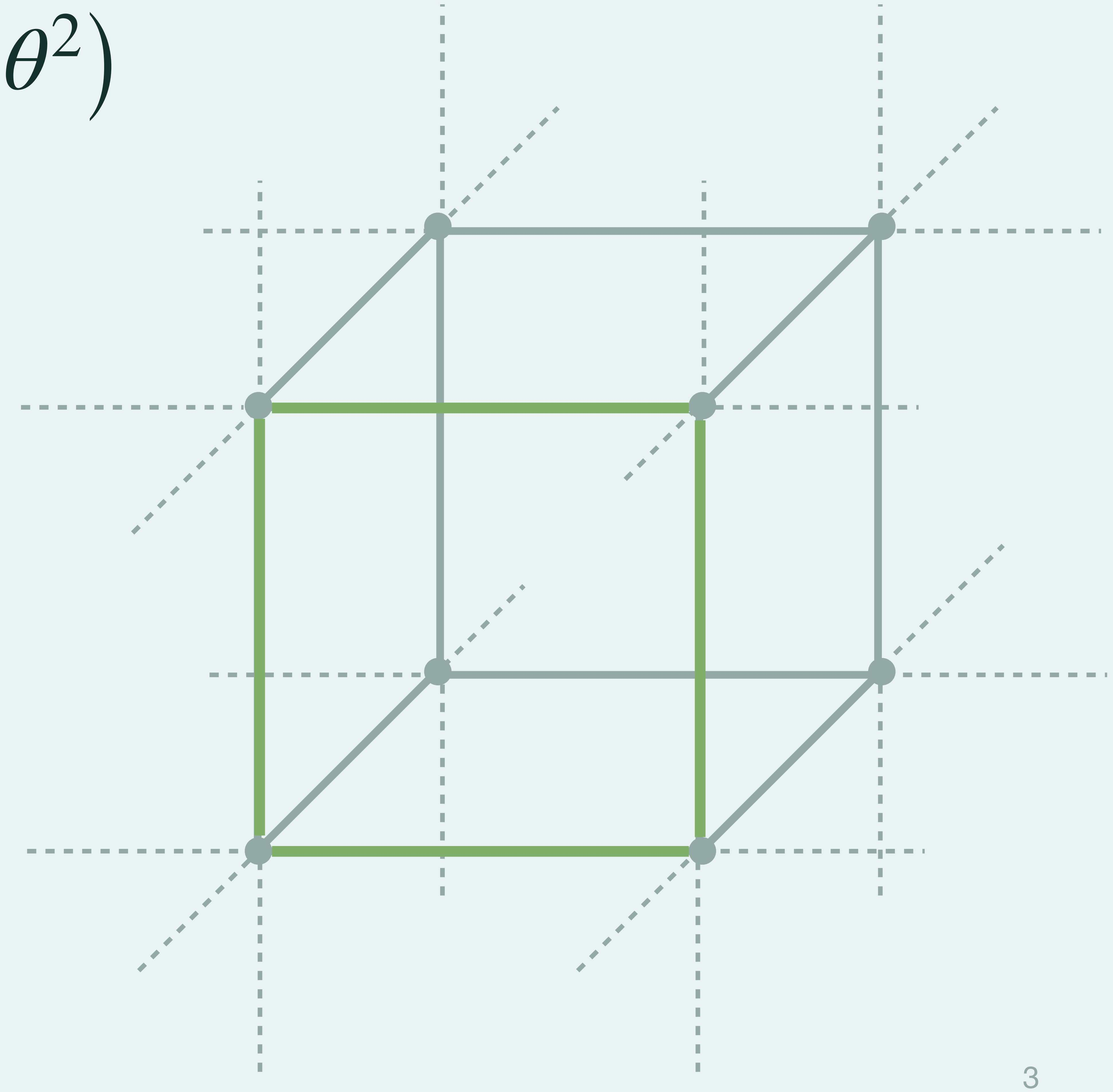
$$\hat{H} = \hat{H}_E + \hat{H}_B + \tilde{\theta}\hat{Q} + \mathcal{O}\left(\theta^2\right)$$

$$\hat{H}_B = -\frac{\beta}{2}\sum_{\vec{n},j>k}\left(\hat{U}_{\vec{n},jk} + \hat{U}^{\dagger}_{\vec{n},jk}\right)$$



# The Hamiltonian

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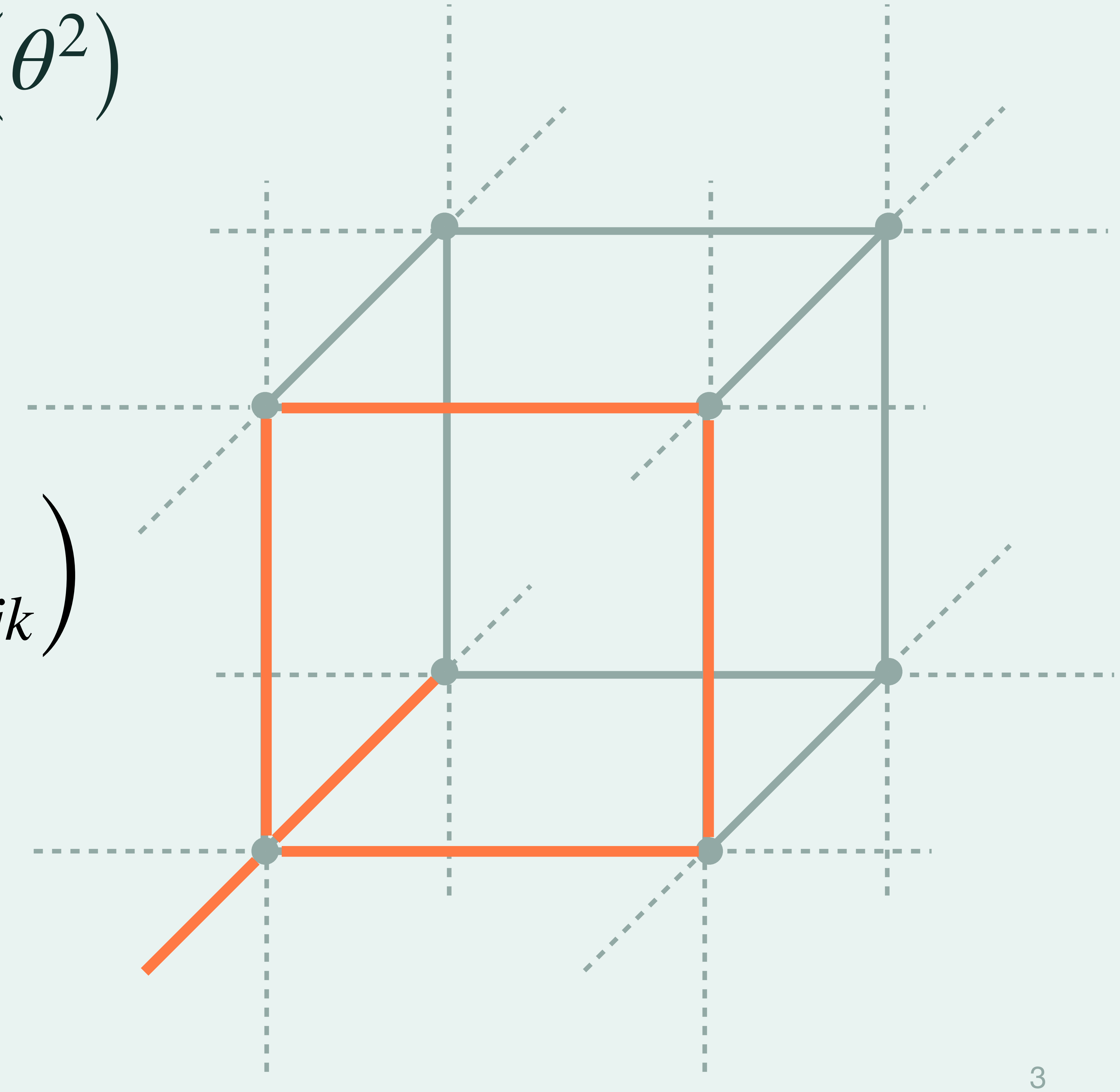
$$\hat{H} = \hat{H}_E + \hat{H}_B + \tilde{\theta}\hat{Q} + \mathcal{O}\left(\theta^2\right)$$

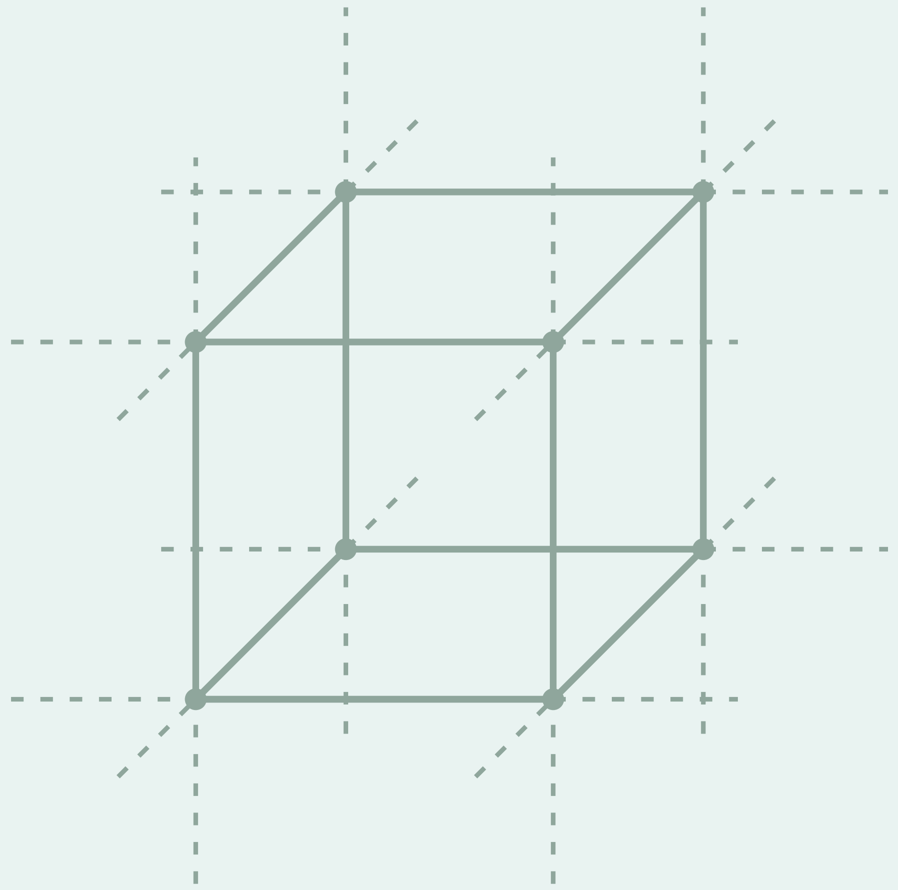
$$\tilde{\theta}\hat{Q} = -i\frac{\tilde{\theta}}{\beta} \sum_{\vec{n},(i,j,k)\in\text{even}} \left(\hat{E}_{\vec{n}-\hat{i},i} + \hat{E}_{\vec{n},i}\right)\left(\hat{U}_{\vec{n},jk} - \hat{U}^{\dagger}_{\vec{n},jk}\right)$$

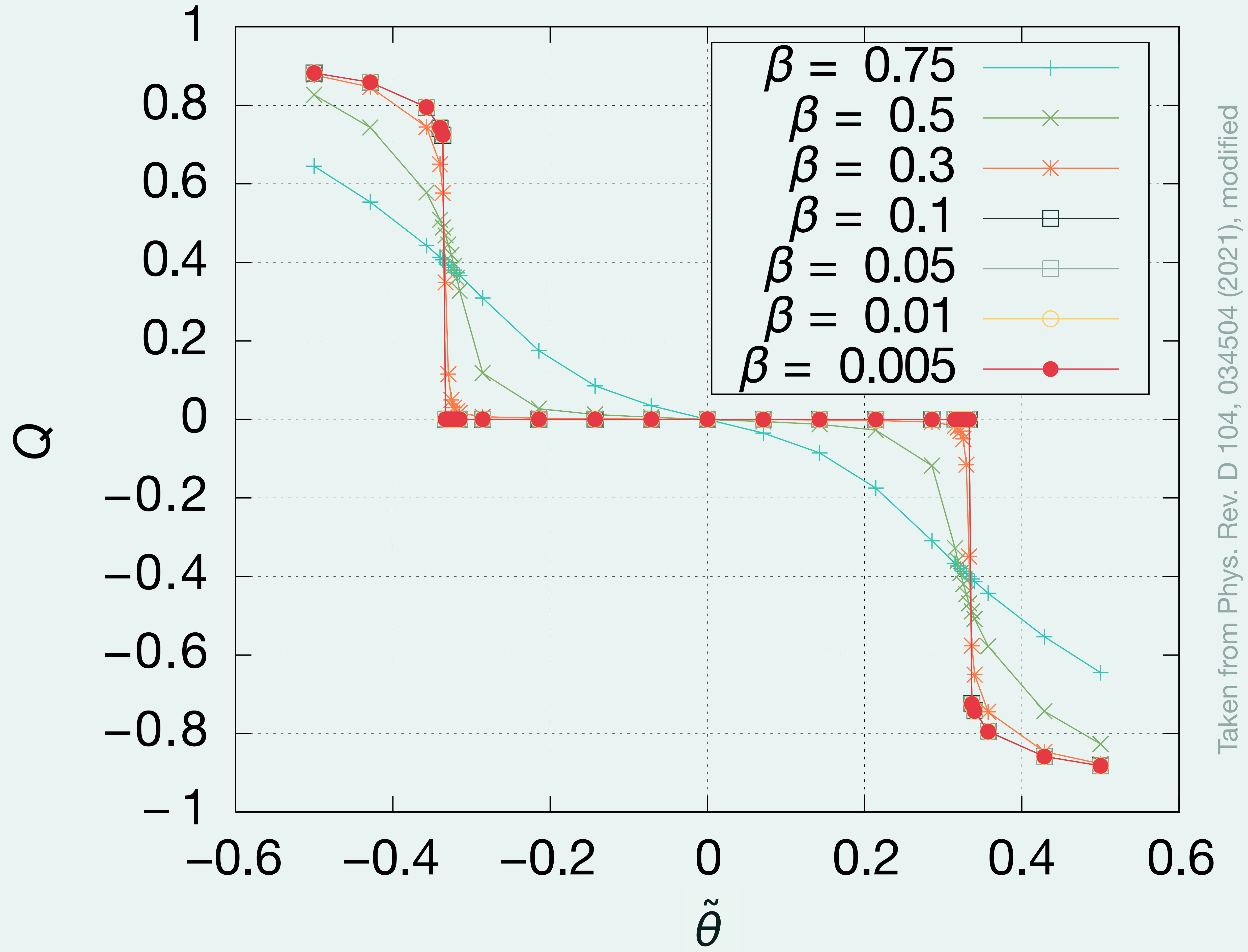
$$\tilde{\theta} = \frac{\theta}{8\pi^2}$$



# Results from Kan et al.

## (3+1)D, all directions periodic $E \in [-1,0,1]$, non-cyclic

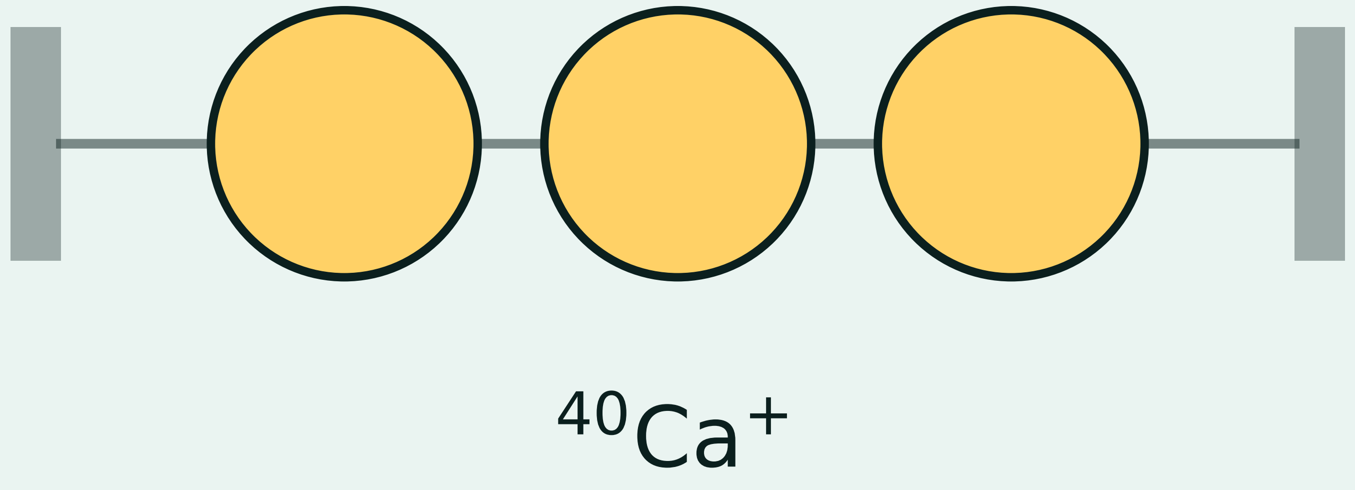





Taken from Phys. Rev. D 104, 034504 (2021), modified

# The Qudit Machine (University of Innsbruck)

## John et al., arXiv:2605.05841 (2026)

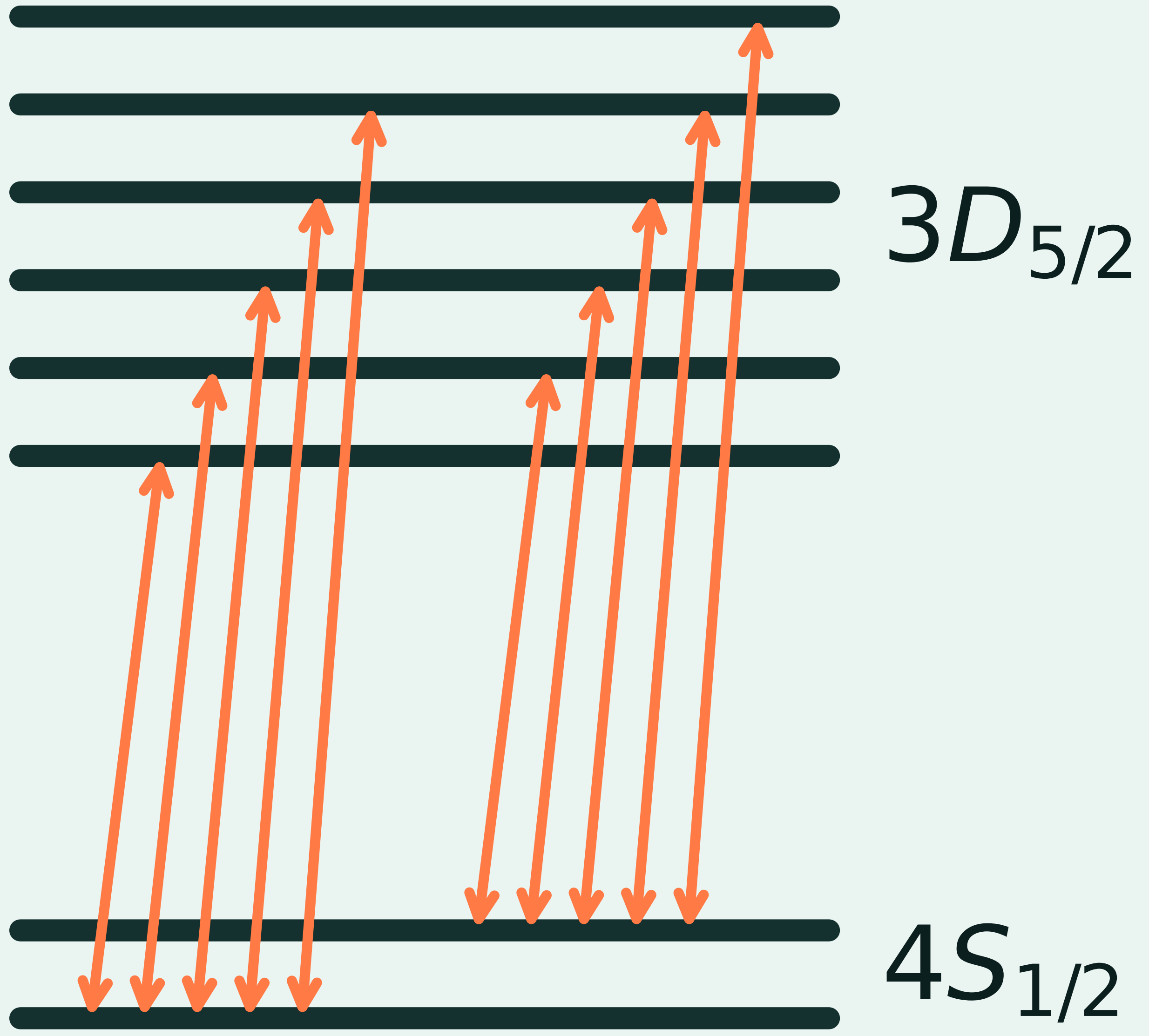



**In this paper:**

Trapped Ions $\left(^{40}\mathrm{Ca}^+\right)$

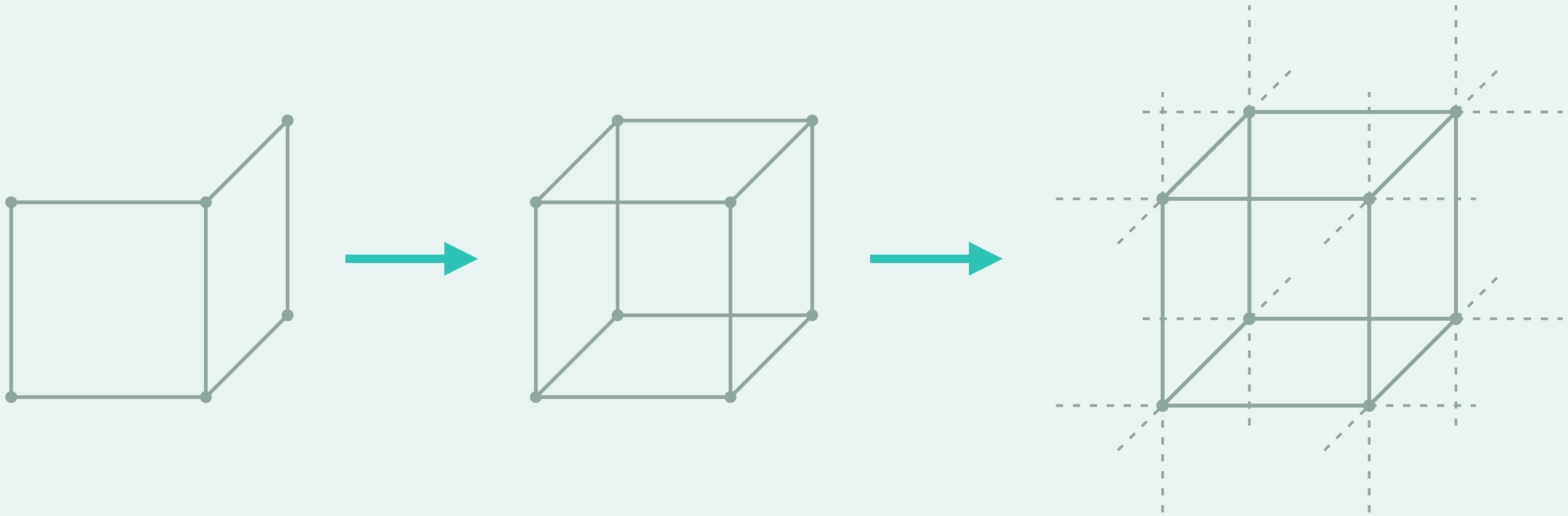
3 Qudits $\rightarrow$ up to 8 states per Qudit

Qudits allow for high truncations



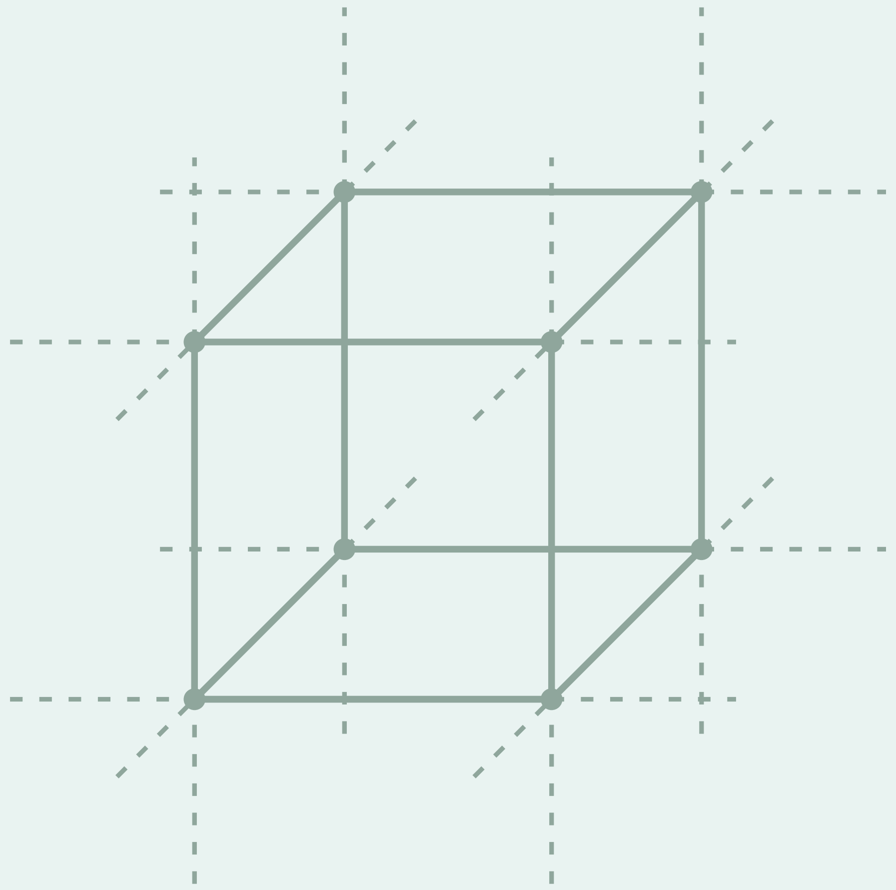
# Overview

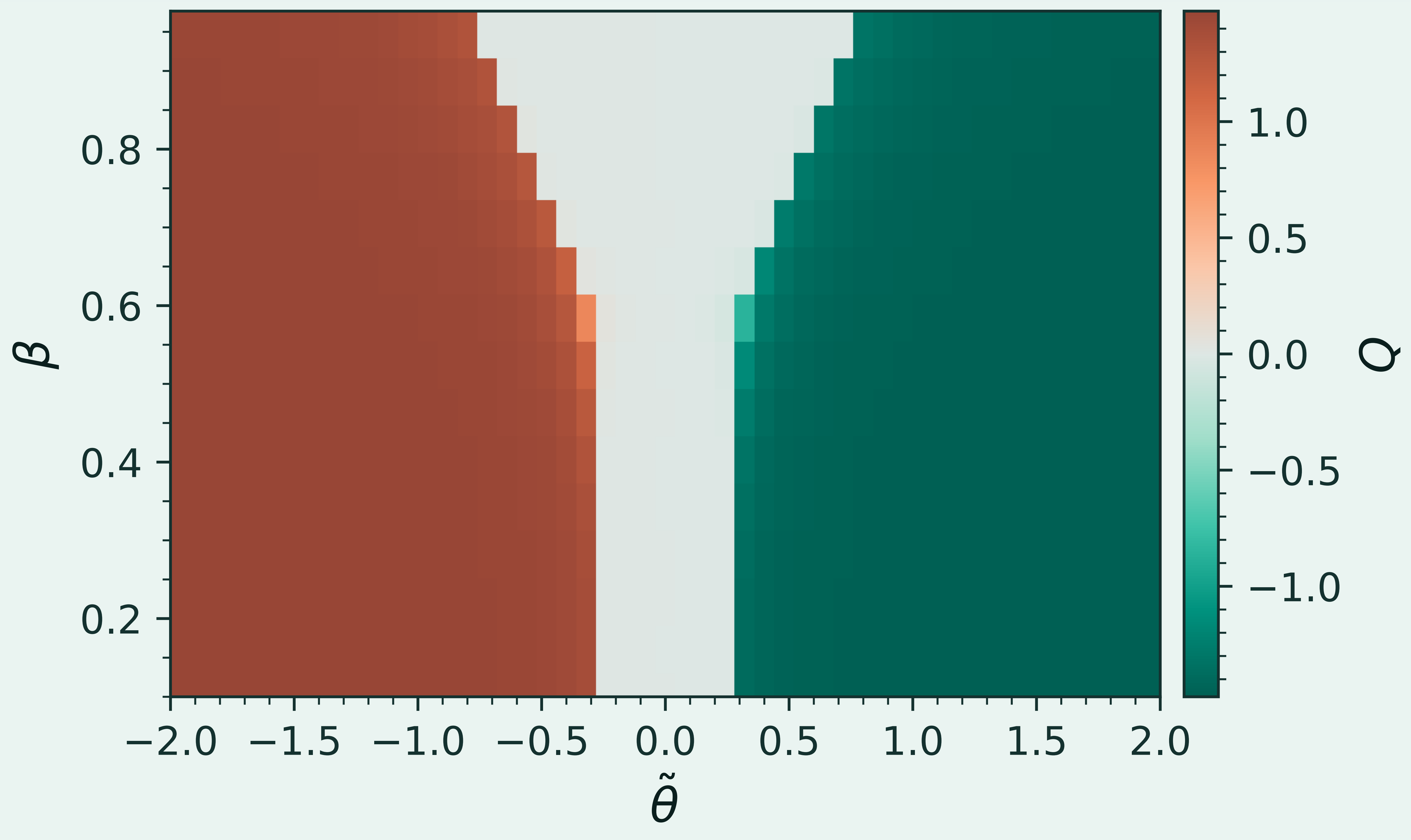
## Steps necessary for the simulation of the full theory



# A Periodic Cube

## 3+1D, All directions periodic, $E \in [-1,0,1]$, cyclic





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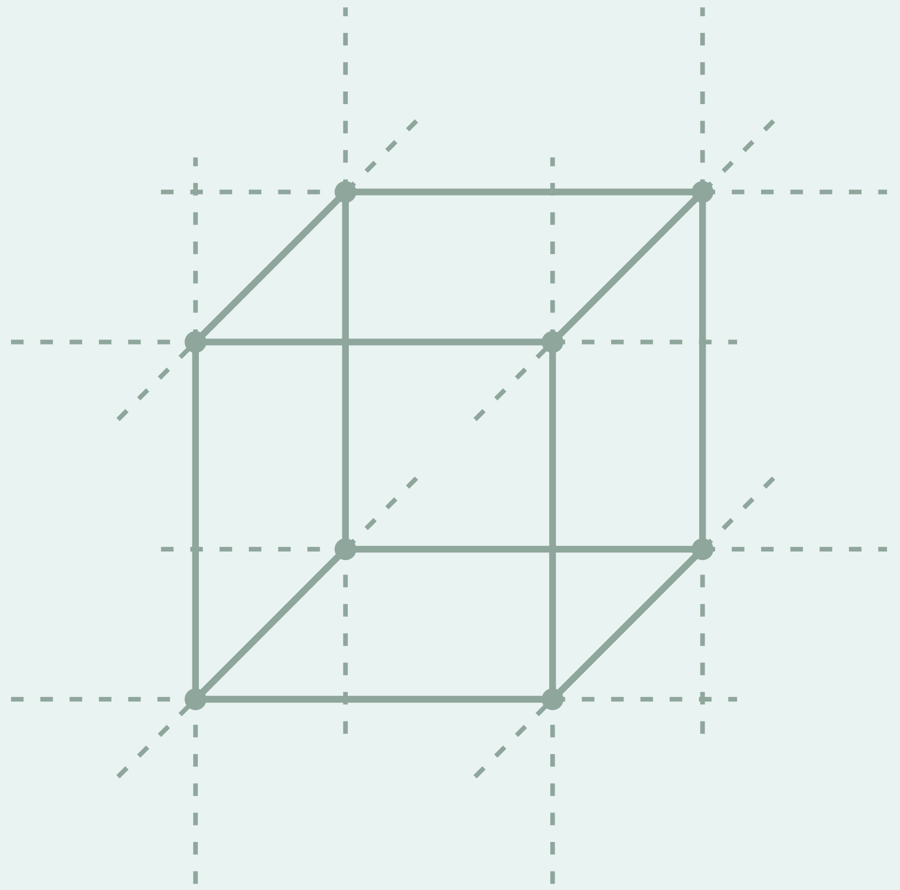

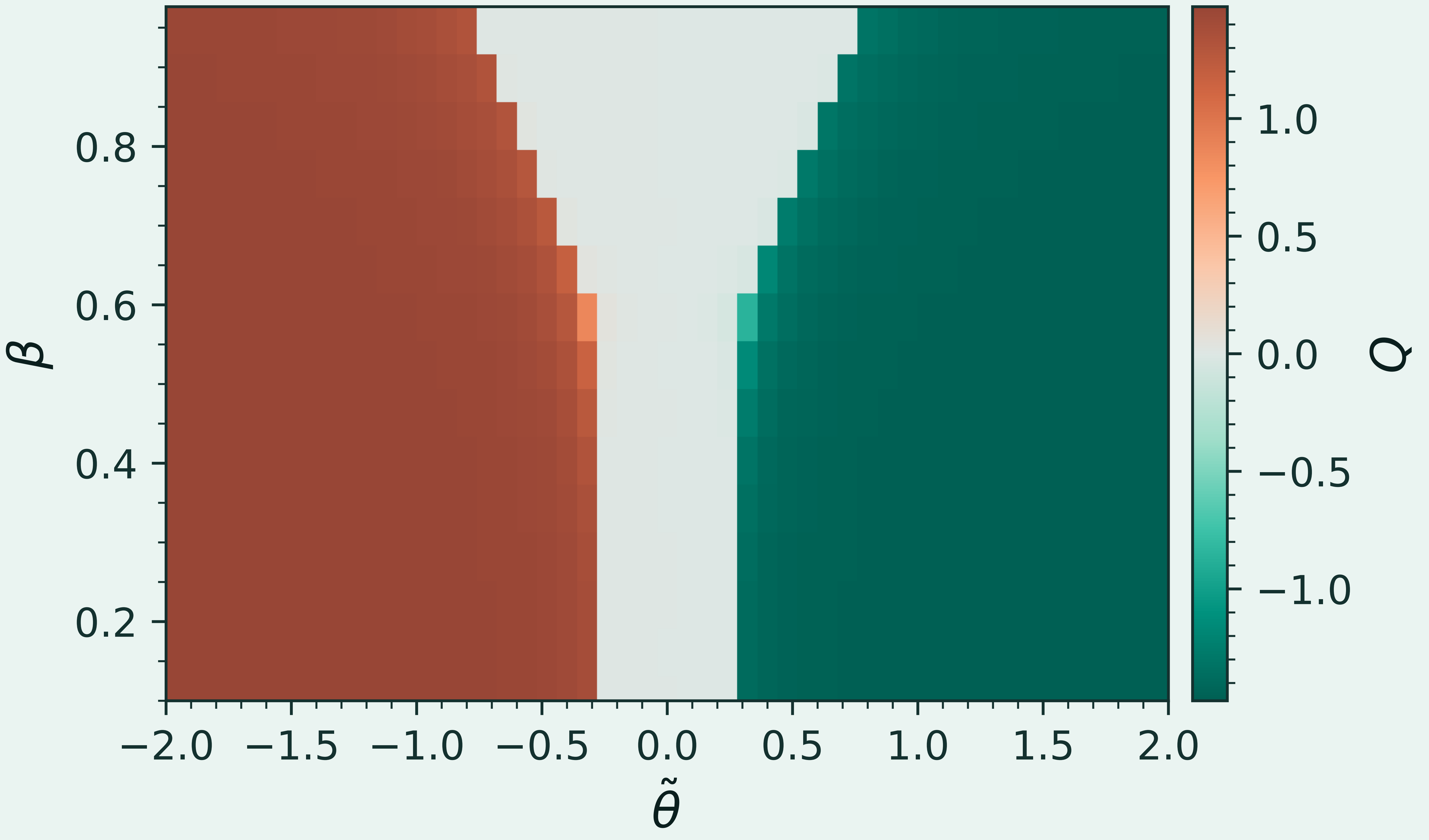


- 24 links
- 24 plaquettes
- Cyclic $\hat{E}$ $(\hat{U}\,|\,1\rangle = |\,-1\rangle)$

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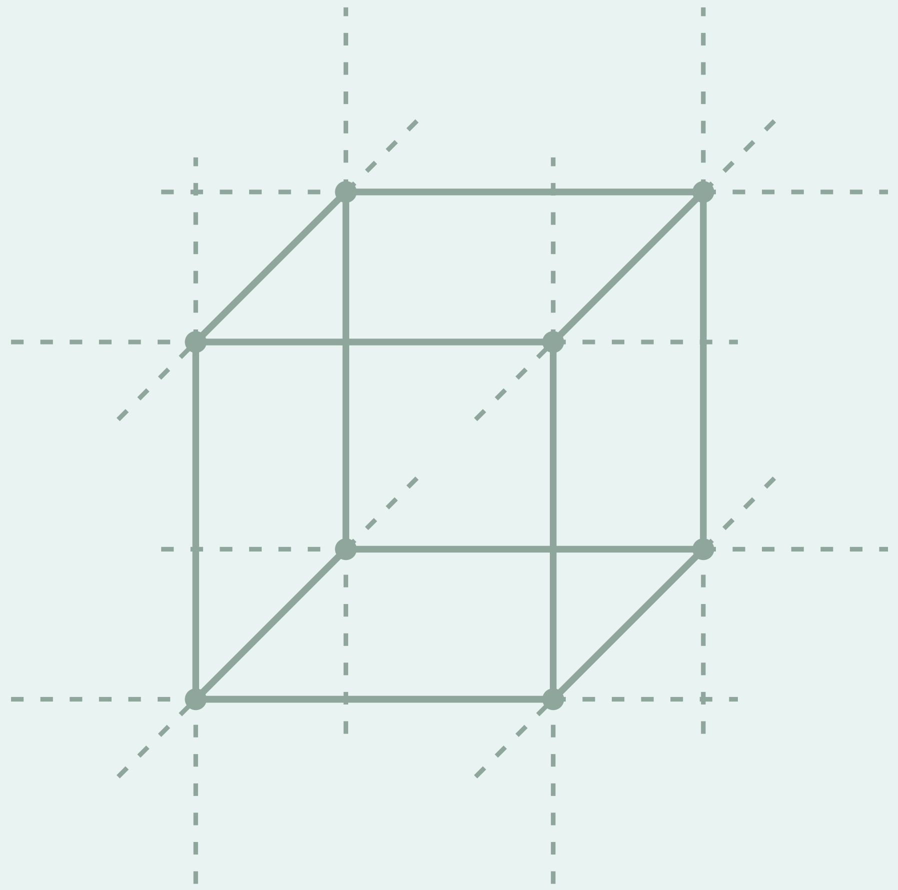

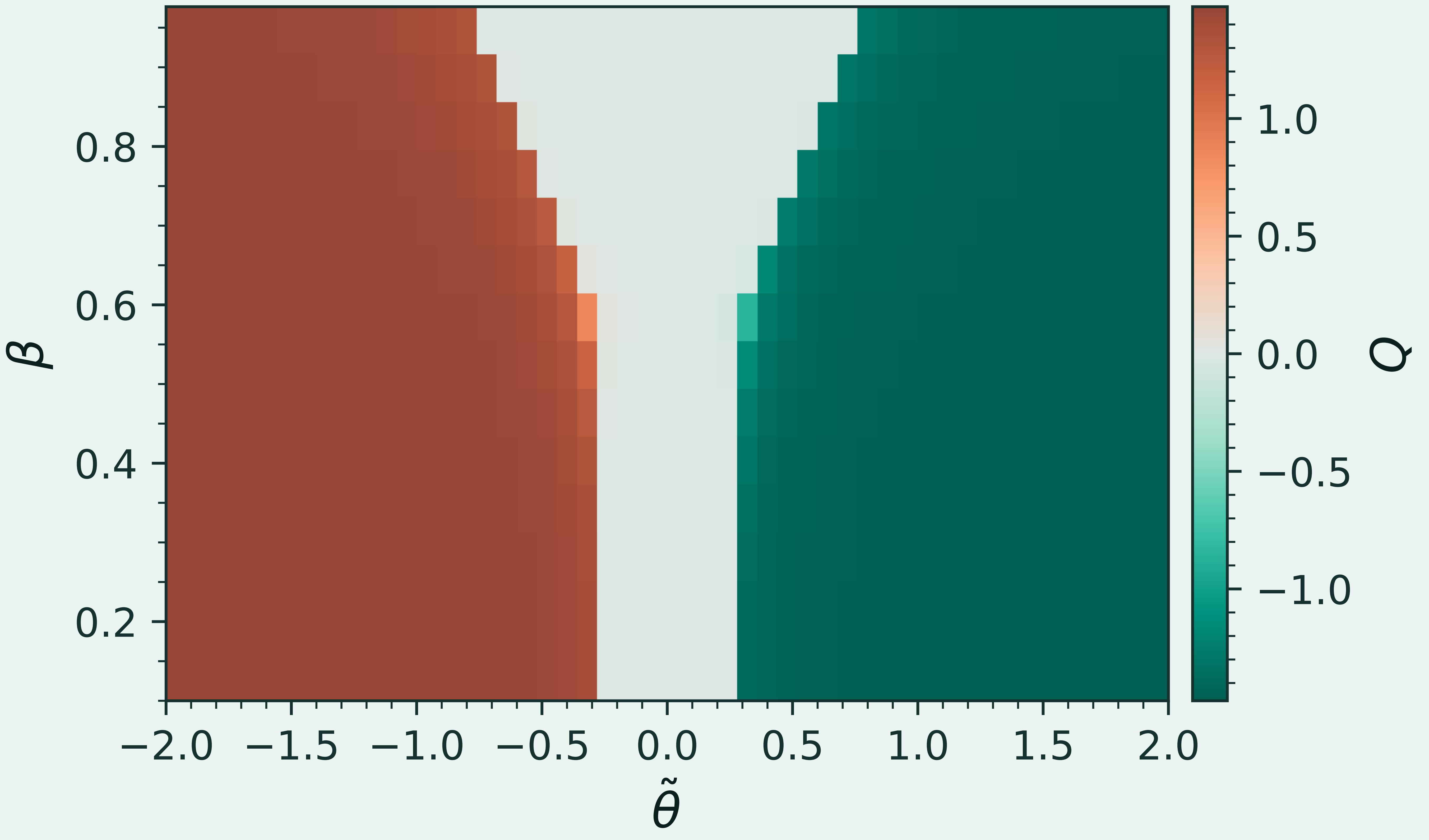


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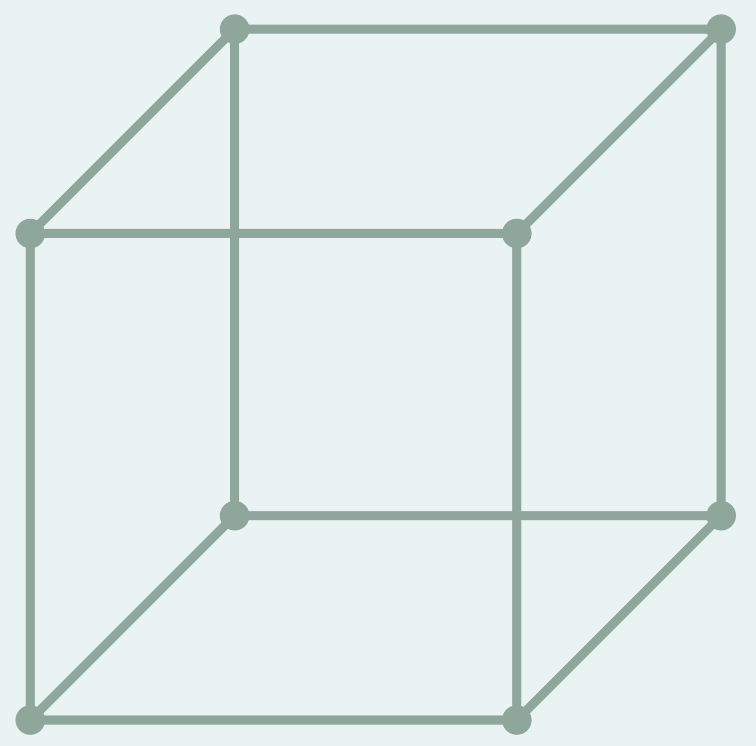
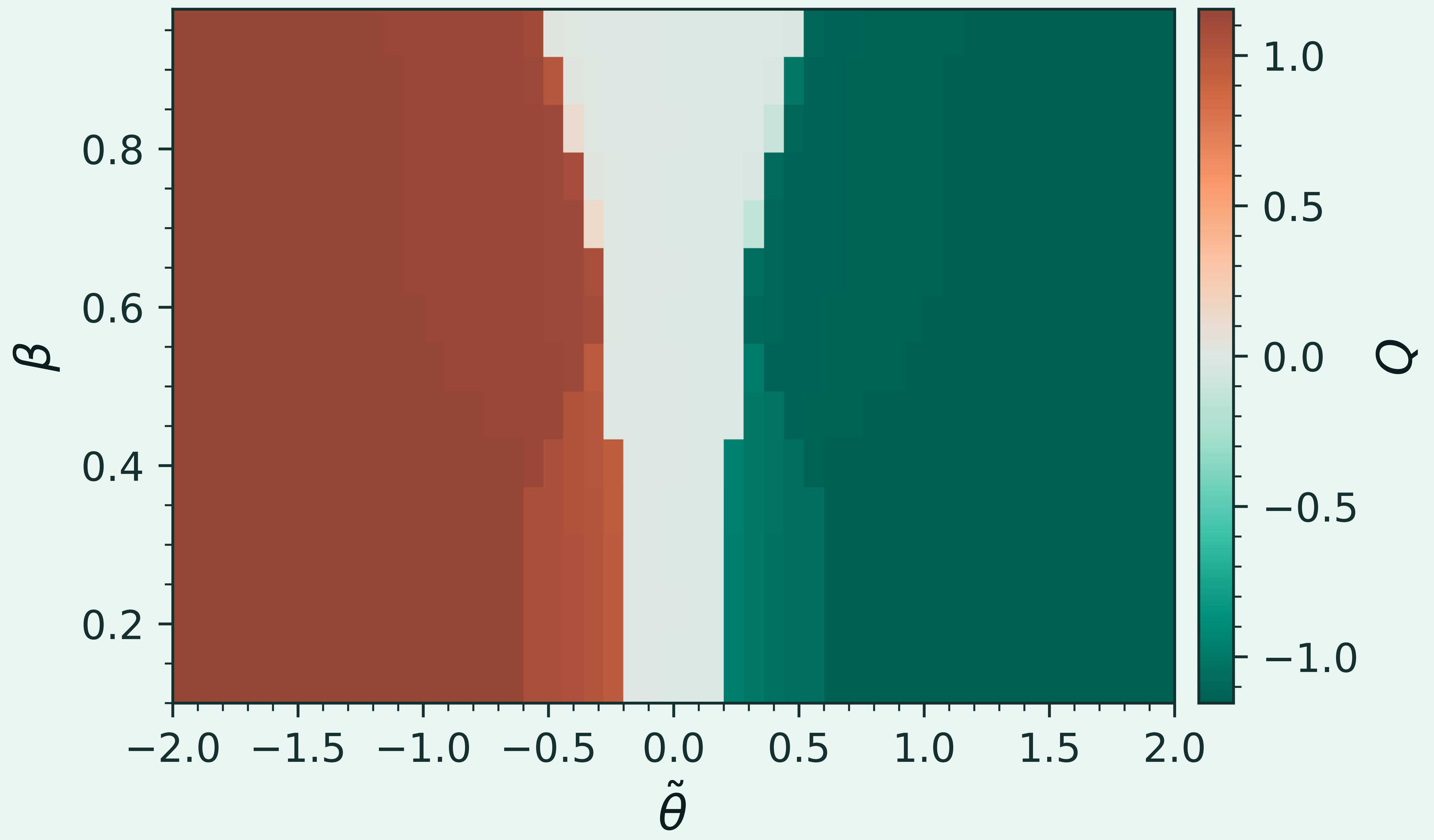
- 24 plaquettes
- Cyclic $\hat{E}$ $(\hat{U}\,|\,1\rangle = |-1\rangle)$

$\Rightarrow$Reduce costs for QC

# A Cube with Open Boundaries

## 3+1D, $E \in [-1,0,1]$, cyclic





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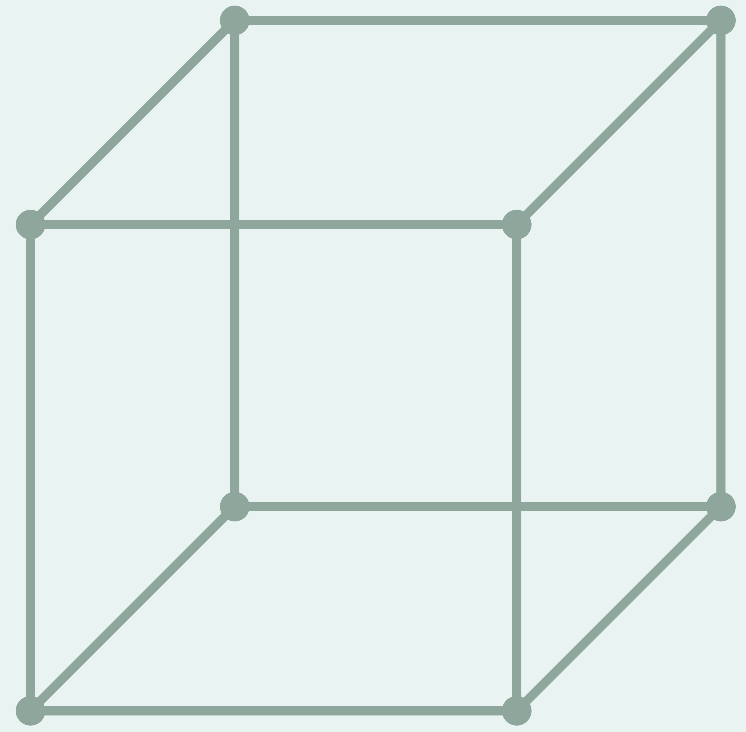
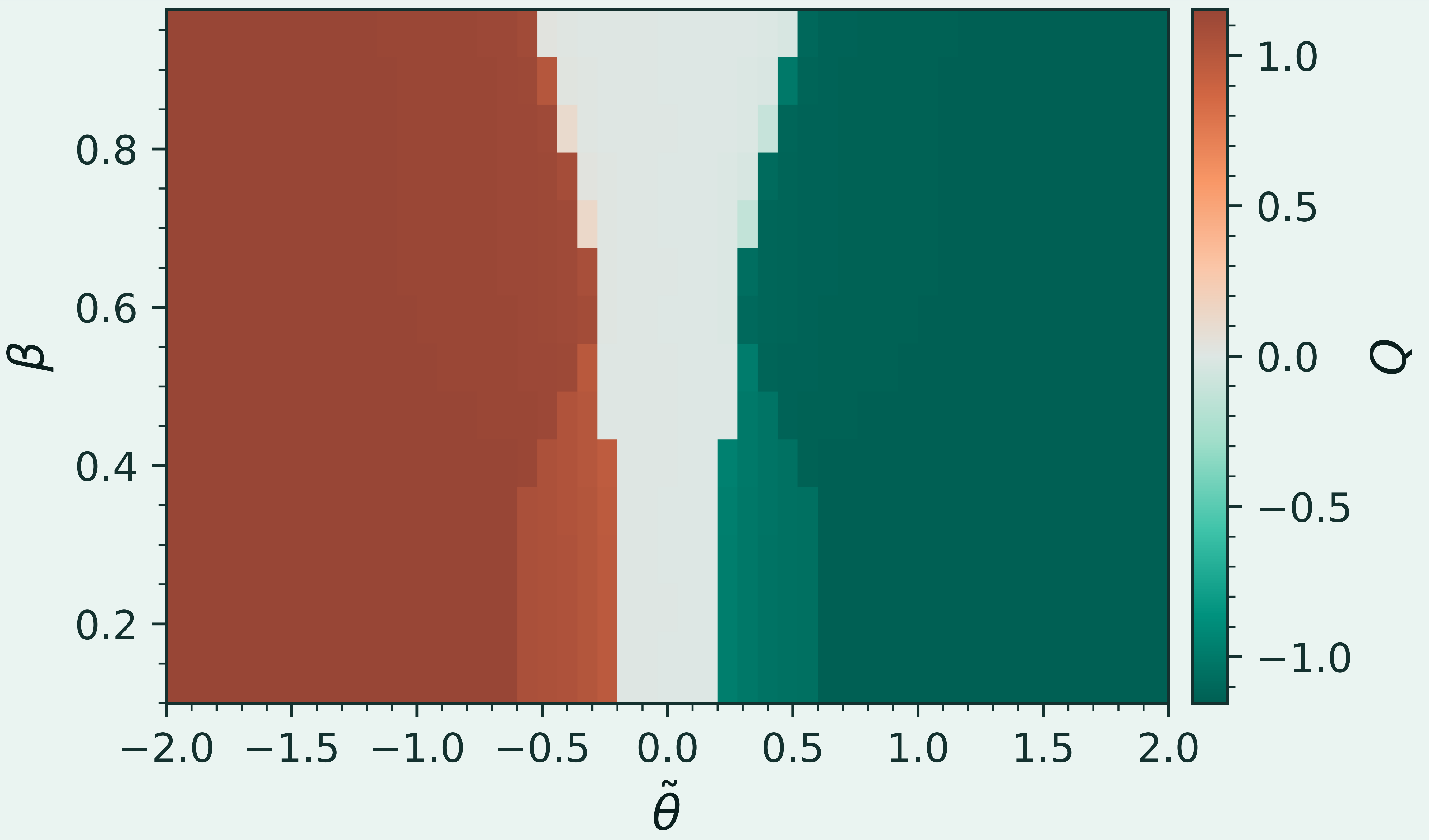
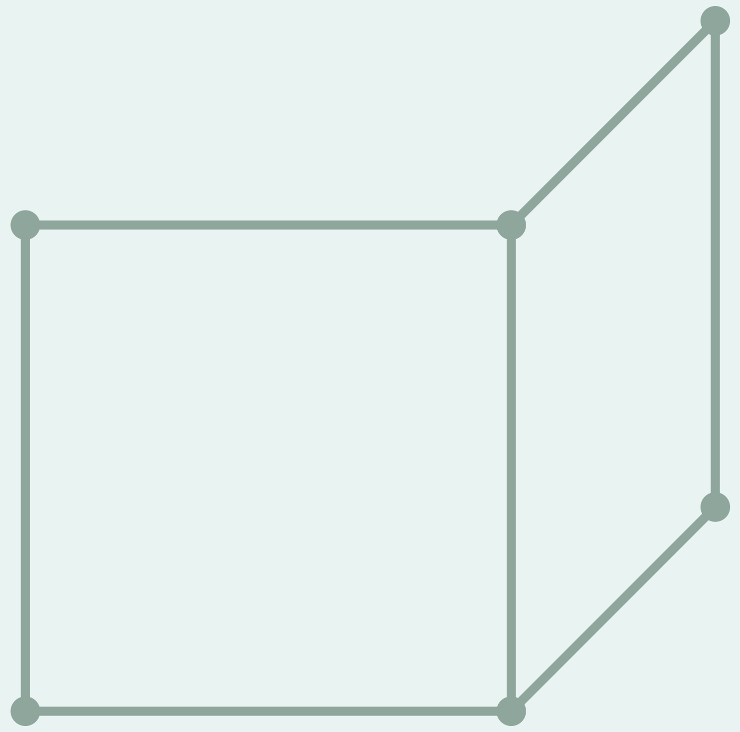


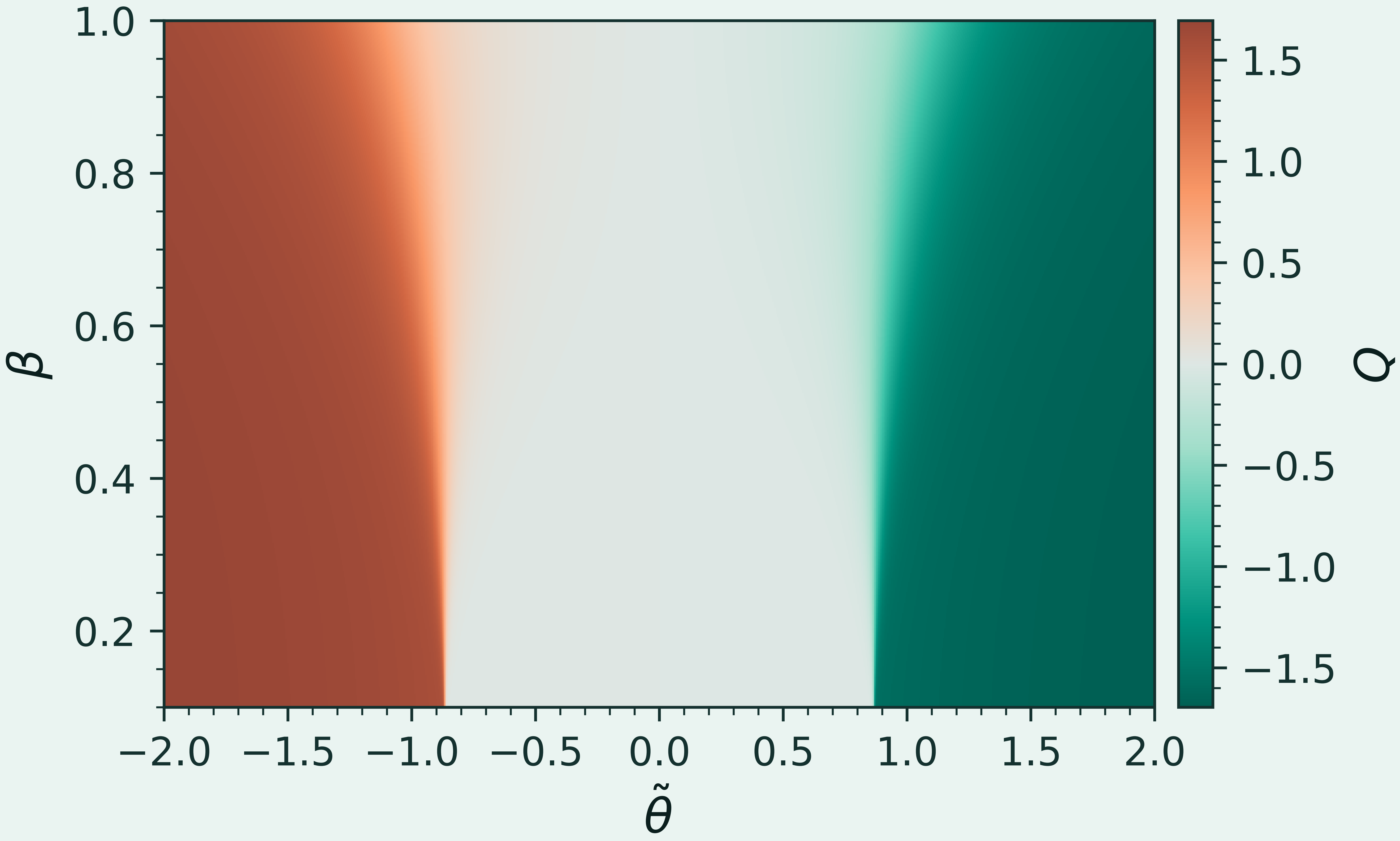



- 12 links
- 6 plaquettes
- $$\sum_{\vec{n},(i,j,k)\in\text{even}} \hat{E}_{\vec{n},i}\left(\hat{U}_{\vec{n},jk} - \hat{U}^{\dagger}_{\vec{n},jk}\right)$$

# Two Plaquettes

## 3+1D, but reduced to minimal system, $E \in [-1,0,1]$





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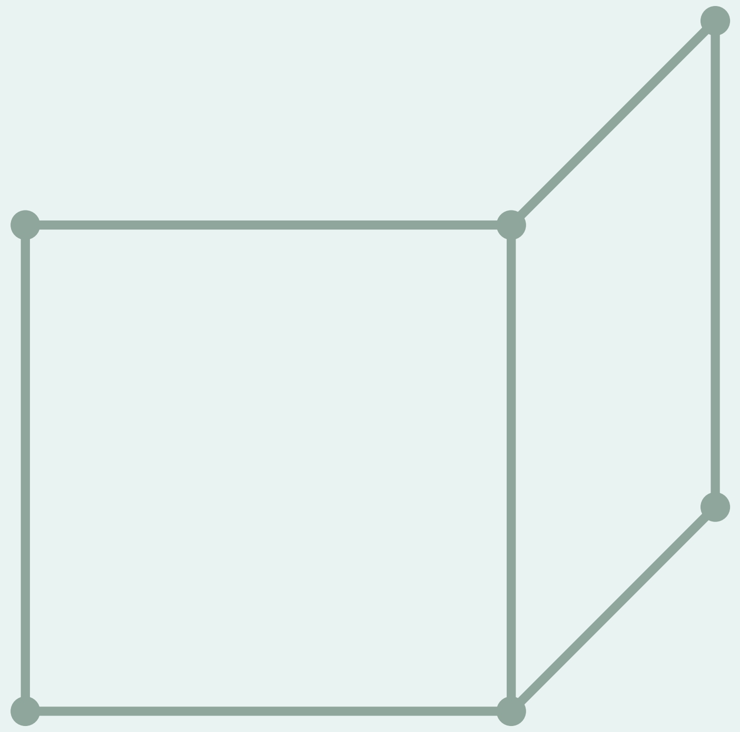
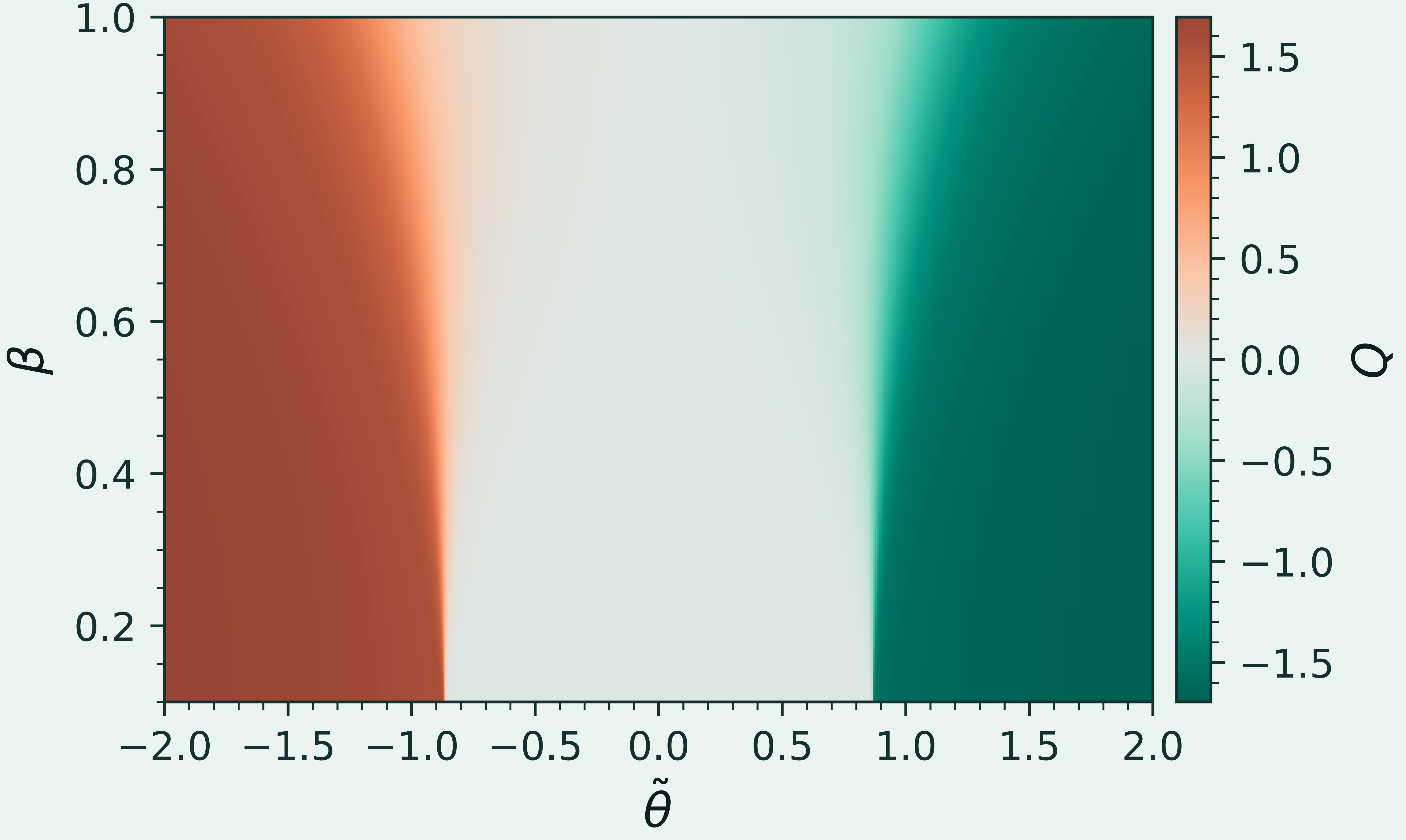
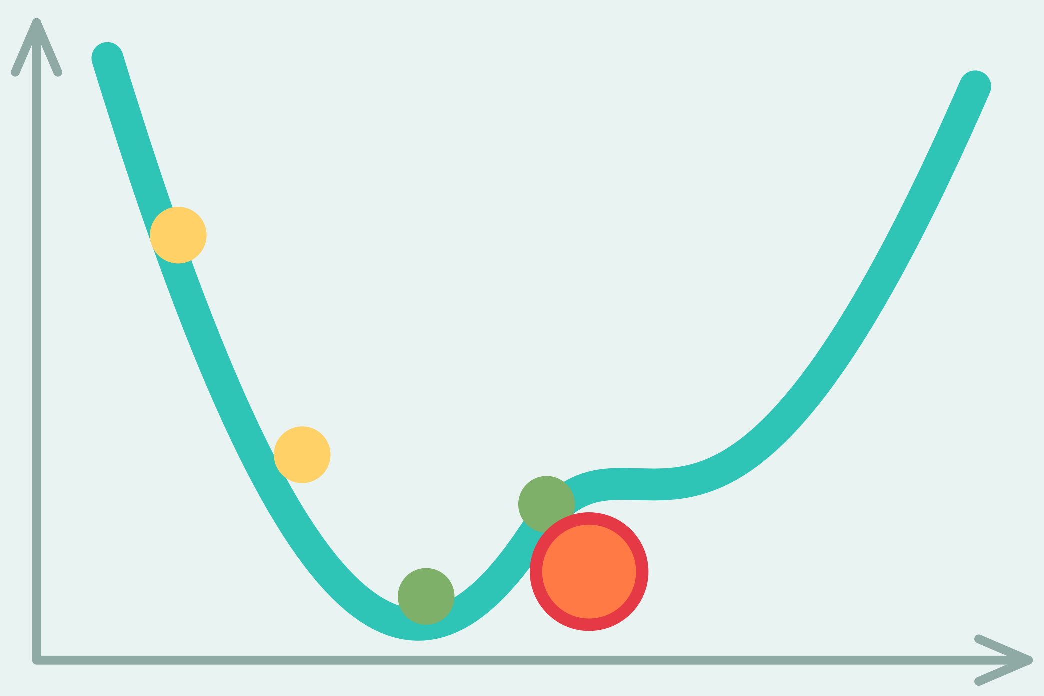





- 7 links
- With Gauss law: 2 qudits

# Obtaining the Ground State

## Searching for a suitable quantum algorithm



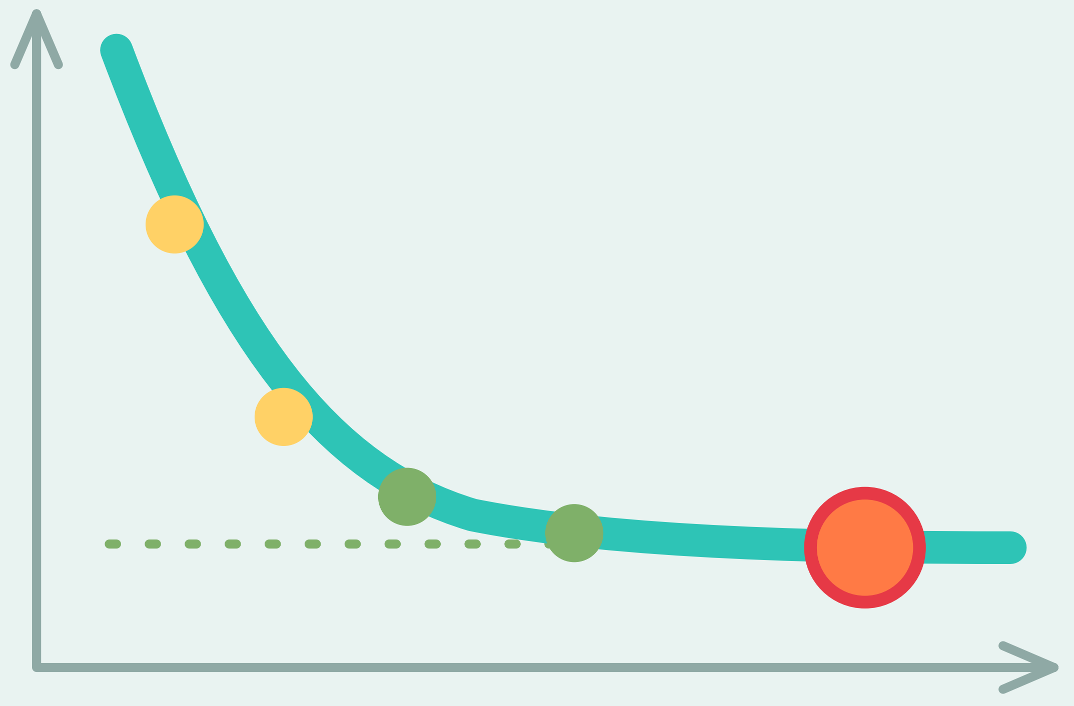
### Variational Quantum Eigensolver

- Needs specific ansatz
- No convergence guarantee
- Barren plateaus might become challenging

$\Rightarrow$ We want another algorithm

# Obtaining the Ground State

## Searching for a suitable quantum algorithm



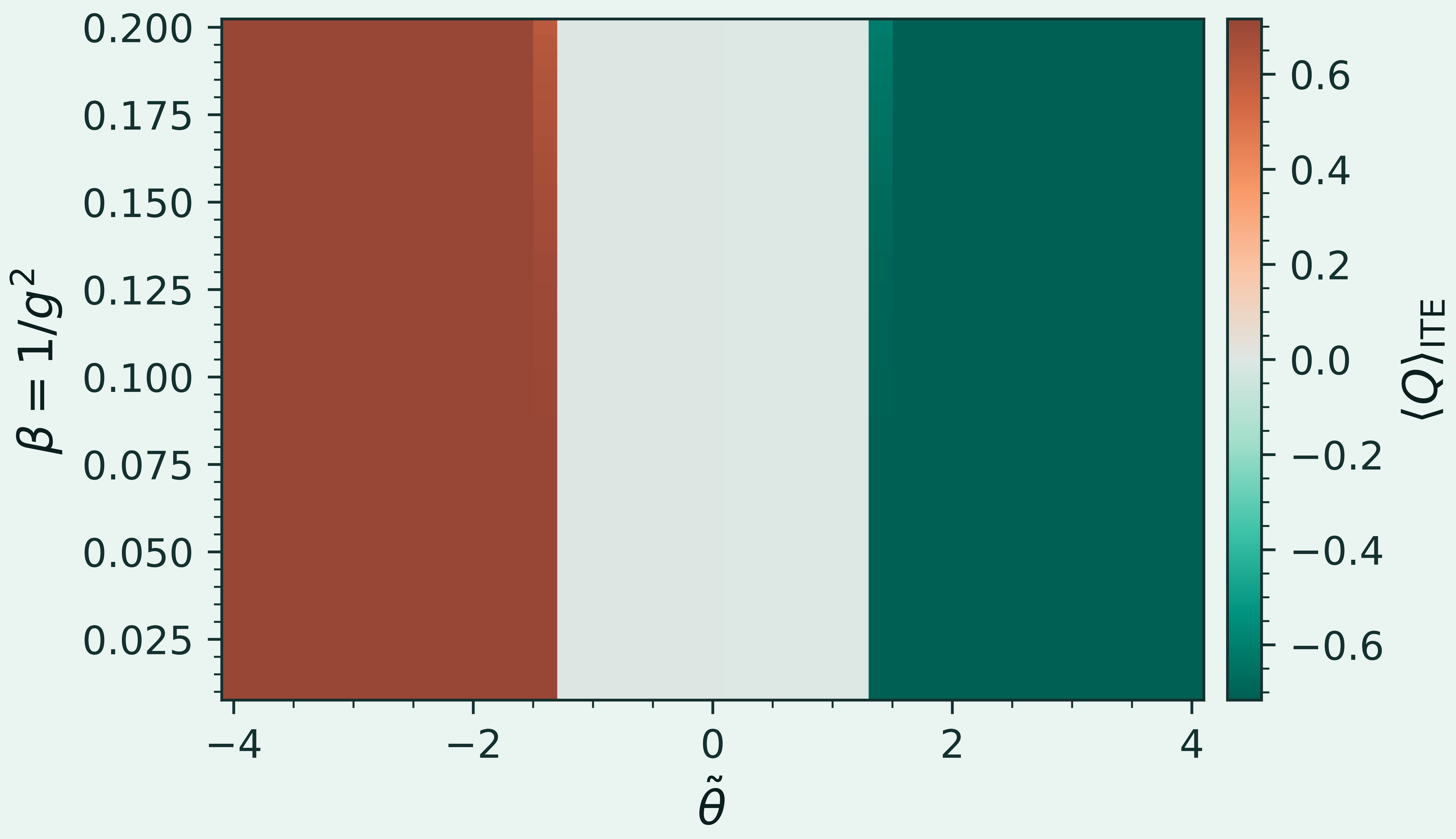
### Quantum Imaginary Time Evolution

- Has convergence guarantee
- No expressivity issues
- Resources challenging

$\Rightarrow$ Focus on minimal setups

# One Step of Imaginary Time Evolution

## Approximating the Ground State




$$\boxed{e^{-\hat{H}\tau} \approx e^{-\hat{H}_\theta \tau} e^{-\hat{H}_B \tau} e^{-\hat{H}_E \tau}}$$

# From Non-Unitary to Unitary

## Mapping ITE to unitary operators (Motta et al., *Nat. Phys.* 16, 205–210 (2020))

$$\frac{e^{-\tau H_k}}{||e^{-\tau H_k}|\psi\rangle||}|\psi\rangle \longrightarrow e^{-iA\tau}|\psi\rangle \qquad A = \sum_\mu a_\mu P_\mu$$

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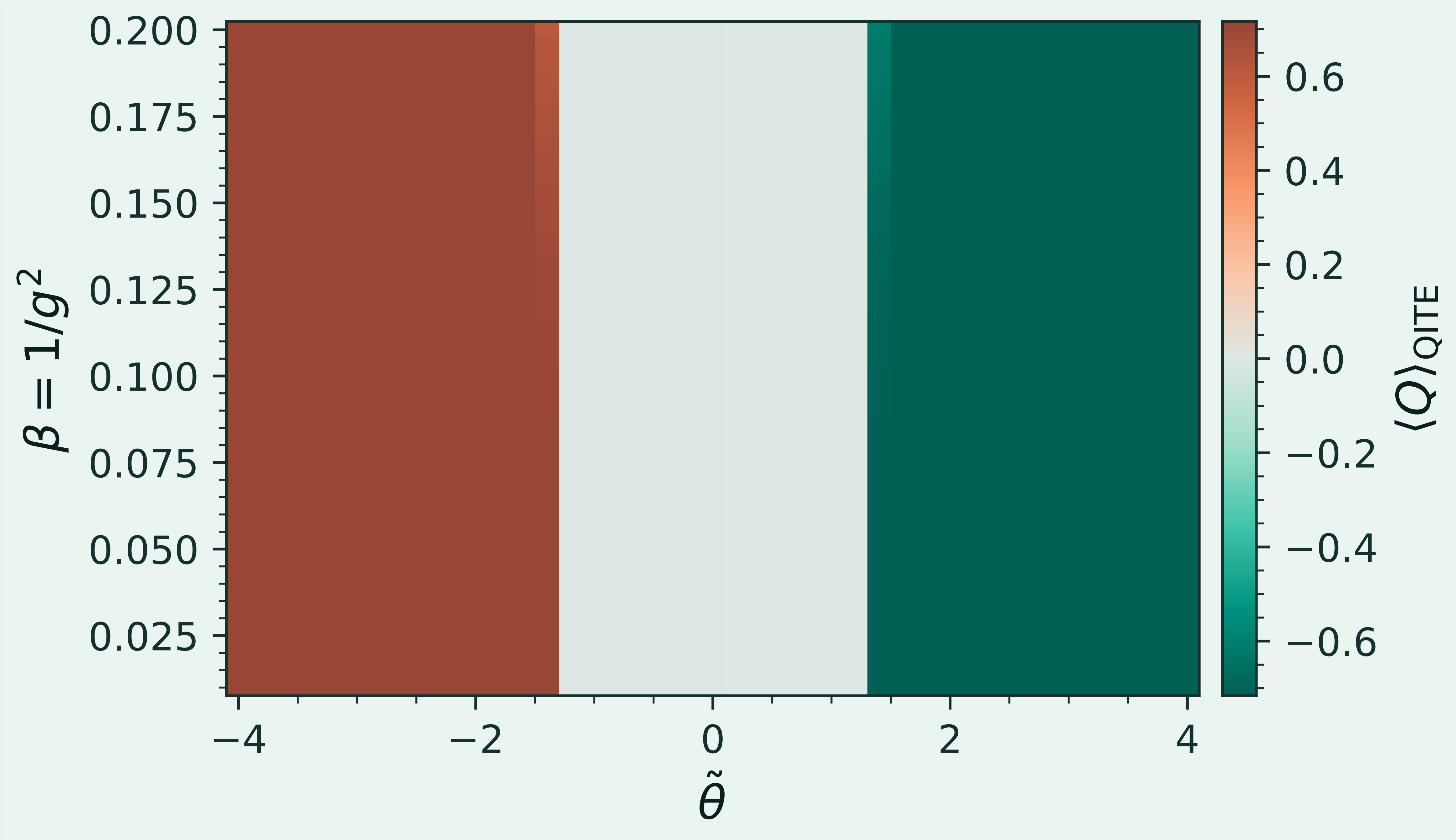
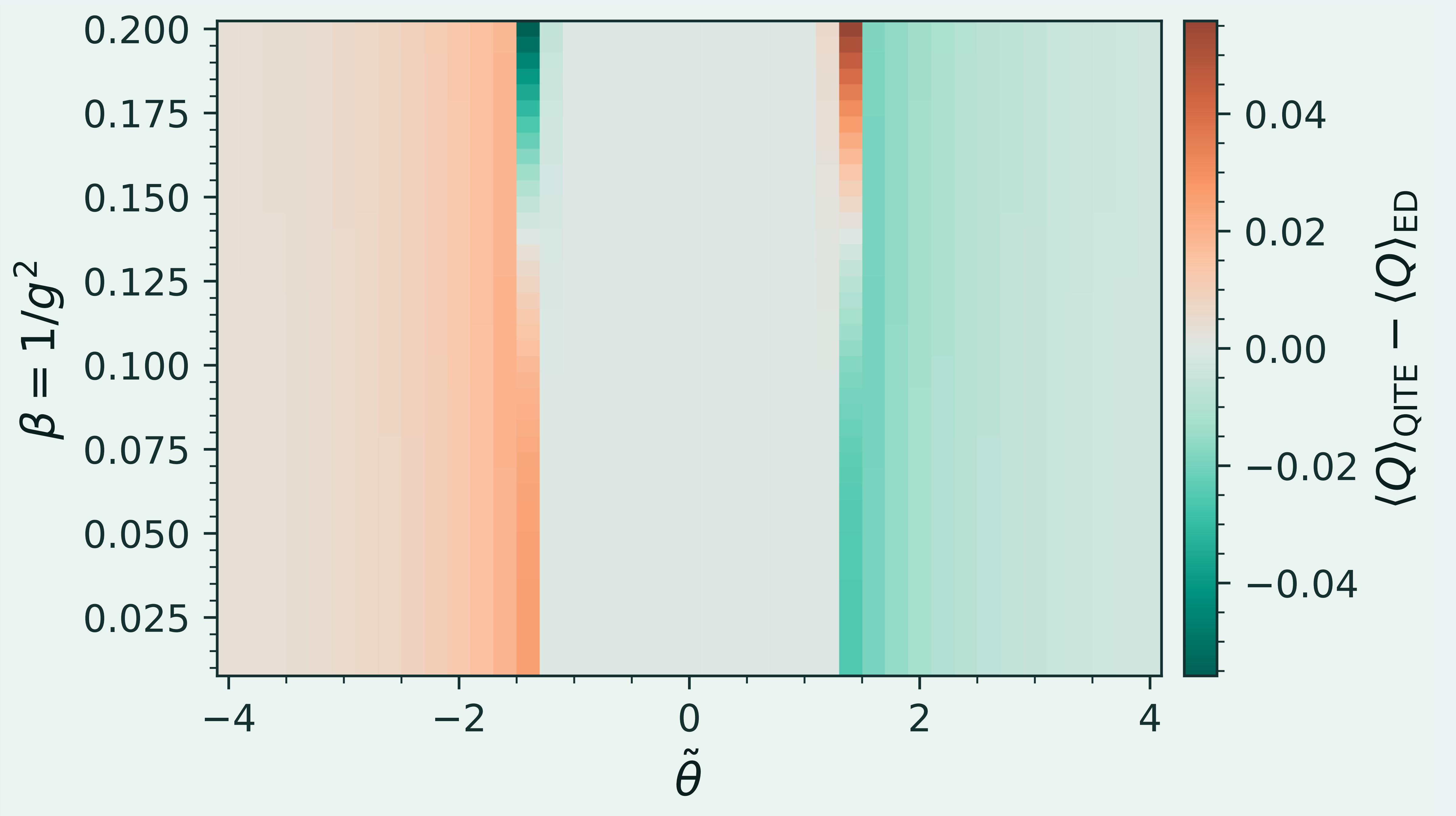
$$\frac{e^{-\tau H_k}}{||e^{-\tau H_k}|\psi\rangle||}|\psi\rangle \longrightarrow e^{-iA\tau}|\psi\rangle \qquad A = \sum_{\mu} a_{\mu} P_{\mu}$$

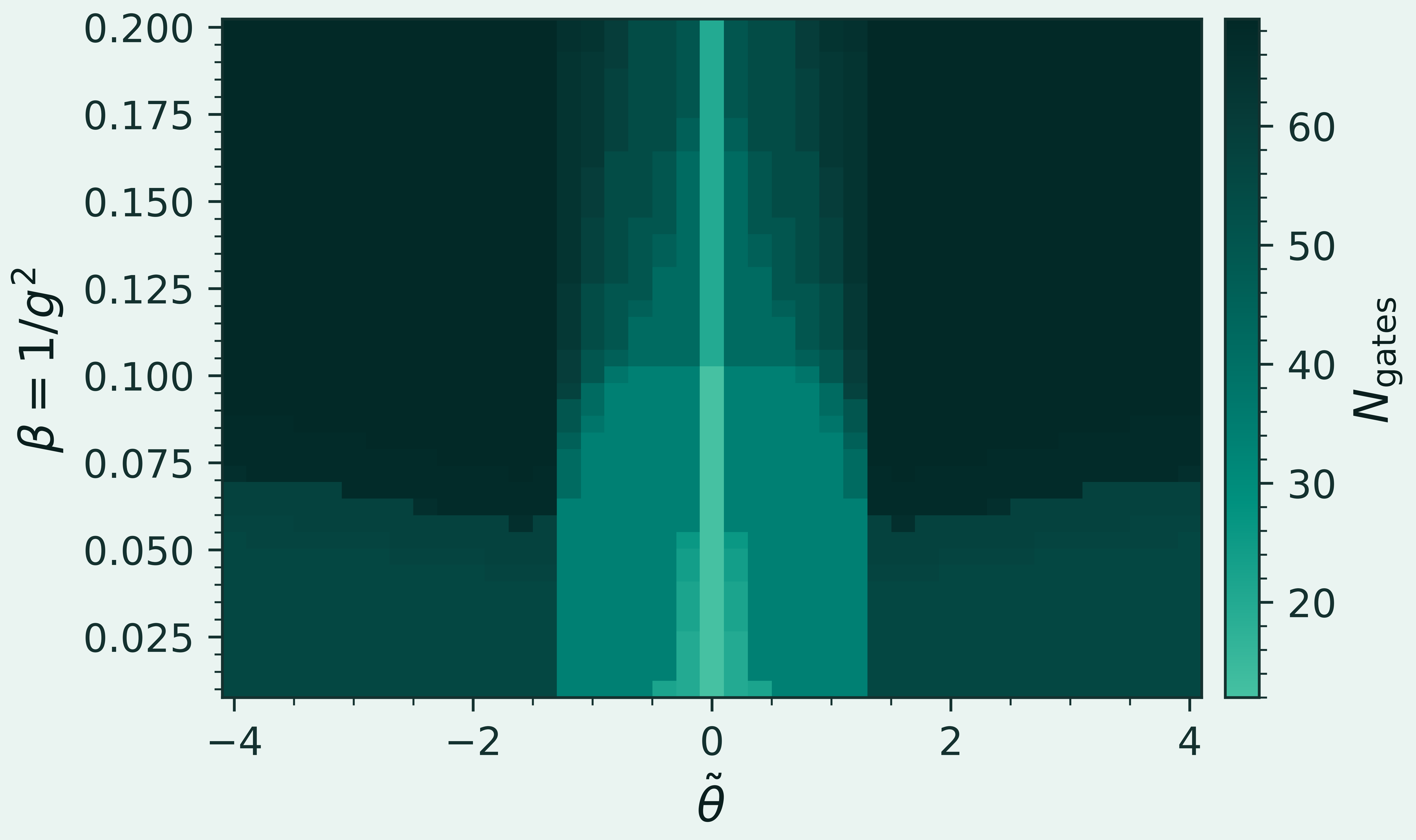
This can either be mapped in first order of the time or exactly in special cases (e.g. $H_k$ being a Pauli string for qubits)

# Mapping One Time Step



# Resources for the Mapping



# Summary And Outlook

**What has been done**

Phase diagram for cube with PBC/OBC and two plaquettes

Estimation of resources for all sizes

Mapping of ITE to Qudits

Classical simulations of minimal system

**The next steps**

Inference run for smallest system on real machine

Estimation of resources for larger systems

Simulation for larger systems

# Collaborators

Lena Funcke

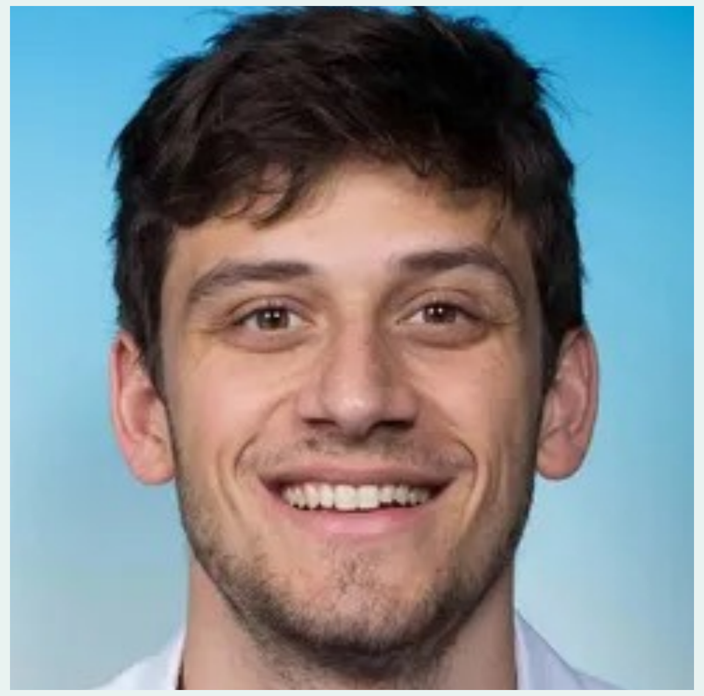
Karl Jansen

Martin Ringbauer

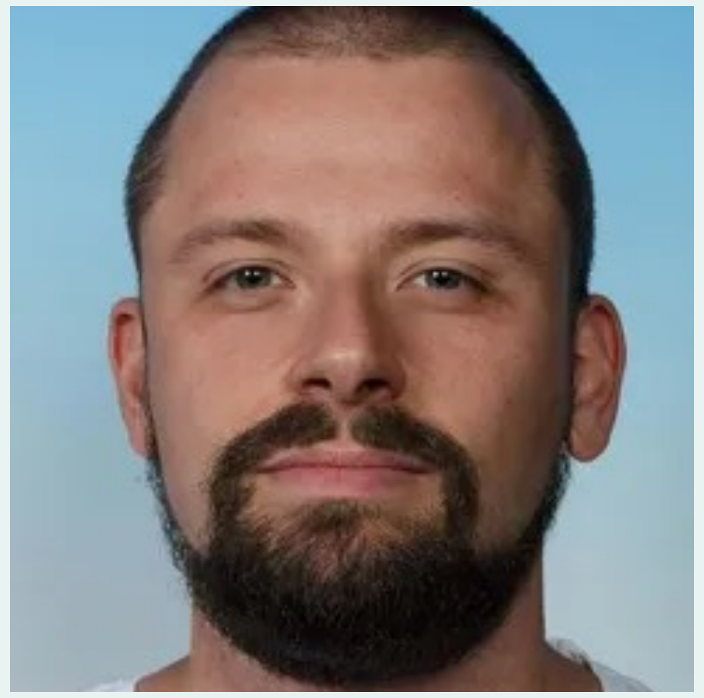
Rainer Blatt

Tobias Hartung

Torsten Zache

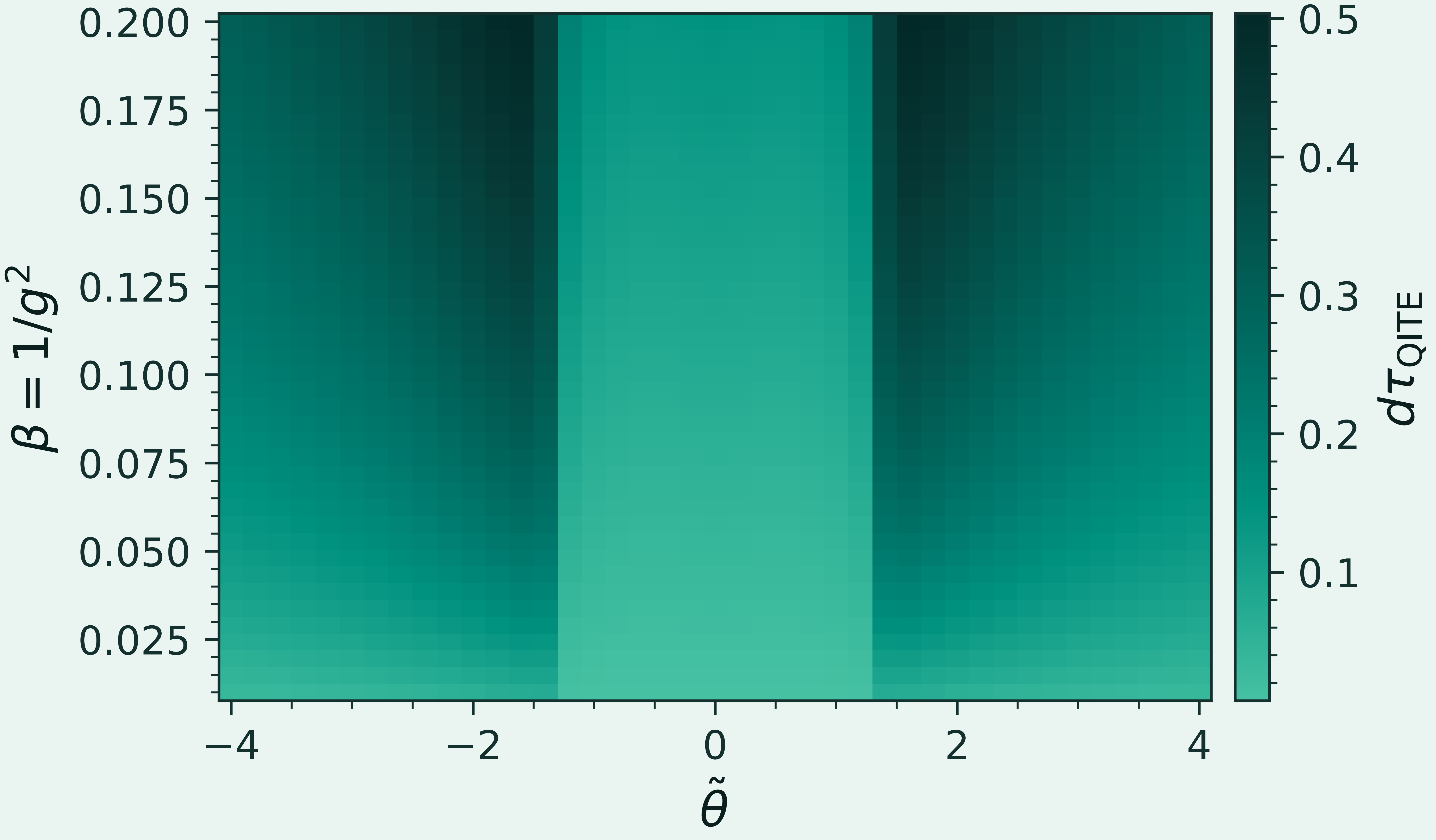


Manuel John



Michael Meth

# The Time Step



# Exact Generator

## How it can be done efficiently

**Generic: expensive**

Rotation in initial-/final-state plane

One needs $|\phi\rangle = e^{-\tau H_k}|\psi\rangle$

**The case here:**

$$H_k = f(P), \quad P^d = 1$$

$$e^{-\tau H_k} = \sum_{j=0}^{d-1} e^{-\tau f(\omega^j)} \Pi_j$$

$$\omega = e^{2\pi i/d}$$

# Matching First Order

$$e^{-i\,dt\,A}\,|\psi\rangle = |\psi\rangle - i\,dt\,A\,|\psi\rangle + O(dt^2) \text{ and } |\phi(dt)\rangle = |\psi\rangle - dt\,\Delta H_k\,|\psi\rangle + O(dt^2)$$

$\Delta H_k = H_k - \langle\psi|H_k|\psi\rangle$

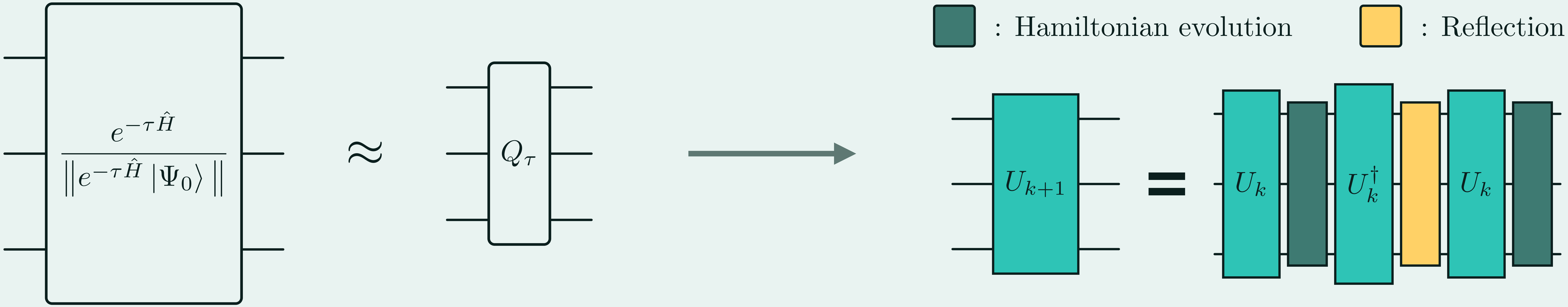
$$-i\,A\,|\psi\rangle \overset{!}{=} -\Delta H_k\,|\psi\rangle \quad \Longleftrightarrow \quad i\,A\,|\psi\rangle = \Delta H_k\,|\psi\rangle$$

Minimize: $f(\mathbf{a}) = \left\| -i\sum_\mu a_\mu P_\mu\,|\psi\rangle + \Delta H_k\,|\psi\rangle \right\|^2$

Leads to: $S\,\mathbf{a} = \mathbf{b}$ with $S_{\mu\nu} = \mathrm{Re}\,\langle\psi|P_\mu P_\nu|\psi\rangle$ and $b_\mu = \mathrm{Im}\,\langle\psi|P_\mu\,\Delta H_k|\psi\rangle$

# DB-QITE

## Proposed in Gluza et al., Phys. Rev. Lett. 136, 020601 (2026)



# Mapping to Qudits

$$\boxed{\hat{E}_{\vec{n},i}}$$

$$\boxed{\hat{U}_{\vec{n},i}}$$

# Mapping to Qudits

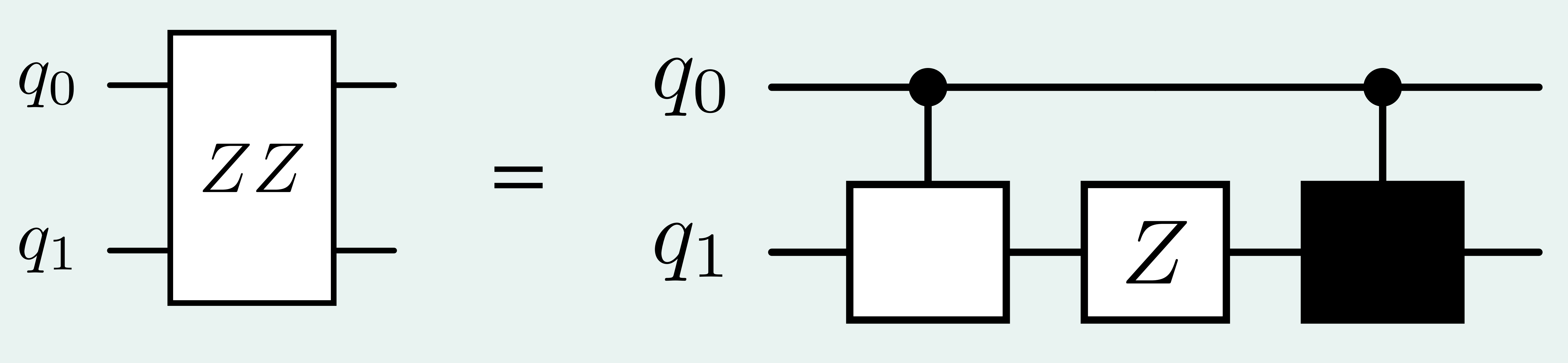
$$\boxed{\hat{E}_{\vec{n},i}} \longrightarrow \hat{Z}|n\rangle = e^{i\delta n}|n\rangle \text{ with } \delta = \frac{2\pi}{N}$$

$$\boxed{\hat{U}_{\vec{n},i}} \longrightarrow X|n\rangle = |(n+1) \bmod N\rangle$$

# Localizing the Terms

The CSUM Gate: $\Theta\,|n_1\rangle\,|n_2\rangle = |n_1\rangle\,|(n_1+n_2) \bmod N\rangle$

$$\boxed{Z_1 Z_2 = \Theta_{12}^{\dagger} Z_2 \Theta_{12}}$$



# One Plaquette Quantum Circuit

## One Trotter step of magnetic and $\theta$-term in real time evolution