Testing strong isospin breaking effects in QCD thermodynamics

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Outline

Introduction

Strong isospin breaking (SIB)

- Standard lattice simulations set $m_u = m_d \equiv m_l$ for efficiency
- SIB: $m_d - m_u \ll \Lambda_{\rm QCD}$: expect SIB effects are negligible
- Rarely explicitly tested in thermodynamics
  - Earlier lattice study finds no discernible impact of SIB on $T_{\rm pc}$[1]
  - $N_f = 1+1$ unimproved staggered
  - $N_\tau = 4$
  - $N_f = 1+1$ has lower mean $T_{\rm pc}$ than $N_f = 2$
  - Both $T_{\rm pc}$ fully compatible within uncertainties
- **This work:** $N_f = 1+1+1$ lattice study of thermodynamic observables at physical quark masses, $\vec{\mu} = 0$

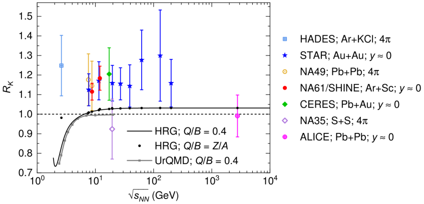
[1] R. V. Gavai and S. Gupta, Phys. Rev. D, 66, 094510 (2002).

Surprising kaon yields in heavy-ion collisions[2]

- Collision experiments consistently find more charged kaons than neutral:

$$R_K \equiv \frac{\langle K^+ \rangle + \langle K^- \rangle}{\langle K^0 \rangle + \langle \bar{K}^0 \rangle} > 1$$

- NA61/SHINE: newest evidence for isospin-symmetry violation in nucleus-nucleus collisions
- Can we understand $R_K > 1$ from first-principles QCD?



[2]H. Adhikary et al., Nature Commun. 16.1, 2849 (2025).

Setup

Computational setup

- $N_f = 1+1+1$ stout-staggered fermions
- Bare quark mass ratios from FLAG[3]:

$$m_d = \frac{m_s}{20.1}\,, \quad m_u = 0.49\, m_d \quad \Longrightarrow \quad m_u : m_d : m_s \approx 1 : 2 : 40$$

- $T \approx 0$ ensembles yield m_K compatible with PDG
- **No continuum limit** $(N_\tau = 8)$
- $T \in [114,\, 174]$ MeV
- For comparison: $T_{\rm pc} = 157.3(2.3)$ MeV[4,5]
- For comparison: $N_f = 2+1$ confs at same β values
- HRG curves: QMHRG2020[6]

[3] Y. Aoki et al., Phys. Rev. D, 113.1, 014508 (2026).
[4] A. Bazavov et al., Phys. Lett. B, 795, 15–21 (2019).
[5] S. Borsanyi et al., Phys. Rev. Lett. 125.5, 052001 (2020).
[6] D. Bollweg et al., Phys. Rev. D, 104.7 (2021).

Chiral condensate

Renormalized chiral observables

Look at renormalized condensate[7]:

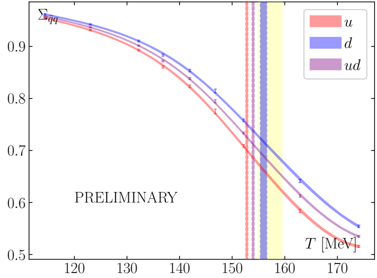
$$\Sigma_{\bar{q}q} \equiv \frac{m_l}{m_\pi^2 f_\pi^2}\big(\langle\bar{q}q\rangle_T - \langle\bar{q}q\rangle_0\big) + 1, \qquad m_l \equiv \frac{1}{2}(m_u + m_d)$$

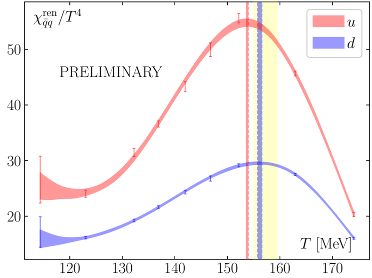
GMOR-inspired definition, roughly keeps $\Sigma \in [0,1]$. Similarly renormalize chiral susceptibility:

$$\frac{\chi_{\bar{q}q}^{\mathrm{ren}}}{T^4} = \frac{m_s^2}{T^4}\left(\chi_{\bar{q}q}(T) - \chi_{\bar{q}q}(0)\right)$$

[7] B. B. Brandt, G. Endrodi, and S. Schmalzbauer, Phys. Rev. D, 97.5, 054514 (2018).

Estimate $T_{\rm pc}$ from different quark condensates.
Previous lattice study found negligible impact of SIB on $T_{\rm pc}$.[8]





Separation of $T_{\rm pc}$ from different quark condensates.
At-most-small effect compared to standard definition of $T_{\rm pc}$.

[8] R. V. Gavai and S. Gupta, Phys. Rev. D, 66, 094510 (2002).

Probing abundances

Two renormalized abundances measuring $\langle\bar{d}d\rangle$ vs. $\langle\bar{u}u\rangle$:

Strategy 1 — subtract $T=0$, normalize with GMOR:

$$A_1 \equiv \frac{m_l(\Delta_d-\Delta_u)}{m_l(\Delta_d+\Delta_u)+2m_\pi^2 f_\pi^2}\,, \qquad \Delta_q \equiv \langle\bar{q}q\rangle_T - \langle\bar{q}q\rangle_0$$

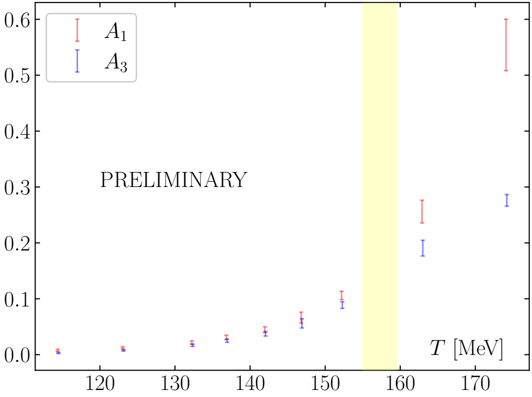
Disadvantage: $A_1=0$ at $T=0$

Strategy 2 — subtract $\bar{s}s$:

$$A_3 \equiv \frac{\delta_d-\delta_u}{\delta_d+\delta_u}\,, \qquad \delta_q \equiv \langle\bar{q}q\rangle - \frac{m_q}{m_s}\langle\bar{s}s\rangle$$

Disadvantage: A_3 has remaining logarithmic divergence

Abundances A_1 and A_3



$A_1 \approx A_3$ below $T_{\rm pc}$.

Abundance of $\bar{d}d$ over $\bar{u}u$ at all T.

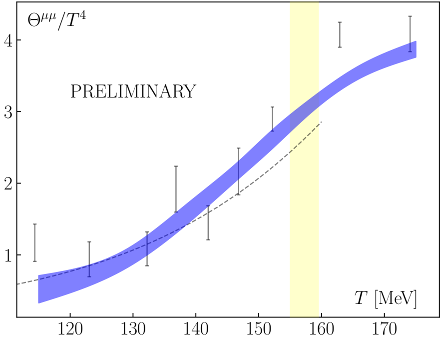
Magnitude increases dramatically around $T_{\rm pc}$.

Equation of state

Trace anomaly

$$\frac{p(T)}{T^4} - \frac{p(T_0)}{T_0^4} = \int_{T_0}^{T} \frac{\Theta^{\mu\mu}(T')}{T'^5}\,\mathrm{d}T'$$

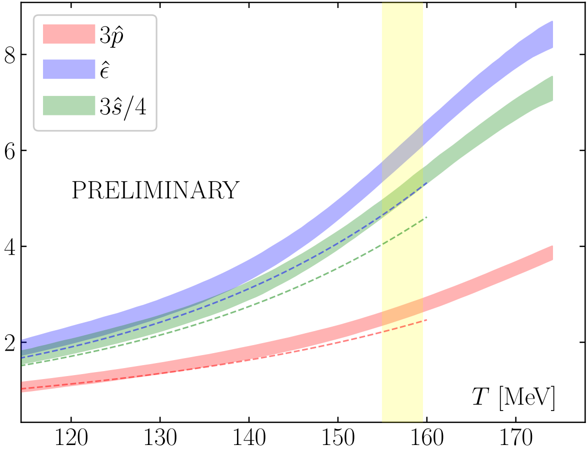
Start integration using HRG at $T = 100$ MeV



Similar to past $N_f = 2+1$ study with same action.[9]

[9] S. Borsanyi et al., JHEP, 11, 077 (2010).

Equation of state at $\mu_B = 0$



Results for thermodynamic observables work in progress.

Second-order cumulants

Pressure $p(\vec{\mu})$ expands into conserved-charge cumulants:

$$\frac{p(\vec{\mu})}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!\,j!\,k!}\,\hat{\mu}_B^i\,\hat{\mu}_Q^j\,\hat{\mu}_S^k$$

with $\hat{\mu}_X = \mu_X/T$ and

$$\chi_{ijk}^{BQS} \equiv \partial^i_{\hat{\mu}_B}\,\partial^j_{\hat{\mu}_Q}\,\partial^k_{\hat{\mu}_S}\,\frac{p}{T^4}\bigg|_{\vec{\mu}=0}$$

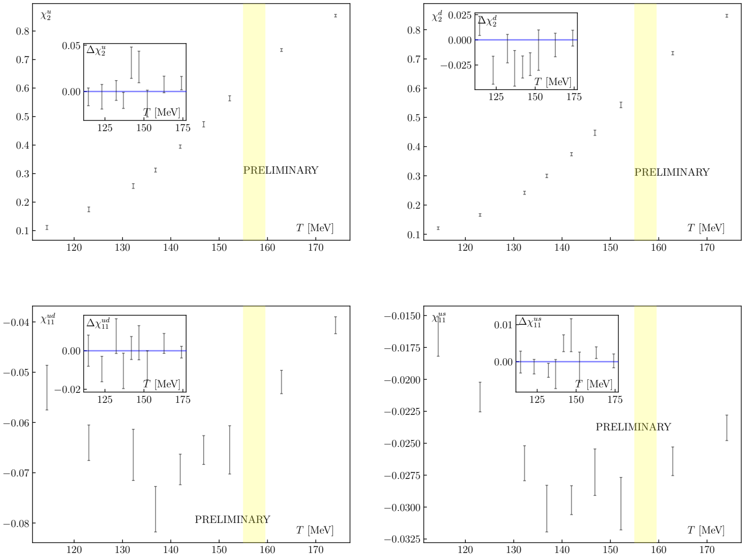
Can also consider flavor basis, where

$$\chi_{ijk}^{uds} \equiv \partial^i_{\hat{\mu}_u}\,\partial^j_{\hat{\mu}_d}\,\partial^k_{\hat{\mu}_s}\,\frac{p}{T^4}\bigg|_{\vec{\mu}=0}$$

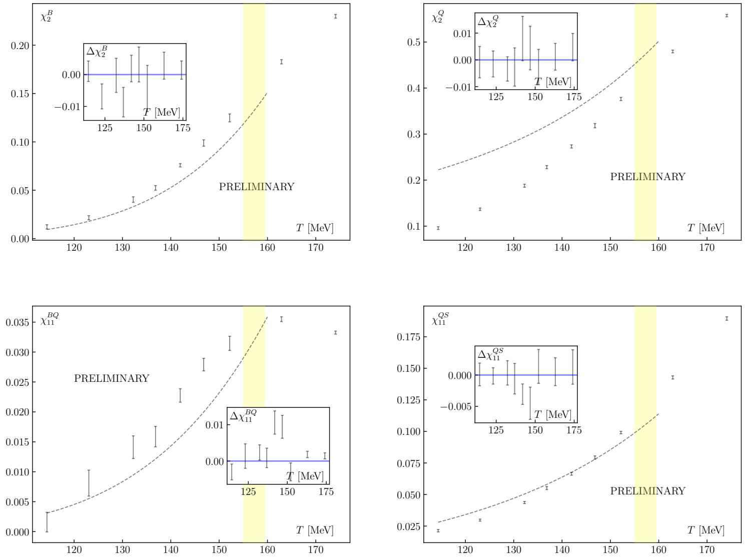
Cumulants directly enter EoS at finite μ. Want to see effect of SIB:

$$\Delta\chi \equiv \chi(m_u \neq m_d) - \chi(m_u = m_d)$$

Quark-flavor cumulants

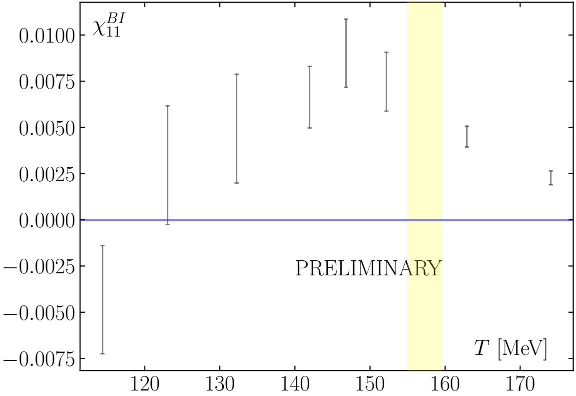


Conserved-charge cumulants



Isospin cumulants

PNJL model calculation with SIB predicts $\chi_{11}^{BI} > 0$ near $T_{\rm pc}$.[10]



We also find $\chi_{11}^{BI} > 0$.

[10] A. Bhattacharyya et al., Phys. Rev. C, 89.6, 064905 (2014).

Summary

Summary

With the caveat that we lack a continuum extrapolation...

- T_{pc} has at-most-mild sensitivity to SIB
- Find $\langle \bar{d}d \rangle$ abundance over $\langle \bar{u}u \rangle$
- Abundance increases with T; implications for R_K?
- $\Theta^{\mu\mu}$ roughly insensitive to SIB
- Most cumulants insensitive to SIB
- Signal in χ_{11}^{BQ} below T_{pc}?
- Signal for χ_{11}^{BI} in isobroken sea
- Further thermodynamic observables forthcoming
- Statistics are increasing

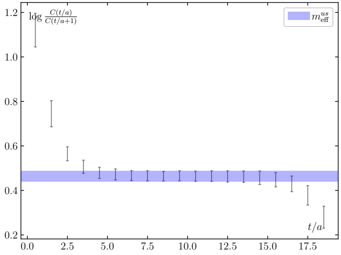
Thanks for listening!

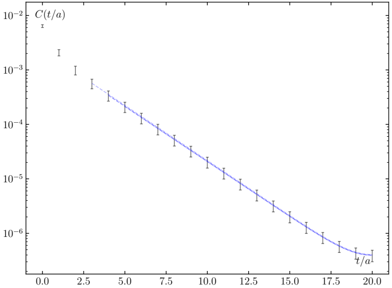
Appendix

Pseudoscalar masses (zero-temperature ensembles)

Extracted masses $m_{qq'}$ [MeV] for $T \approx 0$ ensembles

β	m_{uu}	m_{ud}	m_{dd}	m_{us}	m_{ds}
3.45	109.6(2.0)	134.6(2.0)	155.4(1.9)	479.7(6.0)	484.9(5.2)
3.55	110.6(2.8)	136.6(2.8)	158.2(2.7)	488.8(7.5)	493.8(6.2)
3.60	111.6(2.9)	137.8(2.7)	159.5(2.5)	492.5(7.4)	497.9(6.1)
3.67	112.3(3.0)	138.6(3.0)	160.3(3.0)	494.5(9.5)	500.3(7.8)





$$\frac{\Theta^{\mu\mu}_G}{T^4} = R_\beta N_\tau^4 \left(\langle s_G\rangle_0 - \langle s_G\rangle_T\right)$$
$$\frac{\Theta^{\mu\mu}_F}{T^4} = -R_\beta R_m N_\tau^4 \sum_q m_q \left(\langle \bar{q}q\rangle_0 - \langle \bar{q}q\rangle_T\right)$$